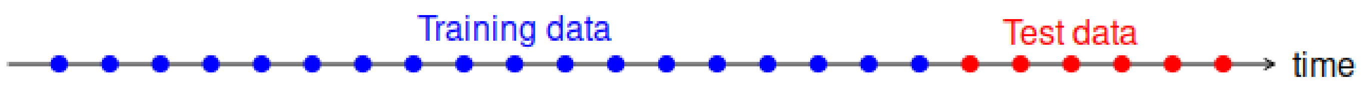
3.4 Evaluating forecast accuracy

Training and test sets

It is important to evaluate forecast accuracy using genuine forecasts. Consequently, the size of the residuals is not a reliable indication of how large true forecast errors are likely to be. The accuracy of forecasts can only be determined by considering how well a model performs on new data that were not used when fitting the model.

When choosing models, it is common practice to separate the available data into two portions, **training** and **test** data, where the training data is used to estimate any parameters of a forecasting method and the test data is used to evaluate its accuracy. Because the test data is not used in determining the forecasts, it should provide a reliable indication of how well the model is likely to forecast on new data.



The size of the test set is typically about 20% of the total sample, although this value depends on how long the sample is and how far ahead you want to forecast. The test set should ideally be at least as large as the maximum forecast horizon required. The following points should be noted.

- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.

Some references describe the test set as the "hold-out set" because these data are "held out" of the data used for fitting. Other references call the training set the "in-sample data" and the test set the "out-of-sample data." We prefer to use "training data" and "test data" in this book.

Functions to subset a time series

The `window()` function introduced in Chapter 2 is useful when extracting a portion of a time series, such as we need when creating training and test sets. In the `window()` function, we specify the start and/or end of the portion of time series required using time values. For example,

```
window(ausbeer, start=1995)
```

extracts all data from 1995 onward.

Another useful function is `subset()` which allows for more types of subsetting. A great advantage of this function is that it allows the use of indices to choose a subset. For example,

```
subset(ausbeer, start=length(ausbeer)-4*5)
```

extracts the last 5 years of observations from `ausbeer`. It also allows extracting all values for a specific season. For example,

```
subset(ausbeer, quarter = 1)
```

extracts the first quarters for all years.

Finally, `head` and `tail` are useful for extracting the first few or last few observations. For example, the last 5 years of `ausbeer` can also be obtained using

```
tail(ausbeer, 4*5)
```

Forecast errors

A forecast "error" is the difference between an observed value and its forecast. Here "error" does not mean a mistake, it means the unpredictable part of an observation. It can be written as

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$ and the test data is given by $\{y_{T+1}, y_{T+2}, \dots\}$.

Note that forecast errors are different from residuals in two ways. First, residuals are calculated on the *training* set while forecast errors are calculated on the *test* set. Second, residuals are based on *one-step* forecasts while forecast errors can involve *multi-step* forecasts.

We can measure forecast accuracy by summarising the forecast errors in different ways.

Scale-dependent errors

The forecast errors are on the same scale as the data. Accuracy measures that are based only on e_t are therefore scale-dependent and cannot be used to make comparisons between series that involve different units.

The two most commonly used scale-dependent measures are based on the absolute errors or squared errors:

$$\begin{aligned}\text{Mean absolute error: MAE} &= \text{mean}(|e_t|),\\ \text{Root mean squared error: RMSE} &= \sqrt{\text{mean}(e_t^2)}.\end{aligned}$$

When comparing forecast methods applied to a single time series, or to several time series with the same units, the MAE is popular as it is easy to both understand and compute. A forecast method that minimises the MAE will lead to forecasts of the median, while minimising the RMSE will lead to forecasts of the mean. Consequently, the RMSE is also widely used, despite being more difficult to interpret.

Percentage errors

The percentage error is given by $p_t = 100e_t/y_t$. Percentage errors have the advantage of being unit-free, and so are frequently used to compare forecast performances between data sets. The most commonly used measure is:

$$\text{Mean absolute percentage error: MAPE} = \text{mean}(|p_t|).$$

Measures based on percentage errors have the disadvantage of being infinite or undefined if $y_t = 0$ for any t in the period of interest, and having extreme values if any y_t is close to zero. Another problem with percentage errors that is often overlooked is that they assume the unit of measurement has a meaningful zero.[2] For example, a percentage error makes no sense when measuring the accuracy of temperature forecasts on either the Fahrenheit or Celsius scales, because temperature has an arbitrary zero point.

