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Surrogate-assisted Differential Evolution with Adaptation of Training Data Selection Criterion

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Background

- **Surrogate-assisted Evolutionary Algorithm (SAEA)** [Jin 11]
  - SAEA prescreens candidate solutions by surrogates
    - Machine learning models are used to estimate fitness values of solutions
    - SAEA can prevent the large consumption of the number of fitness evaluations (FEs)
  - SAEAs are representative methodology to solve Expensive Optimization Problems (EOP)

    In EOP…
    - FEs are computationally or financially expensive
    - Thus, the number of FEs are strictly limited
- **It is crucial to use proper surrogate options to particular problems**
  - The model accuracy should be improved to enhance the performance of SAEAs [Tong+ 21]
    - The performance of SAEAs depends on the settings of surrogates and training data [Díaz-Manríquez+ 21]

Related Work 1/2

Training Data Selection Criterion

- Various criteria have proposed and improved the performance of SAEAs

Criterion	Definition	Comment	Example
All Data	all data in archive (Archive is the place where all evaluated data are stored)	**Global model** is suitable to reflect the entire fitness landscape and thus helps SAEAs select promising solutions [F.Yu+ 22]	S-JADE [Cai+ 19], RBFBS [Li+ 19], SACOBRA MQ-Cubic [Bagheri+ 16]
Current Population	current population and corresponding fitness values	**Local model** enhances the model accuracy in the corresponding search space [F.Yu+ 22]	iDEaSm [Awad+ 18], aRBF-NFO [M.Yu+ 21]
Recent Data	certain number of recently evaluated data and corresponding fitness values		GPEME [Liu+ 14]
Neighbor	data in the archive that is close to the population	**Global-oriented local model** raises the model accuracy in a marginal area [Pan+ 21]	S-JADE, SAHO [Pan+ 21], GORS-SSLPSO [H.Yu+ 19]

  - Some research attempts to ensemble two different criteria [Wang+ 19] [Cai+ 19]
    - The ensemble can take advantage of the strength of different criteria
- Proper criterion can change dependent on the problems and/or search situations since surrogates with different criteria have different properties [F.Yu+ 22]
  - However, the criterion is typically fixed through the search process in most SAEAs

Related Work 2/2

- **Adaptive SAEA**: adjusts surrogate settings during a single run

Classification of single-objective adaptive SAEAs

Adaptation Target	Algorithm	Detailed Target	Comment
Model Type	ASAGA [shi+ 08]	model type (6 candidates)	select model with correlation coefficient
	SUMO [Gorissen+ 09]	model type (7 candidates)	select model with RMSE
	ASMDE, ASMPSO [M.Yu+ 20]	model type (5 candidates)	
Model Settings	RBFBS [Li+ 19]	spread parameter ε of RBF model	select model with RMSE
	SACOBRA MQ-Cubic [Bagheri+ 16]	kernel function & ε of RBF	select model with MAE
	aRBF-NFO [M.Yu+ 21]	kernel function of RBF	select model with RMSE
	EAs with DACE model [Lophaven+ 02]	length scale parameter θ of Kriging model	optimize θ by Hookes-Jeeves method [Hooke+ 61]
	Sa-DE-DPS [Elsayed+ 14]		optimize θ by Differential Evolution [Storn 97]
	iDEaSm [Awad+ 18]		optimize θ using covariant matrix
Training Data	HESNFO [M.Yu+ 22]	Training data and kernel function of RBF	**use bootstrap sampling to construct multiple RBF models**
	DSS-DE [Ma+ 22]	Training data	

- Few works have attempted to adapt the training data
  - and they simply pick data at random

Research Purpose	To propose a novel SAEA with adaptation of training data selection criterion

Components

- **Differential Evolution (DE)** [Storn 97]
  - population-based evolutionary algorithm

Algorithm 1 DE/best/1/bin

1: Initialize $\mathcal{P} = \{\boldsymbol{x}_i\}_{i=1}^N$ and Evaluate all $\boldsymbol{x}_i$ — Initialization

2: **while** termination criteria are not met **do**

3:   **for** $i = 1$ **to** N **do**

4:     $\boldsymbol{v}_i = \boldsymbol{x}_{best} + F(\boldsymbol{x}_{r_1} - \boldsymbol{x}_{r_2})$ — *best/1* mutation

5:     **for** $j = 1$ **to** D **do**

6:       $u_{i,j} = \begin{cases} v_{i,j} & \text{if } rand(0,1) \le CR \text{ or } j = j_{rand}, \\ x_{i,j} & \text{otherwise} \end{cases}$

