A Visual Analytics System for Oil Spill Response and Recovery

Yuxin Ma
Arizona State University

Prannoy Chandra Pydi Medini
Arizona State University

Jake R. Nelson
University of Texas

Ran Wei
University of California, Riverside

Tony H. Grubesic
University of Texas

Jorge A. Sefair
Arizona State University

Ross Maciejewski
Arizona State University

Extensive research has been done on oil spill simulation techniques, spatial optimization models, and oil collection strategies. This article presents a visual analytics system which efficiently integrates the independent facets of spill modeling techniques and spatial optimization to enable inspection, exploration and decision making for offshore oil spill response.

Offshore oil exploration and production is a large and important component of the United States energy sector. The majority of offshore oil activity takes place within the Gulf of Mexico (GOM), which has continued to record high levels of oil production for many decades. Many of the coastal ecosystems and biologic communities within and around the GOM can be negatively affected by oil spills, even in small amounts. With sustained offshore oil exploration and extraction in the GOM, combined with the actual and potential threat of oil spills, it is important to continue to enhance methods for spill response and impact mitigation efforts.

Following an oil spill, response coordinators are tasked with evaluating the potential consequences of the spill and deploying response resources in a timely and efficient manner. This requires decision makers to evaluate complex spatial problems and the weighing of different response objectives, which tend to consider the trade-off between costs and benefits (e.g., economic, environmental, social). One mechanism for evaluating the outcomes of an oil spill with various mitigation efforts is the use of a decision support system (DSS). DSS are designed to support complex decision-making problems and have been used in a variety of application areas including chemical emergencies in urban environments[1], land use planning for rural areas[2], transportation infrastructure and management[3], and disaster management[4]. Where oil spill response is concerned, a DSS requires some indication of the location and size of a spill, the ability to track the oil plume through time and space, and a mechanism to evaluate the consequences of

Ma, Y., Medini, P. C. P., Nelson, J. R., Wei, R., Grubesic, T. H., Sefair, J., & Maciejewski, R. (2020). A Visual Analytics System for Oil Spill Response and Recovery. *IEEE Computer Graphics and Applications*.

the spill with respect to the costs and benefits of different response plans. Currently, several studies[5,6] have proposed fully operational DSS with automated models for oil spill response. However, those systems do not provide a quantitative evaluation of the consequences, nor do they evaluate different response options. Further, missing from these systems is a mechanism for evaluating response options in relation to an oil spill. If response options are considered, current systems provide no indication of how efficient the response can/will be relative to an optimal solution balancing cost versus benefit.

The goal of this research is to design a visual analytics system capable of providing response teams with the geospatial intelligence required to evaluate the costs and benefits of different tactical response options to control an oil spill. This includes decisions about where equipment should be deployed from, how much equipment to send, and how much oil cleanup can be done with a given budget. The solution proposed in this paper combines advanced spatial optimization and oil spill modeling into a system with three unique components. The first is the Blowout and Spill Occurrence Model (BLOSOM)[12] leveraged for its ability to simulate the fate and transport of the oil plume. The second is the Oil Spill Cleanup Operations Model (OSCOM)[10], a spatial optimization model for determining where response equipment should be dispatched from and how much equipment to dispatch to achieve a user-specified cleanup target. Finally, we incorporate outputs from BLOSOM and OSCOM into an interactive system to analyze and visualize different outcomes simultaneously, allowing responders and decisions makers to compare different response options and their performance concerning coastal impacts.

RELATED WORK: DECISION SUPPORT SYSTEMS IN MULTI-CRITERIA DECISION MAKING

Pajer et al.[7], Dimara et al.[8], and Kostuk et al.[9] have developed DSSs to deal with multi-criteria decision making problems in which a multitude of alternatives can arise while trying to optimize an objective under multiple constraints. Typically, a DSS is supported by an optimization model in order to identify and evaluate multiple alternatives for further analysis. Mixed integer programming models are among the most commonly used optimization tools in these optimization-based DSS solutions (Kostuk et al.[9]). In our system, the main goal is to minimize the total dispatch time and cost of the cleanup equipment by recommending equipment, vessel, and crews discrete decisions in space and time. As a result, we formulate a set of objectives and constraints as a mixed integer programming problem, which is the core of OSCOM[8].

Uran et al.[11] suggests that a spatial decision support system should have a user-friendly interface and simultaneously be comprehensive enough to cover a wide range of possible scenarios. In our visual analytics system, the design and development of the user interface involves a combination of multiple models, and sufficient care has been taken such that it is easily understood by the targeted user.