They also have the disadvantage that they put a heavier penalty on negative errors than on positive errors. This observation led to the use of the so-called "symmetric" MAPE (sMAPE) proposed by Armstrong (1978, p. 348), which was used in the M3 forecasting competition. It is defined by

$$\text{sMAPE} = \text{mean}\left(200|y_t - \hat{y}_t|/(y_t + \hat{y}_t)\right).$$

However, if y_t is close to zero, $\hat{y}_t$ is also likely to be close to zero. Thus, the measure still involves division by a number close to zero, making the calculation unstable. Also, the value of sMAPE can be negative, so it is not really a measure of "absolute percentage errors" at all.

Hyndman & Koehler (2006) recommend that the sMAPE not be used. It is included here only because it is widely used, although we will not use it in this book.

Scaled errors

Scaled errors were proposed by Hyndman & Koehler (2006) as an alternative to using percentage errors when comparing forecast accuracy across series with different units. They proposed scaling the errors based on the *training* MAE from a simple forecast method.

For a non-seasonal time series, a useful way to define a scaled error uses naïve forecasts:

$$q_j = \frac{e_j}{\displaystyle\frac{1}{T-1}\sum_{t=2}^{T}|y_t - y_{t-1}|}.$$

Because the numerator and denominator both involve values on the scale of the original data, q_j is independent of the scale of the data. A scaled error is less than one if it arises from a better forecast than the average naïve forecast computed on the training data. Conversely, it is greater than one if the forecast is worse than the average naïve forecast computed on the training data.

For seasonal time series, a scaled error can be defined using seasonal naïve forecasts:

$$q_j = \frac{e_j}{\frac{1}{T-m}\sum_{t=m+1}^{T}|y_t - y_{t-m}|}.$$

The *mean absolute scaled error* is simply

$$\text{MASE} = \text{mean}(|q_j|).$$

Examples

```
beer2 <- window(ausbeer,start=1992,end=c(2007,4))
beerfit1 <- meanf(beer2,h=10)
beerfit2 <- rwf(beer2,h=10)
beerfit3 <- snaive(beer2,h=10)
autoplot(window(ausbeer, start=1992)) +
  autolayer(beerfit1, series="Mean", PI=FALSE) +
  autolayer(beerfit2, series="Naïve", PI=FALSE) +
  autolayer(beerfit3, series="Seasonal naïve", PI=FALSE) +
  xlab("Year") + ylab("Megalitres") +
  ggtitle("Forecasts for quarterly beer production") +
  guides(colour=guide_legend(title="Forecast"))
```

