TDT4171 Exercice 1

Harald Husum

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1 Counting and basic laws of probability

1.1 5-card Poker Hands (Ex. 13.7 Russel & Norvig)

A 5-card hand can be thought of as the 5-combination of the set of 52 different cards. The number of atomic events is thus given by:

$$|\Omega| = \binom{52}{5} = 2'598'960.$$

Under the assumption that the dealer is fair, all events are equally likely, and has a probability defined by:

$$P(\omega|\omega \in \Omega) = \binom{52}{5}^{-1} \approx 3.85 \times 10^{-7}.$$

There are exactly four possible royal straight flushes, one for each color. The probability of getting one is thus:

$$P(rsf) = \binom{4}{1} \times P(\omega) = \binom{4}{1} \times \binom{52}{5}^{-1} \approx 1.54 \times 10^{-6}.$$

Four of a kind implies that four of our cards have the same number value. Thus there are 13 different numbering possibilities. For any instance of four of a kind, we also have a last card. This card has no restrictions and can take on any remaining value. The probability is then given by:

$$P(fok) = \binom{13}{1} \times \binom{12}{1} \times \binom{4}{1} \times \binom{52}{5}^{-1} \approx 0.024\%.$$

1.2 Two cards in a deck

Pulling one card out of a deck, leaves three of the same value among the remaining 51 cards. Drawing a pair in the first try has probability equal to drawing one of the three remaining cards of same value in the second draw:

$$P(pair) = \frac{3}{51}$$

Given that the second draw has a different color than the first, we still are left with three possible cards of the same value, but now out of a 39-card subset.

$$P(pair|Card_1.color \neq Card_2.color) = \frac{3}{39}$$

1.3 Conditional probability

If the occurrence of B makes A more likely, the occurrence of A make B more likely. A proof follows:

$$P(a|b) > P(a) \Leftrightarrow \frac{P(a \wedge b)}{P(b)} > P(a) \Leftrightarrow P(a \wedge b) > P(a)P(b) \Leftrightarrow$$

$$\frac{P(b \wedge a)}{P(a)} > P(b) \Leftrightarrow P(b|a) > P(b).$$

2 Bayesian Network Construction

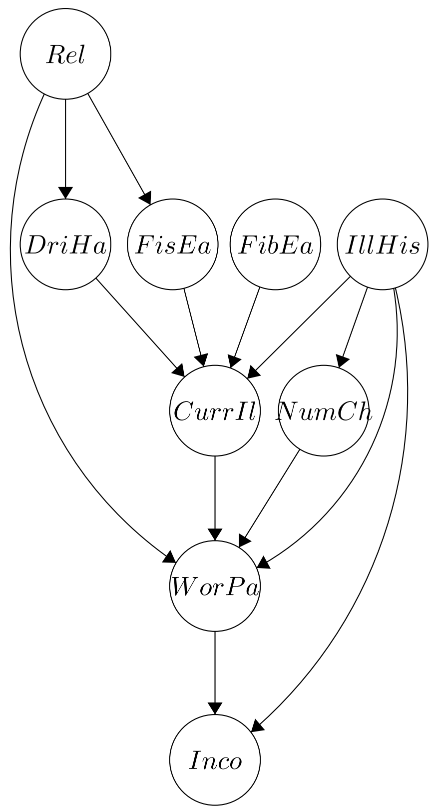

Figure 1: Bayesian network for the specified world

2.1 World assumptions

The above Bayesian network is constructed under assumptions described here. Religion directly influences drinking habits and fish eating, as some religions consider alcohol consumption or eating of animals as unethical. Religion also influences the number of working parents, as some conservative strains will fell that it is inappropriate for one of the sexes to seek work outside the home.

Furthermore, alcohol consumption, fish eating and fiber eating all impact on the likelihood of current illness.

I also assume a history of illness increases the likelihood of current illness, as a illness history could allude to chronic disease. Illness history also affects the number of children, through a number of vectors. Illness could mean infertility or serve as motivation for not taking on the responsibility of children. A history of illness could also increase the likelihood of one or more parents being out of work, as long periods of debilitation can be hard to come back from. For the same reason, a history of illness will affect income negatively. Having spent long periods incapacitated, means lower average education and work experience.

Current illness also influences the number of working parents; a severely sick parent is unable to work. The same is the case for number of children. It is hard to say what way it will influence; for some families, having children will mean both parent need to work extra hard to make due, while for others, it might mean that one parent desires to stay home.

Finally the number of parents working directly influences the income of the family.

2.2 Difficulties

Further causal claims can be made. I find it likely that level of income would affect both religion, drinking habits, eating habits and health history. It is tempting to include this in the network, but it unfortunately causes cycles, and I have to leave it out. This is not to say I don't think causal cycles exist. Feedback loops are definitely a real phenomena. This does however illustrate the limitations in designing a Bayesian network solely based on assumed causal relations. It seems as if this might be an issue that is symptomatic for designing complex networks.

2.3 Conditional independence properties

Let us look at a couple of conditional independencies given by the network. Given religion, drinking habits and fish eating are independent. This seems reasonable. Furthermore, current illness and number of children should be independent given illness history. This also seems reasonable.

3 Bayesian Network Application

Following the recommended variable choices, i set up my network. It is quite simple, stating only that $OpenedByOfficial$ is dependent on both $MyChoice$ and $ContainsPrice$. The conditional probability table as well as the network are appended. By adding evidence to $MyChoice$, we see no change in the distribution for $ContainsPrice$. But after adding evidence for $OpenedByOfficial$, we see a pattern emerging. The probability for the door of $MyChoice$ also being the door of $ContainsPrice$ remains at one third. But the probability of the remaining unopened and unselected door being the right one, jumps to two thirds, implying that switching doors is the rational choice.

This makes intuitive sense. We expect the probability of the prize being among the unselected doors to be two thirds initially. Removing one empty door from the set doesn't change this, and only results in the remaining door "inheriting" the probability from the opened door, as we remove a choice.