Columbia University

MATH G6071 Spring 2015
Numerical Methods in Finance
Professor Tat Sang Fung
Homework 1: Excel practice and Warm up

	Distribution date	Due date
Homework	Jan 28, 2015 (Wed)	Feb 18, 2015 (Wed) 7:30pm

Note: please send all homework solution to TA before the due date and time. Spreadsheet should be uploaded from CourseWorks. For theory part, please deposit it into the homework box in MATH building 4/F. If you suspect there are typos in this homework, or some questions are wrong, please feel free to email the instructor

QUESTION 1

If you have forgotten the Black-Scholes formula for currency call option[1], you may want to review [Hull] 9th edition chapter 17. The Greeks formulas are somehow given in Chapter 19.

Make an Excel spreadsheet that does the following:

1. User inputs spot, domestic interest rate, foreign interest rate, volatility, strike, today's date, expiration date
2. The spreadsheet outputs (simultaneously or based on a user choice): Value, Delta, Gamma, Vega, Theta, Phi (rate of change of value with respect to change of foreign interest rate), Rho (rate of change of value with respect to change of domestic interest rate) of the call option[2]
3. The user can select one Greek from the list of the Greeks above, the starting spot, the end spot and the number of spots between the start and end value, (maybe then press a button), the spreadsheet gives the spot-Greek graph.

Save your spreadsheet with the following data while exhibiting the spot-Theta graph (a USD/JPY call option, valued in JPY):

Spot is 120, USD zero rate is 4%, JPY zero rate is 0.5%, strike is 119, vol is **10%**, time to expiration is 92 calendar days. The user then wants to see the spot-Theta graph from spot = 100 to spot = 150 with 99 points in between. So graph is between from Theta with spot = 100, 100.5, 101, 101.5, …, 149.5, 150.

THEORY

QUESTION 1

Since a call option can be thought of, roughly, as insurance, it seems natural to expect that when tomorrow comes, its offers one less day's protection and hence it should lose value. In finance terms, this means its theta should be negative.

If you read [Hull] 9th edition chapter 19 page 410, or 8th edition chapter 18 page 388, it says "Theta is usually negative for an option". And then the footnote says "An exception to this could be an in-the-money

[1] You should have learned it from the class "introduction to math finance" in the fall semester

[2] If you are in doubt, in this homework a USD/JPY call with strike 119 is an option such that on expiration the holder has the right but not the obligation to pay JPY 119 and receive USD 1. Sometimes in the market it is called a "Call USD/Put JPY" option but we don't need to confuse ourselves here.

European put option on a non-dividend paying stock or an in-the-money European call option on a currency with a very high interest rate."

a) Use the graph you built in the practice section to see that it is true that for an in-the-money European call option on a currency with an interest rate larger than the domestic currency (e.g. USD zero rate > JPY zero rate when the USD/JPY call is valued in JPY). Print the graph and hand it in.

b) Suppose the USD zero rate is less than JPY zero rate (e.g. USD zero rate is 0.5% and JPY zero rate is 4%), do you think the call option theta can be positive?

c) A Trader tells you "for any European call option, if it is sufficiently deep in the money, its theta (given by the BS formula) will become positive." Do you think the statement is true? If yes, prove it. If no, give a counter example

Please note that, as we discussed in class, that interest rates may be zero or even negative nowadays when answering this question.

QUESTION 2

In the market it is no longer sufficient just to monitor Vega $\frac{\partial V}{\partial \sigma}$ (the rate of change of value with respect to the change of volatility). People need to monitor the second derivative $\frac{\partial^2 V}{\partial \sigma^2}$. It is known as Volga in the case of FX Options. But they cannot seem to find the formula in [Hull].

A risk manager comes to you and says "$\frac{\partial^2 V}{\partial \sigma^2}$ is the same for a (European) call and a put[3]. The formula is given by $\frac{\partial V}{\partial \sigma} \cdot \frac{d_1 d_2}{\sigma}$"

a) Do you think what the risk manager says is true? If yes, prove it; if no, give a counter example.

QUESTION 3

We discussed in class the change of swap valuation convention and its effect on interest rate curve bootstrapping since 2008 or so: Currently market conventions is such that for a USD LIBOR swap the discounting may not be using USD LIBOR (for example, it may be using USD OIS). In [MA], the author proved the valuation formula when collateral currency is in currency j while the payoff currency is in currency i with his formula (5) below. For notations meaning please refer to [MA]

$$h^{(i)}(t) = E_t^{Q^i}\left[e^{-\int_t^T r^{(i)}(s)ds}\left(e^{\int_t^T y^{(j)}(s)ds}\right)h^{(i)}(T)\right] \qquad (4)$$

$$= D^{(i)}(t,T)E_t^{T^i}\left[e^{-\int_t^T y^{(i,j)}(s)ds}h^{(i)}(T)\right] \qquad (5)$$

(i) Deduce that the valuation formula when payoff currency is the same as the payoff currency is indeed given by his formula 8:

[3] Assuming the call and the put has the same strike, days to maturity… etc

$$h(t) = E_t^Q\left[e^{-\int_t^T c(s)ds} h(T)\right] = D(t,T) E_t^T\left[h(T)\right] \qquad (8)$$

(ii) Show that the swap valuation formula when a USD LIBOR fixed-floating swap, with collateral currency in USD, is given as below.

$$IRS_M \sum_{m=1}^{M} \Delta_m D(0,T_m) = \sum_{m=1}^{M} \delta_m D(0,T_m) E^{T_m}\left[L(T_{m-1},T_m;\tau)\right]$$

(iii) What does the left hand side of the formula above stand for?

(iv) What does the right hand side of the formula above stand for?

REFERENCES

[Hull] John Hull, *Options, Futures and other derivatives*, Prentice Hall, 9th Edition

[MA] Masaaki Fujii and Akihiko Takahashi, *Choice of collateral currency*, RISK Jan 2011