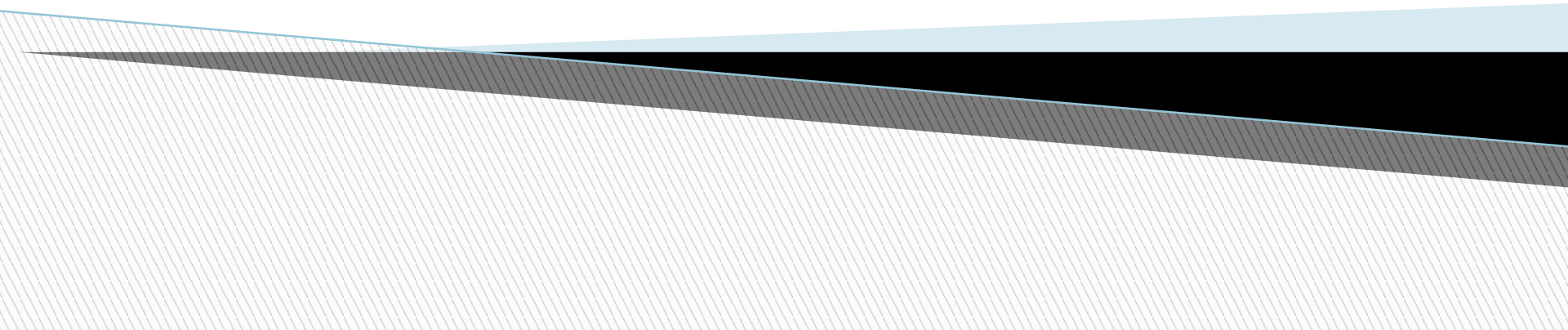


# Pertemuan 2

# LOGIC

Naim Rochmawati



# Proporsitional logic

- **The simplest logic**

- **Definition:**

**A proposition is a statement that is either true or false.**

- **Examples:**

- Penny stays in Sidoarjo (T)
- $5 + 2 = 8$  (F)
- It is raining today (**either T or F**)

# Propositional Logic

## Examples (cont.):

- How are you?
- **a question is not a proposition**
- $x + 5 = 3$
- **since  $x$  is not specified, neither true nor false**
- 2 is a prime number (T)
- There are other life forms on other planets in the universe **either T or F**

# Composite Statements

More complex propositional statements can be build from elementary statements using **logical connectives**.

## **Example:**

- Proposition A: It rains outside
- Proposition B: We will see a movie
- A new (combined) proposition:

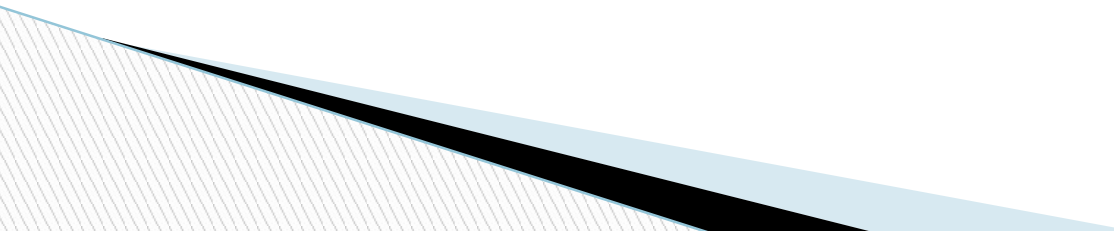
If it rains outside then we will see a movie



# Composite Statements

More complex propositional statements can be build from elementary statements using **logical connectives**.

- **Logical connectives:**

- Negation
  - Conjunction
  - Disjunction
  - Exclusive or
  - Implication
  - Biconditional
- 

# Negation

**Definition:** Let  $p$  be a proposition. The statement "It is not the case that  $p$ ." is another proposition, called the **negation of  $p$** . The negation of  $p$  is denoted by  $\neg p$  and read as "not  $p$ ."

**Example:**

- Pitt is located in the Oakland section of Pittsburgh.  
→
- It is **not the case** that Pitt is located in the Oakland section of Pittsburgh.

**Other examples:**

- $5 + 2 \neq 8$ .
- 10 is **not** a prime number.
- It is **not** the case that buses stop running at 9:00pm.

