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# Fast Prediction of Equivalent Model of Installed Patch Antenna Radiation Pattern

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**Abstract**—This paper proposes an efficient equivalent model of patch antenna for the fast prediction of its installed radiation pattern. More CPU time and memory cost are required for accurate prediction of installed radiation pattern of patch antenna on different platforms. However, a fast and efficient prediction can save CPU time and memory cost when constructing an equivalent model of patch antenna that can reproduce a similar radiation pattern to that of the patch antenna. A code is developed to determine the electric field of a magnetic dipole based on Green function derivation. The result of the radiation pattern for the far-field and near-field are computed and validated with the result using commercial software tool (FEKO). The magnetic dipole is used to construct the equivalent model of patch antenna based on the radiation mechanism to predict its installed radiation pattern. The numbers of design parameters needed to be optimized are reduced to only two parameters which are the spacing distance between the dipoles in the  $x$ - and  $y$ -directions. The height of the dipole is kept at a fixed value above the same ground plane as that of the patch antenna. This makes it more computational efficient by reducing the CPU time and memory cost. After the equivalent model is optimized with FEKO optimization tool, it is further installed on a platform to compute the installed radiation pattern. The simulation results show that the proposed equivalent model based on a magnetic dipole with only two design parameters can obtain a fast prediction of installed radiation pattern of patch antenna when mounted on a platform. The equivalent model does not require detailed geometry and material information of the patch antenna.

**Keywords**— Equivalent model, magnetic dipole, optimization, patch antenna, radiation pattern.

## I. INTRODUCTION

The applications of patch antennas are expanding across various sectors and therefore adequate and efficient techniques are necessary and important to perform accurate predictions of electromagnetic devices for the development of modern electronics and telecommunication systems [1]-[5]. Due to the low profile characteristics of patch antennas, they are widely used on wearable devices, biomedical devices satellites and aircraft [6]-[10]. Different platforms on which a patch antenna is seated on, considerably changes its radiation pattern [11]-[15]. Therefore, it is important to examine the performance of the installed radiation pattern of

the patch antenna. However, the size of the general problem makes it cumbersome to carry out such examination. To carry out an effective examination of patch antenna on different large platforms, a comprehensive electromagnetic method can be considered as a technique for constructing a problem equivalent to the actual antenna under test (AUT) [16]-[19]. The model should match with the physical problem by generating similar electromagnetic fields when the AUT and its equivalent model are interacting with the same environment [20]-[23]. However, equivalent models based on elementary sources of infinitesimal dipoles have been successfully applied to predict both the far and near field in any direction [24]-[28]. The methods stated presume that the geometry information of a patch antenna is known. More so, geometry information of antennas is barely provided by dealers except for the antenna performance [29]-[32]. Optimization method was employed to construct a fast prediction equivalent model of the patch antenna based on a magnetic dipole in this paper. Simulation results show that the radiation pattern of the equivalent model matches optimally with those of the patch antenna. The rest of this paper is organized as follows. The theory and formulation is detailed in section II. Numerical results are presented in section III. Optimization method is detailed in section IV. Optimization workflow is presented in section V. Simulation results are presented in section VI to illustrate the advantages of the proposed method. Patch antenna on a car is presented in section VII to show the radiation pattern of both the patch antenna and the equivalent model. The comparison efficiency is detailed in section VIII to illustrate the efficiency of the proposed model. Finally, the work enumerated in this paper is concluded in section IX.

## II. THEORY AND FORMULATION

Numerical Green's function is widely used in E-field radiation calculation due to its contribution to the impulse response of antennas. The expression is given below;

$$\vec{E}(\vec{r}) = -\nabla \times \iint M(\vec{r}') G(\vec{r}, \vec{r}') \partial r' \quad (1)$$

The green's function is defined below

$$G(\vec{r}, \vec{r}') = \nabla \left( \frac{e^{-jk_r}}{4\pi r} \right) = -\frac{\partial}{\partial r} \left( \frac{e^{-jk_r}}{4\pi r} \right) \cdot \frac{\partial r}{\partial x} \quad (2)$$

