

Hui Lin and Ming Li

Introduction to Data Science



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Preface

During the first couple years of our career as data scientists, we were bewildered by all kinds of data science hype. There is a lack of definition of many basic terminologies such as “Big Data” and “Data Science.” How big is big? If someone ran into you asked what data science was all about, what would you tell them? What is the difference between the sexy role “Data Scientist” and the traditional “Data Analyst”? How suddenly came all kinds of machine algorithms? All those struck us as confusing and vague as real-world data scientists! But we always felt that there was something real there. After applying data science for many years, we explored it more and had a much better idea about data science. And this book is our endeavor to make data science to a more legitimate field.

Goal of the Book

This is an introductory book to data science with a specific focus on the application. Data Science is a cross-disciplinary subject involving hands-on experience and business problem-solving exposures. The majority of existing introduction books on data science are about the modeling techniques and the implementation of models using R or Python. However, they fail to introduce data science in a context of the industrial environment. Moreover, a crucial part, the art of data science in practice, is often missing. This book intends to fill the gap.

Some key features of this book are as follows:

- It is comprehensive. It covers not only technical skills but also soft skills and big data environment in the industry.
- It is hands-on. We provide the data and repeatable R and Python code. You can repeat the analysis in the book using the data and code provided. We also suggest you perform the analyses with your data whenever possible. You can only learn data science by doing it!
- It is based on context. We put methods in the context of industrial data science questions.
- Where appropriate, we point you to more advanced materials on models to dive deeper

Who This Book Is For

Non-mathematical readers will appreciate the emphasis on problem-solving with real data across a wide variety of applications and the reproducibility of the companion R and python code.

Readers should know basic statistical ideas, such as correlation and linear regression analysis. While the text is biased against complex equations, a mathematical background is needed for advanced topics.

What This Book Covers

Based on industry experience, this book outlines the real world scenario and points out pitfalls data science practitioners should avoid. It also covers big data cloud platform and the art of data science such as soft skills. We use R as the main tool and provide code for both R and Python.

Conventions

Acknowledgements



About the Authors

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1

Introduction

Interest in data science is at an all-time high and has exploded in popularity in the last couple of years. Data scientists today are from various backgrounds. If someone ran into you asked what data science was all about, what would you tell them? It is not easy to answer. Data science is one of the areas where if you ask ten people you get ten different answers. It is not well-defined as an academic subject but broadly used in the industry. Media has been hyping about “Data Science” “Big Data” and “Artificial Intelligence” over the fast few years. With the data science hype picking up steam, many professionals changed their titles to “Data Scientist” without any of the necessary qualifications. Your first reaction to all of this might be some combination of skepticism and confusion. We want to address this up front that: we had that exact reaction. To make things clear, let’s start with the fundamental question.

1.1 What is data science?

David Donoho ([Donoho, 2015](#)) summarizes in “50 Years of Data Science” the main recurring “Memes” about data sciences:

1. The ‘Big Data’ Meme
2. The ‘Skills’ Meme
3. The ‘Jobs’ Meme

Everyone should have heard about big data. Data science trainees now need the skills to cope with such big data sets. What are those

skills? You may hear about: Hadoop, a system using Map/Reduce to process large data sets distributed across a cluster of computers. The new skills are for dealing with organizational artifacts of large-scale cluster computing but not for better solving the real problem. A lot of data on its own is worthless. It isn't the size of the data that's important. It's what you do with it. The big data skills that so many are touting today are not skills for better solving the real problem of inference from data.

We are transiting to universal connectivity with a deluge of data filling telecom servers. But these facts don't immediately create a science. The statisticians and computer scientists have been laying the groundwork for data science for at least 50 years. Today's data science is an enlargement and combination of statistics and computer science rather than a brand new discipline.

Data Science doesn't come out of the blue. Its predecessor is data analysis. Back in 1962, John Tukey wrote in "The Future of Data Analysis":

For a long time I have thought I was a statistician, interested in inferences from the particular to the general. But as I have watched mathematical statistics evolve, I have had cause to wonder and to doubt. ...All in all, I have come to feel that my central interest is in data analysis, which I take to include, among other things: procedures for analyzing data, techniques for interpreting the results of such procedures, ways of planning the gathering of data to make its analysis easier, more precise or more accurate, and all the machinery and results of (mathematical) statistics which apply to analyzing data.

It deeply shocked his academic readers. Aren't you supposed to present something mathematically precise, such as definitions, theorems, and proofs? If we use one sentence to summarize what John said, it is:

data analysis is more than mathematics.

In September 2015, the University of Michigan made plans to invest \$100 million over the next five years in a new Data Science Initiative (DSI) that will enhance opportunities for student and faculty researchers across the university to tap into the enormous potential of big data. How does DSI define Data science? Their website gives us an idea:

“This coupling of scientific discovery and practice involves the collection, management, processing, analysis, visualization, and interpretation of vast amounts of heterogeneous data associated with a diverse array of scientific, translational, and interdisciplinary applications.”

How about data scientist? Here is a list of somewhat whimsical definitions for a “data scientist”:

- “A data scientist is a data analyst who lives in California”
- “A data scientist is someone who is better at statistics than any software engineer and better at software engineering than any statistician.”
- “A data scientist is a statistician who lives in San Francisco.”
- “Data Science is statistics on a Mac.”

There is lots of confusion between Data Scientist, Statistician, Business/Financial/Risk(etc.) Analyst and BI professional due to the apparent intersections among skillsets. We see data science as a discipline to make sense of data. The techniques and methodologies of data science stem from the fields of computer science and statistics. One of the most well-cited diagrams describing the

area comes from Drew Conway where he suggested data science is the intersection of hacking skills, math and stats knowledge, and substantial expertise. This diagram might be a bit of an oversimplification, but it's a great start.

There are almost as many definitions of data science as there are data scientists. Instead of listing some of these definitions, it might be more informative to let the subject matter define the field.

Let's start from a brief history of data science. If you hit up the Google Trends website which shows search keyword information over time and check the term “data science,” you will find the history of data science goes back a little further than 2004. From the way media describes it, you may feel machine learning algorithms were just invented last month, and there was never “big” data before Google. That is not true. There are new and exciting developments of data science, but many of the techniques we are using are based on decades of work by statisticians, computer scientists, mathematicians and scientists of all types.

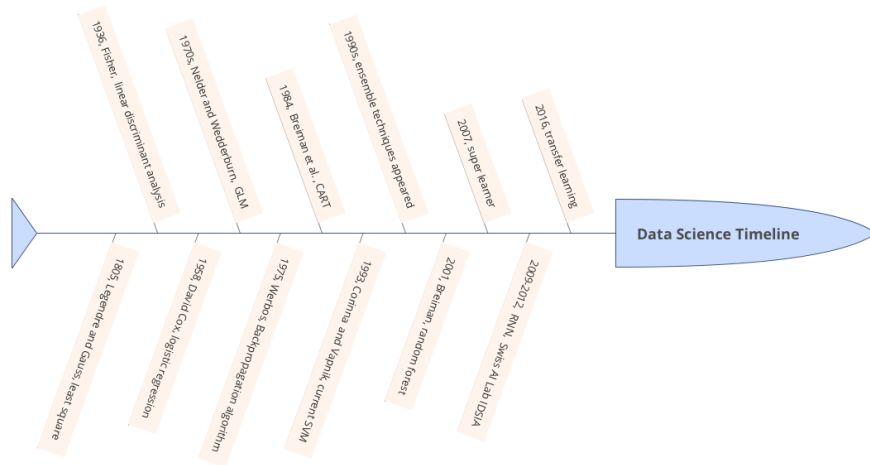


FIGURE 1.1: Data Science Timeline

In the early 19th century when Legendre and Gauss came up the least squares method for linear regression, only physicists would use it to fit linear regression. Now, even non-technical people can fit

linear regressions using excel. In 1936 Fisher came up with linear discriminant analysis. In the 1940s, we had another widely used model – logistic regression. In the 1970s, Nelder and Wedderburn formulated “generalized linear model (GLM)” which:

“generalized linear regression by allowing the linear model to be related to the response variable via a link function and by allowing the magnitude of the variance of each measurement to be a function of its predicted value.” [from Wikipedia]

By the end of the 1970s, there was a range of analytical models and most of them were linear because computers were not powerful enough to fit non-linear model until the 1980s.

In 1984 Breiman et al. introduced classification and regression tree (CART) which is one of the oldest and most utilized classification and regression techniques. After that Ross Quinlan came up with more tree algorithms such as ID3, C4.5, and C5.0. In the 1990s, ensemble techniques (methods that combine many models’ predictions) began to appear. Bagging is a general approach that uses bootstrapping in conjunction with any regression or classification model to construct an ensemble. Based on the ensemble idea, Breiman came up with random forest in 2001. Later, Yoav Freund and Robert Schapire came up with the AdaBoost.M1 algorithm. Benefiting from the increasing availability of digitized information, and the possibility to distribute that via the internet, the toolbox has been expanding fast. The applications include business, health, biology, social science, politics, etc.

John Tukey identified four forces driving data analysis (there was no “data science” then):

1. The formal theories of math and statistics
2. Acceleration of developments in computers and display devices

3. The challenge, in many fields, of more and ever larger bodies of data
4. The emphasis on quantification in an ever wider variety of disciplines

Tukey's 1962 list is surprisingly modern. Let's inspect those points in today's context. There is always a time gap between a theory and its application. We had the theories much earlier than application. Fortunately, for the past 50 years, statisticians have been laying the theoretical groundwork for constructing "data science" today. The development of computers enables us to calculate much faster and deliver results in a friendly and intuitive way. The striking transition to the internet of things generates vast amounts of commercial data. Industries have also sensed the value of exploiting that data. Data science seems certain to be a major preoccupation of commercial life in coming decades. All the four forces John identified exist today and have been driving data science.

1.2 What kind of questions can data science solve?

1.2.1 Prerequisites

Data science is not a panacea, and data scientists are not magicians. There are problems data science can't help. It is best to make a judgment as early in the analytical cycle as possible. Tell your clients honestly and clearly when you figure data analytics can't give the answer they want. What kind of questions can data science solve? What are the requirements for our question?

1. Your question needs to be specific enough

Look at two examples:

- Question 1: How can I increase product sales?

- Question 2: Is the new promotional tool introduced at the beginning of this year boosting the annual sales of P1197 in Iowa and Wisconsin? (P1197 is an impressive corn seed product from DuPont Pioneer)

It is easy to see the difference between the two questions. Question 1 is a grammatically correct question, but it is proper for data analysis to answer. Why? It is too general. What is the response variable here? Product sales? Which product? Is it annual sales or monthly sales? What are the candidate predictors? You nearly can't get any useful information from the questions. In contrast, question 2 is much more specific. From the analysis point of view, the response variable is clearly "annual sales of P1197 in Iowa and Wisconsin". Even we don't know all the predictors, but the variable of interest is "the new promotional tool introduced early this year." We want to study the impact of the promotion of the sales. You can start from there and move on to figure out other variables need to include in the model by further communication.

As a data scientist, you may start with something general and unspecific like question 1 and eventually get to question 2. Effective communication and in-depth domain knowledge about the business problem are essential to convert a general business question into a solvable analytical problem. Domain knowledge helps data scientist communicate with the language the other people can understand and obtain the required information.

However, defining the question and variables involved don't guarantee that you can answer it. I have encountered a well-defined supply chain problem. My client asked about the stock needed for a product in a particular area. Why can not this question be answered? I did fit a Multivariate Adaptive Regression Spline (MARS) model and thought I found a reasonable solution. But it turned out later that the data they gave me was inaccurate. In some areas, only estimates of past supply figures were available. The lesson lends itself to the next point.

2. You need to have sound and relevant data

One cannot make a silk purse out of a sow's ear. Data scientists need data, sound and relevant data. The supply problem is a case in point. There was relevant data, but not sound. All the later analytics based on that data was a building on sand. Of course, data nearly almost have noise, but it has to be in a certain range. Generally speaking, the accuracy requirement for the independent variables of interest and response variable is higher than others. In question 2, it is data related to the "new promotion" and "sales of P1197".

The data has to be helpful for the question. If you want to predict which product consumers are most likely to buy in the next three months, you need to have historical purchasing data: the last buying time, the amount of invoice, coupons and so on. Information about customers' credit card number, ID number, the email address is not going to help.

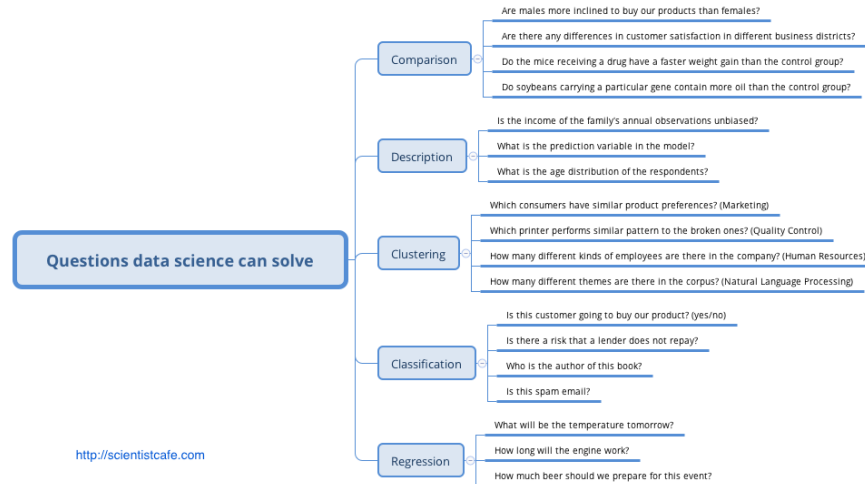
Often the quality of the data is more important than the quantity, but the quantity cannot be overlooked. In the premise of guaranteeing quality, usually the more data, the better. If you have a specific and reasonable question, also sound and relevant data, then congratulations, you can start playing data science!

1.2.2 Problem type

Many of the data science books classify the various models from a technical point of view. Such as supervised vs. unsupervised models, linear vs. nonlinear models, parametric models vs. non-parametric models, and so on. Here we will continue on "problem-oriented" track. We first introduce different groups of real problems and then present which models can be used to answer the corresponding category of questions.

1. Comparison

The first common problem is to compare different groups. Such as: Is A better in some way than B? Or more comparisons: Is there

**FIGURE 1.2:** Data Science Questions

any difference among A, B, C in a certain aspect? Here are some examples:

- Are the purchasing amounts different between consumers receiving coupons and those without coupons?
- Are males more inclined to buy our products than females?
- Are there any differences in customer satisfaction in different business districts?
- Do the mice receiving a drug have a faster weight gain than the control group?
- Do soybeans carrying a particular gene contain more oil than the control group?

For those problems, it is usually to start exploring from the summary statistics and visualization by groups. After a preliminary visualization, you can test the differences between treatment and control group statistically. The commonly used statistical tests are chi-square test, t-test, and ANOVA. There are also methods using Bayesian methods. In biology industry, such as new drug development, crop breeding, mixed effect models are the dominant technique.

2. Description

In the problem such as customer segmentation, after you cluster the sample, the next step is to figure out the profile of each class by comparing the descriptive statistics of the various variables. Questions of this kind are:

- Is the income of the family's annual observations unbiased?
- What is the mean/variance of the monthly sales volume of a product in different regions?
- What is the difference in the magnitude of the variable? (Decide whether the data needs to be standardized)
- What is the prediction variable in the model?
- What is the age distribution of the respondents?

Data description is often used to check data, find the appropriate data preprocessing method, and demonstrate the model results.

3. Clustering

Clustering is a widespread problem, which is usually related to classification. Clustering answers questions like:

- Which consumers have similar product preferences? (Marketing)
- Which printer performs similar pattern to the broken ones? (Quality Control)
- How many different kinds of employees are there in the company? (Human Resources)
- How many different themes are there in the corpus? (Natural Language Processing)

Note that clustering is unsupervised learning. The most common clustering algorithms include K-Means and Hierarchical Clustering.

4. Classification

Usually, a labeled sample set is used as a training set to train the classifier. Then the classifier is used to predict the category of a future sample. Here are some example questions:

- Is this customer going to buy our product? (yes/no)
- Is there a risk that a lender does not repay?
- Who is the author of this book?
- Is this spam email?

There are hundreds of classifiers. In practice, we do not have to try all the models as long as we fit in several of the best models in most cases.

5. Regression

In general, regression deals with the problem of “how much is it?” and return a numerical answer. In some cases, it is necessary to coerce the model results to be 0, or round the result to the nearest integer. It is the most common problem.

- What will be the temperature tomorrow?
- What will be the company’s sales in the fourth quarter of this year?
- How long will the engine work?
- How much beer should we prepare for this event?

1.3 Data Scientist Skill Set

We talked about the bewildering definitions of data scientist. What are the required skills for a data scientist?

- Educational Background

Most of the data scientists today have undergraduate or higher degree from one of the following areas: computer science, electronic engineering, mathematics or statistics. According to a 2017 survey, 25% of US data scientists have a Ph.D. degree, 64% have a Master’s degree, and 11% are Bachelors.

- Database Skills

Data scientists in the industry need to use SQL to pull data from the database. So it is necessary to be familiar with how data is structured and how to do basic data manipulation using SQL. Many statistics/mathematics students do not have experience with SQL in school. Don't worry. If you are proficient in one programming language, it is easy to pick up SQL. The main purpose of graduate school should be to develop the ability to learn and analytical thinking rather than the technical skills. Even the technical skills are necessary to enter the professional area. Most of the skills needed at work are not taught in school.

- Programming Skills

Programming skills are critical for data scientists. According to a 2017 survey from Burtch Works¹, 97% of the data scientists today using R or Python. We will provide exemplary code for both in this book. There is not one “have-to-use” tool. The goal is to solve the problem not which tool to choose. However, a good tool needs to be flexible and scalable.

- Modeling Skills

Data scientists need to know statistical and machine learning models. There is no clear line separating these two. Many statistical models are also machine learning models, vice versa. Generally speaking, a data scientist is familiar with basic statistical tests such as t-test, chi-square test, and analysis of variance. They can explain the difference between Spearman rank correlation and Pearson correlation, be aware of basic sampling schemes, such as Simple Random Sampling, Stratified Random Sampling, and Multi-Stage Sampling. Know commonly used probability distributions such as Normal distribution, Binomial distribution, Poisson distribution, F distribution, T distribution, and Chi-square distribution. Experimental design plays a significant role in the biological study. Understanding the main tenants of Bayesian methods is necessary (at least be able to write the Bayes theorem on the whiteboard and know what does it mean). Know the difference between supervised and unsupervised learning. Understand commonly used cluster al-

¹<http://www.burtchworks.com/2017/06/19/2017-sas-r-python-flash-survey-results/>

gorithms, classifiers, and regression models. Some powerful tools in predictive analytics are tree models (such as random forest and AdaBoost) and penalized model (such as lasso and SVM). Data scientist working on social science (such as consumer awareness surveys), also needs to know the latent variable model, such as exploratory factor analysis, confirmatory factor analysis, structural equation model.

Is the list getting a little scary? It can get even longer. Don't worry if you don't know all of them now. You will learn as you go. Standard mathematics, statistics or computer science training in graduate school can get you started. But you have to learn lots of new skills after school. Learning is happening increasingly outside of formal educational settings and in unsupervised environments. An excellent data scientist must be a lifetime learner. Fortunately, technological advantages provide new tools and opportunities for lifetime learners, MOOC, online data science workshops and various online tutorials. So above all, being a **life-time learner** is the most critical.

- Soft Skills

In addition to technical knowledge, there are some critical soft skills. These include the ability to translate practical problems into data problems, excellent communication skill, attention to detail, storytelling and so on. We will discuss it in a later chapter in more detail.

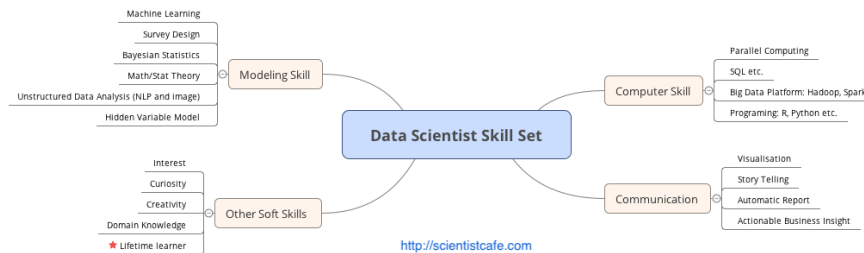


FIGURE 1.3: Data Scientist Skill Set

1.4 Types of Learning

There are three broad groups of styles: supervised learning, reinforcement learning, and unsupervised learning.

In supervised learning, each observation of the predictor measurement(s) corresponds to a response measurement. There are two flavors of supervised learning: regression and classification. In regression, the response is a real number such as the total net sales in 2017, or the yield of corn next year. The goal is to approximate the response measurement as much as possible. In classification, the response is a class label, such as dichotomous response such as yes/no. The response can also have more than two categories, such as four segments of customers. A supervised learning model is a function that maps some input variables with corresponding parameters to a response y . Modeling tuning is to adjust the value of parameters to make the mapping fit the given response. In other words, it is to minimize the discrepancy between given response and the model output. When the response y is a real value, it is intuitive to define discrepancy as the squared difference between model output and given the response. When y is categorical, there are other ways to measure the difference, such as AUC or information gain.

In reinforcement learning, the correct input/output pairs are not present. The model will learn from a sequence of actions and select the action maximizing the expected sum of the future rewards. There is a discount factor that makes future rewards less valuable than current rewards. Reinforcement learning is difficult for the following reasons:

- (1) The rewards are not instant. If the action sequence is long, it is hard to know which action was wrong.
- (2) The rewards are occasional. Each reward does not supply much information, so its impact of parameter change is limited. Typically, it is not likely to learn a large number

of parameters using reinforcement learning. However, it is possible for supervised and unsupervised learning. The number of parameters in reinforcement learning usually range from dozens to maybe 1,000, but not millions.

In unsupervised learning, there is no response variable. For a long time, the machine learning community overlooked unsupervised learning except for one called clustering. Moreover, many researchers thought that clustering was the only form of unsupervised learning. One reason is that it is hard to define the goal of unsupervised learning explicitly. Unsupervised learning can be used to do the following:

- (1) Identify a good internal representation or pattern of the input that is useful for subsequent supervised or reinforcement learning, such as finding clusters.
- (2) It is a dimension reduction tool that is to provide compact, low dimensional representations of the input, such as factor analysis.
- (3) Provide a reduced number of uncorrelated learned features from original variables, such as principal component regression.

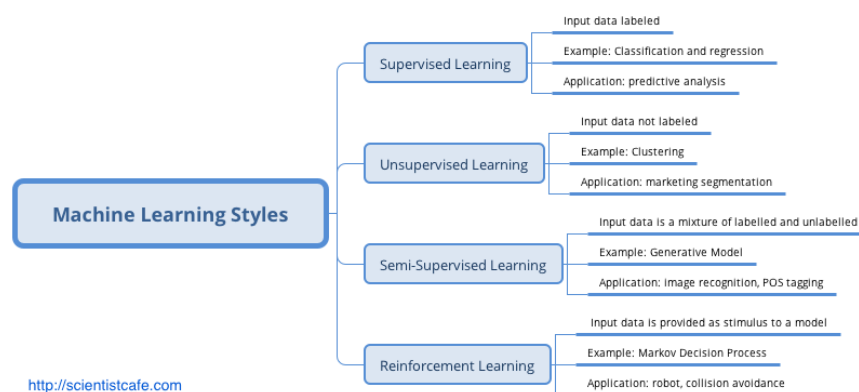


FIGURE 1.4: Machine Learning Styles

1.5 Types of Algorithm

The categorization here is based on the structure (such as tree model, Regularization Methods) or type of question to answer (such as regression).² It is far less than perfect but will help to show a bigger map of different algorithms. Some can be legitimately classified into multiple categories, such as support vector machine (SVM) can be a classifier, and can also be used for regression. So you may see other ways of grouping. Also, the following summary does not list all the existing algorithms (there are just too many).

1. Regression

Regression can refer to the algorithm or a particular type of problem. It is supervised learning. Regression is one of the oldest and most widely used statistical models. It is often called the statistical machine learning method. Standard regression models are:

- Ordinary Least Squares Regression
- Logistic Regression
- Multivariate Adaptive Regression Splines (MARS)
- Locally Estimated Scatterplot Smoothing (LOESS)

The least squares regression and logistic regression are traditional statistical models. Both of them are highly interpretable. MARS is similar to neural networks and partial least squares (PLS) in the respect that they all use surrogate features instead of original predictors.

They differ in how to create the surrogate features. PLS and neural networks use linear combinations of the original predictors as

²The summary of various algorithms for data science in this section is based on Jason Brownlee's blog "(A Tour of Machine Learning Algorithms)[<http://machinelearningmastery.com/a-tour-of-machine-learning-algorithms/>]." We added and subtracted some algorithms in each category and gave additional comments.

surrogate features³. MARS creates two contrasted versions of a predictor by a truncation point. And LOESS is a non-parametric model, usually only used in visualization.

2. Similarity-based Algorithms

This type of model is based on a similarity measure. There are three main steps: (1) compare the new sample with the existing ones; (2) search for the closest sample; (3) and let the response of the nearest sample be used as the prediction.

- K-Nearest Neighbour [KNN]
- Learning Vector Quantization [LVQ]
- Self-Organizing Map [SOM]

The biggest advantage of this type of model is that they are intuitive. K-Nearest Neighbour is generally the most popular algorithm in this set. The other two are less common. The key to similarity-based algorithms is to find an appropriate distance metric for your data.

3. Feature Selection Algorithms

The primary purpose of feature selection is to exclude non-information or redundant variables and also reduce dimension. Although it is possible that all the independent variables are significant for explaining the response. But more often, the response is only related to a portion of the predictors. We will expand the feature selection in detail later.

- Filter method
- Wrapper method
- Embedded method

Filter method focuses on the relationship between a single feature

³To be clear on neural networks, the linear combinations of predictors are put through non-linear activation functions, deeper neural networks have many layers of non-linear transformation

and a target variable. It evaluates each feature (or an independent variable) before modeling and selects “important” variables.

Wrapper method removes the variable according to particular law and finds the feature combination that optimizes the model fitting by evaluating a set of feature combinations. In essence, it is a searching algorithm.

Embedding method is part of the machine learning model. Some model has built-in variable selection function such as lasso, and decision tree.

4. Regularization Method

This method itself is not a complete model, but rather an add-on to other models (such as regression models). It appends a penalty function on the criteria used by the original model to estimate the variables (such as likelihood function or the sum of squared error). In this way, it penalizes model complexity and contracts the model parameters. That is why people call them “shrinkage method.” This approach is advantageous in practice.

- Ridge Regression
- Least Absolute Shrinkage and Selection Operator (LASSO)
- Elastic Net

5. Decision Tree

Decision trees are no doubt one of the most popular machine learning algorithms. Thanks to all kinds of software, implementation is a no-brainer which requires nearly zero understanding of the mechanism. The followings are some of the common trees:

- Classification and Regression Tree (CART)
- Iterative Dichotomiser 3 (ID3)
- C4.5
- Random Forest
- Gradient Boosting Machines (GBM)

6. Bayesian Models

People usually confuse Bayes theorem with Bayesian models. Bayes theorem is an implication of probability theory which gives Bayesian data analysis its name.

$$Pr(\theta|y) = \frac{Pr(y|\theta)Pr(\theta)}{Pr(y)}$$

The actual Bayesian model is not identical to Bayes theorem. Given a likelihood, parameters to estimate, and a prior for each parameter, a Bayesian model treats the estimates as a purely logical consequence of those assumptions. The resulting estimates are the posterior distribution which is the relative plausibility of different parameter values, conditional on the observations. The Bayesian model here is not strictly in the sense of Bayesian but rather model using Bayes theorem.

- Naïve Bayes
- Averaged One-Dependence Estimators (AODE)
- Bayesian Belief Network (BBN)

7. Kernel Methods

The most common kernel method is the support vector machine (SVM). This type of algorithm maps the input data to a higher order vector space where classification or regression problems are easier to solve.

- Support Vector Machine (SVM)
- Radial Basis Function (RBF)
- Linear Discriminate Analysis (LDA)

8. Clustering Methods

Like regression, when people mention clustering, sometimes they mean a class of problems, sometimes a class of algorithms. The clustering algorithm usually clusters similar samples to categories in a centroidal or hierarchical manner. The two are the most common clustering methods:

- K-Means
- Hierarchical Clustering

9. Association Rule

The basic idea of an association rule is: when events occur together more often than one would expect from their rates of occurrence, such co-occurrence is an interesting pattern. The most used algorithms are:

- Apriori algorithm
- Eclat algorithm

10. Artificial Neural Network

The term neural network has evolved to encompass a repertoire of models and learning methods. There has been lots of hype around the model family making them seem magical and mysterious. A neural network is a two-stage regression or classification model. The basic idea is that it uses linear combinations of the original predictors as surrogate features, and then the new features are put through non-linear activation functions to get hidden units in the 2nd stage. When there are multiple hidden layers, it is called deep learning, another over hyped term. Among varieties of neural network models, the most widely used “vanilla” net is the single hidden layer back-propagation network.

- Perceptron Neural Network
- Back Propagation
- Hopfield Network
- Self-Organizing Map (SOM)
- Learning Vector Quantization (LVQ)

11. Deep Learning

The name is a little misleading. As mentioned before, it is multilayer neural network. It is hyped tremendously especially after AlphaGO defeated Li Shishi at the board game Go. We don't have too much experience with the application of deep learning and are

not in the right position to talk more about it. Here are some of the common algorithms:

- Restricted Boltzmann Machine (RBN)
- Deep Belief Networks (DBN)
- Convolutional Network
- Stacked Autoencoders
- Long short-term memory (LSTM)

12. Dimensionality Reduction

Its purpose is to construct new features that have significant physical or statistical characteristics, such as capturing as much of the variance as possible.

- Principle Component Analysis (PCA)
- Partial Least Square Regression (PLS)
- Multi-Dimensional Scaling (MDS)
- Exploratory Factor Analysis (EFA)

PCA attempts to find uncorrelated linear combinations of original variables that can explain the variance to the greatest extent possible. EFA also tries to explain as much variance as possible in a lower dimension. MDS maps the observed similarity to a low dimension, such as a two-dimensional plane. Instead of extracting underlying components or latent factors, MDS attempts to find a lower-dimensional map that best preserves all the observed similarities between items. So it needs to define a similarity measure as in clustering methods.

13. Ensemble Methods

Ensemble method made its debut in the 1990s. The idea is to build a prediction model by combining the strengths of a collection of simpler base models. Bagging, originally proposed by Leo Breiman, is one of the earliest ensemble methods. After that, people developed Random Forest (T, 1998; Y and D, 1997) and Boosting method (L, 1984; M and L, 1989). This is a class of powerful and effective algorithms.