7:     **end for**

8:   **end for** — *binomial* crossover

9:   **for** $i = 1$ **to** N **do**

10:     Evaluate $\boldsymbol{u}_i$

11:     $\boldsymbol{x}_i = \begin{cases} \boldsymbol{u}_i & \text{if } f(\boldsymbol{u}_i) \le f(\boldsymbol{x}_i), \\ \boldsymbol{x}_i & \text{otherwise} \end{cases}$ — Selection

12:   **end for**

13: **end while**

- **RBF model** Referring to the implementation of [Regis 14]
  - nonparametric fitting method

training data

$$\mathcal{X} = \{\boldsymbol{x}_i\}_{i=1}^n \quad \boldsymbol{f} = [f(\boldsymbol{x}_1), f(\boldsymbol{x}_2), \ldots, f(\boldsymbol{x}_n)]^\mathsf{T}$$

approximation of $f(\boldsymbol{x})$

$$\hat{f}(\boldsymbol{x}) = \sum_{i=1}^{n} \lambda_i \phi\left(\|\boldsymbol{x} - \boldsymbol{x}_i\|\right) + p(\boldsymbol{x})$$

weight vector

$$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_n]^\mathsf{T}$$

cubic kernel as a Radial Basis Function (RBF)

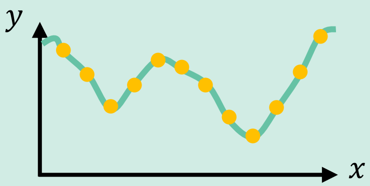
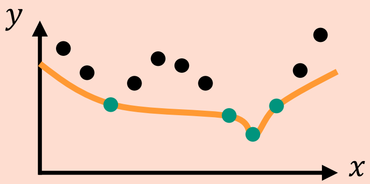
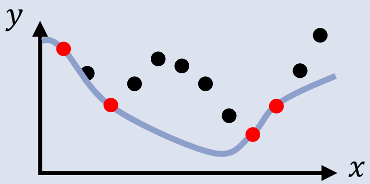
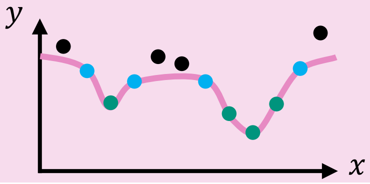
$$\phi(r) = r^3$$

polynomial function as a regularization term

$$p(\boldsymbol{x}) = \boldsymbol{c}^\mathsf{T}\boldsymbol{x} + c_0 \ (\boldsymbol{c} \in \mathbb{R}^D, c_0 \in \mathbb{R})$$

SADE-ATDSC: Preliminary

- **Training Data Selection Criteria as adaptation candidates**
  - SADE-ATDSC adapts the criterion depending on problems and search situations
  - Definitions

m	Criterion	Definition	Comment	Image in 1D
1	**All Data**	all data in $\mathcal{A}$	suitable to reflect the entire fitness landscape	
2	**Current Population**	$\mathcal{P}$ and $\mathcal{F}$	enhance the model accuracy in the corresponding search space	
3	**Recent Data**	n recently evaluated data and their fitness values		
4	**Neighbor**	union of selected n data near each $\boldsymbol{x}_i \in \mathcal{P}$ from $\mathcal{A}$	raise the model accuracy in a marginal area	

FE	The number of FEs	$\mathcal{A}$	Archive
N	Population size	$\mathcal{P}$	DE population
$\mathcal{F}$	A set of corresponding fitness values of $\boldsymbol{x}_i \in \mathcal{P}$		
M	The number of criteria and candidate RBF models		
n	Hyper-parameter to define data size		
$\mathcal{D}_m$	Training data obtained by m-th criterion		

