

# Battery Management System in the Bayesian Paradigm: Part I: SOC Estimation

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**Abstract**—Accurate State-of-Charge (SOC) estimation of Li-ion batteries is essential for effective battery control and energy management of electric and hybrid electric vehicles. To this end, first, the battery is modelled by an OCV-R-RC equivalent circuit. Then a dual Bayesian estimation scheme is developed: The battery model parameters are identified online and fed to the SOC estimator, the output of which is then fed back to the parameter identifier. Both parameter identification and SOC estimation are treated in the Bayesian framework. The square-root recursive least-squares estimator and the extended Kalman-Bucy filter are systematically paired up for the first time in the battery management literature to tackle the SOC estimation problem. The proposed method is finally compared with the conventional Coulomb counting method. The results indicate that the proposed method significantly outperforms the Coulomb counting method in terms of accuracy and robustness.

**Index Terms**—Battery Management System, Coulomb Counting, Kalman Filter, Parameter Identification, State-of-Charge (SOC)

## I. INTRODUCTION

The state of charge (SOC) is defined as a ratio of the remaining capacity to the nominal capacity. The SOC is a critical parameter for control and energy management of battery systems. Accurate SOC estimation is crucial for prolonged battery life and improved fuel economy of electric vehicles. However, the battery SOC is not directly measurable during vehicle operation. Therefore, an efficient and effective algorithm is required to estimate the SOC from measured signals such as battery current and terminal voltage.

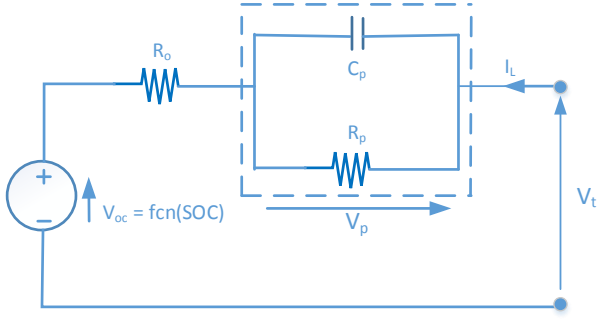
Current-based SOC estimation is obtained through conventional Coulomb (Ampere-hour) integration. However, this method suffers from the unknown initial value of the SOC. The initial SOC value is difficult to be determined accurately after key-on due to battery self-discharge during rest periods and the accumulated SOC estimation error over time from previous vehicle operation. Although current sensors with high accuracy are available, errors can be introduced by different sources such as data acquisition process, noise and analog to digital resolution. Moreover, the current-based SOC estimation method gives a worse SOC estimate when batteries age, because the battery capacity tends to decrease due to aging. On the other hand, the voltage-based SOC estimation method inferred from the open-circuit voltage (OCV) is independent of an initial SOC value and more robust to aging effects. To obtain the voltage-based SOC estimation method, the

battery state estimator estimates the OCV through a regression algorithm [17]. Then the estimated OCV is used to indicate the SOC. However, the estimation of SOC requires an accurate OCV-SOC relationship— If the OCV is flat in the mid SOC range, the prediction of the OCV even with small errors will cause highly erroneous SOC estimates.

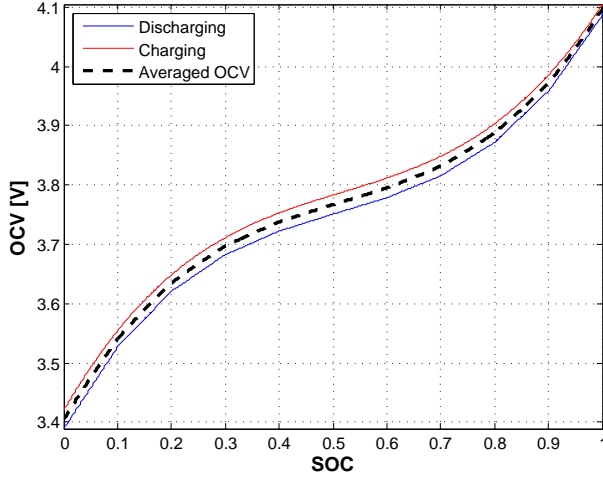
In order to extract the benefits of both the current and voltage based methods, they must be systematically combined. To this end, model-based techniques are widely used, e.g., the Kalman filter [2], [3], [10], [13], the sliding mode observer [8], [9] and  $H_\infty$  filters [21]. Although these techniques are capable of tackling nonlinearities and uncertainties in the battery model, they are all designed based on an off-line parameter identification of a battery model. Assuming fixed model parameters often leads to divergence whenever the field test results deviate from analytical results at different SOC and various environmental conditions.

