TDT4171 Ex2

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1 Part A

For the Umbrella domain, our only hidden variable is whether it rains or not. Let us call this boolean variable R_t. It follows that;

$$\mathbf{X}_t = \{R_t\}.$$

Similarly our set of observable variables also only holds one boolean variable. It describes whether or not we see our director bring an umbrella to work in the morning. Let us call it U_t, and we have;

$$\mathbf{E}_t = \{U_t\}.$$

The dynamic model of the domain is given by a 0.7 probability for a given day having the same weather as the day preceding it. Assuming a time step of one day we state the model as follows;

$$\mathbf{D} = \mathbf{P}(\mathbf{X}_t|\mathbf{X}_{t-1}) = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}.$$

The observation model is similarly given by;

$$\mathbf{O} = \mathbf{P}(\mathbf{E}_t|\mathbf{X}_t) = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}.$$

That is to say that we expect the umbrella to be brought with a probability of 0.9 if it rains and 0.2 if it does not.

In describing the Umbrella domain, several assumptions are made. We assume the weather to be a first order Markov process. This is technically not accurate, but given that the only observational evidence we have access to is the umbrella, describing the weather in a more detailed manner does not add any information. A more dubious assumption is that we are dealing with a stationary process. I have no doubt that a more accurate dynamic model would change with the seasons. Consecutive rainy days might be less likely in summer than in winter. Of course it is not stated that we have access to anything more than a relative date, so this is not possible to implement in our domain. Finally we have the sensor Markov assumption. We assume that the umbrella is only dependent on today's weather. One could argue that this is a mistake. Could it be assumed that the director gets more adept at remembering to bring his umbrella as a consecutive rainy period progresses? I find this highly likely, but at the same time, I don't regard the information loss to be significant in this example.

2 Part B

Applying the forward method consecutively, using the specified observations, we get the following message sequence:

$$\mathbf{f}_{1:0} \approx \langle 0.500, 0.500 \rangle$$

$$\mathbf{f}_{1:1} \approx \langle 0.818, 0.182 \rangle$$
$$\mathbf{f}_{1:2} \approx \langle 0.883, 0.117 \rangle$$
$$\mathbf{f}_{1:3} \approx \langle 0.191, 0.809 \rangle$$
$$\mathbf{f}_{1:4} \approx \langle 0.731, 0.269 \rangle$$
$$\mathbf{f}_{1:5} \approx \langle 0.867, 0.133 \rangle.$$

Implying a probability of rain on day 5 of 0.867.

3 Part C

Applying the forward-backward algorithm, we get the following backward messages:

$$\mathbf{b}_{6:6} \approx \langle 1.000, 1.000 \rangle$$
$$\mathbf{b}_{5:6} \approx \langle 0.690, 0.410 \rangle$$
$$\mathbf{b}_{4:6} \approx \langle 0.459, 0.244 \rangle$$
$$\mathbf{b}_{3:6} \approx \langle 0.091, 0.150 \rangle$$
$$\mathbf{b}_{2:6} \approx \langle 0.066, 0.046 \rangle.$$

And finally a smoothed probability estimate for the first day:

$$\mathbf{P}(\mathbf{X}_1 | \mathbf{e}_{1:5}) = \langle 0.867, 0.133 \rangle.$$

The code implemented for this purpose can be found in the appended archive.