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Original Article

Implementation of Strength Pareto Evolutionary Algorithm II in the Multiobjective Burnable Poison Placement Optimization of KWU Pressurized Water Reactor

Rahman Gharari [a], *Navid Poursalehi* [b,*], *Mohammadreza Abbasi* [b], *and Mahdi Aghaie* [b]

[a] *Nuclear Science and Technology Research Institute (NSTRI), P.O. Box 14395-836, North Kargar Street, Tehran, Iran*
[b] *Nuclear Engineering Department, Shahid Beheshti University, G.C., P.O. Box 1983963113, Evin, Tehran, Iran*

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ABSTRACT

In this research, for the first time, a new optimization method, i.e., strength Pareto evolutionary algorithm II (SPEA-II), is developed for the burnable poison placement (BPP) optimization of a nuclear reactor core. In the BPP problem, an optimized placement map of fuel assemblies with burnable poison is searched for a given core loading pattern according to defined objectives. In this work, SPEA-II coupled with a nodal expansion code is used for solving the BPP problem of Kraftwerk Union AG (KWU) pressurized water reactor. Our optimization goal for the BPP is to achieve a greater multiplication factor (K_{eff}) for gaining possible longer operation cycles along with more flattening of fuel assembly relative power distribution, considering a safety constraint on the radial power peaking factor. For appraising the proposed methodology, the basic approach, i.e., SPEA, is also developed in order to compare obtained results. In general, results reveal the acceptance performance and high strength of SPEA, particularly its new version, i.e., SPEA-II, in achieving a semi-optimized loading pattern for the BPP optimization of KWU pressurized water reactor.

1. Introduction

For a commercial nuclear power plant, maximization of benefits is an important goal to improve the economics while satisfying the safety constraints such as maximum burn-up, radial power peaking factor (PPF), and moderator temperature coefficient [1]. The development of a burnable poison (BP) loading map for a core is also one of the optimization problems in the nuclear engineering field. The main problem in the determination of the optimized BP position in a reactor core is

* *Corresponding author.*
E-mail address: n_poursalehi@sbu.ac.ir (N. Poursalehi).

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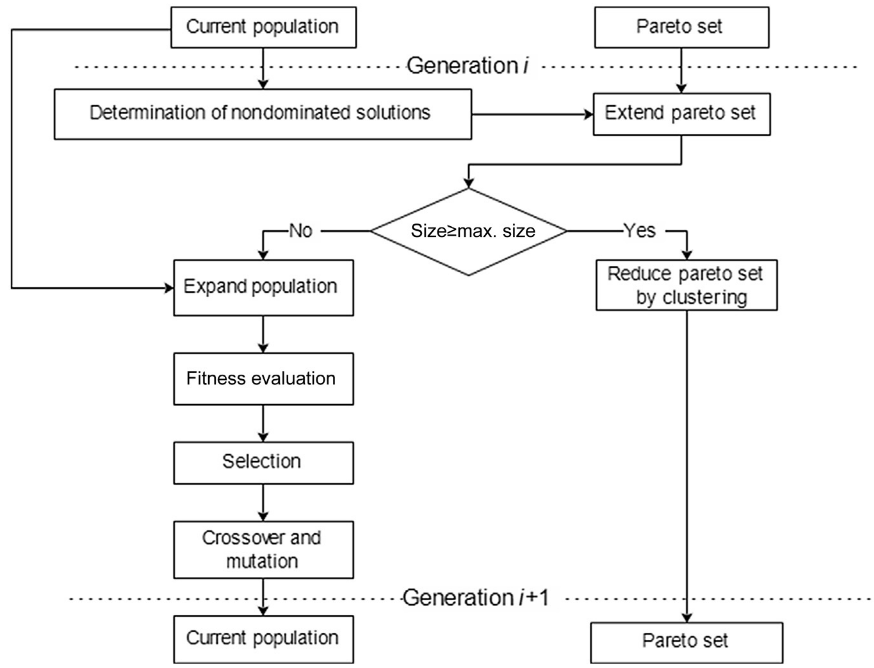

Fig. 1 – Simple schematic of SPEA. SPEA, strength Pareto evolutionary algorithm.

the large number of feasible combinations for the loading of various fuel assemblies (FAs) including BP in the core. In addition, this is a multiobjective optimization problem that creates complications in the utilization of conventional optimization techniques.

A burnable absorber is commonly used in pressurized water reactors (PWRs) and boiling water reactors to control core reactivity and enhance fuel performance. It captures neutrons during its burnout cycle and thus reduces the reactivity of an FA for a certain length of time that is proportional to the rate at which the primary absorbing isotopes are transformed to other isotopes by absorption and decay. This rate is obviously dependent on the cross sections of each of the highly absorbent isotopes. Being able to absorb more neutrons at the beginning of a cycle allows loading, e.g., more ^{235}U in an FA, so that the FA's cycle time can be enhanced [2]. Thus, the BP has several desirable properties such as being able to increase the core lifetime without any abatement in the control safety and also improving FA power distribution in the core by suppressing the reactivity in high-flux regions.

The burnable poison placement (BPP) optimization problem has been investigated earlier by some researchers. For example, Yilmaz et al. [3] applied a genetic algorithm to optimize BPP in PWRs. Rogers et al. [4] implemented the annealing algorithm in optimizing the radial enrichment of PWR FA and BP location, and also Alim et al. [5] simultaneously optimized the core loading pattern and the BPP in a PWR by developing an innovative genetic algorithm.

In multiobjective evolutionary algorithms, the optimal solution set is found by coordinating the relationship of the objectives in a fitness function. Strength Pareto evolutionary algorithm (SPEA) is a well-known evolutionary algorithm for multiobjective evolutionary algorithms in the past few years and it is a very efficient algorithm in the development of multiobjective evolutionary algorithms. In addition, SPEA was among the first approaches that were likened extensively to several existing evolution-based techniques [6]. As it clearly outperformed the alternative approaches under consideration, it has been exploited as a point of reference by various researchers [7–9].

SPEA-II is the improved version of SPEA, which was proposed by Zitzler et al. [10]. It can obtain orderly distributed Pareto solution by truncation and managing the archive set. SPEA-II is a multiobjective optimization algorithm, which has few configuration parameters, fast converging speed, good strength, and orderly distributed solution sets. In addition, it has been used for various domains of multiobjective design in both academic and industrial fields.

In the current work, SPEA-II is developed and applied to the multiobjective optimization of BPP design, based on flattening of the FA radial power distribution in the core along with maximization of the effective multiplication factor (K_{eff}). In order to evaluate the performance of the proposed methodology, we provided a calculator package for the BPP optimization of a nuclear reactor core using the SPEA-II module for the optimization process and the nodal expansion code for neutronic calculation. The neutronic module, which solves the two-dimensional (2D) multigroup neutron diffusion equation exploiting the second-order nodal expansion method, has been verified earlier [11,12]. Moreover, the primary method, i.e., SPEA, has also been developed in order to compare the numerical results with SPEA-II for the BPP optimization of KWU PWR core as a test case. In regard to the results obtained by SPEA and SPEA-II, SPEA-II appears to function better in comparison with the basic SPEA for the BPP design of KWU PWR.

2. Strength Pareto evolutionary algorithm

SPEA and SPEA-II have been presented in 1999 and 2001 by Zitzler and Thiele [13] and Zitzler et al. [10], respectively. As SPEA forms the basis for SPEA-II, a brief summary of the SPEA is given here. For obtaining more detailed explanation, the interested reader is referred to [13]. SPEA is a multiple objective optimization algorithm, and it also belongs to the field of evolutionary multiple objective algorithms. SPEA is an extension of the original genetic algorithm [3], for multiple objective optimization problems. Strength Pareto has an important role in SPEA because this shows how solutions close to the first rank. The objective of the algorithm is to identify and preserve a set of nondominated solutions, ideally a set of Pareto optimal solutions. All the Pareto optimal solutions are called the Pareto optimal set. The Pareto optimal set is made up of the best nondominated solutions in the objective space. Two main parameters are considered for each solution: the first case is the strength Pareto that is defined as follows:

$$S(i) = \frac{n}{N+1} \quad (1)$$

where N is the population size and n the number of individuals (each individual represents a solution vector) that are dominated by or equal to individual i. Therefore, dominated solutions are the responses that have lower strength than others, according to Eq. (1). Moreover, the second parameter is the fitness value for individual j with the following definition [14]:

$$F(j) = 1 + \sum_{i \prec j} S(i) \quad (2)$$

According to Eq. (2), $F(j)$ of individual j in the population is calculated using the summation of the strength values $S(i)$ of all external population members that dominate ($\prec$) or are equal to individual j. This method of fitness assignment proposes that a solution with lower fitness is better [14].

