

1.  $y = 4.9 t^2$  公尺

$$(a) \frac{\Delta y}{\Delta t} = \frac{4.9(2^2) - 4.9(0)^2}{2 - 0} = 9.8$$

$$(b) \frac{\Delta y}{\Delta t} = \frac{4.9(2^2) - 4.9(1^2)}{2 - 1} = 14.7$$

2.

$$(a) \frac{\Delta y}{\Delta t} = \frac{4.9(1+h)^2 - 4.9(1)^2}{h} = 9.8 + 4.9h$$

$$\lim_{h \rightarrow 0} 9.8 + 4.9h = 9.8$$

$$(b) \frac{\Delta y}{\Delta t} = \frac{4.9(2+h)^2 - 4.9(2)^2}{h} = 19.6 + 4.9h$$

$$\lim_{h \rightarrow 0} 19.6 + 4.9h = 19.6$$

3.  $y = x^2$ ,  $(2, 4)$

$$\frac{\Delta y}{\Delta x} = \frac{(2+h)^2 - (2)^2}{(2+h) - (2)} = \frac{(2+h)^2 - (2)^2}{h} = h + 4$$

$$\lim_{h \rightarrow 0} h + 4 = 4$$

$$y = ax + b, \quad a = 4$$

$$y = 4x + b, \quad (2, 4)$$

$$4 = 8 + b$$

$$b = -4 \rightarrow y = 4x - 4$$

# 習題

1.  $f(x) = x^3 + 1$

$$(a) \frac{\Delta y}{\Delta x} = \frac{(3^3+1) - (2^3+1)}{3-2} = \frac{28-9}{1} = 19$$

$$(b) \frac{\Delta y}{\Delta x} = \frac{(1^3+1) - [(-1)^3+1]}{1 - (-1)} = \frac{2-0}{2} = 1$$

2.  $h(t) = \cot t$

$$(a) \frac{\Delta y}{\Delta t} = \frac{(-1) - (1)}{\frac{3}{4}\pi - \frac{1}{4}\pi} = \frac{-2}{\frac{2}{4}\pi} = -\frac{4}{\pi}$$

$$(b) \frac{\Delta y}{\Delta t} = \frac{0 - \sqrt{3}}{\frac{1}{2}\pi - \frac{1}{6}\pi} = \frac{-\sqrt{3}}{\frac{1}{3}\pi} = -\frac{3\sqrt{3}}{\pi}$$

$$\cot \frac{\pi}{4} = 1$$

$$\cot \frac{3\pi}{4} = -1$$

$$\cot \frac{\pi}{6} = \sqrt{3}$$

$$\cot \frac{\pi}{2} = 0$$

3.  $R(\theta) = \sqrt{4\theta+1}$

$$\frac{\Delta y}{\Delta \theta} = \frac{(\sqrt{4 \cdot 2 + 1}) - (\sqrt{4 \cdot 0 + 1})}{2 - 0} = \frac{3-1}{2} = 1$$

4.  $P(\theta) = \theta^3 - 4\theta^2 + 5\theta$

$$\frac{\Delta y}{\Delta \theta} = \frac{(2^3 - 4 \cdot 2^2 + 5 \cdot 2) - (1^3 - 4 \cdot 1^2 + 5 \cdot 1)}{2 - 1}$$

$$= \frac{(8 - 16 + 10) - (1 - 4 + 5)}{1}$$

$$= 2 - 2$$

$$= 0$$

5.  $y = x^2 - 5$ ,  $P(2, -1)$

$$\frac{\Delta y}{\Delta x} = \frac{[(2+h)^2 - 5] - (-1)}{h} = \frac{h^2 + 4h}{h} = h + 4$$

$$\lim_{h \rightarrow 0} h + 4 = 4$$

$$y = 4x + b, (2, -1)$$

$$-1 = 8 + b$$

$$b = -9 \rightarrow y = 4x - 9$$

6.  $y = 7 - x^2$ ,  $P(2, 3)$

$$\frac{\Delta y}{\Delta x} = \frac{[7 - (2+h)^2] - (3)}{h} = \frac{-h^2 - 4h}{h} = -h - 4$$

$$\lim_{h \rightarrow 0} -h - 4 = -4$$

$$y = -4x + b, (2, 3)$$

$$3 = -8 + b$$

$$b = 11 \rightarrow y = -4x + 11$$

7.  $y = x^3$ ,  $P(2, 8)$

$$\frac{\Delta y}{\Delta x} = \frac{(2+h)^3 - (8)}{h} = \frac{h^3 + 6h^2 + 12h}{h} = h^2 + 6h + 12$$

$$\lim_{h \rightarrow 0} h^2 + 6h + 12 = 12$$

$$y = 12x + b, (2, 8)$$

$$8 = 24 + b$$

$$b = -16 \rightarrow y = 12x - 16$$

8.  $y = x^3 - 3x^2 + 4$ ,  $P(2,0)$

$$\frac{\Delta y}{\Delta x} = \frac{[(2+h)^3 - 3(2+h)^2 + 4] - (0)}{h} = \frac{h^3 - 3h^2}{h} = h^2 - 3h$$

$$\lim_{h \rightarrow 0} h^2 - 3h = 0$$

$$y = 0x + b, (2,0)$$

$$0 = 0 + b$$

$$b = 0 \rightarrow y = 0$$