

Columbia University
MATH G6071 Spring 2015
Numerical Methods in Finance
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Homework3: Implied Calculations / Root solving

	Distribution date	Due date
Homework	Mar 25, 2015 (Wed)	April 8, 2015 (Wed) 7:30pm

Note: please send all homework solution to TA before the due date and time. Spreadsheet should be uploaded from CourseWorks. For theory part, please deposit it into the homework box in MATH building 4/F. If you suspect there are typos in this homework, or some questions are wrong, please feel free to email the instructor.

PRACTICE

QUESTION 1

Given the market value of a option (either a European call or a European put), the implied volatility of that option is obtained by finding the volatility such that the standard Black Scholes formula returns the given market value.

Using Excel, implement implied vol functionality for a call option, which the user can choose among the following methods of root solving we covered in class

1. Newton Raphson method
2. Regula Falsi method
3. Secant method

In each of the above method, the initial guess(es) are user inputs.

Consider the case when domestic zero rate is 0.5%, foreign interest rate is 4%, strike of option is 120, spot is 120, and expiration is 92 calendar days.

Using the spreadsheet you made in Question 1, suppose user always use the following initial guesses

- Using Newton's method, initial guess is 20%
- Using Secant method, initial guess pair is 19% and 20%
- Using Regula Falsi, initial guess pair is 1% and 30%

Discuss your experience with the following situations trying to imply a vol for the target of 1, 3, and 10 (in domestic currency). Do they all converge?

QUESTION 2

A bull spread is a package which you buy a call and then sell another call of a higher strike. You can find more details in [Hull] (you may look it up from index). We already came across it as call spread before.

Since your spreadsheet in question 1 can imply the vol given the value for one call, enhance your spreadsheet now so that it can also imply the vol given the value of a bull spread.

Consider the same market data, and the bull spread is such that you buy a call at strike 118 and sell another call at strike 122. The target value for the bull spread is 1.7

Suppose user always use the following initial guesses

- Using Newton's method, initial guess is 20%
- Using Secant method, initial guess pair is 19% and 20%
- Using Regula Falsi, initial guess pair is 1% and 30%

Do all three methods converge to an implied vol¹? Why and why not?

Do you think that by "well-chosen" initial guesses, all three methods work? Please explain.

THEORY

QUESTION 3

Suppose you want to find a zero for the cubic polynomial $f(x) = x^3 - 3x + 6$.

1. Explain why there is at least one zero for f
2. With initial guess $x_0 = 1$, apply Newton's method to find a zero for f . Does it work? If not, explain why not.

QUESTION 4

1. Let $f(x) = e^{-x^2}$. Compute its general n -term Taylor's expansion $T_n(x)$ at $c = 0$. Is it true that $\lim_{n \rightarrow \infty} T_n(x) = f(x)$?
2. Let $g(x) = \begin{cases} e^{\frac{-1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Compute its general n -term Taylor's expansion $T_n(x)$ at $c = 0$. Is it true that $\lim_{n \rightarrow \infty} T_n(x) = g(x)$?

REFERENCES

[StBu] J. Stoer, R. Bulirsch, *Introduction to Numerical Analysis*, Springer, 3rd edition

[Hull] John Hull, *Options, Futures and other derivatives*,