$M(\vec{r})$  is the magnetic field, Hence, the E-field is given as;

$$G(\vec{r}, \vec{r}') \times M(\vec{r}') = \begin{bmatrix} a_x & a_y & a_z \\ G_x & G_y & G_z \\ M_x & M_y & M_z \end{bmatrix} \quad (3)$$

$M(\vec{r}')$  is the magnetic current source while  $G(\vec{r}, \vec{r}')$  is given as the Green's function which denotes the impulse response and  $\vec{E}(\vec{r})$  is given as the E-field of the magnetic dipole. It's difficult to analyze radiation pattern of patch antennas [33]-[36]. Various methods have been proposed to predict radiation pattern of path antennas [37]-[39]. A recent method was proposed in a prediction based equivalent model using electric dipole with three optimized design parameters [40]. In this paper, magnetic dipole was used with only two design parameters needed to be optimized (pertaining to the position spacing between the dipoles in the  $x$ - and  $y$ -directions) as an advantage over the previous method which makes it more computational efficient than those equivalent dipole model. Considering a dipole that is placed very close to a PEC finite sized ground plane, it does not radiate effectively along the plane comprising the PEC. To get rid of this effect, this paper proposed to place the dipole at a reasonable distance of 4 times the thickness of the substrate from the ground plane to obtain a good radiation match. This minimizes the negative effect of the PEC ground plane.

#### A. Initial Patch Antenna Model

This paper considered a rectangular patch antenna for simplicity sake. Table 1 shows the geometry and material details of the patch antenna. The perfect electric contacting patch had a dimension of  $W_p \times L_p$ . It was on a thin substrate of dimensions  $W_s \times L_s$ . The thickness and relative permittivity of the substrate are denoted by  $H_s$  and  $\epsilon_r$ , respectively. The antenna was probe fed at a point  $W/2$  and  $F$  away from the left and lower end of the patch antenna. A three-dimensional radiation pattern is shown in Figure 2. However, the goal of this research is to construct a fast prediction model that will generate similar radiation pattern to that of the patch antenna.

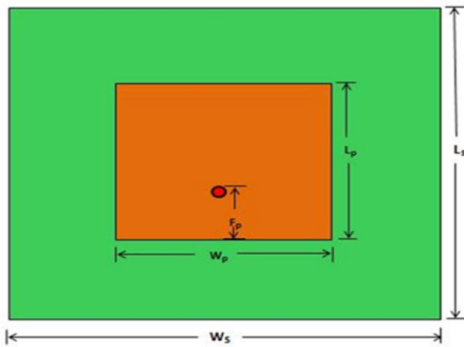


Fig. 1. Schematic of the patch antenna

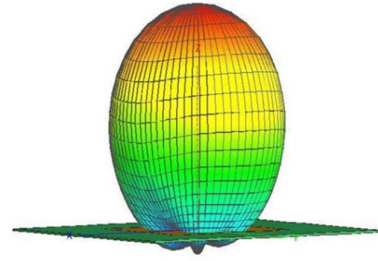


Fig. 2. Three-dimensional radiation pattern of the patch antenna

#### B. Initial Equivalent Model of Patch Antenna

The magnetic current model can generate similar radiation pattern to that of the patch antenna. However, to match the polarization, the magnetic dipoles were placed above the ground plane and aligned in horizontal direction. The size of the ground plane was equal to that of the patch antenna. 2X2 dipole array model employed as shown in figure 3. Adjusting the distance between the dipoles, the E-plane beam width can be changed. To further broaden the beam width, the positions of the dipoles were introduced as optimization parameters. The concept of the equivalent model is basically replacing the patch with a 2x2 magnetic dipole above the same ground plane arranged to be pointing in the  $x$ -direction. The height of the dipoles was place approximately four times the height of the substrate of the patch antenna above the same ground plane. The spacing parameters in both  $x$ - and  $y$ - directions were optimized to further obtain the value of best fit which is then used to determine the positions of the dipole array. Figure 4 shows the arrangement of the optimized parameters of the position spacing in both  $x$ - and  $y$ -directions.