SIMULATION OF OIL SPILL AND CLEANUP

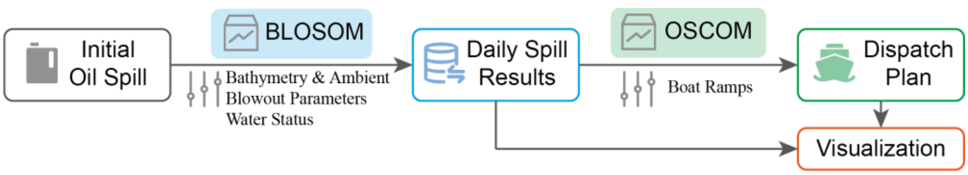

Figure 1: Overview of the data processing pipeline.

The goal of this Spatial Decision Support System is to assist decision makers in exploring the various possibilities that can arise when an oil spill occurs. Using this system, analysts can make informed decisions on the type of vessels, equipment, and response crews that should be dispatched, in what quantity, and from which boat ramps, to optimally reduce the operational costs while maintaining a high cleanup rate.

The overview of our system is presented in Figure 1. In our system, the temporal dynamics of oil spread and drift is modeled by the Blowout and Spill Occurrence Model (BLOSOM) simulation package[12]. Oceanic factors, such as the temperature, pressure, salinity, winds, and many other factors are taken into account and used for predicting the movement of the oil in the ocean. When all the required parameters are set, the simulator is executed to generate a series of shapefiles at fixed time intervals. In each shapefiles, the original crude oil is represented by parcels (points), which describe the affected area by the oil spill at the end of the corresponding time interval. Once the shapefiles are produced by the simulation, they are sent as inputs to the OSCOM model. OSCOM utilizes mathematical programming to minimize the total time and cost spent in dispatching the cleanup equipment. The OSCOM output includes specific information on the boat ramps used to dispatch vessels and the spill sites to which they have been dispatched. The oil parcels which have been cleaned up are also reported.

Data Generation with BLOSOM

BLOSOM is executed to simulate offshore spills that occur in deep and ultra-deep water blowouts, but is also capeable of simulating surface spills. This model generates simulation statistics and key attribute values upon adding a recorder at specified time intervals. The generated data files include statistical values and shapefiles that describe the area of oil spill. For the simulations to run, there are three categories of parameters that can be tuned by experts:

- Bathymetry and Ambient ocean conditions of the target sea;
- Blowout parameters, including location, duration, type of crude oil and corresponding physical properties;
- Water diffusion, dispersion, and weather.

"Bathymetry" and "Ambient" indicate the depth of the water at a given point and the ambient conditions of the ocean at that point of time, respectively. The blowout parameters, such as the duration of the blowout, the location where it happens and the type of crude oil emitted and their corresponding physical properties can also be specified. The values for diffusion, dispersion and weather conditions can also be specified in the simulation model. Once all the required parameters are set, the simulator is run and the shape and statistical files are generated to describe the status of the individual oil parcels. The generated statistics contain 41 attributes. The key attributes used in our system are listed in Table 1.

Table 1: Key attributes of parcels generated by BLOSOM.

Name	Description
Current Time	The last time that the parcel had an active status
Parcel ID	ID of the oil parcel
Status	Floating status of the parcel, including "water column", "surfaced", "beached", and "cleaned"
Longitude	Longitude of the parcel
Latitude	Latitude of the parcel

Spatial Optimization with OSCOM

Once the shapefiles are generated from the simulation, they are sent as inputs to a spatial optimization model called "Oil Spill Cleanup and Operational Model" (OSCOM)[10]. OSCOM utilizes mathematical programming to minimize the total time and cost spent in dispatching the cleanup equipment, which is abstracted as "boat ramps". The boat ramp dataset, whose attributes are listed in Table 2, describes the capability of handling spilled oil near its location.

Table 2: Attributes of boat ramps.

Name	Description
Ramp ID	Unique ID of the boat ramp
# Vessels	Number of vessels in each boat ramp
Vessel Capacity	Amount of oil the vessel can hold

Mathematically, we use I to denote the set of of staging areas (i.e., boat ramps) where vessels are located and J to denote the set of spill locations (i.e., parcels). To reflect a realistic operation, spills sites are grouped into containment areas (i.e., using containment booms), which are typically deployed to facilitate recovery operations. We use Ω_j to denote the set of spill locations enclosed by containment booms centered at location j. We assume that if a vessel is sent to location $j \in J$, then it has the capacity to recover all the oil in the containment area Ω_j within a day, after which the vessel must return to the ramp to unload the recovered oil. The oil volume at location $j \in J$ is denoted by v_j and the cost of sending a vessel from boat ramp $i \in I$ to location $j \in J$ is given by d_{ij}.We use n_i to denote the number of vessels available in boat ramp $i \in I$. The cleanup operation must recover a target value of Γ% of the oil spill. For example, a cleanup target of Γ =95% indicates that 95% of the oil ejected into the ocean must be recovered by the vessel operations reccomended by this model.

