Theory of Computer Science

M. Helmert
F. Pommerening
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University of Basel
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Exercise Sheet 2

Due: Wednesday, March 9, 2016

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 2.1 (Syntax; 0.5+0.5+0.5+0.5 Points)
Formalize the following statements as propositional formulas. In order to do so, also define appropriate atomic propositions. Take care to fully parenthesize all formulas.

(a) If it does not rain, it is warm.

(b) If Bob is going for a swim, then he always eats ice cream and it is not raining.

(c) Bob is going for a swim exactly if he eats eats ice cream and it is warm or does not rain.

(d) Either the sun is shining or it is raining (but not both).

Exercise 2.2 (Truth tables; 1+1+1+1 Points)
Let $A = \{\mathrm{X}, \mathrm{Y}\}$ be a set of propositional variables. Specify a propositional formula φ over A for each of the following properties and then use a truth table to prove that the formula has the property.

(a) φ is satisfiable and falsifiable.

(b) φ has exactly three models.

(c) φ is valid and uses both variables.

(d) φ is unsatisfiable.

Exercise 2.3 (Semantics; 0.5+0.25+1+1+0.5 Points)
Consider the propositional formula φ over $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}$:

$$\varphi = ((\mathrm{F} \lor ((\neg\mathrm{B} \leftrightarrow ((\mathrm{C} \land \mathrm{A}) \rightarrow \neg\mathrm{B})) \lor (\mathrm{D} \rightarrow \mathrm{E}))) \rightarrow (\mathrm{A} \rightarrow \neg\mathrm{F}))$$

(a) How many lines would be needed for a truth table for φ?

(b) Formula φ is an implication. Specify the truth table for the general implication formula $\varphi \rightarrow \psi$ (see chapter B1, slide 30). Attention: You should **not** specify the truth table of φ.

(c) Specify a model $\mathcal{I}$ for φ and prove without truth table that $\mathcal{I} \models \varphi$.

(d) Specify an assignment $\mathcal{I}$ with $\mathcal{I} \not\models \varphi$ and prove that $\mathcal{I}$ has the desired property without a truth table.

(e) Which of the properties *satisfiable*, *unsatisfiable*, *valid*, and *falsifiable* are true for φ? Justify your answer for each of the four properties.

Hint: The proofs for this exercises are fairly short (4 and 6 steps, respectively). If you need a considerably larger amount of steps, rethink your solution and try to find an easier proof. The solution of part (b) may help you identify the requirements for $\mathcal{I}$.