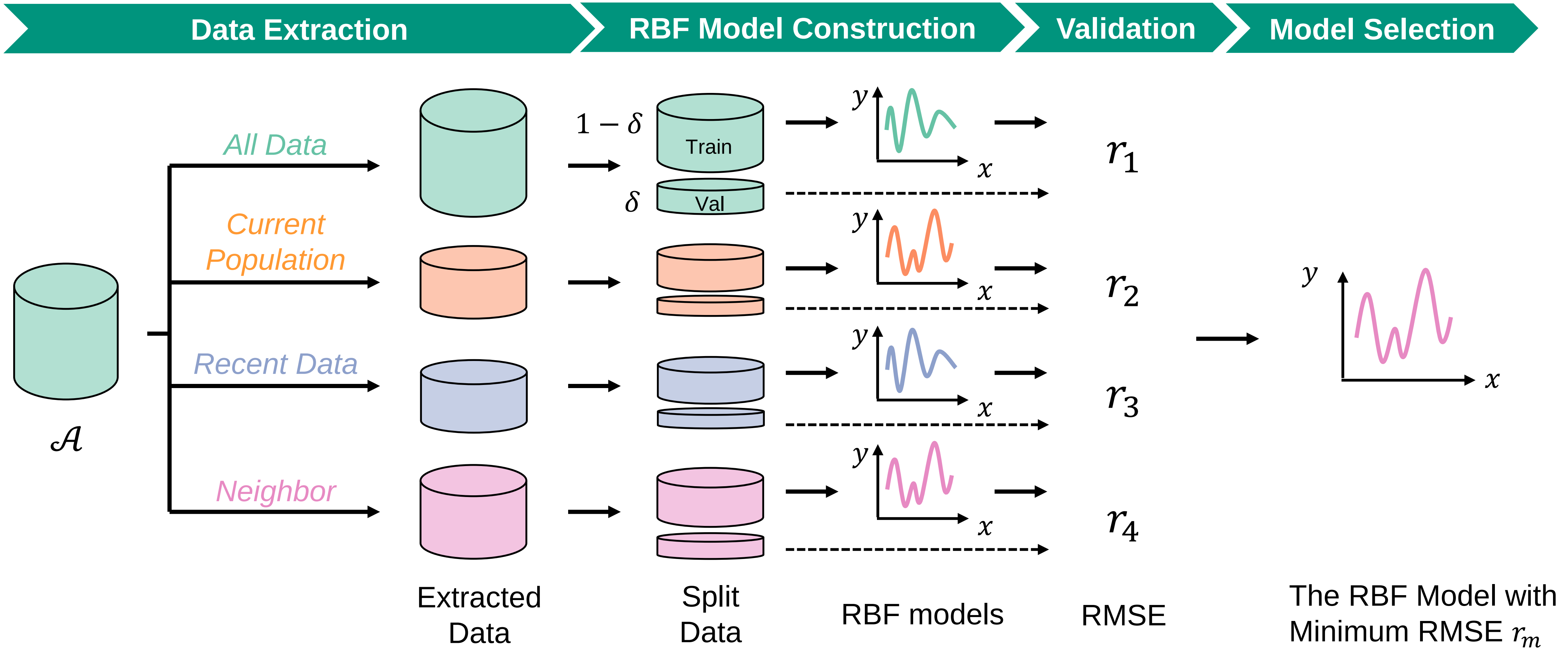
Detailed procedure to obtain training data

Algorithm 2 *Get-Training-Data*$(\mathcal{A}, \mathcal{P}, \mathcal{F}, FE, N, n, m)$

switch m **do**
  case 1 // *All Data*
    return $\mathcal{D}_{AD} = \mathcal{A}$
  case 2 // *Current Population*
    return $\mathcal{D}_{CP} = \{(\mathcal{P}, \mathcal{F})\}$
  case 3 // *Recent Data*
    return $\mathcal{D}_{RD} = \{(\boldsymbol{x}_i, f(\boldsymbol{x}_i))\}_{i=FE-n+1}^{FE}$
  case 4 // *Neighbor*
    $\mathcal{D}_{NB} = \emptyset$
    for $i = 1$ **to** N **do**
      $\mathcal{D}_{temp} \leftarrow$ Select the Nearest n data of $\boldsymbol{x}_i \in \mathcal{P}$ from $\mathcal{A}$
      $\mathcal{D}_{NB} = \mathcal{D}_{NB} \cup \mathcal{D}_{temp}$, $\mathcal{D}_{temp} = \emptyset$
    end for
    return $\mathcal{D}_{NB}$
end switch

