Pertemuan 2
LOGIC

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Proporsitional logic

- **The simplest logic**
- **Definition:**

  A proposition is a statement that is either true or false.
- **Examples:**

– Penny stays in Sidoarjo **(T)**

– 5 + 2 = 8 **(F)**

– It is raining today **(either T or F)**

Propositional Logic

Examples (cont.):

– How are you?

• **a question is not a proposition**

– $x + 5 = 3$

• **since x is not specified, neither true nor false**

– 2 is a prime number **(T)**

– There are other life forms on other planets in the universe **either T or F**

Composite Statements

More complex propositional statements can be build from elementary statements using **logical connectives.**

Example:

- Proposition A: It rains outside
- Proposition B: We will see a movie
- A new (combined) proposition:

If it rains outside then we will see a movie

Composite Statements

More complex propositional statements can be build from elementary statements using **logical connectives.**

- **Logical connectives:**

– Negation
– Conjunction
– Disjunction
– Exclusive or
– Implication
– Biconditional

Negation

Definition: Let p be a proposition. The statement "It is not the case that p." is another proposition, called the **negation of p**. The negation of p is denoted by $\neg p$ and read as "not p."

Example:

- Pitt is located in the Oakland section of Pittsburgh.

  →

- It is not the case that Pitt is located in the Oakland section of Pittsburgh.

Other examples:

- $5 + 2 \neq 8$.
- 10 is not a prime number.
- It is not the case that buses stop running at 9:00pm.

Negation

- **<u>Negate the following propositions:</u>**
  - It is raining today.
    - It is not raining today.
  - 2 is a prime number.
    - 2 is not a prime number
  - There are other life forms on other planets in the universe.
    - It is not the case that there are other life forms on other planets in the universe.

Negation

- A **truth table displays the relationships between truth values (T or F) of different propositions.**

p	¬p
T	F
F	T

Rows: all possible values of elementary propositions:

Conjunction

- **Definition**: Let p and q be propositions. The proposition "**p and q**" denoted by $p \wedge q$, is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the **conjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh and $5 + 2 = 8$
  - It is raining today and 2 is a prime number.
  - 2 is a prime number and $5 + 2 \neq 8$.
  - 13 is a perfect square and 9 is a prime.

Disjunction

- **Definition**: Let p and q be propositions. The proposition "**p or q**" denoted by $p \vee q$, is false when both p and q are false and is true otherwise. The proposition $p \vee q$ is called the **disjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh or $5 + 2 = 8$.
  - It is raining today or 2 is a prime number.
  - 2 is a prime number or $5 + 2 \neq 8$.
  - 13 is a perfect square or 9 is a prime.

Truth tables

Conjunction and disjunction

• Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T		
T	F		
F	T		
F	F		

Rows: all possible combinations of values for elementary propositions: 2^n values

Truth table

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).

Exclusive or

Definition: Let p and q be propositions. The proposition "p exclusive or q" denoted by p   q, is true when exactly one of p and q is true and it is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication

- **Definition**: Let p and q be propositions. The proposition "**p implies q**" denoted by $p \rightarrow q$ is called **implication**. It is false when p is true and q is false and is true otherwise.

- In $p \rightarrow q$, p is called the **hypothesis** and q is called the **conclusion**.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication

- $p \to q$ is read in a variety of equivalent ways:
  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p

- **Examples:**
  - if Steelers win the Super Bowl in 2013 then 2 is a prime.
    - If F then T ?

Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
    - if p then q
    - p only if q
    - p is sufficient for q
    - q whenever p

- **Examples:**
    - if Steelers win the Super Bowl in 2013 then 2 is a prime.
        - T
    - if today is Tuesday then $2 * 3 = 8$.
        - What is the truth value ?

Biconditional

- **Definition**: Let p and q be propositions. The biconditional $p \leftrightarrow q$ (**read p if and only if q**), is true when p and q have the same truth values and is false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- **Note:** two truth values always agree.

Case

- **Example: Construct a truth table for**
  $(p \to q) \land (\neg p \leftrightarrow q)$
- Simpler if we decompose the sentence to elementary and intermediate propositions

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p \to q) \land (\neg p \leftrightarrow q)$
T	T				
T	F				
F	T				
F	F				

Answer

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p\to q) \wedge (\neg p \leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F