HW1 TJ Jiang

Load necessary libraries

```r
library(fields)
```

```
## Warning: package 'fields' was built under R version 3.2.5
## Loading required package: spam
## Warning: package 'spam' was built under R version 3.2.5
## Loading required package: grid
## Spam version 1.4-0 (2016-08-29) is loaded.
## Type 'help( Spam)' or 'demo( spam)' for a short introduction
## and overview of this package.
## Help for individual functions is also obtained by adding the
## suffix '.spam' to the function name, e.g. 'help( chol.spam)'.
##
## Attaching package: 'spam'
## The following objects are masked from 'package:base':
##
##     backsolve, forwardsolve
## Loading required package: maps
## Warning: package 'maps' was built under R version 3.2.5
```

```r
library(SpatialTools)
```

```
## # This research was partially supported under NSF Grant ATM-0534173
```

```r
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.2.5
```

Problem 1

Chapter 2 problem 2

a)

Regression, inference, n = 500, p = 3

b)

Classification, prediction, n = 20, p = 13

c)

Regression, prediction, p = 3, n = 52

Problem 2

Exercise 3 from section 2.4 (p. 52)

a)

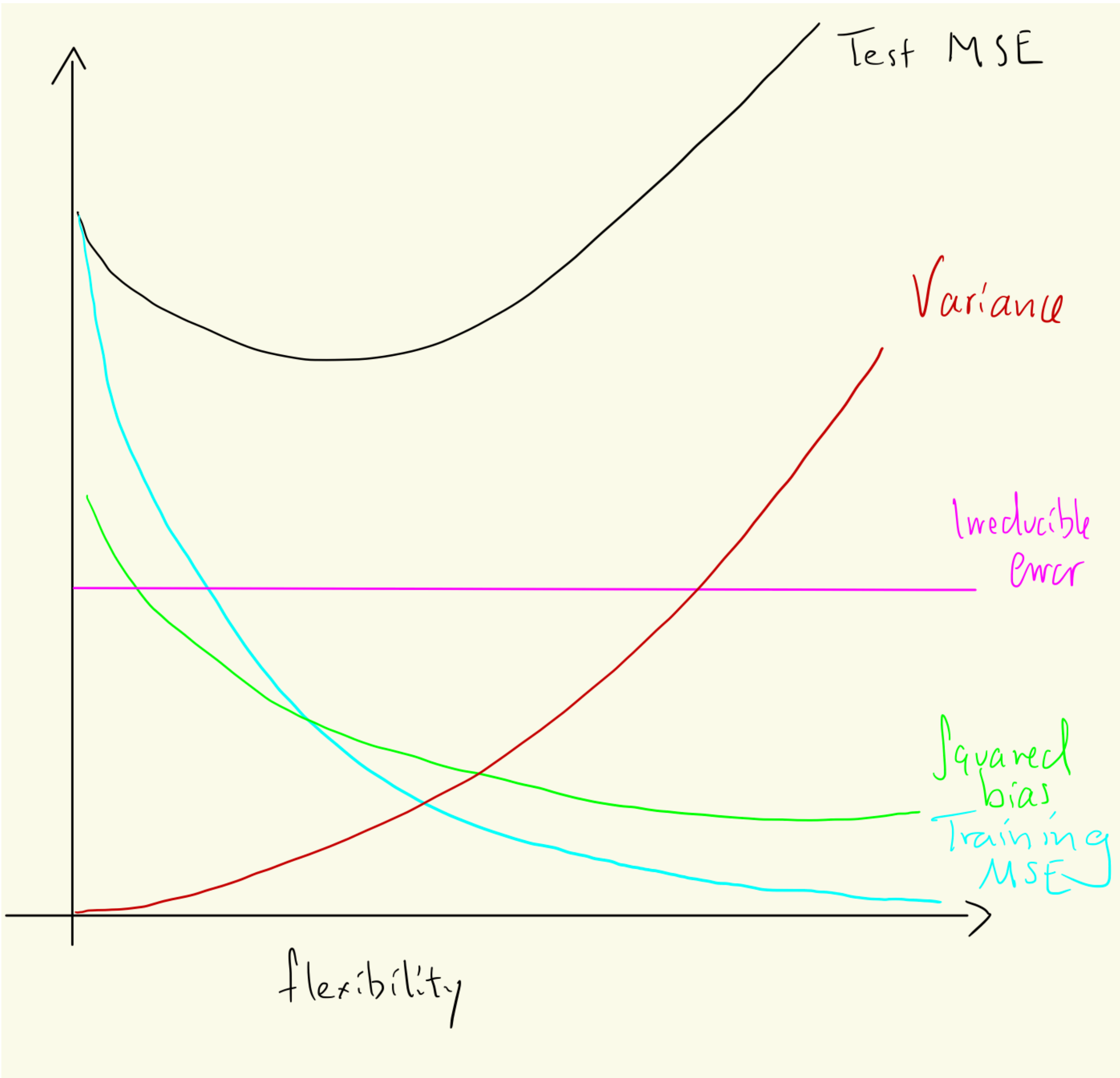


Figure 1: Problem 2a Sketch

b)

Irreducible error cannot be predicted using the input predictor variables. No matter how well the model fits it will not be attenuated. This term could be due to unmeasured quantities or unmeasurable variation. Training error will always decrease as the model becomes more flexible, but will eventually lead to detriment in generalizability Test error curve goes from an underfit model to a minima where the model fit well with low variance and low bias, but as flexibility increases more the model becomes overfit Variance will increase as model becomes more flexible as overfitting becomes an issue and small changes in predictor variables leads to potentially large changes in prediced response Bias will decrease as model becomes more flexible but will reach an asymptote (diminishing returns) at some point depending on the true model coefficients

Problem 3

Complete Exercise 7 from section 2.4 of the textbook (p. 53).

```
Obs <- seq(1,6,length=6)
X1 <- c(0,2,0,0,-1,1)
X2 <- c(3,0,1,1,0,1)
X3 <- c(0,0,3,2,1,1)
Y <- c('Red', 'Red', 'Red', 'Green', 'Green', 'Red')
df <- data.frame(Obs, X1, X2, X3, Y)
(df)
```

```
##   Obs X1 X2 X3     Y
## 1   1  0  3  0   Red
## 2   2  2  0  0   Red
## 3   3  0  1  3   Red
## 4   4  0  1  2 Green
## 5   5 -1  0  1 Green
## 6   6  1  1  1   Red
```

```
coords_mat <- data.matrix(df[2:4])
coords_mat
```

```
##      X1 X2 X3
## [1,]  0  3  0
## [2,]  2  0  0
## [3,]  0  1  3
## [4,]  0  1  2
## [5,] -1  0  1
## [6,]  1  1  1
```

a)

```
dist <- NULL
for(i in 1:nrow(coords_mat)) {
    dist[i] <- dist(rbind(coords_mat[i,], c(0,0,0)))
}
df$dist <- dist

df
```

```
##   Obs X1 X2 X3     Y     dist
## 1   1  0  3  0   Red 3.000000
## 2   2  2  0  0   Red 2.000000
## 3   3  0  1  3   Red 3.162278
## 4   4  0  1  2 Green 2.236068
## 5   5 -1  0  1 Green 1.414214
## 6   6  1  1  1   Red 1.732051
```

b)

If the only datapoint we care about is the one nearest neighbor, then the prediction will be Green (Obs 5)

c)

Obs 2, 5, 6 are the closest 3 neighbors for [0,0,0], which corresponds with a Y of Red, Green and Red respectively; thus the prediction would be red.

d)

If the actual Bayes decision boundary is highly non-linear, we would want a small k allowing a more flexible decision boundary. This corresponds to a classifier with low bias but high variance.

Problem 4: Exercise 1 (p. 413)

a)

$$\frac{1}{|C_k|}\sum_{i,i'\in C}\sum_{j=1}^{p}(x_{ij}-x_{i'j})^2 = \frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}\left((x_{ij}-\bar{x}_{ij})-(x_{i'j}-\bar{x}_{kj})\right)^2$$

subtract and add $\bar{x}_{ij}$ term, net 0

$$= \frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}\left((x_{ij}-\bar{x}_{kj})^2 - 2(x_{ij}-\bar{x}_{kj})(x_{i'j}-\bar{x}_{kj}) + (x_{i'j}-\bar{x}_{kj})^2\right)$$

multiply out

Summing over i and i'; expressions w/o i' will be repeated for each i', gaining a Ck term.

$$= \sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2 - \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})(x_{i'j}-\bar{x}_{kj}) + \sum_{i\in C_k}\sum_{j=1}^{p}(x_{i'j}-\bar{x}_{kj})^2$$

As $\bar{x}_{kj} = \frac{1}{|C_k|}\sum_{i\in C_k} x_{ij}$ e.g. mean for feature j in cluster Ck

$$= 2\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2$$

Figure 2: Proof of 10.12

b)

On the basis of this identity, argue that the K-means clustering algorithm (Algorithm 10.1) decreases the objective (10.11) at each iteration.

The left hand term (10.10) defines the within-cluster variation using squared Euclidian distance . In k-means clustering, we want to partition the observations into k clusters such that the total within-cluster variation summed over all clusters is minimized. 10.12 shows that as we minimize 10.10 we also minimize the within-cluster variation for each cluster.

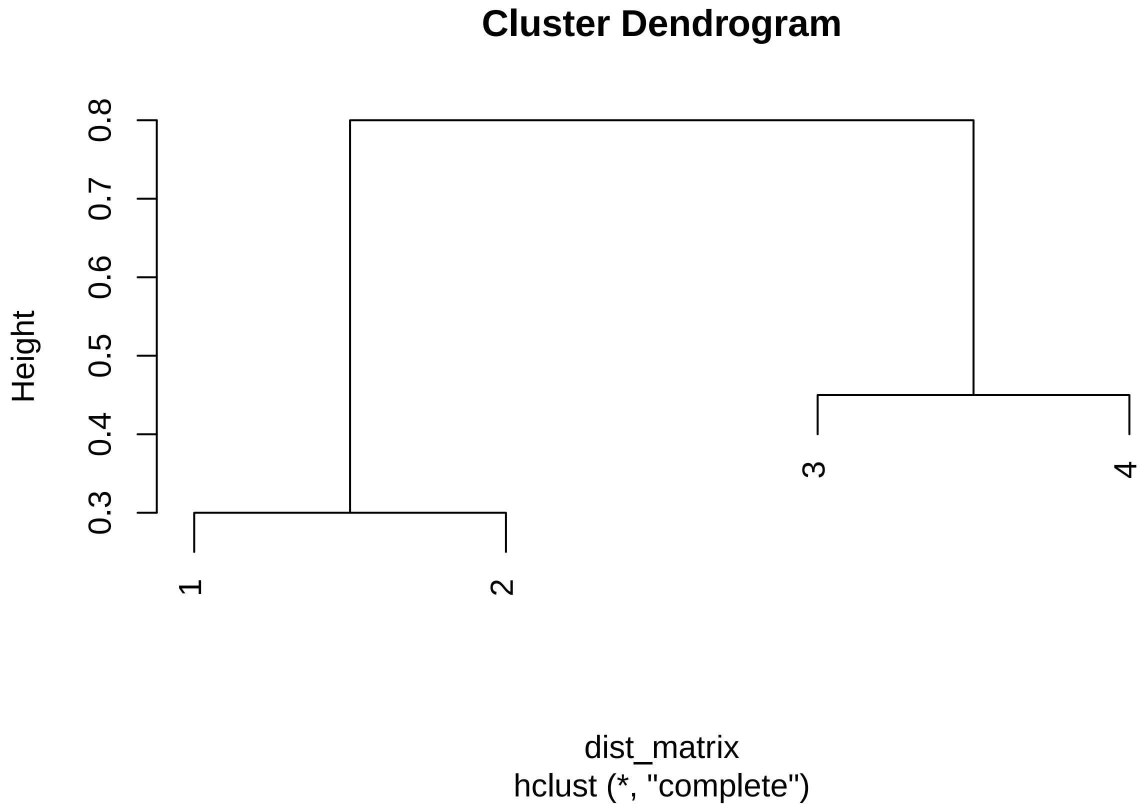
Problem 5

Exercise 2 (p. 413)

a)

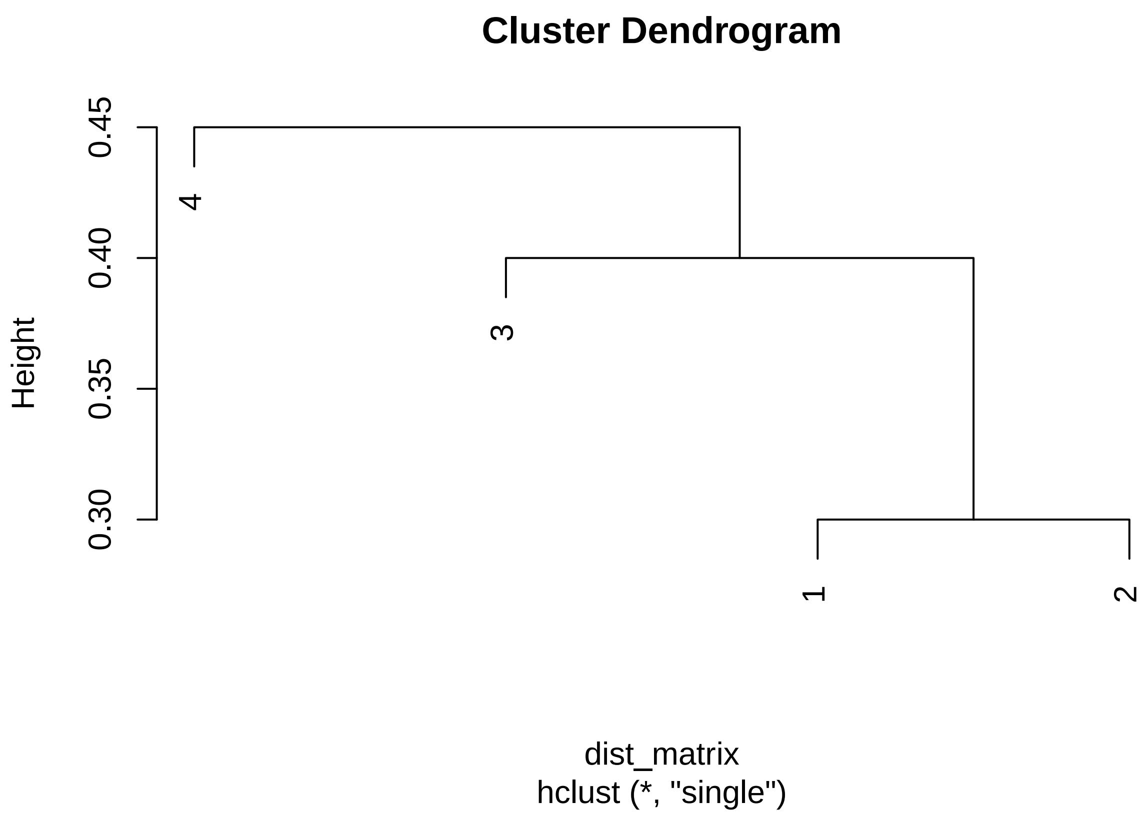
On the basis of this dissimilarity matrix, sketch the dendrogram that results from hierarchically clustering these four observa- tions using complete linkage. Be sure to indicate on the plot the height at which each fusion occurs, as well as the observations corresponding to each leaf in the dendrogram.

```
data <-c (0, 0.3, 0.4, 0.7, 0.3, 0, 0.5, 0.8, 0.4, 0.5, 0.0, 0.45, 0.7, 0.8, 0.45, 0.0)
data_mat <- matrix(data, nrow=4)
dist_matrix <- as.dist(data_mat)
plot(hclust(dist_matrix, method="complete"))
```



b)

```
plot(hclust(dist_matrix, method="single"))
```



c)

