

# Tracking the Mode of Operation of Multi-Function Radars

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**Abstract**—One of the important objectives of a Radar Warning Receiver (RWR) aboard a tactical aircraft is to evaluate the level of threat posed by hostile radars in an extremely complex Electronic Warfare (EW) environment in reliable, robust and timely manner. For the RWR objective to be achieved, it passively collects electromagnetic signals emitted from potentially hostile radars. One class of such radar systems is the Multi-Function Radar (MFR) which presents a serious threat from the standpoint of a RWR. MFRs perform multiple functions simultaneously employing complex hierarchical signal architecture. The purpose of this paper is to uncover the evolution of the operational mode (radar function) from the view point of a target carrying the RWR when provided with noisy observations and some prior knowledge about how the observed radar functions. The RWR estimates the radar's threat which is directly dependant on its current mode of operation. This paper presents a grid filter approach to estimate operational mode probabilities accurately with the aid of pre-trained Observable Operator Models (OOMs) and Hidden Markov Models (HMMs). Subsequently, the current mode of operation of a radar is estimated in the *maximum a posteriori* (MAP) sense. Practicality of this novel approach is tested for an EW scenario in this paper by means of a hypothetical MFR example. Finally, we conclude that the OOM-based grid filter tracks the mode of operation of a MFR more accurately than the corresponding HMM-based grid filter.

## I. INTRODUCTION

Electronic Warfare (EW) is a dynamically changing field due to continually changing threats in the battlefield. The EW problem may, in general, be viewed in two ways:

- From a radar's view point, the main focus is to detect the target, track and identify its critical parameters. For the radar's objective to be achieved, it needs to perform satisfactorily even in the presence of electronic counter measures (ECM) while exhibiting a low probability of intercept. A low probability of intercept is achieved by employing the techniques such as random frequency hopping, chirped and direct sequence spread spectrum transmission, dynamic transmit power control, infrequent scanning and antennas with suppressed side-lobes [9].
- By contrast a Radar Warning Receiver (RWR) aboard a tactical aircraft needs to generate warnings to the operator due to threats posed by potentially hostile radars in a reliable, robust and timely manner. The warnings generated by the RWR facilitate an aircraft to control its behavior based on what is going on within the hostile radar. The design of the RWR is therefore heavily influenced by the design of the radar systems. In the context of this

work, a radar system that is of particular interest is a Multi-Function Radar (MFR). The MFR is capable of performing multiple tasks related to several targets simultaneously in time-multiplexed fashion. These tasks extend from generalized ones such as search, acquisition, tracking and target-illumination to counter attacks such as initiation of electronic counter-counter measures (ECCM), missile launching etc. For the objective of a RWR to be achieved, it has to perform the following two tasks:

- 1) Classify observed radars.
- 2) Estimate the mode of operation of the observed radar.

Based on radar signal characteristics, we assume that we have classified radars responsible for the emission successfully [6]. We therefore focus our attention on the operational mode estimation problem from the point of view of a RWR aboard a tactical airborne platform. The level of threat posed by a hostile radar directly depends on its current mode of operation. For example, the level of threat posed by the radar becomes higher and higher as the radar mode evolves from search to target acquisition and then to track mode which may be the precursor to the missile engagement. Estimation of the current operational mode therefore gives clues about the level of instantaneous threat posed by potentially hostile radar so that the aircraft under scrutiny can prepare for the next tactic in advance. For example, a target can deploy counter measures or perform evasive maneuvers if it becomes engaged.

Since MFRs exhibit a sophisticated pulse structure, signal processing at pulse level is no longer as simple as the signal processing of conventional radars. In order to circumvent this problem, a layered signal architecture for a MFR signal is proposed and the mode estimation of the MFR is performed at a higher level of the layered structure.