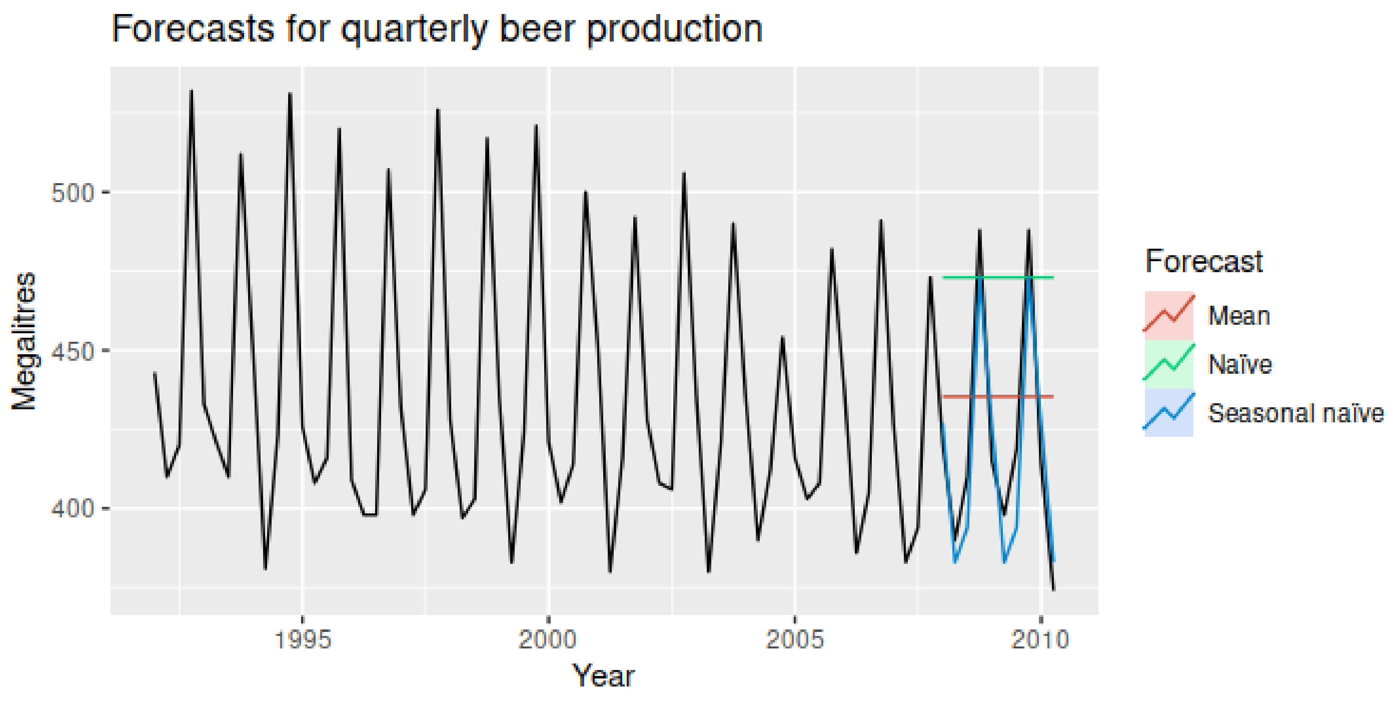


Figure 3.9: Forecasts of Australian quarterly beer production using data up to the end of 2007.

Figure 3.9 shows three forecast methods applied to the quarterly Australian beer production using data only to the end of 2007. The actual values for the period 2008–2010 are also shown. We compute the forecast accuracy measures for this period.

```
beer3 <- window(ausbeer, start=2008)
accuracy(beerfit1, beer3)
accuracy(beerfit2, beer3)
accuracy(beerfit3, beer3)
```

	RMSE	MAE	MAPE	MASE
Mean method	38.45	34.83	8.28	2.44
Naïve method	62.69	57.40	14.18	4.01
Seasonal naïve method	14.31	13.40	3.17	0.94

It is obvious from the graph that the seasonal naïve method is best for these data, although it can still be improved, as we will discover later. Sometimes, different accuracy measures will lead to different results as to which forecast method is best. However, in this case, all of the results point to the seasonal naïve method as the best of these three methods for this data set.

To take a non-seasonal example, consider the Google stock price. The following graph shows the 200 observations ending on 6 Dec 2013, along with forecasts of the next 40 days obtained from three different methods.

```
googfc1 <- meanf(goog200, h=40)
googfc2 <- rwf(goog200, h=40)
googfc3 <- rwf(goog200, drift=TRUE, h=40)
autoplot(subset(goog, end = 240)) +
  autolayer(googfc1, PI=FALSE, series="Mean") +
  autolayer(googfc2, PI=FALSE, series="Naïve") +
  autolayer(googfc3, PI=FALSE, series="Drift") +
  xlab("Day") + ylab("Closing Price (US$)") +
  ggtitle("Google stock price (daily ending 6 Dec 13)") +
  guides(colour=guide_legend(title="Forecast"))
```