- Bootstrapped Aggregation (Bagging)
- Random Forest
- Gradient Boosting Machine (GBM)

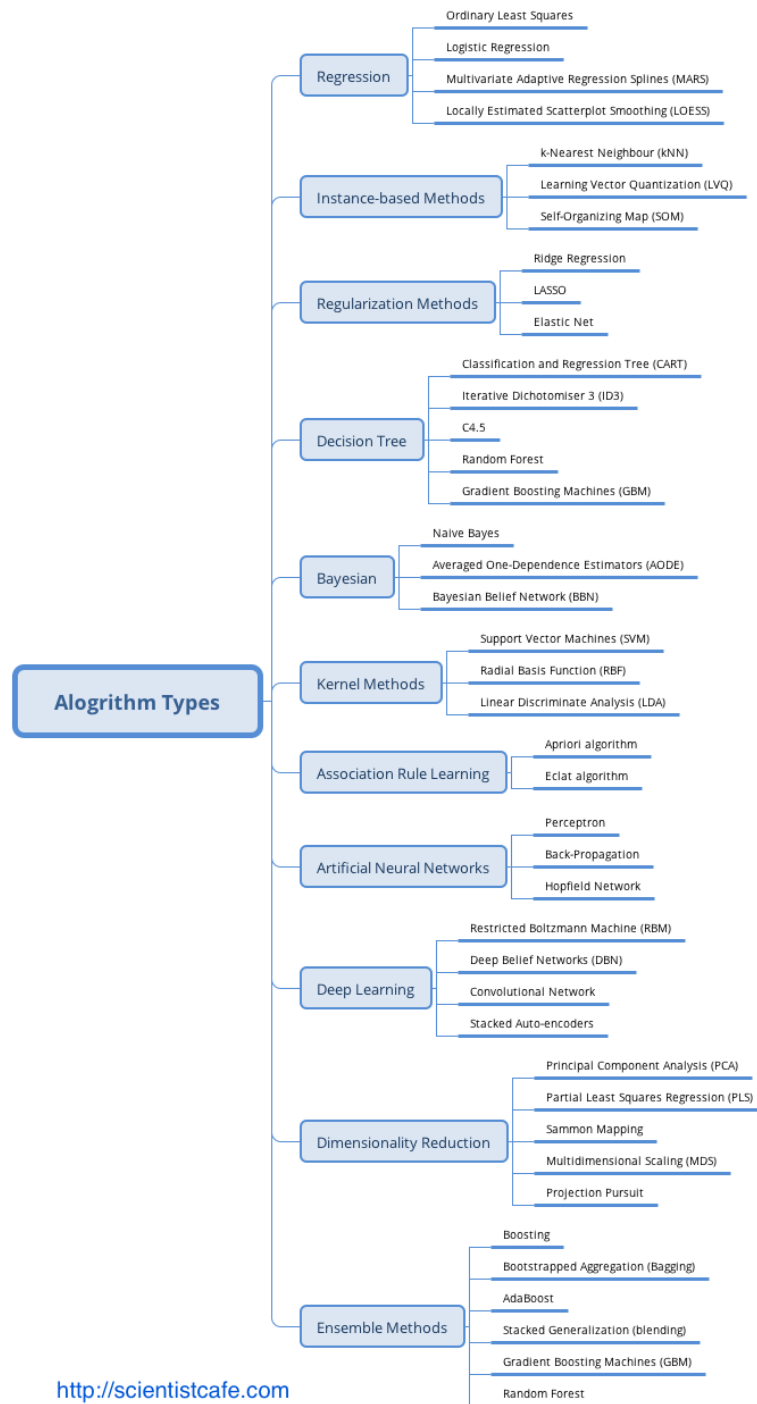


FIGURE 1.5: Machines Learning Algorithms



2

Introduction to the data

Before tackling analytics problem, we start by introducing data to be analyzed in later chapters.

2.1 Customer Data for Clothing Company

Our first data set represents customers of a clothing company who sells products in stores and online. This data is typical of what one might get from a company's marketing data base (the data base will have more data than the one we show here). This data includes 1000 customers for whom we have 3 types of data:

1. Demography
 - age: age of the respondent
 - gender: male/female
 - house: 0/1 variable indicating if the customer owns a house or not
2. Sales in the past year
 - store_exp: expense in store
 - online_exp: expense online
 - store_trans: times of store purchase
 - online_trans: times of online purchase
3. Survey on product preference

It is common for companies to survey their customers and draw insights to guide future marketing activities. The survey is as below:

How strongly do you agree or disagree with the following statements:

1. Strong disagree
 2. Disagree
 3. Neither agree nor disagree
 4. Agree
 5. Strongly agree
- Q1. I like to buy clothes from different brands
 - Q2. I buy almost all my clothes from some of my favorite brands
 - Q3. I like to buy premium brands
 - Q4. Quality is the most important factor in my purchasing decision
 - Q5. Style is the most important factor in my purchasing decision
 - Q6. I prefer to buy clothes in store
 - Q7. I prefer to buy clothes online
 - Q8. Price is important
 - Q9. I like to try different styles
 - Q10. I like to make a choice by myself and don't need too much of others' suggestions

There are 4 segments of customers:

1. Price
2. Conspicuous
3. Quality
4. Style

Let's check it:

```
str(sim.dat,vec.len=3)
```

```
## 'data.frame':   1000 obs. of  19 variables:
## $ age          : int  57 63 59 60 51 59 57 57 ...
## $ gender       : Factor w/ 2 levels "Female","Male": 1 1 2 2 2 2 2 2 ...
## $ income       : num  120963 122008 114202 113616 ...
## $ house        : Factor w/ 2 levels "No","Yes": 2 2 2 2 2 2 2 2 ...
## $ store_exp    : num  529 478 491 348 ...
```



```
## $ online_exp : num 304 110 279 142 ...
## $ store_trans : int 2 4 7 10 4 4 5 11 ...
## $ online_trans: int 2 2 2 2 4 5 3 5 ...
## $ Q1          : int 4 4 5 5 4 4 4 5 ...
## $ Q2          : int 2 1 2 2 1 2 1 2 ...
## $ Q3          : int 1 1 1 1 1 1 1 1 ...
## $ Q4          : int 2 2 2 3 3 2 2 3 ...
## $ Q5          : int 1 1 1 1 1 1 1 1 ...
## $ Q6          : int 4 4 4 4 4 4 4 4 ...
## $ Q7          : int 1 1 1 1 1 1 1 1 ...
## $ Q8          : int 4 4 4 4 4 4 4 4 ...
## $ Q9          : int 2 1 1 2 2 1 1 2 ...
## $ Q10         : int 4 4 4 4 4 4 4 4 ...
## $ segment     : Factor w/ 4 levels "Conspicuous",...: 2 2 2 2 2 2 2 2 ...
```

2.2 Customer Satisfaction Survey Data from Airline Company

This data set is from a customer satisfaction survey for three airline companies. There are $N=1000$ respondents and 15 questions. The market researcher asked respondents to recall the experience with different airline companies and assign a score (1-9) to each airline company for all the 15 questions. The higher the score, the more satisfied the customer to the specific item. The 15 questions are of 4 types (the variable names are in the parentheses):

- How satisfied are you with your_____?
 1. Ticketing
 - Ease of making reservation Easy_Reservation
 - Availability of preferred seats Preferred_Seats
 - Variety of flight options Flight_Options
 - Ticket prices Ticket_Prices
 2. Aircraft
 - Seat comfort Seat_Comfort

- Roominess of seat area Seat_Roominess
 - Availability of Overhead Overhead_Storage
 - Cleanliness of aircraft Clean_Aircraft
3. Service
 - Courtesy of flight attendant Courtesy
 - Friendliness Friendliness
 - Helpfulness Helpfulness
 - Food and drinks Service
 4. General
 - Overall satisfaction Satisfaction
 - Purchase again Fly_Again
 - Willingness to recommend Recommend

Now check the data frame we have:

```
str(rating,vec.len=3)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':  3000 obs. of  17 variables:
## $ Easy_Reservation: int  6 5 6 5 4 5 6 4 ...
## $ Preferred_Seats : int  5 7 6 6 5 6 6 6 ...
## $ Flight_Options  : int  4 7 5 5 3 4 6 3 ...
## $ Ticket_Prices   : int  5 6 6 5 6 5 5 5 ...
## $ Seat_Comfort    : int  5 6 7 7 6 6 6 4 ...
## $ Seat_Roominess  : int  7 8 6 8 7 8 6 5 ...
## $ Overhead_Storage: int  5 5 7 6 5 4 4 4 ...
## $ Clean_Aircraft  : int  7 6 7 7 7 7 6 4 ...
## $ Courtesy        : int  5 6 6 4 2 5 5 4 ...
## $ Friendliness    : int  4 6 6 6 3 4 5 5 ...
## $ Helpfulness     : int  6 5 6 4 4 5 5 4 ...
## $ Service         : int  6 5 6 5 3 5 5 5 ...
## $ Satisfaction    : int  6 7 7 5 4 6 5 5 ...
## $ Fly_Again       : int  6 6 6 7 4 5 3 4 ...
## $ Recommend       : int  3 6 5 5 4 5 6 5 ...
## $ ID              : int  1 2 3 4 5 6 7 8 ...
## $ Airline         : chr  "AirlineCo.1" "AirlineCo.1" "AirlineCo.1" ...
```

3

Data Pre-processing

Many data analysis related books focus on models, algorithms and statistical inferences. However, in practice, raw data is usually not directly used for modeling. Data preprocessing is the process of converting raw data into clean data that is proper for modeling. A model fails for various reasons. One is that the modeler doesn't correctly preprocess data before modeling. Data preprocessing can significantly impact model results, such as imputing missing value and handling with outliers. So data preprocessing is a very critical part.

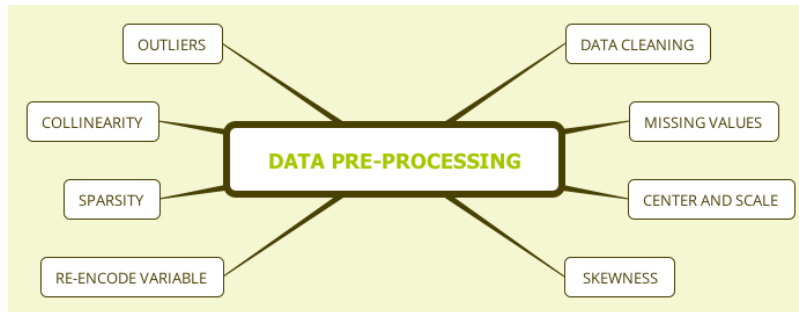


FIGURE 3.1: Data Pre-processing Outline

In real life, depending on the stage of data cleanup, data has the following types:

1. Raw data
2. Technically correct data
3. Data that is proper for the model
4. Summarized data
5. Data with fixed format

The raw data is the first-hand data that analysts pull from the database, market survey responds from your clients, the experimental results collected by the R & D department, and so on. These data may be very rough, and R sometimes can't read them directly. The table title could be multi-line, or the format does not meet the requirements:

- Use 50% to represent the percentage rather than 0.5, so R will read it as a character;
- The missing value of the sales is represented by “-” instead of space so that R will treat the variable as character or factor type;
- The data is in a slideshow document, or the spreadsheet is not “.csv” but “.xlsx”
- ...

Most of the time, you need to clean the data so that R can import them. Some data format requires a specific package. Technically correct data is the data, after preliminary cleaning or format conversion, that R (or another tool you use) can successfully import it.

Assume we have loaded the data into R with reasonable column names, variable format and so on. That does not mean the data is entirely correct. There may be some observations that do not make sense, such as age is negative, the discount percentage is greater than 1, or data is missing. Depending on the situation, there may be a variety of problems with the data. It is necessary to clean the data before modeling. Moreover, different models have different requirements on the data. For example, some model may require the variables are of consistent scale; some may be susceptible to outliers or collinearity, some may not be able to handle categorical variables and so on. The modeler has to preprocess the data to make it proper for the specific model.

Sometimes we need to aggregate the data. For example, add up the daily sales to get annual sales of a product at different locations. In customer segmentation, it is common practice to build a profile for each segment. It requires calculating some statistics such as average age, average income, age standard deviation, etc.

Data aggregation is also necessary for presentation, or for data visualization.

The final table results for clients need to be in a nicer format than what used in the analysis. Usually, data analysts will take the results from data scientists and adjust the format, such as labels, cell color, highlight. It is important for a data scientist to make sure the results look consistent which makes the next step easier for data analysts.

It is highly recommended to store each step of the data and the R code, making the whole process as repeatable as possible. The R markdown reproducible report will be extremely helpful for that. If the data changes, it is easy to rerun the process. In the remainder of this chapter, we will show the most common data preprocessing methods.

Load the R packages first:

```
source("https://raw.githubusercontent.com/happyrabbit/CE_JSM2017/master/Rcode/00-course-setup.R")
```

3.1 Data Cleaning

After you load the data, the first thing is to check how many variables are there, the type of variables, the distributions, and data errors. Let's read and check the data:

```
sim.dat <- read.csv("https://raw.githubusercontent.com/happyrabbit/DataScientistR/master/Data/sim.dat")
summary(sim.dat)
```

```
##      age      gender      income
##  Min.   : 16.0   Female:554   Min.     : 41776
##  1st Qu.: 25.0   Male  :446   1st Qu.: 85832
##  Median : 36.0                      Median : 93869
##  Mean   : 38.8                      Mean    :113543
##  3rd Qu.: 53.0                      3rd Qu.:124572
##  Max.   :300.0                      Max.     :319704
```

```

##                                     NA's   :184
## house          store_exp          online_exp
## No :432   Min.    : -500   Min.    : 69
## Yes:568   1st Qu.: 205   1st Qu.: 420
##           Median : 329   Median :1942
##           Mean   : 1357   Mean   :2120
##           3rd Qu.: 597   3rd Qu.:2441
##           Max.   :50000   Max.   :9479
##
## store_trans      online_trans      Q1
## Min.    : 1.00   Min.    : 1.0   Min.    :1.0
## 1st Qu.: 3.00   1st Qu.: 6.0   1st Qu.:2.0
## Median : 4.00   Median :14.0   Median :3.0
## Mean   : 5.35   Mean   :13.6   Mean   :3.1
## 3rd Qu.: 7.00   3rd Qu.:20.0   3rd Qu.:4.0
## Max.   :20.00   Max.   :36.0   Max.   :5.0
##
##           Q2           Q3           Q4
## Min.    :1.00   Min.    :1.00   Min.    :1.00
## 1st Qu.:1.00   1st Qu.:1.00   1st Qu.:2.00
## Median :1.00   Median :1.00   Median :3.00
## Mean   :1.82   Mean   :1.99   Mean   :2.76
## 3rd Qu.:2.00   3rd Qu.:3.00   3rd Qu.:4.00
## Max.   :5.00   Max.   :5.00   Max.   :5.00
##
##           Q5           Q6           Q7
## Min.    :1.00   Min.    :1.00   Min.    :1.00
## 1st Qu.:1.75   1st Qu.:1.00   1st Qu.:2.50
## Median :4.00   Median :2.00   Median :4.00
## Mean   :2.94   Mean   :2.45   Mean   :3.43
## 3rd Qu.:4.00   3rd Qu.:4.00   3rd Qu.:4.00
## Max.   :5.00   Max.   :5.00   Max.   :5.00
##
##           Q8           Q9           Q10
## Min.    :1.0   Min.    :1.00   Min.    :1.00
## 1st Qu.:1.0   1st Qu.:2.00   1st Qu.:1.00
## Median :2.0   Median :4.00   Median :2.00

```

```
## Mean      :2.4   Mean      :3.08   Mean      :2.32
## 3rd Qu.:3.0   3rd Qu.:4.00   3rd Qu.:3.00
## Max.      :5.0   Max.      :5.00   Max.      :5.00
##
##          segment
## Conspicuous:200
## Price       :250
## Quality     :200
## Style       :350
##
##
##
```

Are there any problems? Questionnaire response Q1-Q10 seem reasonable, the minimum is 1 and maximum is 5. Recall that the questionnaire score is 1-5. The number of store transactions (`store_trans`) and online transactions (`store_trans`) make sense too. Things need to pay attention are:

- There are some missing values.
- There are outliers for store expenses (`store_exp`). The maximum value is 50000. Who would spend \$50000 a year buying clothes? Is it an imputation error?
- There is a negative value (-500) in `store_exp` which is not logical.
- Someone is 300 years old.

How to deal with that? Depending on the real situation, if the sample size is large enough, it will not hurt to delete those problematic samples. Here we have 1000 observations. Since marketing survey is usually expensive, it is better to set these values as missing and impute them instead of deleting the rows.

```
# set problematic values as missings
sim.dat$age[which(sim.dat$age>100)]<-NA
sim.dat$store_exp[which(sim.dat$store_exp<0)]<-NA
# see the results
summary(subset(sim.dat,select=c("age","income")))
```

```
##          age          income
## Min.      :16.0   Min.      : 41776
```

```
## 1st Qu.:25.0    1st Qu.: 85832
## Median :36.0    Median : 93869
## Mean   :38.6    Mean   :113543
## 3rd Qu.:53.0    3rd Qu.:124572
## Max.   :69.0    Max.   :319704
## NA's   :1       NA's   :184
```

Now we will deal with the missing values in the data.

3.2 Missing Values

You can write a whole book about missing value. This section will only show some of the most commonly used methods without getting too deep into the topic. Chapter 7 of the book by De Waal, Pannekoek and Scholtus ([de Waal et al., 2011](#)) makes a concise overview of some of the existing imputation methods. The choice of specific method depends on the actual situation. There is no best way.

One question to ask before imputation: Is there any auxiliary information? Being aware of any auxiliary information is critical. For example, if the system set customer who did not purchase as missing, then the real purchasing amount should be 0. Is missing a random occurrence? If so, it may be reasonable to impute with mean or median. If not, is there a potential mechanism for the missing data? For example, older people are more reluctant to disclose their ages in the questionnaire, so that the absence of age is not completely random. In this case, the missing values need to be estimated using the relationship between age and other independent variables. For example, use variables such as whether they have children, income, and other survey questions to build a model to predict age.

Also, the purpose of modeling is important for selecting imputation methods. If the goal is to interpret the parameter estimate or statistical inference, then it is important to study the missing

mechanism carefully and to estimate the missing values using non-missing information as much as possible. If the goal is to predict, people usually will not study the absence mechanism rigorously (but sometimes the mechanism is obvious). If the absence mechanism is not clear, treat it as missing at random and use mean, median, or k-nearest neighbor to impute. Since statistical inference is sensitive to missing values, researchers from survey statistics have conducted in-depth studies of various imputation schemes which focus on valid statistical inference. The problem of missing values in the prediction model is different from that in the traditional survey. Therefore, there are not many papers on missing value imputation in the prediction model. Those who want to study further can refer to Saar-Tsechansky and Provost's comparison of different imputation methods (M and F, 007b) and De Waal, Pannekoek and Scholtus' book (de Waal et al., 2011).

3.2.1 Impute missing values with median/mode

In the case of missing at random, a common method is to impute with the mean (continuous variable) or median (categorical variables). You can use `impute()` function in `imputeMissings` package.

```
# save the result as another object
demo_imp<-impute(sim.dat,method="median/mode")
# check the first 5 columns, there is no missing values in other columns
summary(demo_imp[,1:5])
```

```
##      age      gender      income
##  Min.   :16.0  Female:554  Min.    : 41776
##  1st Qu.:25.0  Male  :446  1st Qu.: 87896
##  Median :36.0                      Median : 93869
##  Mean   :38.6                      Mean   :109923
##  3rd Qu.:53.0                      3rd Qu.:119456
##  Max.   :69.0                      Max.    :319704
##  house      store_exp
##  No :432  Min.    : 156
##  Yes:568  1st Qu.: 205
##                      Median : 330
```

```
##          Mean   : 1358
##          3rd Qu.:  597
##          Max.    :50000
```

After imputation, `demo_imp` has no missing value. This method is straightforward and widely used. The disadvantage is that it does not take into account the relationship between the variables. When there is a significant proportion of missing, it will distort the data. In this case, it is better to consider the relationship between variables and study the missing mechanism. In the example here, the missing variables are numeric. If the missing variable is a categorical/factor variable, the `impute()` function will impute with the mode.

You can also use `preProcess()` function, but it is only for numeric variables, and can not impute categorical variables. Since missing values here are numeric, we can use the `preProcess()` function. The result is the same as the `impute()` function. `preProcess()` is a powerful function that can link to a variety of data preprocessing methods. We will use the function later for other data preprocessing.

```
imp<-preProcess(sim.dat,method="medianImpute")
demo_imp2<-predict(imp,sim.dat)
summary(demo_imp2[,1:5])
```

```
##          age          gender          income
## Min.      :16.0    Female:554    Min.       : 41776
## 1st Qu.:25.0    Male  :446    1st Qu.: 87896
## Median :36.0                                Median : 93869
## Mean      :38.6                                Mean   :109923
## 3rd Qu.:53.0                                3rd Qu.:119456
## Max.      :69.0                                Max.    :319704
## house          store_exp
## No :432    Min.      : 156
## Yes:568    1st Qu.:  205
##                      Median :  330
##                      Mean    : 1358
##                      3rd Qu.:  597
```

```
##               Max.      :50000
```

3.2.2 K-nearest neighbors

K-nearest neighbor (KNN) will find the k closest samples (Euclidean distance) in the training set and impute the mean of those “neighbors.”

Use `preProcess()` to conduct KNN:

```
imp<-preProcess(sim.dat,method="knnImpute",k=5)
# need to use predict() to get KNN result
demo_imp<-predict(imp,sim.dat)
```

```
Error in `[.data.frame`(old, , non_missing_cols, drop = FALSE) :
  undefined columns selected
```

Now we get an error saying “undefined columns selected.” It is because `sim.dat` has non-numeric variables. The `preProcess()` in the first line will automatically ignore non-numeric columns, so there is no error. However, there is a problem when using `predict()` to get the result. Removing those variable will solve the problem.

```
# find factor columns
imp<-preProcess(sim.dat,method="knnImpute",k=5)
idx<-which(lapply(sim.dat,class)== "factor")
demo_imp<-predict(imp,sim.dat[, -idx])
summary(demo_imp[,1:3])
```

```
##      age      income      store_exp
##  Min.   :-1.591   Min.   :-1.440   Min.   :-0.433
##  1st Qu.: -0.957   1st Qu.: -0.537   1st Qu.: -0.416
##  Median :-0.182   Median :-0.376   Median :-0.371
##  Mean    : 0.000   Mean    : 0.024   Mean    : 0.000
##  3rd Qu.: 1.016   3rd Qu.: 0.215   3rd Qu.: -0.274
##  Max.    : 2.144   Max.    : 4.136   Max.    :17.527
```

`lapply(data,class)` can return a list of column class. Here the data frame is `sim.dat`, and the following code will give the list of column class:

```
# only show the first three elements
lapply(sim.dat,class)[1:3]
```

```
## $age
## [1] "integer"
##
## $gender
## [1] "factor"
##
## $income
## [1] "numeric"
```

Comparing the KNN result with the previous median imputation, the two are very different. This is because when you tell the `preProcess()` function to use KNN (the option `method = "knnImpute"`), it will automatically standardize the data. Another way is to use Bagging tree (in the next section). Note that KNN can not impute samples with the entire row missing. The reason is straightforward. Since the algorithm uses the average of its neighbors if none of them has a value, what does it apply to calculate the mean? Let's append a new row with all values missing to the original data frame to get a new object called `temp`. Then apply KNN to `temp` and see what happens:

```
temp<-rbind(sim.dat,rep(NA,ncol(sim.dat)))
imp<-preProcess(sim.dat,method="knnImpute",k=5)
idx<-which(lapply(temp,class)=="factor")
demo_imp<-predict(imp,temp[,-idx])
```

```
Error in FUN(newX[, i], ...) :
  cannot impute when all predictors are missing in the new data point
```

There is an error saying “cannot impute when all predictors are missing in the new data point”. It is easy to fix by finding and removing the problematic row:

```
idx<-apply(temp,1,function(x) sum(is.na(x)) )
as.vector(which(idx==ncol(temp)))
```

```
## [1] 1001
```

It shows that row 1001 is problematic. You can go ahead to delete it.

3.2.3 Bagging Tree

Bagging (Bootstrap aggregating) was originally proposed by Leo Breiman. It is one of the earliest ensemble methods (L, 966a). When used in missing value imputation, it will use the remaining variables as predictors to train a bagging tree and then use the tree to predict the missing values. Although theoretically, the method is powerful, the computation is much more intense than KNN. In practice, there is a trade-off between computation time and the effect. If a median or mean meet the modeling needs, even bagging tree may improve the accuracy a little, but the upgrade is so marginal that it does not deserve the extra time. The bagging tree itself is a model for regression and classification. Here we use `preProcess()` to impute `sim.dat`:

```
imp<-preProcess(sim.dat,method="bagImpute")
demo_imp<-predict(imp,sim.dat)
summary(demo_imp[,1:5])
```

age	gender	income	house	store_exp
Min. :16.00	Female:554	Min. : 41776	No :432	Min. : 155.8
1st Qu.:25.00	Male :446	1st Qu.: 86762	Yes:568	1st Qu.: 205.1
Median :36.00		Median : 94739		Median : 329.0
Mean :38.58		Mean :114665		Mean : 1357.7
3rd Qu.:53.00		3rd Qu.:123726		3rd Qu.: 597.3
Max. :69.00		Max. :319704		Max. :50000.0

3.3 Centering and Scaling

It is the most straightforward data transformation. It centers and scales a variable to mean 0 and standard deviation 1. It ensures that the criterion for finding linear combinations of the predictors is based on how much variation they explain and therefore improves the numerical stability. Models involving finding linear combinations of the predictors to explain response/predictors variation need data centering and scaling, such as PCA (Jolliffe, 2002), PLS (Geladi P, 1986) and EFA (Mulaik, 2009). You can quickly write code yourself to conduct this transformation.

Let's standardize the variable `income` from `sim.dat`:

```
income<-sim.dat$income
# calculate the mean of income
mux<-mean(income,na.rm=T)
# calculate the standard deviation of income
sdx<-sd(income,na.rm=T)
# centering
tr1<-income-mux
# scaling
tr2<-tr1/sdx
```

Or the function `preProcess()` in package `caret` can apply this transformation to a set of predictors.

```
sdat<-subset(sim.dat,select=c("age","income"))
# set the "method" option
trans<-preProcess(sdat,method=c("center","scale"))
# use predict() function to get the final result
transformed<-predict(trans,sdat)
```

Now the two variables are in the same scale:

```
summary(transformed)
```

```
##          age          income
```

```
## Min.    :-1.591    Min.    :-1.44
## 1st Qu.: -0.957    1st Qu.: -0.56
## Median : -0.182    Median : -0.39
## Mean   :  0.000    Mean   :  0.00
## 3rd Qu.:  1.016    3rd Qu.:  0.22
## Max.    :  2.144    Max.    :  4.14
## NA's    :1         NA's    :184
```

Sometimes you only need to scale the variable. For example, if the model adds a penalty to the parameter estimates (such as L_2 penalty is ridge regression and L_1 penalty in LASSO), the variables need to have a similar scale to ensure a fair variable selection. I am a heavy user of this kind of penalty-based model in my work, and I used the following quantile transformation:

$$x_{ij}^* = \frac{x_{ij} - \text{quantile}(x_{.j}, 0.01)}{\text{quantile}(x_{.j} - 0.99) - \text{quantile}(x_{.j}, 0.01)}$$

The reason to use 99% and 1% quantile instead of maximum and minimum values is to resist the impact of outliers.

It is easy to write a function to do it:

```
qscale<-function(dat){
  for (i in 1:ncol(dat)){
    up<-quantile(dat[,i],0.99)
    low<-quantile(dat[,i],0.01)
    diff<-up-low
    dat[,i]<-(dat[,i]-low)/diff
  }
  return(dat)
}
```

In order to illustrate, let's apply it to some variables from 'demo_imp2':

```
demo_imp3<-qscale(subset(demo_imp2,select=c("income","store_exp","online_exp")))
summary(demo_imp3)
```

```
##      income      store_exp      online_exp
```

```
## Min.    :-0.0578   Min.    :-0.003   Min.    :-0.0060
## 1st Qu.: 0.1574   1st Qu.: 0.004   1st Qu.: 0.0427
## Median : 0.1852   Median : 0.023   Median : 0.2537
## Mean    : 0.2601   Mean     : 0.177   Mean     : 0.2784
## 3rd Qu.: 0.3046   3rd Qu.: 0.063   3rd Qu.: 0.3229
## Max.    : 1.2386   Max.     : 7.477   Max.     : 1.2988
```

After transformation, most of the variables are between 0-1.

3.4 Resolve Skewness

Skewness¹ is defined to be the third standardized central moment. The formula for the sample skewness statistics is:

$$skewness = \frac{\sum (x_i - \bar{x})^3}{(n-1)v^{3/2}}$$

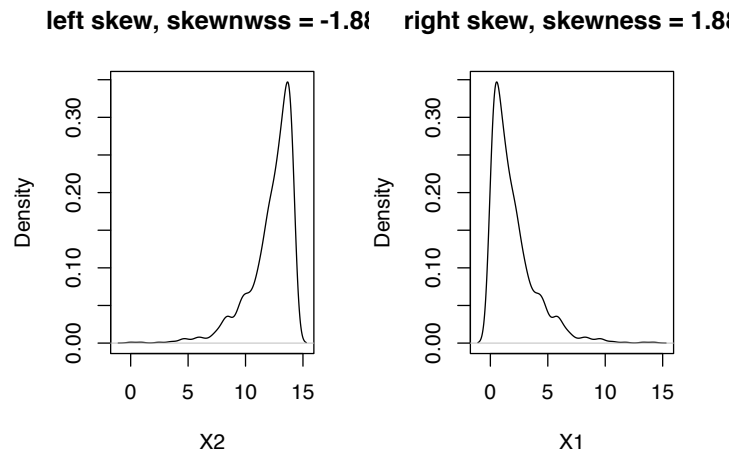
$$v = \frac{\sum (x_i - \bar{x})^2}{(n-1)}$$

Skewness=0 means that the distribution is symmetric, i.e. the probability of falling on either side of the distribution's mean is equal.

```
# need skewness() function from e1071 package
set.seed(1000)
par(mfrow=c(1,2), oma=c(2,2,2,2))
# random sample 1000 chi-square distribution with df=2
# right skew
x1<-rchisq(1000,2, ncp = 0)
# get left skew variable x2 from x1
x2<-max(x1)-x1
plot(density(x2),main=paste("left skew, skewnwss =",round(skewness(x2),2)), xlab="X2")
plot(density(x1),main=paste("right skew, skewness =",round(skewness(x1),2)), xlab="X1")
```

You can easily tell if a distribution is skewed by simple visualization(Figure3.2). There are different ways may help to remove

¹<https://en.wikipedia.org/wiki/Skewness>

**FIGURE 3.2:** Shewed Distribution

skewness such as log, square root or inverse. However, it is often difficult to determine from plots which transformation is most appropriate for correcting skewness. The Box-Cox procedure automatically identified a transformation from the family of power transformations that are indexed by a parameter λ (Box & G, 1964).

$$x^* = \begin{cases} \frac{x^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \log(x) & \text{if } \lambda = 0 \end{cases}$$

It is easy to see that this family includes log transformation ($\lambda = 0$), square transformation ($\lambda = 2$), square root ($\lambda = 0.5$), inverse ($\lambda = -1$) and others in-between. We can still use function `preProcess()` in package `caret` to apply this transformation by changing the `method` argument.

```
describe(sim.dat)
```

##	vars	n	mean	sd	median
## age	1	999	38.58	14.19	36.0
## gender*	2	1000	1.45	0.50	1.0
## income	3	816	113543.07	49842.29	93868.7
## house*	4	1000	1.57	0.50	2.0
## store_exp	5	999	1358.71	2775.17	329.8

```

## online_exp      6 1000    2120.18  1731.22  1941.9
## store_trans     7 1000      5.35    3.70    4.0
## online_trans    8 1000    13.55    7.96   14.0
## Q1              9 1000     3.10    1.45    3.0
## Q2             10 1000     1.82    1.17    1.0
## Q3             11 1000     1.99    1.40    1.0
## Q4             12 1000     2.76    1.16    3.0
## Q5             13 1000     2.94    1.28    4.0
## Q6             14 1000     2.45    1.44    2.0
## Q7             15 1000     3.43    1.46    4.0
## Q8             16 1000     2.40    1.15    2.0
## Q9             17 1000     3.08    1.12    4.0
## Q10            18 1000     2.32    1.14    2.0
## segment*       19 1000     2.70    1.15    3.0
##               trimmed      mad      min      max  range
## age              37.67    16.31    16.00     69    53
## gender*          1.43     0.00     1.00     2     1
## income          104841.94 28989.47 41775.64 319704 277929
## house*           1.58     0.00     1.00     2     1
## store_exp        845.14   197.47   155.81  50000  49844
## online_exp       1874.51  1015.21   68.82   9479   9411
## store_trans       4.89     2.97     1.00    20    19
## online_trans     13.42    10.38     1.00    36    35
## Q1                3.13     1.48     1.00     5     4
## Q2                1.65     0.00     1.00     5     4
## Q3                1.75     0.00     1.00     5     4
## Q4                2.83     1.48     1.00     5     4
## Q5                3.05     0.00     1.00     5     4
## Q6                2.43     1.48     1.00     5     4
## Q7                3.54     0.00     1.00     5     4
## Q8                2.36     1.48     1.00     5     4
## Q9                3.23     0.00     1.00     5     4
## Q10               2.27     1.48     1.00     5     4
## segment*         2.75     1.48     1.00     4     3
##               skew kurtosis      se
## age              0.47    -1.18    0.45
## gender*          0.22    -1.95    0.02

```

```
## income      1.69      2.57 1744.83
## house*     -0.27     -1.93   0.02
## store_exp   8.08   115.04  87.80
## online_exp  1.18     1.31  54.75
## store_trans 1.11     0.69   0.12
## online_trans 0.03    -0.98   0.25
## Q1         -0.12    -1.36   0.05
## Q2          1.13    -0.32   0.04
## Q3          1.06    -0.40   0.04
## Q4         -0.18    -1.46   0.04
## Q5         -0.60    -1.40   0.04
## Q6          0.11    -1.89   0.05
## Q7         -0.90    -0.79   0.05
## Q8          0.21    -1.33   0.04
## Q9         -0.68    -1.10   0.04
## Q10         0.39    -1.23   0.04
## segment*   -0.20    -1.41   0.04
```

It is easy to see the skewed variables. If `mean` and `trimmed` differ a lot, there is very likely outliers. By default, `trimmed` reports mean by dropping the top and bottom 10%. It can be adjusted by setting argument `trim=`. It is clear that `store_exp` has outliers.