Cluster1: Observations 1 and 2; Cluster2: Observations 3 and 4

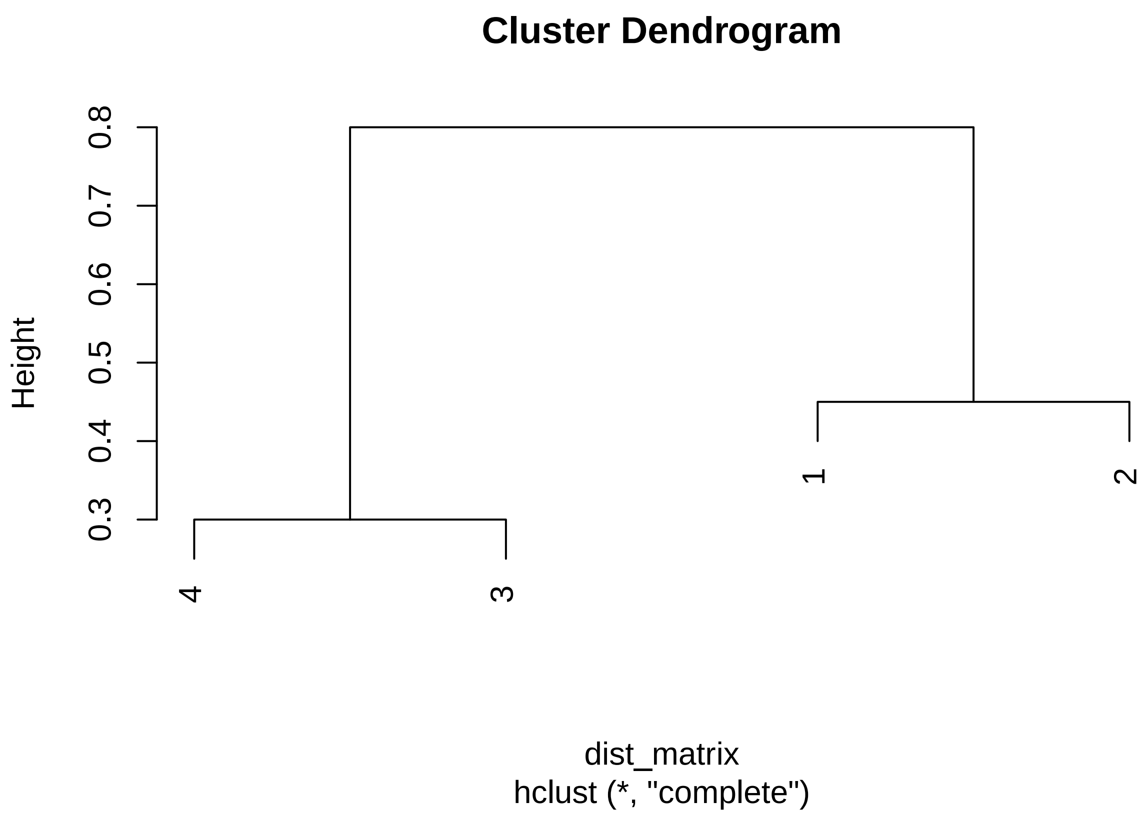
d)

Cluster1: Observations 1,2, and 3; Cluster2: Observation 4

e)

The following dendrogram swaps positions of the two clusters without changing the meaning

```
plot(hclust(dist_matrix, method="complete"), labels=c(4,3,1,2))
```



Problem 6

Exercise 4 (p. 414)

Suppose that for a particular data set, we perform hierarchical clus- tering using single linkage and using complete linkage. We obtain two dendrograms.

a)

At a certain point on the single linkage dendrogram, the clus- ters {1,2,3} and {4,5} fuse. On the complete linkage dendro- gram, the clusters {1, 2, 3} and {4, 5} also fuse at a certain point. Which fusion will occur higher on the tree, or will they fuse at the same height, or is there not enough information to tell?

This question requires more information to answer and is dependent on both the organization of information as well as the dissimilarity measure (euclidian distance, correlation etc). Complete linkage joins on maximal intercluster dissimilarity, while single linknage joins on minimal intercluster dissimilarity; were these to be equal, then the two clusters in question would fuse at the same height. Otherwise, a dendrogram formed with complete linkage would fuse them at a greater height than a dendrogram formed with single linkage.

b)

At a certain point on the single linkage dendrogram, the clusters {5} and {6} fuse. On the complete linkage dendrogram, the clus- ters {5} and {6} also fuse at a certain point. Which fusion will occur higher on the tree, or will they fuse at the same height, or is there not enough information to tell?

They would fuse at the same height because the choice of complete vs single linkage operates on observations between two clusters instead of the clustering of two individual observations that are not yet clustered.

Problem 7

Exercise 9 (p. 416)

```r
data("USArrests")
names(USArrests)
```

```
## [1] "Murder"   "Assault"  "UrbanPop" "Rape"
```

```r
dim(USArrests)
```

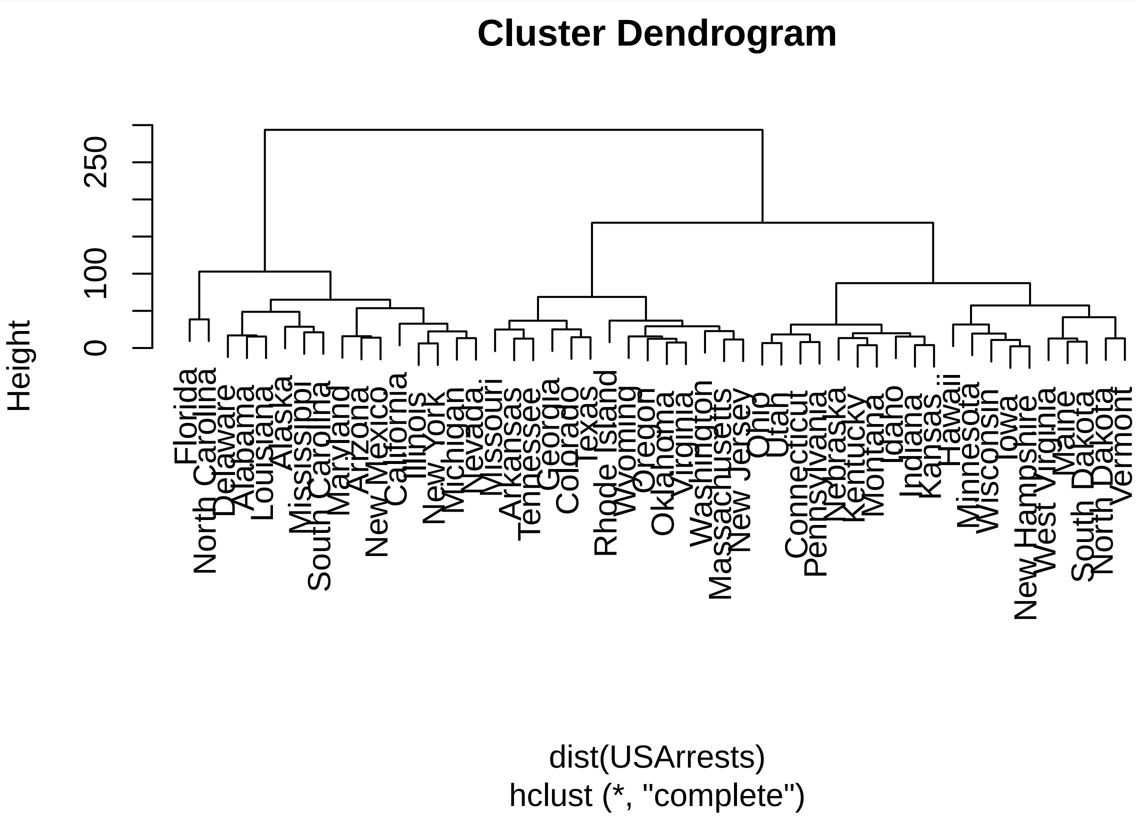
```
## [1] 50  4
```

```r
class(USArrests)
```

```
## [1] "data.frame"
```

a)

```r
cluster_USArrests <- hclust(dist(USArrests), method="complete")
plot(cluster_USArrests)
```



b)

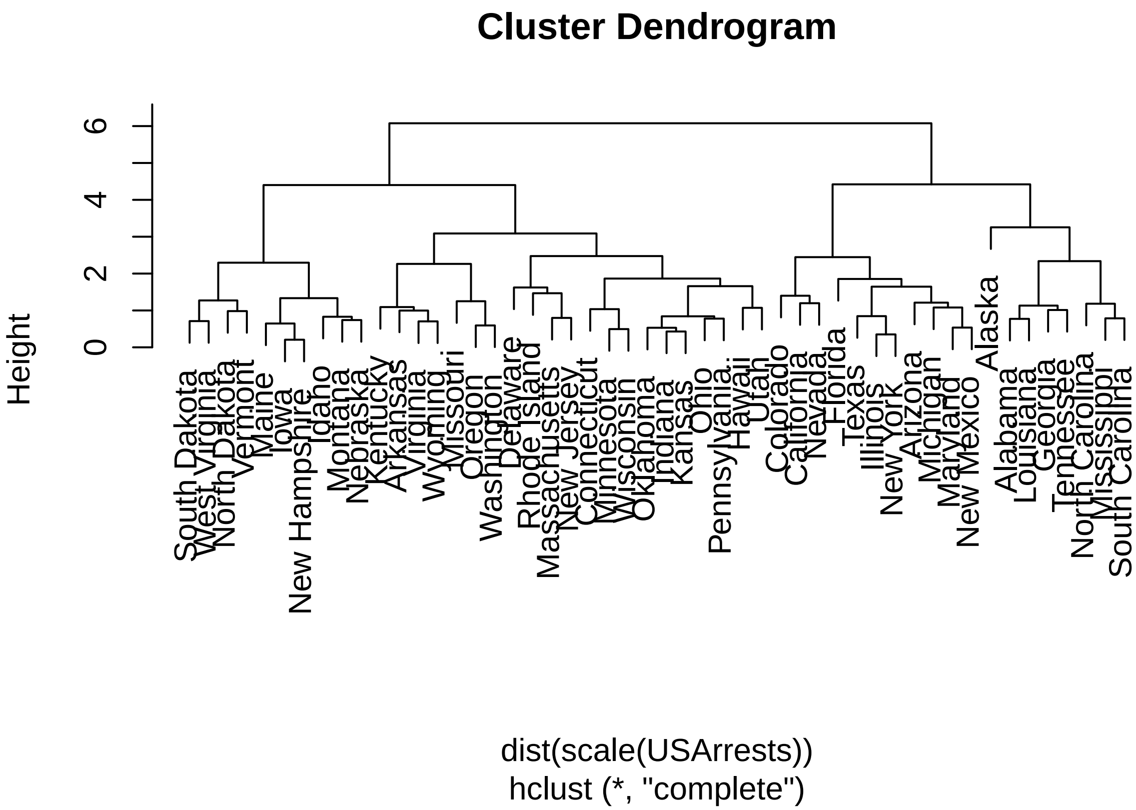
```r
cutree(cluster_USArrests, 3)
```

```
##        Alabama        Alaska       Arizona      Arkansas    California
##              1             1             1             2             1
##       Colorado   Connecticut      Delaware       Florida       Georgia
```

```
##              2            3            1            1            2
##         Hawaii        Idaho     Illinois      Indiana         Iowa
##              3            3            1            3            3
##         Kansas     Kentucky    Louisiana        Maine     Maryland
##              3            3            1            3            1
##  Massachusetts     Michigan    Minnesota  Mississippi     Missouri
##              2            1            3            1            2
##        Montana     Nebraska       Nevada New Hampshire   New Jersey
##              3            3            1            3            2
##     New Mexico     New York North Carolina North Dakota         Ohio
##              1            1            1            3            3
##       Oklahoma       Oregon  Pennsylvania Rhode Island South Carolina
##              2            2            3            2            1
##   South Dakota    Tennessee        Texas         Utah      Vermont
##              3            2            2            3            3
##       Virginia   Washington West Virginia    Wisconsin      Wyoming
##              2            2            3            3            2
```

c)

```r
cluster_USArrests_scaled = hclust(dist(scale(USArrests)), method="complete")
plot(cluster_USArrests_scaled)
```



d)

Number of states in each cluster without scaling USArrests:

```r
table(cutree(cluster_USArrests, 3))
```

```
##
##  1  2  3
## 16 14 20
```

Number of states in each cluster after scaling USArrests:

```r
table(cutree(cluster_USArrests_scaled, 3))
```

```
##
##  1  2  3
##  8 11 31
```

The overall height and spread of the dendrogram was not dramatically altered after scaling the input dataset. The states that ended up in each 3 cluster did change. In general, scaling each variable vector to standardize variance makes sense. The variables in USArrests dataset have different units with different inherent variance. Units with a larger variance has a greater effect on euclidian distance, and thus have a greater influence on how clusters are formed.

