1.

(a) False – Set A has just elements however Set B has sets as elements

(b) True – Because C has the set {1,2,3} while A is the set {1,2,3}

(c) True - every element in A is also in C

(d) True – Set C has the set {1, 2, 3}

(e) False – Set A has just elements, but Set B has sets as elements. $a \in A\ but\ a\ \notin B$

(f) False – 1, 2, 3 elements are not in Set D

(g) True – Every element within B is within D

(h) False – Because B is a set that contains the sets {1},{2},{3} while

(i) True – Because in Set D it has an element {1, 2, 3} different from 1, 2, 3

2.

(a)

Reflexive – False because **a** can't run faster than **a**, also **b** can't ran faster than **b**

Symmetric – False because if **a** run faster than **b** then **b** can't run faster than **a**

Antisymmetric – True, because this relationship never shows symmetry and therefore never shows symmetry on two elements that are different

Transitive – True, because **a** run faster than **b** and **b** runs faster than **c** → a run faster than **c**

(b)

Reflexive – True because **a** and **a** would still have the same fur color. Also, **b** and **b** would have the same fur color

Symmetric – True, because **a** and **b** have the same fur color as **b** and **a** has the same fur color.

Antisymmetric – False, **a** and **b** are symmetric.

Transitive – True, because **a** and **b** have the same fur color and **b** and **c** have the same fur color → **a** and **c** have the same fur color.

(c)

Reflexive – True, because **a** ate from the same bowl as **a.** Also, **b** and **b** would have eaten from the same bowl.

Symmetric – True, because **a** ate from same bowl as **b** and **b** ate from the same bowl as **a**

Antisymmetric – False, **a** and **b** are symmetric

Transitive – True, because **a** and **b** ate from the same bowl and **b** and **c** ate from the same bowl → **a** and **c** ate from the same bowl.

3.

(a)

A = [0, 2) Set of real numbers between 0 to 2 but not including 2

B = (-5, 0) Set of real numbers between -5 to 0 but not including -5 and 0

C = [1, 3] Set of real numbers between 1 to 3 including 1 and 3

D = $A \cup B = (-5, 2)$

E = $B \cup C = (-5, 0) \cup [1, 3]$

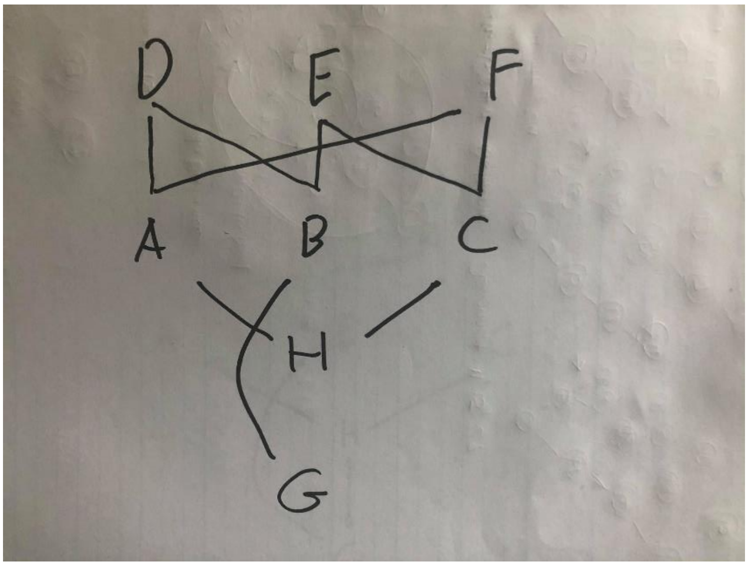
F = $A \cup C = [0, 3]$

G = $A \cap B = B \cap C = \emptyset$

H = $A \cap C = [1, 2)$

(b)

Relation 'P' = { (A,A), (B,B), (C,C), (D,D), (E,E), (F,F), (G,G), (H,H), (A,D), (A,F), (B,D), (B,E), (C,E), (C,F), (G,A), (G,B), (G,C), (G,D), (G,E), (G,F), (G,H), (H,A), (H,C), (H,D), (H,E), (H,F) }



