Assignment-6

1) Write the step-by-step process used in logistic regression model.

→ Logistic Regression is a statistical method used for binary classification, where the dependent variable is categorical and typically has two outcomes, i.e 0 or 1, true or False, below is the step by step process used in logistic regression model.

1) Logistic Regression Assumptions:

- Binary target: The dependent variable should be binary
- Independence of observations: observations should be independent of each other.
- Initialize 0 to each coefficient i.e two input variable & Output variables.

2) Logistic Function (Sigmoid Function)

$$f = \frac{1}{1 + e^{-y}} \quad \text{where} \quad y = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_n x_n$$

This function outputs a value between 0 & 1, which represents the probability of the event occurring.

- if probability > 0.5, the predicted class is 1
- if probability < 0.5, the predicted class is 0

3) Model Estimation & Training

Set up the Hypothesis: The logistic regression hypothesis is that the log-odds of the probability of the event is a linear combination of probabilities.

$$\text{log-odds} = \log\left(\frac{p}{1-p}\right) = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_n x_n$$

where 'p' is the probability of the event, and $b_0, b_1, b_2 \dots b_n$ are coefficients to be eliminated.

4) Using co-efficient new values in calculating weight:-

$$W_{new} = W_{old} + \alpha(x-t)(1-t) + x,$$

where α = learning rate
t = threshold function

5) Linear combination of weighted sum.

Each input feature vector $x = [x_1, x_2 \dots x_n]$ compute the linear regression using the below formula.

$$z = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n$$

6) Repeat the sigmoid process

$$t = \frac{1}{1+e^{-z}}$$ to map linear combination 'z' into 0 probability between '0' & '1'

7) Repeat for multiple epochs.

* The process of combining to compute Linear combination, apply sigmoid function

Calculate the loss & update the weights is repeated over multiple iterations with each epoch the weights become more refined and the cost gradually decreases.

2) Dataset given below has two input variables (x_1 & x_2) and one output variable (y) Apply logistic regression model to make prediction after two epoch for each data point, Use 0.5 as the learn rate.

Input		Actual output
x_1	x_2	y
2.7810836	2.550537	0
8.675341	−0.2420	1

epoch - 01.

$x_1 = 2.7810836$ $x_2 = 2.550537$

$z = b_0 + b_1 x_1 + b_2 x_2$ or $z = w_0 + w_1 x_1 + w_2 x_2$

$w_0 = w_1 = w_2 = 0$

i.e $z = w_0 + w_1 x_1 + w_2 x_2$

$= 0$

$\hat{y} = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{0}} = 0.5$ $\alpha = 0.5$

$w_{(new)} = w_{(old)} + \alpha (y - \hat{y}) \hat{y} (1 - \hat{y}) x_1$

$w_0 (new) = 0 + (0.5)(0.5)(0 - 0.5)(1 - 0.5) \cdot 1$

$= -0.0625$

$w_1(new) = 0 + (0.1)(0.1)(0-0.0)*(1-0.0)*$

$= -0.1738$

$w_2(new) = 0 + 0.1 * 0.5 \quad (1-0.0) *$

$= -0.1594$

$x = w_0 + w_1 x_1 + w_2 x_2$

$= -0.0625 + (-0.1738 * 0.7810836) +$

$(-0.1594 * 2.550837003)$

$= -0.9524$

Epoch-2

$x = -0.9524$

$w_0 = -0.0620$

$w_1 = -0.1738$

$w_2 = -0.1594$

$$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-(-0.9524)}} = 0.2784$$

$\alpha = 0.5$

$w(new) = w(old) + \alpha t (x-t)(1-t) \cdot x_i$

$w_0(new) = -0.0620 + 0.5 * 0.2784 * (-0.9524 - 0.2784)$

$* (1-0.2784) * 1$

$= -0.1261$

$w_1(new) = -0.1738 + 0.1 * 0.2784 * (-0.9524 - 0.2784)$

$* (1-0.2784) * 0.7810836$

$= -0.5176$

$w_2(new) = -0.1594 + 0.5 * 0.2784 (1-0.9524-0.2784) *$

$(1-0.2784) * 2.550837003$

$= -0.8868$

$x = w_0 + w_1 x_1 + w_2 x_2$

$= -0.1861 + (-0.1196 \times 3.7410836) +$

(-0.1197×2.5603)

$= -2.8363$

$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-(-2.8363)}} = 0.055$

2) $x_1 = 8.67541865\,1$ $\quad x_2 = -0.242068$

epochs :-

$X = w_0 + w_1 x_1 + w_2 x_2$

Initially $w_0 = w_1 = w_2 = 0$

$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-0}} = 0.5 \qquad \alpha = 0.5$

$w_{(new)} = w_{(old)} + \alpha t\,(x-t)\,(1-t)\,(x_i)$

$w_{0\,(new)} = 0.0 + (0.5)(0.5)\,(0-0.5)\,(1-0.5) \times 1$

$= -0.0625$

$w_{1\,(new)} = 0 + 0.5 \times 0.5 \times (0-0.5) \times (1-0.5) \times$
8.67541

$= -0.5422$

$w_{2\,(new)} = 0 + 0.5 \times 0.5\,(0-0.5) \times (1-0.5) \times$
(-0.242068)

$= 0.0151$

$X = w_0 + w_1 x_1 + w_2 x_2$

$= -0.0625 + (-0.5422 \times 8.67541865\,1) +$
$(0.0151 \times -0.242068) = -4.7690$

epoch-2

$x = -4.7699$

$w_0 = -0.0625$

$w_1 = -0.5422$

$w_2 = 0.0151$

$$t = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-(-4.7699)}} = 0.0084$$

$$\alpha = 0.5$$

$$w_{0(new)} = -0.0625 + 0.5 * 0.0084 (-4.7699 - 0.0084)(1 - 0.0084) * 1$$

$$= -0.0629$$

$$w_{1(new)} = -0.5422 + 0.5 * 0.0084 (-4.7699 - 0.0084)(1 - 0.0084) * 5.678$$

$$= -0.7148$$

$$w_{2(new)} = (0.0151 + 0.5 * 0.0084)(-4.7699 - 0.0084)(1 - 0.0084) * 0.$$

$$= 0.0199$$

$$y = w_0 + w_1 x_1 + w_2 x_2$$

$$= -0.0629 + (-0.7148 * 5.678) + (0.0199 * 0.)$$

$$= -6.266$$

$$t = \frac{1}{1+e^{y}} = \frac{1}{1+e^{6.266}}$$

$$= 0.0021$$