Problem 8

Exercise 4 (p. 120)

I collect a set of data (n = 100 observations) containing a single predictor and a quantitative response. I then fit a linear regression model to the data, as well as a separate cubic regression, i.e. Y = 0 + 1X + 2X2 + 3X3 + .

a)

Suppose that the true relationship between X and Y is linear, i.e. Y = 0 + 1X + . Consider the training residual sum of squares (RSS) for the linear regression, and also the training RSS for the cubic regression. Would we expect one to be lower than the other, would we expect them to be the same, or is there not enough information to tell? Justify your answer.

Adding more variables to the least squares equations always improves the fit to the training data; thus, the RSS to training data should decrease

b)

Answer (a) using test rather than training RSS.

test RSS should decrease due to the overfitting and failing to generalize overfit model to test dataset

c)

Suppose that the truerelationshipbetweenXandYisnotlinear, but we don't know how far it is from linear. Consider the training RSS for the linear regression, and also the training RSS for the cubic regression. Would we expect one to be lower than the other, would we expect them to be the same, or is there not enough information to tell? Justify your answer.

The increased flexibility from polynomial regression will lead to a better fit to training data over a linear regression.

d)

Answer (c) using test rather than training RSS.

Since the true relationship is not known, there is not enough information to exactly tell whether test dataset RSS will be better with a polynomial fit

Problem 9

Exercise 9 (p. 122). In parts (e) and (f), you need only try a few interactions and transformations.

```
Auto = na.omit(read.csv("Auto.csv", na.strings="?")) #massage out question marks and lists with missing
Auto
```

```
##      mpg cylinders displacement horsepower weight acceleration year origin
## 1   18.0         8        307.0        130   3504         12.0   70      1
## 2   15.0         8        350.0        165   3693         11.5   70      1
## 3   18.0         8        318.0        150   3436         11.0   70      1
## 4   16.0         8        304.0        150   3433         12.0   70      1
## 5   17.0         8        302.0        140   3449         10.5   70      1
## 6   15.0         8        429.0        198   4341         10.0   70      1
## 7   14.0         8        454.0        220   4354          9.0   70      1
## 8   14.0         8        440.0        215   4312          8.5   70      1
## 9   14.0         8        455.0        225   4425         10.0   70      1
## 10  15.0         8        390.0        190   3850          8.5   70      1
## 11  15.0         8        383.0        170   3563         10.0   70      1
## 12  14.0         8        340.0        160   3609          8.0   70      1
## 13  15.0         8        400.0        150   3761          9.5   70      1
## 14  14.0         8        455.0        225   3086         10.0   70      1
## 15  24.0         4        113.0         95   2372         15.0   70      3
## 16  22.0         6        198.0         95   2833         15.5   70      1
## 17  18.0         6        199.0         97   2774         15.5   70      1
## 18  21.0         6        200.0         85   2587         16.0   70      1
## 19  27.0         4         97.0         88   2130         14.5   70      3
## 20  26.0         4         97.0         46   1835         20.5   70      2
## 21  25.0         4        110.0         87   2672         17.5   70      2
## 22  24.0         4        107.0         90   2430         14.5   70      2
## 23  25.0         4        104.0         95   2375         17.5   70      2
## 24  26.0         4        121.0        113   2234         12.5   70      2
## 25  21.0         6        199.0         90   2648         15.0   70      1
## 26  10.0         8        360.0        215   4615         14.0   70      1
## 27  10.0         8        307.0        200   4376         15.0   70      1
## 28  11.0         8        318.0        210   4382         13.5   70      1
## 29   9.0         8        304.0        193   4732         18.5   70      1
## 30  27.0         4         97.0         88   2130         14.5   71      3
## 31  28.0         4        140.0         90   2264         15.5   71      1
## 32  25.0         4        113.0         95   2228         14.0   71      3
## 34  19.0         6        232.0        100   2634         13.0   71      1
## 35  16.0         6        225.0        105   3439         15.5   71      1
## 36  17.0         6        250.0        100   3329         15.5   71      1
## 37  19.0         6        250.0         88   3302         15.5   71      1
## 38  18.0         6        232.0        100   3288         15.5   71      1
## 39  14.0         8        350.0        165   4209         12.0   71      1
## 40  14.0         8        400.0        175   4464         11.5   71      1
## 41  14.0         8        351.0        153   4154         13.5   71      1
```

```
## 42  14.0         8       318.0        150  4096          13.0  71       1
## 43  12.0         8       383.0        180  4955          11.5  71       1
## 44  13.0         8       400.0        170  4746          12.0  71       1
## 45  13.0         8       400.0        175  5140          12.0  71       1
## 46  18.0         6       258.0        110  2962          13.5  71       1
## 47  22.0         4       140.0         72  2408          19.0  71       1
## 48  19.0         6       250.0        100  3282          15.0  71       1
## 49  18.0         6       250.0         88  3139          14.5  71       1
## 50  23.0         4       122.0         86  2220          14.0  71       1
## 51  28.0         4       116.0         90  2123          14.0  71       2
## 52  30.0         4        79.0         70  2074          19.5  71       2
## 53  30.0         4        88.0         76  2065          14.5  71       2
## 54  31.0         4        71.0         65  1773          19.0  71       3
## 55  35.0         4        72.0         69  1613          18.0  71       3
## 56  27.0         4        97.0         60  1834          19.0  71       2
## 57  26.0         4        91.0         70  1955          20.5  71       1
## 58  24.0         4       113.0         95  2278          15.5  72       3
## 59  25.0         4        97.5         80  2126          17.0  72       1
## 60  23.0         4        97.0         54  2254          23.5  72       2
## 61  20.0         4       140.0         90  2408          19.5  72       1
## 62  21.0         4       122.0         86  2226          16.5  72       1
## 63  13.0         8       350.0        165  4274          12.0  72       1
## 64  14.0         8       400.0        175  4385          12.0  72       1
## 65  15.0         8       318.0        150  4135          13.5  72       1
## 66  14.0         8       351.0        153  4129          13.0  72       1
## 67  17.0         8       304.0        150  3672          11.5  72       1
## 68  11.0         8       429.0        208  4633          11.0  72       1
## 69  13.0         8       350.0        155  4502          13.5  72       1
## 70  12.0         8       350.0        160  4456          13.5  72       1
## 71  13.0         8       400.0        190  4422          12.5  72       1
## 72  19.0         3        70.0         97  2330          13.5  72       3
## 73  15.0         8       304.0        150  3892          12.5  72       1
## 74  13.0         8       307.0        130  4098          14.0  72       1
## 75  13.0         8       302.0        140  4294          16.0  72       1
## 76  14.0         8       318.0        150  4077          14.0  72       1
## 77  18.0         4       121.0        112  2933          14.5  72       2
## 78  22.0         4       121.0         76  2511          18.0  72       2
## 79  21.0         4       120.0         87  2979          19.5  72       2
## 80  26.0         4        96.0         69  2189          18.0  72       2
## 81  22.0         4       122.0         86  2395          16.0  72       1
## 82  28.0         4        97.0         92  2288          17.0  72       3
## 83  23.0         4       120.0         97  2506          14.5  72       3
## 84  28.0         4        98.0         80  2164          15.0  72       1
## 85  27.0         4        97.0         88  2100          16.5  72       3
## 86  13.0         8       350.0        175  4100          13.0  73       1
## 87  14.0         8       304.0        150  3672          11.5  73       1
## 88  13.0         8       350.0        145  3988          13.0  73       1
## 89  14.0         8       302.0        137  4042          14.5  73       1
## 90  15.0         8       318.0        150  3777          12.5  73       1
## 91  12.0         8       429.0        198  4952          11.5  73       1
## 92  13.0         8       400.0        150  4464          12.0  73       1
## 93  13.0         8       351.0        158  4363          13.0  73       1
## 94  14.0         8       318.0        150  4237          14.5  73       1
## 95  13.0         8       440.0        215  4735          11.0  73       1
```

```
## 96  12.0          8       455.0       225   4951         11.0   73      1
## 97  13.0          8       360.0       175   3821         11.0   73      1
## 98  18.0          6       225.0       105   3121         16.5   73      1
## 99  16.0          6       250.0       100   3278         18.0   73      1
## 100 18.0          6       232.0       100   2945         16.0   73      1
## 101 18.0          6       250.0        88   3021         16.5   73      1
## 102 23.0          6       198.0        95   2904         16.0   73      1
## 103 26.0          4        97.0        46   1950         21.0   73      2
## 104 11.0          8       400.0       150   4997         14.0   73      1
## 105 12.0          8       400.0       167   4906         12.5   73      1
## 106 13.0          8       360.0       170   4654         13.0   73      1
## 107 12.0          8       350.0       180   4499         12.5   73      1
## 108 18.0          6       232.0       100   2789         15.0   73      1
## 109 20.0          4        97.0        88   2279         19.0   73      3
## 110 21.0          4       140.0        72   2401         19.5   73      1
## 111 22.0          4       108.0        94   2379         16.5   73      3
## 112 18.0          3        70.0        90   2124         13.5   73      3
## 113 19.0          4       122.0        85   2310         18.5   73      1
## 114 21.0          6       155.0       107   2472         14.0   73      1
## 115 26.0          4        98.0        90   2265         15.5   73      2
## 116 15.0          8       350.0       145   4082         13.0   73      1
## 117 16.0          8       400.0       230   4278          9.5   73      1
## 118 29.0          4        68.0        49   1867         19.5   73      2
## 119 24.0          4       116.0        75   2158         15.5   73      2
## 120 20.0          4       114.0        91   2582         14.0   73      2
## 121 19.0          4       121.0       112   2868         15.5   73      2
## 122 15.0          8       318.0       150   3399         11.0   73      1
## 123 24.0          4       121.0       110   2660         14.0   73      2
## 124 20.0          6       156.0       122   2807         13.5   73      3
## 125 11.0          8       350.0       180   3664         11.0   73      1
## 126 20.0          6       198.0        95   3102         16.5   74      1
## 128 19.0          6       232.0       100   2901         16.0   74      1
## 129 15.0          6       250.0       100   3336         17.0   74      1
## 130 31.0          4        79.0        67   1950         19.0   74      3
## 131 26.0          4       122.0        80   2451         16.5   74      1
## 132 32.0          4        71.0        65   1836         21.0   74      3
## 133 25.0          4       140.0        75   2542         17.0   74      1
## 134 16.0          6       250.0       100   3781         17.0   74      1
## 135 16.0          6       258.0       110   3632         18.0   74      1
## 136 18.0          6       225.0       105   3613         16.5   74      1
## 137 16.0          8       302.0       140   4141         14.0   74      1
## 138 13.0          8       350.0       150   4699         14.5   74      1
## 139 14.0          8       318.0       150   4457         13.5   74      1
## 140 14.0          8       302.0       140   4638         16.0   74      1
## 141 14.0          8       304.0       150   4257         15.5   74      1
## 142 29.0          4        98.0        83   2219         16.5   74      2
## 143 26.0          4        79.0        67   1963         15.5   74      2
## 144 26.0          4        97.0        78   2300         14.5   74      2
## 145 31.0          4        76.0        52   1649         16.5   74      3
## 146 32.0          4        83.0        61   2003         19.0   74      3
## 147 28.0          4        90.0        75   2125         14.5   74      1
## 148 24.0          4        90.0        75   2108         15.5   74      2
## 149 26.0          4       116.0        75   2246         14.0   74      2
## 150 24.0          4       120.0        97   2489         15.0   74      3
```

