6.7 Measuring strength of trend and seasonality

A time series decomposition can be used to measure the strength of trend and seasonality in a time series (Wang, Smith, & Hyndman, 2006). Recall that the decomposition is written as

$$y_t = T_t + S_t + R_t,$$

where T_t is the smoothed trend component, S_t is the seasonal component and R_t is a remainder component. For strongly trended data, the seasonally adjusted data should have much more variation than the remainder component. Therefore $\text{Var}(R_t)/\text{Var}(T_t + R_t)$ should be relatively small. But for data with little or no trend, the two variances should be approximately the same. So we define the strength of trend as:

$$F_T = \max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(T_t + R_t)}\right).$$

This will give a measure of the strength of the trend between 0 and 1. Because the variance of the remainder might occasionally be even larger than the variance of the seasonally adjusted data, we set the minimal possible value of F_T equal to zero.

The strength of seasonality is defined similarly, but with respect to the detrended data rather than the seasonally adjusted data:

$$F_S = \max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t + R_t)}\right).$$

A series with seasonal strength F_S close to 0 exhibits almost no seasonality, while a series with strong seasonality will have F_S close to 1 because $\text{Var}(R_t)$ will be much smaller than $\text{Var}(S_t + R_t)$.

These measures can be useful, for example, when there you have a large collection of time series, and you need to find the series with the most trend or the most seasonality.