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Research Team Project Report on the Topic:

Interacting Black–Scholes and Heston Agents in an Agent-Based Market

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Abstract

Agent-based modelling has been employed to significant effect in the area of financial mathematics. The objective of this research is to utilise agent-based modelling (ABM) to observe how agents who use the two most important models in financial mathematics, Heston and Black-Scholes, will interact and what market phenomena will ensue as a result, particularly volatility behaviour

Аннотация

Агентное моделирование с успехом применяется в области финансовой математики. Цель данного исследования — с помощью агентного моделирования проанализировать, как агенты, использующие две наиболее важные модели в финансовой математике — Хестона и Блэка-Шоулза, — будут взаимодействовать и какие рыночные явления возникнут в результате, в частности, поведение волатильности.

Keywords

Agent-base modeling(ABM), Black-Sholes model, Heston model, volatility, regime switching

1 Intoduction

For many years classical models with representative agents were the prevailing paradigm of financial mathematics. However Agent-based modelling has gained renewed interest and has become increasingly popular due to its capacity to capture complex market dynamics arising from the interaction of heterogeneous agents

In real markets, traders and investors rely on different models to make financial decisions and trades. Two fundamental models that are widely used even today are The Black–Scholes model and Heston model. The Black–Scholes model assumes constant volatility, while the Heston model incorporates stochastic volatility dynamics. The coexistence of agents employing these distinct approaches may create feedback effects through trading and hedging activity, potentially leading to interesting market behavior.

The objective of this study is to use an agent-based framework to investigate how interactions between agents using the Black–Scholes and Heston models influence market dynamics, with a particular focus on volatility.

2 Literature review

During the second part of the 20th century theoretical pricing models were on the peak. However, the academic literature has raised concerns regarding the issue of market efficiency and the relevance of the theoretical background related to this issue. Consequently, agent-based modelling has emerged as a more appealing approach for this domain. In his 2006 work [1], LeBaron B. provides a comprehensive overview of the justifications for utilising ABM models in the financial world. In our research we also utilize the ABM framework on account of the fact that it is highly suitable for the observation of market phenomena.

The objective of this study was to develop a model that would illustrate empirical facts observed in real-life markets. As demonstrated in the works of Rama Cont (2001) [2] and Chakraborti et al. (2011) [3], stylised facts and other properties were thoroughly examined. These studies indicated that any model that aims to be realistic should explain this. In our present study we will concentrate on volatility.

In order to observe the complex market dynamics in question, it is necessary to observe the interaction of heterogeneous agents. Lux & Marchesi (1999) and Hommes (2006) in their research ([4] and [5] accordingly) have studied complexity and non-linearity of these models and how they can explain important observed stylized facts that we mentioned earlier. We adapt their strategy in aim to capture complex trends due resulting from interaction of Black-sholes and Heston agents.

The selection of agents is based on the prevalence of these models in the financial sector. The primary distinction between these approaches lies in their respective interpretations of volatility. In his work [6] Heston used another stochastic process to model volatility. Furthermore, it is evident that both Black-sholes and Heston exert a significant influence on market volatility. This was previously examined in a 1997 study [7] by Sircar & Papanicolaou. Traders actively hedge their portfolios, so their trade activity greatly impacts price and volatility of underlying assets, resulting in endogenous volatility. This is a crucial observation, not only in favour of using the Heston model, but also with the potential to result in regime switching in volatility.

The idea of regime-switching was studied by James D. Hamilton in his 1989 paper [8]. The model of regime switching by Markov process is employed to interpret the real-world behaviour of business cycles. This concept can be easily transformed into a financial-world, where different market cycles are also evident. Nevertheless, in contrast to his work, this phenomenon will be observed by itself, as opposed to being explicitly modelled.

We can see that in present Academica a significant number of stylised facts have been examined through the utilisation of ABM and interaction of Heterogeneous agents. Also, the regime-switch was a part of studying. However, the interaction between agents with different volatility models and the market-phenomenons that can occur remain relatively unexplored. Our research aims to contribute in filling this research gap.

3 Hypothesis

The main hypothesis in this study is

"*The interaction between agents employing the Black–Scholes and Heston volatility models leads to endogenous regime switching in market volatility.*"

BS agents assume constant volatility, while Heston agents assume volatility as a stochastic process. Their interaction leads to regime switching in volatility.