One of the main motivations of this paper is to estimate the SOC accurately in a wide range of operating conditions throughout the battery lifetime. To this end, a dual estimation method based on an OCV-R-RC (OCV- Open Circuit Voltage, R- Resistance, C- Capacitance) equivalent circuit model is proposed. Although there exist dual estimation methods for SOC estimation [18], [19], the main novelty of this paper lies in the formulation of the state and parameter estimation and the choice of estimators— For online battery parameter estimation, the Square-Root Recursive Least Squares (SR-RLS) estimator is employed whereas the Extended Kalman-Bucy Filter (EKF) is applied to estimate the SOC for the first time ever in the battery management literature. These two Bayesian estimators operate cooperatively feeding each other with their latest estimates.

This rest of this paper is organized as follows: In section II, an electrical equivalent circuit model of a battery is presented. Section III presents a dual estimation strategy for battery SOC and parameter estimation. In this section, the recursive Square-Root Recursive Least Squares algorithm is adopted for the online parameter estimation of the battery model, whereas the extended Kalman-Bucy filter is used for SOC estimation. In section IV, the performance of the proposed algorithm is demonstrated using UDDS test profiles. Finally, the paper concludes with remarks.



(a) OCV-R-RC equivalent circuit model



(b) OCV-SOC relationship

Fig. 1.

## II. STATE-SPACE MODELING

In this section, the battery is modelled by the OCV-R-RC equivalent circuit as shown in Fig. 1(a). The OCV-R-RC model includes four key elements:

- Ohmic resistance ( $R_o$ ): This is associated with electrolyte resistance, plate resistance and fluid resistance.
- Charge transfer resistance ( $R_p$ ): This resistance is largely attributed the electrolyte diffusion during charging and discharging.
- Double layer capacitance ( $C_p$ ): This is the capacitance used to model the chemical diffusion of the electrolyte within the battery.
- Open circuit voltage ( $V_{oc}$ ): This is the battery voltage under the equilibrium condition and depends on the SOC and temperature.

Inside the RC network, the double layer capacitor is in parallel with the resistor due to the charge transfer reaction. The electrical behaviour of the OCV-R-RC circuit can be expressed

by the following equations:

$$\dot{V}_p = \frac{1}{C_p} I_L - \frac{1}{C_p R_p} V_p \quad (1)$$

$$V_t = V_{oc} + V_p + R_o I_L, \quad (2)$$

where  $V_p$  is the polarization voltage across the RC circuit,  $V_t$  is the terminal voltage and  $I_L$  is the load current. Positive current denotes the charging process.

As shown in Fig. 1(b), the OCV ( $V_{oc}$ ) is a nonlinear of function of the SOC and includes a hysteresis due to charging and discharging. In this paper, the final OCV is obtained by averaging the charge and discharge curves at the same SOC and approximately written in the form of a third order polynomial:

$$V_{oc} = \text{poly}(SOC) \approx p_0 + p_1 SOC + p_2 SOC^2 + p_3 SOC^3, \quad (3)$$

where the polynomial coefficients  $p_0, p_1, p_2$  and  $p_3$  can be determined via the least-squares algorithm off-line.

