SIT191 Problem Solving Task 3
Due: by 8pm Monday October 4th

Total marks: 80, Weighting: 20%

3.1 The International Tennis Federation (ITF) requires that tennis balls have an average diameter of 6.70 centimeters. Tennis balls being produced by one manufacturer are tested to see if they meet the ITF standard. A random sample of 15 tennis balls is taken and the diameters (in cm) are shown below.

6.74 6.64 6.71 6.57 6.91 6.65 6.80 6.72 6.50 6.62 6.54 6.85 6.73 6.79 6.67

$$\bar{\boldsymbol{x}} = \frac{100.44}{15} = 6.696, \boldsymbol{n} = 15, \boldsymbol{S} = \sqrt{\frac{672.7296 - \frac{(100.44)^2}{15}}{14}} = 0.1144$$

a) Create a 98% confidence interval for the mean diameter of tennis balls produced by the manufacturer and interpret your interval in context.

$$6.696 \pm 2.624 * \frac{0.1144}{\sqrt{15}}$$
$$= 6.618, 6.774$$

We are 98% confident that mean diameter of tennis balls produced by the manufacturers is between 6.618% and 6.774%.

b) Perform a hypothesis test to see if there is evidence that the manufacturer's tennis balls are smaller than the ITF standard. Use α=0.01.

$H_0: \mu = 6.70$
$H_A: \mu < 6.70$

$$t = \frac{6.696 - 6.7}{0.1144 \div \sqrt{15}} = -0.135$$

$P - value = 0.4471$
$P - value \geq 0.01$
$Fail\ to\ reject\ H_0$
$Therefore, there\ isn't\ enough\ evidence\ to\ claim\ that\ the\ population\ mean\ is\ less\ then\ 6.7,$
$at\ the\ 0.01\ significance\ level.$

c) Explain how your confidence interval and hypothesis test conclusion support each other.

The confidence interval supports the hypothesis test result since isn't 0 (no difference) is contained in the interval and we failed to reject the null hypothesis of difference of manufacturer's tennis balls.

d) Are the assumptions required for the test you performed met? Discuss briefly.

The assumptions required are met because the p – value is greater than our alpha value and we fail to reject the null hypothesis.

[4+5+2+2 = 13 marks]

3.2 Exercise vigorous enough to increase heart rate to between 50% and 85% of maximum heart rate is recommended for good health. Does boxing high intensity interval training (HIIT) or cardio tennis increase heart rate the most?

Ten subjects participated in two different 45-minute exercise classes – boxing HIIT and cardio tennis. At the end of each class their heart rate (beats per minute) was recorded.

Heart rate (bpm)

Subject	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Boxing HIIT	145	168	157	129	189	135	165	172	127	148
Cardio tennis	152	159	143	122	162	130	154	175	132	146

a) Should the results best be analyzed as paired data or two independent means? Explain.
Paired-t test would be the best for this data, as Boxing HIIT and Cardio tennis are the variables that are the same subject.

b) Briefly describe how this experiment could be designed to help avoid biased results.
To avoid biased results the subjects should be randomly selected from the population. The matched pairs should be normally distributed and no extreme outliers.

c) Is there a significant difference in heart rate between boxing HIIT and cardio tennis?
Carry out a hypothesis test using α = 0.05, showing all steps.
$H_0: \mu: there\ is\ no\ significant\ difference\ in\ heart\ rate\ between\ boxing\ and\ cardio$
$H_A: \mu: there\ is\ a\ significant\ difference\ in\ heart\ rate\ between\ boxing\ and\ cardio$

$Mean\ of\ difference = 6$
$SD\ of\ difference = 10.1544$
$t = \dfrac{6}{10.1544/\sqrt{10}} = 1.87$
$P(1.87,9) = 0.095$

$P - value > 0.05, therefore\ we\ fail\ to\ reject\ H_0$
Therefore, there isn't a significant difference in heart rate between boxing and cardio.

d) Calculate a 95% confidence interval for the difference in heart rate between the two classes and interpret your interval in context.
Based on your hypothesis test result, did you expect to find 0 in the interval? Explain.
$6 \pm 2.2622 * \dfrac{10.1544}{\sqrt{10}}$
$= (-1.26, 13.26)$

The 95% confidence mean difference of heart rate of boxing and cardio is $(-1.26, 13.26)$
Yes, we expected to find 0, as 0 lays within the confidence interval and fail to reject the null.