Auxiliary hypotheses are

"An increase in the proportion of Heston-based agents amplifies volatility smile compared to markets dominated by Black–Scholes agents."

"Hedging activity by agents using different volatility models increases the divergence between realized volatility and implied volatility"

As BS and heston agents hedge from using different volatilities, systematic difference between RV and IV is observed.

4 Model description

This study employs an agent-based model (ABM) of a financial market with an explicit option market in order to analyze the interaction between heterogeneous volatility beliefs and their impact on market dynamics. The model is designed to investigate how different assumptions about volatility formation affect realized volatility, implied volatility, and the emergence of volatility regimes.

4.1 Model architecture

The model consists of three core components: agents, market environment, and dynamic updating rules. Time is discrete and indexed by $t = 0, 1, 2, \ldots, T$. At each time step, agents interact through trading in the underlying asset and its associated options.

4.1.1 Agents

The market is populated by heterogeneous agents who differ in their trading objectives, information sets, and assumptions about volatility dynamics. Each agent type represents a stylized group of market participants and follows a rule-based decision process.

Fundamental traders. Fundamental traders base their decisions on deviations of the market price from an exogenously specified fundamental value. When the market price is below the fundamental value, they submit buy orders; when the price exceeds the fundamental value, they submit sell orders. Their trading activity introduces long-term mean reversion and stabilizing forces into the market.

Chartists (momentum traders). Chartist agents condition their decisions on past price movements. They extrapolate recent trends and buy in rising markets while selling in declining markets. This behavior amplifies short-term price trends and can lead to excess volatility and endogenous cycles.

Noise traders. Noise traders submit random buy or sell orders independently of fundamentals or price trends. Although individually uninformed, their aggregate behavior contributes to market liquidity and introduces stochastic disturbances into the price formation process.

Option traders. Option traders operate in the derivatives market and trade options written on the underlying asset. Their decisions depend on option prices, volatility expectations, and hedging considerations. Option traders link expectations about future volatility to current market outcomes through option demand and hedging activity.

Black–Scholes agents. A subset of option traders assumes that volatility is constant over time. These agents price and hedge options using the Black–Scholes framework with a fixed volatility parameter. Their hedging strategies are based on static volatility assumptions and do not account for stochastic volatility dynamics.

Heston agents. Another subset of option traders assumes that volatility follows a stochastic mean-reverting process. These agents form expectations using a Heston-type volatility model and update their volatility estimates dynamically. As a result, their option pricing and hedging behavior responds more strongly to changes in market conditions.

Let $p_{\text{Heston}} \in [0, 1]$ denote the proportion of Heston-based option traders in the population. The remaining fraction $1 - p_{\text{Heston}}$ follows the Black–Scholes framework. This parameter is treated as an exogenous control variable and is varied across simulation scenarios.

At each time step, agents observe the current market price and submit orders according to their individual rules. Agents do not observe the true volatility process directly. Instead, realized volatility, implied volatility, and volatility regimes emerge endogenously from market interactions.

4.1.2 Market environment

The market environment consists of a simplified price formation mechanism that aggregates order flow from all agents. Prices are updated based on excess demand. An option pricing module computes option prices and implied volatilities using either the Black–Scholes or Heston framework, depending on agent type.

The model records key state variables at each time step, including asset price, returns, realized volatility, implied volatility at different strikes, and volatility smile measures.

4.2 Model dynamics and algorithms

The simulation evolves according to a discrete-time updating scheme summarized below.

4.2.1 Main simulation loop

```
Initialize market price S_0
Initialize agent population and parameters
Set random seed for reproducibility
```

```
for t = 1 to T do
    for each agent do
        Observe current price S_t
        Form expectations about returns and volatility
        Submit trading orders
    end for

    Aggregate order flow
    Update market price S_{t+1}
    Compute returns
    Update realized volatility
    Price options and compute implied volatility
    Store simulation data
end for
```

4.2.2 Option pricing and volatility updating

```
for each option trader do
    Observe current underlying price

    if trader uses Black-Scholes then
        Assume constant volatility
    else if trader uses Heston then
        Update stochastic volatility via mean reversion
    end if

    Price options at multiple strikes
    Compute implied volatility
    Adjust hedge positions
end for
```

