Computer Project 4: Image Restoration using 2-D Wiener Filtering

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1. **Introduction**

1.1. Purpose of the computer project

The goal of this computer assignment is to study the performance of the frequency domain 2-D Wiener filter for de-blurring and noise removal applications.

1.2. What will be accomplished or carried out?

De-Blurring: In this part, to investigate the use of Wiener filter for de-blurring, I will first generate a blurred version of the image by introducing horizontal motion blur (discrete version) with a small region of support (truncated). I will then apply a 2-D frequency domain Wiener filter to this blurred image and comment on the performance of this filter for removing motion blur.

Noise Removal: In this part, to investigate the noise removal ability of the Wiener filter, I will apply this frequency domain filter to the noisy "Lena" image that you generated in Computer Assignment 3. Comment on the performance of the filtered image of the spatial average filters and the wiener filter in terms of SNR and visual appearance. I will develop an algorithm to approximately estimate the variance from the observed noisy image.

Image Restoration: In this part, I will add white Gaussian noise to the blurred Lena image in Part 1 to generate SNR=5 dB and then apply the 2-D Wiener filter in the frequency domain. Then I will comment on the simultaneous noise removal and de-blurring (i.e. restoration) abilities of the filter.

2. **Theory**

2.1. 2-D Wiener Filter

2-D extension is straightforward leading to 2-D Wiener transfer function.

$$G(k,l) = \frac{H^*(k,l)S_{xx}(k,l)}{H^*(k,l)^2 S_{xx}(k,l) + S_{\eta\eta}(k,l)}$$

Where $H(k,l) = 2 - D\ DFT\{h(m,n)\}$, and $S_{xx}(k,l)$ and $S_{\eta\eta}(k,l)$ are power spectra of the original image $x(m,n)$ and additive noise, $\eta(m,n)$, respectively. The process is depicted in Fig.1.

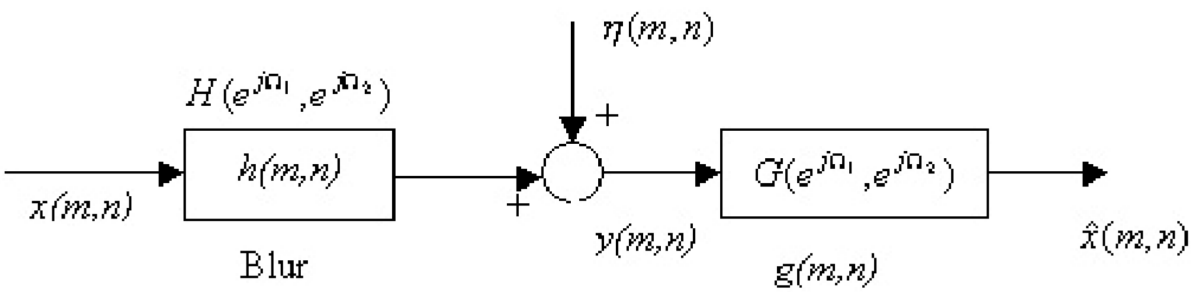

Figure 1: Image formation system and 2-D Wiener filtering.

2.2. De-Blurring

Since this part is all about testing the de-blurring property of the wiener filter, there's no noise and $\eta(m,n)$ in Figure 1 is equal to zero. The model function for de-blurring can be written as:

$$g(m,n) = x(m,n) * h(m,n)$$

where * is 2-D convolution, $h(m,n)$ is the point-spread function (PSF), $x(m,n)$ is the original image.

To generate a blurred version of the Lena image by introducing horizontal motion blur with a small region of support, we can use a function called $fspecial$ to generate a PSF and use a chebesvy low pass filter with 10th order to truncate the signal.

Even if there's no noise in this part, the term $S_{\eta\eta}(k,l)/S_{xx}(k,l)$ in the equation below should be a nonzero small constant because of quantization.

$$G(k,l) = \frac{H^*(k,l)}{H^*(k,l)^2 + S_{\eta\eta}(k,l)/S_{xx}(k,l)}$$

2.3. Noise Removal

Since this part is all about testing the noise removal property of the wiener filter, there's no blur. Now we can use the following version of the wiener filter transformation $h(m,n) = \delta(m,n) \rightarrow H(k,l) = 1$, then

$$G(k,l) = \frac{S_{xx}(k,l)}{S_{xx}(k,l) + S_{\eta\eta}(k,l)} = \frac{S_{xx}(k,l)}{S_{xx}(k,l) + \sigma_{noise}^2}$$

Where $S_{xx}(k,l)$ is the power spectrum of the image and $S_{\eta\eta}(k,l)$ is the power spectrum of the additive noise.