Binary decision variable x_{ij} is equal to 1 if a vessel from boat ramp $i \in I$ is sent to spill location $j \in J$ and is equal to 0 otherwise. If the spill at location $j \in J$ is cleaned up, that is, a vessel is sent to j or any other parcel whose containment area includes location j, then the binary decision variable u_j is equal to 1, and is equal to 0 otherwise. Using these elements, the OSCOM model is defined as

$$\min \sum_{i\in I} \sum_{j\in J} d_{ij} x_{ij} \tag{1}$$

$$\text{s.t. } \sum_{j\in J} x_{ij} \leq n_i, \forall i \in I \tag{2}$$

$$\sum_{i\in I} \sum_{l\in \Omega_j} x_{il} \geq u_j, \forall j \in J \tag{3}$$

$$\sum_{j\in J} v_j u_j \geq \Gamma \tag{4}$$

$$x_{ij} \in \{0,1\}, \forall i \in I, j \in J \tag{5}$$

$$u_j \in \{0,1\}, \forall j \in J. \tag{6}$$

The objective function (1) minimizes the total cost of dispatching vessels from boat ramps to spill sites. Constraints (2) denote that no more than n_i vessels can be sent out of boat ramp $i \in I$. Constraints (3) enforce that the oil in location $j \in J$ is fully recovered only if a vessel is sent to j or any other parcel whose containment area cover location j. Constraint (4) states that the total amount oil removed should achieve the pre-specified oil cleanup target. Constraints (5) and (6) define the nature of the decision variables.

The formulation in (1)—(6) admits customized containment areas (i.e., Ω-sets) to reflect the area or depth covered by each vessel according to their capacity, which is a realistic operational feature. Moreover, this formulation can be used prospectively to plan operations for more than one day at a time by calculating the remaining oil left at each location after the optimal cleanup. Specifically, we BLOSOM to estimate the location and quantity of oil at each location for the next day. These subsequent locations and quantitites can be used as the new v-parameters in a new run of the model in (1)—(6). The same procedure can be repeated for any number of days. From an algorithmic point of view, note that Constraint (6) can be replaced by $u_j \in [0,1], \forall j \in J$, as u-variables need not be binary. If an optimal solution to (1)—(6) contains a fractional solution for any u_j, then an alternative optimal solution can be constructed by rounding up the fractional value. The new solution is clearly feasible for Constraints (3) and (4), and produces the same (optimal) objective function value as the x-variables remain unchanged. This observation reduces the number of binary variables required in the model.

This spatial optimization model is embedded into the proposed visual analytics system as a model file linked to the shapefiles generated by BLOSOM and the boat ramp shapefiles provided as inputs. These model files are sent to Gurobi[13], a mixed-integer programming solver that optimizes the model files and generates a log file as well as a solution file. The log file contains the entire report of the solution process, while the solution file contains information on vessels dispatched and their assigned spill sites. The cleaned oil parcels are also listed in the solution file.

VISUAL ANALYTICS SYSTEM

Analytical Tasks

The purpose of this visual analytics system is to assist the user in exploring the various response and associated impactpossibilities which can arise when an oil spill occurs. The user can then make informed decisions about the quantity of response vessels to dispatch and where those vessels should be dispatched from in order to optimally reduce the operational costs while maintaining a target cleanup rate. The system supports:

- Visualization: Because of the complexity and large-scale output generated by the oil spill simulation (BLOSOM) and cleanup optimization (OSCOM) models, effective visualization of basic units coming from the models should be carefully designed, including oil parcels and corresponding attributes, boat ramps, cleanup status, etc.
- Visual Comparison: There are several aspects of comparison for the oil spill data. First and foremost, the system is required to provide spatiotemporal comparison where simulation steps from the oil spill and cleanup models are involved. A major requirement is to show how the data vary with time and space in both backward and forward directions. Additionally, the comparison between simulation runs with different parameter settings should be supported as well.
- Decision Making Support: By setting different initial states, spill parameters, and cleanup capabilities, the user needs to search for optimized solutions in various oil spill conditions with given cleanup constraints. Thus, the real-time response of parameter adjustment and effective presentation of oil impact and cleaning progress is essential to user's analysis.