(c)

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
A	A	D	E	D	E	NULL	A	A
B		B						
C			C					
D				D				

E					E			
F						F		
G							G	
H								H

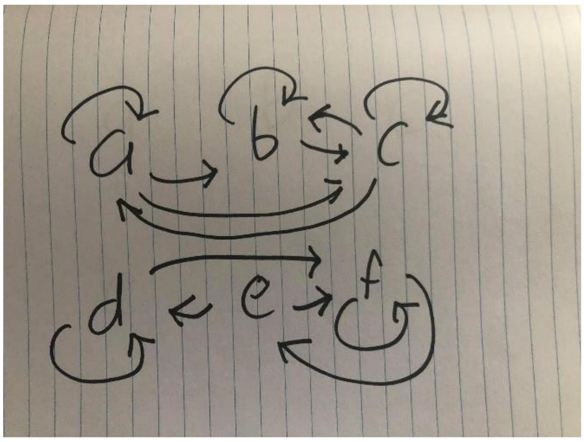
No, because a poset in which every pairs of elements have both least upper bound and greatest lower bound is called lattice. However, there isn't a relationship between A and F. No (A, F)

(d)

The least upper bound of set {A, B, C} is G but the set {A, B, C} does not have greatest lower bound.

4.

(a)



(b)

Reflective – False, because there is not (e,e)

Symmetric – False, because there isn't (b,a), (d,e) and (f,d)

Transitive – False, because there isn't (d,e). Also there isn't a (b,a) and (e,e) to make it transitive.

(a,a), (b,b), (c,c), (d,d), (e,e), (f,f), (a,b), (b,a), (a,c), (c,a), (b,c), (c,b), (e,d), (d,e), (d,f), (e,f), (f,e)

[a] = {a, b, c,}

[b] = {b, a, c} = [a]

[c] = {c, a, b} = [a] = [b]

[d] = {d, e, f}

[e] = {e, d, f} = [d]