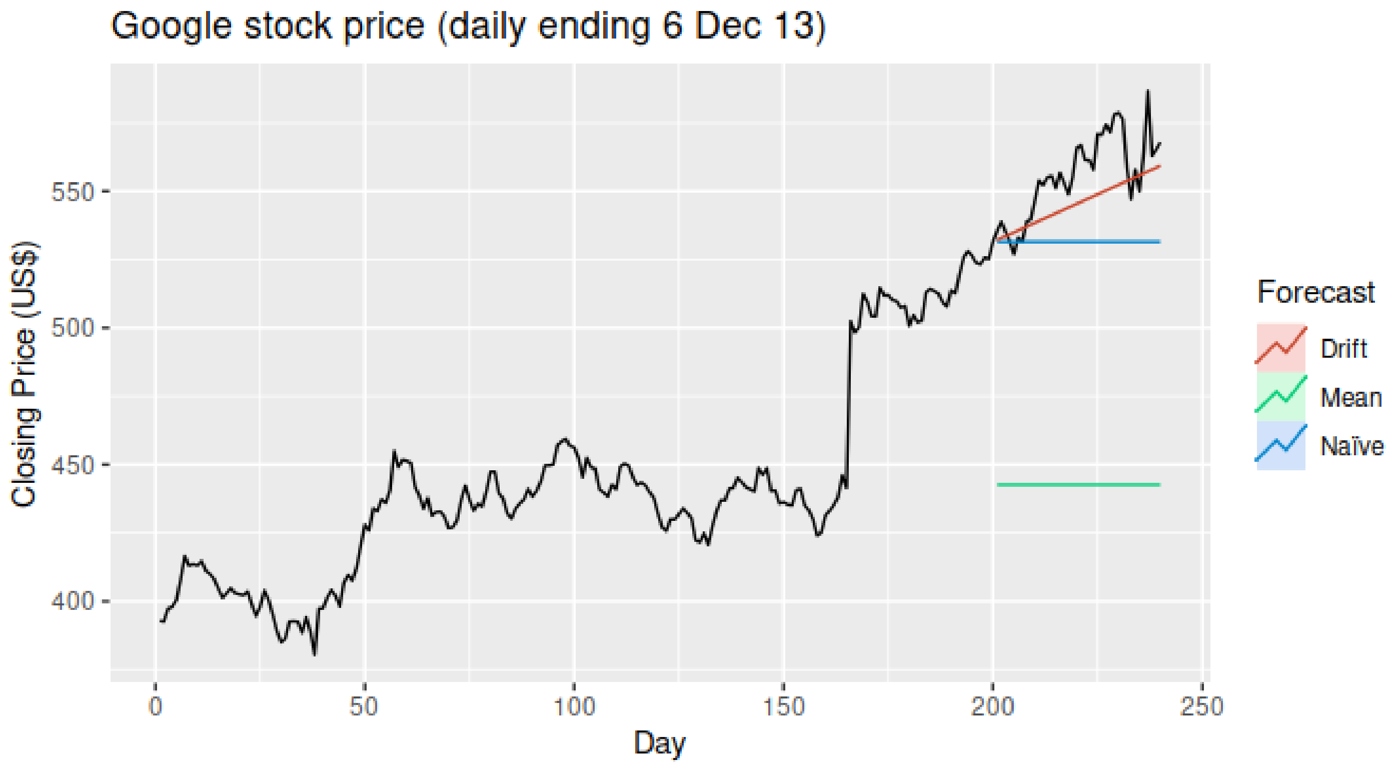


Figure 3.10: Forecasts of the Google stock price from 7 Dec 2013.

```
googtest <- window(goog, start=201, end=240)
accuracy(googfc1, googtest)
accuracy(googfc2, googtest)
accuracy(googfc3, googtest)
```