To estimate the noise variance from the observed noisy image, I will estimate the local mean and variance around each pixel using $wiener2$ function in matlab.

$$\mu = \frac{1}{NM} \sum_{n_1,n_2 \in \eta} a(n_1,n_2)$$

And

$$\sigma^2 = \frac{1}{NM} \sum_{n_1,n_2 \in \eta} a^2(n_1,n_2) - \mu^2$$

Where η is the N-by-M local neighborhood of each pixel in the image A. wiener2 then creates a pixelwise Wiener filter using these estimates,

$$b(n_1,n_2) = \mu + \frac{\sigma^2 - v^2}{\sigma^2}(a(n_1,n_2) - \mu)$$

where v^2 is the noise variance. If the noise variance is not given, wiener2 uses the average of all the local estimated variances.

2.4. Image restoration

Image restoration is a combination process of de-blurring and noise removal mentioned above.

3. Results and Discussions

3.1. De-Blurring

Figs. 2(a)-(c) show original, blurred (horizontal motion blur size 15), and finally the Wiener filtered Lena. As you can notice between Fig 2(a) and (b), the circular convolution caused the face to be ambiguous and blur into the environment. Comparing Fig. 2(c) with Fig. 2(b), the wiener filter shows a pretty good restoration visually for most parts of the blurred image except that you can still notice some edge smearing artifacts in the restoration image because the wiener filter assumes WSS for image which is not true.

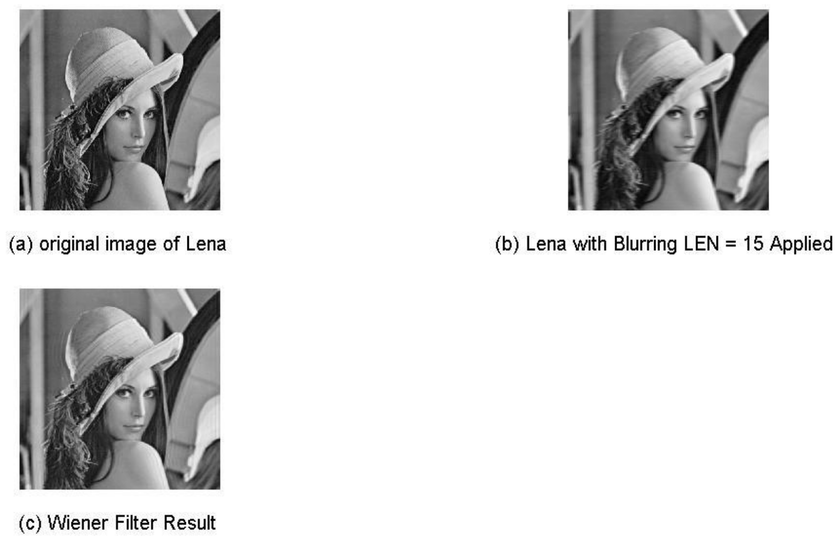

Figure 2: original Lena and Blurred Lena.

3.2. Noise Removal

Fig. 3(a)-(b) show the original and noisy version of Lena image that will be processed

Figure 3: original Lena and noisy Lena

Fig 4(a)-(d) show the spatial average filtered images with different sizes of mask and the filtered image with wiener filter. Note that there is a trade-off between noise removal ability and edge smearing artifacts among spatial average filters shown in Fig. (a)-(c). The restoration image in Fig. (d) filtered by wiener filter shows a better quality visually and also a better SNR=21.1125dB comparing with the ones by spatial average filters.

The variance of noise is $v_{noise} = 0.0149$ and the estimated variance of noise is $v_{noise\_estimated} = 0.0173$. Therefore, the estimated result shows that the relative error is equal to $\delta = \left|\frac{v_{noise\_estimated} - v_{noise}}{v_{noise}}\right| \times 100\% = 16.1\% < 20\%$. The estimated variance of noise is acceptable.