As an example, we will apply Box-Cox transformation on `store_trans` and `online_trans`:

```
# select the two columns and save them as dat_bc
dat_bc<-subset(sim.dat,select=c("store_trans","online_trans"))
(trans<-preProcess(dat_bc,method=c("BoxCox")))
```

```
## Created from 1000 samples and 2 variables
##
## Pre-processing:
##   - Box-Cox transformation (2)
##   - ignored (0)
##
## Lambda estimates for Box-Cox transformation:
## 0.1, 0.7
```

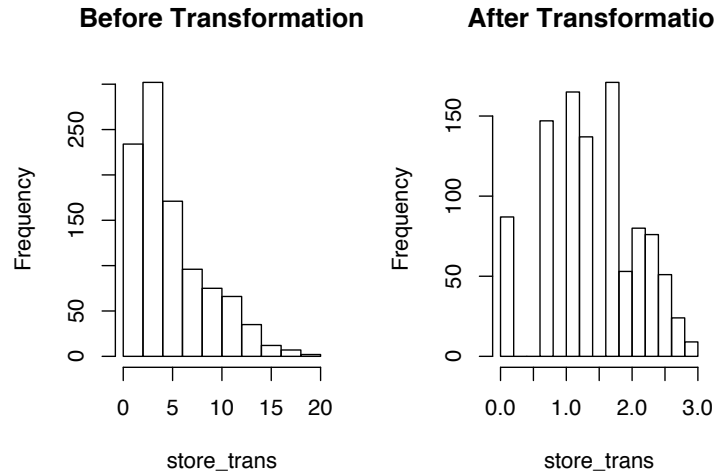


FIGURE 3.3: Box-Cox Transformation

The last line of the output shows the estimates of λ for each variable. As before, use `predict()` to get the transformed result:

```
transformed<-predict(trans,dat_bc)
par(mfrow=c(1,2),oma=c(2,2,2,2))
hist(dat_bc$store_trans,main="Before Transformation",xlab="store_trans")
hist(transformed$store_trans,main="After Transformation",xlab="store_trans")
```

Before the transformation, the `store_trans` is skewed right. The situation is significantly improved after (figure 3.3). `BoxCoxTrans()` can also conduct Box-Cox transform. But note that `BoxCoxTrans()` can only be applied to a single variable, and it is not possible to transform different columns in a data frame at the same time.

```
(trans<-BoxCoxTrans(dat_bc$store_trans))

## Box-Cox Transformation
##
## 1000 data points used to estimate Lambda
##
## Input data summary:
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      1.00   3.00   4.00   5.35   7.00  20.00
##
```

```
## Largest/Smallest: 20
## Sample Skewness: 1.11
##
## Estimated Lambda: 0.1
## With fudge factor, Lambda = 0 will be used for transformations
transformed<-predict(trans,dat_bc$store_trans)
skewness(transformed)

## [1] -0.2155
```

The estimate of λ is the same as before (0.1). The skewness of the original observation is 1.1, and -0.2 after transformation. Although it is not strictly 0, it is greatly improved.

3.5 Resolve Outliers

Even under certain assumptions we can statistically define outliers, it can be hard to define in some situations. Box plot, histogram and some other basic visualizations can be used to initially check whether there are outliers. For example, we can visualize numerical non-survey variables in `sim.dat`:

```
# select numerical non-survey data
sdat<-subset(sim.dat,select=c("age","income","store_exp","online_exp","store_trans","online_tr
# use scatterplotMatrix() function from car package
par(oma=c(2,2,1,2))
scatterplotMatrix(sdat,diagonal="boxplot",smoother=FALSE)
```

As figure 3.4 shows, `store_exp` has outliers. It is also easy to observe the pair relationship from the plot. `age` is negatively correlated with `online_trans` but positively correlated with `store_trans`. It seems that older people tend to purchase from the local store. The amount of expense is positively correlated with income. Scatterplot matrix like this can reveal lots of information before modeling.

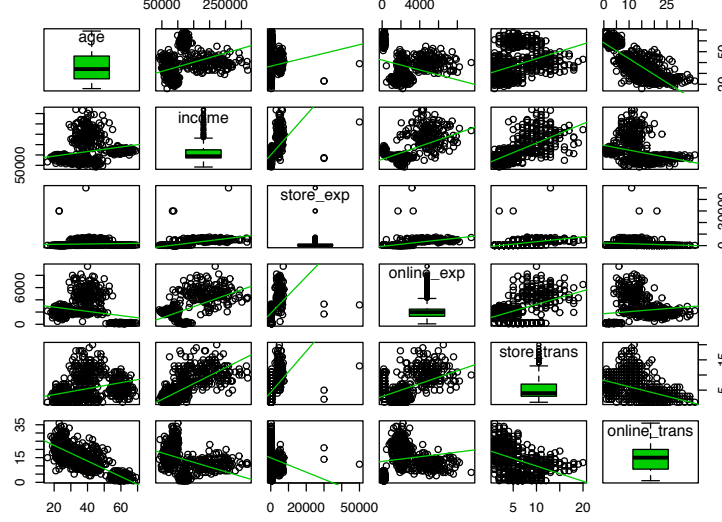


FIGURE 3.4: Use basic visualization to check outliers

In addition to visualization, there are some statistical methods to define outliers, such as the commonly used Z-score. The Z-score for variable \mathbf{Y} is defined as:

$$Z_i = \frac{Y_i - \bar{Y}}{s}$$

where \bar{Y} and s are mean and standard deviation for Y . Z-score is a measurement of the distance between each observation and the mean. This method may be misleading, especially when the sample size is small. Iglewicz and Hoaglin proposed to use the modified Z-score to determine the outlier (Iglewicz and Hoaglin, 1993)

$$M_i = \frac{0.6745(Y_i - \bar{Y})}{MAD}$$

Where MAD is the median of a series of $|Y_i - \bar{Y}|$, called the median of the absolute dispersion. Iglewicz and Hoaglin suggest that the points with the Z-score greater than 3.5 corrected above are possible outliers. Let's apply it to income:

```
# calculate median of the absolute dispersion for income
ymad<-mad(na.omit(sdat$income))
# calculate z-score
zs<-(sdat$income-mean(na.omit(sdat$income)))/ymad
# count the number of outliers
sum(na.omit(zs>3.5))
```

```
## [1] 59
```

According to modified Z-score, variable income has 59 outliers. Refer to (Iglewicz and Hoaglin, 1993) for other ways of detecting outliers.

The impact of outliers depends on the model. Some models are sensitive to outliers, such as linear regression, logistic regression. Some are pretty robust to outliers, such as tree models, support vector machine. Also, the outlier is not wrong data. It is real observation so cannot be deleted at will. If a model is sensitive to outliers, we can use *spatial sign transformation* (Serneels S, 2006) to minimize the problem. It projects the original sample points to the surface of a sphere by:

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^p x_{ij}^2}}$$

where x_{ij} represents the i^{th} observation and j^{th} variable. As shown in the equation, every observation for sample i is divided by its square mode. The denominator is the Euclidean distance to the center of the p -dimensional predictor space. Three things to pay attention here:

1. It is important to center and scale the predictor data before using this transformation
2. Unlike centering or scaling, this manipulation of the predictors transforms them as a group
3. If there are some variables to remove (for example, highly correlated variables), do it before the transformation

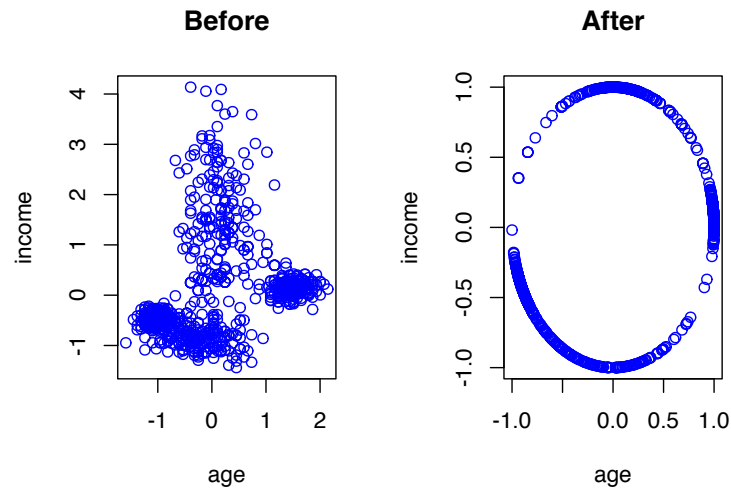


FIGURE 3.5: Spatial sign transformation

Function `spatialSign()` caret package can conduct the transformation. Take income and age as an example:

```
# KNN imputation
sdat<-sim.dat[,c("income", "age")]
imp<-preProcess(sdat,method=c("knnImpute"),k=5)
sdat<-predict(imp,sdat)
transformed <- spatialSign(sdat)
transformed <- as.data.frame(transformed)
par(mfrow=c(1,2),oma=c(2,2,2,2))
plot(income ~ age,data = sdat,col="blue",main="Before")
plot(income ~ age,data = transformed,col="blue",main="After")
```

Some readers may have found that the above code does not seem to standardize the data before transformation. Recall the introduction of KNN, `preProcess()` with `method="knnImpute"` by default will standardize data.

3.6 Collinearity

It is probably the technical term known by the most un-technical people. When two predictors are very strongly correlated, including both in a model may lead to confusion or problem with a singular matrix. There is an excellent function in `corrplot` package with the same name `corrplot()` that can visualize correlation structure of a set of predictors. The function has the option to reorder the variables in a way that reveals clusters of highly correlated ones.

```
# select non-survey numerical variables
sdat<-subset(sim.dat,select=c("age","income","store_exp","online_exp","store_trans","online_tr
# use bagging imputation here
imp<-preProcess(sdat,method="bagImpute")
sdat<-predict(imp,sdat)
# get the correlation matrix
correlation<-cor(sdat)
# plot
par(oma=c(2,2,2,2))
corrplot.mixed(correlation,order="hclust",tl.pos="lt",upper="ellipse")
```

Here use `corrplot.mixed()` function to visualize the correlation matrix (figure 3.6). The closer the correlation is to 0, the lighter the color is and the closer the shape is to a circle. The elliptical means the correlation is not equal to 0 (because we set the `upper = "ellipse"`), the greater the correlation, the narrower the ellipse. Blue represents a positive correlation; red represents a negative correlation. The direction of the ellipse also changes with the correlation. The correlation coefficient is shown in the lower triangle of the matrix. The variables relationship from previous scatter matrix (figure @ref(fig: scm)) are clear here: the negative correlation between age and online shopping, the positive correlation between income and amount of purchasing. Some correlation is very strong (such as the correlation between `online_trans` and `age` is -0.85) which means the two variables contain duplicate information.



FIGURE 3.6: Correlation Matrix

Section 3.5 of “Applied Predictive Modeling” (Kuhn and Johnston, 2013) presents a heuristic algorithm to remove a minimum number of predictors to ensure all pairwise correlations are below a certain threshold:

- (1) Calculate the correlation matrix of the predictors.
- (2) Determine the two predictors associated with the largest absolute pairwise correlation (call them predictors A and B).
- (3) Determine the average correlation between A and the other variables. Do the same for predictor B.
- (4) If A has a larger average correlation, remove it; otherwise, remove predictor B.
- (5) Repeat Step 2-4 until no absolute correlations are above the threshold.

The `findCorrelation()` function in package `caret` will apply the above algorithm.

```
(highCorr<-findCorrelation(cor(sdat),cutoff=.75))
```

```
## [1] 1
```

It returns the index of columns need to be deleted. It tells us that we need to remove the first column to make sure the correlations are all below 0.75.

```
# delete highly correlated columns
sdat<-sdat[-highCorr]
# check the new correlation matrix
cor(sdat)
```

```
##           income store_exp online_exp store_trans
## income      1.0000    0.6004    0.5199    0.7070
## store_exp    0.6004    1.0000    0.5350    0.5399
## online_exp   0.5199    0.5350    1.0000    0.4421
## store_trans  0.7070    0.5399    0.4421    1.0000
## online_trans -0.3573   -0.1367    0.2256   -0.4368
##           online_trans
## income             -0.3573
## store_exp          -0.1367
## online_exp           0.2256
## store_trans        -0.4368
## online_trans        1.0000
```

The absolute value of the elements in the correlation matrix after removal are all below 0.75. How strong does a correlation have to get, before you should start worrying about multicollinearity? There is no easy answer to that question. You can treat the threshold as a tuning parameter and pick one that gives you best prediction accuracy.

3.7 Sparse Variables

Other than the highly related predictors, predictors with degenerate distributions can cause the problem too. Removing those variables can significantly improve some models' performance and stability (such as linear regression and logistic regression but the tree based model is impervious to this type of predictors). One extreme example is a variable with a single value which is called zero-variance variable. Variables with very low frequency of unique values are near-zero variance predictors. In general, detecting those variables follows two rules:

- The fraction of unique values over the sample size
- The ratio of the frequency of the most prevalent value to the frequency of the second most prevalent value.

`nearZeroVar()` function in the `caret` package can filter near-zero variance predictors according to the above rules. In order to show the usage of the function, let's arbitrarily add some problematic variables to the original data `sim.dat`:

```
# make a copy
zero_demo<-sim.dat
# add two sparse variable
# zero1 only has one unique value
# zero2 is a vector with the first element 1 and the rest are 0s
zero_demo$zero1<-rep(1,nrow(zero_demo))
zero_demo$zero2<-c(1,rep(0,nrow(zero_demo)-1))
```

The function will return a vector of integers indicating which columns to remove:

```
nearZeroVar(zero_demo,freqCut = 95/5, uniqueCut = 10)
```

As expected, it returns the two columns we generated. You can go ahead to remove them. Note the two arguments in the function `freqCut =` and `uniqueCut =` are corresponding to the previous two rules.

- `freqCut`: the cutoff for the ratio of the most common value to the second most common value
- `uniqueCut`: the cutoff for the percentage of distinct values out of the number of total samples

3.8 Re-encode Dummy Variables

A dummy variable is a binary variable (0/1) to represent subgroups of the sample. Sometimes we need to recode categories to smaller bits of information named “dummy variables.” For example, some questionnaires have five options for each question, A, B, C, D, and E. After you get the data, you will usually convert the corresponding categorical variables for each question into five nominal variables, and then use one of the options as the baseline.

Let’s encode `gender` and `house` from `sim.dat` to dummy variables. There are two ways to implement this. The first is to use `class.ind()` from `nnet` package. However, it only works on one variable at a time.

```
dumVar<-nnet::class.ind(sim.dat$gender)
head(dumVar)
```

```
##      Female Male
## [1,]      1    0
## [2,]      1    0
## [3,]      0    1
## [4,]      0    1
## [5,]      0    1
## [6,]      0    1
```

Since it is redundant to keep both, we need to remove one of them when modeling. Another more powerful function is `dummyVars()` from `caret`:

```
dumMod<-dummyVars(~gender+house+income,
                  data=sim.dat,
                  # use "original variable name + level" as new name
                  levelsOnly=F)
head(predict(dumMod,sim.dat))
```

```
##   gender.Female gender.Male house.No house.Yes income
## 1             1           0       0         1 120963
## 2             1           0       0         1 122008
## 3             0           1       0         1 114202
## 4             0           1       0         1 113616
## 5             0           1       0         1 124253
## 6             0           1       0         1 107661
```

`dummyVars()` can also use formula format. The variable on the right-hand side can be both categorical and numeric. For a numerical variable, the function will keep the variable unchanged. The advantage is that you can apply the function to a data frame without removing numerical variables. Other than that, the function can create interaction term:

```
dumMod<-dummyVars(~gender+house+income+income:gender,
                  data=sim.dat,
                  levelsOnly=F)
head(predict(dumMod,sim.dat))
```

```
##   gender.Female gender.Male house.No house.Yes income
## 1             1           0       0         1 120963
## 2             1           0       0         1 122008
## 3             0           1       0         1 114202
## 4             0           1       0         1 113616
## 5             0           1       0         1 124253
## 6             0           1       0         1 107661
##   gender.Female:income gender.Male:income
## 1             120963                0
## 2             122008                0
## 3                0             114202
## 4                0             113616
## 5                0             124253
```

```
## 6                                0                107661
```

If you think the impact income levels on purchasing behavior is different for male and female, then you may add the interaction term between `income` and `gender`. You can do this by adding `income:gender` in the formula.

3.9 Python Computing

Environmental Setup

```
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"

import numpy as np
import scipy as sp
import pandas as pd
import math

from sklearn.preprocessing import Imputer
from sklearn.preprocessing import StandardScaler

from pandas.plotting import scatter_matrix
import matplotlib.pyplot as plt
```

3.9.1 Data Cleaning



4

Data Wrangling

This chapter focuses on some of the most frequently used data manipulations and shows how to implement them in R and Python. It is critical to explore the data with descriptive statistics (mean, standard deviation, etc.) and data visualization before analysis. Transform data so that the data structure is in line with the requirements of the model. You also need to summarize the results after analysis.

4.1 Data Wrangling Using R

4.1.1 Read and write data

4.1.1.1 `readr`

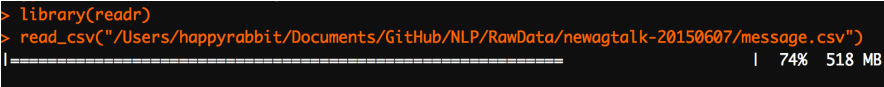
You must be familiar with `read.csv()`, `read.table()` and `write.csv()` in base R. Here we will introduce a more efficient package from RStudio in 2015 for reading and writing data: `readr` package. The corresponding functions are `read_csv()`, `read_table()` and `write_csv()`. The commands look quite similar, but `readr` is different in the following respects:

1. It is 10x faster. The trick is that `readr` uses C++ to process the data quickly.
2. It doesn't change the column names. The names can start with a number and "." will not be substituted to "_". For example:

```
library(readr)
read_csv("2015,2016,2017
1,2,3
4,5,6")
```

```
## # A tibble: 2 x 3
##   `2015` `2016` `2017`
##   <int>  <int>  <int>
## 1       1       2       3
## 2       4       5       6
```

3. readr functions do not convert strings to factors by default, are able to parse dates and times and can automatically determine the data types in each column.
4. The killing character, in my opinion, is that readr provides **progress bar**. What makes you feel worse than waiting is not knowing how long you have to wait.



```
> library(readr)
> read_csv("/Users/happyrabbit/Documents/GitHub/NLP/RawData/newagtalk-20150607/message.csv")
|=====| 74% 518 MB
```

The major functions of readr is to turn flat files into data frames:

- read_csv(): reads comma delimited files
- read_csv2(): reads semicolon separated files (common in countries where , is used as the decimal place)
- read_tsv(): reads tab delimited files
- read_delim(): reads in files with any delimiter
- read_fwf(): reads fixed width files. You can specify fields either by their widths with fwf_widths() or their position with fwf_positions()
- read_table(): reads a common variation of fixed width files where columns are separated by white space
- read_log(): reads Apache style log files

The good thing is that those functions have similar syntax. Once

you learn one, the others become easy. Here we will focus on `read_csv()`.

The most important information for `read_csv()` is the path to your data:

```
library(readr)
sim.dat <- read_csv("https://raw.githubusercontent.com/happyrabbit/DataScientistR/master/Data/
head(sim.dat)
```

```
## # A tibble: 6 x 19
##   age gender income house store_exp online_exp
##   <int> <chr> <dbl> <chr> <dbl> <dbl>
## 1    57 Female 120963   Yes    529.1    303.5
## 2    63 Female 122008   Yes    478.0    109.5
## 3    59   Male 114202   Yes    490.8    279.2
## 4    60   Male 113616   Yes    347.8    141.7
## 5    51   Male 124253   Yes    379.6    112.2
## 6    59   Male 107661   Yes    338.3    195.7
## # ... with 13 more variables: store_trans <int>,
## #   online_trans <int>, Q1 <int>, Q2 <int>, Q3 <int>,
## #   Q4 <int>, Q5 <int>, Q6 <int>, Q7 <int>, Q8 <int>,
## #   Q9 <int>, Q10 <int>, segment <chr>
```

The function reads the file to R as a `tibble`. You can consider `tibble` as next iteration of the data frame. They are different with data frame for the following aspects:

- It never changes an input's type (i.e., no more `stringsAsFactors = FALSE`!)
- It never adjusts the names of variables
- It has a refined print method that shows only the first 10 rows and all the columns that fit on the screen. You can also control the default print behavior by setting options.

Refer to <http://r4ds.had.co.nz/tibbles.html> for more information about 'tibble'.

When you run `read_csv()` it prints out a column specification that gives the name and type of each column. To better understanding

how `readr` works, it is helpful to type in some baby data set and check the results:

```
dat=read_csv("2015,2016,2017
100,200,300
canola,soybean,corn")
print(dat)
```

```
## # A tibble: 2 x 3
##   `2015` `2016` `2017`
##   <chr>  <chr>  <chr>
## 1    100    200    300
## 2 canola soybean  corn
```

You can also add comments on the top and tell R to skip those lines:

```
dat=read_csv("# I will never let you know that
# my favorite food is carrot
Date,Food,Mood
Monday,carrot,happy
Tuesday,carrot,happy
Wednesday,carrot,happy
Thursday,carrot,happy
Friday,carrot,happy
Saturday,carrot,extremely happy
Sunday,carrot,extremely happy", skip = 2)
print(dat)
```

```
## # A tibble: 7 x 3
##       Date   Food      Mood
##       <chr> <chr>    <chr>
## 1  Monday  carrot   happy
## 2  Tuesday  carrot   happy
## 3 Wednesday  carrot   happy
## 4  Thursday  carrot   happy
## 5   Friday  carrot   happy
## 6 Saturday  carrot extremely happy
## 7   Sunday  carrot extremely happy
```

If you don't have column names, set `col_names = FALSE` then R will assign names "x1", "x2"... to the columns:

```
dat=read_csv("Saturday,carrot,extremely happy
              Sunday,carrot,extremely happy", col_names=FALSE)
print(dat)
```

```
## # A tibble: 2 x 3
##       X1      X2      X3
##   <chr> <chr>   <chr>
## 1 Saturday carrot extremely happy
## 2   Sunday carrot extremely happy
```

You can also pass `col_names` a character vector which will be used as the column names. Try to replace `col_names=FALSE` with `col_names=c("Date","Food","Mood")` and see what happen.

As mentioned before, you can use `read_csv2()` to read semicolon separated files:

```
dat=read_csv2("Saturday; carrot; extremely happy \n Sunday; carrot; extremely happy", col_names=FALSE)
print(dat)
```

```
## # A tibble: 2 x 3
##       X1      X2      X3
##   <chr> <chr>   <chr>
## 1 Saturday carrot extremely happy
## 2   Sunday carrot extremely happy
```

Here "\n" is a convenient shortcut for adding a new line.

You can use `read_tsv()` to read tab delimited files

```
dat=read_tsv("every\tman\tis\ta\tpoet\twhen\the\tis\tin\tlove\n", col_names = FALSE)
print(dat)
```

```
## # A tibble: 1 x 10
##       X1      X2      X3      X4      X5      X6      X7      X8
##   <chr> <chr> <chr> <chr> <chr> <chr> <chr> <chr>
## 1 every   man   is    a     poet  when  he    is
## # ... with 2 more variables: X9 <chr>, X10 <chr>
```

Or more generally, you can use `read_delim()` and assign separating character

```
dat=read_delim("THE|UNBEARABLE|RANDOMNESS|OF|LIFE\n", delim = "|", col_names = FALSE)
print(dat)
```

```
## # A tibble: 1 x 5
##       X1       X2       X3    X4    X5
##   <chr>   <chr>   <chr> <chr> <chr>
## 1  THE UNBEARABLE RANDOMNESS   OF  LIFE
```

Another situation you will often run into is the missing value. In marketing survey, people like to use “99” to represent missing. You can tell R to set all observation with value “99” as missing when you read the data:

```
dat=read_csv("Q1,Q2,Q3
              5, 4,99",na="99")
print(dat)
```

```
## # A tibble: 1 x 3
##       Q1    Q2    Q3
##   <int> <int> <chr>
## 1     5     4 <NA>
```

For writing data back to disk, you can use `write_csv()` and `write_tsv()`. The following two characters of the two functions increase the chances of the output file being read back in correctly:

- Encode strings in UTF-8
- Save dates and date-times in ISO8601 format so they are easily parsed elsewhere

For example:

```
write_csv(sim.dat, "sim_dat.csv")
```

For other data types, you can use the following packages:

- Haven: SPSS, Stata and SAS data
- Readxl and xlsx: excel data(.xls and .xlsx)

- DBI: given data base, such as RMySQL, RSQLite and RPostgreSQL, read data directly from the database using SQL

Some other useful materials:

- For getting data from the internet, you can refer to the book “XML and Web Technologies for Data Sciences with R”.
- R data import/export manual¹
- rio package <https://github.com/leeper/rio>

4.1.1.2 data.table— enhanced data.frame

What is `data.table`? It is an R package that provides an enhanced version of `data.frame`. The most used object in R is `data.frame`. Before we move on, let’s briefly review some basic characters and manipulations of `data.frame`:

- It is a set of rows and columns.
- Each row is of the same length and data type
- Every column is of the same length but can be of differing data types
- It has characteristics of both a matrix and a list
- It uses `[]` to subset data

We will use the clothes customer data to illustrate. There are two dimensions in `[]`. The first one indicates the row and second one indicates column. It uses a comma to separate them.

```
# read data
sim.dat<-readr::read_csv("https://raw.githubusercontent.com/happyrabbit/DataScientistR/master/
# subset the first two rows
sim.dat[1:2,]
```

```
## # A tibble: 2 x 19
##   age gender income house store_exp online_exp
##   <int> <chr> <dbl> <chr>    <dbl>    <dbl>
## 1    57 Female 120963 Yes      529.1     303.5
```

¹<https://cran.r-project.org/doc/manuals/r-release/R-data.html#>

```
## 2    63 Female 122008    Yes      478.0      109.5
## # ... with 13 more variables: store_trans <int>,
## #   online_trans <int>, Q1 <int>, Q2 <int>, Q3 <int>,
## #   Q4 <int>, Q5 <int>, Q6 <int>, Q7 <int>, Q8 <int>,
## #   Q9 <int>, Q10 <int>, segment <chr>
```

```
# subset the first two rows and column 3 and 5
```

```
sim.dat[1:2,c(3,5)]
```

```
## # A tibble: 2 x 2
##   income store_exp
##   <dbl>     <dbl>
## 1 120963     529.1
## 2 122008     478.0
```

```
# get all rows with age>70
```

```
sim.dat[sim.dat$age>70,]
```

```
## # A tibble: 1 x 19
##   age gender income house store_exp online_exp
##   <int> <chr>  <dbl> <chr>     <dbl>     <dbl>
## 1   300   Male 208017    Yes      5077      6053
## # ... with 13 more variables: store_trans <int>,
## #   online_trans <int>, Q1 <int>, Q2 <int>, Q3 <int>,
## #   Q4 <int>, Q5 <int>, Q6 <int>, Q7 <int>, Q8 <int>,
## #   Q9 <int>, Q10 <int>, segment <chr>
```

```
# get rows with age> 60 and gender is Male
```

```
# select column 3 and 4
```

```
sim.dat[sim.dat$age>68 & sim.dat$gender == "Male", 3:4]
```

```
## # A tibble: 2 x 2
##   income house
##   <dbl> <chr>
## 1 119552    No
## 2 208017    Yes
```

Remember that there are usually different ways to conduct the same manipulation. For example, the following code presents three

ways to calculate an average number of online transactions for male and female:

```
tapply(sim.dat$online_trans, sim.dat$gender, mean )

## Female    Male
##  15.38   11.26

aggregate(online_trans ~ gender, data = sim.dat, mean)

##   gender online_trans
## 1 Female          15.38
## 2  Male           11.26

library(dplyr)
sim.dat%>%
  group_by(gender)%>%
  summarise(Avg_online_trans=mean(online_trans))

## # A tibble: 2 x 2
##   gender Avg_online_trans
##   <chr>         <dbl>
## 1 Female          15.38
## 2  Male           11.26
```

There is no gold standard to choose a specific function to manipulate data. The goal is to solve the real problem, not the tool itself. So just use whatever tool that is convenient for you.

The way to use `[]` is straightforward. But the manipulations are limited. If you need more complicated data reshaping or aggregation, there are other packages to use such as `dplyr`, `reshape2`, `tidyr` etc. But the usage of those packages are not as straightforward as `[]`. You often need to change functions. Keeping related operations together, such as subset, group, update, join etc, will allow for:

- concise, consistent and readable syntax irrespective of the set of operations you would like to perform to achieve your end goal
- performing data manipulation fluidly without the cognitive burden of having to change among different functions

- by knowing precisely the data required for each operation, you can automatically optimize operations effectively

`data.table` is the package for that. If you are not familiar with other data manipulating packages and are interested in reducing programming time tremendously, then this package is for you.

Other than extending the function of `[]`, `data.table` has the following advantages:

Offers fast import, subset, grouping, update, and joins for large data files It is easy to turn data frame to data table Can behave just like a data frame

You need to install and load the package:

```
# If you haven't install it, use the code to instal
# install.packages("data.table")
# load packagw
library(data.table)
```

Use `data.table()` to covert the existing data frame `sim.dat` to data table:

```
dt <- data.table(sim.dat)
class(dt)
```

```
## [1] "data.table" "data.frame"
```

Calculate mean for counts of online transactions:

```
dt[, mean(online_trans)]
```

```
## [1] 13.55
```

You can't do the same thing using data frame:

```
sim.dat[,mean(online_trans)]
```

```
Error in mean(online_trans) : object 'online_trans' not found
```

If you want to calculate mean by group as before, set “by =” argument:

```
dt[ , mean(online_trans), by = gender]
```

```
##      gender    V1  
## 1: Female 15.38  
## 2:   Male 11.26
```

You can group by more than one variables. For example, group by “gender” and “house”:

```
dt[ , mean(online_trans), by = .(gender, house)]
```

```
##      gender house    V1  
## 1: Female   Yes 11.312  
## 2:   Male   Yes  8.772  
## 3: Female   No 19.146  
## 4:   Male   No 16.486
```

Assign column names for aggregated variables:

```
dt[ , .(avg = mean(online_trans)), by = .(gender, house)]
```

```
##      gender house    avg  
## 1: Female   Yes 11.312  
## 2:   Male   Yes  8.772  
## 3: Female   No 19.146  
## 4:   Male   No 16.486
```

`data.table` can accomplish all operations that `aggregate()` and `tapply()` can do for data frame.

- General setting of `data.table`

Different from data frame, there are three arguments for data table:



DT[i, j, by]

It is analogous to SQL. You don't have to know SQL to learn data table. But experience with SQL will help you understand data table. In SQL, you select column *j* (use command `SELECT`) for row *i* (using command `WHERE`). `GROUP BY` in SQL will assign the variable to group the observations.

