CNOT Gate Implemented by Quantum Teleportation

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Outline

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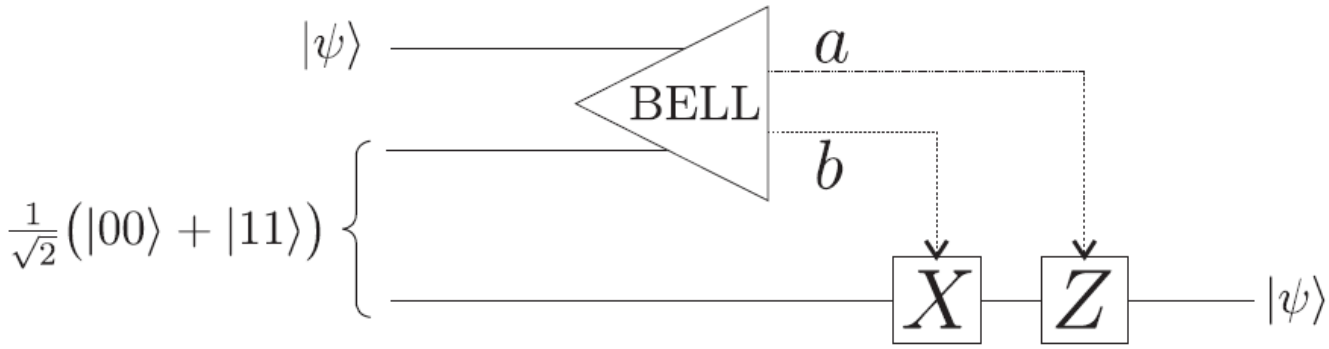
Circuit for quantum teleportation



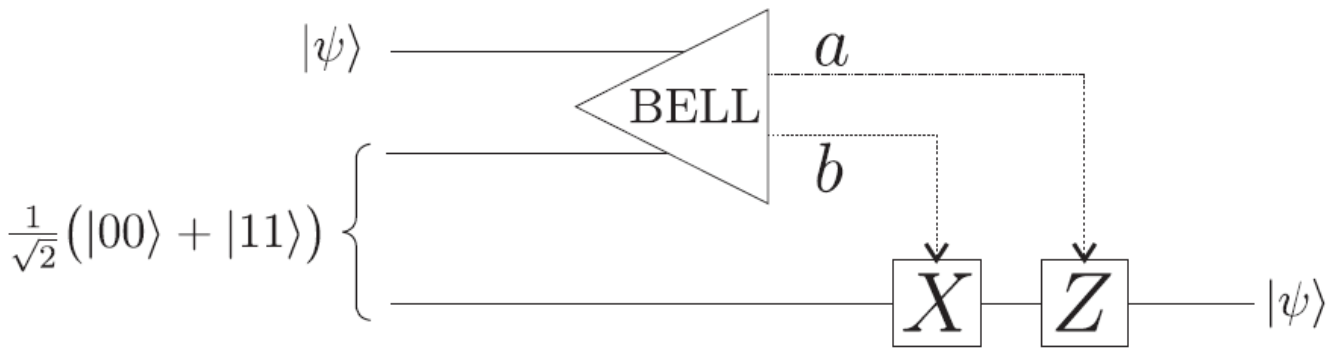
- The 3-qubit state possessed by Alice and Bob is

