

$$1. \quad f(x) = \frac{x}{x-1}$$

$$f(x) = \frac{x-1+1}{x-1}$$

$$= \frac{x-1}{x-1} + \frac{1}{x-1}$$

$$= 1 + \frac{1}{x-1}$$

$$= 1 + (x-1)^{-1}$$

$$f'(x) = -1(x-1)^{-2}$$

$$= \frac{-1}{(x-1)^2}$$

$$2. \quad f(x) = \sqrt{x}, \quad x = 4$$

$$(a) \quad f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$(b) \quad f(4) = 2 = y$$

$$f'(4) = \frac{1}{4} = a$$

$$y = ax + b$$

$$2 = \frac{1}{4} \cdot 4 + b$$

$$2 = 1 + b$$

$$b = 1$$

$$y = \frac{1}{4}x + 1$$

習題

1. $f(x) = 4 - x^2$

$$f'(x) = -2x$$

$$f'(-3) = 6$$

$$f'(0) = 0$$

$$f'(1) = -2$$

2. $g(t) = \frac{1}{t^2}$

$$g(t) = t^{-2}$$

$$g'(t) = -2t^{-3}$$
$$= \frac{-2}{t^3}$$

$$g'(-1) = 2$$

$$g'(2) = -\frac{1}{4}$$

$$g'(\sqrt{3}) = -\frac{2}{3\sqrt{3}}$$

3. $y = 2x^3$

$$y' = 6x^2$$

$$\frac{dy}{dx} = 6x^2$$

$$4. \quad s = \frac{t}{2t+1}$$

$$s = t \cdot (2t+1)^{-1}$$

$$s' = t' \cdot (2t+1)^{-1} + t \cdot [(2t+1)^{-1}]'$$

$$= 1 \cdot (2t+1)^{-1} + t \cdot (-1)(2t+1)^{-2} \cdot (2)$$

$$= \frac{1}{2t+1} - \frac{2t}{(2t+1)^2}$$

$$= \frac{2t+1-2t}{(2t+1)^2}$$

$$= \frac{1}{(2t+1)^2}$$

$$\frac{ds}{dt} = \frac{1}{(2t+1)^2}$$

$$5. \quad v = t - \frac{1}{t}$$

$$v = t - t^{-1}$$

$$v' = 1 - (-1)t^{-2}$$

$$= 1 + \frac{1}{t^2}$$

$$\frac{dv}{dt} = 1 + \frac{1}{t^2}$$

$$6. \quad p = q^{\frac{3}{2}}$$

$$p' = \frac{3}{2} q^{\frac{1}{2}}$$

$$= \frac{3}{2} \sqrt{q}$$

$$\frac{dp}{dq} = \frac{3}{2} \sqrt{q}$$

$$7. f(x) = x + \frac{9}{x}, \quad x = -3$$

$$f(x) = x + 9x^{-1}$$

$$f'(x) = 1 + 9 \cdot (-1) x^{-2}$$

$$= 1 - \frac{9}{x^2}$$

$$f'(3) = 1 - \frac{9}{9}$$

$$= 1 - 1$$

$$= 0$$

$$8. k(x) = \frac{1}{2+x}, \quad x = 2$$

$$k(x) = (2+x)^{-1}$$

$$k'(x) = -1 \cdot (2+x)^{-2}$$

$$= -\frac{1}{(2+x)^2}$$

$$k'(2) = -\frac{1}{4^2}$$

$$= -\frac{1}{16}$$

$$9. s = 1 - 3t^2, \quad t = -1$$

$$s' = -6t, \quad t = -1$$

$$s' = 6$$

$$10. \quad W = Z + \sqrt{Z} \quad , \quad Z = 4$$

$$W = Z + Z^{\frac{1}{2}}$$

$$W' = 1 + \frac{1}{2} Z^{-\frac{1}{2}}$$

$$= 1 + \frac{1}{2\sqrt{Z}} \quad , \quad Z = 4$$

$$W' = 1 + \frac{1}{4}$$

$$= \frac{5}{4}$$

$$11. \quad f(x) = \frac{1}{x+2}$$

$$f(x) = (x+2)^{-1}$$

$$f'(x) = -(x+2)^{-2}$$

$$= -\frac{1}{(x+2)^2}$$