## III. BATTERY PARAMETER AND SOC ESTIMATION

### A. SOC Estimation

From Fig. 1(a), the state-space model of the equivalent circuit can be written as

$$\begin{pmatrix} \dot{SOC} \\ \dot{V}_p \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{\tau} \end{pmatrix} \begin{pmatrix} SOC \\ V_p \end{pmatrix} + \begin{pmatrix} \frac{1}{Q_{batt}} \\ \frac{1}{C_p} \end{pmatrix} I_L \quad (4)$$

$$V_t = V_{oc}(SOC) + V_p + R_o I_L, \quad (5)$$

where the states are assumed to be the SOC and  $V_p$ ; the time constant  $\tau$  is denoted as  $\tau = R_p C_p$ ; and  $Q_{batt}$  is the nominal capacity of the battery. The terminal voltage  $V_t$  is the measurement whereas the load current  $I_L$  is treated as an input to the model.

Based on (4)-(5), the extended Kalman-Bucy filter (EKBF) can be designed as follows:

$$\hat{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}I_L + \mathbf{K}(V_t - \hat{V}_t) \quad (6)$$

$$\hat{V}_t = V_{oc}(\hat{SOC}) + \hat{V}_p + \hat{R}_o I_L \quad (7)$$

where the state vector  $\mathbf{x}$  is defined as  $\mathbf{x} = [SOC \ V_p]^T$  and the matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by the following equations:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{\tau} \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} \frac{1}{Q_{batt}} \\ \frac{1}{C_p} \end{pmatrix}.$$

The EKBF gain  $\mathbf{K}$  is defined as

$$\mathbf{K} = \mathbf{P}\mathbf{C}\mathbf{R}^{-1}, \quad (8)$$

where  $\mathbf{C}$  is the Hessian of the measurement function,  $\mathbf{C} = \frac{\partial V_t}{\partial \mathbf{x}}$  and  $\mathbf{P}$  is the solution of the Ricatti equation:

$$\mathbf{P} = \mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T - \mathbf{K}\mathbf{R}\mathbf{K}^T + \mathbf{Q} \quad (9)$$

$$\mathbf{P}(0) = \mathbf{P}_0 \quad (10)$$

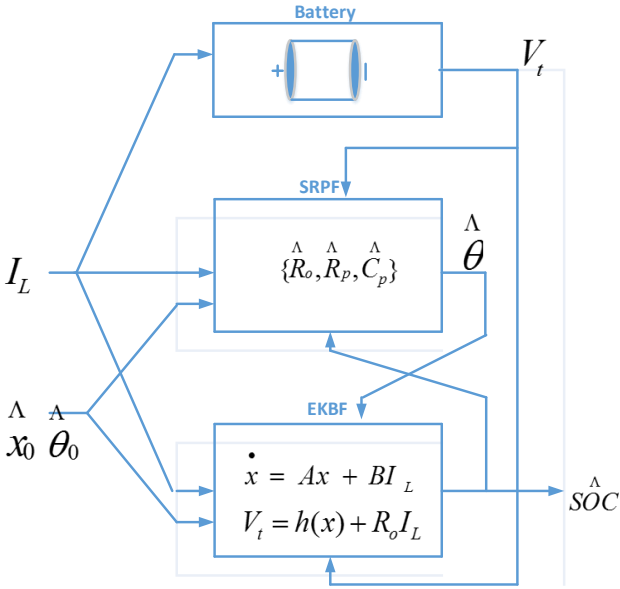


Fig. 2. Dual Bayesian Estimator for State and Parameter Estimation

Here  $\mathbf{Q}$  and  $\mathbf{R}$  are weighting matrices and can be appropriately tuned to minimize the quadratic battery voltage error.