$$|\psi\rangle|\beta_{00}\rangle$$

- This state can be rewritten as

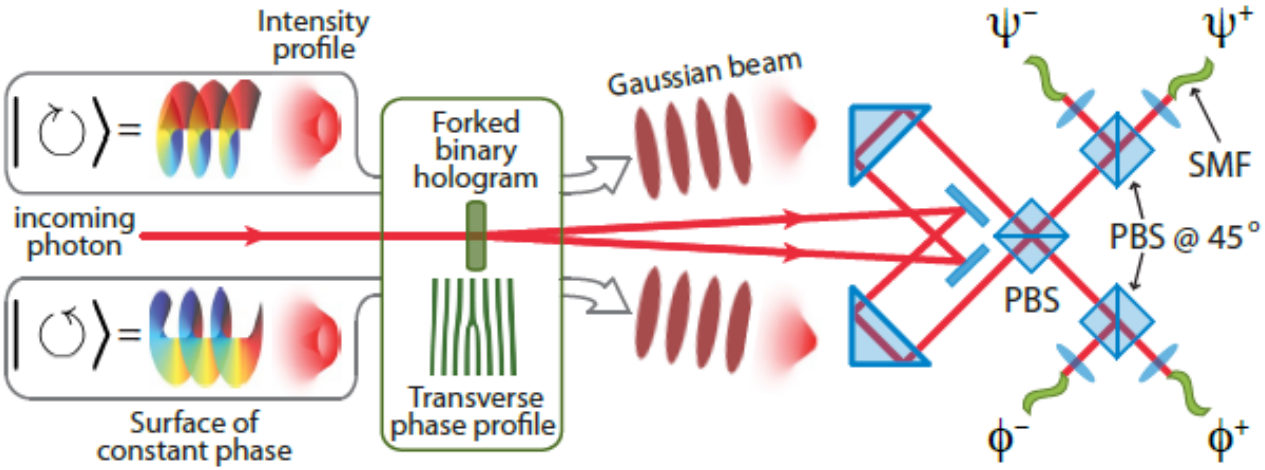
$$\begin{aligned} |\psi\rangle|\beta_{00}\rangle &= \frac{1}{2}|\beta_{00}\rangle|\psi\rangle + \frac{1}{2}|\beta_{01}\rangle(X|\psi\rangle) \\ &+ \frac{1}{2}|\beta_{10}\rangle(Z|\psi\rangle) + \frac{1}{2}|\beta_{11}\rangle(XZ|\psi\rangle) \end{aligned}$$

Circuit for quantum teleportation (cont.)



- After the Bell measurement by Alice, Bob conditionally applies X and Z to his qubit (classically) conditioned on the measurement results a and b.

Bell measurement



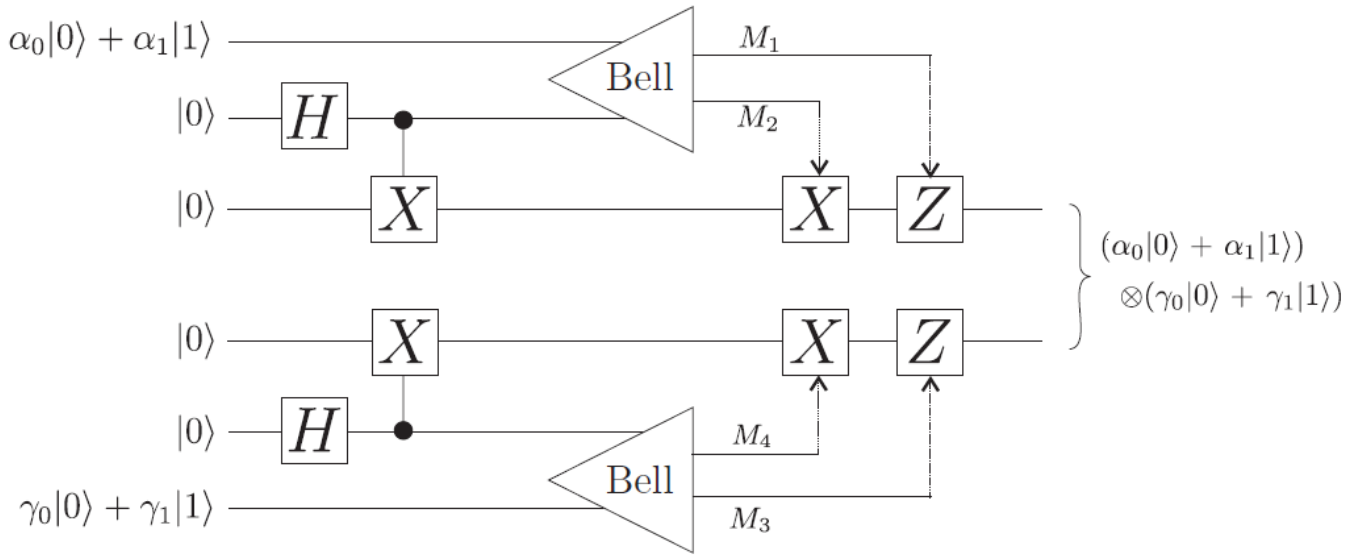
- Linear optics can only produce a partial bell measurement, allowing $\log_2(3) = 1.58$ bits per photon[MWKZ96].
- Barreiro et al. uses hyperentanglement to assist in the Bell-state measurement in linear-optical experiment[BWK08], which has succeeded in surpassing the linear-optics limit of 1.58 bits per photon.

