1.

a) (b,a), (c,a), (d,a), (e,a), (f,a), (g,a)
, (e,c), (e,d), (d,e), (c,e).

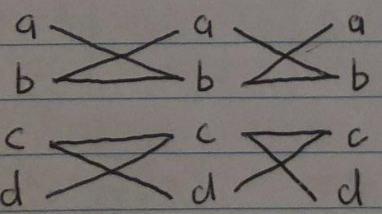
b)

	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b		b	c	d	e	f	g
c			c	d	f	f	g
d				d	f	f	g
e					e	f	g
f						f	g
g							g

c) No, because there is no relationship between ~~(c,e)~~ and ~~(d,e)~~
C and E $\boxed{D}$ and E

d)

p^2 =

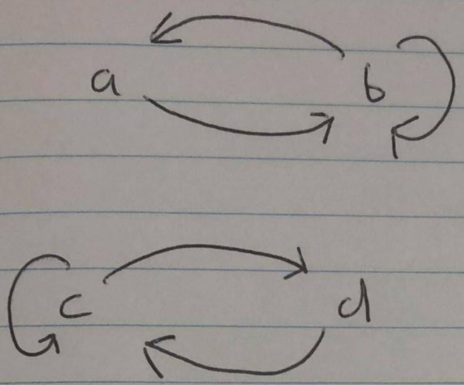


(a,a)
~~(a,a)~~, (b,a), (b,b), (a,b)
, (c,c), (c,d), (d,c)
, (d,d)

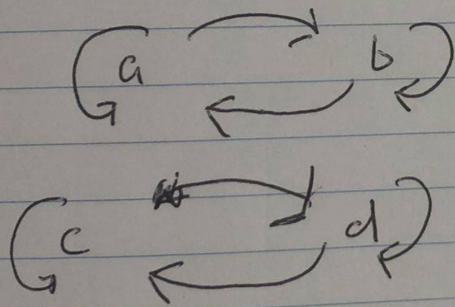
~~e)~~

e)

$p =$



$p^2 =$



No, ordered pairs need to be adjoint, p^2 is a equivalence relation already.

2.

a)

$$\left[\begin{array}{cccc|c} -1 & 2 & -1 & 1 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 6 & -3 & 1 & 7 \\ 0 & 3 & 0 & 2 & 14 \end{array}\right] \quad \left\{\begin{array}{l} R2 = R2 + R1 \\ R3 = R3 + 2R1 \\ R4 = R4 + R1 \end{array}\right.$$

$$\left[\begin{array}{cccc|c} -1 & 2 & -1 & 1 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & -3 & 1 & -5 \\ 0 & 0 & 0 & 2 & 8 \end{array}\right] \quad \left\{\begin{array}{l} R3 = R3 - 6R2 \\ R4 = R4 - 3R2 \end{array}\right.$$

b)

$$\left[\begin{array}{cccc|c} +1 & -2 & +1 & -1 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & -3 & 1 & -5 \\ 0 & 0 & 0 & 2 & 8 \end{array}\right] \quad \left\{\begin{array}{l} R1 = R1 \times -1 \end{array}\right.$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 2 & 8 \end{array}\right] \quad \left\{\begin{array}{l} R1 = R1 + 2R2 \\ R3 = R3 \times -\frac{1}{3} \end{array}\right.$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{2}{3} & -\frac{5}{3} \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad \left\{\begin{array}{l} R1 = R1 - R3 \\ R4 = R4 \times \frac{1}{2} \end{array}\right.$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad \left\{ \begin{array}{l} R1 = R1 + R4 \times \frac{2}{3} \\ R3 = R3 + R4 \times \frac{1}{3} \end{array} \right.$$

c)

$x_1 = 1$

$x_2 = 2$

$x_3 = 3$

$x_4 = 4$

d)

$$\begin{bmatrix} -1 & 2 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 2 & 2 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -1 \\ 10 \end{bmatrix} \quad \begin{array}{l} -1+4-3+4=4 \\ 1+-2+3-4=-2 \\ 2+4-3-4=-1 \\ 1+2+3+4=10 \end{array}$$