R	:	i	j	by
SQL	:	WHERE	SELECT	GROUP BY

Let's review our previous code:

```
dt[ , mean(online_trans), by = gender]
```

```
##      gender      V1
## 1: Female 15.38
## 2:   Male 11.26
```

The code above is equal to the following SQL

```
SELECT gender, avg(online_trans) FROM sim.dat GROUP BY gender
```

R code:

```
dt[ , .(avg = mean(online_trans)), by = .(gender, house)]
```

```
##      gender house      avg
## 1: Female   Yes 11.312
## 2:   Male   Yes  8.772
## 3: Female   No 19.146
## 4:   Male   No 16.486
```

is equal to SQL

```
SELECT gender, house, avg(online_trans) AS avg FROM sim.dat GROUP BY gender, house
```

R code

```
dt[ age < 40, .(avg = mean(online_trans)), by = .(gender, house)]
```

```
##      gender house   avg
## 1:   Male   Yes 14.46
## 2: Female   Yes 18.14
## 3:   Male    No 18.24
## 4: Female    No 20.10
```

is equal to SQL

```
SELECT gender, house, avg(online_trans) AS avg FROM sim.dat WHERE age < 40 GROUP BY gender, house
```

You can see the analogy between `data.table` and `SQL`. Now let's focus on operations in data table.

- select row

```
# select rows with age<20 and income > 80000
dt[age < 20 & income > 80000]
```

```
##      age gender income house store_exp online_exp
## 1:   19 Female  83535    No    227.7        1491
## 2:   18 Female  89416   Yes    209.5        1926
## 3:   19 Female  92813    No    186.7        1042
##      store_trans online_trans Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9
## 1:             1           22 2  1  1  2  4  1  4  2  4
## 2:             3           28 2  1  1  1  4  1  4  2  4
## 3:             2           18 3  1  1  2  4  1  4  3  4
##      Q10 segment
## 1:    1   Style
## 2:    1   Style
## 3:    1   Style
```

```
# select the first two rows
dt[1:2]
```

```
##      age gender income house store_exp online_exp
```

```
## 1:  57 Female 120963   Yes    529.1    303.5
## 2:  63 Female 122008   Yes    478.0    109.5
##   store_trans online_trans Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9
## 1:         2           2 4 2 1 2 1 4 1 4 2
## 2:         4           2 4 1 1 2 1 4 1 4 1
##   Q10 segment
## 1:   4   Price
## 2:   4   Price
```

- select column

Selecting columns in `data.table` don't need `$`:

```
# select column "age" but return it as a vector
# the argument for row is empty so the result will return all observations
ans <- dt[, age]
head(ans)
```

```
## [1] 57 63 59 60 51 59
```

To return `data.table` object, put column names in `list()`:

```
# Select age and online_exp columns and return as a data.table instead
ans <- dt[, list(age, online_exp)]
head(ans)
```

```
##   age online_exp
## 1:  57    303.5
## 2:  63    109.5
## 3:  59    279.2
## 4:  60    141.7
## 5:  51    112.2
## 6:  59    195.7
```

Or you can also put column names in `.`():

```
ans <- dt[, .(age, online_exp)]
# head(ans)
```

To select all columns from “age” to “income”:

```
ans <- dt[, age:income, with = FALSE]
head(ans, 2)
```

```
##      age gender income
## 1:   57 Female 120963
## 2:   63 Female 122008
```

Delete columns using - or !:

```
# delete columns from age to online_exp
ans <- dt[, -(age:online_exp), with = FALSE]
ans <- dt[, !(age:online_exp), with = FALSE]
```

- tabulation

In data table. .N means to count

```
# row count
dt[, .N]
```

```
## [1] 1000
```

If you assign the group variable, then it will count by groups:

```
# counts by gender
dt[, .N, by= gender]
```

```
##      gender    N
## 1: Female  554
## 2:   Male  446
```

```
# for those younger than 30, count by gender
dt[age < 30, .(count=.N), by= gender]
```

```
##      gender count
## 1: Female    292
## 2:   Male     86
```

Order table:

```
# get records with the highest 5 online expense:
head(dt[order(-online_exp)], 5)
```

```
##      age gender income house store_exp online_exp
```

```
## 1: 40 Female 217600 No 7024 9479
## 2: 41 Female NA Yes 3787 8638
## 3: 36 Male 228550 Yes 3280 8221
## 4: 31 Female 159508 Yes 5177 8006
## 5: 43 Female 190407 Yes 4695 7876
## store_trans online_trans Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9
## 1: 10 6 1 4 5 4 3 4 4 1 4
## 2: 14 10 1 4 4 4 4 4 4 1 4
## 3: 8 12 1 4 5 4 4 4 4 1 4
## 4: 11 13 1 4 4 4 4 4 4 1 4
## 5: 6 11 1 4 5 4 4 4 4 1 4
## Q10 segment
## 1: 2 Conspicuous
## 2: 2 Conspicuous
## 3: 1 Conspicuous
## 4: 2 Conspicuous
## 5: 2 Conspicuous
```

Since data table keep some characters of data frame, they share some operations:

```
dt[order(-online_exp)][1:5]
```

```
## age gender income house store_exp online_exp
## 1: 40 Female 217600 No 7024 9479
## 2: 41 Female NA Yes 3787 8638
## 3: 36 Male 228550 Yes 3280 8221
## 4: 31 Female 159508 Yes 5177 8006
## 5: 43 Female 190407 Yes 4695 7876
## store_trans online_trans Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9
## 1: 10 6 1 4 5 4 3 4 4 1 4
## 2: 14 10 1 4 4 4 4 4 4 1 4
## 3: 8 12 1 4 5 4 4 4 4 1 4
## 4: 11 13 1 4 4 4 4 4 4 1 4
## 5: 6 11 1 4 5 4 4 4 4 1 4
## Q10 segment
## 1: 2 Conspicuous
## 2: 2 Conspicuous
## 3: 1 Conspicuous
```



```
## 4: 2 Conspicuous
## 5: 2 Conspicuous
```

You can also order the table by more than one variable. The following code will order the table by `gender`, then order within `gender` by `online_exp`:

```
dt[order(gender, -online_exp)][1:5]
```

```
##   age gender income house store_exp online_exp
## 1:  40 Female 217600    No      7024      9479
## 2:  41 Female    NA   Yes      3787      8638
## 3:  31 Female 159508   Yes      5177      8006
## 4:  43 Female 190407   Yes      4695      7876
## 5:  50 Female 263858   Yes      5814      7449
##   store_trans online_trans Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9
## 1:          10           6  1  4  5  4  3  4  4  1  4
## 2:          14          10  1  4  4  4  4  4  4  1  4
## 3:          11          13  1  4  4  4  4  4  4  1  4
## 4:           6          11  1  4  5  4  4  4  4  1  4
## 5:          11          11  1  4  5  4  4  4  4  1  4
##   Q10      segment
## 1:  2 Conspicuous
## 2:  2 Conspicuous
## 3:  2 Conspicuous
## 4:  2 Conspicuous
## 5:  1 Conspicuous
```

- Use `fread()` to import data

Other than `read.csv` in base R, we have introduced ‘`read_csv`’ in ‘`readr`’. `read_csv` is much faster and will provide progress bar which makes user feel much better (at least make me feel better). `fread()` in `data.table` further increase the efficiency of reading data. The following are three examples of reading the same data file `topic.csv`. The file includes text data scraped from an agriculture forum with 209670 rows and 6 columns:

```
system.time(topic<-read_csv("https://raw.githubusercontent.com/happyrabbit/DataScientistR/master/topic.csv"))
```

```
user system elapsed
4.313 0.027 4.340
```

```
system.time(topic<-readr::read_csv("https://raw.githubusercontent.com/happyrabbit/DataScientis"))
```

```
user system elapsed
0.267 0.008 0.274
```

```
system.time(topic<-data.table::fread("https://raw.githubusercontent.com/happyrabbit/DataScientis"))
```

```
user system elapsed
0.217 0.005 0.221
```

It is clear that `read_csv()` is much faster than `read.csv()`. `fread()` is a little faster than `read_csv()`. As the size increasing, the difference will become for significant. Note that `fread()` will read file as `data.table` by default.

4.1.2 Summarize data

4.1.2.1 `apply()`, `lapply()` and `sapply()` in base R

There are some powerful functions to summarize data in base R, such as `apply()`, `lapply()` and `sapply()`. They do the same basic things and are all from “apply” family: apply functions over parts of data. They differ in two important respects:

1. the type of object they apply to
2. the type of result they will return

When do we use `apply()`? When we want to apply a function to margins of an array or matrix. That means our data need to be structured. The operations can be very flexible. It returns a vector or array or list of values obtained by applying a function to margins of an array or matrix.

For example you can compute row and column sums for a matrix:

```
## simulate a matrix
x <- cbind(x1 = 1:8, x2 = c(4:1, 2:5))
```

```
dimnames(x)[[1]] <- letters[1:8]
apply(x, 2, mean)
```

```
## x1 x2
## 4.5 3.0
```

```
col.sums <- apply(x, 2, sum)
row.sums <- apply(x, 1, sum)
```

You can also apply other functions:

```
ma <- matrix(c(1:4, 1, 6:8), nrow = 2)
ma
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    3    1    7
## [2,]    2    4    6    8
```

```
apply(ma, 1, table) #--> a list of length 2
```

```
## [[1]]
##
## 1 3 7
## 2 1 1
##
## [[2]]
##
## 2 4 6 8
## 1 1 1 1
```

```
apply(ma, 1, stats::quantile) # 5 x n matrix with rownames
```

```
##      [,1] [,2]
## 0%      1  2.0
## 25%      1  3.5
## 50%      2  5.0
## 75%      4  6.5
## 100%     7  8.0
```

Results can have different lengths for each call. This is a trickier example. What will you get?

```
## Example with different lengths for each call
z <- array(1:24, dim = 2:4)
zseq <- apply(z, 1:2, function(x) seq_len(max(x)))
zseq          ## a 2 x 3 matrix
typeof(zseq)  ## list
dim(zseq)     ## 2 3
zseq[1,]
apply(z, 3, function(x) seq_len(max(x)))
```

- `lapply()` applies a function over a list, data.frame or vector and returns a list of the same length.
- `sapply()` is a user-friendly version and wrapper of `lapply()`. By default it returns a vector, matrix or if `simplify = "array"`, an array if appropriate. `apply(x, f, simplify = FALSE, USE.NAMES = FALSE)` is the same as `lapply(x, f)`. If `simplify=TRUE`, then it will return a data.frame instead of list.

Let's use some data with context to help you better understand the functions.

- Get the mean and standard deviation of all numerical variables in the dataset.

```
# Read data
sim.dat<-read.csv("https://raw.githubusercontent.com/happyrabbit/DataScientistR/master/Data/Se
# Get numerical variables
sdat<-sim.dat[,!lapply(sim.dat,class)=="factor"]
## Try the following code with apply() function
## apply(sim.dat,2,class)
## What is the problem?
```

The data frame `sdat` only includes numeric columns. Now we can go head and use `apply()` to get mean and standard deviation for each column:

```
apply(sdat, MARGIN=2,function(x) mean(na.omit(x)))
```

```
##          age          income    store_exp    online_exp
##    3.884e+01    1.135e+05    1.357e+03    2.120e+03
## store_trans online_trans          Q1          Q2
```

```
##      5.350e+00    1.355e+01    3.101e+00    1.823e+00
##           Q3           Q4           Q5           Q6
##      1.992e+00    2.763e+00    2.945e+00    2.448e+00
##           Q7           Q8           Q9          Q10
##      3.434e+00    2.396e+00    3.085e+00    2.320e+00
```

Here we defined a function using `function(x) mean(na.omit(x))`. It is a very simple function. It tells R to ignore the missing value when calculating the mean. `MARGIN=2` tells R to apply the function to each column. It is not hard to guess what `MARGIN=1` mean. The result show that the average online expense is much higher than store expense. You can also compare the average scores across different questions. The command to calculate standard deviation is very similar. The only difference is to change `mean()` to `sd()`:

```
apply(sdat, MARGIN=2,function(x) sd(na.omit(x)))
```

```
##      age      income  store_exp  online_exp
##      16.417  49842.287   2774.400   1731.224
## store_trans online_trans      Q1      Q2
##      3.696      7.957      1.450      1.168
##           Q3           Q4           Q5           Q6
##      1.402      1.155      1.284      1.439
##           Q7           Q8           Q9          Q10
##      1.456      1.154      1.118      1.136
```

Even the average online expense is higher than store expense, the standard deviation for store expense is much higher than online expense which indicates there is very likely some big/small purchase in store. We can check it quickly:

```
summary(sdat$store_exp)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##      -500    205     329    1360    597   50000
```

```
summary(sdat$online_exp)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##       69    420    1940    2120    2440    9480
```

There are some odd values in store expense. The minimum value is -500 which is a wrong imputation which indicates that you should preprocess data before analyzing it. Checking those simple statistics will help you better understand your data. It then gives you some idea how to preprocess and analyze them. How about using `lapply()` and `sapply()`?

Run the following code and compare the results:

```
lapply(sdat, function(x) sd(na.omit(x)))  
sapply(sdat, function(x) sd(na.omit(x)))  
sapply(sdat, function(x) sd(na.omit(x)), simplify = FALSE)
```

4.1.3 `dplyr` package

`dplyr` provides a flexible grammar of data manipulation focusing on tools for working with data frames (hence the `d` in the name). It is faster and more friendly:

- It identifies the most important data manipulations and make they easy to use from R
- It performs faster for in-memory data by writing key pieces in C++ using `Rcpp`
- The interface is the same for data frame, data table or database.

We will illustrate the following functions in order:

1. Display
2. Subset
3. Summarize
4. Create new variable
5. Merge

Display

- `tbl_df()`: Convert the data to `tibble` which offers better checking and printing capabilities than traditional data frames. It will adjust output width according to fit the current window.

```
library(dplyr)
tbl_df(sim.dat)
```

- `glimpse()`: This is like a transposed version of `tbl_df()`

```
glimpse(sim.dat)
```

Subset

Get rows with income more than 300000:

```
library(magrittr)
filter(sim.dat, income >300000) %>%
  tbl_df()
```

```
## # A tibble: 4 x 19
##   age gender income  house store_exp online_exp
##   <int> <fctr>  <dbl> <fctr>      <dbl>      <dbl>
## 1   40   Male 301398   Yes      4840       3618
## 2   33   Male 319704   Yes      5998       4396
## 3   41   Male 317476   Yes      3030       4180
## 4   37 Female 315697   Yes      6549       4284
## # ... with 13 more variables: store_trans <int>,
## #   online_trans <int>, Q1 <int>, Q2 <int>, Q3 <int>,
## #   Q4 <int>, Q5 <int>, Q6 <int>, Q7 <int>, Q8 <int>,
## #   Q9 <int>, Q10 <int>, segment <fctr>
```

Here we meet a new operator `%>%`. It is called “Pipe operator” which pipes a value forward into an expression or function call. What you get in the left operation will be the first argument or the only argument in the right operation.

```
x %>% f(y) = f(x, y)
y %>% f(x, ., z) = f(x, y, z )
```

It is an operator from `magrittr` which can be really beneficial. Look at the following code. Can you tell me what it does?

```
ave_exp <- filter(
  summarise(
    group_by(
```

```

    filter(
      sim.dat,
      !is.na(income)
    ),
    segment
  ),
  ave_online_exp = mean(online_exp),
  n = n()
),
n > 200
)

```

Now look at the identical code using “%>%”:

```

ave_exp <- sim.dat %>%
  filter(!is.na(income)) %>%
  group_by(segment) %>%
  summarise(
    ave_online_exp = mean(online_exp),
    n = n() ) %>%
  filter(n > 200)

```

Isn't it much more straightforward now? Let's read it:

1. Delete observations from `sim.dat` with missing income values
2. Group the data from step 1 by variable `segment`
3. Calculate mean of online expense for each segment and save the result as a new variable named `ave_online_exp`
4. Calculate the size of each segment and saved it as a new variable named `n`
5. Get segments with size larger than 200

You can use `distinct()` to delete duplicated rows.

```
dplyr::distinct(sim.dat)
```

`sample_frac()` will randomly select some rows with a specified per-

centage. `sample_n()` can randomly select rows with a specified number.

```
dplyr::sample_frac(sim.dat, 0.5, replace = TRUE)
dplyr::sample_n(sim.dat, 10, replace = TRUE)
```

`slice()` will select rows by position:

```
dplyr::slice(sim.dat, 10:15)
```

It is equivalent to `sim.dat[10:15,]`.

`top_n()` will select the order top n entries:

```
dplyr::top_n(sim.dat, 2, income)
```

If you want to select columns instead of rows, you can use `select()`. The following are some sample codes:

```
# select by column name
dplyr::select(sim.dat, income, age, store_exp)

# select columns whose name contains a character string
dplyr::select(sim.dat, contains("_"))

# select columns whose name ends with a character string
# similar there is "starts_with"
dplyr::select(sim.dat, ends_with("e"))

# select columns Q1, Q2, Q3, Q4 and Q5
select(sim.dat, num_range("Q", 1:5))

# select columns whose names are in a group of names
dplyr::select(sim.dat, one_of(c("age", "income")))

# select columns between age and online_exp
dplyr::select(sim.dat, age:online_exp)

# select all columns except for age
dplyr::select(sim.dat, -age)
```

Summarize

A standard marketing problem is customer segmentation. It usually starts with designing survey and collecting data. Then run a cluster analysis on the data to get customer segments. Once we have different segments, the next is to understand how each group of customer look like by summarizing some key metrics. For example, we can do the following data aggregation for different segments of clothes customers.

```
sim.dat%>%
  group_by(segment)%>%
  summarise(Age=round(mean(na.omit(age)),0),
            FemalePct=round(mean(gender=="Female"),2),
            HouseYes=round(mean(house=="Yes"),2),
            store_exp=round(mean(na.omit(store_exp),trim=0.1),0),
            online_exp=round(mean(online_exp),0),
            store_trans=round(mean(store_trans),1),
            online_trans=round(mean(online_trans),1))

## # A tibble: 4 x 8
##   segment Age FemalePct HouseYes store_exp
##   <fctr> <dbl>   <dbl>   <dbl>   <dbl>
## 1 Conspicuous 42    0.32    0.86    4990
## 2 Price      60    0.45    0.94     501
## 3 Quality   35    0.47    0.34     301
## 4 Style     24    0.81    0.27     200
## # ... with 3 more variables: online_exp <dbl>,
## #   store_trans <dbl>, online_trans <dbl>
```

Now, let's peel the onion in order.

The first line `sim.dat` is easy. It is the data you want to work on. The second line `group_by(segment)` tells R that in the following steps you want to summarise by variable `segment`. Here we only summarize data by one categorical variable, but you can group by multiple variables, such as `group_by(segment, house)`. The third argument `summarise` tells R the manipulation(s) to do. Then list the exact actions inside `summarise()`. For example, `Age=round(mean(na.omit(age)),0)` tell R the following things:

1. Calculate the mean of column `age` ignoring missing value for each customer segment
2. Round the result to the specified number of decimal places
3. Store the result in a new variable named `Age`

The rest of the command above is similar. In the end, we calculate the following for each segment:

1. `Age`: average age for each segment
2. `FemalePct`: percentage for each segment
3. `HouseYes`: percentage of people who own a house
4. `store_exp`: average expense in store
5. `online_exp`: average expense online
6. `store_trans`: average times of transactions in the store
7. `online_trans`: average times of online transactions

There is a lot of information you can extract from those simple averages.

- **Conspicuous**: average age is about 40. It is a group of middle-age wealthy people. 1/3 of them are female, and 2/3 are male. They buy regardless the price. Almost all of them own house (0.86). It makes us wonder what is wrong with the rest 14%?
- **Price**: They are older people with average age 60. Nearly all of them own a house(0.94). They are less likely to purchase online (`store_trans=6` while `online_trans=3`). It is the only group that is less likely to buy online.
- **Quality**: The average age is 35. They are not way different with Conspicuous regarding age. But they spend much less. The percentages of male and female are similar. They prefer online shopping. More than half of them don't own a house (0.66).
- **Style**: They are young people with average age 24. The majority of them are female (0.81). Most of them don't own a house (0.73). They are very likely to be digital natives and prefer online shopping.

You may notice that Style group purchase more frequently online

(online_trans) but the expense (online_exp) is not higher. It makes us wonder what is the average expense each time, so you have a better idea about the price range of the group.

The analytical process is aggregated instead of independent steps. The current step will shed new light on what to do next. Sometimes you need to go back to fix something in the previous steps. Let's check average one-time online and instore purchase amounts:

```
sim.dat%>%
  group_by(segment)%>%
  summarise(avg_online=round(sum(online_exp)/sum(online_trans),2),
            avg_store=round(sum(store_exp)/sum(store_trans),2))
```

```
## # A tibble: 4 x 3
##       segment avg_online avg_store
##       <fctr>    <dbl>    <dbl>
## 1 Conspicuous  442.27    479.2
## 2 Price        69.28     81.3
## 3 Quality     126.05    105.1
## 4 Style       92.83    121.1
```

Price group has the lowest averaged one-time purchase. The Conspicuous group will pay the highest price. When we build customer profile in real life, we will also need to look at the survey summarization. You may be surprised how much information simple data manipulations can provide.

Another common task is to check which column has missing values. It requires the program to look at each column in the data. In this case you can use `summarise_all`:

```
# apply function anyNA() to each column
# you can also assign a function vector such as: c("anyNA","is.factor")
dplyr::summarise_all(sim.dat, funs_(c("anyNA")))
```

```
##      age gender income house store_exp online_exp
## 1 FALSE  FALSE   TRUE  FALSE      FALSE      FALSE
##  store_trans online_trans    Q1    Q2    Q3    Q4
## 1      FALSE          FALSE FALSE FALSE FALSE FALSE
##      Q5    Q6    Q7    Q8    Q9   Q10 segment
```

```
## 1 FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

The above code returns a vector indicating if there is any value missing in each column.

Create new variable

There are often situations where you need to create new variables. For example, adding online and store expense to get total expense. In this case, you will apply **window function** to the columns and return a column with the same length. `mutate()` can do it for you and append one or more new columns:

```
dplyr::mutate(sim.dat, total_exp = store_exp + online_exp)
```

##	age	gender	income	house	store_exp	online_exp
## 1	57	Female	120963	Yes	529.1	303.51
## 2	63	Female	122008	Yes	478.0	109.53
## 3	59	Male	114202	Yes	490.8	279.25
## 4	60	Male	113616	Yes	347.8	141.67
## 5	51	Male	124253	Yes	379.6	112.24
## 6	59	Male	107661	Yes	338.3	195.69
## 7	57	Male	120483	Yes	482.5	284.54
## 8	57	Male	110542	Yes	340.7	135.26
## 9	61	Female	132061	Yes	608.2	142.55
## 10	60	Male	105049	Yes	470.3	163.47
## 11	58	Male	107197	Yes	366.6	170.13
## 12	59	Male	NA	Yes	674.9	310.27
## 13	64	Male	119020	Yes	613.9	160.85
## 14	57	Female	NA	Yes	737.0	224.53
## 15	64	Male	114539	Yes	402.5	241.83
## 16	61	Female	NA	Yes	615.1	238.10
## 17	57	Female	133078	Yes	429.4	262.66
## 18	63	Male	115709	Yes	552.6	187.52
## 19	57	Female	113211	No	540.3	254.58
## 20	57	Female	129774	No	384.3	311.03
## 21	58	Male	NA	Yes	372.4	296.83
## 22	58	Male	124357	Yes	535.5	205.54
## 23	59	Female	123117	Yes	481.3	157.29
## 24	61	Female	NA	Yes	546.2	161.23

## 25	60	Female	NA	Yes	411.1	123.78
## 26	56	Male	NA	Yes	492.7	128.02
## 27	58	Male	127887	Yes	519.3	142.63
## 28	64	Male	115925	Yes	627.7	240.98
## 29	63	Female	NA	Yes	511.1	203.49
## 30	56	Female	112621	Yes	699.3	223.44
## 31	57	Male	NA	Yes	530.6	217.92
## 32	59	Female	121773	Yes	532.1	176.88
## 33	63	Male	126903	Yes	601.6	257.84
## 34	55	Male	128254	Yes	595.3	248.06
## 35	60	Female	NA	Yes	403.6	236.96
## 36	64	Female	NA	Yes	611.7	102.24
## 37	57	Female	118170	Yes	482.7	183.36
## 38	57	Female	119148	Yes	412.9	246.87
## 39	58	Male	125844	Yes	473.8	261.67
## 40	55	Male	128194	Yes	595.7	156.93
## 41	59	Female	NA	Yes	548.3	186.50
## 42	61	Male	NA	Yes	597.2	209.68
## 43	59	Female	122338	Yes	455.3	205.16
## 44	58	Female	114519	Yes	493.5	271.67
## 45	62	Male	123459	Yes	561.7	209.23
## 46	59	Male	125626	Yes	518.2	261.31
## 47	56	Male	100583	Yes	633.0	195.95
## 48	66	Male	NA	Yes	510.3	198.46
## 49	62	Female	128606	Yes	499.7	244.05
## 50	54	Male	114337	Yes	518.0	217.25
## 51	59	Male	121889	Yes	490.4	120.95
## 52	57	Female	125566	Yes	506.2	155.84
## 53	52	Female	NA	Yes	529.3	193.48
## 54	56	Male	111594	Yes	567.7	195.55
## 55	60	Female	109520	Yes	594.7	212.07
## 56	55	Female	NA	Yes	321.2	203.29
## 57	64	Female	124792	Yes	594.9	190.81
## 58	56	Male	117325	Yes	525.8	211.02
## 59	58	Female	NA	Yes	392.1	226.47
## 60	58	Male	121033	Yes	451.6	157.40
## 61	62	Female	117474	Yes	483.6	213.19