### B. Battery Parameter Estimation

In this subsection, parameter estimation will be treated in an on-line estimation context. To set the stage, from (5), the discrete-time measurement equation is written as

$$V_{t,k+1} = V_{oc}(SOC_k) + V_{p,k+1} + R_o I_{L,k+1} \quad (11)$$

Let the output signal  $y$  denote the difference between  $V_{oc}$  and  $V_t$ . From (11), we write

$$y_{k+1} = V_{t,k+1} - V_{oc}(SOC_{k+1}) = V_{p,k+1} + R_o I_{L,k+1} \quad (12)$$

Consider the second differential equation of (4):

$$\dot{V}_p = -\frac{1}{\tau} V_p + \frac{1}{C_p} I_L \quad (13)$$

Solving the ordinary differential equation (13) for  $V_p$  yields

$$V_{p,k+1} = e^{-T_s/\tau} V_{p,k} + (1 - e^{-T_s/\tau}) R_p I_{L,k}, \quad (14)$$

where  $T_s$  is the sampling period. Substituting (14) into (12) yields

$$y_{k+1} = e^{-T_s/\tau} V_{p,k} + (1 - e^{-T_s/\tau}) R_p I_{L,k} + R_o I_{L,k+1} \quad (15)$$

Substituting the time index,  $k$ , in place of  $(k+1)$  in (12) yields

$$y_k = V_{p,k} + R_o I_{L,k} \quad (16)$$

Substituting  $V_{p,k}$  from (16) into (15) yields

$$y_{k+1} = e^{-T_s/\tau} y_k + (R_p - e^{-T_s/\tau} (R_p + R_o)) I_{L,k} + R_o I_{L,k+1} \quad (17)$$

Equation (17) now looks similar to an ARX (AutoRegression with eXogenous inputs) model:

$$y_{k+1} = a_0 y_k + b_0 I_{L,k} + b_1 I_{L,k+1}, \quad (18)$$

where the coefficients of the above ARX model are given by the following set of equations:

$$a_0 = e^{-T_s/\tau} \quad (19)$$

$$b_0 = -e^{-T_s/\tau} R_o + (1 - e^{-T_s/\tau}) R_p \quad (20)$$

$$b_1 = R_o \quad (21)$$

Let the vector-valued parameter  $\theta$  be defined as

$$\theta \triangleq [a_0 \ b_0 \ b_1]^T$$

Using (18), the linear dynamic state-space model for estimating the battery parameters can be expressed by the following state-space model:

$$\theta_{k+1} = \theta_k \quad (22)$$

$$y_{k+1} = [y_k \ I_k \ I_{k+1}] \theta_{k+1} \quad (23)$$

From the estimate of  $\theta$ , the battery parameters  $R_o$ ,  $R_p$  and  $C_p$  can be calculated using (19)-(21):

$$\hat{R}_o = b_1 \quad (24)$$

$$\hat{R}_p = \frac{b_0 + a_0 b_1}{1 - a_0} \quad (25)$$

$$\hat{C}_p = \frac{(a_0 - 1)T_s}{(a_0 b_1 + b_0) \log(a_0)} \quad (26)$$

### C. Dual Bayesian Estimation

This paper proposes a dual estimation scheme, which uses an iterative procedure to estimate the state (SOC) and battery parameters alternately [1], [7]. The idea of dual estimation is illustrated in Fig. 2. Dual estimation can be viewed as a generalized EM algorithm: E-step uses a state estimator whereas M- step performs model parameter estimation. The iterative optimization process guarantees the algorithm to converge to a suboptimal solution. For online battery parameter identification, the Square-Root recursive least squares (SR-RLS) estimator is employed whereas the Extended Kalman-Bucy Filter (EKBF) is adopted to estimate the SOC. We have named this dual estimator as **Parameter Adaptive Coulomb-counting Error compensatoR (PACER)**.

