SIT191 Problem solving task 3 solutions (total marks: 80)

3.1 by hand or with technology

a) Since $df = n$-1 = 14, t = 2.62 for 98% confidence
$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = 6.696 \pm 2.62 \times \frac{0.1144}{\sqrt{15}}$ (By hand or with technology)
$= 6.696 \pm 0.0774$
= (6.62, 6.77) (or, with SPSS: 6.6184 to 6.7736)
We are 98% confident/sure that the mean diameter of tennis balls produced by the manufacturer is between 6.62 and 6.77 cm.

b) H_0: $\mu = 6.70$
H_a: $\mu < 6.70$

$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{6.696-6.7}{0.1144/\sqrt{15}}$
$= -0.135$ (or with SPSS: t = -0.135)
From the t-table with df = 14, P-value > 0.10 (or with SPSS: P-value =0.894/2=0.447)
Since the P-value is greater than $\alpha = 0.01$, fail to reject H_0.
The mean diameter of the manufacturer's tennis balls does not appear to be significantly smaller than the ITF standard of 6.70 cm.

c) Since the confidence interval is from 6.62 to 6.77cm and includes 6.70, the interval supports the conclusion to fail to reject H_0 (that $\mu = 6.70$).

d) Independence: The tennis balls were selected randomly, so diameters should be independent of each other
Normality: the sample size is not large and we don't know if the diameters are normally distributed. The histogram shows a unimodal distribution without outliers or strong skewness so this assumption seems ok.

[4+5+2+2=13 marks]

3.2 by hand or with technology

a) The results should be analysed as paired data since the same subject participated in each activity and heart rate for each subject for both activities would be related due to individual fitness levels.

b) Answers will vary. The subjects should participate in each class on different days or at least with enough time in between classes for heart rate to return to normal, and the order of the classes should be randomised for different subjects. While the subjects can't be blind to the exercise type, the researchers should be blind to each subject's class when measuring heart rate.

c) $H_0 : \mu_d = 0$ (or, no difference in heart rate between boxing HIIT and cardio tennis)
$H_a : \mu_d \neq 0$ (or, there is a significant difference in heart rate between boxing HIIT and cardio tennis)

$t = \frac{\bar{x}_d - 0}{\frac{S_d}{\sqrt{n}}} = \frac{6}{\frac{10.154}{\sqrt{10}}} = 1.87 \; or - 1.87$ (SPSS: 1.869, or by hand, after finding the mean (6.000) and standard deviation (10.154) of all the differences)
P-value = 0.095 (or, t-table: 0.05<P-value<0.1)
P-value > α = 0.05, so fail to reject H_0.
There is no evidence of a significant difference in heart rate between boxing HIIT and cardio tennis.

d) $\bar{x}_d \pm t^* \frac{S_d}{\sqrt{n}} = 6 \pm 2.26 \left(\frac{10.154}{\sqrt{10}}\right)$
= (-1.257, 13.257) **OR** state from SPSS output: -1.264 to 13.264, or rounded
[Or, -13.26, 1.26]
We are 95% confident/sure that the mean difference in heart rate between boxing HIIT and cardio tennis is between -1.26 and 13.26 bpm.
0 was expected in the interval since we failed to reject the null hypothesis that $\mu_d = 0$.

[2+3+5+5=15 marks]

3.3 by hand or with technology

a) The results should be analysed as two independent means since the men and women are different/independent samples.

b) $H_0 : \mu_w = \mu_m$ $(\mu_w - \mu_m = 0)$ w: women, m: men
H_a: $\mu_w > \mu_m$ $(\mu_w - \mu_m > 0)$ or, $\mu_m < \mu_w$

$$t = \frac{\bar{x}_w - \bar{x}_m}{\sqrt{\frac{s_w^2}{n_w} + \frac{s_m^2}{n_m}}}$$

$$= \frac{78-71}{\sqrt{\frac{5^2}{15} + \frac{3^2}{18}}}$$

$$= \frac{7}{1.47196} = 4.76$$ or t=-4.76 for $t = \frac{\bar{x}_m - \bar{x}_w}{\sqrt{\frac{s_m^2}{n_m} + \frac{s_w^2}{n_w}}}$ (technology: t = 4.756)

P-value < 0.005 (t-table with df=31) or SPSS: P-value<0.001 or exact: 0.000022
Since P-value < α = 0.05, reject H_0.
Women of this age appear to have a significantly higher resting heart rate than men.

c) $(\bar{x}_w - \bar{x}_m) \pm t^* \sqrt{\frac{s_w^2}{n_w} + \frac{s_m^2}{n_m}}$

$=(78-71) \pm 1.7\sqrt{\frac{5^2}{15} + \frac{3^2}{18}}$
= 7 ± 2.5023
= (4.50, 9.50) (SPSS: 4.473, 9.527)
[or (-9.527, -4.473) if $\bar{x}_m - \bar{x}_w$]
We are 90% confident/sure that resting heart rate for women is between 4.5 and 9.5 bpm higher on average than resting heart rate for men.

d) The interval is consistent with the hypothesis conclusion from b) since 0 is not within the confidence interval and in b) we rejected the null hypothesis that the difference in means was 0.

e) Yes – since the sample sizes for each group are not large, heart rate can't be assumed to be normally distributed (by the Central Limit Theorem) and normalilty is an assumption for t procedures.