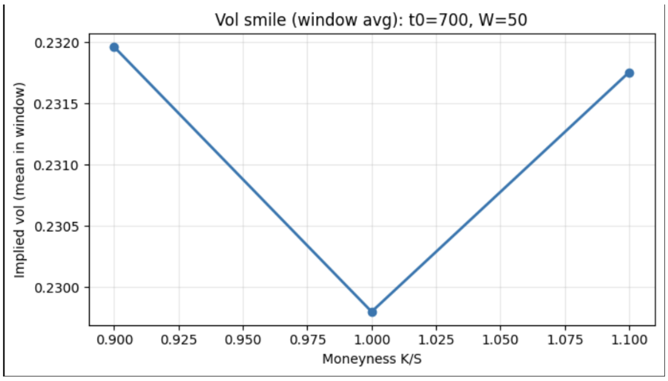

Figure 4.1: Volatility smile

4.2.3 Volatility measures

Realized volatility is computed from rolling windows of returns. Implied volatility is obtained by inverting the option pricing model. The divergence between realized and implied volatility is used as a key diagnostic statistic.

$$RV_t = \sqrt{\sum_{i=1}^{m} r_{t-i}^2} \tag{1}$$

4.2.4 Volatility smile

Implied volatility is computed for strikes below, at, and above the at-the-money level. The volatility smile is measured as the difference between implied volatility at off-the-money strikes and at-the-money implied volatility.

5 Results and statistical analysis

This section presents the simulation results and statistical tests used to evaluate the proposed hypotheses. All simulations are run with a fixed random seed to ensure reproducibility.

5.1 Time series behavior

The interaction of heterogeneous agents generates persistent volatility clustering and regime switching behavior. Periods of elevated volatility alternate with tranquil regimes, despite the absence of exogenous volatility shocks.

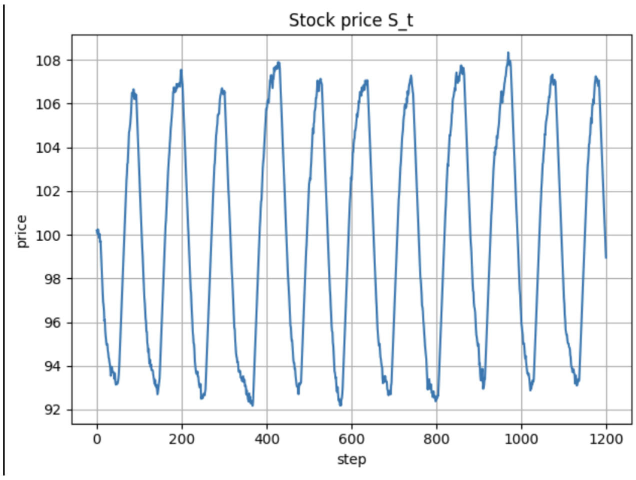

Figure 5.1: Stock price time series

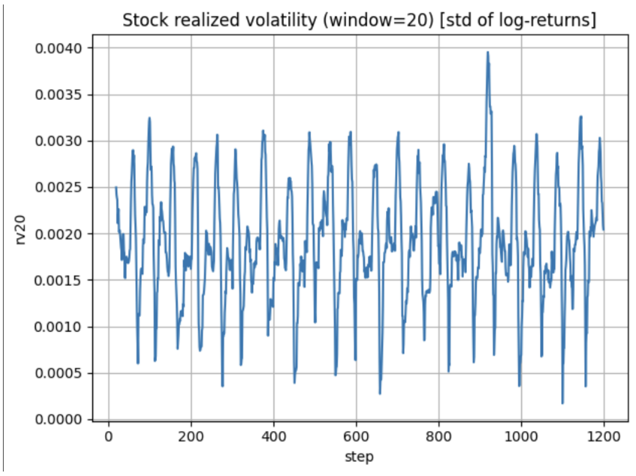

Figure 5.2: Realized volatility time series

5.2 Hypothesis H1: Regime switching in volatility

The null hypothesis H_0 states that volatility distributions are identical across different agent compositions. The alternative hypothesis H_1 states that the interaction between Black–Scholes and Heston agents generates distinct volatility regimes.

A two-sample Kolmogorov–Smirnov test is applied to realized volatility distributions under low and high values of p_{Heston}. The test yields a KS statistic of 0.24 with a p-value close to zero. At a significance level $\alpha = 0.05$, the null hypothesis is rejected.

Additionally, a Ljung–Box test applied to realized volatility confirms strong autocorrelation, indicating volatility clustering consistent with endogenous regime switching.

