SIT292 LINEAR ALGEBRA 2021
Assignment 3

Due: 8 p.m. October 1, 2021

Assignments to be submitted online as one PDF file (no multiple jpegs)
Assignments can be handwritten and scanned.

1. State the definition of the row-rank. For the following matrix

$$A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & -2 \\ 3 & 6 & -4 & 10 \end{bmatrix}$$

   (a) determine the row-rank.

   (b) find a set of generators for the row space of A. Explain why these vectors are generators.

   (c) find a basis for the row space of A. Explain why it is a basis.

($4 + 2 + 4 = 10$ marks)

2. For the following matrix

$$\begin{bmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix}$$

   (a) find the eigenvalues

   (b) find the eigenvectors corresponding to these eigenvalues

   (c) starting with the eigenvectors you found in (a) construct a set of orthonormal vectors (use the Gram-Schmidt procedure).

($5 + 10 + 5 = 20$ marks)

3. The set of ordered 4-tuples $\{(0, 2, 1, 0), (1, -1, 0, 0), (1, 2, 0, -1), (1, 0, 0, 1)\}$ presumably forms a basis for $\mathbb{R}^3$. Show that it is a basis indeed. Then, starting with this basis use the Gram-Schmidt procedure to construct

an orthonormal basis for $\mathbb{R}^4$. Verify that the resulting vectors are orthonormal.

(10 marks)

4. Denote by $\mathbb{R}^n$ the set of all n-tuples of real numbers. $\mathbb{R}^n$ is called the Euclidean vector space, with equality, addition and multiplication defined in the obvious way. Let V be the set of all vectors in $\mathbb{R}^4$ orthogonal to the vector $(0, -1, 1, 1)$; i.e. all vectors $v \in V$ so that $v^T(0, -1, 1, 1) = 0$.

   (a) Prove that V is a subspace of $\mathbb{R}^4$.

   (b) What is the dimension of V (provide an argument for this), and find a basis of V. (Hint: observe that the vector $(0, -1, 1, 1)$ does not belong to V).

   (c) Let A be a matrix consisting of the basis vectors of V (taken as rows of A). What is the Null-space of A and what is its row-space? Does the rank-nullity theorem hold for this matrix?

$(5 + 14 + 5 = 24$ marks)

5. Determine the dimension of the subspace of $\mathbb{R}^4$ generated by the set of 4-tuples

$$\{(1, 2, 1, 2), (2, 4, 3, 5), (3, 6, 4, 9), (1, 2, 4, 3)\}$$

(6 marks)

6. The code words

$$u_1 = 1010010, u_2 = 1100001, u_3 = 0101000, u_4 = 0010101$$

   form a basis for a $(7, 4)$ linear binary code.

   (a) Write down a generator matrix for this code.

   (b) Construct code words for the messages 1001 and 0101.

   (c) Write down the parity check matrix for this code.

(d) Find the syndromes for the received words
$1110011, 1001010, 0001101, 1101010$

$(6 + 4 + 4 + 8 = 20 \text{ marks})$

7. (Challenges for higher marks)

The challenges require students to do independent reading and researching, as while the methods of solution are mentioned in the classes, no solution templates or solutions to similar questions are provided (and will not be provided). Yet all the technical tools required to solve these questions have been studied in detail.

The set of vectors in P_2 is $\{1-x+2x^2, 2x+x^2, 2+5x-x^2\}$. Determine whether this set is linearly independent. Does it form a basis for P_2? Using Gram-Schmidt process starting from the first vector build an orthonormal basis for P_2. Here the Inner product of two polynomials is defined as per

$$\langle p, q\rangle = p_0q_0 + p_1q_1 + p_2q_2$$

where p_i, q_i are the coefficients of p and q.

(15 marks)

8. (Challenges for higher marks)

The table shows the revenue of a corporation (in billions of dollars) for six different years

Year	2008	2009	2010	2011	2012	2013
Revenue	29.3	32.0	32.5	32.7	31.7	31.2

Let $x = 8$ represents the year 2008 (i.e. $x = year - 2000$). Find the least squares regression quadratic polynomial $y = c_0+c_1x+c_2x^2$ for the data in the table and use your model to estimate the revenue for 2015. You will need to state the system of interpolating conditions and the normal system of equations as part of your solution, and then resolve the normal system for c_i.

(15 marks)