## II. LAYERED SIGNAL ARCHITECTURE

The basic building block of a radar signal shall be known as radar *words*. A single word represents a specific pattern of a group of pulses that occurs over a short period of time. Fig. 1 shows an example of a possible hierarchical syntactic structure for a MFR signal. Fig. 1(a) illustrates two words represented by symbols 'a' and 'b' that represent fixed sequences of pulses with their own characteristic set of pulse-to-pulse intervals (PPI). Fig. 1(b) shows a *phrase* made up by the group of words

‘abab’. Table I shows phrases associated with corresponding modes of operation of the hypothetical radar example. Though each phrase can be, in general, mapped into a unique mode, it is not always the case. For example, as shown in Table I, the phrase ‘cccb’ is encountered in three modes of operation.

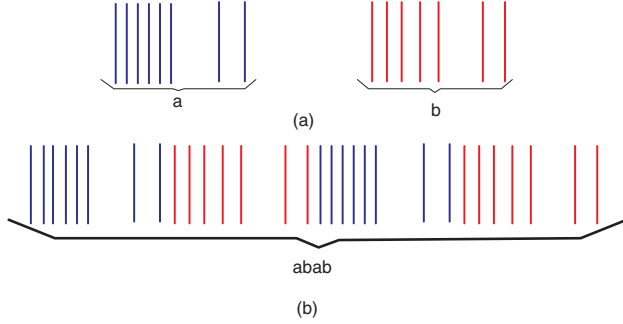


Fig. 1. Layered Signal Structure of a Hypothetical MFR Example

Mode	Phrase Content	Mode	Phrase Content
Search	abca	Range-Resolution (RR)	aabb
	bcab		bbcc
	cabc		ccaa
Acquisition (Acq)	aaab	Track-Maintenance (TM)	aaaa
	bbbc		bbbb
	ccca		cccc
	cccb		cccb
Non-Adaptive Track (NAT)	aabc		
	bbca		
	ccab		
	cccb		

TABLE I

LIST OF MFR PHRASE COMBINATIONS ACCORDING TO ITS OPERATIONAL MODES

Signal processing at word level has the following advantages over the pulse level:

- 1) Since the *a priori* knowledge about a radar signal structure is incorporated in the word extraction process, word level processing is less prone to errors than the signal processing at pulse level. This can also be associated with the well known fact in machine learning that it is always preferable to encode an input sequence with less number of bits for learning a hidden process [3].
- 2) Word level processing is less complex than pulse level.
- 3) It is also possible to analyze the task performed by the MFR for each target under its scrutiny independently.

Since there already exists a technique to extract the words from a sequence of observed pulses [1,10], we focus our attention to develop an accurate filtering methodology to track the current radar mode based on extracted words with the help of pre-trained observable operator models (OOMs).

### III. OBSERVABLE OPERATOR MODELS

An Observable Operator Model (OOM) is mathematical tool for modelling a stationary symbolic process [4], [5]. For example, an OOM can learn the probability distribution of an unknown symbol generator from its training data. OOMs were introduced by Herbert Jaeger recently. As a method of choice for describing an unknown distribution of the stochastic process, an OOM is comparable to a Hidden Markov Model (HMM) [7]. However, the mathematical construction of the OOM can be considered as a generalization of a HMM.

**Definition** A  $m$ -dimensional OOM is a triple,  $A = (\mathbb{R}^m, (\tau_a)_{a \in \Sigma}, \omega_0)$ , where  $\Sigma$  is the vocabulary,  $\omega_0 \in \mathbb{R}^m$  and  $\tau_a : \mathbb{R}^m \rightarrow \mathbb{R}^m$  are linear maps represented by matrices, satisfying

- 1)  $\mathbf{1}\omega_0 = \mathbf{1}$ , where  $\mathbf{1} = (1, \dots, 1) \in \mathbb{R}^m$ , a row vector consisting of all 1's
- 2)  $\mathbf{1}(\sum_{a \in \Sigma} \tau_a) = \mathbf{1}$
- 3)  $\forall a_0 \dots a_r$  it holds that  $\mathbf{1}\tau_{a_r}\omega_0 \geq 0$ , where  $\tau_{a_r} \equiv \tau_{a_r a_{r-1} \dots a_0} \equiv \tau_{a_r} \tau_{a_{r-1}} \dots \tau_{a_0}$ .