# Negation

- **Negate the following propositions:**
  - It is raining today.
    - It is **not** raining today.
  - 2 is a prime number.
    - 2 is **not** a prime number
  - There are other life forms on other planets in the universe.
    - It is **not the case** that there are other life forms on other planets in the universe.

# Negation

- A truth table displays the relationships between truth values (T or F) of different propositions.

$p$	$\neg p$
T	F
F	T

**Rows:** all possible values of elementary propositions:



# Conjunction

- **Definition:** Let  $p$  and  $q$  be propositions. The proposition " **$p$  and  $q$** " denoted by  $p \wedge q$ , is true when both  $p$  and  $q$  are true and is false otherwise. The proposition  $p \wedge q$  is called the **conjunction** of  $p$  and  $q$ .
- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh **and**  $5 + 2 = 8$
  - It is raining today **and** 2 is a prime number.
  - 2 is a prime number **and**  $5 + 2 \neq 8$ .
  - 13 is a perfect square **and** 9 is a prime.

# Disjunction

- **Definition:** Let  $p$  and  $q$  be propositions. The proposition " **$p$  or  $q$** " denoted by  $p \vee q$ , is false when both  $p$  and  $q$  are false and is true otherwise. The proposition  **$p \vee q$**  is called the **disjunction** of  $p$  and  $q$ .
- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh **or**  $5 + 2 = 8$ .
  - It is raining today **or** 2 is a prime number.
  - 2 is a prime number **or**  $5 + 2 \neq 8$ .
  - 13 is a perfect square **or** 9 is a prime.

# Truth tables

## Conjunction and disjunction

- Four different combinations of values for  $p$  and  $q$

$p$	$q$	$p \wedge q$	$p \vee q$
T	T		
T	F		
F	T		
F	F		

**Rows:** all possible combinations of values for elementary propositions:  $2^n$  values

# Truth table

- **Conjunction and disjunction**
- Four different combinations of values for  $p$  and  $q$

$p$	$q$	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

- NB:  $p \vee q$  (the or is used inclusively, i.e.,  $p \vee q$  is true when either  $p$  or  $q$  or both are true).

# Exclusive or

**Definition:** Let  $p$  and  $q$  be propositions. The **proposition "p exclusive or q"** denoted by  $p \oplus q$ , is true when exactly one of  $p$  and  $q$  is true and it is false otherwise.

$p$	$q$	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

# Implication

- **Definition:** Let  $p$  and  $q$  be propositions. The proposition " **$p$  implies  $q$** " denoted by  $p \rightarrow q$  is called **implication**. It is false when  $p$  is true and  $q$  is false and is true otherwise.
- In  $p \rightarrow q$ ,  $p$  is called the **hypothesis** and  $q$  is called the **conclusion**.

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

# Implication

- $p \rightarrow q$  is read in a variety of equivalent ways:
  - if  $p$  then  $q$
  - $p$  only if  $q$
  - $p$  is sufficient for  $q$
  - $q$  whenever  $p$
- **Examples:**
  - if Steelers win the Super Bowl in 2013 then 2 is a prime.
    - If F then T ?



# Implication

- $p \rightarrow q$  is read in a variety of equivalent ways:
  - if  $p$  then  $q$
  - $p$  only if  $q$
  - $p$  is sufficient for  $q$
  - $q$  whenever  $p$
- **Examples:**
  - if Steelers win the Super Bowl in 2013 then 2 is a prime.
    - T
  - if today is Tuesday then  $2 * 3 = 8$ .
    - What is the truth value ?



# Biconditional

- **Definition:** Let  $p$  and  $q$  be propositions. The **biconditional**  $p \leftrightarrow q$  (read  **$p$  if and only if  $q$** ), is true when  $p$  and  $q$  have the same truth values and is false otherwise.

$p$	$q$	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- **Note:** two truth values always agree.

# Case

- **Example: Construct a truth table for**  
 $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
- Simpler if we decompose the sentence to elementary and intermediate propositions

p	q	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
T	T				
T	F				
F	T				
F	F				

# Answer

$p$	$q$	$\neg p$	$p \rightarrow q$	$\neg p \leftrightarrow q$	$(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F