Visualization Design and Interactions

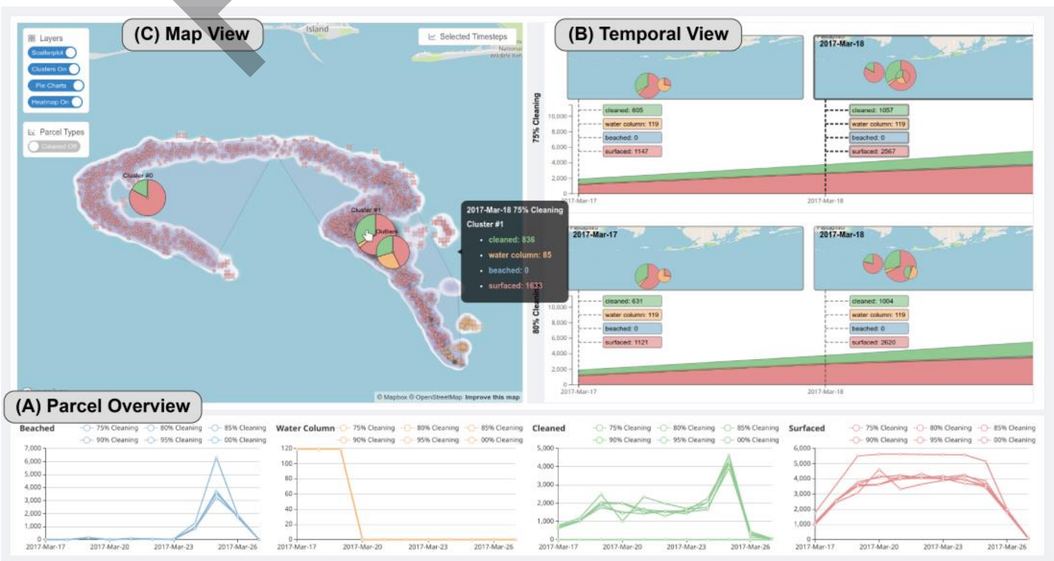

Figure 2: The interface of our system consists of three main views: (A) the map view, (B) the temporal view, and (C) the parcel overview. The parcel types are mapped to four color hues, namely, blue for Beached, yellow for Water Column, green for Cleaned, and red for Surfaced.

The visual analytics system is a combination of the user interactions with the visual interface which draws its data from the optimization module. As shown in Figure 2, the main interface consists of three main features:

A. **Parcel Overview:** the parcel overview presents global distributions of how different types of parcels flow along time, acting as an entry point of the entire analytical process.

B. **Temporal Analysis:** the temporal view, Figure 2 (C), visualizes the temporal dynamics of the oil spill in each scenario and provides comparison between different scenarios. Furthermore, spatial distributions are summarized as glyphs in order to provide partial hints while exploring the temporal flows. Users can explore how the parcels flow along the time and how the oil collection actions were performed in a detailed manner.

C. **Spatial Analysis:** the map view is designed to show the detailed spatial information of the oil spill.

Parcel Overview

By following the visual information seeking mantra[14], the parcel overview is intended to provide an overall temporal distribution of parcel volumes in all scenarios. In Figure 2 (A), the view is split into four panels which are associated to the four parcel types and color hues. In each panel, a line chart is used to present the temporal evolution of parcels where each line represents a scenario. By comparing different lines in a line chart, differences and patterns of flow volumes between scenarios can be revealed. Once the user finds an interesting pattern, such as an extremum at a specific time step, one can locate the corresponding frame in the temporal view to inspect the specific details of that scenario.

Temporal Analysis

After the target time step for detailed analysis is found in the parcel overview, users can direct the analysis flow to the temporal view with comprehensive information on time steps for all scenarios. The goal of the temporal view is to explore the evolution of oil parcels in a scenario where part of the spatial information is added to enhance the understanding of the evolution. In Figure 2 (B), the view consists of multiple rows, each of which corresponds to an individual scenario. A detailed description of the components in a row is illustrated in Figure 3. The identification of scenarios, such as a textual identifier or short description, is placed vertically on the leftmost edge of the row. In the main area, the major component of the temporal view is a

