習題

1. $f(x)=x^2+2x-1$, $[0,1]$

$$\frac{f(1)-f(0)}{1-0}=\frac{2-(-1)}{1}=3$$

$$f'(x)=2x+2$$

$$2x+2=3$$

$$x=\frac{1}{2}$$

2. $f(x)=x^{\frac{2}{3}}$, $[0,1]$

$$\frac{f(1)-f(0)}{1-0}=\frac{1-0}{1}=1$$

$$f'(x)=\frac{2}{3}x^{-\frac{1}{3}}$$

$$\frac{2}{3}x^{-\frac{1}{3}}=1$$

$$x^{-\frac{1}{3}}=\frac{3}{2}$$

$$(x^{-\frac{1}{3}})^{-3}=(\frac{3}{2})^{-3}$$

$$x=\frac{8}{27}$$

3. $f(x) = \sqrt{x-1}$, $[1, 3]$

$$\frac{f(3)-f(1)}{3-1} = \frac{\sqrt{2}-0}{2} = \frac{\sqrt{2}}{2}$$

$$f'(x) = \frac{1}{2\sqrt{x-1}}$$

$$\frac{1}{2\sqrt{x-1}} = \frac{\sqrt{2}}{2}$$

$$2 = 2\sqrt{2x-2}$$

$$1 = \sqrt{2x-2}$$

$$1 = 2x-2$$

$$x = \frac{3}{2}$$

4. $f(x) = x^3 - x^2$, $[-1, 2]$

$$\frac{f(2)-f(-1)}{2-(-1)} = \frac{4+2}{3} = 2$$

$$f'(x) = 3x^2 - 2x$$

$$3x^2 - 2x = 2$$

$$3x^2 - 2x - 2 = 0$$

$$x = \frac{2 \pm \sqrt{4+24}}{6}$$

$$= \frac{1 \pm \sqrt{7}}{3}$$

11. $f(-1)=3$, $f'(x)=0$, $f(x)=?$

$f'(x)=0$

$f(x)=0x+C$

$f(-1)=3$

$C=3$

正確

12. $f'(x)=2x$, $f(2)=?$

(a) $f(0)=0$

$f(x)=x^2+C$

$f(0)=0$

$C=0$

$f(2)=4$

(b) $f(1)=0$

$f(x)=x^2+C$

$0=1+C$

$C=-1$

$f(2)=3$

(c) $f(-2)=3$

$f(x)=x^2+C$

$3=4+C$

$C=-1$

$f(2)=3$

13.

(a) $y' = x$

$y = \frac{1}{2}x^2 + C$

(b) $y' = x^2$

$y = \frac{1}{3}x^3 + C$

(c) $y' = x^3$

$y = \frac{1}{4}x^4 + C$

14.

(a) $y' = -\frac{1}{x^2}$

$= -x^{-2}$

$y = x^{-1} + C$

(b) $y' = 1 - \frac{1}{x^2}$

$y = x + x^{-1} + C$

(c) $y' = 5 + \frac{1}{x^2}$

$y = 5x - x^{-1} + C$

15.

(a) $y' = \sin 2t$

$$y = -\cos 2t \cdot \frac{1}{2}$$

$$= -\frac{1}{2}\cos 2t$$

(b) $y' = \cos \frac{t}{2}$

$$y = \sin \frac{t}{2} \cdot \frac{1}{\frac{1}{2}}$$

$$= 2\sin \frac{t}{2}$$

(c) $y' = \sin 2t + \cos \frac{t}{2}$

$$y = -\cos 2t \cdot \frac{1}{2} + \sin \frac{t}{2} \cdot 2$$

$$= -\frac{1}{2}\cos 2t + 2\sin \frac{t}{2}$$

16. $f'(x) = 2x - 1$, $P(0,0)$

$$f(x) = x^2 - x + C$$

$$f(0) = 0$$

$$C = 0$$

$$f(x) = x^2 - x$$

17. $f(x) = e^{2x}$, $P(0, \frac{3}{2})$

$$f(x) = e^{2x} \cdot \frac{1}{2} + C$$

$$f(0) = \frac{3}{2}$$

$$\frac{3}{2} = \frac{1}{2} + C$$

$$C = 1$$

$$f(x) = \frac{1}{2}e^{2x} + 1$$