

$$\begin{aligned}
7. \quad & \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x} \\
&= \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{x(x-1)} \\
&= \lim_{x \rightarrow 1} \frac{x+2}{x} \\
&= \frac{3}{1} \\
&= 3
\end{aligned}$$

★ 羅必達

$$\begin{aligned}
& \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x} \\
&= \lim_{x \rightarrow 1} \frac{2x + 1}{2x - 1} \\
&= \frac{3}{1} \\
&= 3
\end{aligned}$$

使用時機：求極限的數為不定型

Ex:  $\frac{0}{0}$ ,  $\frac{\infty}{\infty}$ ,  $0 \cdot \infty$ ,  $\infty - \infty$  ...

# 習題

$$3. \lim_{x \rightarrow -3} (x^2 - 13)$$

$$= 9 - 13$$

$$= -4$$

$$4. \lim_{x \rightarrow -2} (x^3 - 2x^2 + 4x + 8)$$

$$= -8 - 8 - 8 + 8$$

$$= -16$$

$$5. \lim_{z \rightarrow 4} (\sqrt{z^2 - 10})$$

$$= \sqrt{16 - 10}$$

$$= \sqrt{6}$$

$$6. \lim_{h \rightarrow 0} \frac{3}{\sqrt{3h+1} + 1}$$

$$= \frac{3}{\sqrt{1} + 1}$$

$$= \frac{3}{2}$$

$$7. \lim_{x \rightarrow 5} \frac{x-5}{x^2-25}$$

$$= \frac{1}{2x}$$

$$= \frac{1}{10}$$

$$8. \lim_{t \rightarrow 1} \frac{t^2+t-2}{t^2-1}$$

$$= \frac{2t+1}{2t}$$

$$= \frac{3}{2}$$

$$9. \lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$$

$$= \lim_{x \rightarrow 9} \frac{x^{\frac{1}{2}}-3}{x-9}$$

$$= \frac{\frac{1}{2}x^{-\frac{1}{2}}}{1}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$= \frac{1}{6}$$

$$\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$$

$$= \lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)}$$

$$= \lim_{x \rightarrow 9} \frac{1}{\sqrt{x}+3}$$

$$= \frac{1}{6}$$

$$\begin{aligned}
 10. \quad & \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{x^2+5}-3} \\
 &= \lim_{x \rightarrow -2} \frac{x+2}{(x^2+5)^{\frac{1}{2}}-3} \\
 &= \lim_{x \rightarrow -2} \frac{1}{\frac{1}{2}(x^2+5)^{-\frac{1}{2}} \cdot 2x} \\
 &= \lim_{x \rightarrow -2} \frac{1}{x \cdot \frac{1}{\sqrt{x^2+5}}} \\
 &= \frac{1}{-2 \cdot \frac{1}{3}} \\
 &= -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 & \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{x^2+5}-3} \\
 &= \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{x^2+5}-3} \cdot \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3} \\
 &= \lim_{x \rightarrow -2} \frac{x+2 \cdot (\sqrt{x^2+5}+3)}{x^2+5-9} \\
 &= \lim_{x \rightarrow -2} \frac{x+2 \cdot (\sqrt{x^2+5}+3)}{x^2-4} \\
 &= \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{(x+2)(x-2)} \\
 &= -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & \lim_{x \rightarrow 0} (2 \sin x - 1) \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & \lim_{x \rightarrow 0} \frac{1+x+\sin x}{3 \cos x} \\
 &= \frac{1+0+0}{3 \cdot 1} \\
 &= \frac{1}{3}
 \end{aligned}$$

$$13. \lim_{x \rightarrow c} f(x) = 5, \quad \lim_{x \rightarrow c} g(x) = -2$$

$$(a) \quad 5 \cdot (-2) = -10$$

$$(b) \quad 2 \cdot 5 \cdot (-2) = -20$$

$$(c) \quad 5 + 3 \cdot (-2) = -1$$

$$(d) \quad \frac{5}{5 - (-2)} = \frac{5}{7}$$

$$14. \lim_{x \rightarrow b} f(x) = 7, \quad \lim_{x \rightarrow b} g(x) = -3$$

$$(a) \quad 7 + (-3) = 4$$

$$(b) \quad 7 \cdot (-3) = -21$$

$$(c) \quad 4 \cdot (-3) = -12$$

$$(d) \quad \frac{7}{-3} = -\frac{7}{3}$$

$$15. \sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$$

$$= \lim_{x \rightarrow 0} \sqrt{5 - 2x^2} \leq \lim_{x \rightarrow 0} f(x) \leq \lim_{x \rightarrow 0} \sqrt{5 - x^2}$$

$$= \sqrt{5} \leq \lim_{x \rightarrow 0} f(x) \leq \sqrt{5}$$

$$\lim_{x \rightarrow 0} f(x) = \sqrt{5}$$

$$16. \quad 2 - x^2 \leq g(x) \leq 2 \cos x$$

$$= \lim_{x \rightarrow 0} (2 - x^2) \leq \lim_{x \rightarrow 0} g(x) \leq \lim_{x \rightarrow 0} (2 \cos x)$$

$$= 2 \leq \lim_{x \rightarrow 0} g(x) \leq 2$$

$$\lim_{x \rightarrow 0} g(x) = 2$$

$$17. \quad \lim_{x \rightarrow 4} \frac{f(x) - 5}{x - 2} = 1$$

$$\Rightarrow \frac{f(4) - 5}{2} = 1$$

$$\lim_{x \rightarrow 4} f(x) = 7$$

$$18. \quad \lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1$$

$$(a) \quad \lim_{x \rightarrow -2} f(x) = 1 \cdot (-2)^2$$

$$= 4$$

$$(b) \quad \lim_{x \rightarrow -2} \frac{f(x)}{x} = \frac{4}{-2}$$

$$= -2$$