Outline

Introduction

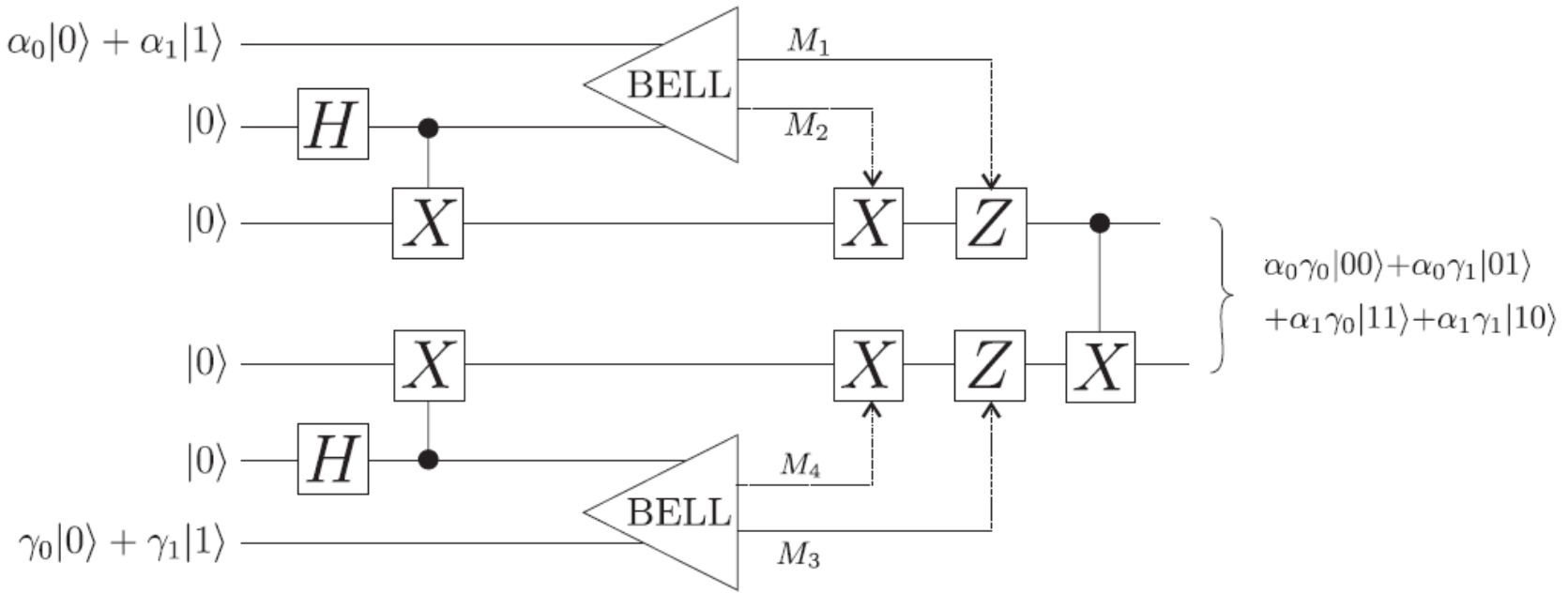
- The idea of this section is from [GC99].
- the CNOT gate is a common choice for a universal set of quantum gates.
- A particular implementation of the CNOT gate may fail some of the time.
- Need a way of non-destructively applying the CNOT gate. If it fails, the relevant qubits are not corrupted, and we can try CNOT gate again.
- Quantum teleportation gives us the ability of doing this, provided that we can
  1. Prepare a Bell state
  2. To do single bit rotations
  3. To do Bell measurement