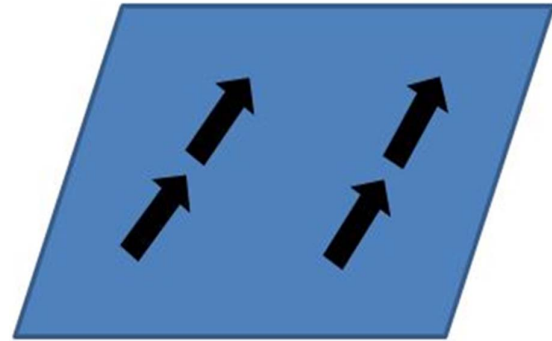


Fig. 3. 2X2 magnetic array dipole

### III. NUMERICAL RESULT

#### A. Array Antenna Radiation Pattern Validation

More so, the results of the magnetic point dipoles in both linear and planar arrays of magnetic dipoles are obtained below with a spacing of half lambda. The simulation is done on a 4-element magnetic dipole array. The dipoles are placed on the  $xy$ -cut planes with  $z = 0$ . Figure 4 shows the accuracy of the algorithm designed to calculate the far-field and generate the radiation pattern of the linear and planar magnetic dipole arrays respectively.

TABLE I. PARAMETERS OF THE PATCH ANTENNA

Patch(cm)	$L_p$	9.9
	$W_p$	11.858
Substrate (cm)	$L_s$	18
	$W_s$	20
	$H_s$	0.1588
Feeding (cm)	F	3.21
Permittivity	$\epsilon_r$	2.2
HPBW(°)	E-	80.2
	H-	74.4
Directivity (dBi)	D	6.75

#### IV. OPTIMIZATION METHOD

According to the proposed initial model, the positions spacing between the dipoles in the x- and y-directions were the only two parameters needed to be optimized. The aim is to determine the optimal values of the parameters in order to lessen the differences between the generated and the desired radiation pattern. To build an equivalent model of the patch antenna with optimization technique, the FEKO optimization tool was employed. The optimization goal was defined by feeding the optimizer with the radiation pattern of the patch antenna which is termed mask. The variables to be optimized were defined. The minimum and maximum values of the variables were chosen as well as the start value. The optimization process performs the optimization search to find the line of best fit by match the values of the equivalent model with that of the patch antenna at every angle in order to obtain an approximate match.

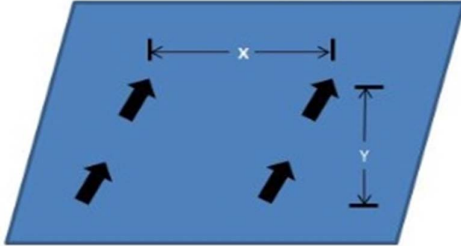


Fig. 4. Arrangement of magnetic dipoles after optimization

#### V. OPTIMIZATION WORKFLOW

In each optimization search, far-field goals were defined to determine the state the optimization process should attempt to achieve. The far-field goal makes provision for optimization relating to all far-field quantities computed as part of the FEKO solution. The focus is identified based on the label of a far-field request in CADFEKO. Figure 5 shows the flowchart of FEKO optimization adopted in this paper.