[1+5+3+2+2=13 marks]

3.4

a) $H_0 : \mu_1 = \mu_2 = \mu_3$ (the average pain relief length is the same for all drugs)
H_a : the average pain relief length is different for at least two drugs (or, not all the drugs result in the same average length of pain relief)

b) Test statistic: F = 2.662
P-value = 0.093
Since P-value > α=0.05, fail to reject H_0.
There is no evidence that any of the drugs are different regarding the average length of pain relief.

c) It would not be appropriate to perform a Bonferroni test to see which drugs differ in the mean hours of pain relief since the conclusion from b) is that none of the drugs result in a different average length of pain relief.

d) Answers will vary. Include reference to random allocation of a number of patients (from different types of surgery, ages, etc) to the drugs, as well as patients being blind to the drug and preferably a double blind design.

[2+2+2+3 = 9 marks]

3.5 use SPSS or other technology

a) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ (average running times are the same for all heats)
H_a : the average running times are not the same for all heats (or, at least two heats differ in average running time)

b) Test statistic: F = 10.670 (output below)
P-value=0.000

c) Since P-value < α=0.05, reject H_0.
The average running time is significantly different for at least two of the heats.

d) Heat: 4 1 3 2
Mean: 51.16 52.18 53.14 53.23 (means connected by lines are not significantly different)

The average running time is significantly faster for heat 4 than heats 2 and 3.

[2+3+2+2=9 marks]

Heat	Mean
1	52.1783
2	53.2338
3	53.1400
4	51.1586

ANOVA

time

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	19.886	3	6.629	10.670	.000
Within Groups	14.289	23	.621		
Total	34.174	26			

Multiple Comparisons

Dependent Variable: time

Bonferroni

(I) heat	(J) heat	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1	2	-1.05542	.42567	.126	-2.2840	.1732
	3	-.96167	.45506	.274	-2.2751	.3518
	4	1.01976	.43851	.175	-.2459	2.2854
2	1	1.05542	.42567	.126	-.1732	2.2840
	3	.09375	.42567	1.000	-1.1348	1.3223
	4	2.07518*	.40793	.000	.8978	3.2526
3	1	.96167	.45506	.274	-.3518	2.2751
	2	-.09375	.42567	1.000	-1.3223	1.1348
	4	1.98143*	.43851	.001	.7158	3.2471
4	1	-1.01976	.43851	.175	-2.2854	.2459
	2	-2.07518*	.40793	.000	-3.2526	-.8978
	3	-1.98143*	.43851	.001	-3.2471	-.7158

*. The mean difference is significant at the 0.05 level.

3.6

a) The relationship between flipper length and body mass appears to be reasonably strong, positive and linear.

b) R=0.878

Since the correlation's magnitude is close to 1, the value supports the strong, positive and linear relationship observed.

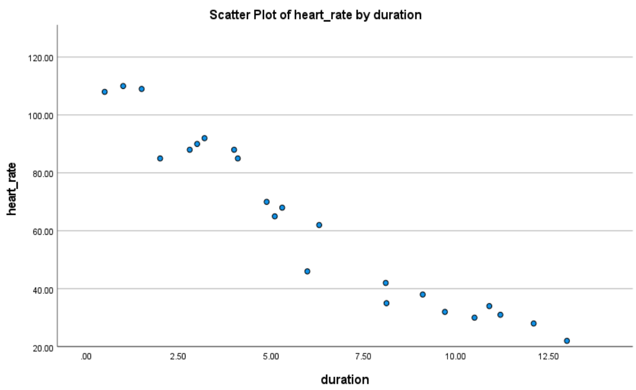
c) $\widehat{mass} = -10.882 + 0.071 \times flipper\_length$ or $\hat{y} = -10.882 + 0.071x$

d) $R^2 = 0.771$ or 77.1%

77.1% of the variation in the body mass of Gentoo penguins is explained by flipper length.

[2+2+2+2=8 marks]

3.7 use SPSS or other technology



a) The relationship between the dive duration and heart rate appears to be negative, linear and quite strong.

b) $\widehat{heart\ rate} = 109.754 - 7.489 \times duration$

or $\hat{y} = 109.754 - 7.489x$

c) It is not meaningful to interpret the y-intercept (109.754) for this study as a dive time of 0 is an extrapolation and would represent the expected heart rate of a penguin before a dive which is not being investigated.

Or: Since a dive time of 0 is not far from the times used in the study, the y-intercept could be considered the heart rate of penguins before diving.

d) For each additional minute of dive duration, heart rate is expected to decrease by 7.489 bpm.

e) $\widehat{heart\ rate} = 109.754 - 7.489 \times 16 \ = -10.07$

This prediction is not reliable because 16 minutes is an extrapolation. The linear relationship may not continue and a negative heart rate isn't possible.

f) $H_0: \beta_1 = 0$ (there is no significant relationship between dive duration and heart rate)

$H_a: \beta_1 \neq 0$ (there is a significant relationship between dive duration and heart rate)

t = -16.632

P-value = 0.000

Since P-value < α = 0.05, reject H_0.

There appears to be a significant relationship between dive duration and heart rate.

[2+2+2+2+2+3=13 marks]

SPSS output:

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.964[a]	.929	.926	8.02617

a. Predictors: (Constant), duration

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	109.754	3.251		33.757	.000
	duration	-7.489	.450	-.964	-16.632	.000

a. Dependent Variable: heart_rate