Circuit for teleporting two states



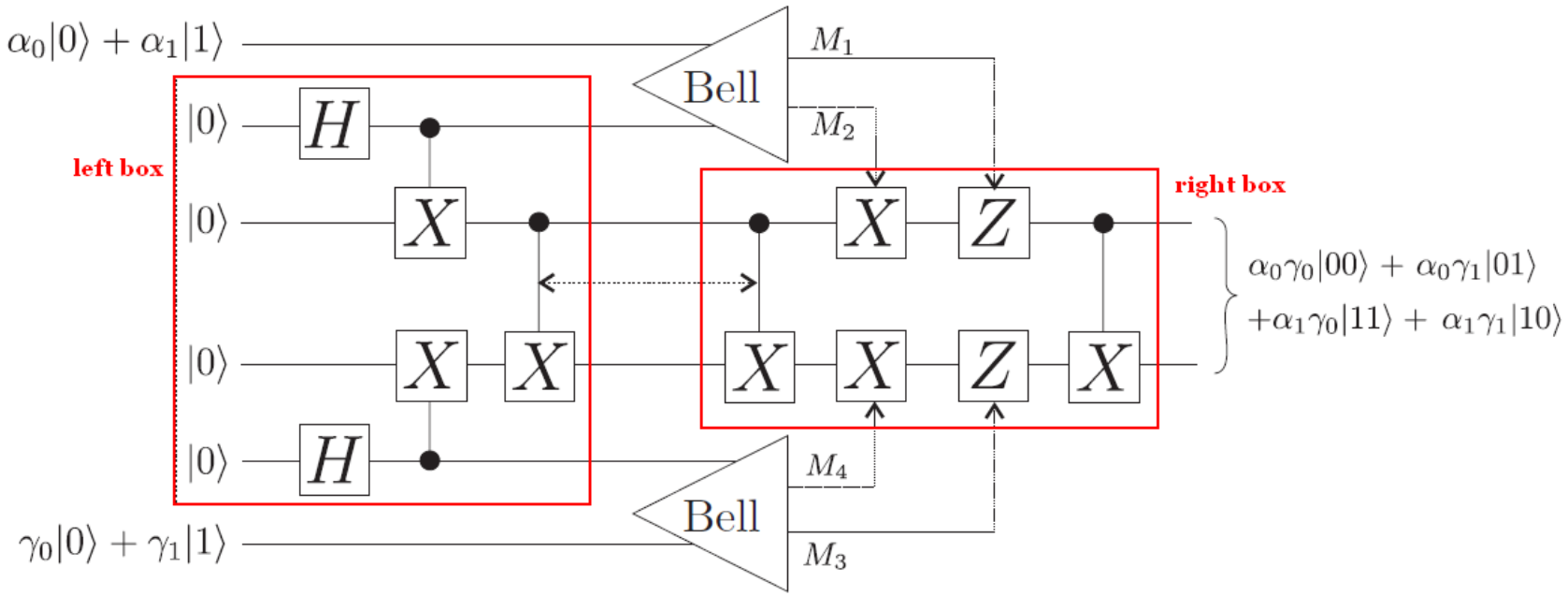
- A circuit to teleport $\alpha_0|0\rangle + \alpha_1|1\rangle$ and $\gamma_0|0\rangle + \gamma_1|1\rangle$.
- Note that the state of two qubits is unaffected by the circuit, thus the circuit can be viewed as an identity operation.