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## 151 26.0         4        108.0          93   2391           15.5   74      3
## 152 31.0         4         79.0          67   2000           16.0   74      2
## 153 19.0         6        225.0          95   3264           16.0   75      1
## 154 18.0         6        250.0         105   3459           16.0   75      1
## 155 15.0         6        250.0          72   3432           21.0   75      1
## 156 15.0         6        250.0          72   3158           19.5   75      1
## 157 16.0         8        400.0         170   4668           11.5   75      1
## 158 15.0         8        350.0         145   4440           14.0   75      1
## 159 16.0         8        318.0         150   4498           14.5   75      1
## 160 14.0         8        351.0         148   4657           13.5   75      1
## 161 17.0         6        231.0         110   3907           21.0   75      1
## 162 16.0         6        250.0         105   3897           18.5   75      1
## 163 15.0         6        258.0         110   3730           19.0   75      1
## 164 18.0         6        225.0          95   3785           19.0   75      1
## 165 21.0         6        231.0         110   3039           15.0   75      1
## 166 20.0         8        262.0         110   3221           13.5   75      1
## 167 13.0         8        302.0         129   3169           12.0   75      1
## 168 29.0         4         97.0          75   2171           16.0   75      3
## 169 23.0         4        140.0          83   2639           17.0   75      1
## 170 20.0         6        232.0         100   2914           16.0   75      1
## 171 23.0         4        140.0          78   2592           18.5   75      1
## 172 24.0         4        134.0          96   2702           13.5   75      3
## 173 25.0         4         90.0          71   2223           16.5   75      2
## 174 24.0         4        119.0          97   2545           17.0   75      3
## 175 18.0         6        171.0          97   2984           14.5   75      1
## 176 29.0         4         90.0          70   1937           14.0   75      2
## 177 19.0         6        232.0          90   3211           17.0   75      1
## 178 23.0         4        115.0          95   2694           15.0   75      2
## 179 23.0         4        120.0          88   2957           17.0   75      2
## 180 22.0         4        121.0          98   2945           14.5   75      2
## 181 25.0         4        121.0         115   2671           13.5   75      2
## 182 33.0         4         91.0          53   1795           17.5   75      3
## 183 28.0         4        107.0          86   2464           15.5   76      2
## 184 25.0         4        116.0          81   2220           16.9   76      2
## 185 25.0         4        140.0          92   2572           14.9   76      1
## 186 26.0         4         98.0          79   2255           17.7   76      1
## 187 27.0         4        101.0          83   2202           15.3   76      2
## 188 17.5         8        305.0         140   4215           13.0   76      1
## 189 16.0         8        318.0         150   4190           13.0   76      1
## 190 15.5         8        304.0         120   3962           13.9   76      1
## 191 14.5         8        351.0         152   4215           12.8   76      1
## 192 22.0         6        225.0         100   3233           15.4   76      1
## 193 22.0         6        250.0         105   3353           14.5   76      1
## 194 24.0         6        200.0          81   3012           17.6   76      1
## 195 22.5         6        232.0          90   3085           17.6   76      1
## 196 29.0         4         85.0          52   2035           22.2   76      1
## 197 24.5         4         98.0          60   2164           22.1   76      1
## 198 29.0         4         90.0          70   1937           14.2   76      2
## 199 33.0         4         91.0          53   1795           17.4   76      3
## 200 20.0         6        225.0         100   3651           17.7   76      1
## 201 18.0         6        250.0          78   3574           21.0   76      1
## 202 18.5         6        250.0         110   3645           16.2   76      1
## 203 17.5         6        258.0          95   3193           17.8   76      1
## 204 29.5         4         97.0          71   1825           12.2   76      2
```

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## 205 32.0         4         85.0        70  1990         17.0  76       3
## 206 28.0         4         97.0        75  2155         16.4  76       3
## 207 26.5         4        140.0        72  2565         13.6  76       1
## 208 20.0         4        130.0       102  3150         15.7  76       2
## 209 13.0         8        318.0       150  3940         13.2  76       1
## 210 19.0         4        120.0        88  3270         21.9  76       2
## 211 19.0         6        156.0       108  2930         15.5  76       3
## 212 16.5         6        168.0       120  3820         16.7  76       2
## 213 16.5         8        350.0       180  4380         12.1  76       1
## 214 13.0         8        350.0       145  4055         12.0  76       1
## 215 13.0         8        302.0       130  3870         15.0  76       1
## 216 13.0         8        318.0       150  3755         14.0  76       1
## 217 31.5         4         98.0        68  2045         18.5  77       3
## 218 30.0         4        111.0        80  2155         14.8  77       1
## 219 36.0         4         79.0        58  1825         18.6  77       2
## 220 25.5         4        122.0        96  2300         15.5  77       1
## 221 33.5         4         85.0        70  1945         16.8  77       3
## 222 17.5         8        305.0       145  3880         12.5  77       1
## 223 17.0         8        260.0       110  4060         19.0  77       1
## 224 15.5         8        318.0       145  4140         13.7  77       1
## 225 15.0         8        302.0       130  4295         14.9  77       1
## 226 17.5         6        250.0       110  3520         16.4  77       1
## 227 20.5         6        231.0       105  3425         16.9  77       1
## 228 19.0         6        225.0       100  3630         17.7  77       1
## 229 18.5         6        250.0        98  3525         19.0  77       1
## 230 16.0         8        400.0       180  4220         11.1  77       1
## 231 15.5         8        350.0       170  4165         11.4  77       1
## 232 15.5         8        400.0       190  4325         12.2  77       1
## 233 16.0         8        351.0       149  4335         14.5  77       1
## 234 29.0         4         97.0        78  1940         14.5  77       2
## 235 24.5         4        151.0        88  2740         16.0  77       1
## 236 26.0         4         97.0        75  2265         18.2  77       3
## 237 25.5         4        140.0        89  2755         15.8  77       1
## 238 30.5         4         98.0        63  2051         17.0  77       1
## 239 33.5         4         98.0        83  2075         15.9  77       1
## 240 30.0         4         97.0        67  1985         16.4  77       3
## 241 30.5         4         97.0        78  2190         14.1  77       2
## 242 22.0         6        146.0        97  2815         14.5  77       3
## 243 21.5         4        121.0       110  2600         12.8  77       2
## 244 21.5         3         80.0       110  2720         13.5  77       3
## 245 43.1         4         90.0        48  1985         21.5  78       2
## 246 36.1         4         98.0        66  1800         14.4  78       1
## 247 32.8         4         78.0        52  1985         19.4  78       3
## 248 39.4         4         85.0        70  2070         18.6  78       3
## 249 36.1         4         91.0        60  1800         16.4  78       3
## 250 19.9         8        260.0       110  3365         15.5  78       1
## 251 19.4         8        318.0       140  3735         13.2  78       1
## 252 20.2         8        302.0       139  3570         12.8  78       1
## 253 19.2         6        231.0       105  3535         19.2  78       1
## 254 20.5         6        200.0        95  3155         18.2  78       1
## 255 20.2         6        200.0        85  2965         15.8  78       1
## 256 25.1         4        140.0        88  2720         15.4  78       1
## 257 20.5         6        225.0       100  3430         17.2  78       1
## 258 19.4         6        232.0        90  3210         17.2  78       1
```

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## 259 20.6          6         231.0         105   3380          15.8   78        1
## 260 20.8          6         200.0          85   3070          16.7   78        1
## 261 18.6          6         225.0         110   3620          18.7   78        1
## 262 18.1          6         258.0         120   3410          15.1   78        1
## 263 19.2          8         305.0         145   3425          13.2   78        1
## 264 17.7          6         231.0         165   3445          13.4   78        1
## 265 18.1          8         302.0         139   3205          11.2   78        1
## 266 17.5          8         318.0         140   4080          13.7   78        1
## 267 30.0          4          98.0          68   2155          16.5   78        1
## 268 27.5          4         134.0          95   2560          14.2   78        3
## 269 27.2          4         119.0          97   2300          14.7   78        3
## 270 30.9          4         105.0          75   2230          14.5   78        1
## 271 21.1          4         134.0          95   2515          14.8   78        3
## 272 23.2          4         156.0         105   2745          16.7   78        1
## 273 23.8          4         151.0          85   2855          17.6   78        1
## 274 23.9          4         119.0          97   2405          14.9   78        3
## 275 20.3          5         131.0         103   2830          15.9   78        2
## 276 17.0          6         163.0         125   3140          13.6   78        2
## 277 21.6          4         121.0         115   2795          15.7   78        2
## 278 16.2          6         163.0         133   3410          15.8   78        2
## 279 31.5          4          89.0          71   1990          14.9   78        2
## 280 29.5          4          98.0          68   2135          16.6   78        3
## 281 21.5          6         231.0         115   3245          15.4   79        1
## 282 19.8          6         200.0          85   2990          18.2   79        1
## 283 22.3          4         140.0          88   2890          17.3   79        1
## 284 20.2          6         232.0          90   3265          18.2   79        1
## 285 20.6          6         225.0         110   3360          16.6   79        1
## 286 17.0          8         305.0         130   3840          15.4   79        1
## 287 17.6          8         302.0         129   3725          13.4   79        1
## 288 16.5          8         351.0         138   3955          13.2   79        1
## 289 18.2          8         318.0         135   3830          15.2   79        1
## 290 16.9          8         350.0         155   4360          14.9   79        1
## 291 15.5          8         351.0         142   4054          14.3   79        1
## 292 19.2          8         267.0         125   3605          15.0   79        1
## 293 18.5          8         360.0         150   3940          13.0   79        1
## 294 31.9          4          89.0          71   1925          14.0   79        2
## 295 34.1          4          86.0          65   1975          15.2   79        3
## 296 35.7          4          98.0          80   1915          14.4   79        1
## 297 27.4          4         121.0          80   2670          15.0   79        1
## 298 25.4          5         183.0          77   3530          20.1   79        2
## 299 23.0          8         350.0         125   3900          17.4   79        1
## 300 27.2          4         141.0          71   3190          24.8   79        2
## 301 23.9          8         260.0          90   3420          22.2   79        1
## 302 34.2          4         105.0          70   2200          13.2   79        1
## 303 34.5          4         105.0          70   2150          14.9   79        1
## 304 31.8          4          85.0          65   2020          19.2   79        3
## 305 37.3          4          91.0          69   2130          14.7   79        2
## 306 28.4          4         151.0          90   2670          16.0   79        1
## 307 28.8          6         173.0         115   2595          11.3   79        1
## 308 26.8          6         173.0         115   2700          12.9   79        1
## 309 33.5          4         151.0          90   2556          13.2   79        1
## 310 41.5          4          98.0          76   2144          14.7   80        2
## 311 38.1          4          89.0          60   1968          18.8   80        3
## 312 32.1          4          98.0          70   2120          15.5   80        1
```

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## 313 37.2           4          86.0           65   2019           16.4   80       3
## 314 28.0           4         151.0           90   2678           16.5   80       1
## 315 26.4           4         140.0           88   2870           18.1   80       1
## 316 24.3           4         151.0           90   3003           20.1   80       1
## 317 19.1           6         225.0           90   3381           18.7   80       1
## 318 34.3           4          97.0           78   2188           15.8   80       2
## 319 29.8           4         134.0           90   2711           15.5   80       3
## 320 31.3           4         120.0           75   2542           17.5   80       3
## 321 37.0           4         119.0           92   2434           15.0   80       3
## 322 32.2           4         108.0           75   2265           15.2   80       3
## 323 46.6           4          86.0           65   2110           17.9   80       3
## 324 27.9           4         156.0          105   2800           14.4   80       1
## 325 40.8           4          85.0           65   2110           19.2   80       3
## 326 44.3           4          90.0           48   2085           21.7   80       2
## 327 43.4           4          90.0           48   2335           23.7   80       2
## 328 36.4           5         121.0           67   2950           19.9   80       2
## 329 30.0           4         146.0           67   3250           21.8   80       2
## 330 44.6           4          91.0           67   1850           13.8   80       3
## 332 33.8           4          97.0           67   2145           18.0   80       3
## 333 29.8           4          89.0           62   1845           15.3   80       2
## 334 32.7           6         168.0          132   2910           11.4   80       3
## 335 23.7           3          70.0          100   2420           12.5   80       3
## 336 35.0           4         122.0           88   2500           15.1   80       2
## 338 32.4           4         107.0           72   2290           17.0   80       3
## 339 27.2           4         135.0           84   2490           15.7   81       1
## 340 26.6           4         151.0           84   2635           16.4   81       1
## 341 25.8           4         156.0           92   2620           14.4   81       1
## 342 23.5           6         173.0          110   2725           12.6   81       1
## 343 30.0           4         135.0           84   2385           12.9   81       1
## 344 39.1           4          79.0           58   1755           16.9   81       3
## 345 39.0           4          86.0           64   1875           16.4   81       1
## 346 35.1           4          81.0           60   1760           16.1   81       3
## 347 32.3           4          97.0           67   2065           17.8   81       3
## 348 37.0           4          85.0           65   1975           19.4   81       3
## 349 37.7           4          89.0           62   2050           17.3   81       3
## 350 34.1           4          91.0           68   1985           16.0   81       3
## 351 34.7           4         105.0           63   2215           14.9   81       1
## 352 34.4           4          98.0           65   2045           16.2   81       1
## 353 29.9           4          98.0           65   2380           20.7   81       1
## 354 33.0           4         105.0           74   2190           14.2   81       2
## 356 33.7           4         107.0           75   2210           14.4   81       3
## 357 32.4           4         108.0           75   2350           16.8   81       3
## 358 32.9           4         119.0          100   2615           14.8   81       3
## 359 31.6           4         120.0           74   2635           18.3   81       3
## 360 28.1           4         141.0           80   3230           20.4   81       2
## 361 30.7           6         145.0           76   3160           19.6   81       2
## 362 25.4           6         168.0          116   2900           12.6   81       3
## 363 24.2           6         146.0          120   2930           13.8   81       3
## 364 22.4           6         231.0          110   3415           15.8   81       1
## 365 26.6           8         350.0          105   3725           19.0   81       1
## 366 20.2           6         200.0           88   3060           17.1   81       1
## 367 17.6           6         225.0           85   3465           16.6   81       1
## 368 28.0           4         112.0           88   2605           19.6   82       1
## 369 27.0           4         112.0           88   2640           18.6   82       1
```

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## 370 34.0         4        112.0          88   2395         18.0   82       1
## 371 31.0         4        112.0          85   2575         16.2   82       1
## 372 29.0         4        135.0          84   2525         16.0   82       1
## 373 27.0         4        151.0          90   2735         18.0   82       1
## 374 24.0         4        140.0          92   2865         16.4   82       1
## 375 36.0         4        105.0          74   1980         15.3   82       2
## 376 37.0         4         91.0          68   2025         18.2   82       3
## 377 31.0         4         91.0          68   1970         17.6   82       3
## 378 38.0         4        105.0          63   2125         14.7   82       1
## 379 36.0         4         98.0          70   2125         17.3   82       1
## 380 36.0         4        120.0          88   2160         14.5   82       3
## 381 36.0         4        107.0          75   2205         14.5   82       3
## 382 34.0         4        108.0          70   2245         16.9   82       3
## 383 38.0         4         91.0          67   1965         15.0   82       3
## 384 32.0         4         91.0          67   1965         15.7   82       3
## 385 38.0         4         91.0          67   1995         16.2   82       3
## 386 25.0         6        181.0         110   2945         16.4   82       1
## 387 38.0         6        262.0          85   3015         17.0   82       1
## 388 26.0         4        156.0          92   2585         14.5   82       1
## 389 22.0         6        232.0         112   2835         14.7   82       1
## 390 32.0         4        144.0          96   2665         13.9   82       3
## 391 36.0         4        135.0          84   2370         13.0   82       1
## 392 27.0         4        151.0          90   2950         17.3   82       1
## 393 27.0         4        140.0          86   2790         15.6   82       1
## 394 44.0         4         97.0          52   2130         24.6   82       2
## 395 32.0         4        135.0          84   2295         11.6   82       1
## 396 28.0         4        120.0          79   2625         18.6   82       1
## 397 31.0         4        119.0          82   2720         19.4   82       1
##                                     name
## 1              chevrolet chevelle malibu
## 2                      buick skylark 320
## 3                     plymouth satellite
## 4                          amc rebel sst
## 5                            ford torino
## 6                       ford galaxie 500
## 7                       chevrolet impala
## 8                      plymouth fury iii
## 9                       pontiac catalina
## 10                    amc ambassador dpl
## 11                   dodge challenger se
## 12                    plymouth 'cuda 340
## 13                 chevrolet monte carlo
## 14               buick estate wagon (sw)
## 15                 toyota corona mark ii
## 16                       plymouth duster
## 17                            amc hornet
## 18                         ford maverick
## 19                          datsun pl510
## 20          volkswagen 1131 deluxe sedan
## 21                           peugeot 504
## 22                           audi 100 ls
## 23                              saab 99e
## 24                              bmw 2002
## 25                           amc gremlin
```

