PROJECT ON

ANALYSIS OF EFFICIENT FRONTIER AND OPTIMAL PORTFOLIO

to be submitted by

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1 Introduction

1.1 What is Portfolio?

Portfolio is group of financial assets like stocks, bond, futures, options, cash etc. Cash is most liquid form of financial assets. However, time value of cash degrades if not invested. Hence financial assets needs to be invested to maximize gains as per investorś risk tolerance. If investor is risk-averse, government bond is the best option as risk associated with government bond is theoretically zero, however the returns will also be minimal. In order to increase returns risk-seeking investors invest in risky assets such as SIP, stock, option etc. Generally, more are the returns in these assets more is the risk associated with the asset. Hence, the portfolio needs to be optimized to maximize the returns for a given level of risk.

Practical Portfolio is the portfolio generated after modelling real world constraints like transaction cost, latency, non-divisibility, liquidity, solvency or other market microstructure issues.

1.2 Portfolio Optimization

Portfolio Optimization is the process of selecting set of best (optimal) portfolios, according to some objective, out of all considered portfolios. Objective can be minimizing risks, maximizing returns while diversifying the portfolio. Return to Risk ratio is a simple objective that is often used to select optimal portfolios. **Sharpe Ratio**, which is defined as the measure of average return, $\vec{r_p}$ earned in excess of the risk-free rate, r_f per unit of volatility or total risk, is a standard benchmark to measure Return to Risk ratio.

$$SharpeRatio = \frac{\vec{r_p} - r_f}{\sigma_p} \tag{1}$$

$$\sigma_p = \sqrt{\vec{w}_{1Xm}\ \mathbf{C}\ \vec{w}_{mX1}} \tag{2}$$

where,
σ_p is portfolio standard deviation,
C is covariance matrix,
$\vec{w}$ is the weight matrix.

2 Background

2.1 Markowitz Portfolio Theory

According to **Modern(Markowitz) Portfolio Theory (MPT)**, it is possible to construct an **efficient frontier** of optimal portfolios, ensuring maximum possible expected return, given investor's risk tolerance.

By diversifying the portfolio, its risk can be reduced to the level below their average weighted risk because the correlation among the assets of portfolio is imperfect i.e. < 1. It will become more clear with this example.

Table 1: *Portfolio diversification*

Asset	Weight	Return	Risk	Sharpe Ratio
A	0.03	0.12	0.25	0.48
B	0.05	0.1	0.30	0.33
C	0.08	0.2	0.45	0.44
Portfolio	0.059	0.116	1	0.509

corr(A,B) = -0.1, corr(A,C) = 0.4, corr(B,C) = 0.3

Clearly the Sharpe ratio of portfolio is more than that of any of the individual assets in the portfolio. This is because assets are not perfectly correlated. This imperfect correlation decrease the risk on diversification.

Efficient frontier is a set of optimal portfolios having highest expected return for a given level of risk.

Figure 1: Efficient frontier

This plot corresponds to the Sharpe Ratio of 20000 randomly regenerated portfolios containing shares of TSLA, GOOG, AMZN and NFLX and efficient frontier marked by red markers. The color of points indicate the Sharpe Ratio.

3 Problem Statement: Portfolio Optimization

To reduce the risk associated with portfolio while ensuring maximum returns.

To reduce the volatility of portfolio, the standard deviation or any other risk measure has to be reduced

$$\text{minimize } \sigma_p^2 = \vec{w}_{1Xm}\ \mathbf{C}\ \vec{w}_{mX1}$$
$$\text{subject to } \Sigma_{i=1}^{i=m} w_i \leq 1$$

All the portfolio below consist of different linear combination of stocks of Apple(AAPL), Advanced Micro Devices(AMD), Google(GOOG), Amazon(AMZN) and netflix(NFLX).

Efficient frontier is the collection of optimal portfolios at all given levels of risk.

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Pseudocode
1. Find maximum return and minimum return portfolio.
2. Generate a series of returns ranging from minimum return and max
3. Find minimum risk portfolio for return corresponding to each ter
4. Plot mean Vs standard deviation to trace out efficient frontier.
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3.1 Using 2 stocks

3.1.1 Random Portfolios

The plot below consist of 5000 random portfolios consisting of different linear combination of stocks of Apple(AAPL), Advanced Micro Devices(AMD)

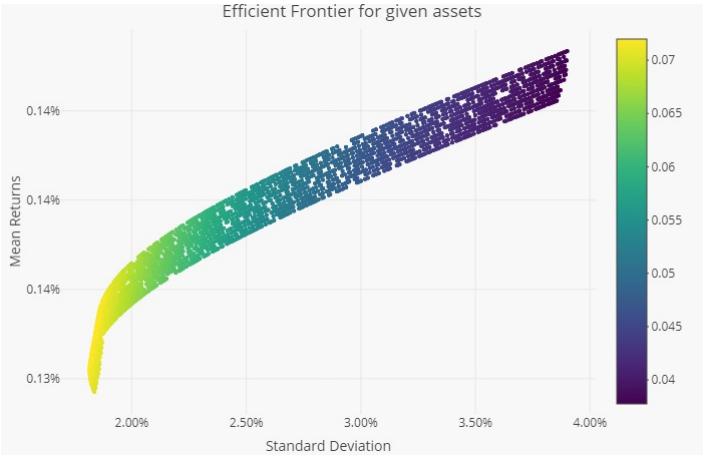

Figure 2: Unconstrained optimization

3.1.2 Efficient Frontier

This is the frontier generated with these constraints.

Figure 3: Unconstrained optimization

3.2 Using 5 stocks

3.2.1 Random Portfolios

The plot below consist of 5000 random portfolios consisting of different linear combination of stocks of Apple(AAPL), Advanced Micro Devices(AMD), Google(GOOG), Amazon(AMZN) and netflix(NFLX).

Figure 4: Unconstrained optimization

3.2.2 Efficient Frontier

This is the frontier generated with these constraints.

Figure 5: Unconstrained optimization

3.3 Conclusion

Table 2: *Risk and Return of various portfolios*

Portfolio	Risk	Return	Sharpe Ratio
Minimum variance	1.14%	0.109%	0.95
Maximum return	1.52%	0.230%	1.51

Portfolios with daily returns ranging from 0.109 % to 0.230 % are achievable using given set of assets. The risk tolerance of investor will determine optimal portfolio for the investor. Assuming risk free rate to be 0, Sharpe ratio by maximum return portfolio is much higher than other portfolios. Hence a risk seeking investor will invest in this portfolio. While a risk averse investor will invest in minimum variance portfolio to minimize risk due to market volatility. However, the extreme portfolios i.e. minimum return and maximum return portfolio in unconstrained optimization will accumulate weights in a single stock which is not a healthy practice as the downtrend in one of the stock will fluctuate the portfolio. Hence it is a good practice to choose a portfolio between minimum return and maximum return to get a well diversified portfolio.