Now, we can note the overall steps of SPEA, which are as follows. Definitions: T is the maximum number of generations, t is the iteration number, and the external set is a set of Pareto optimal solutions. Note that the solutions are stored externally and also upgraded continuously. Finally, the solutions stored in this set represent the Pareto optimal set.

(1) Initialization: the first population is generated and the empty external Pareto optimal set is created [14]. (2) The Pareto optimal set is updated. If the size of the Pareto optimal set exceeds a predefined limit, further Pareto sets are deleted by a clustering technique. An average linkage-based hierarchical clustering algorithm is employed to decrease the Pareto set to a manageable size. It performs iteratively by combining the adjacent clusters until the required number of groups is achieved [14]. (3) The fitness values are calculated for the external Pareto optimal set and population [6]. (4) Binary tournament selection: the population and the external set individuals are joined, and any two individuals at random are chosen. With respect to their fitness function, the better one is moved to the mating pool. Mating pool is a set of populations that cross over, and mutation operations are performed on them in order to produce a new population [6]. (5) A new population is produced by mutation and crossover operations. (6) Set $t = t + 1$. If the stopping criterion ($t > T$) is not satisfied, go to Step 2; or else the archive members are presented as a Pareto optimal set [6].

According to the noted steps, the SPEA process for the i^{th} generation is illustrated in Fig. 1 [14]. In the next section, we will address these issues in more detail and describe the improved algorithm, which is called SPEA-II.

Table 1 – Parameters used in SPEA and SPEA-II.

Population size	100
Probability of crossover	0.8
Probability of mutation	0.2
Type of selection	Binary tournament selection
γ-Crossover	0.1
h-mutation	0.2

SPEA, strength Pareto evolutionary algorithm.

3. SPEA-II

Similar to the steps presented in Section 2 for the basic SPEA, details of the improved algorithm, i.e., SPEA-II, are given in the following paragraphs. Definitions: P_t (main population at iteration t), $\overline{P_t}$ (archive population at iteration t), $\overline{N}$(archive size), and T (maximum number of generations).

(1) Initialization: set $t = 0$ and generate the initial population P_0 and empty archive $\overline{P_t} = \varphi$ [10]. (2) Calculate the fitness values for individuals of P_t and $\overline{P_t}$ (both population and archive sets) [10].

For each individual i in the archive $\overline{P_t}$ and population P_t, the strength value $S(i)$ is calculated using the following equation:

$$S(i) = \left|\left\{ j \middle| j \in p_t + \overline{p_t} \wedge i \succ j \right\}\right| \quad (3)$$

where the symbol $+$ stands for multiset union, the symbol $\succ$ corresponds to the Pareto dominance relation, and the symbol $\wedge$ means AND [10].

For SPEA-II, fitness $F(i)$ is defined as follows:

$$F(i) = R(i) + D(i) \quad (4)$$

The raw fitness$R(i)$ of an individual i is calculated by the following equation:

$$R(i) = \sum_{j \in p_t + \overline{p_t}, j \succ i} S(j) \quad (5)$$

However, if the optimization goal is a minimized search of $F(i)$, the raw fitness should be minimized here, i.e., $R(i) = 0$, which corresponds to a nondominated individual [15].

The individual's density for distinguishing those with the same raw fitness values is calculated by the K-nearest neighbor method, using the following equation:

$$D(i) = \frac{1}{\sigma_i^k + 2} \quad (6)$$

where σ_i^k represents the objective–space distance between the i^{th} and k^{th} nearest neighbors, and we also have$k = \sqrt{N + \overline{N}}$ in Eq. (6) [10].

Table 2 – Characteristics of Ackley function.

Problem	Objective function	Variable bound	f_{min}
Ackley	$-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)+20+\exp(1)$	[−32.768,32.768]	0

(1)Selection: duplicate all the nondominated solutions in both P_t and $\overline{P_t}$ to $\overline{P}_{t+1}$. If the size of$\overline{P}_{t+1}$ exceeds $\overline{N}$, reduce $\overline{P}_{t+1}$ using the truncation operator, i.e.,

$$i\leq_d j,\quad j\in\overline{P}_{t+1}\rightarrow \begin{array}{l} : j\in\overline{P}_{t+1} : \Leftrightarrow \forall\ 0<k<\overline{P}_{t+1} : \sigma_i^k=\sigma_j^k \vee \\ \exists\ 0<k<\overline{P}_{t+1} : \left[\left(\forall 0<l<k : \sigma_i^l=\sigma_j^l\right)\wedge \sigma_i^k<\sigma_j^k\right] \end{array} \tag{7}$$

Otherwise, fill $\overline{P}_{t+1}$ with the dominated individuals in P_t and $\overline{P_t}$. In Eq. (7), i and j are individuals, and also$i\leq_d j$ means that individual i dominated individual j [15].

(2) If the stopping criterion $t \geq T$ is not satisfied, go to Step 2; or else, the archive members are presented as a Pareto optimal set [10]. (3) Create the new population: individuals are selected from $\overline{P}_{t+1}$ by means of binary tournament selection (creating mating pool) [10]. (3) Mutation and crossover operators are applied to the mating pool, and at last the population P_{t+1} are created. Set $t = t + 1$ and go back to Step 2 [10].

3.1. *Selection of parameters and validation of SPEA-II*

In SPEA-II, the user should specify several internal parameters. In addition, solutions slightly depend on initial conditions because of the random nature of the metaheuristic algorithms. However, after carrying out multiple runs of developed approaches for the best performance, parameters have been selected for exploited optimization algorithms, which are given in Table 1.

However, as the investigation space of fuel management optimization for a reactor core is not known earlier, it is necessary to validate SPEA-II. For this purpose, we use the Ackley test function [16]. Table 2 summarizes the information of this test function. The three-dimensional (3D) view of the Ackley function is shown in Fig. 2. In this work, the Ackley function is implemented in two dimensions (i.e., $n = 2$), and the global optimum is located at (0, 0). Table 3 displays the best results of SPEA-II for 10 independent runs; it presents 10 values obtained by SPEA-II in each run along with the used iteration number. SPEA is also developed for the Ackley test function in order to compare its results with SPEA-II; the results of both methods are given in Table 3. According to Table 3, results indicate that both SPEA and SPEA-II can successfully reach the minimum value in all runs, particularly SPEA-II, by earning less (better) fitness relative to the basic SPEA during 30 iterations. Lastly, the optimization process of the Ackley test function using SPEA-II versus iteration is demonstrated in Fig. 3 for 10 successive runs. From Fig. 3, we can find fast convergence of the proposed algorithm for obtaining the optimum (minimum) value of the considered test function.