[2+3+5+5 = 15 marks]

3.3 Resting heart rate was compared for a random sample of healthy adult men and women of the same age. The 15 women sampled had a resting heart rate of 78 beats per minute with standard deviation 5 bpm. The resting heart rate for the sampled 18 men was 71 bpm with standard deviation 3 bpm.

a) Should the results best be analysed as paired data or two independent means? Explain.
Independent means test would be the best for this data, as a requirement for a paired t test is that the sample size of the data should be equal size. There is also two groups which is why we should use the independent t test.

b) Do women of this age have a significantly higher resting heart rate than men?
Carry out a hypothesis test with a significance level of 5%, including all steps.
$\mu_1 = women$
$\mu_2 = men$
$H_0: \mu_1 = \mu_2$
$H_A: \mu_1 > \mu_2$

$$SD = \sqrt{\frac{(15-1)*5^2+(18-1)*3^2}{15+18-2}} = 4.028$$

$$t = \frac{78-71}{\sqrt{\frac{5^2}{15}+\frac{3^2}{18}}} = 4.75556$$

$P - value = 0.00002334$

$P - value < \alpha$

Therefore, we reject the null hypothesis meaning there is a significant difference between the resting heart rate of women and men

c) Create a 90% confidence interval for the mean difference in heart rate and interpret the interval in context.

$$(78-71) \pm 1.696 * \sqrt{\frac{5^2}{15}+\frac{3^2}{18}}$$

$(4.612, 9.388)$

There is a 90% chance that the confidence interval we found will contain the true population mean.

d) Is the confidence interval consistent with your hypothesis test conclusion? Explain.

Since 0 isn't in the confidence interval, we reject null hypothesis.

e) Is it necessary for the test you used that resting heart rate is normally distributed? Explain.

Yes, because the sample sizes of both groups are small.

[1+5+3+2+2 = 13 marks]

3.4 A hospital is comparing three different drugs to determine if there is a difference in the number of hours of pain relief each provides on average for patients with pain after surgery. The mean pain relief (in hours) for each drug and SPSS ANOVA output are given below.

Drug	Mean pain relief
1	8.28
2	7.52
3	8.36

ANOVA

time

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3.385	2	1.693	2.662	.093
Within Groups	13.352	21	.636		
Total	16.737	23			

Multiple Comparisons

Dependent Variable: time

Bonferroni

					95% Confidence Interval	
(I) Drug	(J) Drug	Mean Difference (I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
1.00	2.00	.75903	.38746	.191	-.2489	1.7669
	3.00	-.07937	.40185	1.000	-1.1247	.9660
2.00	1.00	-.75903	.38746	.191	-1.7669	.2489
	3.00	-.83839	.41269	.165	-1.9119	.2352
3.00	1.00	.07937	.40185	1.000	-.9660	1.1247
	2.00	.83839	.41269	.165	-.2352	1.9119

a) Does the average length of pain relief differ between any of the drugs?
State the appropriate hypotheses for this test.

$H_0: There\ no\ significant\ difference\ in\ average\ length\ of\ pain\ relief\ between\ any\ of\ the\ drugs$

$H_A: There\ is\ significant\ difference\ in\ average\ length\ of\ pain\ relief\ between\ any\ of\ the\ drugs$

b) Give the values of the test statistic and P-value, and your conclusion for the test. Use α = 0.05.

$P - value > 0.05, therefore\ we\ fail\ to\ reject\ H_0.$

This shows that there isn't a significant difference in average pain relief length between any of the drugs

c) Would it be appropriate to perform a Bonferroni test to see if any of the drugs differ in the mean hours of pain relief? Explain.