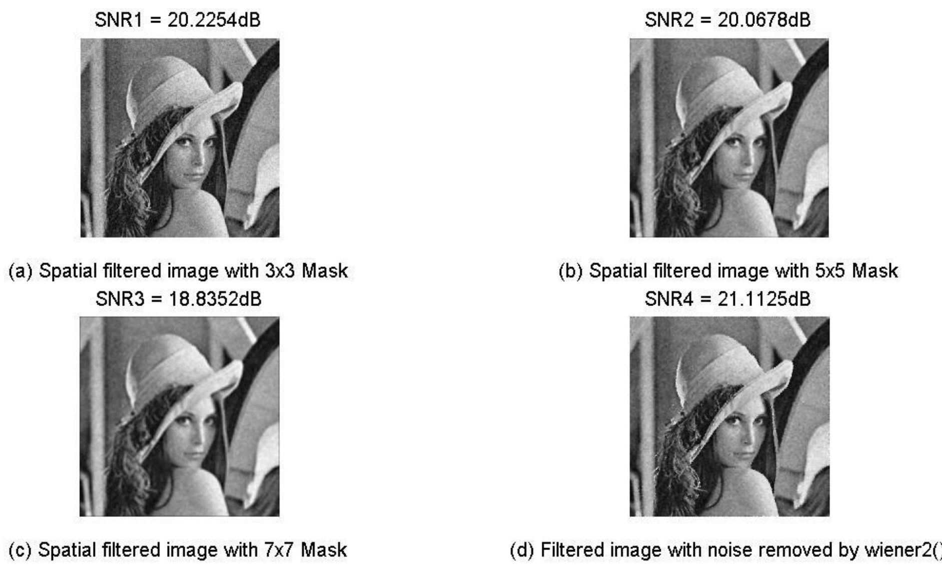

Figure 4: spatial filtered images and noise removal image using wiener2

3.3. Image Restoration

Figs. 5(a)-(d) show original, blurred (horizontal motion blur size 15), blurred and noisy (SNR=5dB) and finally the Wiener filtered Lena. Note that most of the blurring and noise shown in Fig. 5(b)&(c) are removed in the Wiener filter result shown in Fig 5(d). This indicates that the wiener filter can improve the quality of image by de-blurring and removing noise.

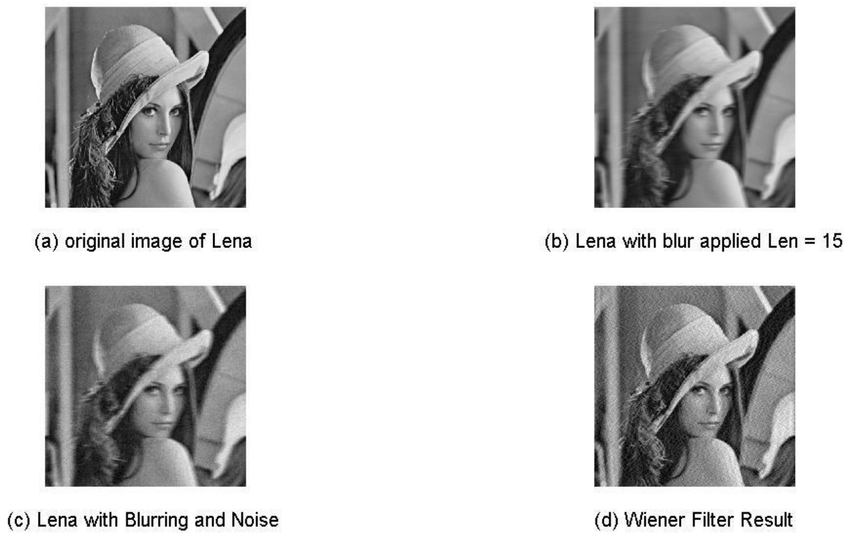

Figure 5: Blurred and Noisy Baboon and Wiener Filter Results.

4. Conclusion

I learn how to apply wiener filter on image restoration from this project. I also know lots of important properties about wiener filter like de-blurring and noise remove. The wiener filter does not have either the singularity problem or ill-conditioning problems of the inverse filter. However, the main disadvantage of wiener filter is the assumption of *wide sense stationary* (WSS). This is not true for image. As a consequence, the filtered images exhibit some edge smearing artifacts.

5. Reference

- Lecture 23-24. ECE513, Spring 2017.
- Steve Eddins, Image deblurring – Introduction, http://blogs.mathworks.com/steve/2007/08/13/image-deblurring-introduction/
- Design and use Kalman filters in MATLAB, https://www.mathworks.com/discovery/kalman-filter.html