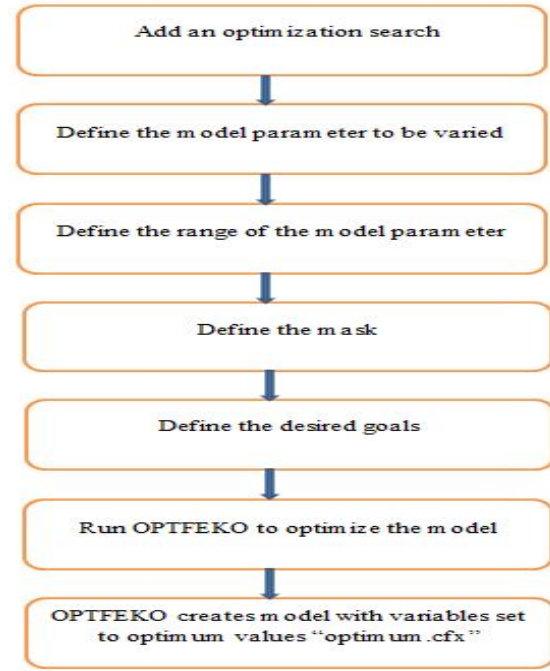


Fig. 5. Optimization workflow in CADFEKO

#### VI. SIMULATION RESULTS

The simulation results show the effectiveness and advantages of the proposed equivalent model based on a magnetic dipole. The radiation pattern of the patch antenna was first generated by FEKO and was further used to construct the equivalent model via optimization technique. The derived equivalent model was finally validated by matching its radiation pattern with that of the patch antenna. The parameters are described in Table II with the operating frequency of 1GHz. The arrangement of the optimized parameters is described in figure 4 above. Good agreement could be observed between the E- and H-plane radiation pattern of the patch antenna and its equivalent model in the forward radiation of the upper hemisphere in figure 6 and 7. It was observed from the result that an optimum width of the backward radiation was obtained at the predetermined height position of the dipoles. However, the discrepancy in the backward radiation of the lower hemisphere can be attributed to the height of the dipole above the ground plane. This effect was not taken into account because the height is a fixed parameter.

TABLE II. PARAMETERS OF THE EQUIVALENT MODEL FOR THE PATCH ANTENNA

Predefined Parameters	Dipole(cm)	Length	1.874
		Height	0.6
	GND Plane(cm)	$L_s$	18
Optimized Parameters	Distance(cm)	$W_s$	20
		X	5.00
		Y	4.50
Radiating Performance	HPBW(°)	E-	80.2
		H-	74.3
	Directivity(dBi)	D	6.75

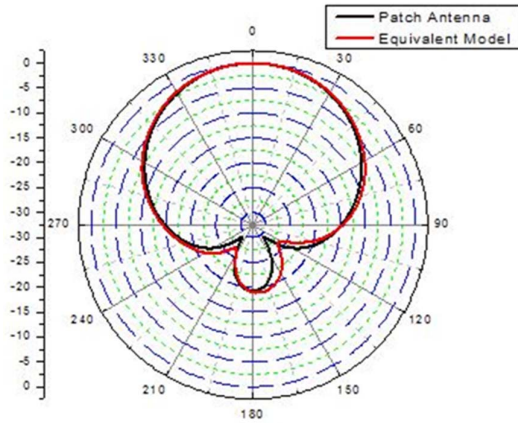


Fig. 6. Normalized radiation pattern of patch antenna and its equivalent model for E-plane

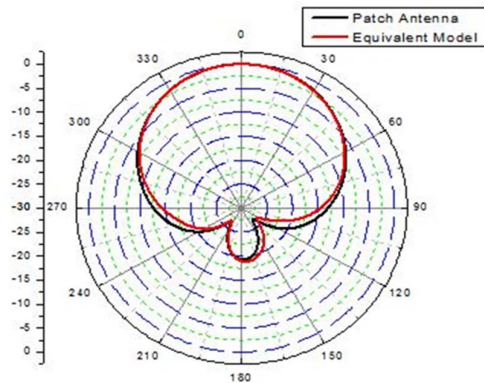


Fig. 7. Normalized radiation pattern of patch antenna and its equivalent model for H-plane

The simulation results show good agreement especially for the forward radiation on the upper direction. Although there is some difference in the backward radiation, the results are still satisfactory in view of the low radiation level.

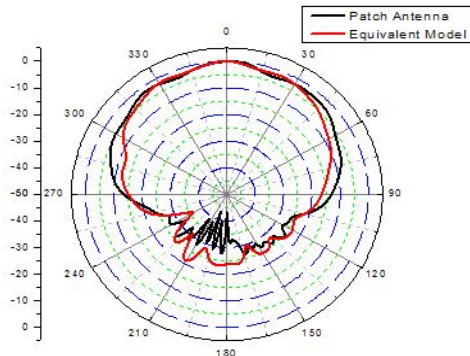


Fig. 10. The normalized radiation pattern of the patch antenna and its equivalent model on a PEC platform of a car model on the E-plane.

## VII. PATCH ANTENNA ON A CAR

The equivalent model was mounted on a car model to further illustrate its efficiency. The placement is shown in Figure 8 and figure 9. The roof of the car is a PEC with a dimension of 40cm x 64cm (operating at 1GHz frequency). The patch antenna was installed on the same car model.