## IV. COMPUTER EXPERIMENTS

To demonstrate the effectiveness of the PACER, we simulated the battery using the OCV-R-RC model as shown by the Matlab/Simulink model in Fig. 3 with the following data:  $R_o = 56m\Omega$ ,  $R_p = 5m\Omega$  and  $C_p = 500$  C. We used a drive cycle created by joining six UDDS cycles together in time. The resulting drive cycle was found to be sufficient to drain the battery SOC from 80% to 34%. Fig. 5 shows the PACER estimate of  $R_o$ . As can be seen from this figure, the estimate quickly converges to a constant value of  $0.056\Omega$ , which closely matches the true value of  $R_o$ . Moreover, of three battery parameters,  $R_o$  seems to be the most observable parameter

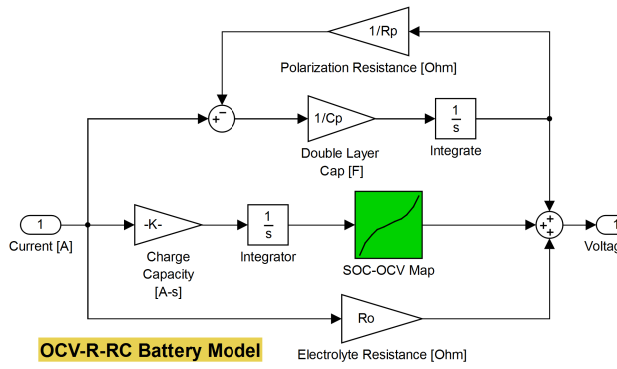


Fig. 3. OCV-R-RC type Battery Model Implemented in Matlab/Simulink

and seems to quickly converge. This is highly desirable from a battery management system perspective because battery aging can be accurately predicted in terms of  $R_o$ .

The PACER was compared with the traditional Coulomb counting method in estimating the battery SOC. The initial SOC estimates of both estimators were set to be 60% whereas the true SOC was set to be 80%. As shown in Fig. 6(a), the Coulomb counting method does not fix the initial estimation error and it is present throughout the test drive cycle. On the other hand, the fusion of the voltage feedback with the input current helps the PACER to quickly catch up with the true SOC. It also tracks the SOC reasonably accurately until the end of the test drive cycle. As shown in Fig. 6(b), the absolute value of the SOC error of the PACER lies within the limit of 3% on average, except a few error overshoots above 3%.

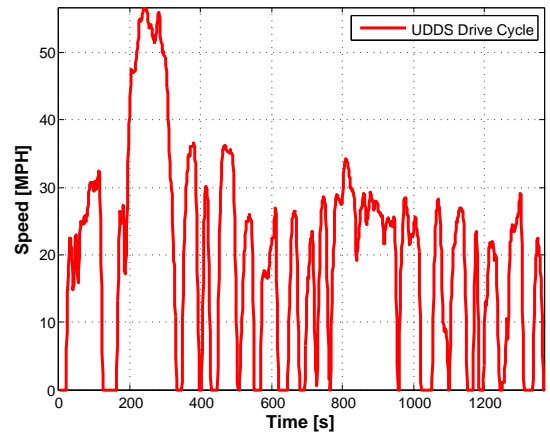
In a nutshell, the PACER is a more accurate SOC estimator than the traditional Coulomb counting method and highly recommended for automotive applications where stringent SOC accuracy requirements have to be met.

## V. CONCLUDING REMARKS

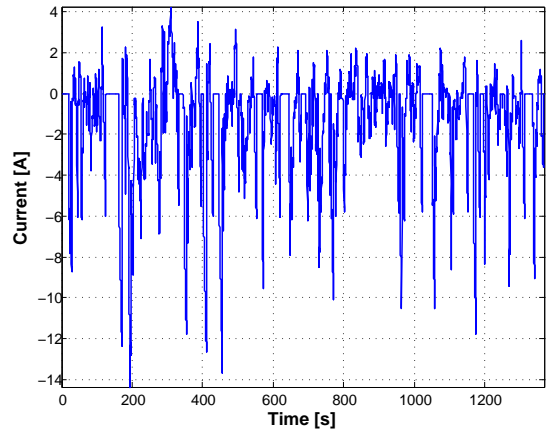
This paper describes the development of a dual estimator for SOC estimation of Li-ion batteries. The proposed dual estimator uses the square-root RLS algorithm for battery model parameter estimation and the extended Kalman-Bucy filter for SOC estimation. The two estimators work cooperatively to produce accurate SOC estimates. The simulation results have proved that the proposed dual estimator is capable of estimating the SOC within 3% error bound on average. During SOC estimation, the battery model parameters are also estimated online, which can be used for predicting the state-of-health and state-of-power of the battery. One of the future interesting research topics could be addressing the effect of current and voltage sensor noise contributions on choosing the weight matrices  $\mathbf{Q}$  and  $\mathbf{R}$  of the extended Kalman-Bucy filter.