4. Application of SPEA-II to KWU PWR BPP optimization problem

In this work, an enhanced optimization approach, i.e., SPEA-II, is used and implemented in the BPP problem of KWU PWR core. For this purpose, a Fortran program has been developed with two sections including the nodal expansion module [11,12], for calculating necessary neutronic parameters and also the optimization module exploiting the proposed SPEA-II approach. It is noted that the nodal expansion code solves the multigroup 2D neutron diffusion equation by coarse meshes with dimensions of an FA, i.e., 23 cm × 23 cm, for the KWU PWR problem. Moreover, we should mention that the burn-up along the operation cycle cannot be performed because the exploited nodal expansion code cannot produce cross sections for fission products. Therefore, the BPP optimization process

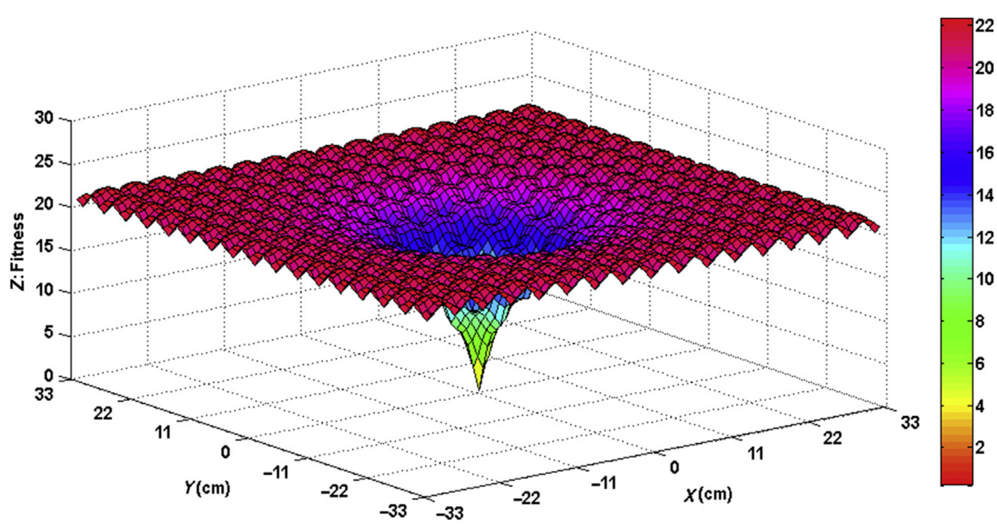

Fig. 2 – Ackley function.

Table 3 – Ackley function optimization results of SPEA and SPEA-II for 10 experiments.

Run	SPEA	SPEA-II
1	0.0001903	0.0000232
2	0.0003623	0.0000249
3	0.0001840	0.0000193
4	0.0000918	0.0000181
5	0.0008756	0.0000150
6	0.0005432	0.0000133
7	0.0000989	0.0000115
8	0.0003547	0.0000103
9	0.0005290	0.0000192
10	0.0001409	0.0000180
Best result	0.0000918	0.0000103
Min. value (reference)	0.0000000	0.0000000
Mean	0.0003371	0.0000172
SD	0.0002518	0.0000069
Iteration	30	30

Min., minimum; SD, standard deviation; SPEA, strength Pareto evolutionary algorithm.

is executed for the beginning of the cycle, and it is also performed for the dislocation of FAs including BP rods. As a result, the optimization process is not in the FA level of BP location and it is not in the scope of this work.

4.1. *Reactor core description*

Our test case is a PWR core type, i.e., Bushehr (old KWU design) with octant symmetry of the core [17,18]. This core has two fuel kinds including FAs, both with and without BP. FAs of KWU PWR have three different enrichments of 1.9%, 2.5%, and 3.2%. The full core aspect of KWU PWR is illustrated in Fig. 4.

According to Fig. 4, the core has six different kinds of FAs, with the number of each type being given in Table 4 for one-eighth of the core. In this study, one-eighth section of KWU PWR core is used for the BPP optimization. In addition, the original (primary) core map of KWU PWR is considered as the reference (designer) solution for comparing its numerical results with the BPP map obtained by SPEA-II. Fig. 5 illustrates the 2D and 3D schematic views of FA's relative power belonging to the KWU original loading pattern obtained by the nodal code.

4.2. *Procedure*

As noted before, we wanted to apply SPEA-II associated with the mentioned evolution operators to the BPP optimization of a reactor core. Here, the length of the solution vector is equal to the number of FAs that can be dislocated in the core. In our work, i.e., BPP optimization, FAs that include burnable absorber rods (i.e., FA Types 2, 4, and 6, according to Table 4) and also FA Types 1 and 5 are chosen for displacement in the one-eighth section of the KWU PWR core. Therefore, positions of FAs (Type 3) are fixed in the core according to the designer scheme shown in Fig. 4. Moreover, the positions of FAs that are located on two symmetry lines (see Fig. 4) also remain unchanged, according to Fig. 6, because the number of FA types should not be varied in the full core pattern and must be equal to the original scheme of KWU for comparison purposes. However, each element of a solution vector represents an FA position in the core; consequently, elements of the solution vector should be integer and nonrepeated numbers. In other words, the solution vector comprises integer and nonrepeated numbers from 1 to n; n equals the number of assemblies that have BP, as well as FA Types 1 and 5 (i.e., $n = 18$) in one-eighth of the core.

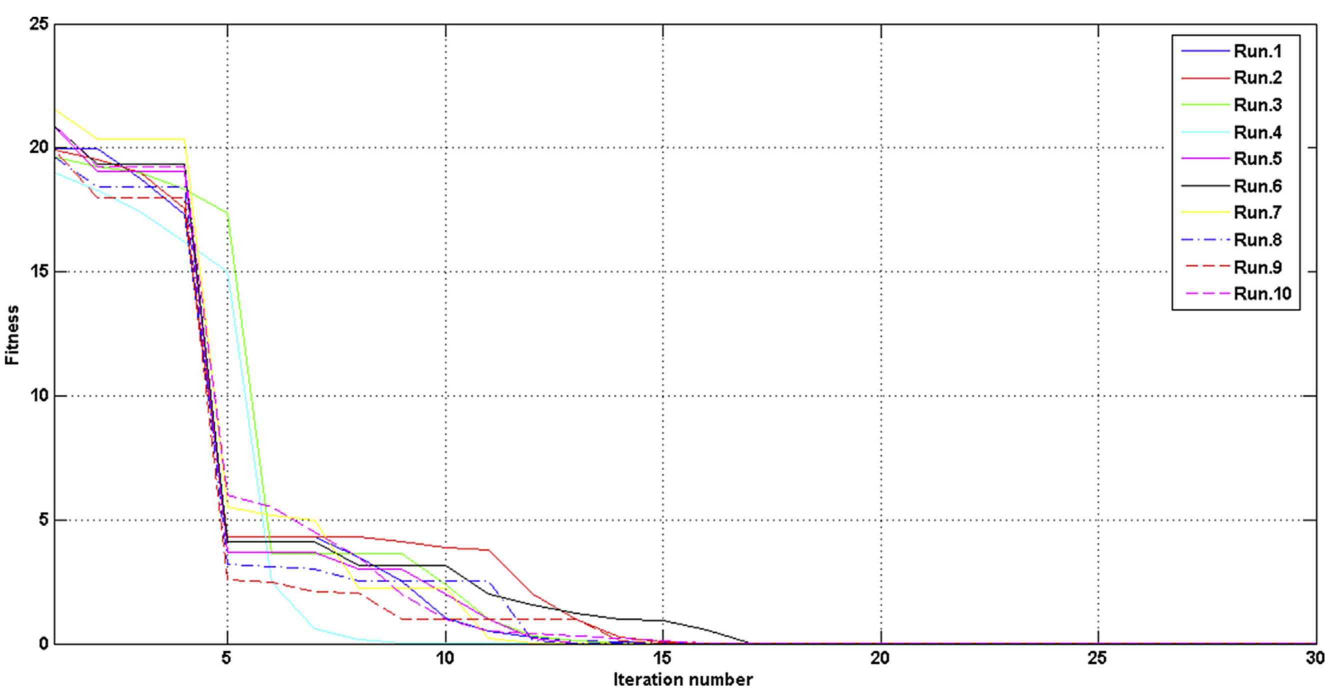

Fig. 3 – Ackley function optimal values versus the iteration number obtained by SPEA-II for 10 runs. SPEA, strength Pareto evolutionary algorithm.

