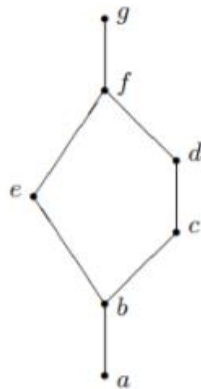


1. Consider the following Hasse diagram



- List the pairs of elements that are not related via this partial order.
- Construct, as far as possible, the table of least upper bounds for pairs of elements under this partial order.
- Is this poset a lattice? (provide an argument for your answer).
- Let $\rho = \{(b, b), (a, b), (b, a), (c, c), (c, d), (d, c)\}$ be a relation on the set $\{a, b, c, d\}$. Construct ρ^2 , list explicitly all the ordered pairs that form ρ^2 .
- Draw a directed graph diagram for ρ and ρ^2 . Which ordered pair(s) must be adjoined to ρ^2 to complete it into an equivalence relation on the set $\{a, b, c, d\}$?

(2 + 5 + 3 + 4 + 4 = 18 marks)

2. For the following system of linear equations:

$$\begin{array}{rrcrcl} -x & + & 2y & - & z & + & w & = & 4 \\ x & - & y & + & z & - & w & = & -2 \\ 2x & + & 2y & - & z & - & w & = & -1 \\ x & + & y & + & z & + & w & = & 10 \end{array}$$

- Use Gaussian elimination to put the system into row echelon form
- Use Gauss-Jordan elimination to change your row echelon form into reduced row echelon form.
- Solve the system of equations using either your answer to part (a) or your answer to part (b).
- Verify that your solutions satisfy this system of equations.

(8 + 5 + 3 + 2 = 18 marks)

3. Consider the following matrix

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

- (a) Confirm that the matrix A is invertible.
- (b) Find all the cofactors C_{ij} of the matrix A and hence find $\text{adj}(A)$.
- (c) Verify that the $\text{adj}(A)$ you obtained is correct by multiplying it with A .

(2 + 6 + 2 = 10 marks)

4. For the following matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

- (a) Find the eigenvalues (one eigenvalue is $\lambda_1 = 2$).
- (b) For each eigenvalue find the corresponding eigenvector(s).
- (c) Determine if possible a matrix P so that $B = P^{-1}AP$ is in diagonal form. Write down B and P .

(2 + 8 + 6 = 16 marks)

5. The vector space of solutions of $\mathbf{Ax} = \mathbf{0}$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{0}$$

is generated by

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}.$$

- (a) Verify that each vector is a solution.
- (b) Show that any solution can be written as a linear combination

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

for a suitable choice of c_1, c_2, c_3 .

- (c) What is the dimension of the row-space of A and the dimension of its nullspace? State one basis for the nullspace of A .

(6 + 8 + 8 = 22 marks)

6. For the following matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 1 & 0 & -2 & 1 \\ 0 & 2 & 2 & -2 \end{bmatrix}$$

- (a) Determine the row-rank.
- (b) Find a set of generators for the row space of A .
- (c) State one possible basis for the row space of A and explain why it is a basis.

(6 + 4 + 6 = 16 marks)

7. The 6-tuples:

$$u_1 = 011100, u_2 = 111010, u_3 = 110011$$

form a basis for a (6,3) linear code.

- (a) Write down the generator matrix for this code.
- (b) Construct code words for the message blocks: 101, 110, 011, 010.
- (c) Construct the parity check matrix for this code.
- (d) Decode if possible

$$010001, 010101, 101101, 000111, 011111, 110011$$

(6 + 4 + 4 + 6 = 20 marks)