習題

6. $\lim\limits_{x \to -0.5^-} \sqrt{\dfrac{x+2}{x+1}}$

$= \sqrt{\dfrac{1.5}{0.5}} = \sqrt{3}$

7. $\lim\limits_{x \to 1^-} \left(\dfrac{1}{x+1}\right)\left(\dfrac{x+6}{x}\right)\left(\dfrac{3-x}{7}\right)$

$= \dfrac{1}{2} \cdot \dfrac{7}{1} \cdot \dfrac{2}{7} = 1$

8. $\lim\limits_{h \to 0^+} \dfrac{\sqrt{h^2+4h+5} - \sqrt{5}}{h}$

$= \dfrac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} \cdot \dfrac{\sqrt{h^2+4h+5} + \sqrt{5}}{\sqrt{h^2+4h+5} + \sqrt{5}}$

$= \dfrac{h^2 + 4h}{h\left(\sqrt{h^2+4h+5} + \sqrt{5}\right)}$

$= \dfrac{h+4}{\sqrt{h^2+4h+5} + \sqrt{5}} = \dfrac{4}{2\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

9. $\lim\limits_{h \to 0^+} \dfrac{\sqrt{6} - \sqrt{5h^2+11h+6}}{h}$

$= \dfrac{\sqrt{6} - \sqrt{5h^2+11h+6}}{h} \cdot \dfrac{\sqrt{6} + \sqrt{5h^2+11h+6}}{\sqrt{6} + \sqrt{5h^2+11h+6}}$

$= \dfrac{-5h^2 - 11h}{h\left(\sqrt{6} + \sqrt{5h^2+11h+6}\right)}$

$= \dfrac{-5h - 11}{\sqrt{6} + \sqrt{5h^2+11h+6}} = \dfrac{-11}{2\sqrt{6}}$

10.

(a) $\lim\limits_{x \to 1^+} \dfrac{\sqrt{2}x(x-1)}{|x-1|}$

$= \dfrac{\sqrt{2}x(x-1)}{(x-1)}$

$= \sqrt{2}x \quad = \sqrt{2}$

(b) $\lim\limits_{x \to 1^-} \dfrac{\sqrt{2}x(x-1)}{|x-1|}$

$= \dfrac{\sqrt{2}x(x-1)}{-(x-1)}$

$= -\sqrt{2}x = -\sqrt{2}$

11. $\lim\limits_{\theta \to 0} \dfrac{\sin(\sqrt{2}\theta)}{\sqrt{2}\,\theta}$

$= \dfrac{\sqrt{2}}{\sqrt{2}} = 1$

12. $\lim\limits_{t \to 0} \dfrac{\sin(kt)}{t}$

$= k$

13. $\lim\limits_{y \to 0} \dfrac{\sin(3y)}{4y}$

$= \dfrac{3}{4}$

☆ 公式： $\lim\limits_{t \to 0} \dfrac{\sin(bt)}{at} = \dfrac{b}{a}$

14. $\lim\limits_{x \to 0} \dfrac{\tan 2x}{x}$

$= \dfrac{\frac{\sin 2x}{\cos 2x}}{x}$

$= \dfrac{\sin 2x}{x \cdot \cos 2x}$

$= \dfrac{\sin 2x}{x} \cdot \dfrac{1}{\cos 2x}$

$= 2 \cdot 1 = 2$

15. $\lim\limits_{x \to 0} \dfrac{x^2 - x + \sin x}{2x}$

$= \dfrac{x^2 - x}{2x} + \dfrac{\sin x}{2x}$

$= \dfrac{x-1}{2} + \dfrac{\sin x}{2x}$

$= -\dfrac{1}{2} + \dfrac{1}{2} = 0$

$\star \quad \sin 2\theta = 2 \cdot \sin\theta \cdot \cos\theta$

16. $\lim\limits_{\theta \to 0} \dfrac{\sin\theta}{\sin 2\theta}$

$= \dfrac{\sin\theta}{2\sin\theta\cos\theta}$

$= \dfrac{1}{2\cos\theta}$

$= \dfrac{1}{2}$