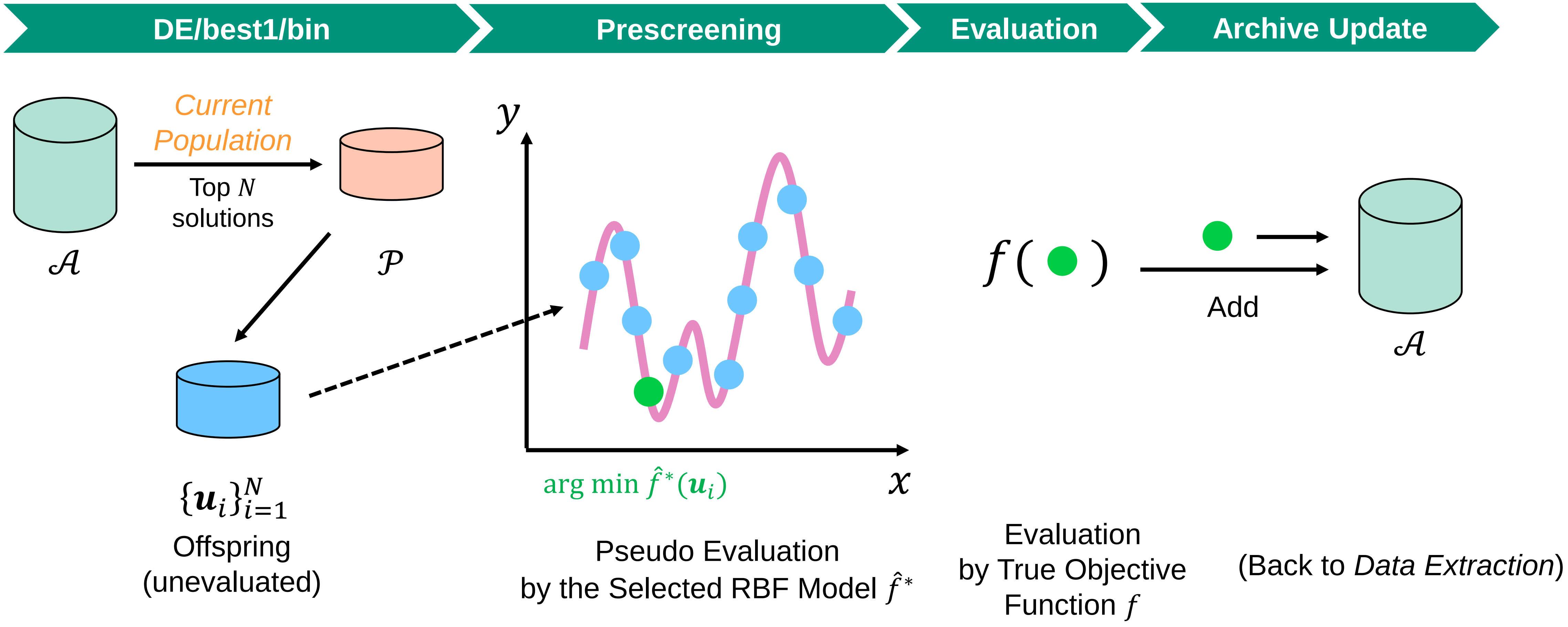
SADE-ATDSC: Mechanism 1/2

- **RBF model adaptation**
  - The model with the best accuracy is selected



SADE-ATDSC: Mechanism 2/2

- **Prescreening of candidate solutions**
  - The solution expected to have the best fitness is selected



SADE-ATDSC: Summary

Whole Procedure

Algorithm 3 SADE-ATDSC

Initialize $\mathcal{P} = \{\boldsymbol{x}_i\}_{i=1}^{N}$ by LHS and Evaluate
$\mathcal{F} = \{f(\boldsymbol{x}_i)\}_{i=1}^{N}$, $\mathcal{A} = \{(\boldsymbol{x}_i, f(\boldsymbol{x}_i))\}_{i=1}^{N}$, $FE = N$
while $FE < FE_{\max}$ **do**
  $\mathcal{S} = \emptyset$, $\mathcal{R} = \emptyset$
  for $m = 1$ **to** M **do**
    $\mathcal{D}_m \leftarrow$ *Get-Training-Data*$(\mathcal{A}, \mathcal{P}, \mathcal{F}, FE, N, n, m)$
    $\mathcal{D}_{m,tra}$, $\mathcal{D}_{m,val} \leftarrow$ Split $\mathcal{D}_m$ by the ratio of $(1-\delta) : \delta$
    $\hat{f}_m \leftarrow$ Build the RBF model with $\mathcal{D}_{m,tra}$
    $r_m \leftarrow$ Get RMSE in Validation of $\hat{f}_m$ with $\mathcal{D}_{m,val}$
    $\mathcal{S} = \mathcal{S} \cup \{\hat{f}_m\}$, $\mathcal{R} = \mathcal{R} \cup \{r_m\}$
  end for
  $\hat{f}^* \leftarrow \hat{f}_m \in \mathcal{S}$ that derived minimum $r_m \in \mathcal{R}$
  $\mathcal{P}$, $\mathcal{F} \leftarrow$ Select top N data and their fitness values from $\mathcal{A}$
  for $i = 1$ **to** N **do**
    $\boldsymbol{v}_i \leftarrow$ *best/1-mutation* with $\mathcal{P}$, F
    $\boldsymbol{u}_i \leftarrow$ *binomial-crossover* with $\boldsymbol{x}_i$, $\boldsymbol{v}_i$, CR
    $\hat{f}^*(\boldsymbol{u}_i) \leftarrow$ Evaluate $\boldsymbol{u}_i$ with $\hat{f}^*$
  end for
  $\boldsymbol{u}^* = \underset{\{\boldsymbol{u}_i\}_{i=1}^{N}}{\arg} \underset{\{\hat{f}^*(\boldsymbol{u}_i)\}_{i=1}^{N}}{\min} \left(\hat{f}^*(\boldsymbol{u}_i)\right)$
  $f(\boldsymbol{u}^*) \leftarrow$ Evaluate $\boldsymbol{u}^*$ with the true function f
  $FE = FE + 1$
  $\mathcal{A} = \mathcal{A} \cup \{(\boldsymbol{u}^*, f(\boldsymbol{u}^*))\}$
end while

