



Linear Algebra for Data Science & Machine Learning in Python

Determinants



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DETERMINANT



Determinant: is function mapping matrix to real scalars



Determinant: can be computed only for square matrices



Determinant of a 2×2 matrix gives the area



Determinant of a 3×3 matrix gives the volume



DETERMINANT

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

2×2

$$\det(M) = |M| = a * d - b * c$$

$$M = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$

2×2

$$\begin{aligned} \det(M) &= 5 * 2 - 1 * 4 \\ &= 10 - 4 \\ &= 6 \end{aligned}$$



Checker Board

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

3×3

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$|M| = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$|M| = a(e * i - f * h) - b(d * i - f * g) + c(d * h - e * g)$$



DETERMINANT

$$M = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 5 & -3 \\ 3 & -1 & 4 \end{bmatrix}$$

$$|M| = 2 \begin{vmatrix} 5 & -3 \\ -1 & 4 \end{vmatrix} - 3 \begin{vmatrix} -1 & -3 \\ 3 & 4 \end{vmatrix} + 4 \begin{vmatrix} -1 & 5 \\ 3 & -1 \end{vmatrix}$$

$$|M| = 2(5 * 4 - (-3 * -1)) - 3(-1 * 4 - (-3 * 3)) + 4(-1 * -1 - 5 * 3)$$

$$|M| = 2(20 - 3) - 3(-4 - (-9)) + 4(1 - 15)$$

$$2(20 - 3) - 3(-4 + 9) + 4(1 - 15)$$

$$2(17) - 3(5) + 4(-14) = 34 - 15 - 56 = -37$$



DETERMINANT



Determinant of 2 x 2 matrix:

$$|M| = a * d - b * c$$



Determinant of 3 x 3 matrix:

using $|M|$ of 2×2 submatrices



Determinant of 4 x 4 matrix:

using $|M|$ of 3×3 submatrices



Determinant of n x n matrix:

using $|M|$ of $(n - 1) \times (n - 1)$ submatrices



DETERMINANT

$$M = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & a_{nn} \end{bmatrix}$$



Its better to select a row or column of matrix which contains many zeros



Determinant: can be positive, negative or zero



A matrix with Determinant as zero is a singular matrix