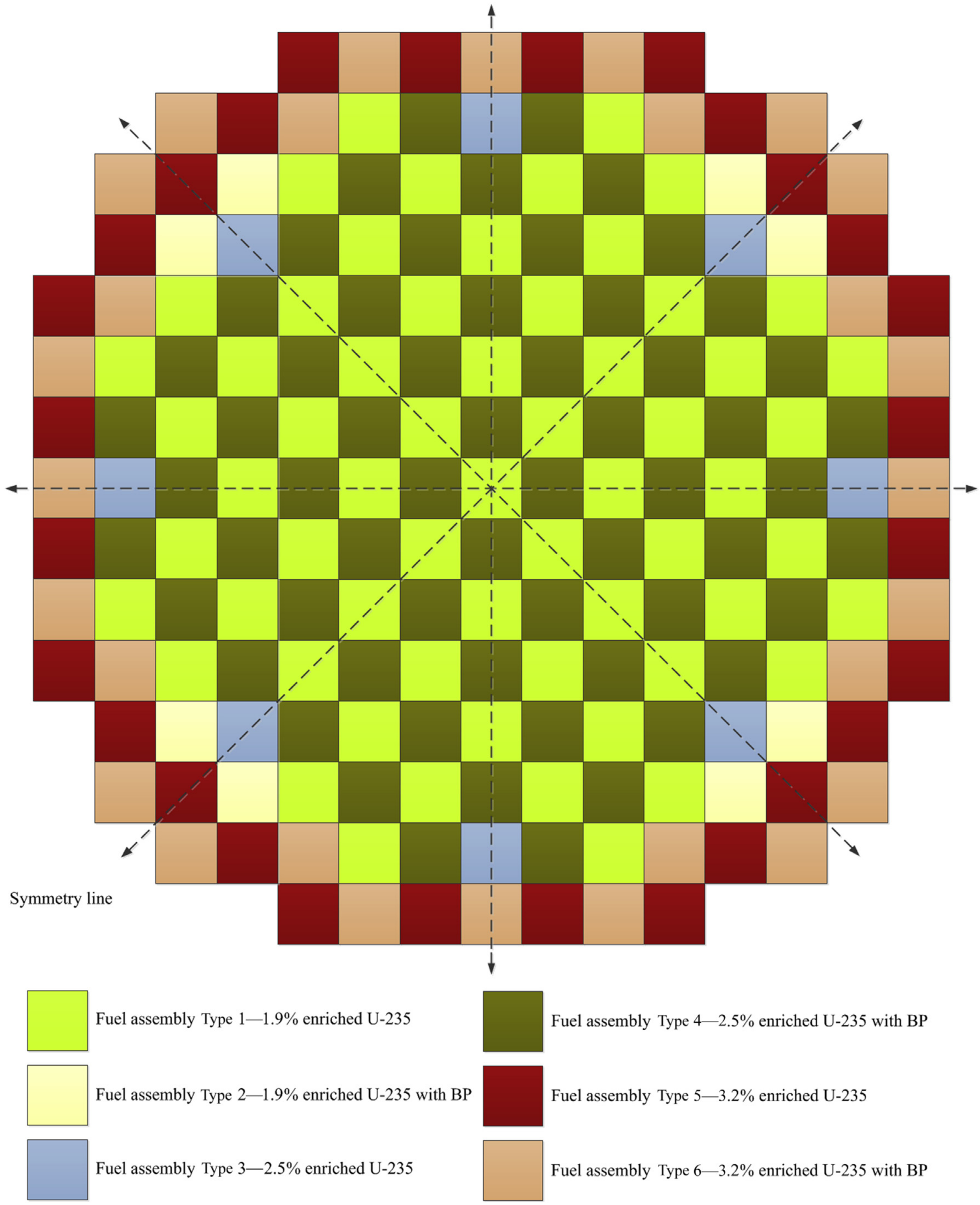

Fig. 4 – A 2D view of KWU PWR full core. BP, burnable poison; PWR, pressurized water reactor; 2D, two dimensional.

For clarifying the optimization process of the proposed method, initial solution vectors are generated, which contain random integer numbers from 1 to *n*. Then, each solution vector must be evaluated by calculating its fitness. Indeed, the solution vector should be converted to the corresponding loading pattern and in the following; the input file of nodal code is prepared for this created LP. The decoding steps of the solution vector to a loading pattern are illustrated in Fig. 6. In regard to Fig. 6, a material vector is introduced, which contains material (FA type) numbers 1, 2, 4, 5, and 6 (see Table 4) for eighth symmetry of the KWU PWR core. For example, Fig. 4 shows that one FA with material number 2 exists in the one-eighth symmetry of the core. As a result, the material vector in Fig. 6 must contain one material number (i.e., 2). Now, the formed solution vector is converted to the corresponding core pattern vector (see Fig. 6) according to the defined material

Table 4 – Various fuel assembly types used in one-eighth symmetry of KWU PWR.

Type	1	2	3	4	5	6
Enrichment (%)	1.9	1.9	2.5	2.5	3.2	3.2
BP	–	BP	–	BP	–	BP
Number	11	1	2	9	4	4

BP, burnable poison; PWR, pressurized water reactor.

vector. As shown in Fig. 6, the first element of the solution vector is 7 and the seventh element of the material vector presents number 4. Therefore, in the first element of the core pattern vector, number 4 is placed. For other elements of the core pattern vector, this process is performed with the abovementioned approach and continued until the last element. After producing the core pattern vector, the new loading pattern is standing in one-eighth of the core from the center to the periphery, element by element, with respect to the position number in a sequence, shown in Fig. 6, from 1 to n. From Fig. 6, it can be seen that FAs with fixed positions, such as FAs located on the symmetry lines (determined by user), do not have numbers. However, this generated loading pattern is replaced in the input file of the neutronic code and then the code is run for obtaining the required neutronic parameters. It should be noted that the core map illustrated in Fig. 6 is the same designer fuel arrangement as that of KWU PWR, which is produced from the solution vector given in Fig. 6 using the aforesaid approach. After obtaining the neutronic parameters for a given loading pattern, the fitness function (defined in the next section) is calculated. However, the aforementioned approach is used for decoding any solution vector into the corresponding loading pattern in any iteration of the optimization process. All the optimization steps for the BPP problem are represented in Fig. 7, according to Section 3 and the abovementioned process. Fig. 7 exhibits the relationship of the optimization process using SPEA-II with the reactor physics code such as the nodal expansion code.

4.3. *Multiobjective fitness function for the BPP optimization problem*

In this section, we define a multiobjective fitness function for the BPP optimization problem. One of our optimization objectives is maximizing the effective multiplication factor (K_{eff}) for a feasible longer operation cycle of the reactor core. Our other objective is to achieve more flattening of the FA relative power distribution for considering safety constraints. For attaining the first objective, i.e., maximization of K_{eff}, FAs, including BP, can be placed in the periphery of the core because the neutron flux is lower in the exterior of the core than in the center. As a result, the BP reactivity worth can further be decreased in this case and K_{eff} can be increased accordingly. However, for achieving the second objective, i.e., more flattening of power distribution, FAs with BP can be placed in regions with a high neutron flux, such as the center of the core, in order to dampen the flux for gaining more power flattening. Therefore, our defined BPP problem is a multiobjective optimization operation with antonym goals. This generates more complexity and difficulty in reaching the global optimum. However, with respect to selected objectives, we delineate the following fitness function (F_i) for the BPP optimization problem:

$$F_i = A \times F_{i1} + B \times F_{i2} \quad (8)$$

where

$$F_{i1} = \begin{cases} \sum_{i=1}^{Z} |P_{ri} - 1| & , \ PPF < P_{rmax} \\ \sum_{i=1}^{Z} |P_{ri} - 1| \times \left(1 + \frac{PPF}{P_{rmax}}\right) & , \ PPF \geq P_{rmax} \end{cases} \quad (9)$$

