8.3 Autoregressive models

In a multiple regression model, we forecast the variable of interest using a linear combination of predictors. In an autoregression model, we forecast the variable of interest using a linear combination of *past values of the variable*. The term *auto*regression indicates that it is a regression of the variable against itself.

Thus, an autoregressive model of order p can be written as

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t,$$

where ε_t is white noise. This is like a multiple regression but with *lagged values* of y_t as predictors. We refer to this as an **AR(p) model**, an autoregressive model of order p.

Autoregressive models are remarkably flexible at handling a wide range of different time series patterns. The two series in Figure 8.5 show series from an AR(1) model and an AR(2) model. Changing the parameters $\phi_1, \dots, \phi_p$ results in different time series patterns. The variance of the error term ε_t will only change the scale of the series, not the patterns.

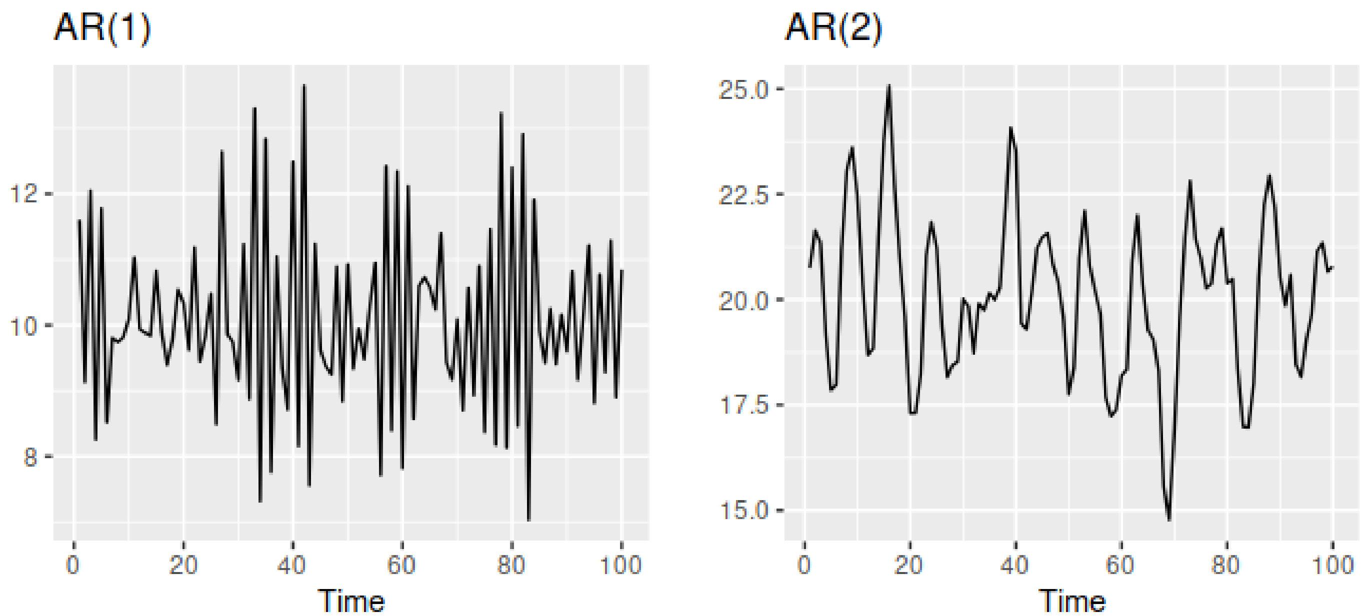

Figure 8.5: Two examples of data from autoregressive models with different parameters. Left: AR(1) with $y_t = 18 - 0.8 y_{t-1} + \varepsilon_t$. Right: AR(2) with $y_t = 8 + 1.3 y_{t-1} - 0.7 y_{t-2} + \varepsilon_t$. In both cases, ε_t is normally distributed white noise with mean zero and variance one.

For an AR(1) model:

- when $\phi_1 = 0$, y_t is equivalent to white noise;
- when $\phi_1 = 1$ and $c = 0$, y_t is equivalent to a random walk;
- when $\phi_1 = 1$ and $c \neq 0$, y_t is equivalent to a random walk with drift;
- when $\phi_1 < 0$, y_t tends to oscillate around the mean.

We normally restrict autoregressive models to stationary data, in which case some constraints on the values of the parameters are required.

- For an AR(1) model: $-1 < \phi_1 < 1$.
- For an AR(2) model: $-1 < \phi_2 < 1$, $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$.

When $p \geq 3$, the restrictions are much more complicated. R takes care of these restrictions when estimating a model.