5.3 Hypothesis H2: Volatility smile amplification

Hypothesis H2 states that a higher proportion of Heston agents increases the strength of the volatility smile. The null hypothesis assumes no difference in smile strength across agent compositions.

An ANOVA test is conducted on the volatility smile metric across scenarios with varying p_{Heston}.

$$F = \frac{MS_B}{MS_W} = \frac{SS_B/(g-1)}{SS_W/(N-g)} \tag{2}$$

The test yields a large F-statistic with a p-value effectively equal to zero. The null hypothesis is rejected, confirming that higher shares of Heston agents amplify the volatility smile.

5.4 Hypothesis H3: Divergence between realized and implied volatility

Hypothesis H3 states that heterogeneous volatility models increase the divergence between realized and implied volatility. A two-sample t-test is applied to the RV–IV spread under low and high values of p_{Heston}.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{3}$$

The test yields a large positive t-statistic and a p-value close to zero. At the 5% significance level, the null hypothesis of equal means is rejected. This result indicates that volatility heterogeneity among agents increases the divergence between realized and implied volatility.

5.5 Discussion

Overall, the results demonstrate that heterogeneous volatility beliefs are sufficient to generate realistic market features such as volatility clustering, regime switching, volatility smiles, and persistent RV–IV spreads. These patterns emerge endogenously from agent interactions rather than from exogenous shocks.

The findings support the view that option markets play a central role in volatility formation and that model heterogeneity among market participants is a key driver of observed volatility phenomena.

6 Conclusion

6.1 Results

The results obtained from the simulation of agent-based financial markets populated by agents employing different volatility models, namely Black–Scholes and Heston, have confirmed the hypothesis. Of particular significance is the interaction between agents with different option pricing models, which has resulted in the emergence of regime switching of volatility without any exogenous shocks. In addition, the model has exhibited market characteristics that are consistent with reality, including volatility smiles and persistent divergences between realised and implied volatility.

6.2 Constrains

However, this model faces serious constraints.Firstly, for the purposes of simplification, discrete time was employed, as well as a more simplified market microstructure. Also the number of agents was equal to 140. Nevertheless, with these simplifications the model still produces statistically significant observations, even though it can not be as representable as real-world data. It also should be noticed that, many parameters like: choice of volatility parameters of Heston model or balance between different agent strategies could have influenced the final result.

6.3 Practical significance

The findings of the study indicated that a switch of volatility can be caused by not only exogenous shocks, but market activity by itself. Furthermore, the use of different models by market participants can amplify the market distortions. It is suggested that these observations can be used by market participants, risk managers and regulators to better understand and control risks, especially those related to model risk.

6.4 Future work

Future work may extend our model with more realistic market structure by implementing limit order book, transaction costs and liquidity constraints. In addition, the implementation of an endogenous switch between the BS and Heston models can make the conflict between models more realistic. Finally, it should be noted that model parameters can be calibrated with real-life data.

7 Bibliography

- **LeBaron, B.** (2006). *Agent-based computational finance.* **Handbook of Computational Economics**, 2, 1187–1233. [1]
- **Cont, R.** (2001). *Empirical properties of asset returns: Stylized facts and statistical issues.* **Quantitative Finance**, 1(2), 223–236. [2]
- **Chakraborti, A., Toke, I. M., Patriarca, M., & Abergel, F.** (2011). *Econophysics review: I. Empirical facts.* **Quantitative Finance**, 11(7), 991–1012. [3]
- **Lux, T., & Marchesi, M.** (1999). *Scaling and criticality in a stochastic multi-agent model of a financial market.* **Nature**, 397(6719), 498–500.[4]
- **Hommes, C. H.** (2006). *Heterogeneous agent models in economics and finance.* **Handbook of Computational Economics**, 2, 1109–1186. [5]
- **Heston, S. L.** (1993). *A closed-form solution for options with stochastic volatility with applications to bond and currency options.* **The Review of Financial Studies**, 6(2), 327–343. [6]
- **Sircar, R., & Papanicolaou, G.** (1998). *General Black–Scholes models accounting for increased market volatility from hedging strategies.* **Applied Mathematical Finance**, 5(1), 45–82. [7]
- **Hamilton, J. D.** (1989). *A new approach to the economic analysis of nonstationary time series and the business cycle.* **Econometrica**, 57(2), 357–384. [8]

8 Appendix

Github code