and

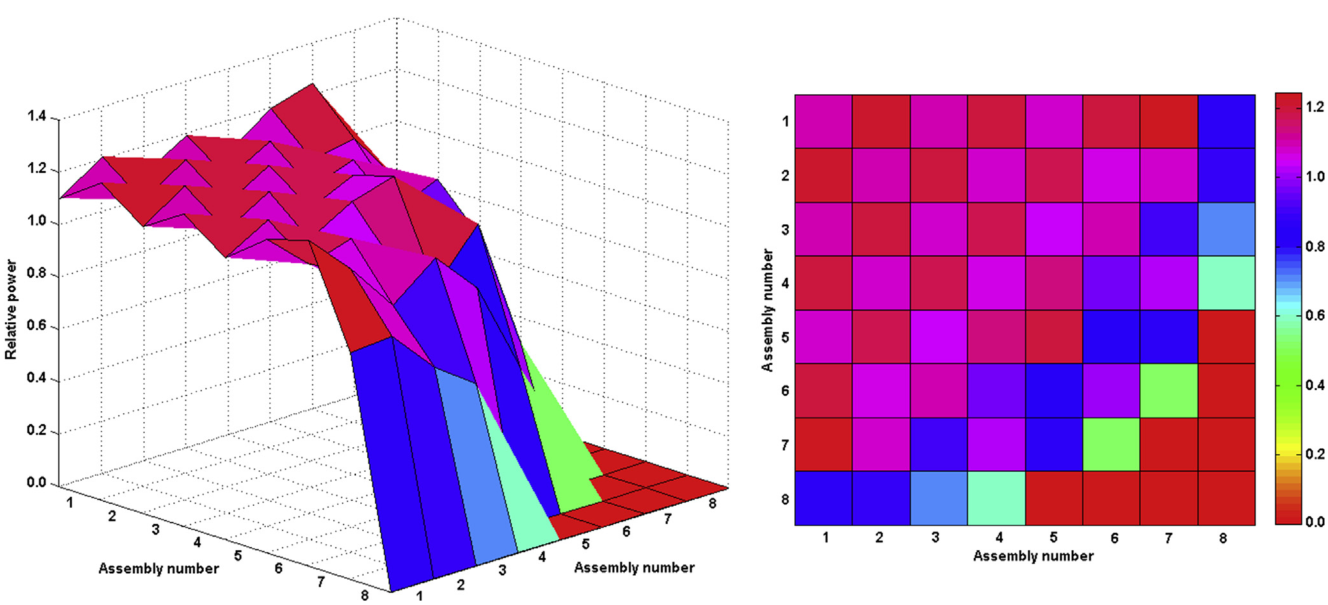

Fig. 5 – Fuel assembly relative power distribution of the reference (designer) loading pattern in 2D and 3D views. 2D, two dimensional; 3D, three dimensional.

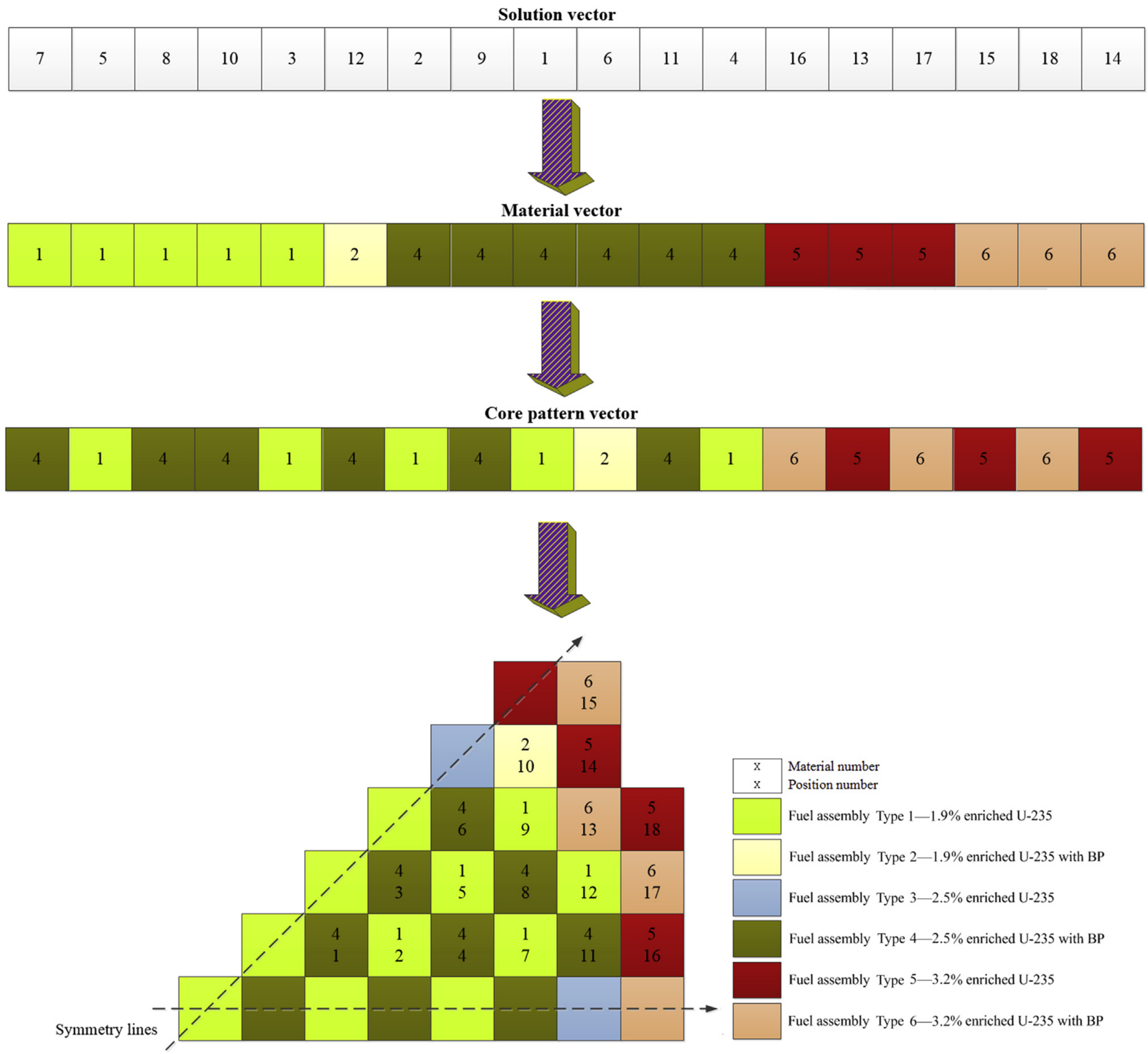

Fig. 6 – Producing a loading pattern from a solution vector for KWU PWR optimization problem. BP, burnable poison; PWR, pressurized water reactor.

$$F_{i2} = \frac{1}{K_{eff}} \tag{10}$$

In Eq. (8), A and B are constant coefficients, which are selected as 1 and 5, respectively. Also in Eq. (9), P_{ri} and P_{rmax} are the relative power of FA (i) and the maximum value permitted for FA relative power, respectively. P_{rmax} in this work is considered to be 1.25. PPF is the maximum FA relative power and Z is the number of all FAs located in the core. For having an efficient search performance in the presence of a constraint, the fitness function is required to be carefully considered, which penalizes unsuitable solutions with appropriate severity. A simple form of fitness is defined by considering an additional term to Eq. (8) for the situation that the PPF value is larger than P_{rmax}. In this situation, the relative power in an FA of the core is greater than the allowable limit, and as a result, the considered fitness function is enhanced. Then the accordant core pattern has a lower chance of proceeding further in the computational procedure and may be rejected in comparison with the best patterns [18].

However, the target of our optimization process is to obtain the minimum value of Eq. (8). According to Eq. (8), the value of defined F_i is decreased when F_{i1} is diminished by generating a core map with high flattening of FA power and also by having a small value of F_{i2} when K_{eff} is large. In this case, the corresponding fuel arrangement with F_i is more suitable and optimized in regard to the considered objectives. Consequently, our BPP optimization goal using SPEA-II is a minimum search of F_i value.