```
## 62    62 Female 119597   Yes    697.5    169.40
## 63    58 Female 116507   Yes    527.7    192.62
## 64    58  Male 125974   Yes    350.5    151.46
## 65    61  Male 111027   Yes    606.8    278.53
## 66    64  Male      NA   Yes    689.6    239.12
## 67    59  Male 125334   Yes    524.4    190.49
## 68    56 Female 115479   Yes    438.6    109.31
## 69    58 Female      NA   No    293.1    253.49
## 70    62  Male 116434   Yes    360.1    274.21
## 71    62 Female      NA   Yes    534.9    181.33
## 72    61 Female 127498   Yes    644.1    238.12
## 73    54  Male 117859   No    570.4    262.01
## 74    59  Male 133587   Yes    624.6    206.62
## 75    59  Male      NA   Yes    610.5    171.44
## 76    62 Female 130151   Yes    704.6    225.27
## 77    60  Male 125086   Yes    388.2    163.91
## 78    62  Male      NA   Yes    377.2    259.30
## 79    59  Male      NA   Yes    723.8    179.83
## 80    58  Male 118653   Yes    659.0    208.69
## 81    61  Male      NA   Yes    358.2    315.42
## 82    61 Female 117817   Yes    607.8    207.02
## 83    61  Male      NA   Yes    409.4    341.66
## 84    57  Male      NA   Yes    382.9    171.65
## 85    55 Female 127716   Yes    494.9    183.64
## 86    65  Male      NA   Yes    633.5    202.92
## 87    58  Male      NA   Yes    528.4    202.01
## 88    62  Male 121172   Yes    652.8    185.31
## 89    59  Male 115396   Yes    622.8    192.09
## 90    60 Female 123867   Yes    441.3    166.77
## 91    64  Male 113301   Yes    460.0    252.77
## 92    61  Male 133464   Yes    627.1    151.89
## 93    56 Female 119472   Yes    515.7    197.70
## 94    62 Female 119461   Yes    532.3    176.85
## 95    58 Female 119233   Yes    568.6    253.94
## 96    58 Female      NA   Yes    420.7    208.86
## 97    60 Female      NA   Yes    478.0    198.95
## 98    59 Female      NA   No    434.7    239.83
```

## 99	53	Female	121089	No	556.9	163.28
## 100	63	Male	125302	Yes	544.1	233.97
## 101	61	Male	NA	Yes	416.5	235.55
## 102	56	Female	NA	Yes	427.0	279.10
## 103	56	Male	106712	Yes	410.6	132.02
## 104	64	Female	NA	Yes	302.4	112.59
## 105	60	Male	127820	Yes	423.2	255.74
## 106	62	Male	129258	No	555.3	142.39
## 107	66	Male	117317	Yes	499.1	158.88
## 108	67	Female	113786	Yes	504.5	193.11
## 109	60	Female	115340	Yes	538.2	144.37
## 110	63	Female	122790	Yes	341.8	158.82
## 111	60	Female	126579	Yes	257.9	222.29
## 112	67	Male	126651	Yes	430.9	177.61
## 113	58	Male	NA	No	302.8	181.84
## 114	60	Female	NA	Yes	582.6	220.78
## 115	65	Female	111526	Yes	406.4	189.85
## 116	59	Female	122550	No	620.8	185.13
## 117	61	Male	116833	Yes	311.9	254.31
## 118	65	Male	127422	Yes	567.8	199.96
## 119	64	Female	130856	Yes	399.4	217.99
## 120	56	Female	124593	Yes	212.9	192.38
## 121	60	Male	117161	Yes	465.0	246.81
## 122	60	Male	112290	Yes	430.5	136.68
## 123	68	Male	124419	Yes	389.2	206.57
## 124	55	Female	NA	Yes	475.6	163.12
## 125	62	Male	NA	Yes	518.9	229.38
## 126	54	Male	127204	Yes	597.5	155.14
## 127	59	Female	116489	Yes	469.3	205.45
## 128	60	Male	121120	No	452.6	256.68
## 129	60	Female	121269	Yes	471.1	233.39
## 130	62	Male	NA	Yes	406.9	159.74
## 131	59	Male	132961	Yes	518.5	208.18
## 132	59	Male	NA	Yes	471.2	252.30
## 133	59	Male	NA	Yes	390.8	217.02
## 134	60	Female	128337	Yes	518.9	202.17
## 135	62	Male	121456	Yes	410.1	68.82

## 136	60	Male	125824	Yes	510.7	335.14
## 137	61	Male	120403	Yes	519.1	251.42
## 138	54	Female	108161	Yes	377.9	176.64
## 139	59	Female	129715	Yes	566.9	130.03
## 140	55	Male	NA	Yes	482.0	212.63
## 141	55	Male	121293	No	413.7	213.29
## 142	62	Male	124251	Yes	427.9	94.00
## 143	54	Female	NA	Yes	375.4	130.70
## 144	61	Male	NA	Yes	566.9	224.30
## 145	59	Female	116347	Yes	428.8	79.40
## 146	61	Male	114550	Yes	648.0	172.58
## 147	62	Female	122186	Yes	512.3	215.44
## 148	59	Female	124382	Yes	498.0	244.65
## 149	62	Female	119771	Yes	464.3	237.74
## 150	64	Female	NA	Yes	499.3	208.47
## 151	64	Male	114683	Yes	521.7	203.82
## 152	59	Male	NA	Yes	306.2	161.16
## 153	60	Female	113287	Yes	677.1	243.17
## 154	57	Male	129904	Yes	675.8	240.78
## 155	61	Male	118475	Yes	209.3	142.54
## 156	56	Male	NA	Yes	324.6	183.98
## 157	56	Female	126413	Yes	602.6	129.45
## 158	66	Female	NA	Yes	517.0	152.26
## 159	63	Male	NA	Yes	593.8	225.88
## 160	65	Female	123786	Yes	493.2	196.80
## 161	56	Male	111408	Yes	630.6	275.48
## 162	53	Female	NA	Yes	526.5	145.41
## 163	62	Female	129469	Yes	688.1	189.40
## 164	63	Female	NA	Yes	314.4	145.21
## 165	56	Male	NA	Yes	468.0	150.58
## 166	59	Male	126272	Yes	377.3	234.59
## 167	62	Female	108396	Yes	446.1	238.23
## 168	58	Male	NA	Yes	405.5	119.23
## 169	54	Male	108699	Yes	574.9	245.28
## 170	59	Female	112985	Yes	456.9	181.70
## 171	61	Male	NA	Yes	701.4	254.80
## 172	61	Female	NA	Yes	608.4	301.55

## 173	61	Female	NA	Yes	539.7	223.32
## 174	56	Male	NA	Yes	517.9	233.06
## 175	57	Female	131449	Yes	390.1	147.85
## 176	59	Male	115900	Yes	528.1	186.37
## 177	56	Female	NA	Yes	602.2	191.31
## 178	62	Female	110510	Yes	473.7	253.33
## 179	61	Female	NA	Yes	458.8	156.64
## 180	57	Male	NA	Yes	317.5	187.41
## 181	63	Female	124568	Yes	471.6	202.65
## 182	58	Male	119448	Yes	466.1	249.35
## 183	59	Female	123706	Yes	314.3	212.45
## 184	63	Female	134146	Yes	549.9	241.30
## 185	54	Male	111547	Yes	540.6	189.29
## 186	61	Male	119540	Yes	700.9	247.79
## 187	59	Male	123976	Yes	526.7	198.84
## 188	59	Male	119461	Yes	587.8	181.95
## 189	63	Female	134418	Yes	488.7	254.34
## 190	61	Female	121046	Yes	330.4	168.52
## 191	60	Female	119772	Yes	747.0	127.50
## 192	60	Female	106708	No	418.6	244.42
## 193	56	Male	NA	Yes	419.7	192.37
## 194	63	Male	NA	Yes	584.1	320.96
## 195	58	Male	116516	Yes	588.9	110.15
## 196	61	Male	124903	Yes	390.9	109.30
## 197	58	Male	111364	Yes	402.1	179.17
## 198	57	Female	119470	Yes	515.5	153.16
## 199	63	Male	118613	Yes	571.6	230.38
## 200	60	Female	118609	Yes	645.1	169.31
## 201	65	Male	99409	Yes	526.5	270.56
## 202	57	Female	124554	Yes	307.6	310.26
## 203	53	Male	NA	Yes	586.2	195.13
## 204	69	Male	119552	No	603.7	246.00
## 205	59	Female	118707	Yes	618.8	192.68
## 206	60	Male	106360	Yes	658.0	199.56
## 207	60	Male	134577	Yes	339.3	174.01
## 208	60	Male	NA	Yes	562.6	210.15
## 209	62	Male	NA	Yes	503.2	206.86

## 210	67	Female	127121	Yes	348.6	200.55
## 211	64	Male	NA	Yes	426.7	240.03
## 212	62	Female	NA	Yes	639.7	216.00
## 213	65	Male	123845	No	554.0	184.05
## 214	66	Male	133819	Yes	401.0	307.50
## 215	59	Male	104521	Yes	461.7	182.08
## 216	57	Female	114557	No	427.0	247.15
## 217	64	Male	NA	Yes	343.7	254.74
## 218	60	Male	121940	Yes	480.9	230.48
## 219	60	Female	NA	Yes	422.2	270.14
## 220	55	Male	NA	Yes	483.4	203.28
## 221	57	Female	NA	Yes	641.1	187.33
## 222	58	Female	126646	Yes	611.7	222.44
## 223	58	Female	117707	Yes	416.7	236.34
## 224	58	Male	145345	Yes	690.2	164.24
## 225	60	Female	130677	Yes	615.5	171.37
## 226	57	Female	118732	Yes	486.9	224.39
## 227	58	Male	126062	Yes	682.2	208.31
## 228	55	Male	115303	Yes	451.6	213.64
## 229	62	Male	NA	Yes	521.2	273.49
## 230	60	Male	118625	Yes	540.2	238.25
## 231	60	Male	114025	Yes	412.0	252.69
## 232	56	Male	107039	Yes	538.2	275.54
## 233	59	Female	NA	Yes	518.5	161.04
## 234	60	Female	127042	Yes	534.5	273.48
## 235	55	Male	123608	Yes	568.6	253.36
## 236	58	Male	141349	Yes	528.5	269.11
## 237	59	Male	104866	Yes	565.7	144.76
## 238	55	Male	NA	Yes	345.3	207.98
## 239	58	Male	120409	Yes	270.0	198.48
## 240	55	Female	123505	Yes	502.7	250.53
## 241	59	Male	114339	Yes	452.9	309.75
## 242	62	Female	115346	Yes	566.8	77.93
## 243	61	Male	NA	Yes	557.1	279.21
## 244	57	Female	NA	Yes	513.0	112.25
## 245	57	Male	NA	Yes	334.0	261.33
## 246	61	Female	121937	Yes	563.4	177.00

## 247	63	Female	NA	Yes	521.4	264.49
## 248	63	Female	NA	Yes	615.4	226.39
## 249	54	Male	127794	Yes	579.2	172.49
## 250	65	Male	126479	Yes	519.5	157.88
## 251	40	Male	NA	Yes	3562.2	7264.49
## 252	39	Male	180891	Yes	3955.5	4677.19
## 253	38	Male	190941	Yes	5058.1	4499.61
## 254	36	Male	106483	Yes	3891.6	3401.05
## 255	40	Female	227057	Yes	5416.7	2784.96
## 256	45	Female	232140	Yes	3079.0	4515.88
## 257	42	Female	NA	Yes	3905.9	3435.70
## 258	42	Male	NA	Yes	5095.1	2869.87
## 259	41	Male	NA	No	4918.6	6585.00
## 260	40	Male	245176	Yes	5509.3	4742.39
## 261	43	Male	NA	Yes	3835.7	4931.95
## 262	45	Male	197653	No	4541.7	6692.99
## 263	40	Male	301398	Yes	4840.5	3618.21
## 264	38	Female	189199	Yes	5120.1	3398.28
## 265	42	Male	NA	No	6841.4	6070.81
## 266	37	Male	260037	Yes	3724.7	4714.18
## 267	41	Male	186456	Yes	4530.4	5148.72
## 268	55	Female	222738	No	4642.8	4636.42
## 269	45	Male	189379	Yes	3031.9	6259.35
## 270	48	Male	176785	No	6879.0	4208.66
## 271	42	Female	197429	Yes	6033.3	2628.74
## 272	36	Female	NA	Yes	5740.8	2752.77
## 273	47	Male	88844	Yes	4747.8	3655.00
## 274	38	Male	267565	Yes	5335.1	6052.44
## 275	38	Female	179326	Yes	4365.1	4309.18
## 276	42	Male	NA	Yes	5263.2	3924.34
## 277	35	Female	167613	Yes	5375.2	4026.04
## 278	34	Male	176475	Yes	6621.8	4288.32
## 279	42	Male	184352	Yes	6319.1	3765.25
## 280	36	Female	190846	Yes	4806.6	6590.34
## 281	33	Female	217051	Yes	3645.5	2529.51
## 282	39	Female	NA	Yes	3103.4	3070.81
## 283	50	Female	263858	Yes	5813.8	7448.73

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## 284 39 Male 91326 Yes 4960.4 3078.77
## 285 44 Female NA Yes 4842.8 6227.72
## 286 44 Female 206791 Yes 5010.1 4343.07
## 287 42 Male 141526 Yes 5106.5 6590.50
## 288 300 Male 208017 Yes 5076.8 6053.49
## 289 38 Female 164507 Yes 3916.9 5764.12
## 290 49 Male 292446 No 4693.3 4361.46
## 291 31 Female 159508 Yes 5177.1 8005.93
## 292 34 Male 156882 Yes 4792.7 6627.55
## 293 41 Male NA Yes 5302.9 2444.81
## 294 36 Female 189099 Yes 5155.1 4362.71
## 295 48 Male 124586 No 5185.1 6382.02
## 296 36 Male 171877 Yes 5266.5 4713.01
## 297 33 Male 194787 Yes 3224.3 7563.34
## 298 33 Male 319704 Yes 5998.3 4395.92
## 299 49 Male 144164 Yes 6210.9 5148.91
## 300 41 Male 217153 Yes 5081.3 5712.58
## 301 41 Male 135441 Yes 5841.7 2747.00
## 302 36 Male 271401 Yes 4964.9 5478.38
## 303 47 Male 196925 Yes 4525.9 5995.04
## 304 41 Male 172960 Yes 4573.1 5484.87
## 305 39 Male 220098 No 4357.9 4825.81
## 306 37 Male 148274 Yes 4500.0 5083.95
## 307 30 Female 176389 Yes 5698.2 5966.45
## 308 40 Male 258034 Yes 5459.8 4437.28
## 309 48 Male 243304 Yes 4126.0 4016.95
## 310 41 Male 199560 No 3458.3 3757.81
## 311 47 Male 254559 Yes 5139.5 3630.69
## 312 44 Male 295423 No 5460.1 5598.66
## 313 32 Male 160867 Yes 5511.8 7579.47
## 314 41 Male 183991 Yes 4807.5 3598.30
## 315 39 Female NA Yes 5907.9 5462.78
## 316 40 Male 129256 Yes 4257.4 4668.66
## 317 38 Male 154789 Yes 5598.6 4183.51
## 318 41 Male NA Yes 5887.7 3512.45
## 319 53 Female 255254 Yes 4739.7 5385.38
## 320 32 Male NA Yes 4251.6 2421.31
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## 323	43	Male	NA	Yes	4502.9	6887.86
## 324	43	Male	107347	Yes	4465.8	3976.20
## 325	32	Male	73952	Yes	5381.3	4469.66
## 326	37	Female	127373	Yes	5229.7	446.57
## 327	38	Female	232699	Yes	6397.6	5551.22
## 328	48	Male	153518	Yes	4599.7	5291.27
## 329	32	Female	238803	Yes	5065.5	7231.27
## 330	48	Female	192712	Yes	6326.8	2638.37
## 331	41	Female	246952	Yes	3311.1	7178.20
## 332	45	Male	137559	No	6019.4	3425.59
## 333	33	Male	207886	Yes	4309.9	5302.00
## 334	42	Male	168775	Yes	4393.0	5029.97
## 335	40	Male	174716	Yes	4630.6	4760.96
## 336	36	Male	147095	Yes	6434.3	3970.56
## 337	43	Male	185241	Yes	4852.4	4489.94
## 338	40	Male	261916	Yes	4316.9	6350.51
## 339	42	Male	207789	No	3653.7	4952.69
## 340	48	Male	205101	No	5021.9	3391.52
## 341	44	Male	234536	Yes	7074.4	6972.33
## 342	34	Male	131651	No	4848.6	4602.80
## 343	47	Female	180859	Yes	6582.4	3642.05
## 344	40	Male	240776	No	5061.4	4545.37
## 345	40	Male	280653	Yes	5084.8	4167.11
## 346	40	Female	148750	Yes	6807.0	5522.96
## 347	40	Female	217600	No	7023.7	9479.44
## 348	41	Female	247215	Yes	4230.7	6632.27
## 349	37	Male	187063	Yes	5931.7	1942.18
## 350	40	Male	245544	Yes	4935.1	5087.34
## 351	50	Male	198177	Yes	4514.0	1910.46
## 352	36	Male	268528	Yes	6115.8	5396.88
## 353	34	Male	159307	No	4992.9	4036.44
## 354	40	Male	225217	Yes	4791.5	5682.11
## 355	41	Male	158261	Yes	4581.6	4322.50
## 356	35	Female	225379	No	4123.7	3652.63
## 357	48	Male	NA	Yes	4457.0	5722.16

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## 358 30 Male 234691 No 5165.8 5535.35
## 359 43 Female 229674 No 5615.0 4860.80
## 360 44 Male NA Yes 5201.9 2882.03
## 361 42 Female 173379 Yes 5650.2 5332.80
## 362 34 Male NA Yes 5866.4 6598.33
## 363 42 Male 139589 Yes 2364.9 5965.12
## 364 40 Male NA Yes 6905.4 6878.97
## 365 47 Male 140226 No 4387.1 5211.13
## 366 43 Female 217073 Yes 5199.9 6469.65
## 367 35 Male 184216 Yes 2715.1 5972.83
## 368 45 Male 84897 No 4424.8 5621.56
## 369 53 Male 163901 Yes 6513.3 6233.20
## 370 47 Male NA Yes 5372.6 2759.34
## 371 30 Female NA Yes 4361.2 6665.26
## 372 40 Female 149416 No 3515.4 6135.30
## 373 34 Male 257558 Yes 4602.5 5342.44
## 374 41 Male NA Yes 5679.4 3924.42
## 375 48 Female NA Yes 4807.5 4996.92
## 376 42 Male 293010 Yes 5114.8 4798.79
## 377 33 Male 174461 Yes 3916.8 7322.93
## 378 43 Female 190407 Yes 4694.9 7875.56
## 379 50 Female 110149 Yes 5102.8 5478.86
## 380 38 Female 247626 Yes 5731.8 5340.25
## 381 42 Male 205641 Yes 5294.6 4751.09
## 382 51 Male 119454 Yes 4717.1 5245.62
## 383 43 Male 200491 Yes 5411.8 4309.57
## 384 37 Male 197546 Yes 5287.4 4221.31
## 385 46 Male 162741 Yes 4922.5 5346.71
## 386 42 Male 177699 Yes 5135.4 7255.36
## 387 46 Male NA Yes 4117.3 6144.90
## 388 43 Male 130536 Yes 5080.4 4775.69
## 389 47 Female 133038 Yes 5114.7 4685.07
## 390 34 Female 210712 Yes 5257.7 4838.53
## 391 44 Male 166319 Yes 5568.4 5608.00
## 392 41 Male 200071 Yes 7431.2 5726.34
## 393 30 Male NA Yes 5130.7 4546.08
## 394 43 Female 244905 Yes 4345.6 5832.68
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##	396	38	Male	220746	Yes	5318.0	4468.87
##	397	41	Male	198365	Yes	4279.4	2208.08
##	398	38	Female	271750	Yes	5275.0	3813.21
##	399	37	Male	139063	Yes	4070.7	7595.58
##	400	47	Male	228502	Yes	4597.4	4299.84
##	401	33	Male	102390	Yes	4070.8	5683.64
##	402	36	Male	228550	Yes	3279.6	8220.56
##	403	42	Male	163590	Yes	4068.0	4190.33
##	404	44	Male	170380	Yes	4455.9	7750.84
##	405	51	Female	172848	Yes	5931.8	4522.54
##	406	38	Female	181307	Yes	3142.8	3806.81
##	407	41	Male	317476	Yes	3029.8	4179.67
##	408	34	Male	252651	Yes	5310.3	4312.21
##	409	39	Male	259350	Yes	50000.0	3172.26
##	410	44	Female	NA	Yes	4671.9	3766.91
##	411	43	Male	222222	No	4867.5	6509.25
##	412	41	Female	NA	Yes	4369.2	4327.90
##	413	33	Female	150413	Yes	5827.1	3857.16
##	414	39	Female	227953	No	5639.0	6060.18
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##	417	40	Male	NA	Yes	4147.8	1275.30
##	418	42	Female	164408	Yes	5389.0	6134.82
##	419	39	Male	236079	Yes	4487.8	4509.85
##	420	32	Male	170209	Yes	4696.6	4840.47
##	421	48	Male	NA	Yes	4960.4	5512.78
##	422	43	Male	NA	Yes	4120.6	5030.21
##	423	39	Male	254858	Yes	6655.0	3198.15
##	424	32	Male	NA	Yes	7600.8	2911.39
##	425	36	Male	140316	Yes	6040.7	5157.12
##	426	41	Female	NA	No	5022.7	3757.21
##	427	40	Male	224713	Yes	4242.0	5394.22
##	428	42	Female	178619	Yes	5753.2	4099.51
##	429	37	Male	247904	Yes	5448.9	5307.12
##	430	40	Female	255187	Yes	2709.9	2033.85
##	431	46	Male	217732	Yes	3593.9	4398.09

##	432	40	Male	127078	Yes	4471.9	4951.41
##	433	37	Male	208428	Yes	6347.3	5592.74
##	434	42	Male	138567	Yes	4541.1	4949.03
##	435	35	Female	130109	Yes	6155.5	6201.71
##	436	31	Female	179653	Yes	4762.2	3879.54
##	437	42	Female	NA	Yes	3971.5	2802.65
##	438	39	Male	92395	Yes	4822.5	5251.31
##	439	42	Male	186606	Yes	4714.9	4882.97
##	440	44	Female	204052	No	5784.6	2623.46
##	441	41	Male	217297	No	5773.5	4641.28
##	442	38	Male	NA	Yes	5397.9	2587.65
##	443	37	Female	315697	Yes	6549.0	4284.06
##	444	29	Female	247093	Yes	4189.7	6875.14
##	445	41	Male	192074	Yes	7014.7	4817.97
##	446	42	Male	206799	Yes	5480.6	4970.82
##	447	33	Male	251581	Yes	5342.9	5952.70
##	448	37	Male	118144	Yes	6376.3	7152.00
##	449	37	Female	171941	Yes	6676.6	1559.85
##	450	33	Female	197351	Yes	4304.7	3590.20
##	451	30	Female	66317	No	211.0	1864.98
##	452	36	Female	73212	No	278.8	1941.53
##	453	36	Female	NA	No	330.1	1795.23
##	454	40	Female	94279	No	364.6	1521.96
##	455	21	Female	71473	No	349.7	2182.01
##	456	19	Male	79270	Yes	346.6	2124.64
##	457	25	Female	69245	Yes	380.8	2189.33
##	458	38	Female	60945	Yes	296.7	2160.57
##	459	40	Male	NA	No	255.2	2076.83
##	460	33	Female	66346	No	189.1	2005.69
##	461	31	Female	63881	Yes	269.5	1916.12
##	462	35	Male	68918	Yes	201.9	2418.43
##	463	31	Male	77909	Yes	293.2	1745.86
##	464	31	Female	68035	No	381.3	1793.15
##	465	45	Female	41776	No	305.8	2370.60
##	466	35	Female	68203	No	327.6	1701.01
##	467	41	Female	72446	Yes	258.5	1877.09
##	468	33	Female	62383	Yes	314.0	1680.48

## 469	35	Female	85725	No	354.3	1926.27
## 470	33	Male	NA	No	265.7	1892.56
## 471	30	Male	57171	Yes	298.7	2208.77
## 472	32	Male	86840	No	348.1	2029.03
## 473	36	Male	NA	Yes	203.3	2202.51
## 474	34	Female	82647	No	155.8	2214.11
## 475	39	Female	63417	No	362.5	1976.33
## 476	45	Female	54187	No	347.4	2010.68
## 477	27	Male	67893	No	291.5	1752.33
## 478	39	Male	69959	No	251.9	1811.25
## 479	37	Female	62437	Yes	242.2	1846.21
## 480	45	Female	73319	No	266.5	2029.11
## 481	33	Female	NA	No	278.6	1868.19
## 482	43	Male	74626	No	342.8	1933.08
## 483	38	Male	78948	No	268.9	1889.85
## 484	42	Female	77492	Yes	343.3	1852.36
## 485	37	Male	80766	No	340.7	2033.19
## 486	33	Male	67274	Yes	248.2	2075.62
## 487	28	Male	73423	Yes	325.3	2278.97
## 488	41	Female	58605	No	344.8	1660.44
## 489	24	Female	60377	No	338.7	2328.17
## 490	39	Male	86035	Yes	433.6	2205.64
## 491	52	Female	70661	Yes	310.5	1877.82
## 492	33	Male	70123	No	345.3	1843.23
## 493	41	Male	64594	Yes	251.9	1984.13
## 494	28	Male	65864	Yes	330.4	1783.44
## 495	47	Male	52253	No	304.4	1765.03
## 496	49	Female	47103	No	330.9	2191.16
## 497	38	Male	76596	No	257.9	1813.50
## 498	32	Female	78075	No	276.9	2126.37
## 499	35	Female	NA	No	331.7	2240.67
## 500	25	Female	66516	Yes	341.3	2368.39
## 501	39	Female	89220	No	341.9	1573.80
## 502	35	Male	52292	No	221.1	1912.28
## 503	30	Female	NA	No	381.8	2093.38
## 504	30	Female	NA	No	313.1	2153.10
## 505	26	Female	NA	No	263.4	1880.14

## 506	16	Male	66303	No	275.5	2182.04
## 507	32	Male	NA	Yes	316.8	2143.11
## 508	26	Female	65785	Yes	265.5	1855.81
## 509	37	Female	57463	No	339.2	2219.61
## 510	36	Female	64393	No	227.1	1949.13
## 511	38	Male	NA	No	291.1	2153.61
## 512	47	Male	61984	No	340.2	1942.69
## 513	28	Male	89322	No	342.4	1840.74
## 514	42	Female	NA	No	353.4	2765.04
## 515	34	Female	65354	No	225.5	1718.06
## 516	30	Male	65413	Yes	257.3	1735.10
## 517	44	Male	75326	No	249.4	1964.44
## 518	37	Female	76209	Yes	272.2	2035.49
## 519	32	Male	72854	No	282.6	1964.17
## 520	34	Female	71996	No	358.0	2040.19
## 521	35	Male	60287	No	314.2	1950.29
## 522	33	Male	71336	No	241.3	2278.14
## 523	42	Female	71264	No	425.0	1893.20
## 524	44	Male	51281	Yes	274.7	1993.12
## 525	34	Female	73234	No	349.5	2081.45
## 526	40	Male	89468	Yes	290.8	2258.78
## 527	31	Female	74734	Yes	253.3	1941.31
## 528	36	Male	NA	Yes	306.8	2073.36
## 529	40	Female	62844	No	356.3	2081.35
## 530	37	Male	70207	No	305.1	2208.15
## 531	31	Female	71249	No	265.4	1743.05
## 532	26	Male	78942	Yes	299.3	1633.25
## 533	36	Female	NA	No	303.1	2287.73
## 534	41	Male	81855	No	256.3	1805.31
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## 536	36	Male	72641	No	313.4	2063.71
## 537	32	Female	62881	No	270.5	2174.11
## 538	34	Female	60522	No	299.3	2054.17
## 539	43	Male	44160	No	307.6	2103.38
## 540	26	Male	78241	No	430.2	2091.47
## 541	46	Male	60178	Yes	323.4	2099.71
## 542	32	Male	74317	No	308.7	2140.08

## 543	35	Female	79197	No	335.0	2140.34
## 544	37	Male	68483	No	212.3	2107.29
## 545	31	Male	58372	No	291.3	1566.87
## 546	35	Female	86467	No	234.0	1914.89
## 547	31	Female	71550	No	310.9	2166.57
## 548	39	Female	75350	No	341.2	1882.17
## 549	46	Male	78974	Yes	241.0	1576.39
## 550	39	Male	61539	No	269.6	2055.00
## 551	40	Male	71487	Yes	393.0	2358.35
## 552	46	Female	60048	No	299.5	2128.48
## 553	35	Female	70807	No	219.1	2167.16
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## 555	21	Male	59310	Yes	345.3	2167.41
## 556	47	Male	NA	No	297.8	1800.30
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## 558	30	Male	87802	No	311.0	1981.42
## 559	38	Female	76713	No	247.0	2333.38
## 560	34	Male	56927	Yes	325.7	2138.25
## 561	43	Female	NA	No	349.9	2016.89
## 562	46	Male	88624	No	229.0	2137.06
## 563	45	Female	73858	No	306.1	2048.52
## 564	38	Female	NA	Yes	317.6	2308.71
## 565	29	Male	64288	No	367.0	2157.29
## 566	36	Female	77865	Yes	328.2	2159.56
## 567	34	Male	68452	No	237.4	2052.20
## 568	32	Male	NA	No	316.2	2002.27
## 569	37	Male	56282	Yes	252.1	1878.03
## 570	41	Female	57480	No	388.9	2001.53
## 571	48	Female	58364	No	251.0	2097.39
## 572	28	Female	71145	No	316.8	1974.00
## 573	35	Female	NA	No	260.5	1874.39
## 574	30	Male	81359	Yes	331.9	1854.31
## 575	45	Male	48177	Yes	334.3	1896.10
## 576	43	Male	NA	Yes	311.9	2199.05
## 577	39	Male	76509	No	317.3	1808.93
## 578	31	Female	70626	No	359.9	1844.58
## 579	30	Female	75600	No	217.3	2068.10

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## 581	39	Male	81161	Yes	324.7	2123.18
## 582	40	Male	58719	No	263.1	2070.46
## 583	32	Male	83430	No	235.8	1800.12
## 584	34	Female	57762	No	343.1	1863.79
## 585	37	Male	NA	No	280.3	2000.45
## 586	34	Female	69856	No	229.7	1900.11
## 587	27	Male	72062	No	257.2	1914.92
## 588	24	Male	NA	Yes	363.4	2253.95
## 589	35	Female	NA	No	313.6	1800.93
## 590	33	Female	75974	Yes	306.3	1950.07
## 591	31	Female	80539	No	225.9	1975.76
## 592	33	Male	NA	No	388.6	1929.75
## 593	48	Male	NA	No	264.9	1662.88
## 594	38	Male	66167	No	341.0	2619.88
## 595	25	Male	66317	No	277.2	2211.91
## 596	43	Female	58386	No	238.7	2203.26
## 597	36	Male	94470	No	326.2	1734.84
## 598	27	Male	57293	Yes	299.4	2001.80
## 599	38	Male	74080	No	293.0	1995.77
## 600	33	Male	81900	Yes	309.9	2047.93
## 601	49	Female	74314	No	304.2	1582.86
## 602	39	Female	51058	Yes	344.2	2241.17
## 603	39	Male	56819	Yes	303.8	2184.07
## 604	43	Male	73348	Yes	309.3	2175.90
## 605	37	Male	NA	No	227.8	2003.87
## 606	46	Male	71334	Yes	302.9	2155.14
## 607	27	Female	67714	No	259.4	1960.02
## 608	34	Male	74599	No	285.9	1895.79
## 609	36	Female	NA	Yes	254.9	1662.15
## 610	35	Female	54162	No	426.9	1824.69
## 611	44	Male	NA	No	412.5	1761.98
## 612	32	Female	76320	Yes	291.0	2250.24
## 613	35	Female	73300	No	272.4	2059.88
## 614	39	Female	78567	Yes	348.6	2148.27
## 615	40	Male	62515	Yes	346.8	1806.37
## 616	30	Male	80747	Yes	430.2	1795.68

## 617	31	Female	64390	No	298.1	2279.46
## 618	39	Male	94275	No	244.8	1972.00
## 619	35	Male	49074	Yes	255.4	2226.45
## 620	46	Female	88945	Yes	229.0	1881.82
## 621	27	Male	73825	Yes	388.4	2430.97
## 622	36	Female	55672	No	192.4	2102.23
## 623	30	Male	75981	No	314.3	1900.10
## 624	37	Male	NA	No	323.1	1834.27
## 625	30	Male	NA	Yes	394.7	2110.50
## 626	33	Male	85878	No	177.7	2232.62
## 627	37	Male	66628	Yes	333.9	2044.02
## 628	41	Female	69055	No	350.7	2224.51
## 629	25	Male	77831	No	309.0	2062.95
## 630	30	Female	66344	No	187.5	1812.31
## 631	44	Male	66028	Yes	266.6	2173.57
## 632	22	Male	85489	No	329.8	1883.90
## 633	31	Male	59617	No	241.2	2346.53
## 634	38	Male	70418	Yes	257.5	2095.49
## 635	23	Female	79905	No	245.2	2026.54
## 636	38	Male	79704	No	244.5	1931.56
## 637	24	Female	NA	Yes	247.6	1817.30
## 638	23	Male	66349	No	273.3	2054.91
## 639	34	Male	53946	Yes	370.5	2305.34
## 640	31	Female	54923	Yes	319.2	1990.26
## 641	35	Male	57121	No	340.1	2010.43
## 642	28	Male	70499	No	349.5	2126.77
## 643	40	Male	78799	No	292.0	1824.83
## 644	29	Female	NA	No	323.9	2080.41
## 645	30	Female	73364	Yes	380.6	1890.62
## 646	42	Male	77909	Yes	297.5	2052.50
## 647	37	Female	73123	Yes	314.7	2147.49
## 648	32	Male	68784	No	268.3	2330.25
## 649	42	Male	59126	Yes	324.7	1817.53
## 650	40	Female	76242	No	236.9	2098.14
## 651	23	Female	89178	No	205.6	2506.17
## 652	25	Female	87159	Yes	212.5	2069.52
## 653	23	Male	NA	No	193.0	1251.46

## 654	23	Female	89670	No	202.1	1263.24
## 655	26	Male	89644	No	203.3	1033.70
## 656	27	Female	93462	No	192.9	2367.26
## 657	25	Female	88802	No	202.3	1807.88
## 658	26	Female	92634	No	222.5	2268.71
## 659	20	Female	87992	No	185.5	1644.05
## 660	26	Female	83869	No	193.1	1931.37
## 661	21	Female	NA	No	161.8	1797.49
## 662	23	Female	86274	No	198.8	1210.98
## 663	22	Female	91369	Yes	198.7	2150.58
## 664	24	Male	89268	No	193.2	1513.56
## 665	29	Female	88348	Yes	216.3	2006.22
## 666	20	Female	93821	No	198.9	2884.71
## 667	22	Female	91472	No	188.2	2310.93
## 668	27	Female	82967	No	186.7	1824.53
## 669	24	Female	85943	No	212.3	2387.95
## 670	22	Female	102655	Yes	205.7	1871.73
## 671	24	Female	94031	Yes	198.9	1651.71
## 672	22	Female	92386	No	195.7	2600.20
## 673	24	Female	90311	No	200.8	2935.10
## 674	28	Female	90749	No	211.1	1496.41
## 675	26	Female	95957	Yes	196.6	2542.99
## 676	23	Female	89872	No	185.3	1942.55
## 677	24	Female	90547	Yes	194.6	2172.44
## 678	24	Female	87898	No	213.6	873.71
## 679	26	Female	82729	No	194.6	1664.86
## 680	23	Male	NA	No	210.3	1975.99
## 681	22	Female	92494	No	201.1	1919.88
## 682	24	Female	NA	No	203.7	1796.36
## 683	28	Female	90793	Yes	182.5	1524.65
## 684	27	Female	84747	No	190.9	2082.58
## 685	23	Female	89610	No	203.2	1734.34
## 686	23	Female	87122	Yes	193.0	992.96
## 687	27	Female	NA	Yes	205.4	2532.57
## 688	21	Female	86341	No	177.2	2603.03
## 689	25	Female	93571	No	217.2	2229.67
## 690	28	Female	94912	No	200.2	2801.92

## 691	23	Female	91559	Yes	200.4	1896.10
## 692	25	Female	91939	No	223.0	1889.79
## 693	21	Female	84933	No	195.5	2103.33
## 694	22	Male	92386	No	204.2	2237.61
## 695	23	Female	99408	No	200.3	2493.13
## 696	19	Female	83535	No	227.7	1490.72
## 697	23	Female	97857	No	188.2	1551.87
## 698	23	Female	92648	No	197.3	2432.04
## 699	25	Female	90828	Yes	198.1	3072.36
## 700	25	Female	95034	No	202.2	1834.32
## 701	23	Female	78323	No	210.1	2108.84
## 702	26	Female	84832	No	203.4	1236.20
## 703	23	Female	93241	Yes	210.5	1870.61
## 704	26	Female	93656	Yes	203.1	2379.60
## 705	24	Male	87635	No	197.1	1424.30
## 706	24	Female	NA	No	175.9	2412.85
## 707	24	Female	NA	Yes	191.5	1551.18
## 708	22	Female	90081	No	194.1	2035.32
## 709	23	Female	88983	No	202.2	1167.69
## 710	26	Female	NA	No	183.1	2391.23
## 711	23	Female	92176	Yes	213.1	2183.85
## 712	23	Female	NA	No	199.4	2488.37
## 713	26	Female	93115	Yes	203.9	2382.82
## 714	24	Female	81848	Yes	191.0	2334.48
## 715	22	Female	87477	No	195.3	1636.81
## 716	24	Female	92506	No	191.5	1929.63
## 717	26	Female	NA	No	189.6	1107.59
## 718	22	Female	91886	No	199.8	1492.39
## 719	26	Female	NA	No	214.1	2130.79
## 720	23	Female	88386	No	195.1	1740.06
## 721	24	Female	97019	No	188.1	2554.30
## 722	26	Female	96718	No	215.9	1949.94
## 723	24	Female	91834	No	210.3	2448.73
## 724	23	Female	NA	Yes	192.7	2444.43
## 725	26	Female	92346	No	200.6	1388.04
## 726	24	Female	92081	Yes	178.5	2259.28
## 727	25	Male	92088	No	189.3	2082.66

## 728	26	Male	94010	No	203.5	1737.52
## 729	22	Female	90477	No	195.9	2684.44
## 730	25	Female	89653	Yes	222.6	1121.35
## 731	27	Male	93193	No	178.7	2467.77
## 732	26	Female	89056	Yes	193.4	1778.74
## 733	25	Female	88515	No	198.6	1528.57
## 734	24	Male	88930	No	220.7	2069.27
## 735	24	Female	93318	Yes	198.9	2610.17
## 736	23	Male	85384	Yes	192.2	1381.04
## 737	25	Male	86249	No	185.1	3049.03
## 738	23	Female	81764	No	205.7	1040.90
## 739	22	Male	87850	No	201.8	1952.33
## 740	22	Female	93525	No	190.3	2161.74
## 741	30	Female	NA	No	203.9	1812.20
## 742	23	Female	87669	Yes	197.7	2030.82
## 743	24	Female	87565	No	203.6	2072.28
## 744	23	Female	95831	No	199.9	2208.52
## 745	26	Female	93259	No	206.7	2358.84
## 746	22	Female	93569	No	202.4	1575.05
## 747	23	Female	86881	Yes	204.6	2636.66
## 748	26	Female	NA	Yes	178.3	1458.33
## 749	26	Male	97518	No	198.5	1379.55
## 750	22	Female	NA	No	183.8	1812.87
## 751	24	Male	94164	No	201.2	1905.86
## 752	25	Female	87545	No	196.0	1430.16
## 753	25	Female	87509	No	192.5	3075.19
## 754	26	Female	85853	Yes	196.8	1603.84
## 755	27	Male	92512	No	202.8	2179.37
## 756	26	Female	85161	No	203.5	2009.07
## 757	29	Female	85068	No	183.4	1909.02
## 758	24	Female	93131	No	193.0	1791.48
## 759	25	Male	88544	No	192.8	2371.81
## 760	23	Female	87554	Yes	217.2	2037.90
## 761	25	Female	94443	No	206.5	2092.59
## 762	25	Male	87891	No	190.9	1544.12
## 763	33	Female	91748	No	188.8	2216.19
## 764	25	Male	97369	No	166.2	2229.26

## 765	26	Male	86821	Yes	201.2	1560.17
## 766	26	Female	100226	No	209.6	2174.18
## 767	25	Female	88409	Yes	182.4	1304.25
## 768	26	Female	94658	No	212.4	1572.38
## 769	24	Female	100292	No	181.8	2033.15
## 770	25	Female	90913	No	187.3	1669.90
## 771	26	Female	89461	No	200.5	2449.80
## 772	27	Female	99464	No	204.4	1768.27
## 773	26	Female	105529	Yes	186.9	2349.93
## 774	26	Female	93885	No	203.1	2145.69
## 775	25	Female	84813	No	206.0	2273.40
## 776	24	Female	99718	No	203.2	1635.11
## 777	21	Female	93695	No	209.1	1985.93
## 778	23	Female	84628	No	204.8	1753.82
## 779	26	Female	92447	No	192.5	1601.50
## 780	23	Female	90074	No	198.2	1990.16
## 781	27	Male	89414	No	198.4	2207.23
## 782	21	Male	84611	No	185.8	2150.92
## 783	24	Male	98964	No	206.1	1289.37
## 784	24	Female	NA	No	192.1	2197.87
## 785	24	Female	93071	No	202.1	1515.48
## 786	23	Male	100481	Yes	190.5	2214.41
## 787	25	Female	92883	No	208.1	2556.11
## 788	25	Female	80405	No	-500.0	1897.56
## 789	25	Female	88985	Yes	191.9	2735.55
## 790	20	Male	82467	No	205.3	2673.26
## 791	25	Female	93555	Yes	196.8	2152.20
## 792	25	Female	85530	No	209.8	2072.35
## 793	26	Female	93400	No	182.2	1934.79
## 794	23	Female	NA	Yes	195.6	2191.55
## 795	24	Female	82529	No	220.2	2160.57
## 796	24	Female	87792	No	193.3	1829.13
## 797	21	Female	91852	No	200.3	1630.41
## 798	21	Female	101811	No	205.6	1663.38
## 799	25	Female	87634	No	212.1	2495.04
## 800	22	Female	NA	No	215.5	1395.86
## 801	28	Female	NA	No	207.3	2615.89

## 802	27	Female	92935	No	203.8	1885.18
## 803	24	Female	89301	No	223.6	2726.38
## 804	23	Female	NA	No	197.9	2313.23
## 805	21	Female	90184	Yes	197.0	1835.95
## 806	24	Female	NA	No	197.1	1922.65
## 807	24	Female	89718	Yes	188.2	2199.11
## 808	27	Female	88689	Yes	211.2	1673.35
## 809	22	Female	93584	No	206.4	1801.51
## 810	24	Female	89054	No	208.2	2021.27
## 811	26	Female	91257	Yes	195.9	1628.03
## 812	24	Female	95187	No	184.6	1743.79
## 813	26	Male	83997	No	192.3	1822.91
## 814	22	Female	94830	No	199.3	1955.97
## 815	29	Male	87054	No	193.6	2800.16
## 816	23	Male	NA	No	192.2	1531.26
## 817	24	Male	97373	No	185.7	2178.25
## 818	21	Female	NA	Yes	196.6	2250.53
## 819	26	Female	NA	No	200.9	1529.23
## 820	18	Female	89416	Yes	209.5	1926.47
## 821	24	Female	79865	No	212.4	2293.59
## 822	24	Female	NA	Yes	204.8	2007.49
## 823	24	Female	93991	No	210.0	1932.58
## 824	25	Female	89760	No	206.9	1832.92
## 825	23	Female	91634	No	202.1	2082.89
## 826	26	Male	96071	Yes	191.6	1663.72
## 827	26	Female	84007	No	208.1	2181.24
## 828	21	Female	83531	No	215.1	1257.70
## 829	23	Female	90214	No	198.7	2134.74
## 830	23	Female	92445	Yes	187.9	1540.97
## 831	26	Female	88502	No	199.3	1680.43
## 832	25	Female	86094	No	199.8	2297.09
## 833	19	Female	92813	No	186.7	1041.54
## 834	24	Female	95866	Yes	198.1	2098.53
## 835	25	Female	94382	No	210.5	1958.94
## 836	26	Female	92755	No	218.7	1638.54
## 837	26	Male	88378	No	199.5	1504.64
## 838	27	Male	89076	No	204.5	2991.98

## 839	23	Male	94024	No	192.1	1772.65
## 840	25	Female	100952	No	198.8	746.72
## 841	25	Female	84472	No	211.7	1173.95
## 842	24	Female	89810	No	198.9	2665.06
## 843	26	Female	91373	Yes	216.4	2194.18
## 844	25	Female	93085	No	209.4	1167.05
## 845	26	Female	87590	Yes	186.6	2407.72
## 846	24	Female	89558	No	211.7	1910.56
## 847	25	Female	85957	No	186.3	1856.43
## 848	21	Female	89582	Yes	198.1	1986.69
## 849	26	Female	84049	Yes	196.6	1400.31
## 850	26	Male	NA	No	217.2	2018.28
## 851	29	Male	90753	Yes	207.4	2086.91
## 852	26	Female	92267	No	198.7	1122.82
## 853	25	Female	92906	No	204.4	1245.15
## 854	26	Male	95905	No	195.4	2164.48
## 855	25	Female	85401	No	204.1	1614.14
## 856	25	Female	91776	No	201.2	2439.70
## 857	21	Female	88008	No	194.1	1708.93
## 858	27	Female	90734	Yes	210.3	2265.74
## 859	23	Female	86666	Yes	189.6	2127.42
## 860	24	Female	88266	Yes	208.2	1585.71
## 861	24	Female	94957	No	216.2	1239.82
## 862	26	Female	84550	No	209.7	1459.45
## 863	26	Female	85205	Yes	210.1	1933.98
## 864	23	Male	89640	No	194.0	1738.68
## 865	21	Male	87527	No	210.5	2113.96
## 866	25	Female	NA	No	211.1	2412.76
## 867	25	Male	NA	No	201.1	1676.81
## 868	24	Female	88349	Yes	193.6	2528.62
## 869	25	Female	90856	No	178.4	1754.47
## 870	23	Female	82922	No	219.9	1994.64
## 871	23	Female	89201	No	219.8	1376.26
## 872	22	Female	98708	No	207.7	2029.09
## 873	29	Female	77346	No	203.9	2129.71
## 874	20	Female	80078	No	185.9	1334.98
## 875	23	Female	93774	No	191.1	1576.85

## 876	26	Female	98187	Yes	189.5	2033.07
## 877	24	Male	92192	No	203.0	2000.24
## 878	27	Female	92132	No	182.8	996.29
## 879	25	Female	90718	Yes	197.7	1356.00
## 880	24	Male	86815	No	192.6	1225.00
## 881	23	Female	83075	No	30000.0	1678.91
## 882	24	Female	82550	No	195.5	2627.20
## 883	24	Female	96434	No	209.3	2255.37
## 884	25	Female	90856	No	203.8	2228.48
## 885	26	Male	87497	Yes	194.9	1161.85
## 886	24	Female	86970	No	200.1	1680.50
## 887	24	Female	93899	Yes	211.5	1988.68
## 888	23	Male	94526	Yes	212.7	2367.53
## 889	25	Female	93275	No	194.7	1929.53
## 890	25	Female	95425	No	205.1	2729.22
## 891	21	Female	91260	No	198.3	1737.75
## 892	22	Male	82007	No	200.3	1070.51
## 893	24	Female	88170	No	199.9	1284.37
## 894	23	Male	87804	No	202.1	1878.80
## 895	28	Female	92103	No	213.6	1598.35
## 896	27	Male	86813	No	191.1	2047.26
## 897	27	Female	92033	No	209.0	2412.89
## 898	26	Female	90950	No	197.1	1872.80
## 899	25	Female	NA	No	197.7	1713.57
## 900	24	Female	96891	No	208.2	3317.68
## 901	24	Female	81280	Yes	200.6	1583.43
## 902	22	Female	96508	No	203.1	1945.31
## 903	23	Female	95593	No	191.4	2081.22
## 904	24	Male	94335	No	207.6	1760.71
## 905	25	Female	98297	No	178.2	1787.44
## 906	26	Female	87400	No	208.8	2087.68
## 907	26	Female	79641	No	200.0	1558.82
## 908	26	Female	89721	Yes	191.1	1838.62
## 909	24	Female	91569	No	213.4	2196.67
## 910	23	Female	85771	No	191.0	2029.90
## 911	22	Female	88315	No	199.9	2477.18
## 912	26	Female	91320	No	193.3	2424.61

## 913	27	Male	90735	No	184.9	2891.32
## 914	27	Female	91906	No	210.5	2030.71
## 915	24	Female	92975	No	212.8	2329.11
## 916	25	Female	93852	No	192.1	2113.64
## 917	26	Female	92238	No	214.5	1230.39
## 918	24	Male	89175	Yes	181.7	1106.88
## 919	28	Female	NA	No	195.9	2661.26
## 920	26	Female	91589	Yes	216.5	1770.59
## 921	25	Female	82520	Yes	215.6	1788.27
## 922	29	Female	87591	No	205.0	2921.28
## 923	27	Female	90303	No	198.9	1870.39
## 924	26	Female	98462	No	185.7	1906.87
## 925	22	Male	91553	No	200.7	1777.50
## 926	22	Female	93934	Yes	208.8	1520.62
## 927	25	Male	88732	No	204.2	2162.41
## 928	23	Female	87819	No	214.4	2913.31
## 929	21	Female	89840	No	217.2	1357.66
## 930	20	Female	99267	Yes	194.2	2476.18
## 931	27	Female	94708	No	196.4	1773.61
## 932	23	Female	89509	No	176.8	2749.34
## 933	21	Female	95848	Yes	210.3	1907.57
## 934	24	Female	80674	No	208.3	1805.54
## 935	23	Female	95800	No	213.7	1645.49
## 936	25	Female	96685	No	209.7	1385.90
## 937	23	Female	87160	No	30000.0	3298.91
## 938	25	Female	90943	No	202.3	1820.34
## 939	22	Female	88924	No	195.5	2171.47
## 940	20	Female	88201	Yes	200.8	2263.93
## 941	27	Female	94575	No	190.0	2370.88
## 942	24	Female	90619	No	194.2	1930.61
## 943	26	Female	94013	Yes	205.2	1357.30
## 944	23	Female	91746	Yes	198.4	2071.73
## 945	28	Female	95897	No	183.0	1514.87
## 946	23	Female	88709	Yes	186.0	2037.08
## 947	27	Female	85255	Yes	198.5	1609.47
## 948	22	Male	100969	Yes	219.5	2171.74
## 949	23	Female	84444	Yes	181.4	1140.41

##	950	28 Female	83810	Yes	181.0	2888.90
##	951	30 Female	95001	No	182.5	2443.99
##	952	25 Female	82673	No	203.7	2554.74
##	953	23 Female	88359	Yes	200.2	2142.99
##	954	21 Female	99124	No	215.1	2006.19
##	955	26 Female	98547	Yes	189.5	2697.73
##	956	22 Male	94905	Yes	207.1	2171.02
##	957	26 Male	92472	No	180.4	1207.09
##	958	22 Female	NA	Yes	187.2	2742.98
##	959	27 Female	87088	No	207.2	2676.69
##	960	22 Female	90801	No	188.2	470.05
##	961	24 Female	94831	Yes	193.2	1440.43
##	962	28 Male	89949	No	200.2	1859.17
##	963	22 Female	NA	No	194.9	1322.05
##	964	21 Female	89963	No	206.4	3027.63
##	965	22 Female	94185	No	210.1	2344.93
##	966	25 Female	NA	No	218.2	1420.93
##	967	26 Female	NA	No	192.7	1800.12
##	968	24 Male	91300	No	207.1	2188.52
##	969	24 Female	98840	No	196.9	1798.61
##	970	26 Female	87102	No	184.6	1115.42
##	971	26 Female	84162	No	189.8	1441.61
##	972	23 Male	88609	Yes	214.2	2846.34
##	973	22 Female	NA	Yes	202.5	2319.09
##	974	24 Female	84310	Yes	194.2	1424.43
##	975	26 Male	86864	No	189.6	1471.70
##	976	22 Male	81595	No	189.9	1832.26
##	977	26 Female	86016	No	201.2	3210.12
##	978	23 Male	86794	No	202.6	2093.19
##	979	26 Female	95342	No	199.0	2036.47
##	980	28 Female	88251	No	220.3	2097.26
##	981	27 Female	92442	Yes	202.8	1800.57
##	982	21 Female	88723	Yes	202.4	1525.74
##	983	27 Male	100162	Yes	209.5	995.36
##	984	26 Female	84919	Yes	186.4	2216.21
##	985	28 Female	88605	Yes	209.8	2124.79
##	986	27 Male	94157	No	213.9	2618.90

## 987	24	Female	92570	Yes	201.5	2093.85
## 988	22	Female	NA	Yes	190.9	1752.03
## 989	26	Female	86991	No	207.2	1528.81
## 990	23	Male	96072	No	193.9	1128.63
## 991	25	Male	88152	Yes	201.9	2675.73
## 992	22	Male	80839	No	196.0	2621.30
## 993	27	Female	93987	No	202.7	1684.63
## 994	24	Female	94993	No	199.9	2216.16
## 995	21	Male	95579	No	192.6	2489.26
## 996	23	Female	88040	No	206.0	1885.09
## 997	25	Female	91034	No	198.7	2437.62
## 998	26	Female	100724	No	195.0	2360.36
## 999	23	Male	88926	No	190.7	1724.03
## 1000	26	Female	94769	No	199.7	2482.66

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## 966	1	26	2	1	1	1	4	1	5	2
## 967	3	19	3	1	1	2	4	1	5	3
## 968	6	28	2	1	1	2	4	1	4	3
## 969	4	21	3	1	1	2	4	1	5	3
## 970	3	13	2	1	1	2	4	1	4	2
## 971	3	22	2	1	1	1	4	1	5	3
## 972	3	19	2	1	1	2	4	1	5	3
## 973	3	27	3	1	1	1	4	1	4	3
## 974	3	27	3	1	1	1	4	1	4	3
## 975	3	23	2	1	1	1	4	1	4	2
## 976	3	21	3	1	1	1	4	1	4	2
## 977	3	17	2	1	1	2	4	1	5	2
## 978	1	25	2	1	1	2	4	1	5	2
## 979	3	21	3	1	1	2	4	1	5	2
## 980	2	18	3	1	1	1	4	1	5	3
## 981	3	22	3	1	1	1	4	1	4	3
## 982	3	17	3	1	1	1	4	1	4	2
## 983	3	30	3	1	1	1	4	1	5	3
## 984	1	16	3	1	1	2	4	1	5	3

