習題

1. $y = \frac{3}{x} + 5\sin x$

$= 3x^{-1} + 5\sin x$

$y' = -3x^{-2} + 5\cos x$

2. $y = x^2 \cos x$

$y' = (x^2)'(\cos x) + (x^2)(\cos x)'$

$= 2x\cos x + x^2(-\sin x)$

$= 2x\cos x - x^2\sin x$

3. $y = \sqrt{x}\sec x + 3$

$= x^{\frac{1}{2}}\sec x + 3$

$y' = (x^{\frac{1}{2}})'(\sec x) + (x^{\frac{1}{2}})(\sec x)'$

$= \frac{1}{2}x^{-\frac{1}{2}}\sec x + x^{\frac{1}{2}}\sec x\tan x$

$= \frac{1}{2\sqrt{x}}\sec x + \sqrt{x}\sec x\tan x$

4. $y = \csc x - 4\sqrt{x} + \frac{7}{e^x}$

$= \csc x - 4x^{\frac{1}{2}} + 7e^{-x}$

$y' = -\csc x \cot x - 2x^{-\frac{1}{2}} + 7e^{-x}(-1)$

$= -\csc x \cot x - \frac{2}{\sqrt{x}} - 7e^{-x}$

5. $f(x) = \sin x \tan x$

$f'(x) = (\sin x)'(\tan x) + (\sin x)(\tan x)'$

$= \cos x \tan x + \sin x \sec^2 x$

$= \cos x \cdot \frac{\sin x}{\cos x} + \sin x \sec^2 x$

$= \sin x + \sin x \sec^2 x$

6. $y = (\sin x + \cos x)\sec x$

$y' = (\sin x + \cos x)'(\sec x) + (\sin x + \cos x)(\sec x)'$

$= (\cos x - \sin x)(\sec x) + (\sin x + \cos x)(\sec x \tan x)$

$= (\cos x - \sin x)\left(\frac{1}{\cos x}\right) + (\sin x + \cos x)\left(\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}\right)$

$= 1 - \tan x + \tan^2 x + \tan x$

$= 1 + \tan^2 x$

7. $y = \dfrac{\cot x}{1+\cot x}$

$= \dfrac{1+\cot x - 1}{1+\cot x}$

$= 1 - \dfrac{1}{1+\cot x}$

$= 1 - (1+\cot x)^{-1}$

$y' = (1+\cot x)^{-2} \cdot (-\csc^2 x)$

$= -\dfrac{\csc^2 x}{(1+\cot x)^2}$

8. $y = \dfrac{4}{\cos x} + \dfrac{1}{\tan x}$

$= 4\sec x + \cot x$

$y' = 4\sec x \tan x + (-\csc^2 x)$

$= 4\sec x \tan x - \csc^2 x$

9. $s = \tan t - e^{-t}$

$s' = \sec^2 t - e^{-t} \cdot (-1)$

$= \sec^2 t + e^{-t}$

10. $S = \dfrac{\sin t}{1-\cos t}$

$= \sin t\,(1-\cos t)^{-1}$

$S' = (\sin t)'(1-\cos t)^{-1} + (\sin t)\left[(1-\cos t)^{-1}\right]'$

$= \cos t\,(1-\cos t)^{-1} + \sin t \cdot (-1)(1-\cos t)^{-2} \cdot (\sin t)$

$= \dfrac{\cos t}{1-\cos t} - \dfrac{\sin^2 t}{(1-\cos t)^2}$

$= \dfrac{\cos t - \cos^2 t}{(1-\cos t)^2} - \dfrac{\sin^2 t}{(1-\cos t)^2}$

$= \dfrac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^2}$

11. $r = 4 - \theta^2 \sin\theta$

$r' = (-\theta^2)'(\sin\theta) + (-\theta^2)(\sin\theta)'$

$= -2\theta \sin\theta - \theta^2 \cos\theta$

12. $r = (1+\sec\theta)\sin\theta$

$= \left(1 + \dfrac{1}{\cos\theta}\right)\sin\theta$

$= \sin\theta + \tan\theta$

$r' = \cos\theta + \sec^2\theta$

13. $p = 5 + \frac{1}{\cot q}$

$= 5 + \tan q$

$p' = \sec^2 q$

14. $p = \frac{q \sin q}{q^2 - 1}$

$= \frac{\sin q}{q - \frac{1}{q}}$

$= \sin q \left(q - q^{-1}\right)$

$p' = (\sin q)'(q - q^{-1}) + (\sin q)\left[(q - q^{-1})\right]'$

$= \cos q \left(q - q^{-1}\right) + \sin q \,(-1)(q - q^{-1})^{-2}(1 + q^{-2})$

$= \frac{\cos q}{q - \frac{1}{q}} - \frac{\sin q \,(1 + \frac{1}{q^2})}{(q - \frac{1}{q})^2}$

$= \frac{q \cos q}{q^2 - 1} - \frac{\sin q \,(q^2 + 1)}{(q^2 - 1)^2}$

$= \frac{q^3 \cos q - q \cos q}{(q^2 - 1)^2} - \frac{\sin q \,(q^2 + 1)}{(q^2 - 1)^2}$

$= \frac{q^3 \cos q - q \cos q - q^2 \sin q - \sin q}{(q^2 - 1)^2}$

15.

(a) $y = -2\sin x$

$y' = -2\cos x$

$y'' = 2\sin x$

$y''' = 2\cos x$

$y^{(4)} = -2\sin x$

(b) $y = 9\cos x$

$y' = -9\sin x$

$y'' = -9\cos x$

$y''' = 9\sin x$

$y^{(4)} = 9\cos x$