

8. $Y = X^2$

$$F_Y(y) = P(X < y) = P(X^2 < y) = P(-\sqrt{y} < X < \sqrt{y}) = P(X < \sqrt{y}) - P(X < -\sqrt{y})$$
$$= F_X(\sqrt{y}) - F_X(-\sqrt{y})$$

$$f_Y(y) = (F_X(\sqrt{y}) - F_X(-\sqrt{y}))' = f_X(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} - f_X(-\sqrt{y}) \left(-\frac{1}{2\sqrt{y}}\right) =$$
$$= \frac{f_X(\sqrt{y}) + f_X(-\sqrt{y})}{2\sqrt{y}}$$

□