Initialization

RBF Model Selection

DE/best/1/bin

Prescreening of Candidate Solutions

Evaluation & Archive Update

FE	The number of FEs	$\mathcal{A}$	Archive
N	Population size	$\mathcal{P}$	DE population
$\mathcal{F}$	A set of corresponding fitness values of $\boldsymbol{x}_i \in \mathcal{P}$		
M	The number of criteria and candidate RBF models		
n	Hyper-parameter to define data size		
$\mathcal{D}_m$	Training data obtained by m-th criterion		
$\mathcal{S}$	Set of candidate RBF models		
$\mathcal{R}$	Set of RMSE		
δ	Validation data ratio		

Experiment: Settings

- **Real-Parameter Single Objective Optimization Problem**
  - CEC 2013 benchmark suite (28 Problems, $D = \{10, 30, 50, 100\}$, $FE_{\max} = 1{,}000$) [Liang+ 13]
  - Compared Algorithms
    - RBF-based non-adaptive SAEAs
      - **S-JADE**：$N = 30, F_{out} = 0.5, CR_{out} = 0.75, p_{pbest\_out} = 0.05, F_{in} = 0.5, CR_{out} = 0.5, p_{pbest\_in} = 0.1, std_F = 0.1, std_{CR} = 0.1, L = 10, \epsilon = 0.01, c = 0.1, evals_{in} = 2{,}000, kernel_{RBF} = cubic, r = rand(0, 1.25)$ All Data Neighbor
      - **SAHO**：$N = 5D(D < 50)$ or $100 + \lfloor D/10 \rfloor (D \geq 50),, F = 0.5, CR = 0.9, K = 30, neighbour = 5D(D < 50)$ or $D(D \geq 50), kernel_{RBF} = cubic$ Neighbor
    - RBF-based adaptive SAEA
      - **aRBF-NFO**：$N = 5D, c = 1.3, CR = 0.1, |test| = N/100, \epsilon = 1, kernel_{RBF} = cubic$ Current Population
    - **Ours**
      - **SADE-ATDSC**：$N = 100, F = 0.5, CR = 0.9, kernel_{RBF} = cubic, \delta = 0.2$
  - Evaluation Metrics
    - Average over 21 trials of the best fitness value
    - Wilcoxon signed-rank test
    - Average rank

Experiment: Results (Fitness at 1,000FEs)

- **SADE-ATDSC outperforms the compared algorithms**
  - SADE-ATDSC is superior with statistical significance (the number of “-” is 12 to 23)
  - The best average rank is obtained by SADE-ATDSC in all dimensions

+/−/∼ in Wilcoxon test
+ : ours underperforms
− : ours outperforms
∼ : cannot find significance