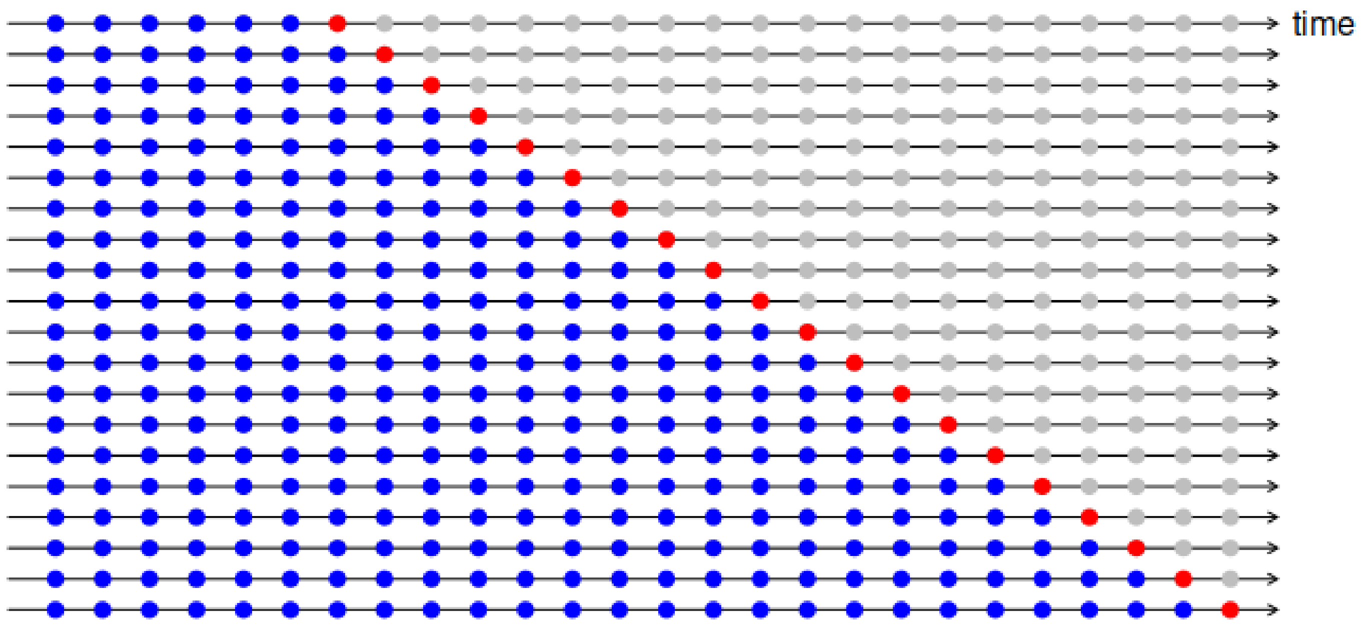
	RMSE	MAE	MAPE	MASE
Mean method	114.21	113.27	20.32	30.28
Naïve method	28.43	24.59	4.36	6.57
Drift method	14.08	11.67	2.07	3.12

Here, the best method is the drift method (regardless of which accuracy measure is used).

Time series cross-validation

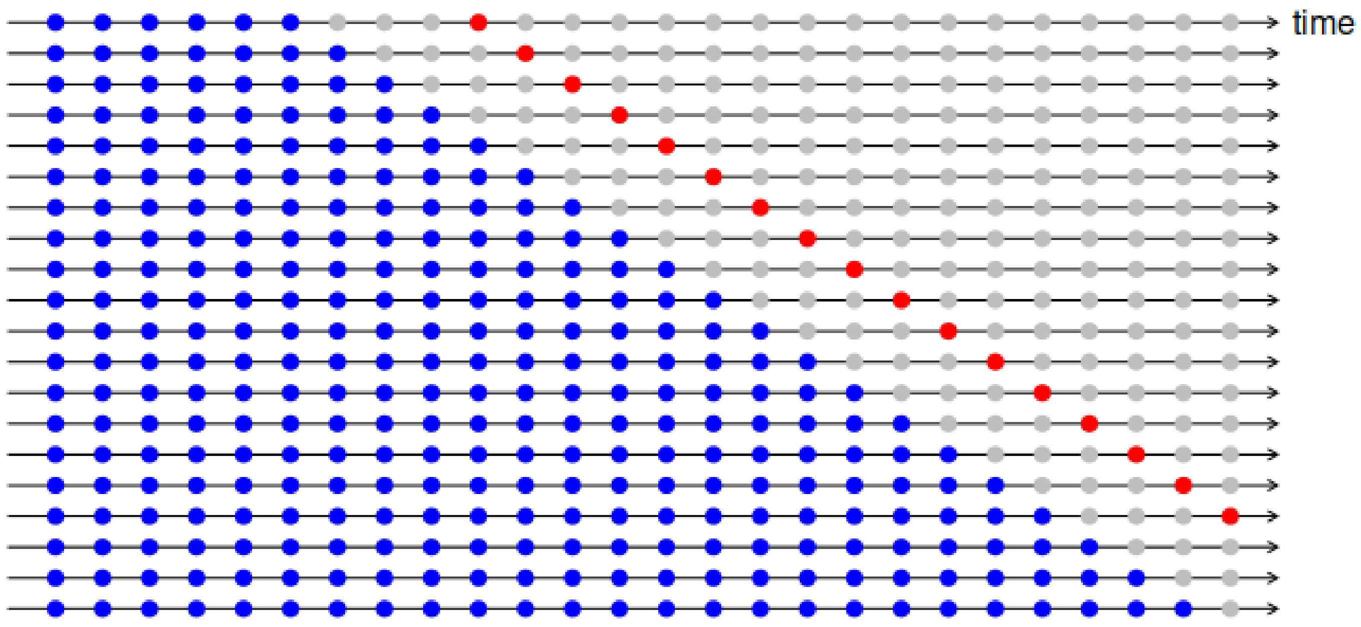
A more sophisticated version of training/test sets is time series cross-validation. In this procedure, there are a series of test sets, each consisting of a single observation. The corresponding training set consists only of observations that occurred *prior* to the observation that forms the test set. Thus, no future observations can be used in constructing the forecast. Since it is not possible to obtain a reliable forecast based on a small training set, the earliest observations are not considered as test sets.