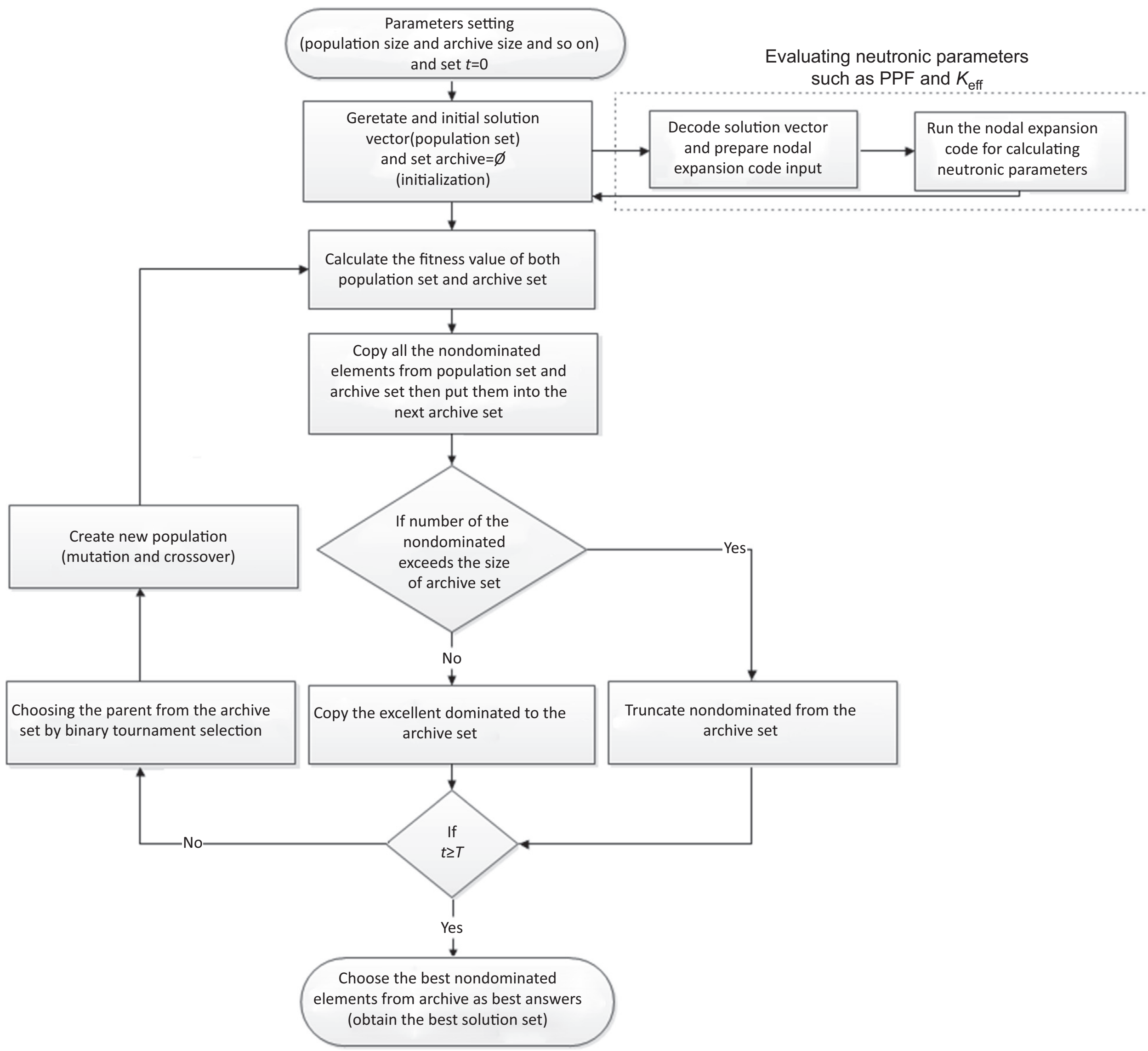

Fig. 7 – Simple view of SPEA-II for the BPP optimization. BP, burnable poison placement; PPF, power peaking factor; SPEA, strength Pareto evolutionary algorithm.

5. BPP optimization results of KWU PWR

The developed program, SPEA-II + nodal expansion code, has been used for our test case, i.e., KWU PWR BPP optimization. An initial population of models is selected at random, to improve the fitness of the population generation after generation. In Table 5, obtained fitness values in 10 runs of the developed code are given for different population sizes, i.e., 80, 100, and 120. According to Table 5, numerical results show some improvements in gaining more optimized fitness by increasing the population size. Of course, one can find the final results with near values of 10 runs for population sizes of 100 and 120, but in this case, the best results belong to the population size of 100 in which these results are selected for more investigations. It must be noted that the used iterations of various population sizes are different because we want the total evaluated fuel arrangements to be equal to 12,000 for comparison of results. Table 6 presents initial and final values of K_{eff}, PPF, and fitness function for 10 independent and successive runs along 120 iterations (for the population size of 100). According to Table 6, we can apprehend improvements of PPF and F_i values from the initial iteration to the last of the optimization process for all 10 experiments. For the population size of 100, fitness values also obtained by the basic approach, i.e., SPEA, and its results for 10 experiments are compared with those of the improved version, i.e., SPEA-II, both given in Table 7. According to Table 7, better results are appertained to SPEA-II with respect to the primary SPEA, for the best, worst, and average obtained fitness values during 10 runs.

A typical convergence process of fitness values gained by SPEA and SPEA-II versus iteration for 10 runs is represented in Figs. 8 and 9, respectively. Relatively near fitness values of

Table 5 – Fitness values obtained by SPEA-II for different population sizes.

	Population size		
	80	100	120
Run 1	9.132	9.087	9.099
Run 2	9.232	9.101	9.177
Run 3	9.192	9.094	9.082
Run 4	9.284	9.110	9.119
Run 5	9.115	9.080	9.195
Run 6	9.107	9.079	9.112
Run 7	9.139	9.099	9.092
Run 8	9.151	9.074	9.169
Run 9	9.104	9.090	9.083
Run 10	9.215	9.098	9.101
Average fitness	9.167	9.091	9.123
Best fitness	9.104	9.074	9.082
Worst fitness	9.284	9.110	9.195
SD	0.061	0.011	0.042
Iteration	150	120	100

SD, standard deviation; SPEA, strength Pareto evolutionary algorithm.

Table 7 – Results obtained by SPEA and SPEA-II during 10 runs for KWU PWR BPP.

	SPEA	SPEA-II
Run 1	9.291	9.087
Run 2	9.336	9.101
Run 3	9.245	9.094
Run 4	9.457	9.110
Run 5	9.331	9.080
Run 6	9.377	9.079
Run 7	9.319	9.099
Run 8	9.254	9.074
Run 9	9.231	9.090
Run 10	9.324	9.098
Average fitness	9.317	9.091
Best fitness	9.231	9.074
Worst fitness	9.457	9.110
SD	0.068	0.011
Max. iteration	120	120
Run time (min)[a]	78.5	80.1

[a] Using a laptop computer with CPU 2.40 GHz.
BPP, burnable poison placement; CPU, central processing unit; Max., maximum; PWR, pressurized water reactor; SD, standard deviation; SPEA, strength Pareto evolutionary algorithm.

runs are seen in these figures, particularly in Fig. 9, for SPEA-II. According to Fig. 9, one can realize that results have converged after 90 iterations. Therefore, the fast convergence of SPEA-II for obtaining minimized fitness is considerable. The semi-optimal (best) arrangement of FAs with/without BP by the lowest fitness value (as F_i defined in Section 4.3) for our test case, obtained by SPEA-II, is depicted in Fig. 10 (gained in the last iteration, i.e., 120). Fig. 10 also represents the FA relative power distribution of the best loading pattern. In addition, values of K_{eff}, PPF, and fitness, and changes of FA arrangement for the best result of SPEA-II, in different iterations of 1, 40, 80 and 120, are also shown in Fig. 10. By increasing the number of iterations, it is apperceived that PPF and fitness become lower by changing the location of some FA types with BP, including those with numbers 2, 4, and 6, until the optimal solution is obtained. Furthermore, the 2D and 3D schematic views of relative power values versus the assembly number for the best result are shown in Fig. 11. As shown in this figure, more flattening of relative power of FAs is found in comparison to the designer scheme in Fig. 5.

