10.6 Mapping matrices

All of the methods considered so far can be expressed using a common notation.

Suppose we forecast all series independently, ignoring the aggregation constraints. We call these the **base forecasts** and denote them by $\hat{\boldsymbol{y}}_h$ where h is the forecast horizon. They are stacked in the same order as the data $\boldsymbol{y}_t$.

Then all forecasting approaches for either hierarchical or grouped structures can be represented as

$$\tilde{\boldsymbol{y}}_h = \boldsymbol{S}\boldsymbol{G}\hat{\boldsymbol{y}}_h, \tag{10.6}$$

where $\boldsymbol{G}$ is a matrix that maps the base forecasts into the bottom-level, and the summing matrix $\boldsymbol{S}$ sums these up using the aggregation structure to produce a set of coherent forecasts $\tilde{\boldsymbol{y}}_h$.

The $\boldsymbol{G}$ matrix is defined according to the approach implemented. For example if the bottom-up approach is used to forecast the hierarchy of Figure 10.1, then

$$\boldsymbol{G} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Notice that $\boldsymbol{G}$ contains two partitions. The first three columns zero out the base forecasts of the series above the bottom-level, while the m-dimensional identity matrix picks only the base forecasts of the bottom-level. These are then summed by the $\boldsymbol{S}$ matrix.

If any of the top-down approaches were used then

$$\boldsymbol{G} = \begin{bmatrix} p_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The first column includes the set of proportions that distribute the base forecasts of the top-level to the bottom-level. These are then summed up the hierarchy by the $\boldsymbol{S}$ matrix. The rest of the columns zero out the base forecasts below the highest level of aggregation.

For a middle out approach, the $\boldsymbol{G}$ matrix will be a combination of the above two. Using a set of proportions, the base forecasts of some pre-chosen level will be disaggregated to the bottom-level, all other base forecasts will be zeroed out, and the bottom-level forecasts will then summed up the hierarchy via the summing matrix.

Forecast reconciliation

We can rewrite Equation (10.6) as

$$\tilde{\boldsymbol{y}}_h = \boldsymbol{P}\hat{\boldsymbol{y}}_h, \tag{10.7}$$

where $\boldsymbol{P} = \boldsymbol{S}\boldsymbol{G}$ is a "projection" or a "reconciliation matrix." That is, it takes the incoherent base forecasts $\hat{\boldsymbol{y}}_h$, and reconciles them to produce coherent forecasts $\tilde{\boldsymbol{y}}_h$.

In the methods discussed so far, no real reconciliation has been done because the methods have been based on forecasts from a single level of the aggregation structure, which have either been aggregated or disaggregated to obtain forecasts at all other levels. However, in general, we could use other $\boldsymbol{G}$ matrices, and then $\boldsymbol{P}$ will be combining and reconciling all the base forecasts in order to produce coherent forecasts.

In fact, we can find the optimal $\boldsymbol{G}$ matrix to give the most accurate reconciled forecasts.