The following diagram illustrates the series of training and test sets, where the blue observations form the training sets, and the red observations form the test sets.



The forecast accuracy is computed by averaging over the test sets. This procedure is sometimes known as "evaluation on a rolling forecasting origin" because the "origin" at which the forecast is based rolls forward in time.

With time series forecasting, one-step forecasts may not be as relevant as multi-step forecasts. In this case, the cross-validation procedure based on a rolling forecasting origin can be modified to allow multi-step errors to be used. Suppose that we are interested in models that produce good 4-step-ahead forecasts. Then the corresponding diagram is shown below.



Time series cross-validation is implemented with the `tsCV()` function. In the following example, we compare the RMSE obtained via time series cross-validation with the residual RMSE.

```
e <- tsCV(goog200, rwf, drift=TRUE, h=1)
sqrt(mean(e^2, na.rm=TRUE))
#> [1] 6.233
sqrt(mean(residuals(rwf(goog200, drift=TRUE))^2, na.rm=TRUE))
#> [1] 6.169
```

As expected, the RMSE from the residuals is smaller, as the corresponding "forecasts" are based on a model fitted to the entire data set, rather than being true forecasts.

A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.

Pipe operator

The ugliness of the above R code makes this a good opportunity to introduce some alternative ways of stringing R functions together. In the above code, we are nesting functions within functions within functions, so you have to read the code from the inside out, making it difficult to understand what is being computed. Instead, we can use the pipe operator `%>%` as follows.

```
goog200 %>% tsCV(forecastfunction=rwf, drift=TRUE, h=1) -> e
e^2 %>% mean(na.rm=TRUE) %>% sqrt()
#> [1] 6.233
goog200 %>% rwf(drift=TRUE) %>% residuals() -> res
res^2 %>% mean(na.rm=TRUE) %>% sqrt()
#> [1] 6.169
```

The left hand side of each pipe is passed as the first argument to the function on the right hand side. This is consistent with the way we read from left to right in English. When using pipes, all other arguments must be named, which also helps readability. When using pipes, it is natural to use the right arrow assignment `->` rather than the left arrow. For example, the third line above can be read as "Take the `goog200` series, pass it to `rwf()` with `drift=TRUE`, compute the resulting residuals, and store them as `res`."