In the above definition,  $\tau_a$  is the operator indexed over the output symbol ‘ $a$ ’,  $\mathbb{R}^m$  is the domain of the operators and  $\omega_0$  is the initial state vector. The OOM allows negative entries in its state vector and operators. The relaxation in sign thus, allows us to develop a better learning algorithm than HMM/EM learning algorithm [7]. The OOM learning shall be known as *efficiency sharpening* (ES) algorithm [4] as it estimates the model with a better statistical efficiency than the previous one in each iteration. Some of the important benefits of OOM Learning algorithm over other learning algorithms of stochastic models (e.g., HMM/EM-algorithm) are its fast convergence to the accurate model, greater accuracy and increased expressiveness [4]. In the context of this work, OOMs play a key role in capturing the underlying true radar word generation mechanism.

### IV. METHODOLOGY

Let the discrete variable  $M(k)$  be the mode of operation at time step  $k$ .  $M(k)$  is in effect during the period starting at time  $(k-1)^+$  and ending at  $k$ . Such systems are called jump linear systems. The mode jump process is assumed to be left-continuous (i.e., the impact of new mode starts at  $k^+$ ). The mode at time  $k$  is assumed to be among the possible  $r$  modes as follows:

$$M(k) \in \{M_i\}_{i=1}^r \quad (1)$$

The prior probability that  $M_i$  is correct or the system is in mode  $i$  is given as

$$p(M(0) = M_i | Y^0) = \omega_{0|0}^i \quad (2)$$

where  $Y^0$  is the prior information at time 0 and

$$\sum_{i=1}^r \omega_{0|0}^i = 1 \quad (3)$$

It is assumed that the mode switching or the mode jump process is a Markovian process (Markov chain) with the known mode transition probabilities as follows.

$$p_{ij} = P(M(k) = M_j | M(k-1) = M_i) \quad (4)$$

Also the mode transition probability will be assumed to be time invariant. In other words, it is a homogenous Markov chain. Fig. 2 shows how an operational mode of a typical radar evolves from one mode to another in a certain pattern.

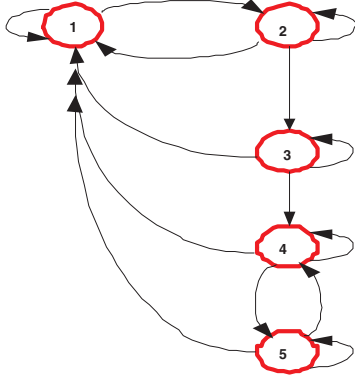


Fig. 2. Operational Mode Transition of the MFR: 1-Search, 2-Acquisition (Acq), 3-Non Adaptive Track (NAT), 4-Range Resolution (RR), 5-Track Maintenance (TM).

The estimation algorithm includes the following three steps:

- step 1: Mode likelihood estimation.
- step 2: Mode probability update.
- step 3: Current mode estimation.

#### A. Mode Likelihood Estimation

The word extractor module at pre-processing stage extracts radar words from a sequence of radar pulse. The stream of extracted words is then fed to a framing module. The framing module breaks the sequence into non-overlapping frames of fixed size. The choice of a frame size is a design parameter. The word sequence received at the framing module may contain erroneous words in the form of mismatched or missing words [1], [10]. The likelihood calculation is carried out on a frame-by-frame basis for the next step known as the mode probability update.

