$$n = c^k$$

$$T(n) = a\,T(n/c) + bn = a\,T(c^{k-1}) + bn = a\left(a\,T(c^{k-2}) + bc^{k-1}\right) + bc^k$$

$$= a^2\,T(c^{k-2}) + abc^{k-1} + bc^k =$$

$$= a^2\left(a\,T(c^{k-3}) + bc^{k-2}\right) + abc^{k-1} + bc^k =$$

$$= a^3\,T(c^{k-3}) + a^2bc^{k-2} + abc^{k-1} + bc^k$$

$$= \dots$$

$$= a^k\,T(c^0) + a^{k-1}bc^1 + a^{k-2}bc^2 + \dots + abc^{k-1} + bc^k$$

$$= a^k \cdot b \cdot c^0 + a^{k-1}bc^1 + a^{k-2}bc^2 + \dots + abc^{k-1} + bc^k$$

$$= b\sum_{i=0}^{k} c^i a^{k-i} = bc^k \sum_{i=0}^{k} \frac{a^{k-i}}{c^{k-i}} = bc^k \sum_{i=0}^{k} \left(\frac{a}{c}\right)^i$$

$a < c \quad \Rightarrow \quad \frac{a}{c} < 1$ i szereg $\sum_{i=0}^{\infty} \left(\frac{a}{c}\right)^i$ jest zbieżny, czyli $O(1)$

$$T(n) = bn\,O(1) = O(n)$$

$a = c \qquad T(n) = bn \sum_{i=0}^{k} 1 = bn \sum_{i=0}^{\log_c n} 1 = bn \log_c n =$

$$= O(n \log n)$$

$a > c \qquad T(n) = bn \sum_{i=0}^{k} \left(\frac{a}{c}\right)^i = bn \dfrac{\left(\frac{a}{c}\right)^{1+\log_c n} - 1}{\frac{a}{c} - 1}$

$$= O\left(\frac{b}{\frac{a}{c}-1} \cdot n \cdot \left(\frac{a}{c}\right)^{\log_c n}\right)$$

$$= O\left(n \cdot \frac{a^{\log_c n}}{c^{\log_c n}}\right) = O\left(n \frac{a^{\log_c n}}{n}\right)$$

$$= O\left(a^{\log_c n}\right)$$

$$a^{\log_c n} = \left(c^{\log_c a}\right)^{\log_c n} = \left(c^{\log_c n}\right)^{\log_c a} = n^{\log_c a}$$