

5.

$$(a) \lim_{x \rightarrow \infty} \sin \frac{1}{x}$$

$$= \sin 0$$

$$= 0$$

$$(b) \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right), t = \frac{1}{x}$$

$$= \lim_{t \rightarrow 0} \frac{1}{t} \cdot \sin t$$

$$= \lim_{t \rightarrow 0} \frac{\sin t}{t}$$

$$= \frac{\cos t}{1} = \frac{1}{1} = 1$$

$\frac{0}{0} = \text{不定型}$

7.  $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 16})$

$$= x - \sqrt{x^2 + 16} \cdot \frac{x + \sqrt{x^2 + 16}}{x + \sqrt{x^2 + 16}}$$

$$= \frac{x^2 - x^2 - 16}{x + \sqrt{x^2 + 16}}$$

$$= \frac{-16}{x + \sqrt{x^2 + 16}}$$

$$= 0$$

$$9. \lim_{x \rightarrow 1^+} \frac{1}{x-1} \quad \text{and} \quad \lim_{x \rightarrow 1^-} \frac{1}{x-1}, \quad t = x-1$$

$$= \lim_{t \rightarrow 0^+} \frac{1}{t} \quad \text{and} \quad \lim_{t \rightarrow 0^-} \frac{1}{t}$$

$$= \infty \quad \text{and} \quad -\infty$$

$$\star \lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

習題

$$2. \quad g(x) = \frac{1}{2 + \frac{1}{x}}$$

$$(a) \quad t = \frac{1}{x}, \quad x \rightarrow \infty$$

$$\lim_{t \rightarrow 0^+} \frac{1}{2+t} = \frac{1}{2}$$

$$(b) \quad t = \frac{1}{x}, \quad x \rightarrow -\infty$$

$$\lim_{t \rightarrow 0^-} \frac{1}{2+t} = \frac{1}{2}$$

$$3. \quad h(x) = \frac{-5 + (\frac{7}{x})}{3 - (\frac{1}{x^2})}$$

$$(a) \quad t = \frac{1}{x}, \quad x \rightarrow \infty$$

$$\lim_{t \rightarrow 0^+} \frac{-5+7t}{3-t^2} = \frac{-5}{3}$$

$$(b) \quad t = \frac{1}{x}, \quad x \rightarrow -\infty$$

$$\lim_{t \rightarrow 0^-} \frac{-5+7t}{3-t^2} = \frac{-5}{3}$$

$$4. \lim_{x \rightarrow \infty} \frac{\sin 2x}{x}$$

$$= 0$$

$$0 \leq \sin 2x \leq 1$$

$$\frac{0}{x} \leq \frac{\sin 2x}{x} \leq \frac{1}{x}, \quad x \rightarrow \infty$$

$$0 \leq \frac{\sin 2x}{x} \leq 0$$

$$5. \lim_{t \rightarrow -\infty} \frac{2-t+\sin t}{t+\cos t}$$

$$= -1$$

$$6. f(x) = \frac{2x+3}{5x+7}$$

$$(a) \frac{2}{5}$$

$$(b) \frac{2}{5}$$

$$7. h(x) = \frac{7x^3}{x^3-3x^2+6x}$$

$$(a) 7$$

$$(b) 7$$

$$8. \lim_{x \rightarrow \infty} \sqrt{\frac{8x^2-3}{2x^2+x}}$$

$$= \sqrt{\frac{8}{2}}$$

$$= \sqrt{4}$$

$$= 2$$

$$9. \lim_{x \rightarrow \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 7}$$

$$= \frac{1}{\infty}$$

$$= 0$$

$$x > \sqrt{x} > x^{-1}$$

$$10. \lim_{x \rightarrow \infty} \frac{2 + \sqrt{x}}{2 - \sqrt{x}}$$

$$= -1$$

$$11. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1}}{x + 1}$$

$$= 1$$

$$12. \lim_{x \rightarrow 0^+} \frac{1}{3x}$$

$$= \infty$$

$$\lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

$$13. \lim_{x \rightarrow 7} \frac{4}{(x-7)^2}$$

$$= \infty$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$