$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\cos^{-1}u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1}u = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\csc^{-1}u = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

背起来

習題

1. (a) $\tan^{-1} 1 = x$

$\tan x = 1$

$x = \frac{\pi}{4}$

(b) $\tan^{-1}(-\sqrt{3}) = x$

$\tan x = -\sqrt{3}$

$x =$

(c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$

$\tan x = \frac{1}{\sqrt{3}}$

$x =$

2. (a) $\sin^{-1}\left(\frac{-1}{2}\right) = x$

$\sin x = -\frac{1}{2}$

$x =$

(b) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = x$

$\sin x = \frac{1}{\sqrt{2}}$

$x =$

(c) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = x$

$\sin x = -\frac{\sqrt{3}}{2}$

$x =$

3. (a) $\cos^{-1}\left(\frac{1}{2}\right) = x$

$\cos x = \frac{1}{2}$

$x =$

(b) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = x$

$\cos x = -\frac{1}{\sqrt{2}}$

$x =$

(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\cos x = \frac{\sqrt{3}}{2}$

$x =$

4. (a) $\csc^{-1}\sqrt{2} = x$

$\csc x = \sqrt{2}$

$x =$

(b) $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right) = x$

$\csc x = -\frac{2}{\sqrt{3}}$

$x =$

(c) $\csc^{-1} 2 = x$

$\csc x = 2$

$x =$

5. (a) $\sec^{-1}(-\sqrt{2}) = x$

$\sec x = -\sqrt{2}$

$x =$

(b) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x$

$\sec x = \frac{2}{\sqrt{3}}$

$x =$

(c) $\sec^{-1}(-2) = x$

$\sec x = -2$

$x =$

6. (a) $\cot^{-1}(-1) = x$

$\cot x = -1$

$x =$

(b) $\cot^{-1}(\sqrt{3}) = x$

$\cot x = \sqrt{3}$

$x =$

(c) $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = x$

$\cot x = -\frac{1}{\sqrt{3}}$

$x =$

7. $\sin\left(\cos^{-1}\frac{\sqrt{2}}{2}\right)$

設 $\cos^{-1}\frac{\sqrt{2}}{2} = x$

$\cos x = \frac{\sqrt{2}}{2}$

$\sin x = ?$

$? = \frac{\sqrt{2^2-\sqrt{2}^2}}{2} = \frac{\sqrt{2}}{2}$

8. $\sec\left(\cos^{-1}\frac{1}{2}\right)$

設 $\cos^{-1}\frac{1}{2} = x$

$\cos x = \frac{1}{2}$

$\sec x = ?$

$? = 2$

9. $y = \cos^{-1}(x^2)$

$$y' = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot (x^2)'$$

$$= \frac{-1}{\sqrt{1-x^4}} \cdot 2x$$

$$= \frac{-2x}{\sqrt{1-x^4}}$$

10. $y = \sin^{-1}\sqrt{2}t$

$$y' = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot (\sqrt{2}t)'$$

$$= \frac{1}{\sqrt{1-2t^2}} \cdot \sqrt{2}$$

$$= \sqrt{\frac{2}{1-2t^2}}$$

11. $y = \sec^{-1}(2s+1)$

$$y' = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \cdot (2s+1)'$$

$$= \frac{1}{|2s+1|\sqrt{4s^2+4s}} \cdot (2)$$

$$= \frac{2}{|2s+1|\sqrt{4s^2+4s}}$$

12. $y = \csc^{-1}(x^2+1)$

$$y' = \frac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}} \cdot (x^2+1)'$$

$$= \frac{-1}{|x^2+1|\sqrt{x^4+2x^2}} \cdot (2x)$$

$$= \frac{-2x}{x^2+1\sqrt{x^4+2x^2}}$$

13. $y = \cot^{-1}\sqrt{t}$

$$y' = \frac{-1}{\sqrt{t}^2+1} \cdot (\sqrt{t})'$$

$$= \frac{-1}{t+1} \cdot \frac{1}{2\sqrt{t}}$$

$$= \frac{-1}{2\sqrt{t}(t+1)}$$

14. $y = \ln(\tan^{-1} x)$

$$y' = \frac{1}{\tan^{-1} x} \cdot (\tan^{-1} x)'$$

$$= \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2} \cdot (1)$$

$$= \frac{1}{(1+x^2)(\tan^{-1} x)}$$

15. $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1} x$

$$y' = \frac{1}{1+x^2-1} \cdot (\sqrt{x^2-1})' - \frac{1}{|x|\sqrt{x^2-1}} \cdot (x)'$$

$$= \frac{1}{x^2} \cdot \frac{x}{\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$$

$$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$$

16. $y = \ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)$

$$y' = \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - x\,\frac{1}{1+\frac{x^2}{4}} \cdot \frac{1}{2}$$

$$= \frac{2x}{x^2+4} - \tan^{-1}\left(\frac{x}{2}\right) - \frac{2x}{4+x^2}$$

$$= -\tan^{-1}\left(\frac{x}{2}\right)$$