```
## 985      1      25  2  1  1  2  4  1  4  3
## 986      2      15  2  1  1  2  4  1  4  3
## 987      3      17  2  1  1  1  4  1  4  3
## 988      3      22  2  1  1  2  4  1  5  2
## 989      2      18  3  1  1  2  5  1  4  3
## 990      6      21  3  1  1  2  4  1  4  3
## 991      3      23  2  1  1  1  4  1  5  2
## 992      1      24  3  1  1  1  4  1  4  2
## 993      3      15  2  1  1  1  4  1  4  3
## 994      4      19  2  1  1  2  4  1  4  2
## 995      4      20  3  1  1  2  4  1  4  3
## 996      2      19  3  1  1  2  4  1  4  2
## 997      2      29  3  1  1  1  4  1  5  3
## 998      2      24  3  1  1  2  4  1  5  2
## 999      1      23  2  1  1  2  4  1  5  3
## 1000     3      16  2  1  1  1  4  1  5  3

##      Q9 Q10      segment total_exp
## 1      2  4      Price      832.6
## 2      1  4      Price      587.5
## 3      1  4      Price      770.1
## 4      2  4      Price      489.5
## 5      2  4      Price      491.9
## 6      1  4      Price      534.0
## 7      1  4      Price      767.1
## 8      2  4      Price      476.0
## 9      1  4      Price      750.8
## 10     1  4      Price      633.8
## 11     2  4      Price      536.8
## 12     1  4      Price      985.1
## 13     2  4      Price      774.8
## 14     1  4      Price      961.6
## 15     1  4      Price      644.3
## 16     1  4      Price      853.2
## 17     1  4      Price      692.1
## 18     2  4      Price      740.1
## 19     2  4      Price      794.9
## 20     2  4      Price      695.4
```

## 21	1	4	Price	669.2
## 22	2	4	Price	741.0
## 23	1	4	Price	638.5
## 24	2	4	Price	707.4
## 25	1	4	Price	534.9
## 26	2	4	Price	620.7
## 27	2	4	Price	661.9
## 28	2	4	Price	868.7
## 29	1	4	Price	714.5
## 30	2	4	Price	922.8
## 31	1	4	Price	748.5
## 32	1	4	Price	709.0
## 33	2	4	Price	859.5
## 34	2	4	Price	843.3
## 35	1	4	Price	640.6
## 36	1	4	Price	713.9
## 37	1	4	Price	666.0
## 38	1	4	Price	659.8
## 39	2	4	Price	735.4
## 40	2	4	Price	752.6
## 41	1	4	Price	734.8
## 42	2	4	Price	806.9
## 43	2	4	Price	660.5
## 44	1	4	Price	765.2
## 45	1	4	Price	771.0
## 46	1	4	Price	779.5
## 47	2	4	Price	829.0
## 48	1	4	Price	708.7
## 49	2	4	Price	743.7
## 50	2	4	Price	735.3
## 51	1	4	Price	611.4
## 52	2	4	Price	662.1
## 53	2	4	Price	722.8
## 54	1	4	Price	763.3
## 55	1	4	Price	806.8
## 56	2	4	Price	524.5
## 57	2	4	Price	785.7

## 58	2	4	Price	736.8
## 59	2	4	Price	618.6
## 60	1	4	Price	609.0
## 61	2	4	Price	696.8
## 62	2	4	Price	866.9
## 63	1	4	Price	720.3
## 64	1	4	Price	502.0
## 65	1	4	Price	885.3
## 66	2	5	Price	928.7
## 67	1	4	Price	714.9
## 68	1	4	Price	547.9
## 69	2	4	Price	546.6
## 70	1	4	Price	634.3
## 71	1	4	Price	716.2
## 72	1	4	Price	882.2
## 73	2	4	Price	832.4
## 74	1	4	Price	831.2
## 75	1	4	Price	782.0
## 76	1	4	Price	929.9
## 77	2	4	Price	552.1
## 78	1	4	Price	636.5
## 79	1	4	Price	903.6
## 80	2	4	Price	867.7
## 81	2	4	Price	673.6
## 82	1	4	Price	814.8
## 83	2	4	Price	751.1
## 84	2	4	Price	554.5
## 85	2	4	Price	678.6
## 86	1	4	Price	836.5
## 87	1	4	Price	730.4
## 88	1	4	Price	838.1
## 89	2	4	Price	814.9
## 90	2	4	Price	608.1
## 91	2	4	Price	712.7
## 92	2	4	Price	779.0
## 93	1	4	Price	713.4
## 94	1	4	Price	709.2

## 95	2	4	Price	822.6
## 96	2	4	Price	629.5
## 97	2	4	Price	676.9
## 98	1	4	Price	674.6
## 99	1	4	Price	720.2
## 100	1	4	Price	778.1
## 101	2	4	Price	652.0
## 102	1	4	Price	706.1
## 103	1	4	Price	542.6
## 104	1	4	Price	415.0
## 105	2	4	Price	678.9
## 106	2	4	Price	697.7
## 107	1	4	Price	658.0
## 108	2	4	Price	697.6
## 109	2	4	Price	682.5
## 110	1	4	Price	500.7
## 111	1	4	Price	480.2
## 112	2	4	Price	608.5
## 113	1	4	Price	484.7
## 114	1	4	Price	803.4
## 115	2	4	Price	596.3
## 116	2	4	Price	805.9
## 117	2	4	Price	566.2
## 118	2	4	Price	767.8
## 119	2	4	Price	617.3
## 120	2	4	Price	405.3
## 121	2	4	Price	711.8
## 122	1	4	Price	567.2
## 123	1	4	Price	595.7
## 124	1	4	Price	638.7
## 125	2	4	Price	748.3
## 126	2	4	Price	752.6
## 127	2	4	Price	674.8
## 128	1	4	Price	709.3
## 129	2	4	Price	704.5
## 130	1	4	Price	566.7
## 131	1	4	Price	726.7

## 132	2	4	Price	723.5
## 133	2	4	Price	607.9
## 134	1	4	Price	721.1
## 135	2	4	Price	479.0
## 136	1	4	Price	845.8
## 137	2	4	Price	770.5
## 138	2	4	Price	554.6
## 139	2	4	Price	696.9
## 140	2	4	Price	694.6
## 141	1	5	Price	626.9
## 142	1	4	Price	521.9
## 143	2	4	Price	506.0
## 144	2	4	Price	791.2
## 145	1	4	Price	508.2
## 146	2	4	Price	820.5
## 147	2	4	Price	727.8
## 148	2	4	Price	742.6
## 149	1	4	Price	702.0
## 150	1	4	Price	707.8
## 151	1	4	Price	725.5
## 152	1	4	Price	467.3
## 153	2	4	Price	920.2
## 154	2	4	Price	916.6
## 155	1	4	Price	351.9
## 156	1	4	Price	508.6
## 157	1	4	Price	732.0
## 158	2	4	Price	669.3
## 159	1	4	Price	819.7
## 160	2	4	Price	690.0
## 161	1	4	Price	906.1
## 162	2	4	Price	671.9
## 163	1	4	Price	877.5
## 164	1	4	Price	459.7
## 165	2	4	Price	618.6
## 166	2	4	Price	611.9
## 167	2	4	Price	684.4
## 168	2	4	Price	524.7

## 169	1	4	Price	820.2
## 170	2	4	Price	638.6
## 171	2	4	Price	956.2
## 172	1	4	Price	910.0
## 173	1	4	Price	763.0
## 174	1	4	Price	751.0
## 175	2	4	Price	538.0
## 176	2	4	Price	714.4
## 177	2	4	Price	793.5
## 178	2	4	Price	727.0
## 179	2	4	Price	615.5
## 180	1	4	Price	504.9
## 181	2	4	Price	674.2
## 182	2	4	Price	715.4
## 183	2	4	Price	526.7
## 184	2	4	Price	791.2
## 185	1	4	Price	729.9
## 186	2	4	Price	948.7
## 187	2	4	Price	725.5
## 188	1	4	Price	769.7
## 189	2	4	Price	743.0
## 190	1	4	Price	498.9
## 191	2	4	Price	874.5
## 192	2	4	Price	663.0
## 193	1	3	Price	612.0
## 194	1	4	Price	905.1
## 195	2	4	Price	699.1
## 196	2	4	Price	500.2
## 197	1	4	Price	581.3
## 198	2	4	Price	668.7
## 199	2	4	Price	802.0
## 200	2	4	Price	814.4
## 201	2	4	Price	797.0
## 202	1	4	Price	617.9
## 203	2	4	Price	781.3
## 204	2	4	Price	849.7
## 205	1	4	Price	811.5

## 206	2	4	Price	857.6
## 207	2	4	Price	513.3
## 208	2	4	Price	772.7
## 209	1	4	Price	710.1
## 210	1	4	Price	549.1
## 211	2	4	Price	666.7
## 212	2	4	Price	855.7
## 213	2	4	Price	738.1
## 214	1	4	Price	708.5
## 215	1	4	Price	643.8
## 216	1	4	Price	674.1
## 217	1	4	Price	598.5
## 218	2	4	Price	711.4
## 219	2	4	Price	692.4
## 220	1	4	Price	686.7
## 221	2	4	Price	828.4
## 222	2	4	Price	834.2
## 223	2	4	Price	653.0
## 224	2	4	Price	854.5
## 225	1	4	Price	786.8
## 226	2	4	Price	711.3
## 227	2	4	Price	890.5
## 228	1	4	Price	665.2
## 229	1	4	Price	794.7
## 230	1	4	Price	778.4
## 231	1	4	Price	664.7
## 232	2	4	Price	813.7
## 233	1	4	Price	679.5
## 234	2	4	Price	808.0
## 235	1	4	Price	821.9
## 236	1	4	Price	797.6
## 237	1	4	Price	710.4
## 238	1	4	Price	553.2
## 239	2	4	Price	468.5
## 240	1	4	Price	753.3
## 241	1	4	Price	762.6
## 242	1	4	Price	644.7

## 243	1	4	Price	836.3
## 244	2	4	Price	625.3
## 245	2	4	Price	595.3
## 246	1	4	Price	740.4
## 247	1	4	Price	785.9
## 248	2	4	Price	841.7
## 249	1	4	Price	751.7
## 250	1	4	Price	677.4
## 251	4	1	Conspicuous	10826.7
## 252	4	1	Conspicuous	8632.7
## 253	4	1	Conspicuous	9557.7
## 254	4	1	Conspicuous	7292.7
## 255	4	1	Conspicuous	8201.7
## 256	4	1	Conspicuous	7594.8
## 257	4	2	Conspicuous	7341.6
## 258	4	1	Conspicuous	7964.9
## 259	4	1	Conspicuous	11503.6
## 260	4	2	Conspicuous	10251.6
## 261	4	2	Conspicuous	8767.7
## 262	4	1	Conspicuous	11234.7
## 263	4	1	Conspicuous	8458.7
## 264	4	2	Conspicuous	8518.4
## 265	4	1	Conspicuous	12912.2
## 266	4	2	Conspicuous	8438.9
## 267	4	1	Conspicuous	9679.1
## 268	4	1	Conspicuous	9279.2
## 269	4	1	Conspicuous	9291.2
## 270	4	1	Conspicuous	11087.6
## 271	4	2	Conspicuous	8662.0
## 272	4	2	Conspicuous	8493.6
## 273	4	1	Conspicuous	8402.8
## 274	4	1	Conspicuous	11387.6
## 275	4	2	Conspicuous	8674.3
## 276	4	1	Conspicuous	9187.5
## 277	4	2	Conspicuous	9401.3
## 278	4	1	Conspicuous	10910.2
## 279	4	1	Conspicuous	10084.3

## 280	4	2	Conspicuous	11397.0
## 281	3	2	Conspicuous	6175.0
## 282	4	1	Conspicuous	6174.2
## 283	4	1	Conspicuous	13262.5
## 284	4	1	Conspicuous	8039.2
## 285	4	1	Conspicuous	11070.5
## 286	4	2	Conspicuous	9353.2
## 287	4	1	Conspicuous	11697.0
## 288	4	2	Conspicuous	11130.3
## 289	4	1	Conspicuous	9681.1
## 290	4	2	Conspicuous	9054.8
## 291	4	2	Conspicuous	13183.0
## 292	4	2	Conspicuous	11420.2
## 293	4	1	Conspicuous	7747.7
## 294	4	2	Conspicuous	9517.8
## 295	4	1	Conspicuous	11567.1
## 296	4	2	Conspicuous	9979.5
## 297	4	1	Conspicuous	10787.6
## 298	4	2	Conspicuous	10394.2
## 299	4	1	Conspicuous	11359.8
## 300	4	1	Conspicuous	10793.9
## 301	4	2	Conspicuous	8588.7
## 302	4	1	Conspicuous	10443.3
## 303	4	1	Conspicuous	10520.9
## 304	4	1	Conspicuous	10058.0
## 305	4	2	Conspicuous	9183.7
## 306	4	2	Conspicuous	9584.0
## 307	4	2	Conspicuous	11664.7
## 308	4	2	Conspicuous	9897.1
## 309	4	1	Conspicuous	8143.0
## 310	4	1	Conspicuous	7216.1
## 311	4	2	Conspicuous	8770.2
## 312	4	2	Conspicuous	11058.8
## 313	4	1	Conspicuous	13091.3
## 314	4	2	Conspicuous	8405.8
## 315	4	1	Conspicuous	11370.7
## 316	4	2	Conspicuous	8926.1

## 317	4	1	Conspicuous	9782.1
## 318	4	1	Conspicuous	9400.1
## 319	4	2	Conspicuous	10125.1
## 320	4	2	Conspicuous	6673.0
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## 323	4	2	Conspicuous	11390.7
## 324	4	1	Conspicuous	8442.0
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## 327	4	2	Conspicuous	11948.9
## 328	4	2	Conspicuous	9890.9
## 329	4	1	Conspicuous	12296.8
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## 332	4	2	Conspicuous	9445.0
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## 334	4	1	Conspicuous	9423.0
## 335	4	1	Conspicuous	9391.5
## 336	4	1	Conspicuous	10404.8
## 337	4	1	Conspicuous	9342.3
## 338	4	1	Conspicuous	10667.4
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## 340	4	1	Conspicuous	8413.4
## 341	4	2	Conspicuous	14046.7
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## 344	4	2	Conspicuous	9606.8
## 345	4	1	Conspicuous	9251.9
## 346	4	1	Conspicuous	12329.9
## 347	4	2	Conspicuous	16503.1
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## 350	4	2	Conspicuous	10022.5
## 351	4	2	Conspicuous	6424.5
## 352	5	2	Conspicuous	11512.6
## 353	4	1	Conspicuous	9029.4

## 354	4	1	Conspicuous	10473.6
## 355	4	1	Conspicuous	8904.1
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## 357	4	1	Conspicuous	10179.2
## 358	4	2	Conspicuous	10701.1
## 359	4	2	Conspicuous	10475.8
## 360	4	2	Conspicuous	8083.9
## 361	4	2	Conspicuous	10983.0
## 362	4	1	Conspicuous	12464.7
## 363	4	2	Conspicuous	8330.0
## 364	4	1	Conspicuous	13784.4
## 365	4	2	Conspicuous	9598.3
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## 367	4	1	Conspicuous	8687.9
## 368	4	1	Conspicuous	10046.4
## 369	4	1	Conspicuous	12746.5
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## 373	4	2	Conspicuous	9944.9
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## 381	4	1	Conspicuous	10045.7
## 382	4	2	Conspicuous	9962.7
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## 386	4	1	Conspicuous	12390.8
## 387	4	2	Conspicuous	10262.2
## 388	4	2	Conspicuous	9856.0
## 389	4	1	Conspicuous	9799.8
## 390	4	1	Conspicuous	10096.2

## 391	4	1	Conspicuous	11176.4
## 392	4	2	Conspicuous	13157.6
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## 394	4	1	Conspicuous	10178.3
## 395	4	2	Conspicuous	9174.1
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## 397	4	2	Conspicuous	6487.5
## 398	4	2	Conspicuous	9088.2
## 399	4	1	Conspicuous	11666.3
## 400	4	1	Conspicuous	8897.2
## 401	4	1	Conspicuous	9754.4
## 402	4	1	Conspicuous	11500.2
## 403	4	2	Conspicuous	8258.4
## 404	4	2	Conspicuous	12206.8
## 405	4	2	Conspicuous	10454.4
## 406	4	1	Conspicuous	6949.6
## 407	4	2	Conspicuous	7209.5
## 408	4	1	Conspicuous	9622.5
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## 410	4	1	Conspicuous	8438.8
## 411	4	2	Conspicuous	11376.7
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## 413	4	1	Conspicuous	9684.2
## 414	4	1	Conspicuous	11699.2
## 415	4	2	Conspicuous	12425.0
## 416	4	2	Conspicuous	10767.0
## 417	4	1	Conspicuous	5423.1
## 418	4	1	Conspicuous	11523.8
## 419	4	1	Conspicuous	8997.7
## 420	4	2	Conspicuous	9537.1
## 421	4	2	Conspicuous	10473.2
## 422	4	2	Conspicuous	9150.8
## 423	4	2	Conspicuous	9853.2
## 424	4	2	Conspicuous	10512.2
## 425	4	2	Conspicuous	11197.8
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## 432	4	2	Conspicuous	9423.4
## 433	4	1	Conspicuous	11940.0
## 434	4	1	Conspicuous	9490.1
## 435	4	1	Conspicuous	12357.2
## 436	4	1	Conspicuous	8641.8
## 437	4	2	Conspicuous	6774.2
## 438	4	2	Conspicuous	10073.8
## 439	4	1	Conspicuous	9597.9
## 440	4	1	Conspicuous	8408.0
## 441	4	1	Conspicuous	10414.8
## 442	4	1	Conspicuous	7985.6
## 443	4	2	Conspicuous	10833.0
## 444	4	2	Conspicuous	11064.9
## 445	4	2	Conspicuous	11832.7
## 446	4	1	Conspicuous	10451.4
## 447	4	2	Conspicuous	11295.6
## 448	4	2	Conspicuous	13528.3
## 449	4	2	Conspicuous	8236.4
## 450	4	1	Conspicuous	7894.9
## 451	2	2	Quality	2076.0
## 452	3	2	Quality	2220.3
## 453	3	3	Quality	2125.3
## 454	3	3	Quality	1886.6
## 455	3	2	Quality	2531.7
## 456	3	3	Quality	2471.2
## 457	3	2	Quality	2570.2
## 458	2	2	Quality	2457.2
## 459	3	2	Quality	2332.0
## 460	2	3	Quality	2194.8
## 461	3	3	Quality	2185.7
## 462	3	3	Quality	2620.3
## 463	2	2	Quality	2039.1
## 464	2	3	Quality	2174.5

## 465	2	3	Quality	2676.4
## 466	2	3	Quality	2028.6
## 467	2	2	Quality	2135.6
## 468	3	2	Quality	1994.5
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## 992  4  2      Style  2817.3
## 993  4  1      Style  1887.3
## 994  4  2      Style  2416.1
## 995  4  2      Style  2681.8
## 996  4  2      Style  2091.1
## 997  4  1      Style  2636.3
## 998  4  1      Style  2555.4
## 999  4  2      Style  1914.7
## 1000 4  2      Style  2682.3
```