A CNOT gate between the pair of teleported states



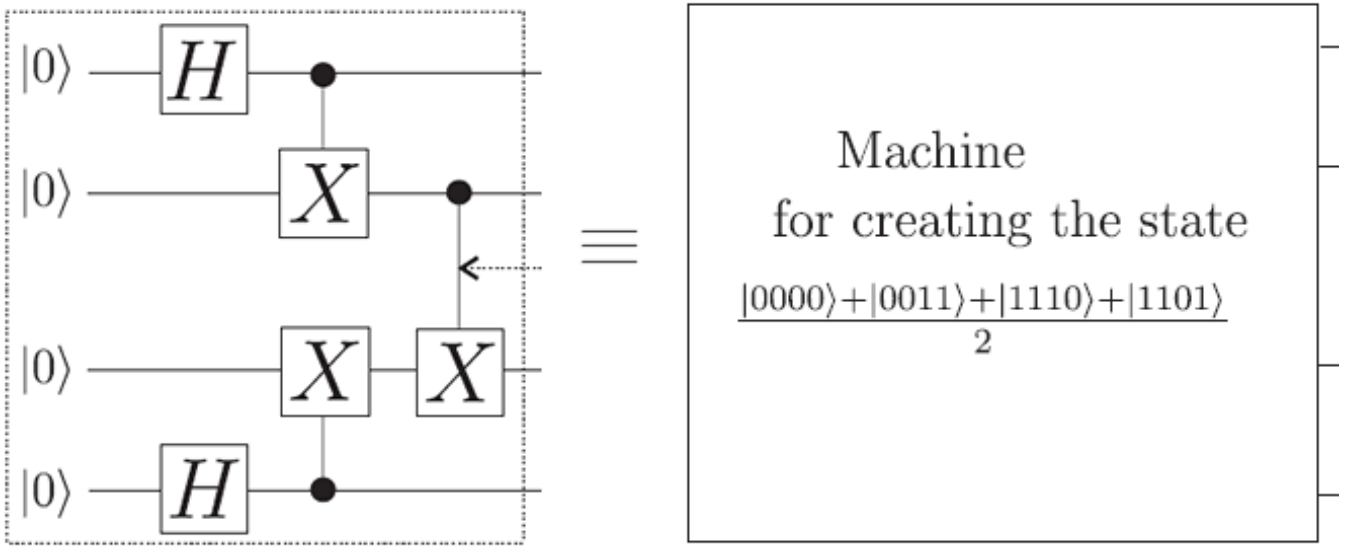
- The overall effect on the two logical qubits is the CNOT operation.

The trick - adding a pair of CNOT gates



- Add a pair of CNOT gates to the middle two qubits. This does not change the overall behavior of the circuit.
- We can regroup the CNOT gates, as shown in two red boxes.

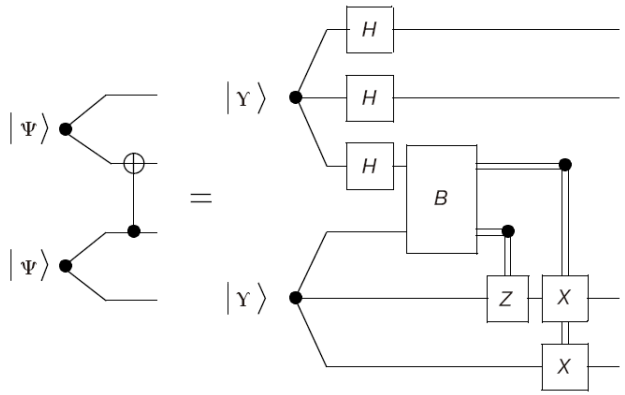
The trick - left box operation



- It's easy to check that the left box creates a 4-state entangled state

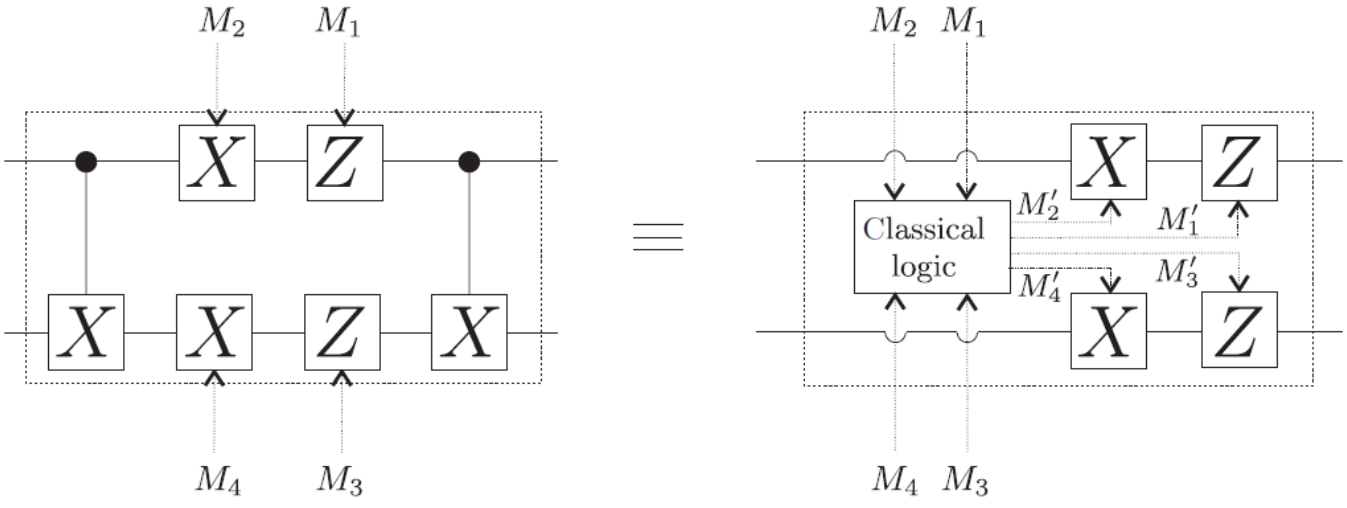
$$|\Psi\rangle = \frac{|0000\rangle + |0011\rangle + |1110\rangle + |1101\rangle}{2}$$