The ANOVA test shows that it is insignificant, therefore we do not need to perform Bonferroni test of multiple of comparison.

d) Describe a well-designed experiment to obtain this data.

A completely randomized design would be best suited. This is where we randomly divide the sample into three groups, and randomly assign the three pain to the three groups (1 of the drug would be assigned to one of the groups). Then we observe the dependent variable, time (in hours) of the pain reliever for all subjects in each group.

[2+2+2+3 = 9 marks]

3.5 At an athletics meet, organizers were interested in whether the standard of the competitors running in the 400m event varied significantly between heats. There were four heats for the 400m event, with 6 competitors in heats 1 and 3, 8 runners in heat 2 and 7 in heat 4. The time (in seconds) each runner took to complete the event was recorded, with the data given in the following table.

Heat			
1	**2**	**3**	**4**
51.92	53.25	53.10	51.22
51.29	52.17	53.20	50.11
52.90	53.10	53.12	51.19
52.97	51.53	52.96	50.34
52.10	54.22	53.91	51.78
51.89	52.85	52.55	52.25
	54.17		51.22
	54.58		

a) State the null and alternative hypotheses required to test if the average 400m running times were significantly different between any of the heats.

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_A: not\ all\ means\ of\ run\ time\ are\ equal(or, at\ least\ two\ mean\ differ)$

b) Carry out the ANOVA (with technology) and report the test statistic and P-value.

ANOVA

VAR00001

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	19.886	3	6.629	10.670	<.001
Within Groups	14.289	23	.621		
Total	34.174	26			

$t = 10.670, p - value < 0.001$

c) Write a brief conclusion of the ANOVA analysis, in context. Use α = 0.05.

$p - value < 0.05, therefore\ we\ reject\ the\ null\ hypothesis.$

Hence, we can conclude that the mean running differs between at least one pair of heats.

d) Carry out the Bonferroni's multiple comparisons (if appropriate) and summarise the results.

Multiple Comparisons

Dependent Variable: VAR00001

Bonferroni

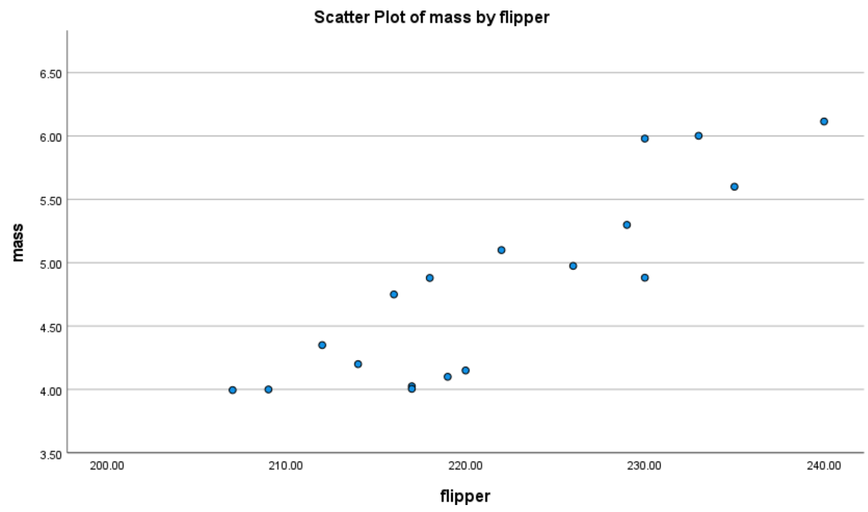
(I) VAR00002	(J) VAR00002	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	2.00	-1.05542	.42567	.126	-2.2840	.1732
	3.00	-.96167	.45506	.274	-2.2751	.3518
	4.00	1.01976	.43851	.175	-.2459	2.2854
2.00	1.00	1.05542	.42567	.126	-.1732	2.2840
	3.00	.09375	.42567	1.000	-1.1348	1.3223
	4.00	2.07518*	.40793	<.001	.8978	3.2526
3.00	1.00	.96167	.45506	.274	-.3518	2.2751
	2.00	-.09375	.42567	1.000	-1.3223	1.1348
	4.00	1.98143*	.43851	<.001	.7158	3.2471
4.00	1.00	-1.01976	.43851	.175	-2.2854	.2459
	2.00	-2.07518*	.40793	<.001	-3.2526	-.8978
	3.00	-1.98143*	.43851	<.001	-3.2471	-.7158

*. The mean difference is significant at the 0.05 level.