Best value

	D = 10				D = 30			
	S-JADE	SAHO	aRBF-NFO	SADE-ATDSC	S-JADE	SAHO	aRBF-NFO	SADE-ATDSC
F1	7.30E-06 −	**2.18E-28** +	1.74E+03 −	5.09E-20	6.78E+00 ∼	**1.20E-15** +	1.87E+04 −	8.43E+01
F2	2.91E+06 −	**9.66E+05** ∼	1.90E+07 −	1.38E+06	8.81E+07 −	**1.16E+07** +	3.75E+08 −	2.88E+07
F3	5.50E+09 −	6.17E+10 −	4.90E+09 −	**6.03E+03**	1.49E+15 −	2.56E+17 −	8.60E+13 −	**7.39E+09**
F4	**1.88E+04** +	2.56E+04 +	6.16E+04 ∼	4.43E+04	**8.68E+04** +	1.22E+05 +	1.52E+05 +	1.88E+05
F5	4.34E+01 −	2.19E-03 −	1.67E+03 −	**5.21E-04**	3.13E+03 −	**1.79E+02** ∼	2.05E+04 −	2.25E+02
F6	**7.97E+00** ∼	9.33E+00 ∼	1.43E+02 −	8.53E+00	1.04E+02 −	**4.79E+01** ∼	1.02E+03 −	5.28E+01
F7	1.07E+02 −	3.37E+02 −	1.12E+02 −	**2.87E+01**	1.53E+04 −	3.00E+05 −	2.47E+03 −	**2.08E+02**
F8	**2.07E+01** ∼	2.08E+01 ∼	2.08E+01 ∼	**2.07E+01**	**2.12E+01** ∼	**2.12E+01** ∼	**2.12E+01** ∼	**2.12E+01**
F9	6.72E+00 ∼	7.42E+00 −	9.16E+00 −	**5.68E+00**	3.66E+01 −	3.13E+01 −	4.35E+01 −	**2.42E+01**
F10	4.36E-01 ∼	6.72E-01 −	2.86E+02 −	**3.29E-01**	5.71E+01 ∼	**1.46E+00** +	2.96E+03 −	7.53E+01
F11	4.08E+01 −	4.75E+01 −	4.13E+01 −	**1.93E+01**	2.85E+02 −	2.48E+02 −	4.75E+02 −	**1.50E+02**
F12	5.33E+01 −	3.50E+01 −	7.90E+01 −	**2.27E+01**	3.14E+02 −	2.21E+02 −	5.56E+02 −	**1.68E+02**
F13	5.49E+01 ∼	6.26E+01 −	8.63E+01 −	**4.75E+01**	3.18E+02 ∼	3.07E+02 ∼	5.93E+02 −	**2.98E+02**
F14	1.75E+03 −	1.23E+03 ∼	**1.04E+03** ∼	1.06E+03	7.82E+03 ∼	**6.22E+03** +	8.05E+03 ∼	7.73E+03
F15	1.97E+03 −	**1.43E+03** ∼	2.03E+03 −	1.59E+03	8.67E+03 ∼	**6.32E+03** +	8.80E+03 ∼	8.77E+03
F16	2.40E+00 ∼	**2.09E+00** +	2.35E+00 +	2.66E+00	4.59E+00 ∼	4.55E+00 ∼	**4.26E+00** ∼	4.55E+00
F17	5.20E+01 −	**3.31E+01** ∼	7.25E+01 −	3.89E+01	**2.74E+02** +	**2.74E+02** +	1.01E+03 −	3.36E+02
F18	5.77E+01 ∼	**3.86E+01** +	1.11E+02 −	6.18E+01	**2.86E+02** +	2.96E+02 +	1.21E+03 −	3.55E+02
F19	8.33E+00 −	1.88E+02 −	1.77E+01 −	**2.29E+00**	3.58E+04 −	3.75E+05 −	7.74E+03 −	**2.12E+01**
F20	4.37E+00 −	4.56E+00 −	4.37E+00 −	**3.92E+00**	1.50E+01 −	1.50E+01 −	1.50E+01 −	**1.48E+01**
F21	4.26E+02 −	4.46E+02 −	4.97E+02 −	**3.91E+02**	2.46E+03 −	4.24E+03 −	2.65E+03 −	**3.24E+02**
F22	1.94E+03 −	1.59E+03 −	1.32E+03 ∼	**1.22E+03**	8.42E+03 ∼	**6.24E+03** +	8.67E+03 ∼	7.77E+03
F23	2.24E+03 −	1.77E+03 −	2.17E+03 −	**1.43E+03**	9.20E+03 ∼	**7.14E+03** +	9.51E+03 −	8.98E+03
F24	2.18E+02 ∼	2.16E+02 ∼	2.24E+02 −	**2.15E+02**	2.98E+02 −	2.84E+02 −	3.18E+02 −	**2.67E+02**
F25	**2.17E+02** ∼	**2.17E+02** ∼	2.25E+02 −	2.18E+02	3.14E+02 −	3.02E+02 −	3.36E+02 −	**2.89E+02**
F26	2.05E+02 ∼	2.11E+02 ∼	2.29E+02 ∼	**1.92E+02**	**3.32E+02** ∼	3.48E+02 ∼	3.61E+02 ∼	3.39E+02
F27	**4.30E+02** +	5.22E+02 ∼	6.00E+02 −	4.86E+02	1.17E+03 −	1.04E+03 −	1.41E+03 −	**9.56E+02**
F28	8.28E+02 −	1.41E+03 −	8.74E+02 −	**4.19E+02**	4.63E+03 −	7.39E+03 −	4.45E+03 −	**7.35E+02**
+/−/∼	2/16/10	4/14/10	1/22/5		3/15/10	10/12/6	1/21/6	
average rank	2.571	2.571	3.393	**1.464**	2.607	2.071	3.429	**1.893**