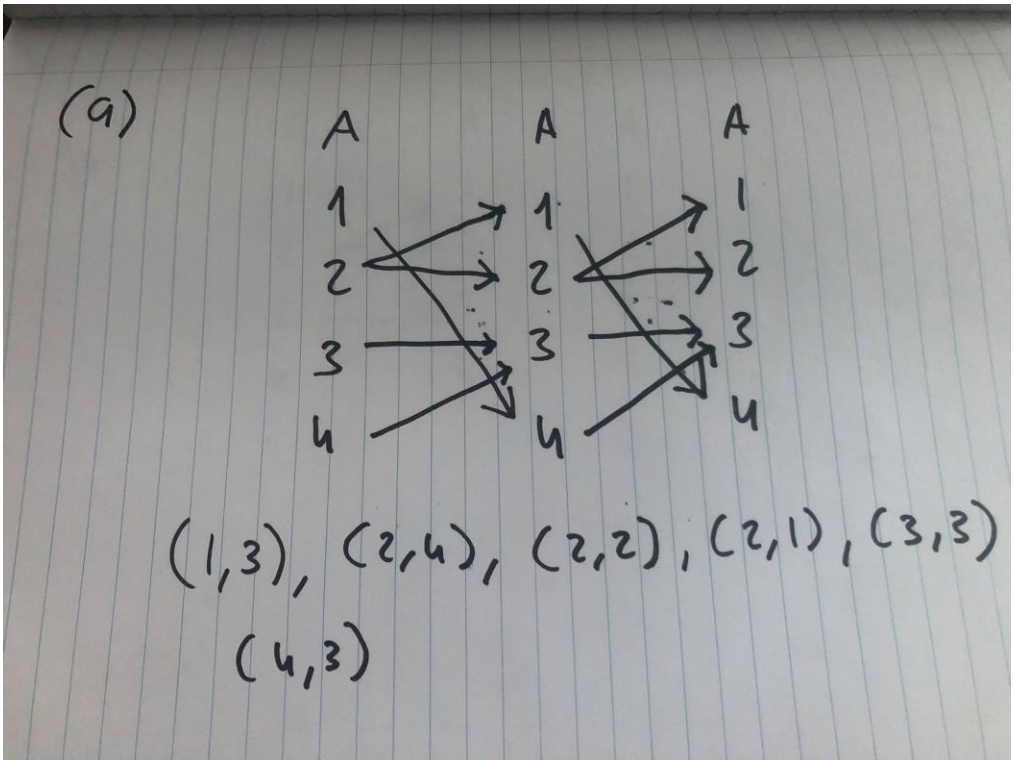
[f] = {f, e}

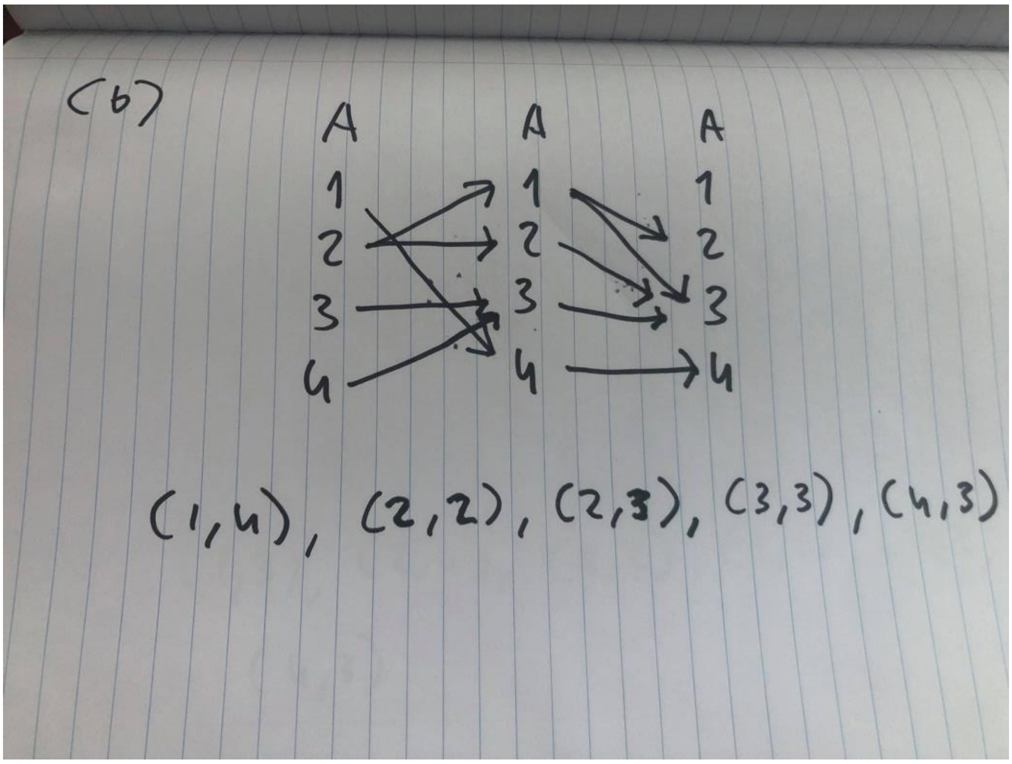
[a] = [b] = [c]

[d] = [e]

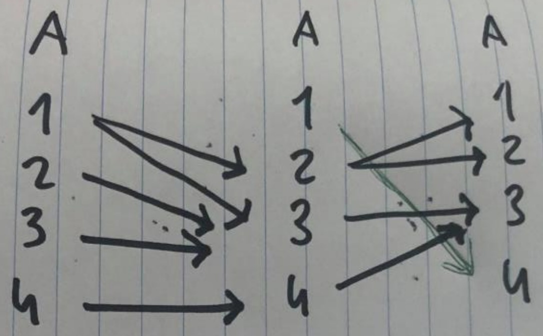
[f]

5.