We can conclude that between heat 2 and heat 3, and heat 3 and heat 4 are significant.

[2+3+2+2 = 9 marks]

3.6 At a station in Antarctica, researchers measured flipper length and body mass for different penguin species. Below are a scatter plot and SPSS regression output for measurements made on 18 Gentoo penguins. Flipper length was measured in millimeters, and body mass in kilograms.



Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.878[a]	.771	.757	.37042

a. Predictors: (Constant), flipper

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-10.882	2.138		-5.089	<.001
	flipper	.071	.010	.878	7.340	<.001

a. Dependent Variable: mass

a) Describe the apparent relationship between the Gentoo penguins' flipper length and body mass.
There is a strong positive linear relation between Gentoo penguins' flippers length and body mass

b) State the value of the correlation between the variables and explain how it supports the relationship you described in a).
$correlation\ value = 0.878$
Meaning there is a strong positive linear relationship between flippers and body mass which supports what was described in a).

c) Give the regression equation.
$Body\ mass = -10.882 + 0.071(flippers\ length)$

d) State the R^2 value and explain what it represents in this context.
$R^2 = 0.771$
This represents that 77.1% of the data is explained by the regression equation.

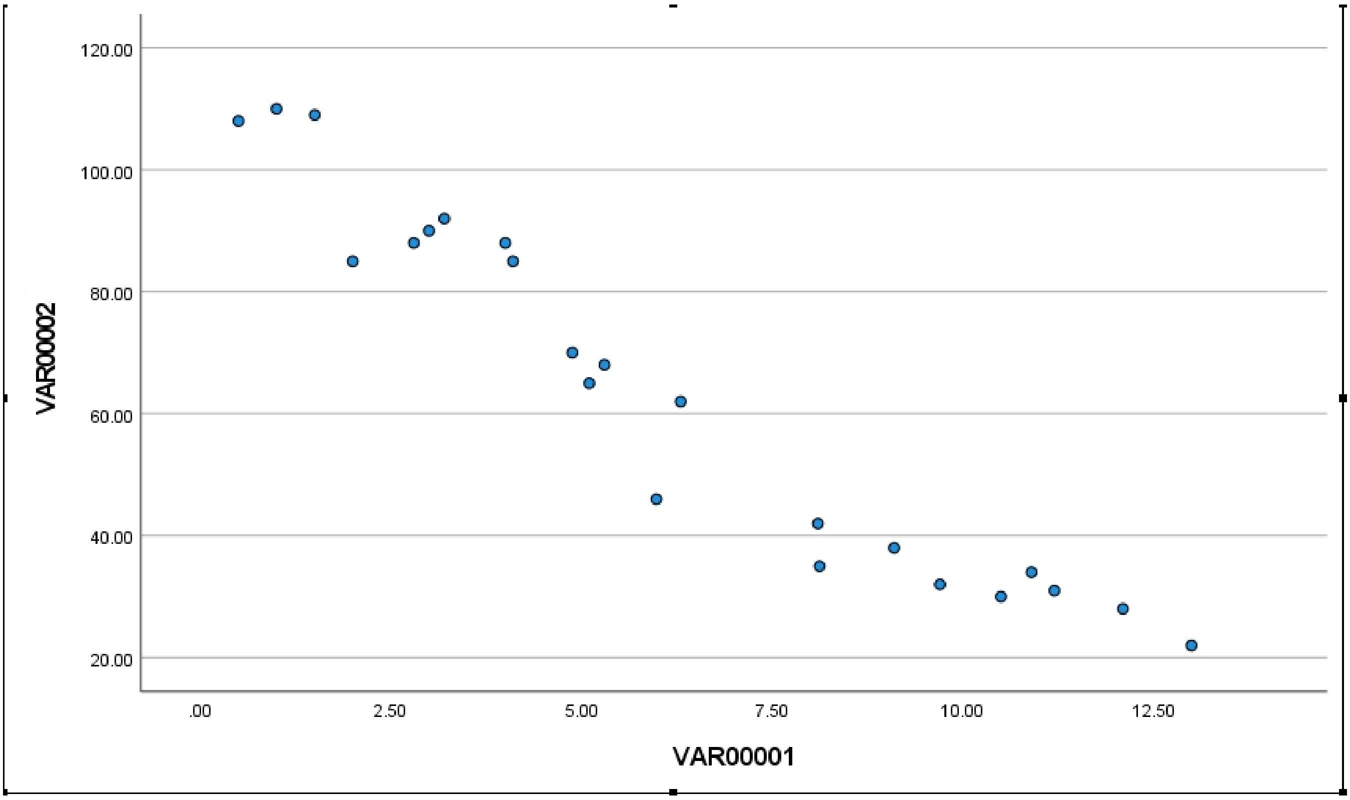
[2+2+2+2 = 8 marks]