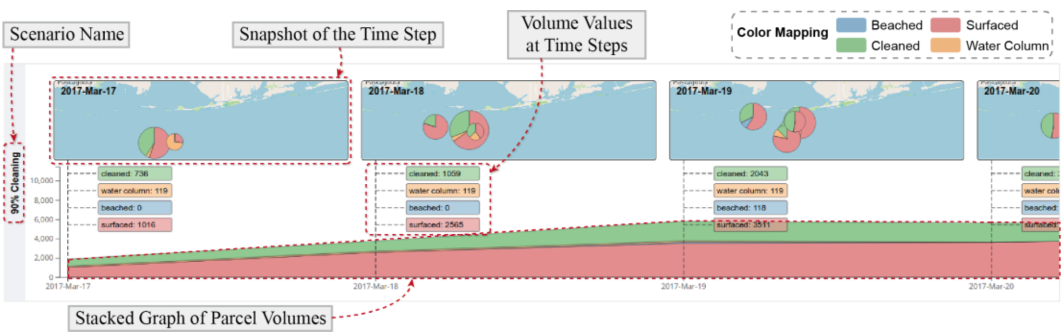

Figure 3: The rows in the temporal view representing a scenario. The temporal volumes of parcel types are visualized by using a stacked graph. The exact volume values are listed beside corresponding time steps. Map snapshots are linked to the time steps to present spatial distributions of the parcels.

stacked graph of the parcel volumes along the horizontal time axis. Vertical dashed gray lines mark the time steps on the time axis with the corresponding parcel volumes of the four types beside the vertical gray lines as well.

One major issue is that the stacked graph with parcel volumes only present the temporal evolution of the oil spill. In order to present spatial information of how the parcels are distributed, we add a map snapshot on top of each time step connected with the vertical gray line. Because of the size limit, it is difficult to directly plot all oil parcels as scatters in the maps; therefore, the parcels are summarized as pie glyphs in the map snapshots with the following two steps:

1. For all non-cleaned parcels on the map, the HDBSCAN clustering method[15] is employed to divide the parcels into densely-distributed clusters. The purpose of not including "Cleaned" parcels in the clustering process is to reveal the oil distribution at the end of each day, which can be used to evaluate the oil collection result. It should be noted that some of the parcels may be marked as outliers since they are not close to any dense clusters.
2. For all parcels in the "Cleaned" type, they are assigned to the closest cluster by measuring the distances to the barycenters of the clusters. In this way, the cleanup progresses in clusters can also be reflected in the map snapshot.
3. For each cluster, a pie chart is generated as the glyph to represent the percentages of parcels in different types. The size of the glyph indicates the total number of parcels inside the cluster. We use the barycenter of the parcels in each cluster as the coordinate of the corresponding pie glyph.

The temporal view supports various interactions. Users can hover the mouse pointer on the pie glyphs to show the parcel volumes inside the cluster. By clicking on the snapshots, the corresponding parcels will be activated in the map view for spatial analysis.

Spatial Analysis

The parcel overview and the temporal view mainly provide temporal information of the oil spill. The map view, on the other hand, is designed for spatial analysis of specific time steps. Initially, the map view is empty without any visual elements. The display of oil parcels in the map view can be toggled by clicking on the map snapshot of the desired time step in the temporal view. For showing different aspects of the oil parcels, the map view supports multiple layers of visualization on the map listed in Figure 4:

- **Parcels.** The oil parcels can be naturally mapped into points whose coordinates are derived from their longitude and latitude values, shown in Figure 4 (A). Colors of the points are assigned to the parcel types. In order to avoid over-plotting, the map view supports semantic zooming where the point sizes do not increase when zooming in. When hovering over the points, a tooltip will pop up with the description of the corresponding time step including the time stamp and number of parcels in all types (Figure 4 (A.1)).
- **Clusters.** The clustering result from the HDBSCAN algorithm in the temporal analysis stage can also be activated in the map view. In Figure 4 (B), the clusters are depicted as alpha shapes when enabling the display of clusters. Instead of using points, the outlying parcels marked by the clustering algorithm is represented by cross marks. Similar to the tooltip for the points, the description of clusters can be highlighted in the same manner, shown in Figure 4 (B.1). Furthermore, we reuse the pie glyphs for the map snapshots in the temporal view in order to provide a instinct for the distributions of parcel types (Figure 4 (B.2)).
- **Density.** Although the semantic zooming is employed to reduce visual clutter, there are still heavy overlaps when the zoom level is low. To this end, a heatmap of the points, Figure 4 (C), can be rendered at the back of the points which shows the density of the parcels.