The above code sums up two columns and appends the result (`total_exp`) to `sim.dat`. Another similar function is `transmute()`. The difference is that `transmute()` will delete the original columns and only keep the new ones.

```
dplyr::transmute(sim.dat, total_exp = store_exp + online_exp)
```

Merge

Similar to SQL, there are different joins in `dplyr`. We create two baby data sets to show how the functions work.

```
(x<-data.frame(cbind(ID=c("A","B","C"),x1=c(1,2,3))))
```

```
##   ID x1
## 1  A  1
## 2  B  2
## 3  C  3
```

```
(y<-data.frame(cbind(ID=c("B","C","D"),y1=c(T,T,F))))
```

```
##   ID  y1
```

```
## 1 B TRUE
## 2 C TRUE
## 3 D FALSE
```

```
# join to the left
# keep all rows in x
left_join(x,y,by="ID")
```

```
## ID x1 y1
## 1 A 1 <NA>
## 2 B 2 TRUE
## 3 C 3 TRUE
```

```
# get rows matched in both data sets
inner_join(x,y,by="ID")
```

```
## ID x1 y1
## 1 B 2 TRUE
## 2 C 3 TRUE
```

```
# get rows in either data set
full_join(x,y,by="ID")
```

```
## ID x1 y1
## 1 A 1 <NA>
## 2 B 2 TRUE
## 3 C 3 TRUE
## 4 D <NA> FALSE
```

```
# filter out rows in x that can be matched in y
# it doesn't bring in any values from y
semi_join(x,y,by="ID")
```

```
## ID x1
## 1 B 2
## 2 C 3
```

```
# the opposite of semi_join()
# it gets rows in x that cannot be matched in y
# it doesn't bring in any values from y
anti_join(x,y,by="ID")
```

```
## ID x1
## 1 A 1
```

There are other functions(`intersect()`, `union()` and `setdiff()`). Also the data frame version of `rbind` and `cbind` which are `bind_rows()` and `bind_col()`. We are not going to go through them all. You can try them yourself. If you understand the functions we introduced so far. It should be easy for you to figure out the rest.

4.2 Tidy and Reshape Data

“Tidy data” represent the information from a dataset as data frames where each row is an observation, and each column contains the values of a variable (i.e., an attribute of what we are observing). Depending on the situation, the requirements on what to present as rows and columns may change. To make data easy to work with for the problem at hand, in practice, we often need to convert data between the “wide” and the “long” format. The process feels like kneading the dough.

There are two commonly used packages for this kind of manipulations: `tidyr` and `reshape2`. We will show how to tidy and reshape data using the two packages. By comparing the functions to show how they overlap and where they differ.

4.2.1 `reshape2` package

It is a reboot of the previous package `reshape`. Take a baby subset of our exemplary clothes consumers data to illustrate:

```
(sdat<-sim.dat[1:5,1:6])

## age gender income house store_exp online_exp
## 1 57 Female 120963 Yes 529.1 303.5
## 2 63 Female 122008 Yes 478.0 109.5
```



```
## 3  59  Male 114202  Yes    490.8    279.2
## 4  60  Male 113616  Yes    347.8    141.7
## 5  51  Male 124253  Yes    379.6    112.2
```

For the above data `sdat`, what if we want to have a variable indicating the purchasing channel (i.e. online or in-store) and another column with the corresponding expense amount? Assume we want to keep the rest of the columns the same. It is a task to change data from “wide” to “long”. There are two general ways to shape data:

- Use `melt()` to convert an object into a molten data frame, i.e., from wide to long
- Use `dcast()` to cast a molten data frame into the shape you want, i.e., from long to wide

```
library(reshape2)
(mdat <- melt(sdat, measure.vars=c("store_exp","online_exp"),
             variable.name = "Channel",
             value.name = "Expense"))
```

```
##   age gender income house   Channel Expense
## 1  57 Female 120963   Yes store_exp   529.1
## 2  63 Female 122008   Yes store_exp   478.0
## 3  59  Male 114202   Yes store_exp   490.8
## 4  60  Male 113616   Yes store_exp   347.8
## 5  51  Male 124253   Yes store_exp   379.6
## 6  57 Female 120963   Yes online_exp   303.5
## 7  63 Female 122008   Yes online_exp   109.5
## 8  59  Male 114202   Yes online_exp   279.2
## 9  60  Male 113616   Yes online_exp   141.7
## 10 51  Male 124253   Yes online_exp   112.2
```

You melted the data frame `sdat` by two variables: `store_exp` and `online_exp` (`measure.vars=c("store_exp","online_exp")`). The new variable name is `Channel` set by command `variable.name = "Channel"`. The value name is `Expense` set by command `value.name = "Expense"`.

You can run a regression to study the effect of purchasing channel as follows:

```
# Here we use all observations from sim.dat
# Don't show result here
mdat<-melt(sim.dat[,1:6], measure.vars=c("store_exp","online_exp"),
           variable.name = "Channel",
           value.name = "Expense")
fit<-lm(Expense~gender+house+income+Channel+age,data=mdat)
summary(fit)
```

You can `melt()` list, matrix, table too. The syntax is similar, and we won't go through every situation. Sometimes we want to convert the data from “long” to “wide”. For example, **you want to compare the online and in-store expense between male and female based on the house ownership.**

```
dcast(mdat, house + gender ~ Channel, sum)
```

```
## Using 'Expense' as value column. Use 'value.var' to override
```

```
##   house gender store_exp online_exp
## 1   Yes Female    1007      413.0
## 2   Yes  Male    1218      533.2
```

In the above code, what is the left side of `~` are variables that you want to group by. The right side is the variable you want to spread as columns. It will use the column indicating value from `melt()` before. Here is “Expense” .

4.2.2 tidyr package

The other package that will do similar manipulations is `tidyr`. Let's get a subset to illustrate the usage.

```
library(dplyr)
# practice functions we learnt before
sdat<-sim.dat[1:5,]%>%
  dplyr::select(age,gender,store_exp,store_trans)
sdat %>% tbl_df()
```

```
## # A tibble: 5 x 4
```

```
##      age gender store_exp store_trans
## * <int> <fctr>      <dbl>      <int>
## 1    57 Female    529.1         2
## 2    63 Female    478.0         4
## 3    59  Male    490.8         7
## 4    60  Male    347.8        10
## 5    51  Male    379.6         4
```

`gather()` function in `tidyr` is analogous to `melt()` in `reshape2`. The following code will do the same thing as we did before using `melt()`:

```
library(tidyr)
msdat<-tidyr::gather(sdat,"variable","value",store_exp,store_trans)
msdat %>% tbl_df()
```

```
## # A tibble: 10 x 4
##      age gender  variable value
##    <int> <fctr>      <chr> <dbl>
## 1    57 Female store_exp 529.1
## 2    63 Female store_exp 478.0
## 3    59  Male store_exp 490.8
## 4    60  Male store_exp 347.8
## 5    51  Male store_exp 379.6
## 6    57 Female store_trans  2.0
## 7    63 Female store_trans  4.0
## 8    59  Male store_trans  7.0
## 9    60  Male store_trans 10.0
## 10   51  Male store_trans  4.0
```

Or if we use the pipe operation, we can write the above code as:

```
sdat%>%gather("variable","value",store_exp,store_trans)
```

It is identical with the following code using `melt()`:

```
melt(sdat, measure.vars=c("store_exp","store_trans"),
      variable.name = "variable",
      value.name = "value")
```

The opposite operation to `gather()` is `spread()`. The previous one stacks columns and the latter one spread the columns.

```
msdat %>% spread(variable,value)

##   age gender store_exp store_trans
## 1  51   Male    379.6           4
## 2  57 Female    529.1           2
## 3  59   Male    490.8           7
## 4  60   Male    347.8          10
## 5  63 Female    478.0           4
```

Another pair of functions that do opposite manipulations are `separate()` and `unite()`.

```
sepdatt<- msdat %>%
  separate(variable,c("Source","Type"))
sepdatt %>% tbl_df()
```

```
## # A tibble: 10 x 5
##   age gender Source Type value
## * <int> <fctr> <chr> <chr> <dbl>
## 1    57 Female store  exp 529.1
## 2    63 Female store  exp 478.0
## 3    59   Male store  exp 490.8
## 4    60   Male store  exp 347.8
## 5    51   Male store  exp 379.6
## 6    57 Female store trans  2.0
## 7    63 Female store trans  4.0
## 8    59   Male store trans  7.0
## 9    60   Male store trans 10.0
## 10   51   Male store trans  4.0
```

You can see that the function separates the original column “variable” to two new columns “source” and “type”. You can use `sep=` to set the string or regular expression to separate the column. By default, it is “_”.

The `unite()` function will do the opposite: combining two columns. It is the generalization of `paste()` to a data frame.

```
sepdatt %>%
  unite("variable",Source,Type,sep="_")
```

```
##   age gender  variable value
## 1  57 Female store_exp 529.1
## 2  63 Female store_exp 478.0
## 3  59  Male store_exp 490.8
## 4  60  Male store_exp 347.8
## 5  51  Male store_exp 379.6
## 6  57 Female store_trans  2.0
## 7  63 Female store_trans  4.0
## 8  59  Male store_trans  7.0
## 9  60  Male store_trans 10.0
## 10 51  Male store_trans  4.0
```

The reshaping manipulations may be the trickiest part. You have to practice a lot to get familiar with those functions. Unfortunately, there is no shortcut.



5

Model Tuning Strategy

When training a machine learning model, there are many decisions to make. For example, when training a random forest, you need to decide the number of trees and the number of variables at each node. For lasso method, you need to determine the penalty parameter. There may be standard settings for some of the parameters, but it's unlikely to guess the right values for all of these correctly. Other than that, making good choices on how you split the data into training and testing sets can make a huge difference in helping you find a high-performance model efficiently.

This chapter will illustrate the practical aspects of model tuning. We will talk about different types of model error, sources of model error, hyperparameter tuning, how to set up your data and how to make sure your model implementation is correct. In practice applying machine learning is a highly iterative process.

5.1 Systematic Error and Random Error

Assume \mathbf{X} is $n \times p$ observation matrix and \mathbf{y} is response variable, we have:

$$\mathbf{y} = f(\mathbf{X}) + \epsilon$$

where ϵ is the random error with a mean of zero. The function $f(\cdot)$ is our modeling target, which represents the information in the response variable that predictors can explain. The main goal of estimating $f(\cdot)$ is inference or prediction, or sometimes both. In

general, there is a trade-off between flexibility and interpretability of the model. So data scientists need to comprehend the delicate balance between these two.

Depending on the modeling purposes, the requirement for interpretability varies. If the prediction is the only goal, then as long as the prediction is accurate enough, the interpretability is not under consideration. In this case, people can use “black box” model, such as random forest, boosting tree, neural network and so on. These models are very flexible but nearly impossible to explain. Their accuracy is usually higher on the training set, but not necessary when it predicts. It is not surprising since those models have a huge number of parameters and high flexibility that they can “memorize” the entire training data. A paper by Chiyuan Zhang et al. in 2017 pointed out that “Deep neural networks (even just two-layer net) easily fit random labels” (Zhang et al., 2017). The traditional forms of regularization, such as weight decay, dropout, and data augmentation, fail to control generalization error. It poses a conceptual challenge to statistical theory and also calls our attention when we use such black-box models.

There are two kinds of application problems: complete information problem and incomplete information problem. The complete information problem has all the information you need to know the correct response. Take the famous cat recognition, for example, all the information you need to identify a cat is in the picture. In this situation, the algorithm that penetrates the data the most wins. There are some other similar problems such as the self-driving car, chess game, facial recognition and speech recognition. But in most of the data science applications, the information is incomplete. If you want to know whether a customer is going to purchase again or not, it is unlikely to have 360-degree of the customer’s information. You may have their historical purchasing record, discounts and service received. But you don’t know if the customer sees your advertisement, or has a friend recommends competitor’s product, or encounters some unhappy purchasing experience somewhere. There could be a myriad of factors that will influence the customer’s purchase decision while what you have as data is only a small part. To

make things worse, in many cases, you don't even know what you don't know. Deep learning doesn't have any advantage in solving those problems. Instead, some parametric models often work better in this situation. You will comprehend this more after learning the different types of model error. Assume we have \hat{f} which is an estimator of f . Then we can further get $\hat{\mathbf{y}} = \hat{f}(\mathbf{X})$. The predicted error is divided into two parts, systematic error, and random error:

$$E(\mathbf{y} - \hat{\mathbf{y}})^2 = E[f(\mathbf{X}) + \epsilon - \hat{f}(\mathbf{X})]^2 = \underbrace{E[f(\mathbf{X}) - \hat{f}(\mathbf{X})]^2}_{(1)} + \underbrace{Var(\epsilon)}_{(2)}$$

It is also called Mean Square Error (MSE) where (1) is the systematic error. It exists because \hat{f} usually does not entirely describe the “systematic relation” between \mathbf{X} and \mathbf{y} which refers to the stable relationship that exists across different samples or time. Model improvement can help reduce this kind of error; (2) is the random error which represents the part of \mathbf{y} that cannot be explained by \mathbf{X} . A more complex model does not reduce the error. There are three reasons for random error:

1. the current sample is not representative, so the pattern in one sample set does not generalize to a broader scale.
2. The information is incomplete. In other words, you don't have all variables needed to explain the response.
3. Measurement error in the variables.

Deep learning has significant success solving problems with complete information and usually low measurement error. As mentioned before, in a task like image recognition, all you need are the pixels in the pictures. So in deep learning applications, increasing the sample size can improve the model performance significantly. But it may not perform well in problems with incomplete information. The biggest problem with the black-box model is that it fits random error, i.e., over-fitting. The notable feature of random error is that it varies over different samples. So one way to determine whether overfitting happens is to reserve a part of the data as the test set and then check the performance of the trained model on

the test data. Note that overfitting is a general problem from which any model could suffer. However, since black-box models usually have a large number of parameters, it is much more susceptible to over-fitting.

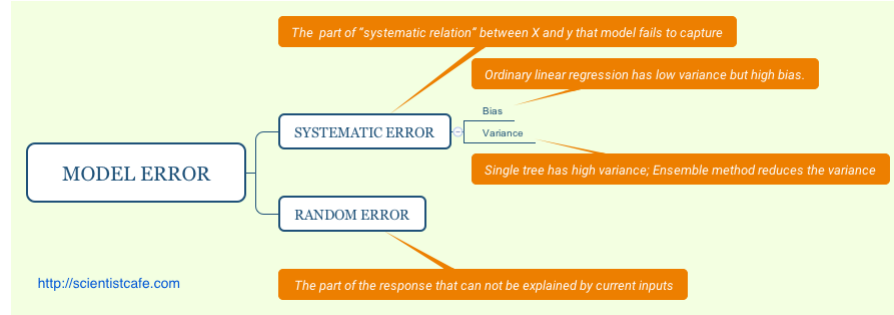


FIGURE 5.1: Types of Model Error

The systematic error can be further decomposed as:

$$\begin{aligned}
 E[f(\mathbf{X}) - \hat{f}(\mathbf{X})]^2 &= E \left(f(\mathbf{X}) - E[\hat{f}(\mathbf{X})] + E[\hat{f}(\mathbf{X})] - \hat{f}(\mathbf{X}) \right)^2 \\
 &= E \left(E[\hat{f}(\mathbf{X})] - f(\mathbf{X}) \right)^2 + E \left(\hat{f}(\mathbf{X}) - E[\hat{f}(\mathbf{X})] \right)^2 \\
 &= [Bias(\hat{f}(\mathbf{X}))]^2 + Var(\hat{f}(\mathbf{X}))
 \end{aligned}$$

The systematic error consists of two parts, $Bias(\hat{f}(\mathbf{X}))$ and $Var(\hat{f}(\mathbf{X}))$. To minimize the systematic error, we need to minimize both. The bias represents the error caused by the model's approximation of the reality, i.e., systematic relation, which may be very complex. For example, linear regression assumes a linear relationship between the predictors and the response, but rarely is there a perfect linear relationship in real life. So linear regression is more likely to have a high bias.

To explore bias and variance, let's begin with a simple simulation. We will simulate a data with a non-linear relationship and fit different models on it. An intuitive way to show these is to compare the plots of various models.

The code below simulates one predictor (x) and one response variable (fx). The relationship between x and fx is non-linear.

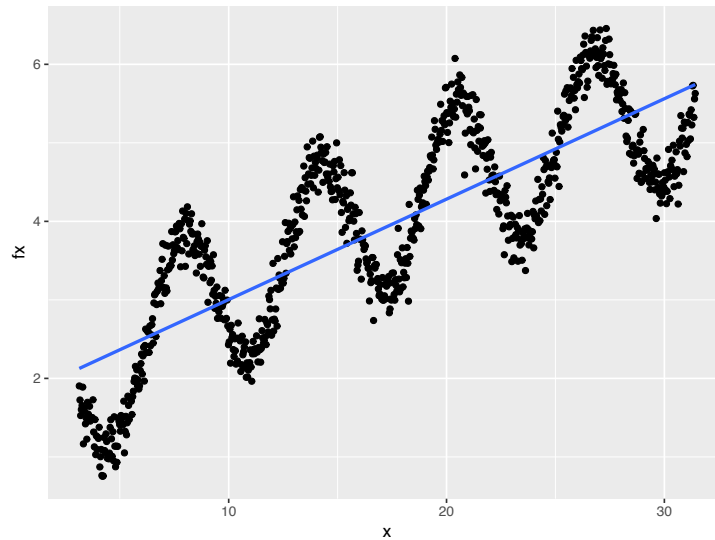


FIGURE 5.2: High bias model

```
source(ids_url('R/multiplot.r'))
# randomly simulate some non-linear samples
x = seq(1, 10, 0.01) * pi
e = rnorm(length(x), mean = 0, sd = 0.2)
fx <- sin(x) + e + sqrt(x)
dat = data.frame(x, fx)
```

Then fit a simple linear regression on these data:

```
# plot fitting result
library(ggplot2)
ggplot(dat, aes(x, fx)) + geom_point() + geom_smooth(method = "lm", se = FALSE)
```

Despite a large sample size, trained linear regression cannot describe the relationship very well. In other words, in this case, the model has a high bias (Fig. 5.2). People also call it underfitting.

Since the estimated parameters will be somewhat different for the different samples, there is the variance of estimates. Intuitively, it gives you some sense that if we fit the same model with different samples (presumably, they are from the same population), how much will the estimates change. Ideally, the change is triv-

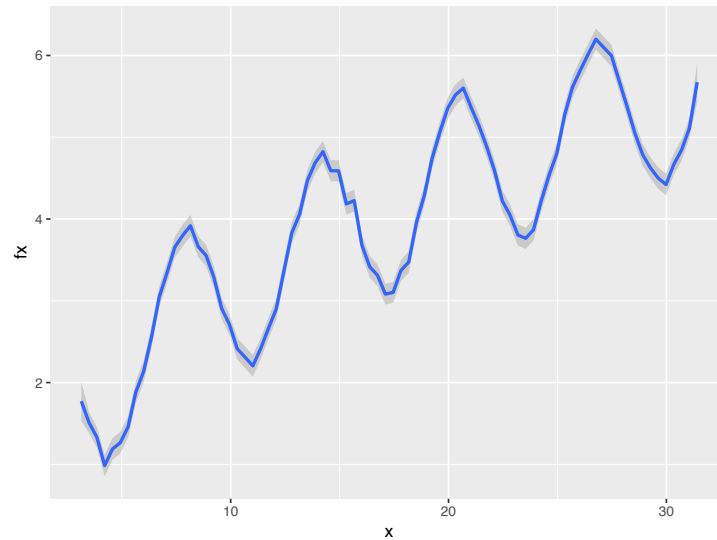


FIGURE 5.3: High variance model

ial. For high variance models, small changes in the training data result in very different estimates. In general, a model with high flexibility also has high variance., such as the CART tree, and the initial boosting method. To overcome that problem, the Random Forest and Gradient Boosting Model aim to reduce the variance by summarizing the results obtained from different samples.

Let's fit the above data using a smoothing method which is highly flexible and can fit the current data tightly:

```
ggplot(dat, aes(x, fx)) + geom_smooth(span = 0.03)
```

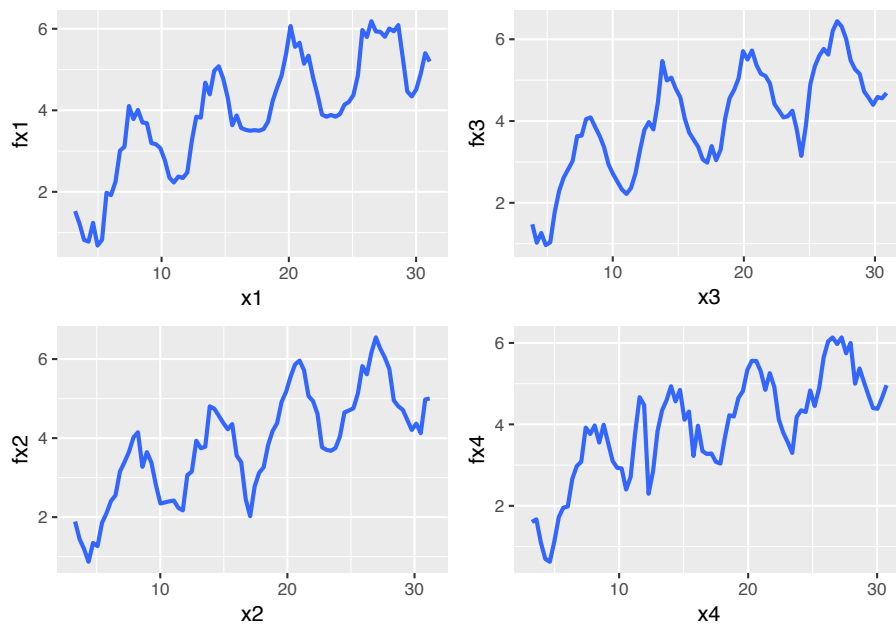
The resulting plot (Fig. 5.3) indicates the smoothing method fit the data much better so it has a much smaller bias. However, this method has a high variance. If we simulate different subsets of the sample, the result curve will change significantly:

```
# set random seed
set.seed(2016)
# sample part of the data to fit model sample 1
idx1 = sample(1:length(x), 100)
dat1 = data.frame(x1 = x[idx1], fx1 = fx[idx1])
```

```

p1 = ggplot(dat1, aes(x1, fx1)) + geom_smooth(span = 0.03)
# sample 2
idx2 = sample(1:length(x), 100)
dat2 = data.frame(x2 = x[idx2], fx2 = fx[idx2])
p2 = ggplot(dat2, aes(x2, fx2)) + geom_smooth(span = 0.03)
# sample 3
idx3 = sample(1:length(x), 100)
dat3 = data.frame(x3 = x[idx3], fx3 = fx[idx3])
p3 = ggplot(dat3, aes(x3, fx3)) + geom_smooth(span = 0.03)
# sample 4
idx4 = sample(1:length(x), 100)
dat4 = data.frame(x4 = x[idx4], fx4 = fx[idx4])
p4 = ggplot(dat4, aes(x4, fx4)) + geom_smooth(span = 0.03)
multiplot(p1, p2, p3, p4, cols = 2)

```



The fitted lines (blue) change over different samples which means it has high variance. People also call it overfitting. Fitting the linear model using the same four subsets, the result barely changes:

```

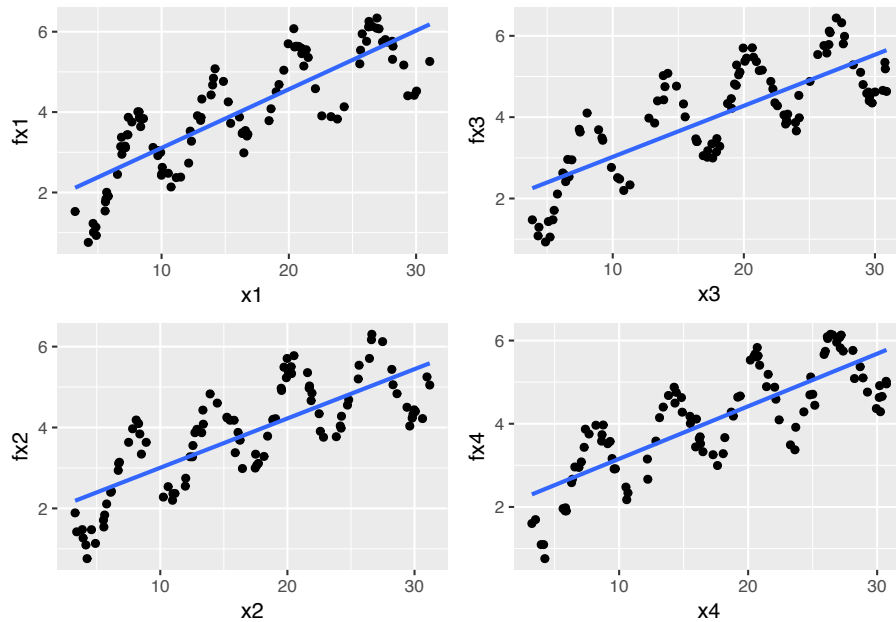
p1 = ggplot(dat1, aes(x1, fx1)) + geom_point() + geom_smooth(method = "lm",
  se = FALSE)

```

```

p2 = ggplot(dat2, aes(x2, fx2)) + geom_point() + geom_smooth(method = "lm",
  se = FALSE)
p3 = ggplot(dat3, aes(x3, fx3)) + geom_point() + geom_smooth(method = "lm",
  se = FALSE)
p4 = ggplot(dat4, aes(x4, fx4)) + geom_point() + geom_smooth(method = "lm",
  se = FALSE)
multiplot(p1, p2, p3, p4, cols = 2)

```



In general, the variance ($Var(\hat{f}(\mathbf{X}))$) **increases** and the bias ($Bias(\hat{f}(\mathbf{X}))$) **decreases** as the model flexibility increases. Variance and bias together determine the systematic error. As we increase the flexibility of the model, at first the rate at which $Bias(\hat{f}(\mathbf{X}))$ decreases is faster than $Var(\hat{f}(\mathbf{X}))$, so the MSE decreases. However, to some degree, higher flexibility has little effect on $Bias(\hat{f}(\mathbf{X}))$ but $Var(\hat{f}(\mathbf{X}))$ increases significantly, so the MSE increases.

5.1.1 Measurement Error in the Response

The measurement error in the response contributes to the random error (ϵ). This part of the error is irreducible if you change the data collection mechanism, and so it makes the root mean square error (RMSE) and R^2 have the corresponding upper and lower limits. RMSE and R^2 are commonly used performance measures for the regression model which we will talk in more detail later. Therefore, the random error term not only represents the part of fluctuations the model cannot explain but also contains measurement error in the response variables. Section 20.2 of Applied Predictive Modeling (Kuhn and Johnston, 2013) has an example that shows the effect of the measurement error in the response variable on the model performance (RMSE and R^2).

The authors increased the error in the response proportional to a base level error which was gotten using the original data without introducing extra noise. Then fit a set of models repeatedly using the “contaminated” data sets to study the change of $RMSE$ and R^2 as the level of noise. Here we use clothing consumer data for a similar illustration. Suppose many people do not want to disclose their income and so we need to use other variables to establish a model to predict income. We set up the following model:

```
# load data
sim.dat <- read.csv("https://raw.githubusercontent.com/happyrabbit/DataScientistR/master/Data")
ymad <- mad(na.omit(sim.dat$income))
# calculate z-score
zs <- (sim.dat$income - mean(na.omit(sim.dat$income)))/ymad
# which(na.omit(zs>3.5)): identify outliers which(is.na(zs)):
# identify missing values
idex <- c(which(na.omit(zs > 3.5)), which(is.na(zs)))
# delete rows with outliers and missing values
sim.dat <- sim.dat[-idex, ]
fit <- lm(income ~ store_exp + online_exp + store_trans + online_trans,
         data = sim.dat)
```

The output shows that without additional noise, the root mean square error (RMSE) of the model is 29567, R^2 is 0.6.