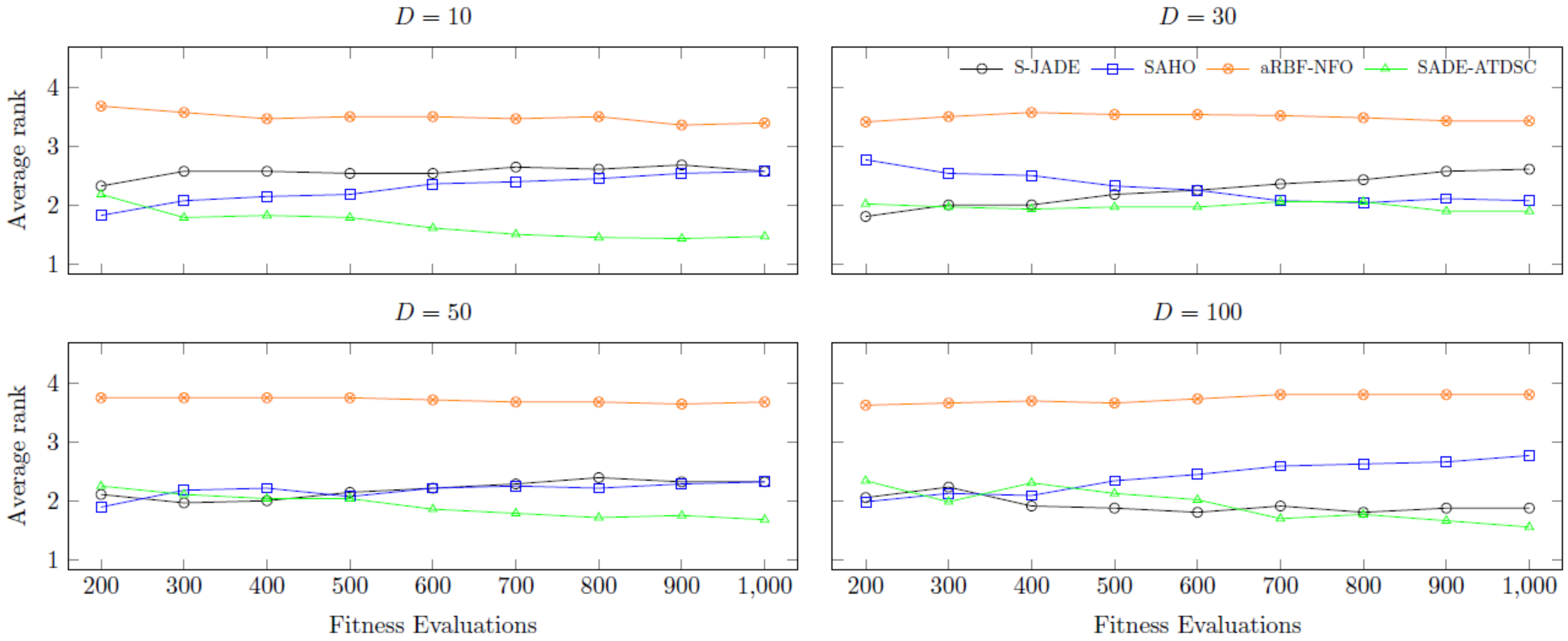
	D = 50				D = 100			
	S-JADE	SAHO	aRBF-NFO	SADE-ATDSC	S-JADE	SAHO	aRBF-NFO	SADE-ATDSC
F1	1.57E+02 ∼	**2.82E-06** +	4.45E+04 −	3.15E+02	3.66E+03 −	6.16E+03 −	3.17E+05 −	**2.66E+03**
F2	1.61E+08 ∼	**5.26E+07** +	1.56E+09 −	1.37E+08	6.39E+08 ∼	9.00E+08 −	1.12E+10 −	**5.62E+08**
F3	7.54E+12 −	3.73E+16 −	9.91E+15 −	**1.24E+11**	5.16E+20 −	9.67E+23 −	4.14E+23 −	**4.69E+19**
F4	**1.26E+05** +	2.07E+05 +	2.36E+05 ∼	2.54E+05	**2.92E+05** +	4.13E+05 ∼	4.61E+05 ∼	4.53E+05
F5	1.13E+04 −	2.42E+03 ∼	5.38E+04 −	**2.32E+03**	2.79E+04 −	2.87E+04 −	3.23E+05 −	**1.59E+04**
F6	2.51E+02 −	**4.74E+01** +	3.92E+03 −	7.16E+01	2.72E+03 −	3.43E+03 −	9.65E+04 −	**1.37E+03**
F7	2.56E+03 −	1.33E+05 −	3.29E+04 −	**2.59E+02**	5.29E+06 −	2.07E+08 −	1.47E+08 −	**9.11E+05**
F8	**2.13E+01** ∼	**2.13E+01** ∼	2.14E+01 ∼	**2.13E+01**	**2.15E+01** ∼	**2.15E+01** ∼	**2.15E+01** ∼	**2.15E+01**
F9	7.01E+01 −	5.78E+01 −	7.97E+01 −	**4.91E+01**	1.60E+02 −	1.54E+02 −	1.71E+02 −	**1.22E+02**
F10	4.69E+02 −	**4.64E+01** +	7.24E+03 −	2.41E+02	3.12E+03 ∼	4.34E+03 −	4.94E+04 −	**2.91E+03**
F11	5.40E+02 −	**2.69E+02** +	1.07E+03 −	3.29E+02	**1.41E+03** ∼	1.73E+03 −	5.00E+03 −	1.45E+03