In order to come up with a more accurate decision on the current mode of operation, a filter in Bayesian framework is applied for the mode probability update. The Bayesian approach to mode estimation normally requires two defining equations. The first is a state model and the second is a measurement model. The state model describes the evolution of the hidden modes with time. In the context of this work, the hidden modes are modes of operation of the radar of interest and the state equation describes the nature of mode switching of MFR state machine which is indeed a Markovian process as described earlier. The measurement equation provides a probabilistic mapping from the unobserved state space to the observable space. In the MFR mode estimation problem,

measurements are radar word sequences (frames of words) coming out from a word extractor. Here we assume that we do not have an access to an optimal measurement model in analytic form. Consequently, the likelihood estimate at  $k$  or  $p(y_k | M(k) = M_i)$ , where  $y_k$  is the frame received at time  $k$ , is unavailable for the update step of a Bayesian filter. To circumvent this difficulty, each operating regime (mode) of the MFR is modelled by a distinct Observable Operator Model (OOM) that captures the underlying actual radar word generation mechanism. The application of OOMs in this context is motivated by virtues of OOM over other stochastic models as described in Section III and the fact that a MFR can be viewed as a stochastic symbol generator [10].

Assume that a set of training sequences is available for each mode of operation. The following tasks are performed in order to estimate the likelihood:

- For each mode of operation  $M_i$  ( $i = 1, \dots, r$ ), we build an OOM  $(\mathbb{R}^m, (\tau_a)_{a \in \Sigma}, \omega_0)$  and estimate the model parameters that optimize the likelihood of the training set of observation for the  $i^{th}$  mode. The OOMs obtained at the end of this step are called *mode-specific pre-trained OOMs*.
- During the actual operation, the word sequences (frames) are recognized (which mode those frames come from) by employing these mode-specific pre-trained OOMs.

This is the very approach taken in an isolated word HMM recognizer in speech recognition [7]. The likelihood of the frame received at time step  $k$  for the model  $i$ ,  $\Omega_i(k)$  is evaluated as [5]

$$\begin{aligned} \Omega_i(k) &= p(y_k | M(k) = M_i) \\ &= \mathbf{1}[\tau_{y(k)} \omega_0]_i \end{aligned} \quad (5)$$

The maximum likelihood (ML) estimate,  $M^{ML}(k)$  is given as

$$M^{ML}(k) = \arg \max_{M_1 \leq M_i \leq M_r} \Omega_i(k) \quad (6)$$

The likelihood estimation on frame-by-frame basis is motivated by the following factors:

- **Timely detection:** Making decisions based on frame-by-frame basis helps detect a mode jump in timely manner. It is noteworthy that the frames with smaller sizes increase the chance of on-time detectability at the expense of computational overhead.
- **Soft switching among operational modes:** Since the likelihood is calculated on the basis of frames, error growth of a mismatched model can be kept under control. This, in turn, allows soft switching when one of the mismatched operational modes in the past is selected as a matched one at present.

In computer simulation, pre-trained HMMs are also employed for the likelihood estimation with a purpose of comparing the performance of OOMs.

### B. Mode Probability Update

As shown in Figs. 4 and 6, the ML estimate is unreliable. Therefore, a grid filter is applied to obtain a more refined version of the likelihood estimate by including the mode switching process as the *a priori* knowledge about a MFR. The grid filter solution is optimal when the following assumptions hold:

- State space model is discrete.
- Number of states in a state space is finite.

The output of the grid filter or the mode probability is in fact a smoother version of the likelihood estimate (see Figs. 4 and 6). The prediction and update equation of the grid filter in a mode estimation frame work are given as follows [8]:

$$\omega_{k|k-1}^i = p(M(k) = M_i | Y^{k-1}) = \sum_{j=1}^r \omega_{k-1|k-1}^j p_{ij} \quad (7)$$

$$\omega_{k|k}^i = p(M(k) = M_i | Y^k) = \frac{\omega_{k|k-1}^i \Omega_i(k)}{\sum_{j=1}^r \omega_{k|k-1}^j \Omega_j(k)} \quad (8)$$

where  $Y^k = \{y_i | i = 1, 2, \dots, k\}$  denotes all the information (measurement) obtained until time  $k$ .