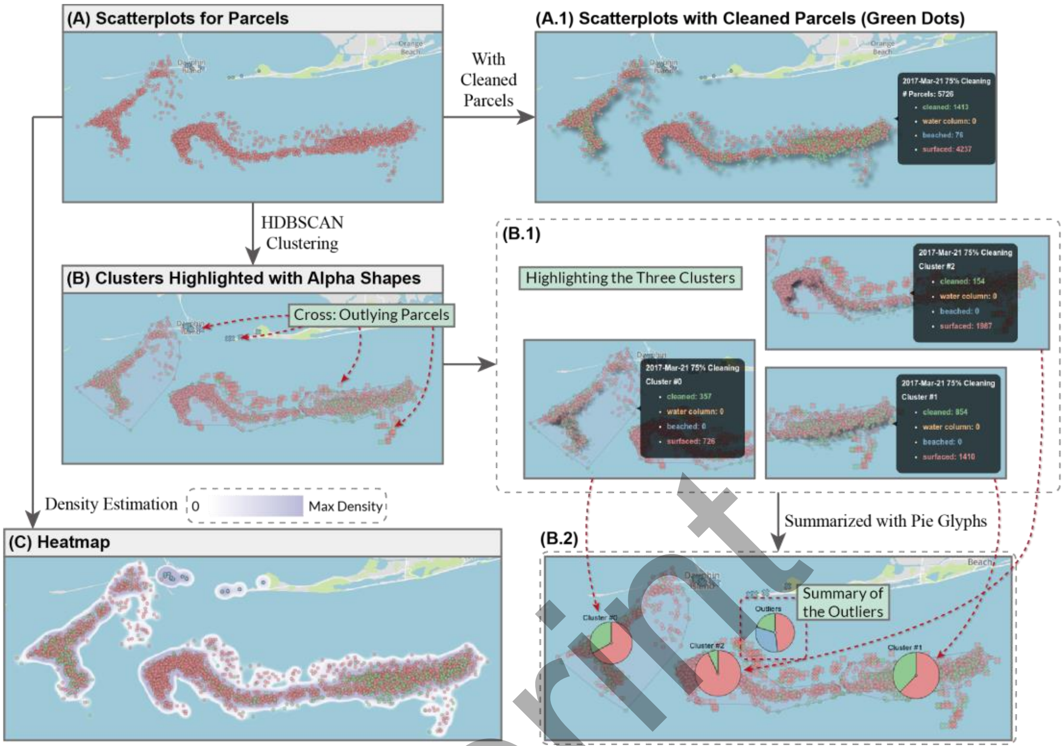

Figure 4: The visual design of the map view. (A and A.1) Parcels are mapped to scatters on the map, and the cleaned parcels can be toggled. (B, B.1 and B.2) Clusters of parcels can be highlighted by alpha hulls and pie glyphs. (C) Heatmaps are employed to show the parcel densities.

CASE STUDY

In this section, we present a case study on the oil spill simulation in the Gulf of Mexico. The simulation starts on 16th March 2017 at 12:00 am (UTC) and lasts for 11 days. The coordinates of the blowout are 30N, -88E at a depth of -25 meters. The blowout diameter is 0.25 m, and the blowout angle is $\angle 0$ altitude, $\angle 90$ azimuth. The total duration of the blowout is three days. The horizontal diffusion scheme of Random Walk has been employed with a Smagorinsky Coefficient of 0.15. The entire simulation is recorded at intervals of 24 hours.

Global Overview of the Temporal Distribution

After starting the system, we first check the parcel overview to see the trends of the oil parcels in different types. From Figure 5, we find that the lines of all six scenarios in “Water Column” and “Cleaned” are similar, while in the “Beached” type there is a significant peak on Mar 24 and an outlying line in “Surfaced”.

Then, we further check the two abnormal patterns in “Beached” and “Surfaced”, respectively. As illustrated in Figure 5 (A), the peak highlighted in the red circle represents the scenario of “0% Cleaning”. This indicates that without proper cleaning, the volume of oil parcels that hit the shore will be almost twice as much as any of the scenarios that have a cleanup target speci-fiedand may have a more significant impact on the coastline. A similar situation is presented in Figure 5 (B) where the outlying line is for “0% Cleaning” as well. The surfaced parcels remain high from the starting date to Mar 25. However, the volume drastically drops on Mar 26. This phenomenon is due to the large amount of parcels that beach on Mar 25 in Figure 5 (A).

There are some other interesting findings in the line charts of “Water Column” and “Cleaned”. The lines for the water column parcels do not vary along the time, indicating that this type of parcels is not affected by cleaning actions. For the cleaned parcels, we also notice that the percentages of cleaning in different scenarios are not held all the time. Figure 5 (C) presents an example that for the scenario of "95% Cleaning", there is a notable drop of the cleanup compared with the other days right before and after Mar 20.