3.7 To conserve oxygen, the heart rate of penguins slows when they dive. For 23 Emperor penguins, researchers have measured the dive heart rate (bpm) and the duration of the dive (minutes) for each penguin. The researchers want to further explore a possible relationship between dive duration and heart rate to predict heart rate from the length of a penguin's dive.
Below are the data for each penguin:

duration	heart rate
2.80	88
1.00	110
1.50	109
2.00	85
0.50	108
3.00	90
3.20	92
4.00	88
4.10	85
4.88	70
5.10	65
5.30	68
6.30	62
5.98	46
8.10	42
8.12	35
12.10	28
9.70	32
10.50	30
11.20	31
10.90	34
13.00	22
9.10	38

Use SPSS or other technology to produce the output necessary to answer the following questions.

a) Produce a scatterplot and use it to describe the apparent relationship between the dive duration and dive heart rate.



$Var2 = heart\ rate$
$Var1 = duration$

b) State the regression equation.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.964[a]	.929	.926	8.02617

a. Predictors: (Constant), VAR00001

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	17820.671	1	17820.671	276.635	<.001[b]
	Residual	1352.808	21	64.419		
	Total	19173.478	22			

a. Dependent Variable: VAR00002

b. Predictors: (Constant), VAR00001

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	109.754	3.251		33.757	<.001
	VAR00001	-7.489	.450	-.964	-16.632	<.001

a. Dependent Variable: VAR00002

$heart\ rate = 109.754 - 7.489(duration)$

c) Is it meaningful to interpret the *y*-intercept for this study? Explain.

Yes, because the y-intercept represents that average heart rate of penguin when they don't dive.

d) Interpret the slope of the regression equation in the context of these variables.
The slope of the regression equation is -7.489, which means each unit increase in the dive duration, there is a 7.489 decrease in the heart rate of the penguin.

e) Predict the heart rate of a penguin that dives for 16 minutes.
Is this prediction reliable? Explain.
$-10.07 = 109.754 - 7.489(16)$
Heart rate of -10.07 is impossible, meaning that the prediction is not reliable.

f) Is the relationship between the variables significant?
Perform the relevant hypothesis test (include all steps) and write your conclusion in context. Use α = 0.05.
$H_0: Slope = 0$
$H_A: slope \neq 0$

$$t = -\frac{7489}{0.4503} = -16.63$$

$p - value = 0.00001$
$p - value < 0.05, therefore\ we\ reject\ H_0$
$We\ can\ conclude\ that\ the\ relationship\ between\ duration\ of\ dive\ and\ heart\ rate\ is\ significant.$

[2+2+2+2+2+3 = 13 marks]