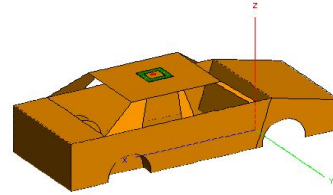


Fig. 8. The placement of the patch antenna on a car model

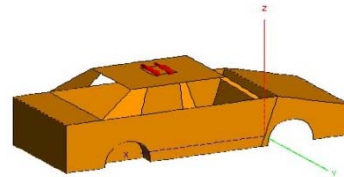


Fig. 9. The placement of the equivalent model on a car model

The radiation pattern of the patch antenna and its equivalent model were compared as shown in figure 10 and 11. Hence, the flat equivalent model is able to represent the flat patch antenna on flat platforms. The proposed equivalent model of patch antenna based on a magnetic dipole is an efficient way to obtain a fast prediction of installed radiation patterns of patch antennas. Two design parameters of the equivalent model are optimized to match the radiation pattern of the patch antenna. The key feature of the proposed model is the implementation of magnetic dipole to construct equivalent model of patch antenna and the reduced number of design parameters needed to be optimized. The important idea of the proposed model is the fast prediction of radiation pattern of patch antenna and its installed radiation pattern on large platforms

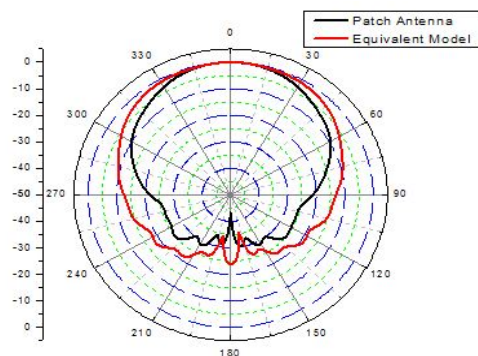


Fig. 11. The normalized radiation pattern of the patch antenna and its equivalent model on a PEC platform of a car model on the H-plane.

## VIII. COMPARISON EFFICIENCY

Summary of the CPU time and memory cost is presented in table 4.2 on the simulation of the original model and proposed model including the installed patch antenna. It is observed that both the CPU time and memory cost is considerable reduced using the equivalent model. However, the complex geometry of the patch antenna may sometimes lead to ill-conditioned matrix during direct modeling. This ill-condition usually increases the memory cost. The equivalent model reduces memory cost and saves time due to its simplicity in geometry and PEC material information. From table 3, it is observed that the number of unknowns in the equivalent model is much less than the others because the equivalent model does not require detailed design geometry and material information of the patch antenna. However, lesser number of unknowns to solve definitely requires lesser CPU time and memory cost as presented in table III below.

TABLE III. MEMORY COST AND CPU TIME BY DIFFERENT METHODS

Antenna Model	Number of Unknown	CPU Time(S)	Memory Cost
Original patch	8,422	21.310	261.189 MB
Equivalent model	237	1.669	515.734 KB
Patch on car	30,417	56.784	156.442 MB
Equivalent model on car	11,736	34.634	32.455 MB

## IX. CONCLUSION AND FUTURE WORK

An equivalent model of patch antenna has been presented. Magnetic dipole has been implemented to construct the equivalent model of patch antenna. Optimization has been performed by the FEKO optimization tool. A fast prediction of installed radiation pattern of patch antenna has been achieved. The following conclusion can be drawn from the advantages of the proposed model. Only two design parameters have been optimized which makes it more computational efficient than the previous methods. The proposed equivalent model method provides an efficient way for fast prediction of radiation pattern of patch antenna and its installed radiation pattern when mounted on a platform compute its installed radiation pattern. For better accuracy and to obtain good agreement in both forward and backward radiation, the height of the dipole above the ground plane could be considered as a design parameter to be optimized especially in the backward radiation.

The above proposed model has a huge application in biomedical and wearable devices. The derived equivalent model based on a magnetic dipole can be mounted on different platforms to compute and predict its installed radiation pattern which will be considered in future work.

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