```
## 26                           ford f250
## 27                           chevy c20
## 28                          dodge d200
## 29                            hi 1200d
## 30                        datsun pl510
## 31                 chevrolet vega 2300
## 32                       toyota corona
## 34                          amc gremlin
## 35           plymouth satellite custom
## 36           chevrolet chevelle malibu
## 37                      ford torino 500
## 38                          amc matador
## 39                    chevrolet impala
## 40           pontiac catalina brougham
## 41                     ford galaxie 500
## 42                    plymouth fury iii
## 43                   dodge monaco (sw)
## 44            ford country squire (sw)
## 45                 pontiac safari (sw)
## 46          amc hornet sportabout (sw)
## 47                 chevrolet vega (sw)
## 48                    pontiac firebird
## 49                        ford mustang
## 50                  mercury capri 2000
## 51                           opel 1900
## 52                         peugeot 304
## 53                           fiat 124b
## 54                 toyota corolla 1200
## 55                         datsun 1200
## 56                volkswagen model 111
## 57                    plymouth cricket
## 58               toyota corona hardtop
## 59                  dodge colt hardtop
## 60                   volkswagen type 3
## 61                      chevrolet vega
## 62                  ford pinto runabout
## 63                    chevrolet impala
## 64                    pontiac catalina
## 65                   plymouth fury iii
## 66                    ford galaxie 500
## 67                  amc ambassador sst
## 68                     mercury marquis
## 69                 buick lesabre custom
## 70          oldsmobile delta 88 royale
## 71               chrysler newport royal
## 72                     mazda rx2 coupe
## 73                    amc matador (sw)
## 74    chevrolet chevelle concours (sw)
## 75               ford gran torino (sw)
## 76       plymouth satellite custom (sw)
## 77                     volvo 145e (sw)
## 78                 volkswagen 411 (sw)
## 79                    peugeot 504 (sw)
## 80                     renault 12 (sw)
```

```
## 81                      ford pinto (sw)
## 82                      datsun 510 (sw)
## 83          toyouta corona mark ii (sw)
## 84                      dodge colt (sw)
## 85             toyota corolla 1600 (sw)
## 86                    buick century 350
## 87                          amc matador
## 88                     chevrolet malibu
## 89                     ford gran torino
## 90                 dodge coronet custom
## 91             mercury marquis brougham
## 92              chevrolet caprice classic
## 93                             ford ltd
## 94             plymouth fury gran sedan
## 95         chrysler new yorker brougham
## 96              buick electra 225 custom
## 97              amc ambassador brougham
## 98                     plymouth valiant
## 99                 chevrolet nova custom
## 100                          amc hornet
## 101                       ford maverick
## 102                     plymouth duster
## 103             volkswagen super beetle
## 104                    chevrolet impala
## 105                        ford country
## 106               plymouth custom suburb
## 107             oldsmobile vista cruiser
## 108                         amc gremlin
## 109                       toyota carina
## 110                      chevrolet vega
## 111                          datsun 610
## 112                           maxda rx3
## 113                          ford pinto
## 114                    mercury capri v6
## 115                fiat 124 sport coupe
## 116             chevrolet monte carlo s
## 117                  pontiac grand prix
## 118                            fiat 128
## 119                          opel manta
## 120                          audi 100ls
## 121                         volvo 144ea
## 122                   dodge dart custom
## 123                           saab 99le
## 124                      toyota mark ii
## 125                    oldsmobile omega
## 126                     plymouth duster
## 128                          amc hornet
## 129                      chevrolet nova
## 130                         datsun b210
## 131                          ford pinto
## 132                 toyota corolla 1200
## 133                      chevrolet vega
## 134   chevrolet chevelle malibu classic
## 135                         amc matador
```

```
## 136          plymouth satellite sebring
## 137                     ford gran torino
## 138            buick century luxus (sw)
## 139           dodge coronet custom (sw)
## 140                ford gran torino (sw)
## 141                     amc matador (sw)
## 142                             audi fox
## 143                   volkswagen dasher
## 144                           opel manta
## 145                        toyota corona
## 146                           datsun 710
## 147                           dodge colt
## 148                             fiat 128
## 149                          fiat 124 tc
## 150                          honda civic
## 151                               subaru
## 152                            fiat x1.9
## 153              plymouth valiant custom
## 154                       chevrolet nova
## 155                      mercury monarch
## 156                        ford maverick
## 157                     pontiac catalina
## 158                    chevrolet bel air
## 159                  plymouth grand fury
## 160                             ford ltd
## 161                        buick century
## 162           chevroelt chevelle malibu
## 163                          amc matador
## 164                        plymouth fury
## 165                        buick skyhawk
## 166                  chevrolet monza 2+2
## 167                      ford mustang ii
## 168                       toyota corolla
## 169                           ford pinto
## 170                          amc gremlin
## 171                        pontiac astro
## 172                        toyota corona
## 173                    volkswagen dasher
## 174                           datsun 710
## 175                           ford pinto
## 176                    volkswagen rabbit
## 177                            amc pacer
## 178                           audi 100ls
## 179                          peugeot 504
## 180                          volvo 244dl
## 181                            saab 99le
## 182                     honda civic cvcc
## 183                             fiat 131
## 184                            opel 1900
## 185                             capri ii
## 186                           dodge colt
## 187                         renault 12tl
## 188   chevrolet chevelle malibu classic
## 189              dodge coronet brougham
```

```
## 190                          amc matador
## 191                     ford gran torino
## 192                     plymouth valiant
## 193                       chevrolet nova
## 194                        ford maverick
## 195                           amc hornet
## 196                   chevrolet chevette
## 197                      chevrolet woody
## 198                            vw rabbit
## 199                          honda civic
## 200                       dodge aspen se
## 201                    ford granada ghia
## 202                   pontiac ventura sj
## 203                        amc pacer d/l
## 204                    volkswagen rabbit
## 205                         datsun b-210
## 206                       toyota corolla
## 207                           ford pinto
## 208                            volvo 245
## 209            plymouth volare premier v8
## 210                          peugeot 504
## 211                       toyota mark ii
## 212                   mercedes-benz 280s
## 213                     cadillac seville
## 214                            chevy c10
## 215                            ford f108
## 216                           dodge d100
## 217                    honda accord cvcc
## 218               buick opel isuzu deluxe
## 219                       renault 5 gtl
## 220                    plymouth arrow gs
## 221                datsun f-10 hatchback
## 222             chevrolet caprice classic
## 223            oldsmobile cutlass supreme
## 224                dodge monaco brougham
## 225               mercury cougar brougham
## 226                   chevrolet concours
## 227                        buick skylark
## 228                plymouth volare custom
## 229                         ford granada
## 230                 pontiac grand prix lj
## 231          chevrolet monte carlo landau
## 232                     chrysler cordoba
## 233                     ford thunderbird
## 234              volkswagen rabbit custom
## 235                 pontiac sunbird coupe
## 236               toyota corolla liftback
## 237                  ford mustang ii 2+2
## 238                   chevrolet chevette
## 239                       dodge colt m/m
## 240                            subaru dl
## 241                    volkswagen dasher
## 242                           datsun 810
## 243                             bmw 320i
```

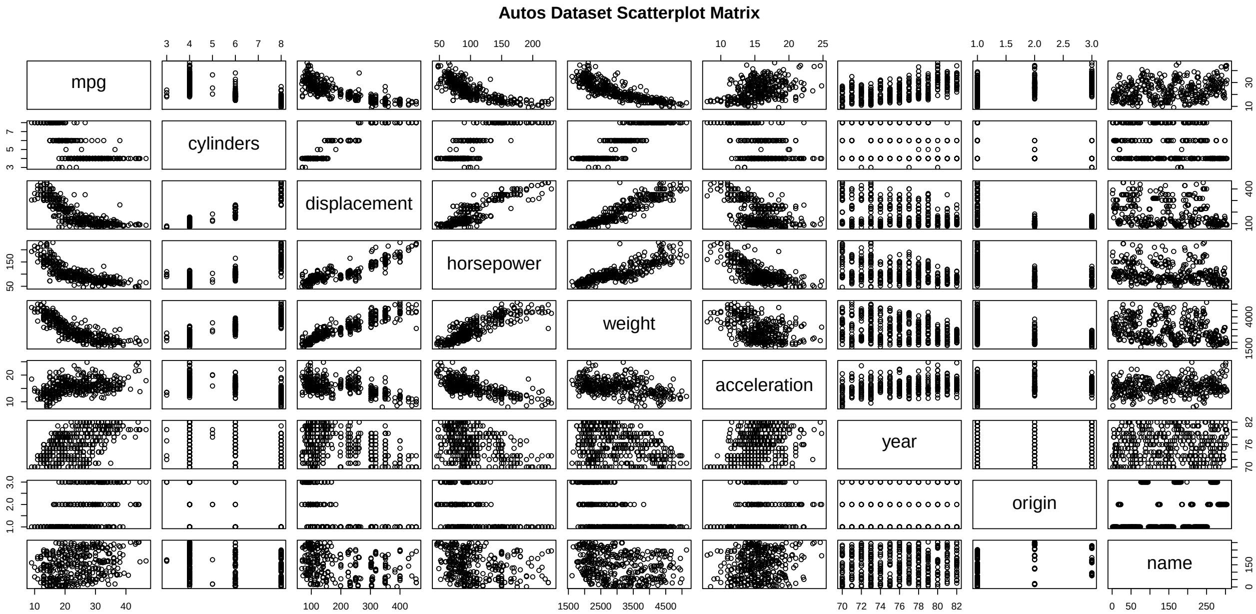
```
## 244                            mazda rx-4
## 245      volkswagen rabbit custom diesel
## 246                           ford fiesta
## 247                      mazda glc deluxe
## 248                        datsun b210 gx
## 249                      honda civic cvcc
## 250    oldsmobile cutlass salon brougham
## 251                        dodge diplomat
## 252                  mercury monarch ghia
## 253                    pontiac phoenix lj
## 254                     chevrolet malibu
## 255                  ford fairmont (auto)
## 256                   ford fairmont (man)
## 257                      plymouth volare
## 258                          amc concord
## 259                buick century special
## 260                       mercury zephyr
## 261                          dodge aspen
## 262                      amc concord d/l
## 263         chevrolet monte carlo landau
## 264      buick regal sport coupe (turbo)
## 265                          ford futura
## 266                      dodge magnum xe
## 267                   chevrolet chevette
## 268                        toyota corona
## 269                           datsun 510
## 270                           dodge omni
## 271            toyota celica gt liftback
## 272                     plymouth sapporo
## 273               oldsmobile starfire sx
## 274                        datsun 200-sx
## 275                            audi 5000
## 276                          volvo 264gl
## 277                           saab 99gle
## 278                        peugeot 604sl
## 279                  volkswagen scirocco
## 280                      honda accord lx
## 281                    pontiac lemans v6
## 282                     mercury zephyr 6
## 283                      ford fairmont 4
## 284                     amc concord dl 6
## 285                        dodge aspen 6
## 286            chevrolet caprice classic
## 287                      ford ltd landau
## 288                mercury grand marquis
## 289                      dodge st. regis
## 290              buick estate wagon (sw)
## 291             ford country squire (sw)
## 292        chevrolet malibu classic (sw)
## 293 chrysler lebaron town @ country (sw)
## 294                     vw rabbit custom
## 295                     maxda glc deluxe
## 296          dodge colt hatchback custom
## 297                        amc spirit dl
```

```
## 298                 mercedes benz 300d
## 299                  cadillac eldorado
## 300                        peugeot 504
## 301    oldsmobile cutlass salon brougham
## 302                   plymouth horizon
## 303               plymouth horizon tc3
## 304                         datsun 210
## 305                 fiat strada custom
## 306              buick skylark limited
## 307                 chevrolet citation
## 308           oldsmobile omega brougham
## 309                    pontiac phoenix
## 310                          vw rabbit
## 311              toyota corolla tercel
## 312                 chevrolet chevette
## 313                         datsun 310
## 314                 chevrolet citation
## 315                      ford fairmont
## 316                        amc concord
## 317                        dodge aspen
## 318                          audi 4000
## 319             toyota corona liftback
## 320                          mazda 626
## 321               datsun 510 hatchback
## 322                     toyota corolla
## 323                          mazda glc
## 324                         dodge colt
## 325                         datsun 210
## 326               vw rabbit c (diesel)
## 327                 vw dasher (diesel)
## 328                audi 5000s (diesel)
## 329                 mercedes-benz 240d
## 330                honda civic 1500 gl
## 332                          subaru dl
## 333                   vokswagen rabbit
## 334                      datsun 280-zx
## 335                      mazda rx-7 gs
## 336                  triumph tr7 coupe
## 338                       honda accord
## 339                   plymouth reliant
## 340                      buick skylark
## 341             dodge aries wagon (sw)
## 342                 chevrolet citation
## 343                   plymouth reliant
## 344                     toyota starlet
## 345                     plymouth champ
## 346                   honda civic 1300
## 347                             subaru
## 348                     datsun 210 mpg
## 349                      toyota tercel
## 350                        mazda glc 4
## 351                 plymouth horizon 4
## 352                     ford escort 4w
## 353                     ford escort 2h
```

```
## 354                   volkswagen jetta
## 356                      honda prelude
## 357                     toyota corolla
## 358                       datsun 200sx
## 359                          mazda 626
## 360          peugeot 505s turbo diesel
## 361                       volvo diesel
## 362                    toyota cressida
## 363                  datsun 810 maxima
## 364                      buick century
## 365              oldsmobile cutlass ls
## 366                    ford granada gl
## 367             chrysler lebaron salon
## 368                 chevrolet cavalier
## 369           chevrolet cavalier wagon
## 370          chevrolet cavalier 2-door
## 371         pontiac j2000 se hatchback
## 372                     dodge aries se
## 373                    pontiac phoenix
## 374               ford fairmont futura
## 375                volkswagen rabbit l
## 376                 mazda glc custom l
## 377                   mazda glc custom
## 378             plymouth horizon miser
## 379                     mercury lynx l
## 380                   nissan stanza xe
## 381                       honda accord
## 382                     toyota corolla
## 383                        honda civic
## 384                 honda civic (auto)
## 385                      datsun 310 gx
## 386              buick century limited
## 387    oldsmobile cutlass ciera (diesel)
## 388         chrysler lebaron medallion
## 389                     ford granada l
## 390                   toyota celica gt
## 391                  dodge charger 2.2
## 392                   chevrolet camaro
## 393                    ford mustang gl
## 394                          vw pickup
## 395                      dodge rampage
## 396                        ford ranger
## 397                         chevy s-10
```

a)

```r
pairs(Auto, main="Autos Dataset Scatterplot Matrix")
```



b)