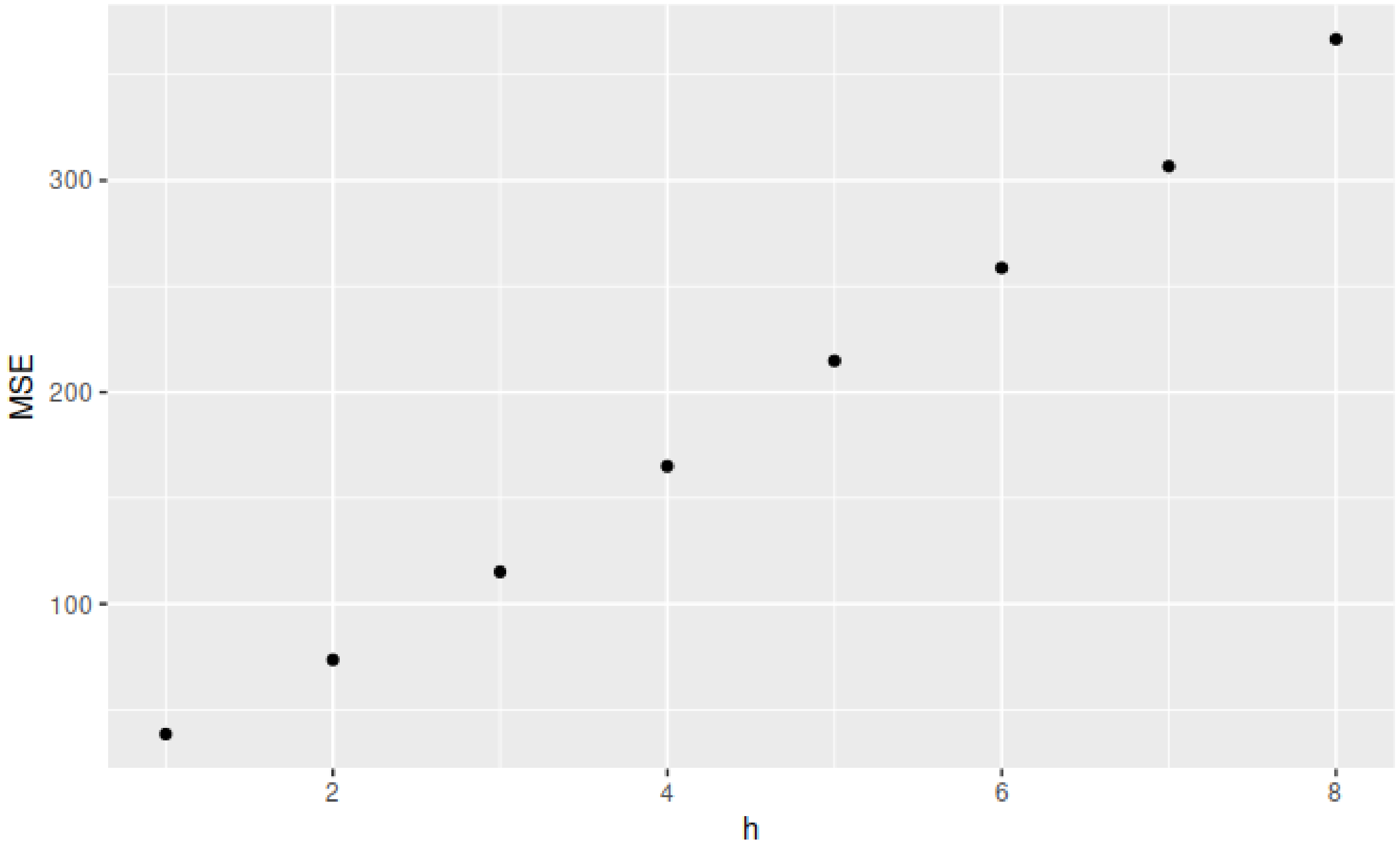
We will use the pipe operator whenever it makes the code easier to read. In order to be consistent, we will always follow a function with parentheses to differentiate it from other objects, even if it has no arguments. See, for example, the use of `sqrt()` and `residuals()` in the code above.

Example: using tsCV()

The `goog200` data, plotted in Figure 3.5, includes daily closing stock price of Google Inc from the NASDAQ exchange for 200 consecutive trading days starting on 25 February 2013.

The code below evaluates the forecasting performance of 1- to 8-step-ahead naïve forecasts with `tsCV()`, using MSE as the forecast error measure. The plot shows that the forecast error increases as the forecast horizon increases, as we would expect.

```
e <- tsCV(goog200, forecastfunction=naive, h=8)
# Compute the MSE values and remove missing values
mse <- colMeans(e^2, na.rm = T)
# Plot the MSE values against the forecast horizon
data.frame(h = 1:8, MSE = mse) %>%
  ggplot(aes(x = h, y = MSE)) + geom_point()
```



2. That is, a percentage is valid on a ratio scale, but not on an interval scale. Only ratio scale variables have meaningful zeros.↩