3.

a)

$$0 \times \det \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} - 2 \times \det \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} + 3 \times \det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= 7 \neq 0$ can be inverted

b)

$$C_{1,1} = \det\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} = 1$$

$$C_{1,2} = \det\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} = -2$$

$$C_{1,3} = \det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$$

$$C_{2,1} = \det\begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix} = -7$$

$$C_{2,2} = \det\begin{pmatrix} 0 & 3 \\ 0 & -2 \end{pmatrix} = 0$$

$$C_{2,3} = \det\begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix} = 0$$

$$C_{3,1} = \det\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} = 2$$

$$C_{3,2} = \det\begin{pmatrix} 0 & 3 \\ 1 & 1 \end{pmatrix} = -3$$

$$C_{3,3} = \det\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} = -2$$

$$= \begin{bmatrix} -1 & -2 & 1 \\ -7 & 0 & 0 \\ 2 & -3 & -2 \end{bmatrix} = \begin{bmatrix} -1^{1+1}(-1) & -1^{1+2}(-2) & -1^{1+3}(1) \\ -1^{2+1}(-7) & -1^{2+2}(0) & -1^{2+3}(0) \\ -1^{3+1}(2) & -1^{3+2}(-3) & -1^{3+3}(-2) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 7 & 0 & 0 \\ 2 & 3 & -2 \end{bmatrix}$$

c)

$$\begin{bmatrix} -1 & 2 & 1 \\ 7 & 0 & 0 \\ 2 & 3 & -2 \end{bmatrix}^T = \begin{bmatrix} -1 & 7 & 2 \\ 2 & 0 & 3 \\ 1 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix} \times \begin{bmatrix} -1 & 7 & 2 \\ 2 & 0 & 3 \\ 1 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= \det(A) \times \underline{I}$$

4)
a)

$$A = \begin{vmatrix} 2-\lambda & 2 & 0 \\ 2 & 0-\lambda & 2 \\ 0 & 2 & 2-\lambda \end{vmatrix}$$

$$= -\lambda^3 + 4\lambda^2 + 4\lambda - 16$$
$$= -(\lambda-4)(\lambda+2)(\lambda-2)$$

$\lambda_1 = -2$
$\lambda_2 = 2$
$\lambda_3 = 4$

b)

$\lambda = -2$

$$\begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 2 & 2 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right] \quad \left\{ R1 = R1 \times \frac{1}{4} \right.$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right] \quad \left\{ R2 = R2 - 2R1 \right.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \left\{ \begin{array}{l} R1 = R1 - R2 \times \frac{1}{2} \\ R3 = R3 - 2R2 \end{array} \right.$$

$x_3 = t$

$x_2 = -2x_3$ $\quad v_1 = \begin{bmatrix} t \\ -2t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$x_1 = x_3$

$\lambda = 2$

$$\begin{bmatrix} 0 & 2 & 0 \\ 2 & -2 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right] \quad \Big\{ \text{Swap 1 and 2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right] \quad \begin{cases} R1 = R1 \times \frac{1}{2} \\ R2 = R2 \times \frac{1}{2} \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{cases} R1 = R1 + R2 \\ R3 = R3 - 2R2 \end{cases}$$

$x_3 = t$

$x_2 = 0$

$x_1 = -x_3$

$$v_2 = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda = 4$

$$\begin{bmatrix} -2 & 2 & 0 \\ 2 & -4 & 2 \\ 0 & 2 & -2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 2 & -4 & 2 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right] \quad \Big\{ R1 = R1 \times -\frac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right] \quad \Big\{ R2 = R2 - 2R1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right] \quad \left\{ R2 = R2 \times -\frac{1}{2} \right.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & 0 \end{array}\right] \quad \left\{ R1 = R1 + R2 \right.$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \left\{ R3 = R3 - 2R2 \right.$$