```r
cor(data.matrix(Auto[,-length(Auto)]))
```

```
##                     mpg  cylinders displacement horsepower     weight
## mpg           1.0000000 -0.7776175   -0.8051269 -0.7784268 -0.8322442
## cylinders    -0.7776175  1.0000000    0.9508233  0.8429834  0.8975273
## displacement -0.8051269  0.9508233    1.0000000  0.8972570  0.9329944
## horsepower   -0.7784268  0.8429834    0.8972570  1.0000000  0.8645377
## weight       -0.8322442  0.8975273    0.9329944  0.8645377  1.0000000
## acceleration  0.4233285 -0.5046834   -0.5438005 -0.6891955 -0.4168392
## year          0.5805410 -0.3456474   -0.3698552 -0.4163615 -0.3091199
## origin        0.5652088 -0.5689316   -0.6145351 -0.4551715 -0.5850054
##              acceleration       year     origin
## mpg             0.4233285  0.5805410  0.5652088
## cylinders      -0.5046834 -0.3456474 -0.5689316
## displacement   -0.5438005 -0.3698552 -0.6145351
## horsepower     -0.6891955 -0.4163615 -0.4551715
## weight         -0.4168392 -0.3091199 -0.5850054
## acceleration    1.0000000  0.2903161  0.2127458
## year            0.2903161  1.0000000  0.1815277
## origin          0.2127458  0.1815277  1.0000000
```

c)

```r
df <- Auto[,-length(Auto)]
# help(lm)
model <- lm(mpg ~ ., data=df)
summary(model)
```

```
##
## Call:
## lm(formula = mpg ~ ., data = df)
##
## Residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -9.5903 -2.1565 -0.1169  1.8690 13.0604
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -17.218435   4.644294  -3.707  0.00024 ***
## cylinders     -0.493376   0.323282  -1.526  0.12780
## displacement   0.019896   0.007515   2.647  0.00844 **
## horsepower    -0.016951   0.013787  -1.230  0.21963
## weight        -0.006474   0.000652  -9.929  < 2e-16 ***
## acceleration   0.080576   0.098845   0.815  0.41548
## year           0.750773   0.050973  14.729  < 2e-16 ***
## origin         1.426141   0.278136   5.127 4.67e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.328 on 384 degrees of freedom
## Multiple R-squared:  0.8215, Adjusted R-squared:  0.8182
## F-statistic: 252.4 on 7 and 384 DF,  p-value: < 2.2e-16
```

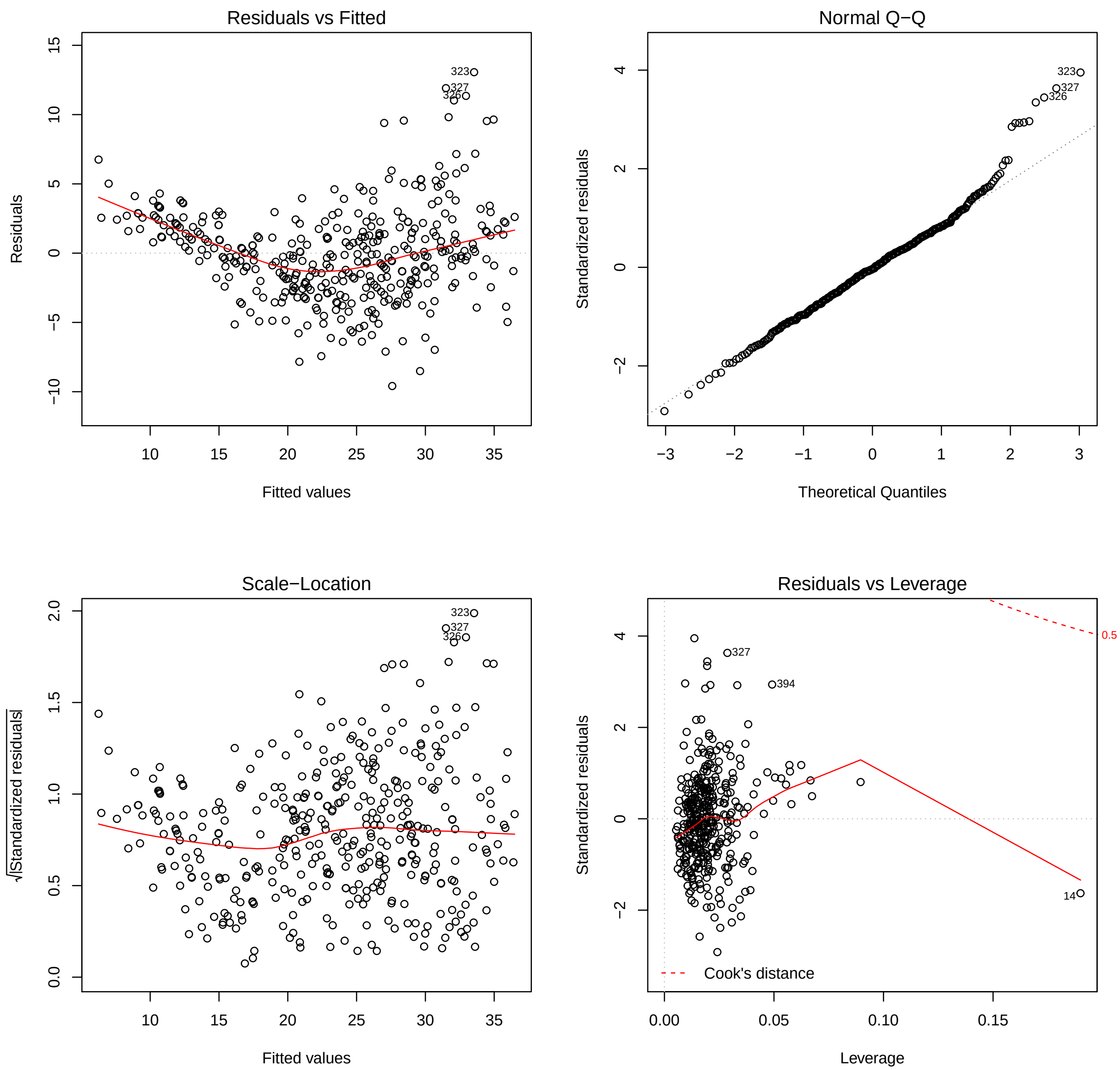
c.i) The model does predict a relationship between the predictors and response; R^2 at 0.903 is high, expressing that the model fits training data well. The F-statistic is high (considering the number of observations is much larger than our number of predictors) and p value is very low, thus the null-hypothesis can be rejected.

c.ii) Based on their low p values (highlighted by the 2-3 stars next to each p-value), displacement, weight, year and origin appear to have a statistically significant relationship to the response

c.iii) With each passing year, mpg increases by ~0.75 mpg

d)

```r
par(mfrow=c(2,2))
plot(model)
```




```r
par(mfrow=c(1,1)) # Change back to 1 x 1
```

The residuals vs fitted plot can be used to diagnose non-linear behavior in residuals. There appears to be a parabolic shape to the residuals curves, where non-linear relationship that was not explained by the model was left out in the residuals. The Normal Q-Q plot shows if residuals are normally distributed; there is some deviation in the points that have gained labels: 326, 327 and 323 The Scale-Location plot shows that the residuals are randomly spread and homoscedastic The residuals vs leverage plot shows that residual 14 has leverage, but all points are within cook's distance, meaning there aren't any particular residual that is highly influential to regression results

e)

with some car knowledge that engine performance depends on an interaction of number of cylinders and total displacement, I believed it would be interesting to see whether this can influence our model:

```r
model_interaction <- lm(mpg~cylinders*displacement, data=df)
summary(model_interaction)
```

```
##
## Call:
## lm(formula = mpg ~ cylinders * displacement, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.0432  -2.4308  -0.2263   2.2048  20.9051
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)             48.22040    2.34712  20.545  < 2e-16 ***
## cylinders               -2.41838    0.53456  -4.524 8.08e-06 ***
## displacement            -0.13436    0.01615  -8.321 1.50e-15 ***
## cylinders:displacement   0.01182    0.00207   5.711 2.24e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.454 on 388 degrees of freedom
## Multiple R-squared:  0.6769, Adjusted R-squared:  0.6744
## F-statistic:   271 on 3 and 388 DF,  p-value: < 2.2e-16
```

Cylinders, displacement as well as their interaction proved to be very statistically significant in predicting mpg, which makes sense from a mechanical standpoint; more cylinders should allow for a more stable engine and higher RPMs, coupled with greater displacement fuel use can increase very quickly

Next I wondered whether there were strong interactions between year and origin, i.e. whether some countries of origin made great improvements overtime or vice veresa

```r
model_interaction <- lm(mpg~year*origin, data=df)
summary(model_interaction)
```

```
##
## Call:
## lm(formula = mpg ~ year * origin, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.3141  -3.7120  -0.6513   3.3621  15.5859
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -83.3809    12.0000  -6.948 1.57e-11 ***
## year          1.3089     0.1576   8.305 1.68e-15 ***
## origin       17.3752     6.8325   2.543   0.0114 *
## year:origin  -0.1663     0.0889  -1.871   0.0621 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.199 on 388 degrees of freedom
## Multiple R-squared:  0.5596, Adjusted R-squared:  0.5562
## F-statistic: 164.4 on 3 and 388 DF,  p-value: < 2.2e-16
```

However, here the significance is rather weak

Finally, knowing that acceleration can have a very high penalty on fuel usage, and that horsepower is a function of RPM:

```
model_interaction <- lm(mpg~acceleration*horsepower, data=df)
summary(model_interaction)
```

```
##
## Call:
## lm(formula = mpg ~ acceleration * horsepower, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.3442  -2.7324  -0.4049   2.4210  15.8840
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             33.512440   3.420187   9.798  < 2e-16 ***
## acceleration             0.800296   0.211899   3.777 0.000184 ***
## horsepower               0.017590   0.027425   0.641 0.521664
## acceleration:horsepower -0.015698   0.002003  -7.838 4.45e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.426 on 388 degrees of freedom
## Multiple R-squared:  0.6809, Adjusted R-squared:  0.6784
## F-statistic: 275.9 on 3 and 388 DF,  p-value: < 2.2e-16
```

It turns out that a fast moving, high revving powerhouse consumes a ton of fuel: the interaction is statistically significacnt.

f)

```
model_transform <- lm(log(mpg)~cylinders+exp(displacement)+log(horsepower)+log(weight)+log(acceleration)
summary(model_transform)
```

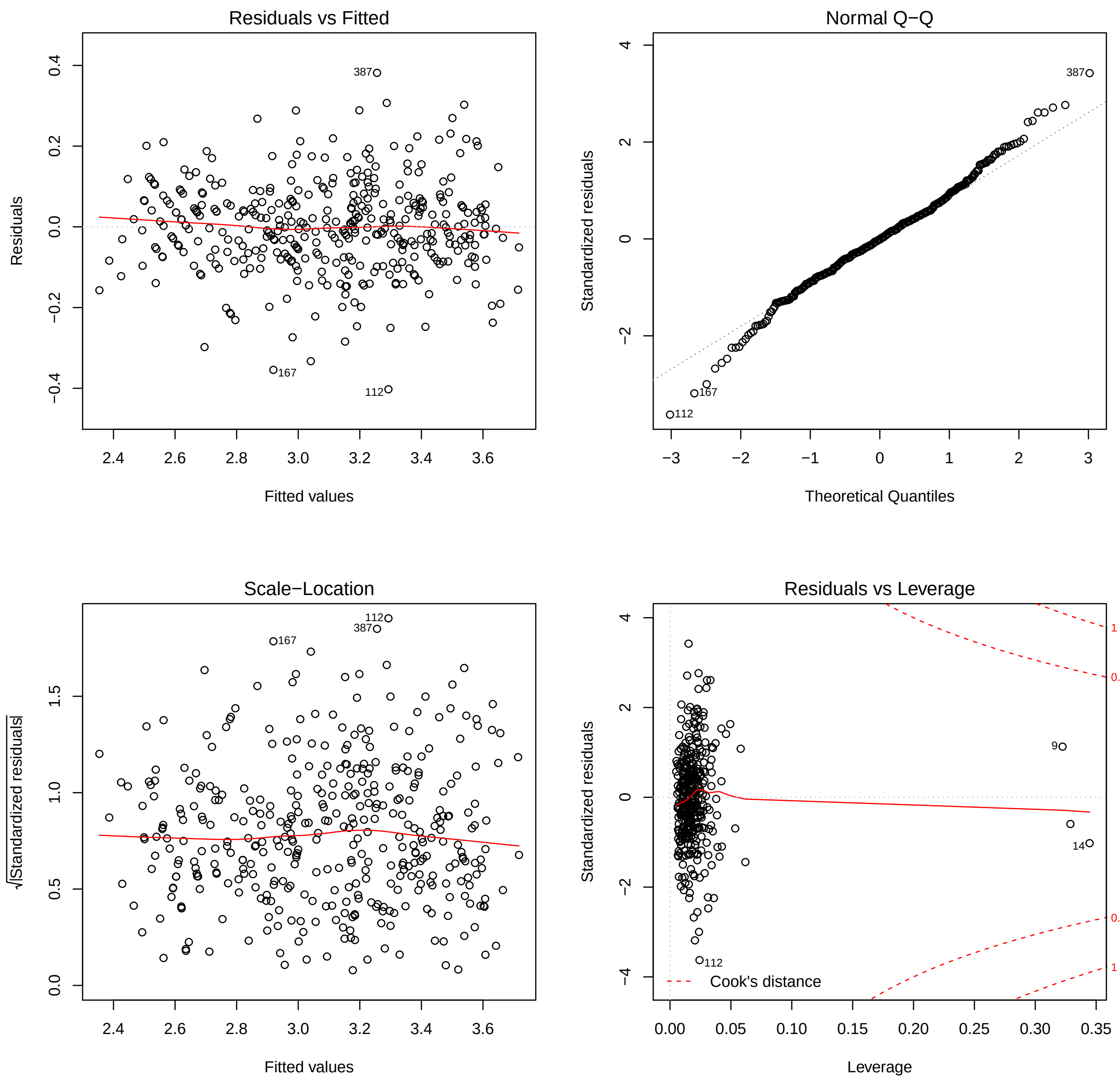
```
##
## Call:
## lm(formula = log(mpg) ~ cylinders + exp(displacement) + log(horsepower) +
##     log(weight) + log(acceleration) + I(year^2) + origin, data = Auto)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.40259 -0.07022 -0.00125  0.06176  0.38161
##
## Coefficients:
##                     Estimate  Std. Error t value Pr(>|t|)
## (Intercept)        8.278e+00   3.461e-01  23.921  < 2e-16 ***
## cylinders         -1.515e-02   7.799e-03  -1.942  0.05284 .
## exp(displacement)  1.922e-199  0.000e+00     Inf  < 2e-16 ***
## log(horsepower)   -3.069e-01   5.805e-02  -5.287 2.09e-07 ***
## log(weight)       -5.475e-01   6.743e-02  -8.119 6.46e-15 ***
## log(acceleration) -1.846e-01   5.754e-02  -3.208  0.00145 **
## I(year^2)          1.959e-04   1.127e-05  17.385  < 2e-16 ***
## origin             2.401e-02   9.074e-03   2.646  0.00847 **
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1124 on 384 degrees of freedom
## Multiple R-squared:  0.8927, Adjusted R-squared:  0.8908
## F-statistic: 456.5 on 7 and 384 DF,  p-value: < 2.2e-16
```

Using a log transform of mpg proved a much better fit on its own; making additional transforms on predictor vectors, especially exp() on displacement and log() on horsepower, weight and acceleration further improved adjusted R^2 and reduced RSE

And now looking at the diagnostic plots:

```r
par(mfrow=c(2,2))
plot(model_transform)
```




```r
par(mfrow=c(1,1)) # Change back to 1 x 1
```

We see that with the aforementioned transforms on our response and predictor variables, the non-linear patterns on residuals greatly decreased (on the residuals vs fitted plot). Deviation of residuals from the straight line on Normal Q-Q plot also decreased, signifying more normally distributed residuals

Problem 10

Exercise 14 (p. 125)

a)

```r
set.seed(1)
x1 <- runif(100)
x2 <- 0.5 * x1 + rnorm(100)/10
y = 2 + 2*x1 + 0.3*x2 + rnorm(100)
```

Form of the model and regression coefficients

$$Y = 2 + 2X_1 + 0.3X_2 + \varepsilon$$

$$B_0 = 2$$

$$B_1 = 2$$

$$B_3 = 0.3$$

Figure 3: problem 10a

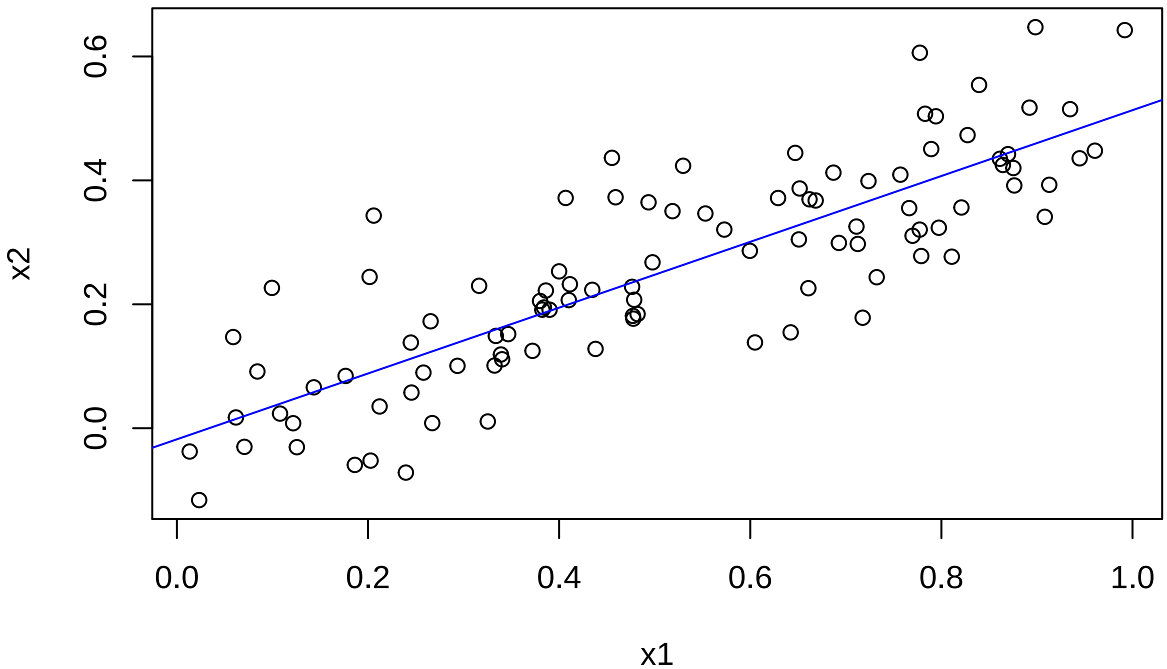
Per 3.36 in the book, the form has a quadratic shape but model is still linear

b)

```r
print(cor(x1, x2))
```

```
## [1] 0.8351212
```

```r
model <- lm(x2~x1)
plot(x1, x2)
abline(model, col='blue')
```



c)

```
model <- lm(y~x1+x2)
coef(model)
```

```
## (Intercept)          x1          x2
##    2.130500    1.439555    1.009674
```

```
summary(model)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8311 -0.7273 -0.0537  0.6338  2.3359
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1305     0.2319   9.188 7.61e-15 ***
## x1            1.4396     0.7212   1.996   0.0487 *
## x2            1.0097     1.1337   0.891   0.3754
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.056 on 97 degrees of freedom
## Multiple R-squared:  0.2088, Adjusted R-squared:  0.1925
## F-statistic:  12.8 on 2 and 97 DF,  p-value: 1.164e-05
```

Only B0 is close to the true coefficients. B1 and B2 are nontriviallly divergent from true coefficient values, with large std. error. We can marginally reject the null hypothesis for B1 as its p-value is below 5%. However, the p-value for B2 is quite high, so we cannot reject the null hypothesis.

d)

```
model_x1only <- lm(y~x1)
summary(model_x1only)
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.89495 -0.66874 -0.07785  0.59221  2.45560
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1124     0.2307   9.155 8.27e-15 ***
## x1            1.9759     0.3963   4.986 2.66e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.055 on 98 degrees of freedom
## Multiple R-squared:  0.2024, Adjusted R-squared:  0.1942
## F-statistic: 24.86 on 1 and 98 DF,  p-value: 2.661e-06
```

Std error is relatively low here as a ratio of the coefficient estimate compared to the multiple linear regression case in 10.c. P-value is very low, thus we can reject the null-hypothesis of B1=0.

e)

```
model_x2only <- lm(y~x2)
summary(model_x2only)
```

```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.62687 -0.75156 -0.03598  0.72383  2.44890
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3899     0.1949   12.26  < 2e-16 ***
## x2            2.8996     0.6330    4.58 1.37e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.072 on 98 degrees of freedom
## Multiple R-squared:  0.1763, Adjusted R-squared:  0.1679
## F-statistic: 20.98 on 1 and 98 DF,  p-value: 1.366e-05
```

Again, std error is relatively low here as a ratio of the coefficient estimate compared to the multiple linear regression case in 10.c. P-value is very low, thus we can reject the null-hypothesis of B2=0.

f)

The results do not contradict each other. From 10.b, we can see that x1 and x2 have significant collinearity and increase together. In a regression problem, this can cause problems due to the difficulty of separating out the individual effects of collinear predictors on the response.

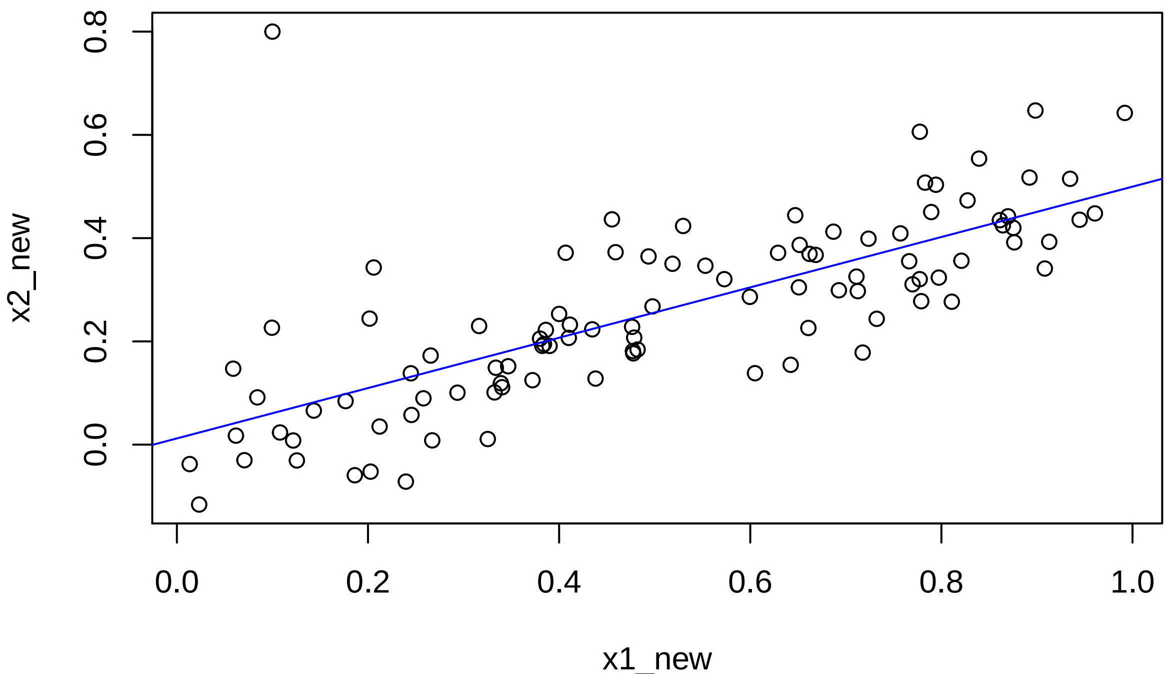
g)

```r
x1_new <- c(x1, 0.1)
x2_new <- c(x2, 0.8)
y_new <- c(y,6)

print(cor(x1_new, x2_new))
```

```
## [1] 0.7392279
```

```r
model = lm(x2_new~x1_new)
plot(x1_new, x2_new)
abline(model, col='blue')
```




```r
model <- lm(y_new~x1_new+x2_new)
summary(model)
```

```
##
## Call:
## lm(formula = y_new ~ x1_new + x2_new)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.73348 -0.69318 -0.05263  0.66385  2.30619
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)   2.2267     0.2314   9.624 7.91e-16 ***
## x1_new        0.5394     0.5922   0.911  0.36458
## x2_new        2.5146     0.8977   2.801  0.00614 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.075 on 98 degrees of freedom
## Multiple R-squared:  0.2188, Adjusted R-squared:  0.2029
## F-statistic: 13.72 on 2 and 98 DF,  p-value: 5.564e-06
```

```r
model_x1only <- lm(y_new~x1_new)
summary(model_x1only)
```

```
##
## Call:
## lm(formula = y_new ~ x1_new)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8897 -0.6556 -0.0909  0.5682  3.5665
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2569     0.2390   9.445 1.78e-15 ***
## x1_new        1.7657     0.4124   4.282 4.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.111 on 99 degrees of freedom
## Multiple R-squared:  0.1562, Adjusted R-squared:  0.1477
## F-statistic: 18.33 on 1 and 99 DF,  p-value: 4.295e-05
```

```r
model_x2only <- lm(y_new~x2_new)
summary(model_x2only)
```

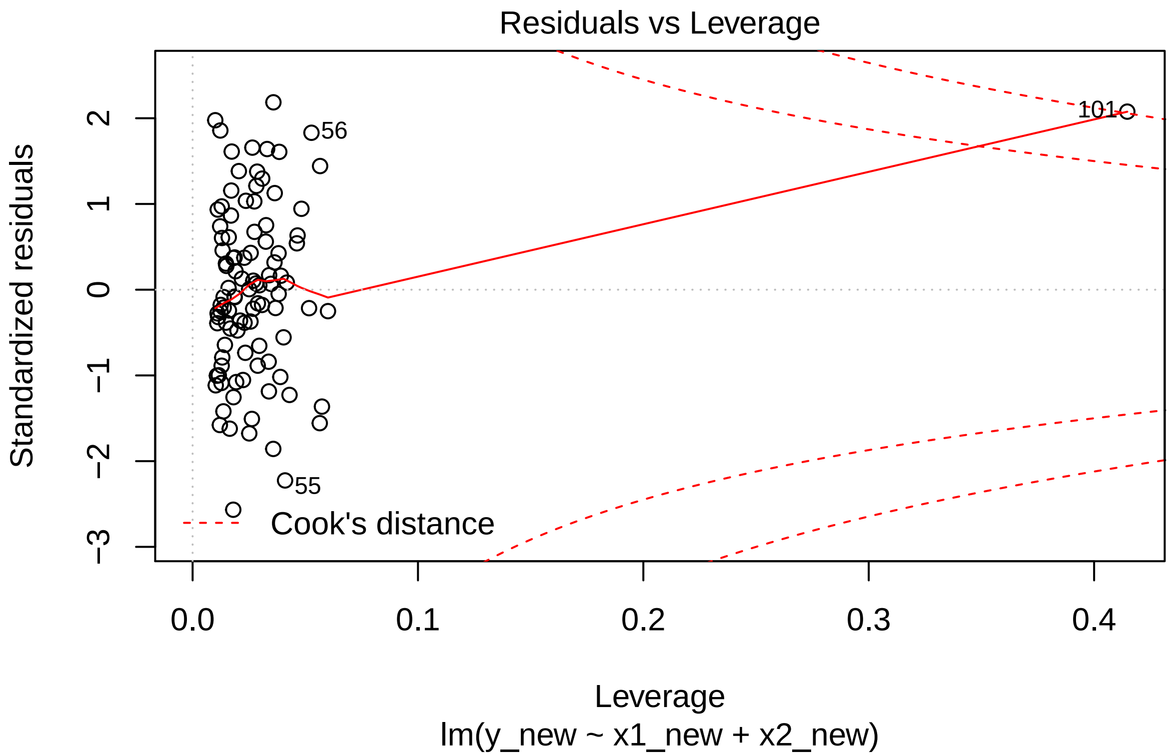
```
##
## Call:
## lm(formula = y_new ~ x2_new)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.64729 -0.71021 -0.06899  0.72699  2.38074
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3451     0.1912  12.264  < 2e-16 ***
## x2_new        3.1190     0.6040   5.164 1.25e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.074 on 99 degrees of freedom
## Multiple R-squared:  0.2122, Adjusted R-squared:  0.2042
## F-statistic: 26.66 on 1 and 99 DF,  p-value: 1.253e-06
```

By adding this point, the amount of response variability explained by the multiple linear regression model improves: adjusted R^2 went from 0.21 to 0.27. Here x1_new is no longer a statistically significant predictor.

The p-value for x2_new shifts down and we can reject the null hypothesis that B2=0.

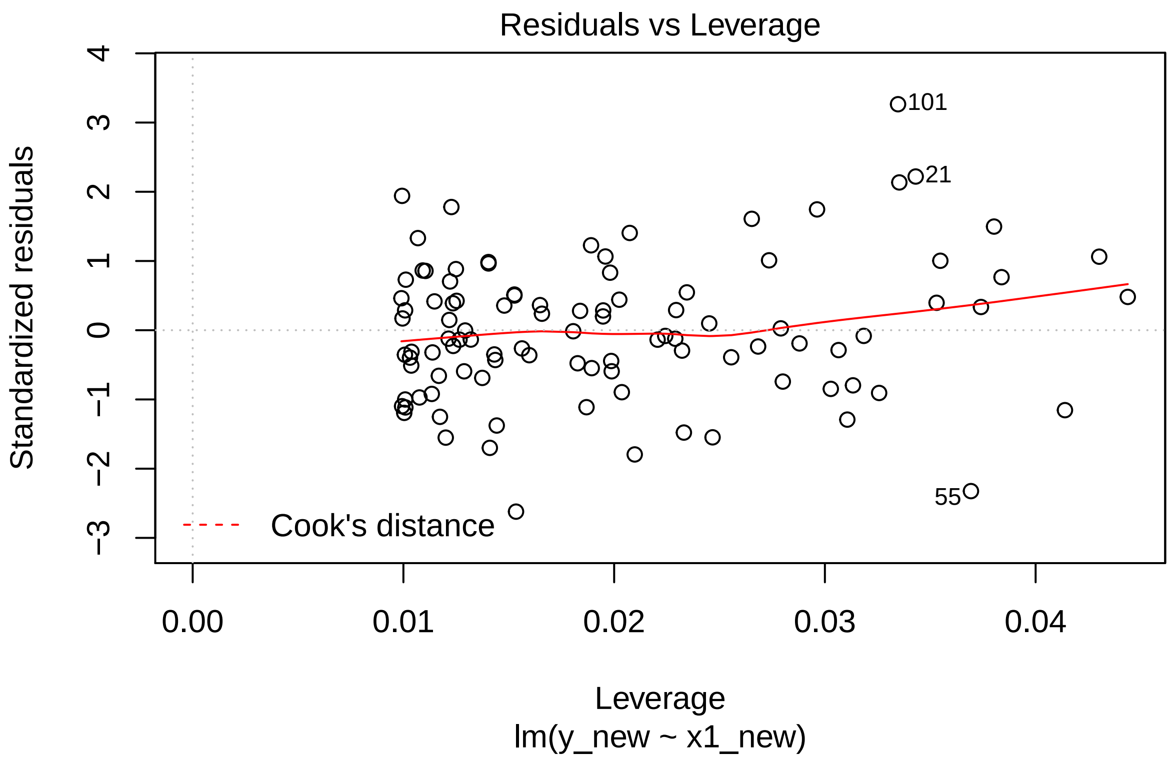
In the both cases of simple linear regression using only either x1_new or x2_new as predictor, the null hypothesis can be rejected based on p-value.