Lastly, Table 8 exhibits the results of comparison of the best core map (BPP) obtained by SPEA and SPEA-II with KWU designer scheme for PPF, K_{eff}, and fitness values. Table 8 shows that the final PPF, K_{eff}, and fitness values obtained by SPEA-II are the best. Of course, SPEA also attained better values of K_{eff} and fitness relative to the reference loading pattern (KWU designer scheme). It should be noted that a major development of fitness is attained using SPEA-II because of more flattening of FA powers belonged to the proposed core map (see the defined F_i). This growth can be found from the comparison of Figs. 5 and 11 due to more flattening of power distribution. Altogether, in regard to numerical results, it can be verified that SPEA-II has an efficient function for the BPP optimization problem.

6. Conclusion

One division of in-core fuel management was considered in this study: optimization of the fresh FA locations with BP in a PWR core. For this purpose, we developed an enhanced

Table 6 – Multiplication factor, PPF, and fitness values of KWU PWR BPP optimization obtained by SPEA-II for 10 runs.

Run	Initial K_{eff}	Final K_{eff}	Initial PPF	Final PPF	Initial fitness	Final fitness	Max. iteration
1	1.0129	1.0054	1.565	1.241	19.427	9.087	120
2	1.0079	1.0059	1.462	1.243	21.703	9.101	120
3	1.0085	1.0053	1.541	1.239	21.754	9.094	120
4	1.0178	1.0050	1.592	1.246	20.602	9.110	120
5	1.0091	1.0059	1.407	1.241	19.590	9.080	120
6	1.0063	1.0058	1.622	1.236	20.949	9.079	120
7	1.0073	1.0058	1.555	1.239	21.538	9.099	120
8	1.0154	1.0059	1.481	1.235	18.396	9.074	120
9	1.0172	1.0054	1.628	1.245	18.677	9.090	120
10	1.0129	1.0056	1.488	1.240	20.699	9.098	120

BPP, burnable poison placement; Max., maximum, PPF, power peaking factor; PWR, pressurized water reactor; SPEA, strength Pareto evolutionary algorithm.

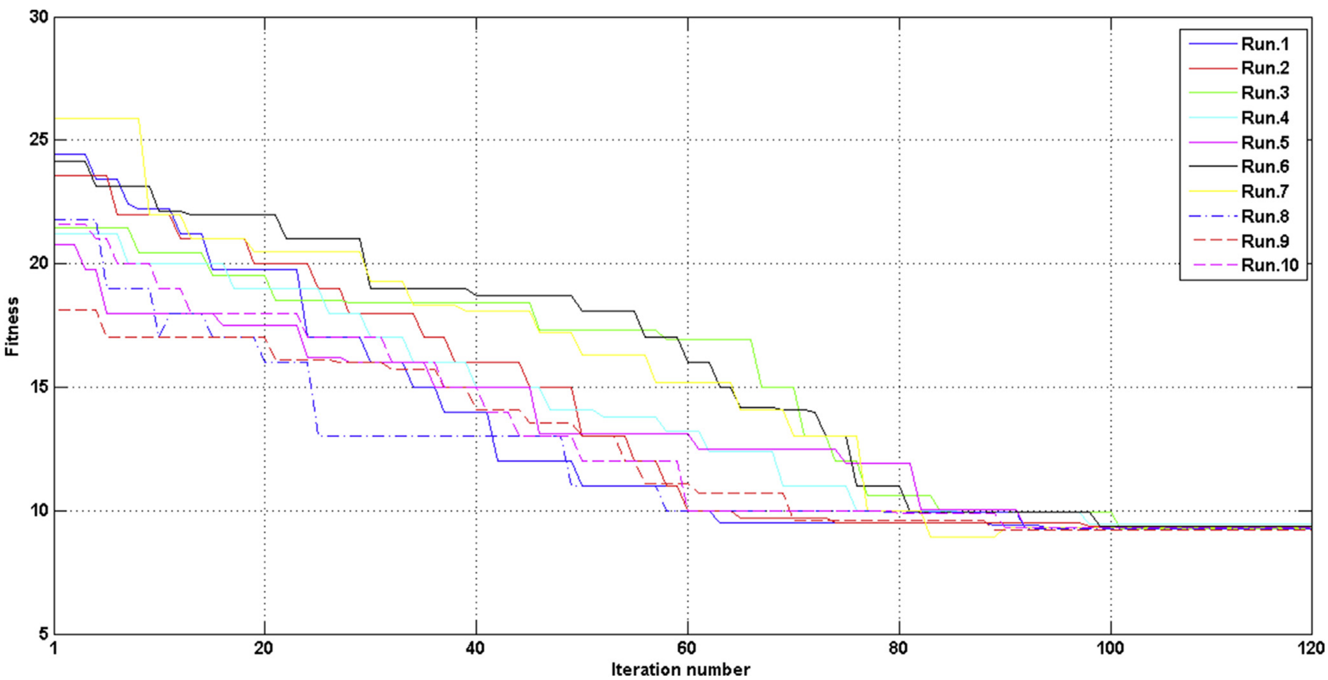

Fig. 8 – Fitness function value obtained by SPEA versus the iteration number for 10 runs. SPEA, strength Pareto evolutionary algorithm.

optimization approach, i.e., SPEA-II, for the BPP problem. In order to evaluate the proposed algorithm, we also developed the basic approach, SPEA, for comparing results. According to numerical results, SPEA-II is exploited successfully for the BPP optimization problem of the KWU PWR core and also SPEA-II can obtain better results relative to SPEA. With respect to neutron economy in the reactor, it is appropriate to have less BP associated with high reactivity worth. In addition, optimization of burnable absorber (BA) placement in the reactor core can be beneficial to this purpose. In this study, the number of FAs including BP has been fixed, but their locations are searched and optimized for attaining the defined optimization results. However, the obtained results indicate that the performance of SPEA-II is convenient, with adequate strength for the BPP optimization of the KWU PWR. In this case, final results, which were obtained by SPEA-II, are closer together

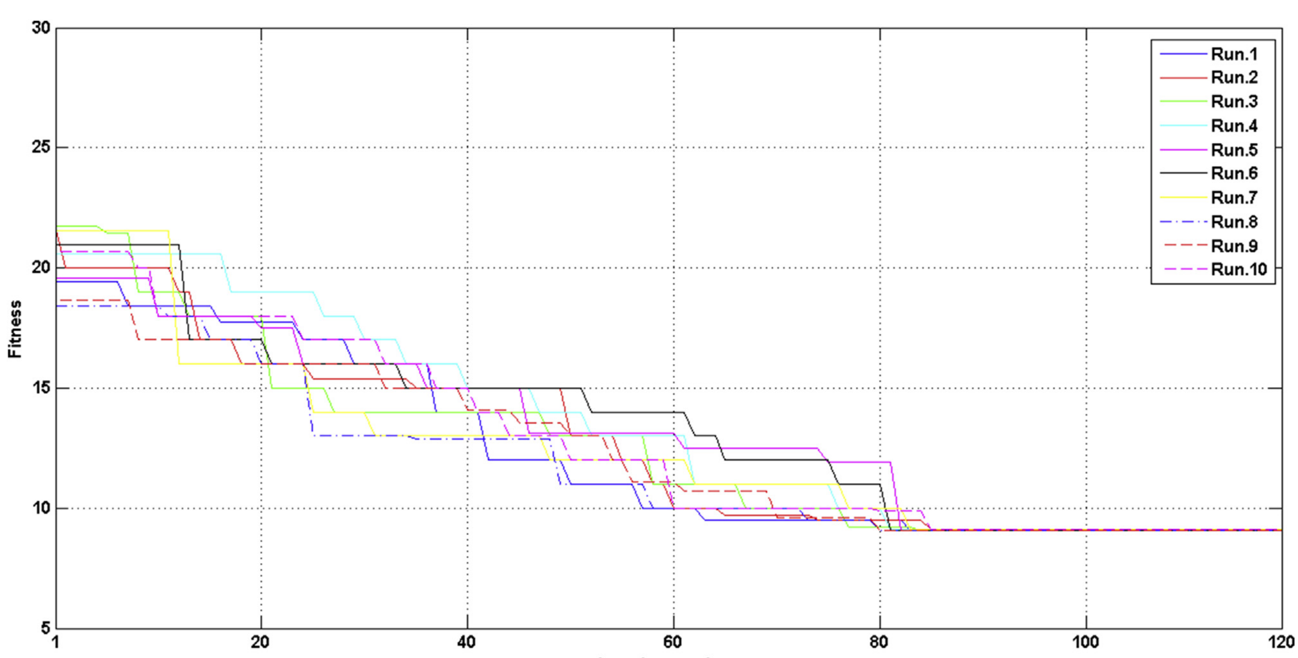

Fig. 9 – Fitness function value obtained by SPEA-II versus the iteration number for 10 runs. SPEA, strength Pareto evolutionary algorithm.