$x_3 = t$

$x_2 = x_3$ $\quad V_3 = \begin{bmatrix} t \\ t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x_1 = x_3$

c)

A is diagonalissabe ~~b~~ cus there are there distinct ~~R~~ eigenvalues

$$P = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & & \\ & 2 & \\ & & 4 \end{bmatrix}$$

5

a)

$$\begin{bmatrix} +1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left\{ \begin{array}{l} R2 = R2 + R1 \\ R3 = R3 - 2R1 \end{array} \right.$$

Rank = 1

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \left\{ R2 = R2 - R1 \right.$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \left\{ R3 = R3 - R2 \right.$$

Rank = 2

dim(nullspace) + Rank(A) = 3

There are 2 linear independent vectors in the set

b)

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & a \\ 0 & 1 & 1 & b \\ 0 & 0 & 0 & c \end{array}\right]$$

$a - b + c = 0$

consistent with $Ax = 0$? Yes

system has no solution

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & a \\ 0 & 1 & 1 & -a+b \\ 0 & 0 & 0 & a-b+c \end{array}\right]$$

c)

$\dim(A) = 1$

$\dim(\text{nullspace}) = 2$

$$\text{basis} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

6.

a)

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 2 & 2 & -2 \end{bmatrix} \quad \Big\{ R2 = R2 - R1$$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \Big\{ R3 = R3 + R2$$

Row-Rank = 2

b)

$V_1 = (1, 2, 0, -1)$

$V_2 = (0, -2, -2, 2)$

c) V_1 can be a basis cus it is a linearly indepent ~~set~~ vector that is generated by A.

7.

a)

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$R3 = R3 + R2$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$G = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) 101

$$G \times \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+0=0 \\ 1+0+0=1 \\ 1+0+1=0 \\ 1+0+0=1 \\ 0+0+0=0 \\ 0+0+1=1 \end{bmatrix}$$

010101

110

$$G \times \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+1+0=1 \\ 1+1+0=0 \\ 1+1+0=0 \\ 1+0+0=1 \\ 0+1+0=1 \\ 0+0+0=0 \end{bmatrix}$$

100110

011

$$G \times \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+1+0=1 \\ 0+1+0=1 \\ 0+1+1=0 \\ 0+0+0=0 \\ 0+1+0=1 \\ 0+0+1=1 \end{bmatrix}$$

110011

010

$$G \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+1+0=1 \\ 0+1+0=1 \\ 0+1+0=1 \\ 0+0+0=0 \\ 0+1+0=1 \\ 0+0+1=1 \end{bmatrix}$$

c)

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} I_3 & P \end{bmatrix}$$

d)

0 1 0 0 0 1

$$H \times \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+0+0+0+0=0 \\ 0+1+0+0+0+0=1 \\ 0+0+0+0+0+1=1 \end{bmatrix}$$

010101

$$H \times \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+0+0+0+0=0 \\ 0+1+0+\overset{1}{\cancel{0}}+0+0=0 \\ 0+0+0+1+0+1=0 \end{bmatrix}$$

a 101101

$$H \cancel{G} \times \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+0+0+0+0+0=1 \\ 0+0+0+1+0+0=1 \\ 0+0+1+1+0+1=1 \end{bmatrix}$$

000111

$$H \cancel{G} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+0+0+1+0=1 \\ 0+0+0+1+1+0=0 \\ 0+0+0+1+1+1=1 \end{bmatrix}$$

011111

$$H \cancel{G} \times \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+0+0+1+0=1 \\ 0+1+0+1+1+0=1 \\ 0+0+1+1+1+1=0 \end{bmatrix}$$

0

G 110011

$$H\,G \times \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+0+0+0+1+0=0 \\ 0+1+0+0+1+0=0 \\ 0+0+0+0+1+1=0 \end{bmatrix}$$