```
plot(model, which=5)
```



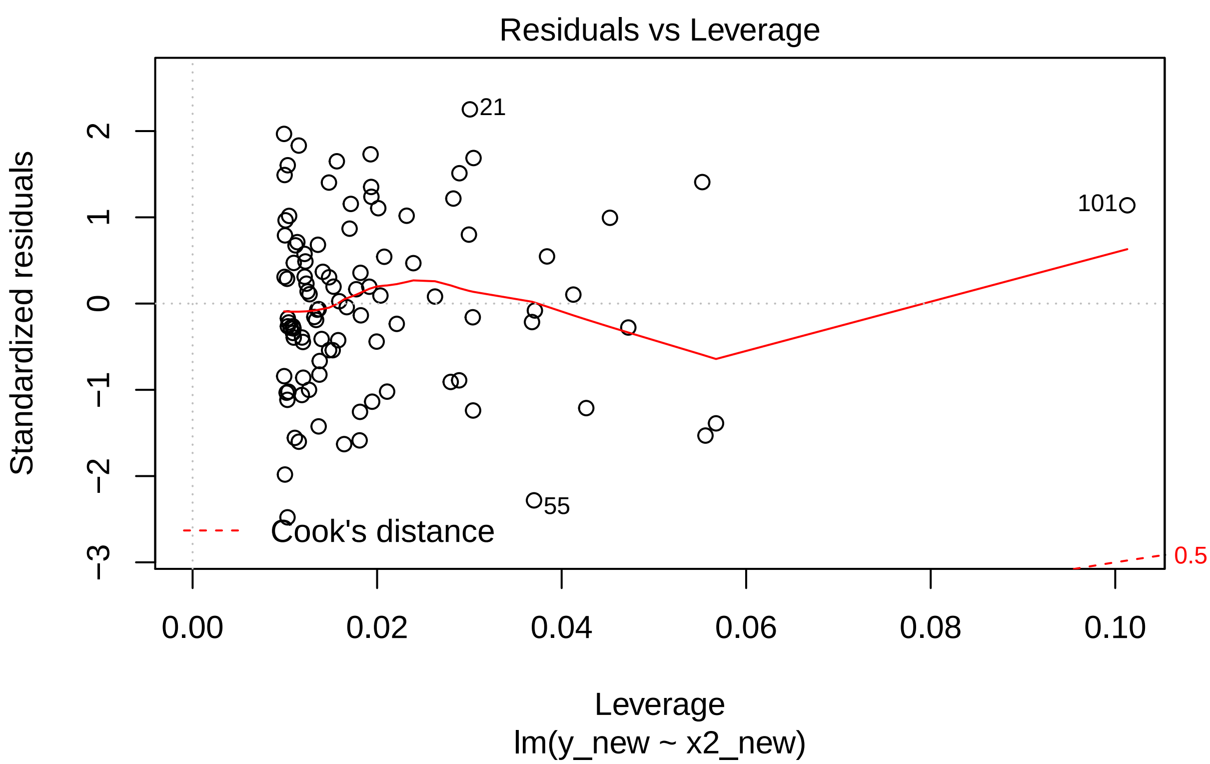
The new point, labeled 101, becomes a high leverage point in the first model using multiple linear regression with both x1 and x2 as predictors.

```
plot(model_x1only, which=5)
```



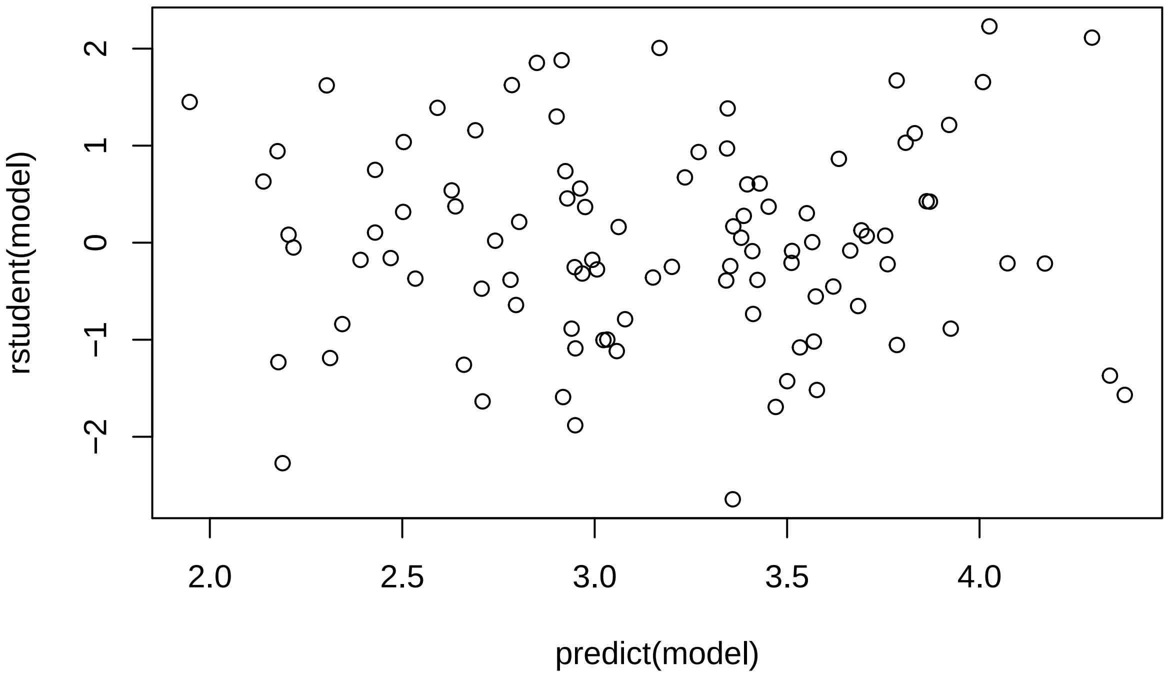
No high leverage points found

```r
plot(model_x2only, which=5)
```



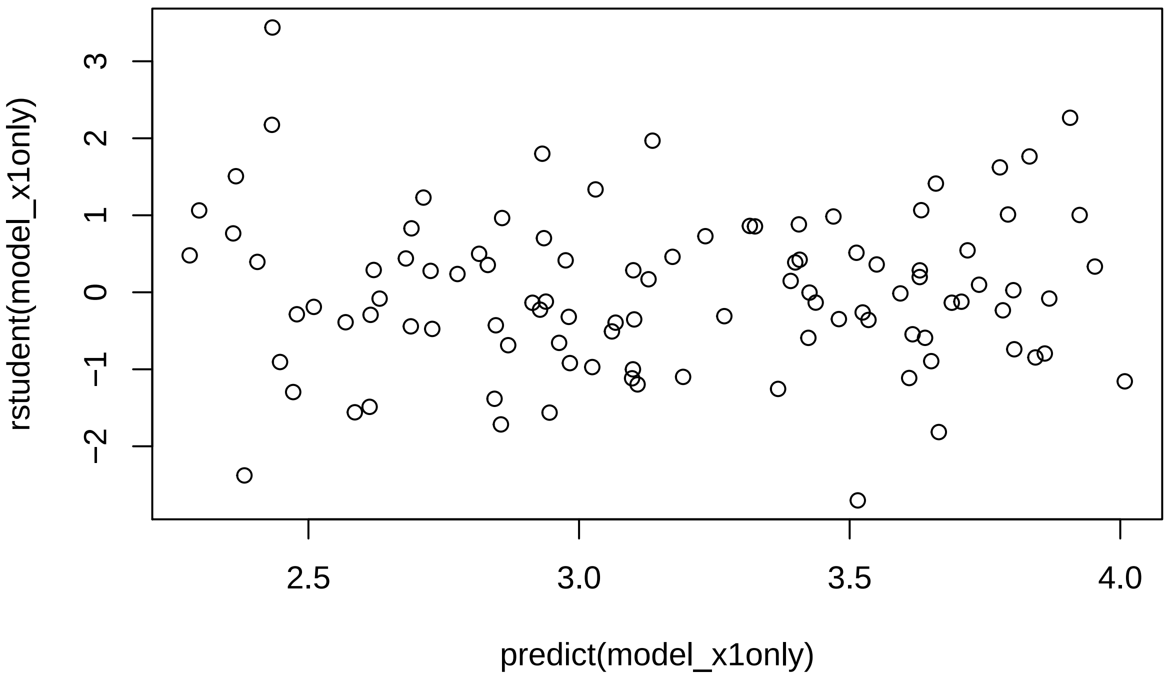
Point 101 is also relatively high leverage in the x2 only model, but is still within Cook's distance.

```r
plot(predict(model), rstudent(model))
```



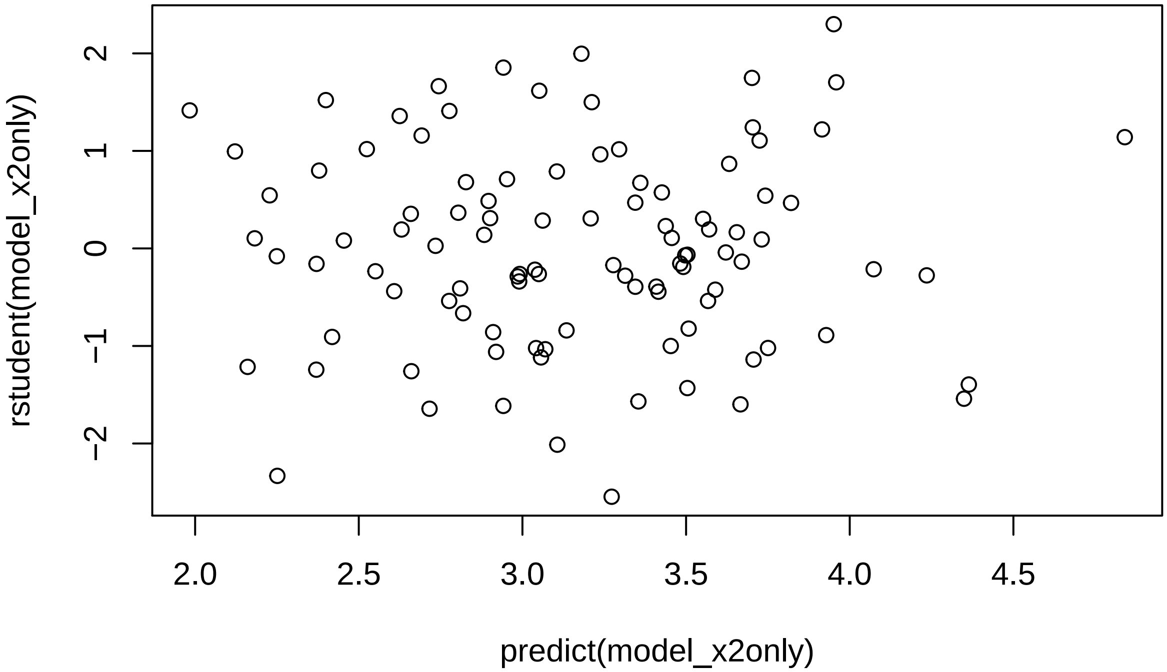
No possible outliers in multiple linear regression model using both x1 and x2

```r
plot(predict(model_x1only), rstudent(model_x1only))
```



Possible outlier lying outside of +3 studentised residual in simple linear regression model using x1

```r
plot(predict(model_x2only), rstudent(model_x2only))
```



No outliers detected for simple linear regression model using x2