(a)
A
A
A
1 2 3 4
1 2 3 4
1 2 3 4
(1,3), (2,4), (2,2), (2,1), (3,3),
(4,3)

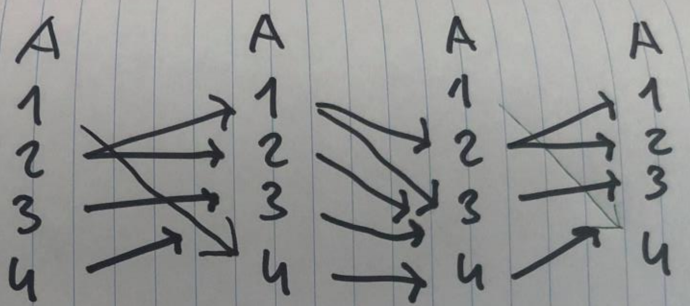

(b)
A
A
A
1 2 3 4
1 2 3 4
1 2 3 4
(1,4), (2,2), (2,3), (3,3), (4,3)

(c)



(1,1), (1,2), (1,3), (2,3), (3,3)

d)



(1,3), (2,1), (2,2), (2,3), (3,3),

(4,3)

6.

i)

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & -3 & -4 & 1 \\ 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix} \quad \begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$$

$$= 0\begin{vmatrix} 2 & 3 & 1 \\ -3 & -4 & 1 \\ 2 & 3 & 2 \end{vmatrix} - 0\begin{vmatrix} 1 & 3 & 1 \\ -1 & -4 & 1 \\ 1 & 3 & 2 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 & 1 \\ -1 & -3 & 1 \\ 1 & 2 & 2 \end{vmatrix} - 5\begin{vmatrix} 1 & 2 & 3 \\ -1 & -3 & -4 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & -3 & -4 \\ 1 & 2 & 3 \end{bmatrix} = 1\begin{bmatrix} -3 & -4 \\ 2 & 3 \end{bmatrix} - 2\begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix} + 3\begin{bmatrix} -1 & -3 \\ 1 & 2 \end{bmatrix}$$

$$= 1(-9-(-8)) - 2(-3-(-4)) + 3(-2-(-3)) = 0$$

$$\det A = 0 - 0 + 0 - 5(0) = 0$$

$$B = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 3 & 2 & 5 & 1 \\ 4 & -1 & 3 & 5 \\ 5 & -4 & 1 & 9 \end{bmatrix}$$ Using LU decomposition method

$$= \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -3 & -2 & 4 \\ 3 & 2 & 5 & 1 \\ 4 & -1 & 3 & 5 \\ 5 & -4 & 1 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -3 & -2 & 4 \\ & & & \\ & & & \\ & & & \end{bmatrix}$$ R2 − 3 x R1

$$= \begin{bmatrix} 1 & & & \\ 3 & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ & & & \\ & & & \end{bmatrix}$$ R3 − 4 x R1

$$= \begin{bmatrix} 1 & & & \\ 3 & 1 & & \\ 4 & & 1 & \\ & & & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ 0 & 11 & 11 & -11 \\ & & & \end{bmatrix}$$ R4 − 5 x R1

$$= \begin{bmatrix} 1 & & & \\ 3 & 1 & & \\ 4 & & 1 & \\ 5 & & & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ 0 & 11 & 11 & -11 \\ 0 & 11 & 11 & -11 \end{bmatrix}$$ R3 − 1 x R2

$$= \begin{bmatrix} 1 & & & \\ 3 & 1 & & \\ 4 & 1 & 1 & \\ 5 & & & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ 0 & 0 & 0 & 22 \\ & & & \end{bmatrix} \quad R4 - 1 \times R2$$

$$= \begin{bmatrix} 1 & & & \\ 3 & 1 & & \\ 4 & 1 & 1 & \\ 5 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ 0 & 0 & 0 & 22 \\ 0 & 0 & 0 & 22 \end{bmatrix} \quad R4 - 0 \times R3$$

$$\det \Delta = 1 \times 11 \times 0 \times 22$$

Det = 1 x 11 x 0 x 22 = 0

ii)