Let's add various degrees of noise (0 to 3 times the RMSE) to the variable `income`:

$$RMSE \times (0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0)$$

```
noise <- matrix(rep(NA, 7 * nrow(sim.dat)), nrow = nrow(sim.dat),
               ncol = 7)
for (i in 1:nrow(sim.dat)) {
  noise[i, ] <- rnorm(7, rep(0, 7), summary(fit)$sigma * seq(0,
    3, by = 0.5))
}
```

We then examine the effect of noise intensity on R^2 for models with different complexity. The models with complexity from low to high are: ordinary linear regression, partial least square regression (PLS), multivariate adaptive regression spline (MARS), support vector machine (SVM, the kernel function is radial basis function), and random forest.

```
# fit ordinary linear regression
rsq_linear <- rep(0, ncol(noise))
for (i in 1:7) {
  withnoise <- sim.dat$income + noise[, i]
  fit0 <- lm(withnoise ~ store_exp + online_exp + store_trans +
    online_trans, data = sim.dat)
  rsq_linear[i] <- summary(fit0)$adj.r.squared
}
```

PLS is a method of linearizing nonlinear relationships through hidden layers. It is similar to the principal component regression (PCR), except that PCR does not take into account the information of the dependent variable when selecting the components, and its purpose is to find the linear combinations (i.e., unsupervised) that capture the most variance of the independent variables. When the independent variables and response variables are related, PCR can well identify the systematic relationship between them. However, when there exist independent variables not associated with response variable, it will undermine PCR's performance. And PLS maximizes the linear combination of dependencies with the

response variable. In the current case, the more complicated PLS does not perform better than simple linear regression.

```
# pls: conduct PLS and PCR
library(pls)
rsq_pls <- rep(0, ncol(noise))
# fit PLS
for (i in 1:7) {
  withnoise <- sim.dat$income + noise[, i]
  fit0 <- plsr(withnoise ~ store_exp + online_exp + store_trans +
    online_trans, data = sim.dat)
  rsq_pls[i] <- max(drop(R2(fit0, estimate = "train", intercept = FALSE)$val))
}
```

```
# earth: fit mars
library(earth)
rsq_mars <- rep(0, ncol(noise))
for (i in 1:7) {
  withnoise <- sim.dat$income + noise[, i]
  fit0 <- earth(withnoise ~ store_exp + online_exp + store_trans +
    online_trans, data = sim.dat)
  rsq_mars[i] <- fit0$rsq
}
```

```
# caret: awesome package for tuning predictive model
library(caret)
rsq_svm <- rep(0, ncol(noise))
# Need some time to run
for (i in 1:7) {
  index <- which(is.na(sim.dat$income))
  withnoise <- sim.dat$income + noise[, i]
  trainX <- sim.dat[, c("store_exp", "online_exp", "store_trans",
    "online_trans")]
  trainY <- withnoise
  fit0 <- train(trainX, trainY, method = "svmRadial", tuneLength = 15,
    trControl = trainControl(method = "cv"))
  rsq_svm[i] <- max(fit0$results$Rsquared)
}
```

```

# randomForest: random forest model
library(randomForest)
rsq_rf <- rep(0, ncol(noise))
# ntree=500 number of trees na.action = na.omit ignore
# missing value
for (i in 1:7) {
  withnoise <- sim.dat$income + noise[, i]
  fit0 <- randomForest(withnoise ~ store_exp + online_exp +
    store_trans + online_trans, data = sim.dat, ntree = 500,
    na.action = na.omit)
  rsq_rf[i] <- tail(fit0$rsq, 1)
}
library(reshape2)
rsq <- data.frame(cbind(Noise = c(0, 0.5, 1, 1.5, 2, 2.5, 3),
  rsq_linear, rsq_pls, rsq_mars, rsq_svm, rsq_rf))
rsq <- melt(rsq, id.vars = "Noise", measure.vars = c("rsq_linear",
  "rsq_pls", "rsq_mars", "rsq_svm", "rsq_rf"))

library(ggplot2)
ggplot(data = rsq, aes(x = Noise, y = value, group = variable,
  colour = variable)) + geom_line() + geom_point() + ylab("R2")

```

Fig. 5.4 shows that:

All model performance decreases sharply with increasing noise intensity. To better anticipate model performance, it helps to understand the way variable is measured. It is something need to make clear at the beginning of an analytical project. A data scientist should be aware of the quality of the data in the database. For data from the clients, it is an important to understand the quality of the data by communication.

More complex model is not necessarily better. The best model in this situation is MARS, not random forests or SVM. Simple linear regression and PLS perform the worst when noise is low. MARS is more complicated than the linear regression and PLS, but it is simpler and easier to explain than random forest and SVM.

When noise increases to a certain extent, the potential structure

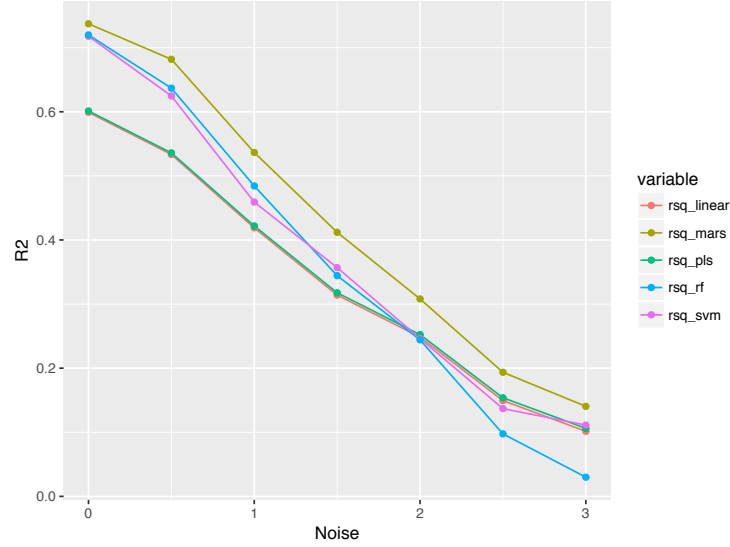


FIGURE 5.4: Test set R^2 profiles for income models when measurement system noise increases. `rsq_linear`: linear regression, `rsq_pls`: Partial Least Square, `rsq_mars`: Multiple Adaptive Regression Spline Regression, `rsq_svm`: Support Vector Machine `rsq_rf`: Random Forest

becomes vaguer, and complex random forest model starts to fail. When the systematic measurement error is significant, a more straightforward but not naive model may be a better choice. It is always a good practice to try different models, and select the simplest model in the case of similar performance. Model evaluation and selection represent the career “maturity” of a data scientist.

5.1.2 Measurement Error in the Independent Variables

The traditional statistical model usually assumes that the measurement of the independent variable has no error which is not possible in practice. Considering the error in the independent variables is necessary. The impact of the error depends on the following factors: (1) the magnitude of the randomness; (2) the importance of the corresponding variable in the model, and (3) the type of model

used. Use variable `online_exp` as an example. The approach is similar to the previous section. Add varying degrees of noise and see its impact on the model performance. We add the following different levels of noise (0 to 3 times the standard deviation) to `online_exp`:

$$\sigma_0 \times (0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0)$$

where σ_0 is the standard error of `online_exp`.

```
noise<-matrix(rep(NA,7*nrow(sim.dat)),nrow=nrow(sim.dat),ncol=7)
for (i in 1:nrow(sim.dat)){
noise[i,]<-rnorm(7,rep(0,7),sd(sim.dat$online_exp)*seq(0,3,by=0.5))
}
```

Likewise, we examine the effect of noise intensity on different models (R^2). The models with complexity from low to high are: ordinary linear regression, partial least square regression (PLS), multivariate adaptive regression spline (MARS), support vector machine (SVM, the Kernel function is radial basis function), and random forest. The code is similar as before so not shown here.

Comparing Fig. 5.5 and Fig. 5.4, the influence of the two types of error is very different. The error in response cannot be overcome for any model, but it is not the case for the independent variables. Imagine an extreme case, if `online_exp` is completely random, that is, no information in it, the impact on the performance of random forest and support vector machine is marginal. Linear regression and PLS still perform similarly. With the increase of noise, the performance starts to decline faster. To a certain extent, it becomes steady. In general, if an independent variable contains error, other variables associated with it can compensate to some extent.

5.2 Data Splitting and Resampling

Those highly adaptable models can model complex relationships. However, they tend to overfit which leads to the poor prediction by

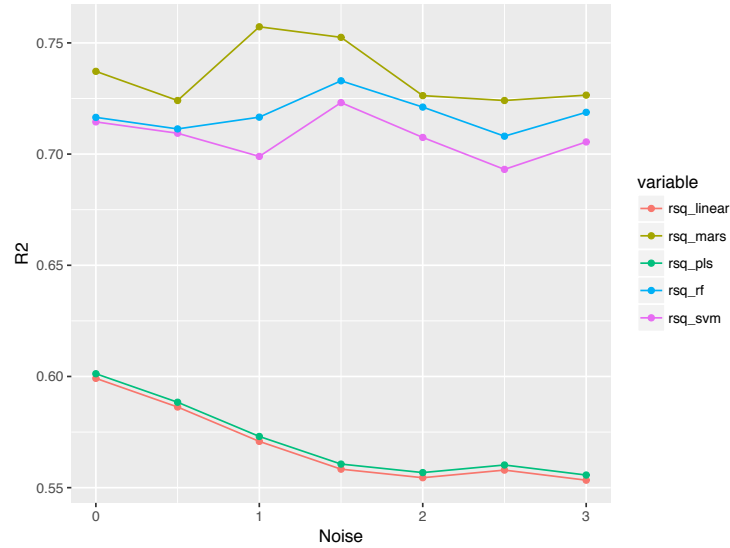


FIGURE 5.5: Test set R^2 profiles for income models when noise in `online_exp` increases. `rsq_linear` : linear regression, `rsq_pls` : Partial Least Square, `rsq_mars`: Multiple Adaptive Regression Spline Regression, `rsq_svm`: Support Vector Machine `rsq_rf`: Random Forest

learning too much from the data. It means that the model is susceptible to the specific sample used to fit it. When future data is not exactly like the past data, the model prediction may have big mistakes. A simple model like ordinary linear regression tends instead to underfit which leads to a bad prediction by learning too little from the data. It systematically over-predicts or under-predicts the data regardless of how well future data resemble past data. Without evaluating models, the modeler will not know about the problem before the future samples. Data splitting and resampling are fundamental techniques to build sound models for prediction.

5.2.1 Data Splitting

Data splitting is to put part of the data aside as testing set (or Hold-outs, out of bag samples) and use the rest for model train-

ing. Training samples are also called in-sample. Model performance metrics evaluated using in-sample are retrodictive, not predictive.

The traditional business intelligence usually handles data description. Answer simple questions by querying and summarizing the data, such as:

- What is the monthly sales of a product in 2015?
- What is the number of visits to our site in the past month?

- What is the sales difference in 2015 for two different product designs?

There is no need to go through the tedious process of splitting the data, tuning and testing model to answer questions of this kind. Instead, people usually use as complete data as possible and then sum or average the parts of interest.

Many models have parameters which cannot be directly estimated from the data, such as λ in the lasso (penalty parameter), the number of trees in the random forest. This type of model parameter is called tuning parameter, and there is no analytical formula available to calculate the optimized value. Tuning parameters often control the complexity of the model. A poor choice can result in over-fitting or under-fitting. A standard approach to estimate tuning parameters is through cross-validation which is a data resampling approach.

To get a reasonable precision of the performance based on a single test set, the size of the test set may need to be large. So a conventional approach is to use a subset of samples to fit the model and use the rest to evaluate model performance. This process will repeat multiple times to get a performance profile. In that sense, resampling is based on splitting. The general steps are:

- Define a set of candidate values for tuning parameter(s)
 - For each candidate value in the set
 - * Resample data
 - * Fit model
 - * Predict hold-out

- * Calculate performance
- Aggregate the results
- Determine the final tuning parameter
- Refit the model with the entire data set

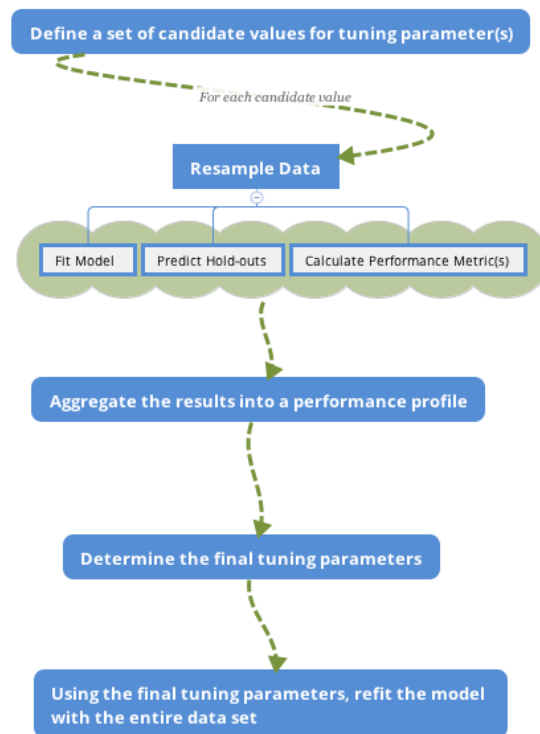


FIGURE 5.6: Parameter Tuning Process

The above is an outline of the general procedure to tune parameters. Now let's focus on the critical part of the process: data splitting. Ideally, we should evaluate model using samples that were not used to build or fine-tune the model. So it provides an unbiased sense of model effectiveness. When the sample size is large, it is a good practice to set aside part of the samples to evaluate the final model. People use “training” data to indicate samples used to fit or fine-tune the model and “test” or “validation” data set is used to validate performance.

The first decision to make for data splitting is to decide the proportion of data in the test set. There are two factors to consider here: (1) sample size; (2) computation intensity. If the sample size is large enough which is the most common situation according to my experience, you can try to use 20%, 30% and 40% of the data as the test set, and see which one works the best. If the model is computationally intense, then you may consider starting from a smaller sample of data to train the model hence will have a higher portion of data in the test set. Depending on how it performs, you may need to increase the training set. If the sample size is small, you can use cross-validation or bootstrap which is the topic in the next section.

The next decision is to decide which samples are in the test set. There is a desire to make the training and test sets as similar as possible. A simple way is to split data by random sampling which, however, does not control for any of the data attributes, such as the percentage of the retained customer in the data. So it is possible that the distribution of outcomes is substantially different between the training and test sets. There are three main ways to split the data that account for the similarity of resulted data sets. We will describe the three approaches using the clothing company customer data as examples.

(1) Split data according to the outcome variable

Assume the outcome variable is customer segment (column `segment`) and we decide to use 80% as training and 20% test. The goal is to make the proportions of the categories in the two sets as similar as possible. The `createDataPartition()` function in `caret` will return a balanced splitting based on assigned variable.

```
# load data
sim.dat <- read.csv("https://raw.githubusercontent.com/happyrabbit/DataScientistR/master/Data/segment.csv")
library(caret)
# set random seed to make sure reproducibility
set.seed(3456)
trainIndex <- createDataPartition(sim.dat$segment, p = 0.8, list = FALSE,
```



```
times = 1)
head(trainIndex)
```

```
##      Resample1
## [1,]         1
## [2,]         2
## [3,]         3
## [4,]         4
## [5,]         6
## [6,]         7
```

The `list = FALSE` in the call to `createDataPartition` is to return a data frame. The `times = 1` tells R how many times you want to split the data. Here we only do it once, but you can repeat the splitting multiple times. In that case, the function will return multiple vectors indicating the rows to training/test. You can set `times=2` and rerun the above code to see the result. Then we can use the returned indicator vector `trainIndex` to get training and test sets:

```
# get training set
datTrain <- sim.dat[trainIndex, ]
# get test set
datTest <- sim.dat[-trainIndex, ]
```

According to the setting, there are 800 samples in the training set and 200 in test set. Let's check the distribution of the two sets:

```
library(plyr)
ddply(datTrain, "segment", summarise, count = length(segment),
      percentage = round(length(segment)/nrow(datTrain), 2))
```

```
##      segment count percentage
## 1 Conspicuous   160         0.20
## 2      Price   200         0.25
## 3    Quality   160         0.20
## 4      Style   280         0.35
```

```
ddply(datTest, "segment", summarise, count = length(segment),
      percentage = round(length(segment)/nrow(datTest), 2))
```

##	segment	count	percentage
## 1	Conspicuous	40	0.20
## 2	Price	50	0.25
## 3	Quality	40	0.20
## 4	Style	70	0.35

The percentages are the same for these two sets. In practice, it is possible that the distributions are not exactly identical but should be close.

(2) Divide data according to predictors

An alternative way is to split data based on the predictors. The goal is to get a diverse subset from a dataset so that the sample is representative. In other words, we need an algorithm to identify the n most diverse samples from a dataset with size N . However, the task is generally infeasible for non-trivial values of n and N (Willett, 2004). And hence practicable approaches to dissimilarity-based selection involve approximate methods that are sub-optimal. A major class of algorithms split the data on *maximum dissimilarity sampling*. The process starts from:

- Initialize a single sample as starting test set
- Calculate the dissimilarity between this initial sample and each remaining samples in the dataset
- Add the most dissimilar unallocated sample to the test set

To move forward, we need to define the dissimilarity between groups. Each definition results in a different version of the algorithm and hence a different subset. It is the same problem as in hierarchical clustering where you need to define a way to measure the distance between clusters. The possible approaches are to use minimum, maximum, sum of all distances, the average of all distances, etc. Unfortunately, there is not a single best choice, and you may have to try multiple methods and check the resulted sample sets. R users can implement the algorithm using `maxDissim()` function from `caret` package. The `obj` argument is to set the definition of dissimilarity. Refer to the help documentation for more details (`?maxDissim`).

Let's use two variables (`age` and `income`) from the customer data as an example to illustrate how it works in R and compare maximum dissimilarity sampling with random sampling.

```
library(lattice)
# select variables
testing <- subset(sim.dat, select = c("age", "income"))
```

Random select 5 samples as initial subset (`start`), the rest will be in `samplePool`:

```
set.seed(5)
# select 5 random samples
startSet <- sample(1:dim(testing)[1], 5)
start <- testing[startSet, ]
# save the rest in data frame 'samplePool'
samplePool <- testing[-startSet, ]
```

Use `maxDissim()` to select another 5 samples from `samplePool` that are as different as possible with the initial set `start`:

```
selectId <- maxDissim(start, samplePool, obj = minDiss, n = 5)
minDissSet <- samplePool[selectId, ]
```

The `obj = minDiss` in the above code tells R to use minimum dissimilarity to define the distance between groups. Next, random select 5 samples from `samplePool` in data frame `RandomSet`:

```
selectId <- sample(1:dim(samplePool)[1], 5)
RandomSet <- samplePool[selectId, ]
```

Plot the resulted set to compare different sampling methods:

```
start$group <- rep("Initial Set", nrow(start))
minDissSet$group <- rep("Maximum Dissimilarity Sampling", nrow(minDissSet))
RandomSet$group <- rep("Random Sampling", nrow(RandomSet))
xyplot(age ~ income, data = rbind(start, minDissSet, RandomSet), grid = TRUE,
        group = group, auto.key = TRUE)
```

The points from maximum dissimilarity sampling are far away from the initial samples (Fig. 5.7, while the random samples are much closer to the initial ones. Why do we need a diverse subset?

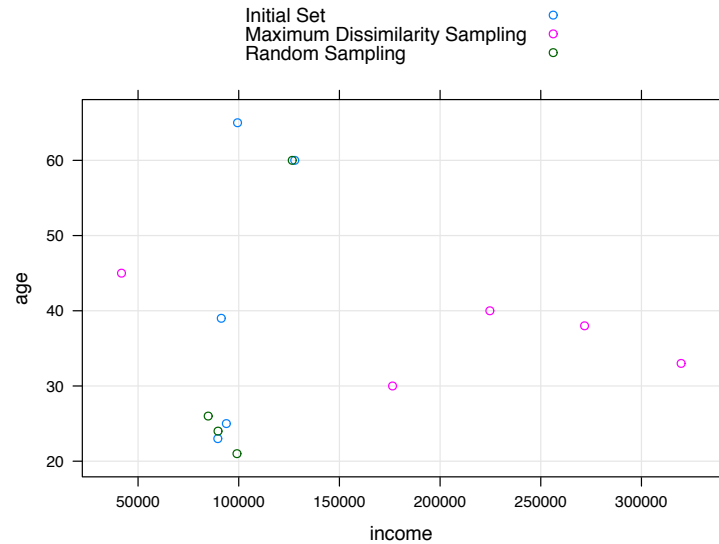


FIGURE 5.7: Compare Maximum Dissimilarity Sampling with Random Sampling

Because we hope the test set to be representative. If all test set samples are from respondents younger than 30, model performance on the test set has a high risk to fail to tell you how the model will perform on more general population.

- Divide data according to time

For time series data, random sampling is usually not the best way. There is an approach to divide data according to time-series. Since time series is beyond the scope of this book, there is not much discussion here. For more detail of this method, see (Hyndman and Athanasopoulos, 2013). We will use a simulated first-order autoregressive model [AR (1)] time-series data with 100 observations to show how to implement using the function `createTimeSlices()` in the `caret` package.

```
# simulate AR(1) time series samples
timedata = arima.sim(list(order=c(1,0,0), ar=-.9), n=100)
# plot time series
plot(timedata, main=(expression(AR(1)~phi==-.9)))
```

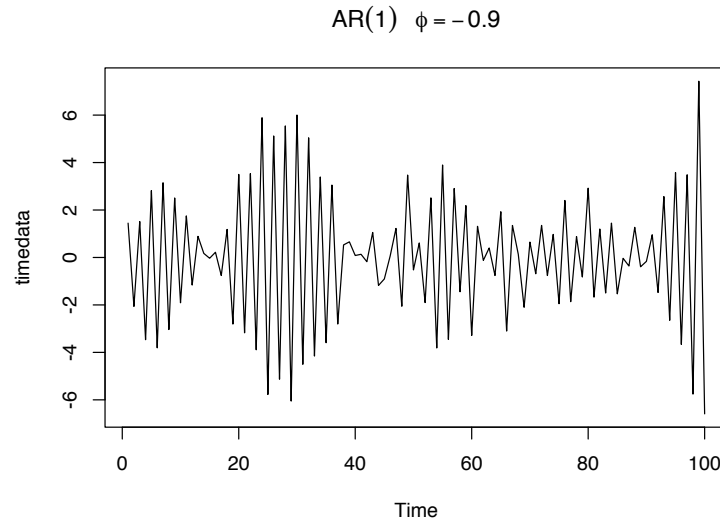
**FIGURE 5.8:** Divide data according to time

Fig. 5.8 shows 100 simulated time series observation. The goal is to make sure both training and test set to cover the whole period.

```
timeSlices <- createTimeSlices(1:length(timedata),
                               initialWindow = 36, horizon = 12, fixedWindow = T)
str(timeSlices,max.level = 1)

## List of 2
## $ train:List of 53
## $ test :List of 53
```

There are three arguments in the above `createTimeSlices()`.

- `initialWindow`: The initial number of consecutive values in each training set sample
- `horizon`: the number of consecutive values in test set sample
- `fixedWindow`: if `FALSE`, all training samples start at 1

The function returns two lists, one for the training set, the other for the test set. Let's look at the first training sample:

```
# get result for the 1st training set
trainSlices <- timeSlices[[1]]
# get result for the 1st test set
```

```
testSlices <- timeSlices[[2]]
# check the index for the 1st training and test set
trainSlices[[1]]

## [1] 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
## [18] 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
## [35] 35 36

testSlices[[1]]

## [1] 37 38 39 40 41 42 43 44 45 46 47 48
```

The first training set is consist of sample 1-36 in the dataset (`initialWindow = 36`). Then sample 37-48 are in the first test set (`horizon = 12`). Type `head(trainSlices)` or `head(testSlices)` to check the later samples. If you are not clear about the argument `fixedWindow`, try to change the setting to be `F` and check the change in `trainSlices` and `testSlices`.

Understand and implement data splitting is not difficult. But there are two things to note:

1. The randomness in the splitting process will lead to uncertainty in performance measurement.
2. When the dataset is small, it can be too expensive to leave out test set. In this situation, if collecting more data is just not possible, the best shot is to use leave-one-out cross-validation which is in the next section.

5.2.2 Resampling

You can consider resampling as repeated splitting. The basic idea is: use part of the data to fit model and then use the rest of data to calculate model performance. Repeat the process multiple times and aggregate the results. The differences in resampling techniques usually center around the ways to choose subsamples. There are two main reasons that we may need resampling:

1. Estimate tuning parameters through resampling. Some

examples of models with such parameters are Support Vector Machine (SVM), models including the penalty (LASSO) and random forest.

2. For models without tuning parameter, such as ordinary linear regression and partial least square regression, the model fitting doesn't require resampling. But you can study the model stability through resampling.

We will introduce three most common resampling techniques: k-fold cross-validation, repeated training/test splitting, and bootstrap.

5.2.2.1 k-fold cross-validation

k-fold cross-validation is to partition the original sample into k equal size subsamples (folds). Use one of the k folds to validate the model and the rest $k - 1$ to train model. Then repeat the process k times with each of the k folds as the test set. Aggregate the results into a performance profile.

Denote by $\hat{f}^{-\kappa}(X)$ the fitted function, computed with the κ^{th} fold removed and x_i^κ the predictors for samples in left-out fold. The process of k-fold cross-validation is as follows:

-
1. Partition the original sample into k equal size folds
 2. for $\kappa = 1k$
 - Use data other than fold κ to train the model $\hat{f}^{-\kappa}(X)$
 - Apply $\hat{f}^{-\kappa}(X)$ to predict fold κ to get $\hat{f}^{-\kappa}(x_i^\kappa)$
 3. Aggregate the results

$$Error = \frac{1}{N} \sum_{\kappa=1}^k \sum_{x_i^\kappa} L(y_i^\kappa, \hat{f}^{-\kappa}(x_i^\kappa))$$
-

It is a standard way to find the value of tuning parameter that gives

you the best performance. It is also a way to study the variability of model performance.

The following figure represents a 5-fold cross-validation example.

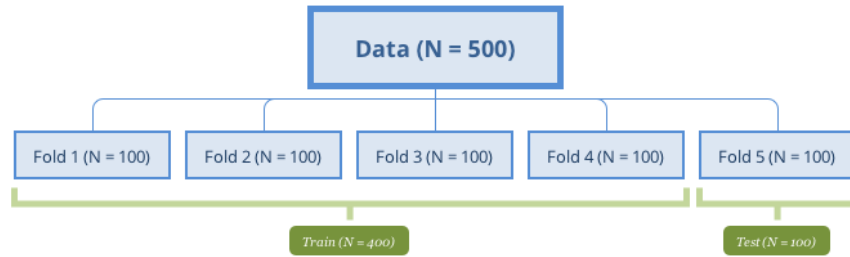


FIGURE 5.9: 5-fold cross-validation

A special case of k -fold cross-validation is Leave One Out Cross Validation (LOOCV) where $k = 1$. When sample size is small, it is desired to use as many data to train the model. Most of the functions have default setting $k = 10$. The choice is usually 5-10 in practice, but there is no standard rule. The more folds to use, the more samples are used to fit model, and then the performance estimate is closer to the theoretical performance. Meanwhile, the variance of the performance is larger since the samples to fit model in different iterations are more similar. However, LOOCV has high computational cost since the number of interactions is the same as the sample size and each model fit uses a subset that is nearly the same size of the training set. On the other hand, when k is small (such as 2 or 3), the computation is more efficient, but the bias will increase. When the sample size is large, the impact of k becomes marginal.

Chapter 7 of (Hastie T, 2008) presents a more in-depth and more detailed discussion about the bias-variance trade-off in k -fold cross-validation.

You can implement k -fold cross-validation using `createFolds()` in `caret`:


```

library(caret)
class<-sim.dat$segment
# creat k-folds
set.seed(1)
cv<-createFolds(class,k=10,returnTrain=T)
str(cv)

## List of 10
## $ Fold01: int [1:900] 1 2 3 4 5 6 7 8 9 10 ...
## $ Fold02: int [1:900] 1 2 3 4 5 6 7 9 10 11 ...
## $ Fold03: int [1:900] 1 2 3 4 5 6 7 8 10 11 ...
## $ Fold04: int [1:900] 1 2 3 4 5 6 7 8 9 11 ...
## $ Fold05: int [1:900] 1 3 4 6 7 8 9 10 11 12 ...
## $ Fold06: int [1:900] 1 2 3 4 5 6 7 8 9 10 ...
## $ Fold07: int [1:900] 2 3 4 5 6 7 8 9 10 11 ...
## $ Fold08: int [1:900] 1 2 3 4 5 8 9 10 11 12 ...
## $ Fold09: int [1:900] 1 2 4 5 6 7 8 9 10 11 ...
## $ Fold10: int [1:900] 1 2 3 5 6 7 8 9 10 11 ...

```

The above code creates ten folds ($k=10$) according to the customer segments (we set `class` to be the categorical variable `segment`). The function returns a list of 10 with the index of rows in training set.

5.2.2.2 Repeated Training/Test Splits

In fact, this method is nothing but repeating the training/test set division on the original data. Fit the model with the training set, and evaluate the model with the test set. Unlike k-fold cross-validation, the test set generated by this procedure may have duplicate samples. A sample usually shows up in more than one test sets. There is no standard rule for split ratio and number of repetitions. The most common choice in practice is to use 75% to 80% of the total sample for training. The remaining samples are for validation. The more sample in the training set, the less biased the model performance estimate is. Increasing the repetitions can reduce the uncertainty in the performance estimates. Of course, it is at the cost of computational time when the model is complex. The number of repetitions is also related to the sample size of

the test set. If the size is small, the performance estimate is more volatile. In this case, the number of repetitions needs to be higher to deal with the uncertainty of the evaluation results.

We can use the same function (`createDataPartition()`) as before. If you look back, you will see `times = 1`. The only thing to change is to set it to the number of repetitions.

```
trainIndex <- createDataPartition(sim.dat$segment, p = .8, list = FALSE, times = 5)
dplyr::glimpse(trainIndex)

##   int [1:800, 1:5] 1 3 4 5 6 7 8 9 10 11 ...
##   - attr(*, "dimnames")=List of 2
##   ..$ : NULL
##   ..$ : chr [1:5] "Resample1" "Resample2" "Resample3" "Resample4" ...
```

Once know how to split the data, the repetition comes naturally.

5.2.2.3 Bootstrap Methods

Bootstrap is a powerful statistical tool (a little magic too). It can be used to analyze the uncertainty of parameter estimates (Efron and Tibshirani, 1986) quantitatively. For example, estimate the standard deviation of linear regression coefficients. The power of this method is that the concept is so simple that it can be easily applied to any model as long as the computation allows. However, you can hardly obtain the standard deviation for some models by using the traditional statistical inference.

Since it is with replacement, a sample can be selected multiple times, and the bootstrap sample size is the same as the original data. So for every bootstrap set, there are some left-out samples, which is also called “out-of-bag samples.” The out-of-bag sample is used to evaluate the model. Efron points out that under normal circumstances (Efron, 1983), bootstrap estimates the error rate of the model with more certainty. The probability of an observation i in bootstrap sample B is:

$$\begin{aligned}
 \text{Pri} \in B &= 1 - \left(1 - \frac{1}{N}\right)^N \\
 &\approx 1 - e^{-1} \\
 &= 0.632
 \end{aligned}$$

On average, 63.2% of the observations appeared at least once in a bootstrap sample, so the estimation bias is similar to 2-fold cross-validation. As mentioned earlier, the smaller the number of folds, the larger the bias. Increasing the sample size will ease the problem. In general, bootstrap has larger bias and smaller uncertainty than cross-validation. Efron came up the following “.632 estimator” to alleviate this bias:

$$(0.632 \gg \text{original bootstrap estimate}) + (0.368 \gg \text{apparent error rate})$$

The apparent error rate is the error rate when the data is used twice, both to fit the model and to check its accuracy and it is apparently over-optimistic. The modified bootstrap estimate reduces the bias but can be unstable with small samples size. This estimate can also be unduly optimistic when the model severely overfits since the apparent error rate will be close to zero. Efron and Tibshirani ([Efron and Tibshirani, 1997](#)) discuss another technique, called the “.632+ method,” for adjusting the bootstrap estimates.



6

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