The mode likelihood,  $\Omega_i(k)$  is obtained from the first step of the estimation algorithm and the mode transition probability  $p_{ij}$  is assumed to be known. Based on simulations carried out considering various scenarios, it is proved that the performance of a grid-based filter is not very sensitive to the choice of the mode transition probabilities. Nevertheless, a reasonably good choice of the mode transition probabilities leads to the accurate and timely detection of mode jump [1], [2]. In addition to some prior knowledge from the analyst, traditional learning techniques such as Baum-Welch algorithm [7] can also be employed in estimating these mode transition probabilities.

### C. Current Mode Estimation

Finally, the current mode at time  $k$  is estimated in the maximum a posteriori (MAP) sense as follows:

$$M^{MAP}(k) = \arg \max_{M_1 \leq M_j \leq M_r} \omega_{k|k}^i \quad (9)$$

Fig. 3 illustrates how the evolution of the current mode of a MFR is tracked by the mode estimator module where the inputs are frames of words and the mode transition probability and the output is the estimate for the current mode of operation in MAP sense.

## V. SIMULATION

The computer experiment simulates a test scenario where a RWR system is fitted on the aircraft that approaches a MFR on the ground. The MFR has five different radar modes denoted as Search (Sea), Acquisition (Acq), Non-Adaptive Track (NAT), Range Resolution (RR) and Track Maintenance (TM) as described in Section II. As the aircraft approaches the radar, it enters the detection zone and the radar initiates the track for that target. The regime of this operation is referred to as target acquisition. After some time, it initiates non-adaptive tracking to track the target of interest. After the non adaptive

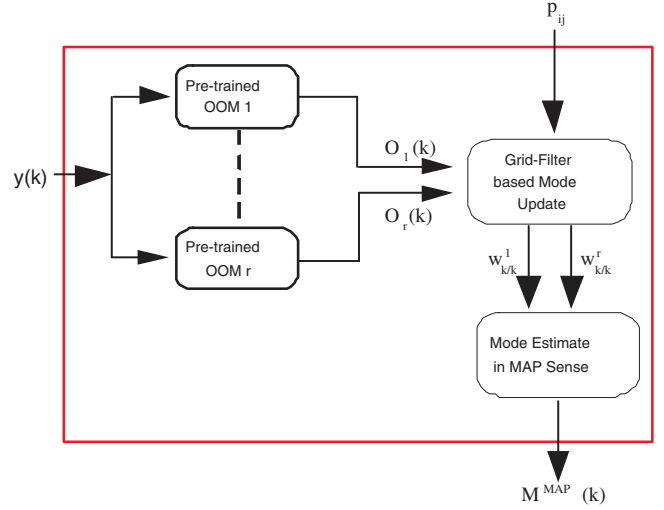


Fig. 3. The Mode Estimator

tracking, it enters into the range resolution mode in order to resolve the range ambiguity. Finally the radar establishes the track for that target. This is known as track maintenance or continuation. Occasionally, it performs a range verification function on the track (see Figs. 4 and 6). As the aircraft flies over and away from the radar, the distance becomes too long for the radar to continue tracking. The target track is finally abandoned and the radar switches back to search mode.

The underlying dynamical behavior of each of the five modes was simulated using a distinct Hidden Markov Model. Each of the five HMMs contained four hidden states and one output symbol for each of the phrases available to its mode, as determined from Table I. For example, since the Search mode admits three possible phrases, its underlying HMM had four states and three output symbols. The five underlying HMMs were created randomly by assigning uniformly distributed random numbers in the interval  $(0, 1)$  to entries of transition and emission probability matrices of HMMs and normalizing them so that the row sum of each matrix was equal to unity.