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & -3 & -4 & 1 \\ 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Reduce to echelon form

$R2 + R1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 2 \\ 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$R3 + (-1)R1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$R2 \times -1$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$R4 + (-5)R3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R2 + (2)R3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R1 + (-1)R3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R1 + (-2)R2$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 3

$$B = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 3 & 2 & 5 & 1 \\ 4 & -1 & 3 & 5 \\ 5 & -4 & 1 & 9 \end{bmatrix} \quad R2 + (-3)R1$$

$$\begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ 4 & -1 & 3 & 5 \\ 5 & -4 & 1 & 9 \end{bmatrix} \quad R3 + (-4)R1$$

$$\begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ 0 & 11 & 11 & -11 \\ 5 & -4 & 1 & 9 \end{bmatrix} \quad R4 + (-5)R1$$

$$\begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 11 & 11 & -11 \\ 0 & 11 & 11 & -11 \\ 0 & 11 & 11 & -11 \end{bmatrix} \quad R2 \times \frac{1}{11}$$

$$\begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 1 & 1 & -1 \\ 0 & 11 & 11 & -11 \\ 0 & 11 & 11 & -11 \end{bmatrix} \quad R3 + (-11)R2$$

$$\begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 11 & 11 & -11 \end{bmatrix} \quad R3 + (-11)R4$$

$$\begin{bmatrix} 1 & -3 & -2 & 4 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R1 + (3) R2

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 2

7.

$$A = \begin{bmatrix} 2 & x & 0 \\ x & 2 & x \\ 0 & x & 2 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= 2 \begin{bmatrix} 2 & x \\ x & 2 \end{bmatrix} - x \begin{bmatrix} x & x \\ 0 & 2 \end{bmatrix} + 0 \begin{bmatrix} x & 2 \\ 0 & x \end{bmatrix}$$

$$= 2(4 - x^2) - x \times 2x + 0 \times x^2$$

$$= -4x^2 + 8$$

$$\det \Delta = -4x^2 + 8$$

$$-4x^2 + 8 = 0$$

$$-4x^2 = -8$$

$$x^2 = 2$$

$$x = \sqrt{2}, -\sqrt{2}$$

$$B = \begin{bmatrix} x-1 & 0 & 4 \\ 0 & x+1 & 3 \\ 0 & 3 & x+1 \end{bmatrix}$$

$$= x-1 \begin{bmatrix} x+1 & 3 \\ 3 & x+1 \end{bmatrix} - 0 \begin{bmatrix} 0 & 3 \\ 0 & x+1 \end{bmatrix} + 4 \begin{bmatrix} 0 & x+1 \\ 0 & 3 \end{bmatrix}$$

$$= (x-1)(x^2+2x-8) - 0 \times 0 + 4 \times 0$$

$$\det \Delta = (x-1)(x^2+2x-8)$$
$$= x^3 + x^2 - 10x + 8$$

$$(x-1)(x^2+2x-8) = 0$$
$$(x-1)(x-2)(x+4) = 0$$

$$x = 1,\ x = 2,\ x = -4$$

8.

$$\det \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

(1)

$$= 1\begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} - x_1\begin{bmatrix} 1 & y_2 \\ 1 & y_3 \end{bmatrix} + y_1\begin{bmatrix} 1 & x_2 \\ 1 & x_3 \end{bmatrix}$$

$$= x_2y_3 - y_2x_3 - x_1y_3 + x_1y_2 + y_1x_3 - y_1x_2$$

$$= x_1y_2 - y_1x_2 + x_2y_3 - y_2x_3 + x_3y_1 - x_1y_3 = 0$$

(2) use $y = mx + c$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$(y_3-y_1)(x_2-x_1) = (y_2=y_1)(x_3-x_1)$$

$$= x_1y_2 - y_1x_2 + x_2y_3 - y_2x_3 + x_3y_1 - y_1y_3 = 0$$

(1) = (2) Proved