Four qubit entangled state



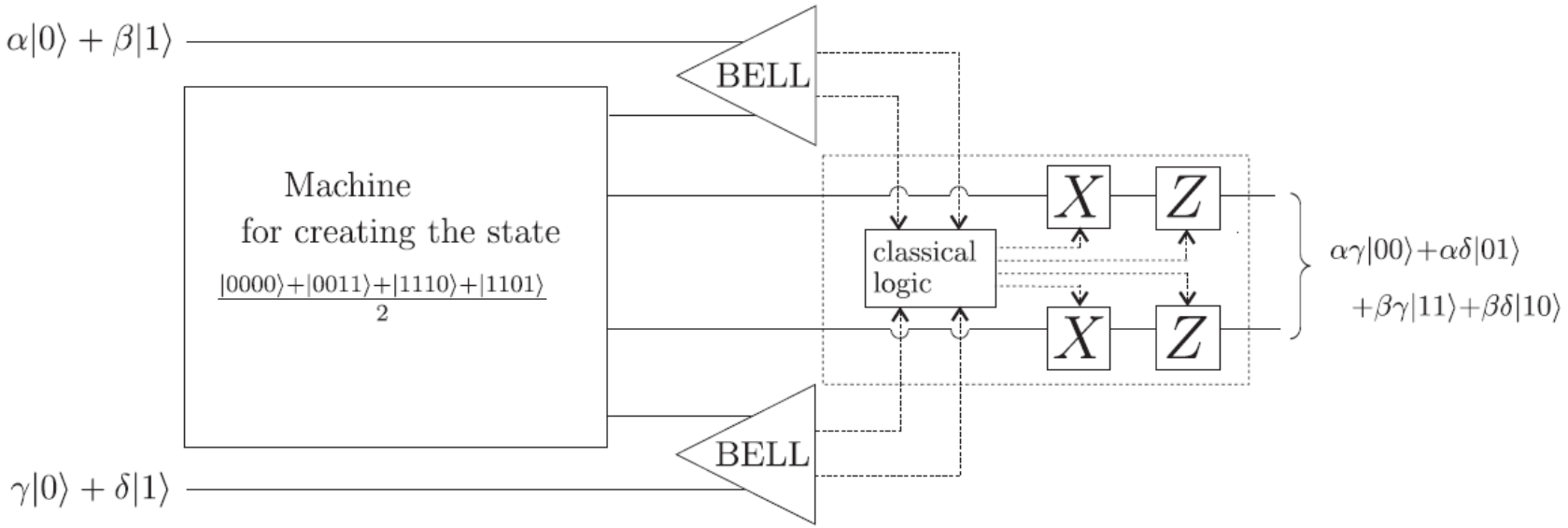
- $|\Psi\rangle = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$, which is Bell state. $|\Upsilon\rangle = \frac{|000\rangle+|111\rangle}{\sqrt{2}}$, which is GHZ state.
- The machine may use CNOT gates, the point is that if it fails to create the state, the verification procedure will tell us the failure, and we can try again with freshly prepared qubits.

The trick - right box operation



- The CNOT gate belongs to **Clifford group**, and thus we can rearrange the order between CNOT gate and X, Z gates.
- We can have a classical logic to control the desired effects.

The trick - whole view



Acknowledgement

Thank you !

References I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