10.3 The bottom-up approach

A simple method for generating coherent forecasts is the bottom-up approach. This approach involves first generating forecasts for each series at the bottom-level, and then summing these to produce forecasts for all the series in the structure.

For example, for the hierarchy of Figure 10.1, we first generate h-step-ahead forecasts for each of the bottom-level series:

$$\hat{y}_{\text{AA},h},\ \hat{y}_{\text{AB},h},\ \hat{y}_{\text{AC},h},\ \hat{y}_{\text{BA},h} \text{ and } \hat{y}_{\text{BB},h}.$$

(We have simplified the previously used notation of $\hat{y}_{T+h|T}$ for brevity.) Summing these, we get h-step-ahead coherent forecasts for the rest of the series:

$$\begin{aligned}
\tilde{y}_h &= \hat{y}_{\text{AA},h} + \hat{y}_{\text{AB},h} + \hat{y}_{\text{AC},h} + \hat{y}_{\text{BA},h} + \hat{y}_{\text{BB},h},\\
\tilde{y}_{\text{A},h} &= \hat{y}_{\text{AA},h} + \hat{y}_{\text{AB},h} + \hat{y}_{\text{AC},h},\\
\text{and}\quad \tilde{y}_{\text{B},h} &= \hat{y}_{\text{BA},h} + \hat{y}_{\text{BB},h}.
\end{aligned}$$

(In this chapter, we will use the "tilde" notation to indicate coherent forecasts.) As in Equation (10.3), we can employ the summing matrix here and write

$$\begin{bmatrix}
\tilde{y}_h\\
\tilde{y}_{\text{A},h}\\
\tilde{y}_{\text{B},h}\\
\tilde{y}_{\text{AA},h}\\
\tilde{y}_{\text{AB},h}\\
\tilde{y}_{\text{AC},h}\\
\tilde{y}_{\text{BA},h}\\
\tilde{y}_{\text{BB},h}
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 & 1\\
1 & 1 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 1\\
1 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{y}_{\text{AA},h}\\
\hat{y}_{\text{AB},h}\\
\hat{y}_{\text{AC},h}\\
\hat{y}_{\text{BA},h}\\
\hat{y}_{\text{BB},h}
\end{bmatrix}.$$

Using more compact notation, the bottom-up approach can be represented as

$$\tilde{\boldsymbol{y}}_h = \boldsymbol{S}\hat{\boldsymbol{b}}_h,$$

where $\tilde{\boldsymbol{y}}_t$ is an n-dimensional vector of coherent h-step-ahead forecasts, and $\hat{\boldsymbol{b}}_h$ is an m-dimensional vector of h-step-ahead forecasts for each of the bottom-level series.

An advantage of this approach is that we are forecasting at the bottom-level of a structure, and therefore no information is lost due to aggregation. On the other hand, bottom-level data can be quite noisy and more challenging to model and forecast.

The hts package for R

Forecasts can be produced using the `forecast()` function applied to objects created by `hts()` or `gts()`. The **hts** package has three in-built options to produce forecasts: ETS models, ARIMA models or random walks; these are controlled by the `fmethod` argument. It also use several methods for producing coherent forecasts, controlled by the `method` argument.

For example, suppose we wanted bottom-up forecasts using ARIMA models applied to the prison data. Then we would use

```
forecast(prison.gts, method="bu", fmethod="arima")
```

which will apply the `auto.arima()` function to every bottom-level series in our collection of time series. Similarly, ETS models would be used if `fmethod="ets"` was used.