1. $\lim\limits_{x \to 3} \dfrac{x^2-9}{x-3}$

$= \dfrac{2x}{1}$

$= 6$

2. $\lim\limits_{x \to 0} \dfrac{x^2+3x+2}{x}$

$= \dfrac{2}{0}$

$= 不存在$

3. $\lim\limits_{x \to 2} \dfrac{x^3-8}{x-2}$

$= \dfrac{3x^2}{1}$

$= 12$

4. $\lim\limits_{x \to \infty} \dfrac{3x^3-5x+4}{x^3+2x^2-7}$

$= \dfrac{3}{1}$

$= 3$

5. $\lim\limits_{x \to 0} \dfrac{\sin 3x - \tan 3x}{x^3}$

$= \dfrac{-\frac{27}{2}x^3}{x^3}$

$= -\dfrac{27}{2}$

$\sin x = x - \dfrac{x^3}{3!}$

$\tan x = x + \dfrac{x^3}{3}$

$\sin 3x = 3x - \dfrac{27x^3}{6}$

$\tan 3x = 3x - \dfrac{27x^3}{3}$

$\sin 3x - \tan 3x = -\dfrac{27x^3}{2}$

6. $\lim\limits_{x \to \infty} \dfrac{2x^2 - 3x + 5}{4x^2 + 7}$

$= \dfrac{2}{4}$

$= \dfrac{1}{2}$

7. $\lim\limits_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

$= \dfrac{2x}{1}$

$= -2$

8. $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$

$= \dfrac{(x+4)^{\frac{1}{2}} - 2}{x}$

$= \dfrac{\frac{1}{2}(x+4)^{-\frac{1}{2}}}{1}$

$= \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{x+4}} = \dfrac{1}{4}$

9.

$$X$$

10. $\displaystyle\lim_{x \to 0^+} \ln(x)$

$$= -\infty$$
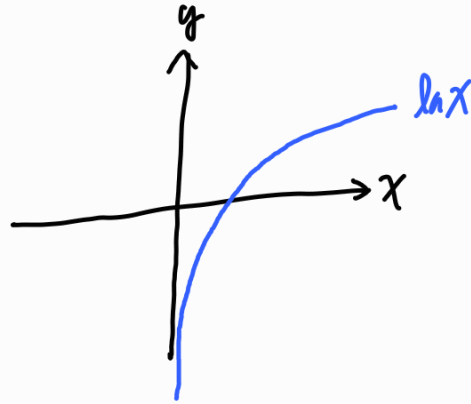
11. $\displaystyle\lim_{x \to 0} \frac{x^3 + 3x^2}{x}$

$$\approx x^2 + 3x$$

$$= 0$$

12. $\displaystyle\lim_{x \to 1^-} \frac{x-1}{|x-1|}$

$$= \frac{(x-1)}{-(x-1)}$$

$$= -1$$

13. $\displaystyle\lim_{x \to 0} \frac{\cos x - 1}{x^2}$

$$\approx \frac{-\sin x}{2x}$$

$$\approx \frac{-\cos x}{2}$$

$$= -\frac{1}{2}$$

14. $\displaystyle\lim_{x\to\infty} \frac{5x-3}{2x+4}$

$= \dfrac{5}{2}$

15. $\displaystyle\lim_{x\to 0} \frac{\tan x}{x}$

$= \dfrac{\sec^2 x}{1}$

$= 1$

16. $f(x) = \begin{cases} x^2+2x+1 & , \; x<1 \\ 3x+a & , \; x\geq 1 \end{cases}$

$\displaystyle\lim_{x\to 1} x^2+2x+1 = \lim_{x\to 1} 3x+a$

$\Rightarrow \quad 4 = 3+a$

$\quad a = 1$

17. $g(x) = \begin{cases} \dfrac{x^2-4}{x+2} & , \; x \neq -2 \\ b & , \; x=-2 \end{cases}$

$\displaystyle\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} b$

$\Rightarrow \quad \dfrac{2x}{1} = b$

$\Rightarrow \quad b = -4$

18. $f(x) = \begin{cases} 4x+3 & , x<0 \\ ax^2+bx+c & , x \geq 0 \end{cases}$

$$\lim_{x \to 0} 4x+3 = \lim_{x \to 0} ax^2+bx+c$$

$\Rightarrow \quad 3 = c$

$c = 3$
$a = 隨便$
$b = 隨便$

19. $h(x) = \begin{cases} \dfrac{\sqrt{x+1}-1}{x} & , x \neq 0 \\ c & , x = 0 \end{cases}$

$$\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} = \lim_{x \to 0} c$$

$\Rightarrow \dfrac{(x+1)^{\frac{1}{2}}-1}{x} = c$

$\Rightarrow \dfrac{\frac{1}{2}(x+1)^{-\frac{1}{2}}}{1} = c$

$\Rightarrow \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{x+1}} = c$

$c = \dfrac{1}{2}$

20. $f(x) = \begin{cases} \sin x & , x < \frac{\pi}{2} \\ 1 & , x = \frac{\pi}{2} \end{cases}$

$\lim\limits_{x \to \frac{\pi}{2}} \sin x$

$= 1$ 連续

$f(\frac{\pi}{2})$

$= 1$

21. $g(x) = \begin{cases} x^2 - 3x + 2 & , x \leq 1 \\ ax + b & , x > 1 \end{cases}$

$\lim\limits_{x \to 1} x^2 - 3x + 2 = \lim\limits_{x \to 1} ax + b$

$\Rightarrow \quad 0 = a + b$

$a = -b$

22. $f(x) = \begin{cases} \dfrac{x^3 - 8}{x - 2} & , x \neq 2 \\ 4 & , x = 2 \end{cases}$

$\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

$= \dfrac{3x^2}{1} = 12$ 不連续

$f(2)$

$= 4$

23. $f(x) = \begin{cases} \ln x & , x > 0 \\ -1 & , x = 0 \end{cases}$

$= \lim\limits_{x \to 0^+} \ln x$

$= -\infty$ 不連續

$f(0)$

$= -1$

24. $f(x) = \begin{cases} x^2 + 3 & , x \neq 1 \\ 4 & , x = 1 \end{cases}$

$\lim\limits_{x \to 1} x^2 + 3$

$= 4$ 連續

$f(1)$

$= 4$

25. $f(x) = \begin{cases} \dfrac{2x^2 - 4x}{x^2 - 4} & , x \neq 2 \\ a & , x = 2 \end{cases}$

$\lim\limits_{x \to 2} \dfrac{2x^2 - 4x}{x^2 - 4}$

$= \dfrac{4x - 4}{2x} = \dfrac{4}{4} = 1$ $a = 1$

$f(2)$

$= a$