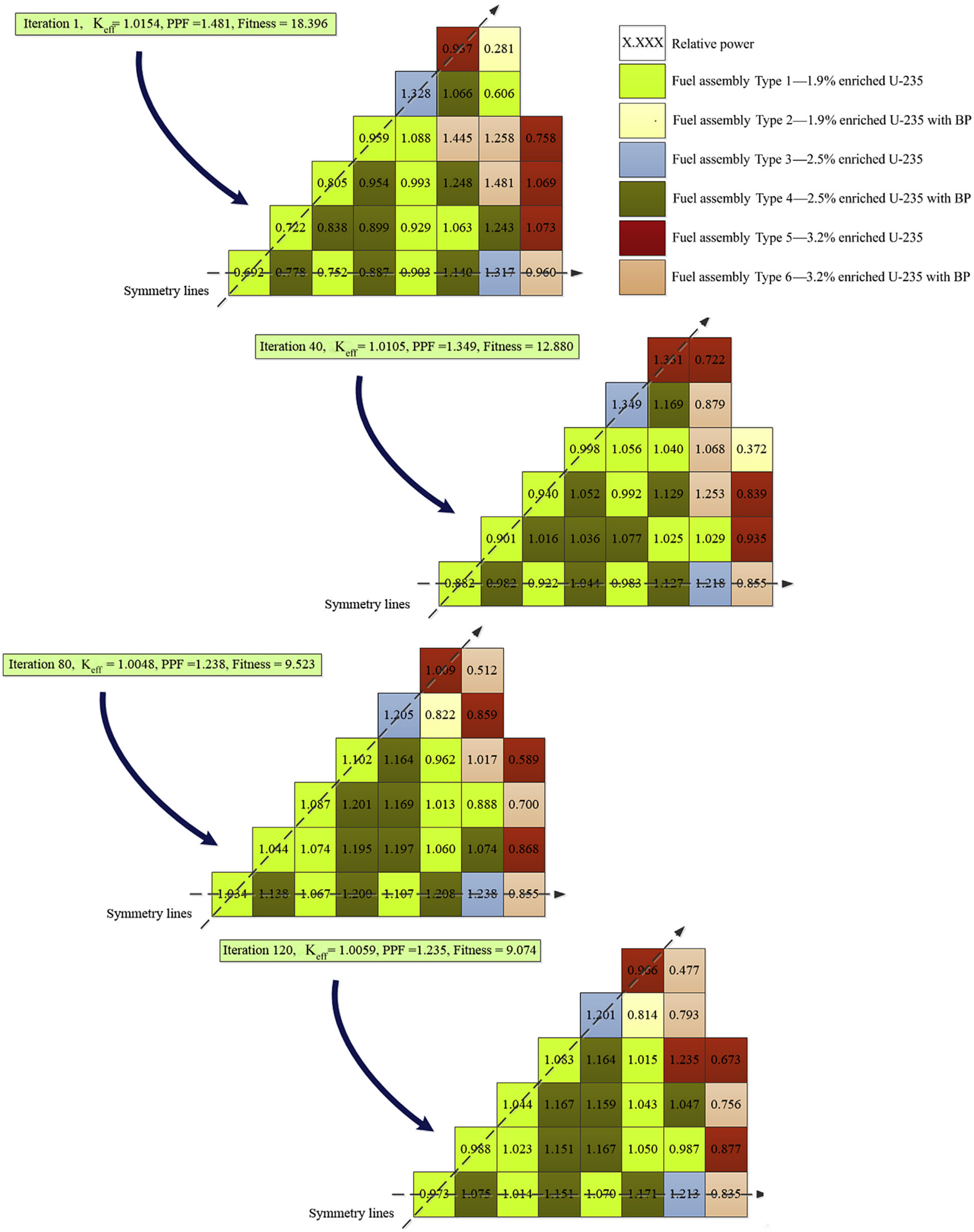

Fig. 10 – Optimization progress of the best loading pattern obtained by SPEA-II from the initial to final states. BP, burnable poison placement; PPF, power peaking factor; SPEA, strength Pareto evolutionary algorithm.

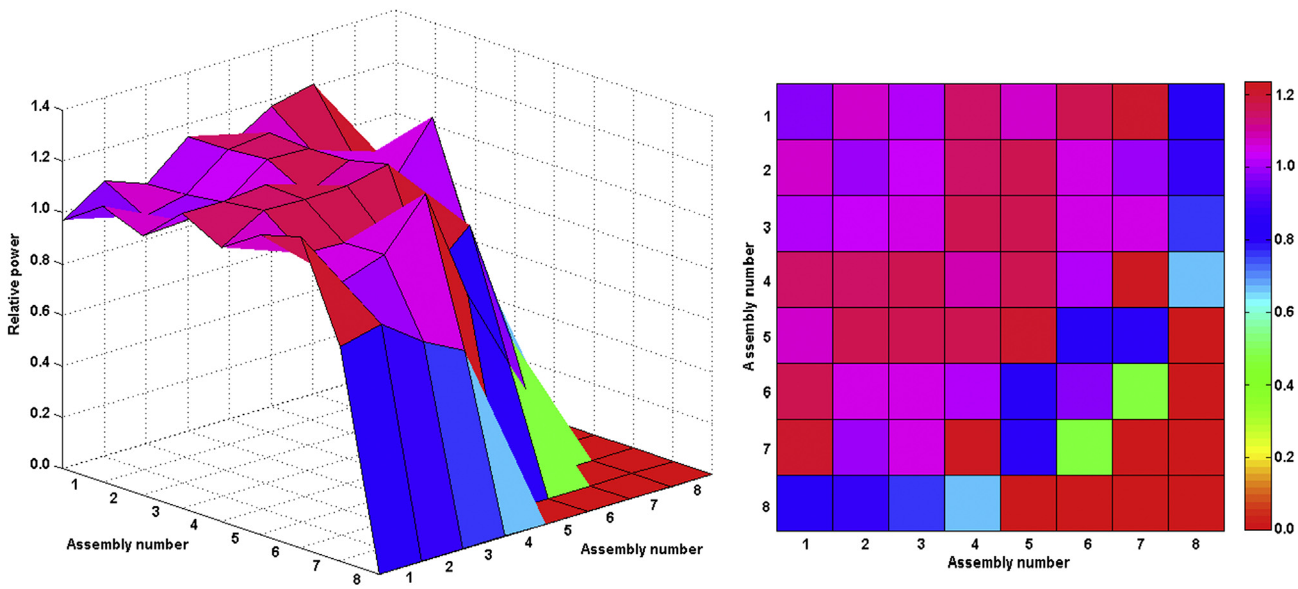

Fig. 11 – Fuel assembly relative power distribution of the best core map obtained by SPEA-II in 2D and 3D views. SPEA, strength Pareto evolutionary algorithm; 2D, two dimensional; 3D, three dimensional.

Table 8 – Comparison of multiplication factor, PPF, and fitness values of SPEA, SPEA-II, and reference schemes.

Scheme	Final Keff	PPF	F_i
Reference	1.0047	1.245	9.523
SPEA	1.0057	1.241	9.231
SPEA-II	1.0059	1.235	9.074

PPF, power peaking factor; SPEA, strength Pareto evolutionary algorithm.

and better than SPEA. Therefore, this merit can verify that SPEA-II is more reliable for the BPP problem. Overall, we can suggest checking the proposed approach with respect to other optimization problems in the nuclear engineering field.

Conflicts of interest

All contributing authors declare no conflicts of interest.

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