For each of the five modes, a training sequence containing 250 phrases was randomly generated using the appropriate underlying HMM and then converted into a word sequence using Table I. We assumed that it is not always possible to find the exact underlying model for the word generation mechanism and therefore each of these word sequences generated from four-state underlying HMMs was used to train a three-dimensional mode-specific OOM using the ES algorithm [5] over the vocabulary  $\Sigma = \{a, b, c\}$ . Practical problems, in general, require a model dimension that gives a good compromise in the bias-variance dilemma. To find an appropriate model dimension, either standard machine learning techniques (e.g., cross-validation [3]) or the condition number of  $V$ -matrix in OOM [5] can be employed. For comparison, the same word sequences were used to construct five mode-specific HMMs over  $\Sigma$ , each containing three states, using the Baum-

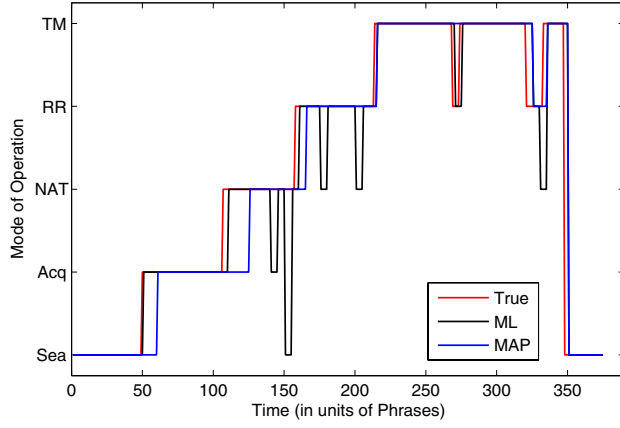


Fig. 4. OOM-Based ML & MAP Estimates  $\forall$ s True Mode Evolution

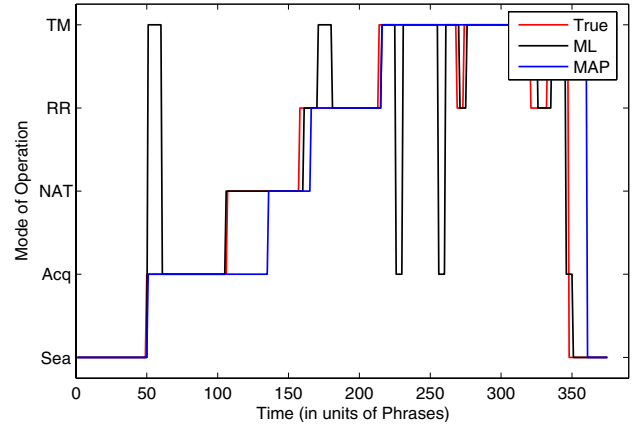


Fig. 6. HMM-Based ML & MAP Estimates  $\forall$ s True Mode Evolution

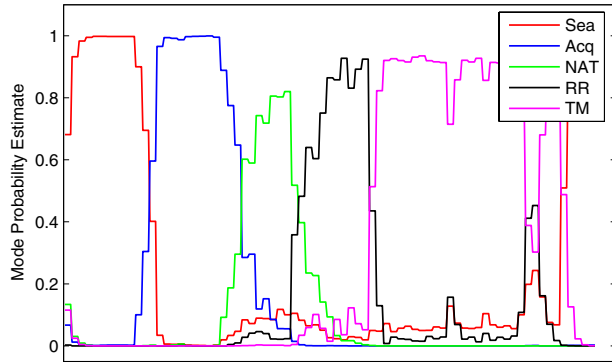


Fig. 5. OOM-Based Mode Probability Estimate for Five Different Modes

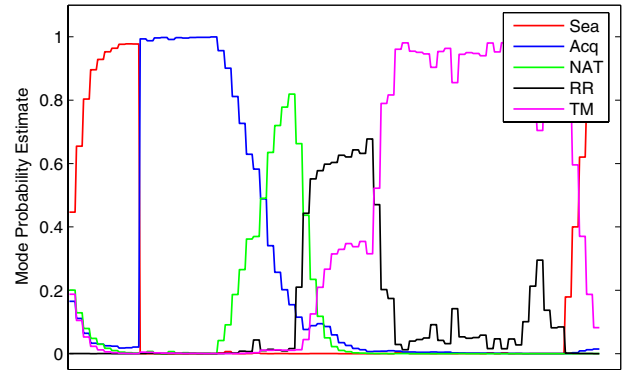


Fig. 7. HMM-Based Mode Probability Estimate for Five Different Modes

Welch algorithm [7]. Both training techniques are iterative and were executed until the ratio between successive training log-likelihoods sank below  $(1 - 1 \times 10^{-5})$ , or until a maximum number of iterations (10 for OOMs and 50 for HMMs) was reached. The true sequence of modes and dwell-times in the test scenario were prescribed manually as illustrated by the red line in Figs. 4 and 6. Within each dwell, a subsequence of phrases was randomly generated using the appropriate underlying HMM and dwell-time. Each subsequence was converted into words using Table I and a test sequence was generated by concatenating together the subsequences for the various dwells. Finally, missing and mismatched words were randomly introduced into the test sequence so that approximately 10% of the words were corrupted.

The test sequence was then partitioned into frames of twenty words and analyzed using the techniques discussed in Section IV. Accounting for the structure of permitted mode transitions (see Fig. 2), the mode transition matrix was taken to be

$$[p_{ij}] = \begin{pmatrix} 0.80 & 0.20 & 0 & 0 & 0 \\ 0.01 & 0.80 & 0.19 & 0 & 0 \\ 0.01 & 0 & 0.80 & 0.19 & 0 \\ 0.01 & 0 & 0 & 0.80 & 0.19 \\ 0.02 & 0 & 0 & 0.03 & 0.95 \end{pmatrix} \quad (10)$$

The initial model probabilities (2) were assigned to be 0.2 each (diffuse uniform prior). The following section summarizes the observations collected from Figs. 4-7.

