Lab 10

Direct methods for solving linear systems

Gauss's method with partial pivoting

Consider the linear system $Ax = b$, with $A = (a(i,j))_{i,j=\overline{1,n}}$ and $b = (b(1), ..., b(n))'$.

Algorithm:

Input: n-order of the system; A-matrix of coefficients; b-vector of free terms;

Output: x-vector of the solutions or a message in case of incompatibility of the system

1. For $p = 1, ..., n-1$

    Let $abs(a(q,p)) = \max(abs(a(p:n,p)))$.

    If $a(q,p) = 0$ then "Message"; Exit

    If $q \neq p$ interchange the lines p and q from A and b.

    Perform the necessary operations for obtaining zeros on the column p, below $a(p,p)$.

    Apply the transformations also to the vector b.

2. If $a(n,n) = 0$ then "Message"; Exit
3. For $i = n : -1 : 1$ do

    Compute $x(i)$.

4. Display x.

Problems:

1. Implement the Gauss method for solving linear systems, using partial elimination. Solve the following system of equations:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 5 \\ -1 & 1 & -5 & 3 \\ 3 & 1 & 7 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 31 \\ -2 \\ 18 \end{bmatrix}.$$

2. Find LU decomposition of the following matrix using Doolittle method.

$$A = \begin{bmatrix} 6 & 2 & 1 & -1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}.$$

For $b = \begin{bmatrix} 8 \\ 7 \\ 5 \\ 1 \end{bmatrix}$, solve the system $Ax = b$.