8.2 Backshift notation

The backward shift operator B is a useful notational device when working with time series lags:

$$By_t = y_{t-1} \, .$$

(Some references use L for "lag" instead of B for "backshift.") In other words, B, operating on y_t, has the effect of shifting the data back one period. Two applications of B to y_t shifts the data back two periods:

$$B(By_t) = B^2 y_t = y_{t-2} \, .$$

For monthly data, if we wish to consider "the same month last year," the notation is $B^{12}y_t$ = y_{t-12}.

The backward shift operator is convenient for describing the process of *differencing*. A first difference can be written as

$$y'_t = y_t - y_{t-1} = y_t - By_t = (1 - B)y_t \, .$$

Note that a first difference is represented by $(1 - B)$. Similarly, if second-order differences have to be computed, then:

$$y''_t = y_t - 2y_{t-1} + y_{t-2} = (1 - 2B + B^2)y_t = (1 - B)^2 y_t \, .$$

In general, a dth-order difference can be written as

$$(1 - B)^d y_t.$$

Backshift notation is particularly useful when combining differences, as the operator can be treated using ordinary algebraic rules. In particular, terms involving B can be multiplied together.

For example, a seasonal difference followed by a first difference can be written as

$$\begin{aligned} (1 - B)(1 - B^m)y_t &= (1 - B - B^m + B^{m+1})y_t \\ &= y_t - y_{t-1} - y_{t-m} + y_{t-m-1}, \end{aligned}$$

the same result we obtained earlier.