## VI. RESULTS

In the simulation, mode estimation was carried out considering four cases: The Maximum Likelihood (ML) and the Maximum A Posteriori (MAP) estimation based on OOMs and HMMs. The ML estimate (6) is an unfiltered estimate obtained directly from the first step of the estimation algorithm. On the other hand, the MAP estimate (9) includes all three steps of the estimation algorithm. The results of the computer simulation are shown in Figs. 4, 5, 6 and 7. The red line in Fig. 4 and Fig. 6 shows the actual mode evolution of the radar. The initial mode of the radar is search. The observations of interest are summarized as follows:

- ML estimator often makes jumps from one mode to another. In other words, ML estimate results seem unreliable.
- In the MAP estimate, there exists a latency associated with the new mode onset time up to some extent.
- MAP estimator fails to detect the mode where the radar stays only for a period less than 8 phrase time units.

- In OOM case, the computation time for the likelihood estimate was 20 seconds on average when implemented on 3.07GHz Intel Pentium IV processor using MATLAB whereas it was 70-75 seconds in HMM case.
- The OOM-based grid filter tracks the mode evolution of the MFR more reliably than the corresponding HMM-based grid filter.

## VII. CONCLUSION

In this paper, the mode probability estimator which assumes that the system to be in one of a finite number of modes at a time is discussed. Each model (mode) yields a model conditioned likelihood estimate evaluated over a sequence of fixed size known as frame. The likelihood estimation is performed using mode specific pre-trained OOMs and HMMs. A mode probability calculator updates the probability of each mode based on the likelihood function of each model and the prior mode probability. The mode estimator finally decides the current mode by taking a mode with the highest mode probability at that time. Computer simulations confirms that the MAP estimator significantly outperforms the ML estimator. Moreover it is observed from the simulations that the performance of OOM is significantly better than that of a HMM given the same scenario. This can be attributed to the fact that the models obtained via the OOM/ES learning algorithm is markedly more accurate than the corresponding models obtained via HMM/EM algorithm. In other words, mode-specific OOMs yield more accurate estimate for a measurement likelihood than corresponding equivalent HMMs. Therefore, we conclude that the OOM-based grid filter outperforms the corresponding HMM-based grid filter in tracking the mode of operation of a MFR.

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