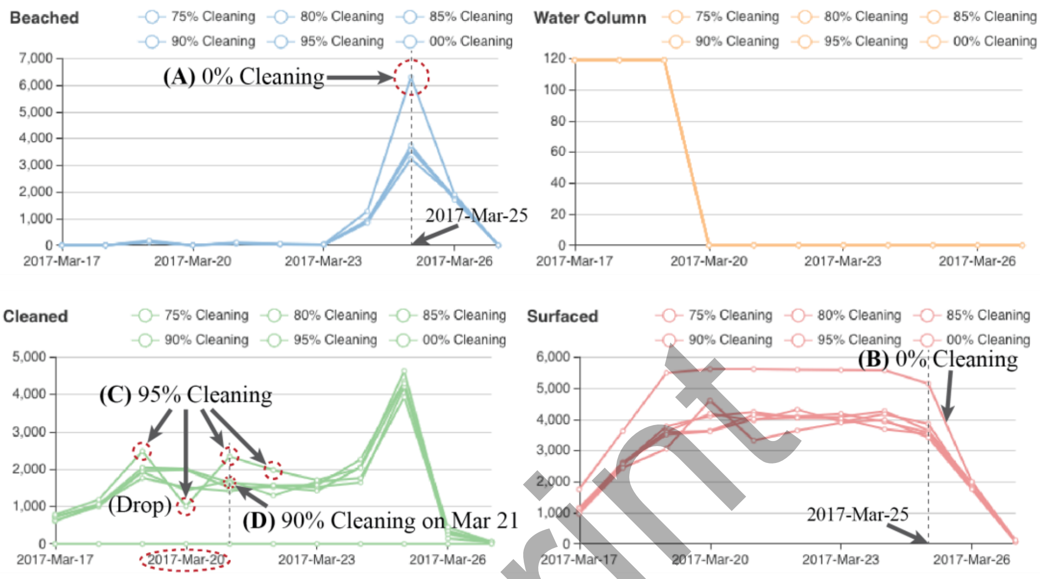

Figure 5: The parcel overview of the simulation.

Comparative Analysis of Differences for Multiple Cleanup Scenarios

After inspecting the parcel overview, we start exploring the details of the temporal evolution in the temporal view. By comparing the map snapshots in the same column, i.e., the same day, there is no significant differences in cluster structures. However, volumes of parcel types may vary in corresponding clusters in different map snapshots. Consider Figure 6,

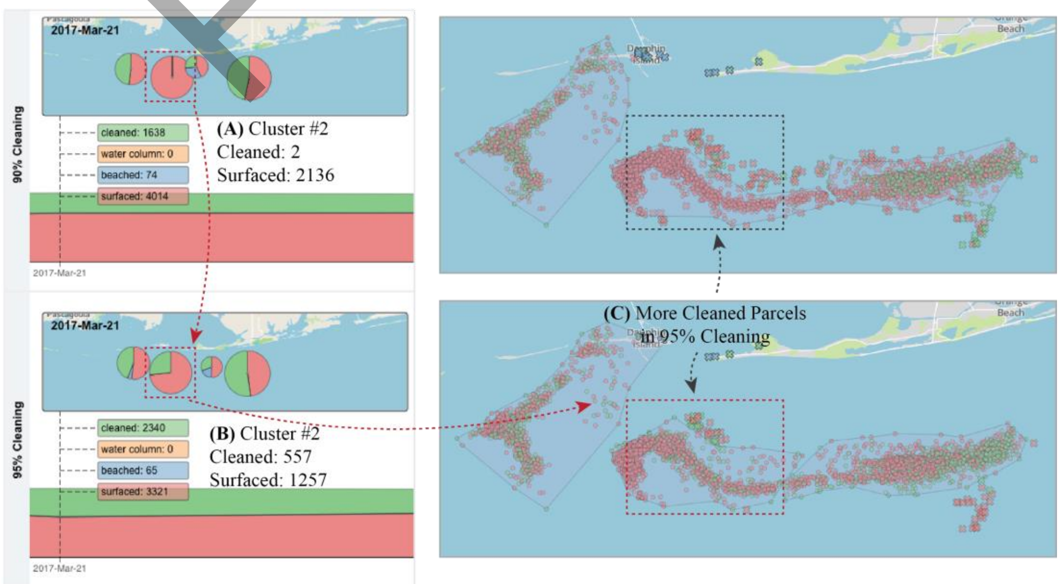

Figure 6: Comparison between "90% Cleaning" and "95% Cleaning" on Mar 21.

where the segments of the stack graphs are placed on the left side with magnified map snapshots on the right side, the cleaned and surfaced parcels in the the two corresponding clusters (A) and (B) are divergent between each other. The difference can be also inspected in Figure 5 (C) and (D) where the “95% Cleaning” holds the largest cleanup on Mar 21. In the rectangle regions in Figure 6 (C), there are more green dots in the map of “95% Cleaning”, which proves that this region obtained more cleanup in the 95% scenario.