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(a) UDDS Speed Profile



(b) Current Profile

Fig. 4. Speed and current profiles for one UDDS drive cycle

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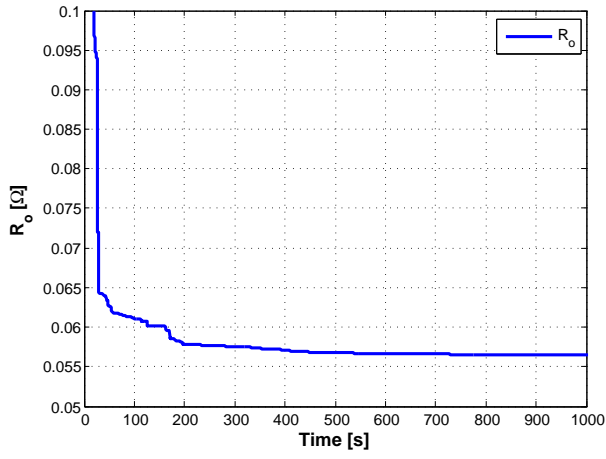
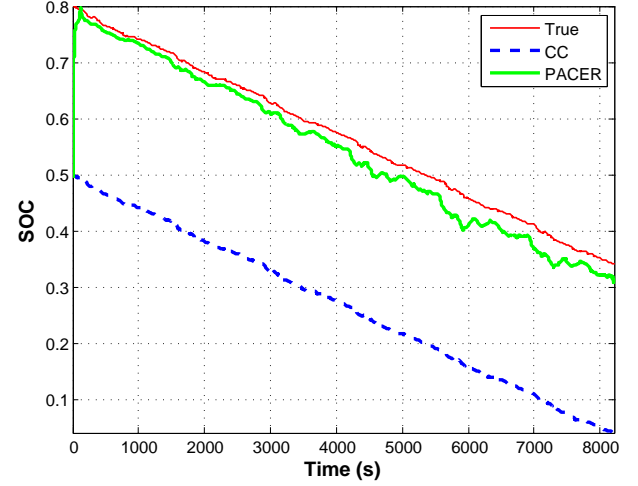
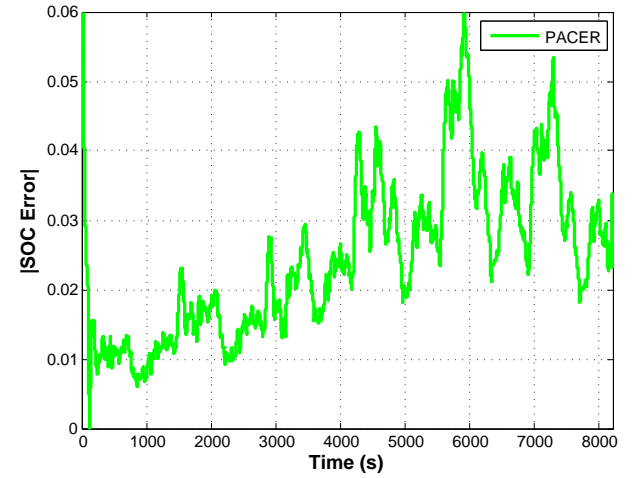


Fig. 5. Battery parameter estimated by PACER

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(a) SOC



(b) SOC estimation error

Fig. 6. SOC estimation results (CC- Coulomb Counting)