CONCLUSION

In this paper, a visual analytics system for oil spill response and recovery for the spills is presented. This system allows users to explore a wide variety of possible outcomes that can arise in different scenarios when an oil spill occurs. It also helps in exploring and visualizing the temporal evolution of the oil spill with the help of geographical maps with respect to change in space and time. The case study presents the analysis of simulated oil spill in the Gulf Of Mexico with six different cleanup scenarios.

ACKNOWLEDGEMENTS

This work was supported by the National Academies of Science Gulf Research Program (grant # 2000007349)

REFERENCES

1. Ni-Bin Chang, Y.L. Wei, C.C. Tseng, C.-Y.J. Kao, “The Design of a GIS-based Decision Support System for Chemical Emergency Preparedness and Response in an Urban Environment,” Computers, Environment and Urban Systems 21(1):67–94, 1997.
2. K. B. Matthews , A. R. Sibbald, S. Craw, “Implementation of a Spatial Decision Support System for Rural Land Use Planning: Integrating Geographic Information System and Environmental Models with Search and Optimisation Algorithms,” Computers and electronics in agriculture 23(1):9–26, 1999.
3. M. DellAmico, G. Fuellerer, G. Höfinger, M. Iori, S. Novellani, “A Decision Support System for Highway Construction: The Autostrada Pedemontana Lombarda,” Interfaces 46(3):245–263, 2016.
4. E. Rolland, R. A. Patterson, K. Ward, B. Dodin, “Decision Support for Disaster Management,” Operations Management Research 3(1-2):68–79, 2010.
5. R. A. Krohling, V. C. Campanharo, “Fuzzy TOPSIS for Group Decision Making: A Case Study for Accidents with Oil Spill in the Sea,” Expert Systems with Applications 38(4):4190–4197, 2011.
6. A. C. Passos, M. G. Teixeira, K. C. Garcia, A. M. Cardoso, L. F. A. M. Gomes, “Using the TODIM-FSE Method as a Decision-making Support Methodology for Oil Spill Response,” Computers & Operations Research 42:40–48, 2014.
7. S. Pajer, M. Streit, T. Torsney-Weir, F. Spechtenhauser, T. Muller, and H. Piringer, “WeightLifter: Visual Weight Space Exploration for Multi-Criteria Decision Making,” IEEE Transactions on Visualization and Computer Graphics 23(1):611–620, 2017.
8. E. Dimara, A. Bezerianos, P. Dragicevic, and A. Austria, “Conceptual and Methodological Issues in Evaluating Multidimensional Visualizations for Decision Support,” IEEE Transactions on Visualization and Computer Graphics 24(1):749–759, 2018.
9. K. J. Kostuk and K. A. Willoughby, “A Decision Support System for Scheduling the Canadian Football League,” Interfaces 42(3):286–295, 2012.
10. T. H. Grubesic, R. Wei, and J. Nelson, “Optimizing Oil Spill Cleanup Efforts: A Tactical Approach and Evaluation Framework,” Marine Pollution Bulletin 125(1–2):318–329, 2017.

11. O. Uran and R. Janssen, “Why Are Spatial Decision Support Systems Not Used? Some Experiences from the Netherlands,” Computers, Environment and Urban Systems 27(5):511–526, 2003.
12. L. Sim, J. Graham, K. Rose, R. Duran, J. Nelson, J. Umhoefer and J. Vielma, “Developing a Comprehensive Deepwater Blowout and Spill Model”, Technical Report, US Department of Energy, National Energy Technology Laboratory, Albany, OR, 2015.
13. Gorubi Optimazation, LLC, “Gurobi Optimizer Reference Manual”, http://www.gurobi.com, 2019.
14. B. Shneiderman, “The Eyes Have It: A Task by Data Type Taxonomy for Information Visualizations.” Proceedings of IEEE Symposium on Visual Languages, 1996.
15. L. McInnes, J. Healy, S. Astels, "hdbscan: Hierarchical Density-based Clustering”, Journal of Open Source Software 2(11):205, 2017.