F12	5.91E+02 −	**2.79E+02** +	1.25E+03 −	3.83E+02	**1.53E+03** ∼	1.67E+03 −	5.21E+03 −	1.54E+03
F13	6.14E+02 +	**6.03E+02** +	1.33E+03 −	6.71E+02	**1.56E+03** +	1.70E+03 ∼	5.30E+03 −	1.70E+03
F14	1.44E+04 −	1.43E+04 −	1.45E+04 −	**1.13E+04**	3.33E+04 −	3.43E+04 −	3.41E+04 −	**3.06E+04**
F15	1.61E+04 ∼	1.62E+04 ∼	1.62E+04 ∼	**1.59E+04**	3.38E+04 ∼	3.40E+04 ∼	3.42E+04 ∼	**3.32E+04**
F16	5.58E+00 ∼	5.66E+00 ∼	**5.22E+00** ∼	5.35E+00	5.86E+00 ∼	5.88E+00 ∼	5.81E+00 ∼	**5.75E+00**
F17	**5.05E+02** +	6.09E+02 +	2.44E+03 −	6.95E+02	**1.35E+03** +	1.89E+03 +	1.01E+04 −	2.21E+03
F18	**5.54E+02** +	6.21E+02 +	2.88E+03 −	7.72E+02	**1.39E+03** +	1.81E+03 +	1.04E+04 −	2.24E+03
F19	4.12E+04 −	3.05E+03 −	2.02E+06 −	**6.79E+01**	1.33E+06 −	8.46E+05 −	5.17E+07 −	**4.51E+04**
F20	2.49E+01 −	2.50E+01 −	2.50E+01 −	**2.48E+01**	**5.00E+01** ∼	**5.00E+01** ∼	**5.00E+01** ∼	**5.00E+01**
F21	4.76E+03 −	7.46E+03 −	7.03E+03 −	**8.62E+02**	9.26E+03 ∼	1.45E+04 −	2.12E+04 −	**8.84E+03**
F22	1.57E+04 −	1.57E+04 −	1.62E+04 −	**1.28E+04**	3.49E+04 −	3.50E+04 −	3.58E+04 −	**2.99E+04**
F23	1.67E+04 ∼	1.68E+04 ∼	1.71E+04 ∼	**1.66E+04**	**3.54E+04** ∼	3.57E+04 ∼	3.59E+04 ∼	**3.54E+04**
F24	3.79E+02 −	3.37E+02 ∼	4.23E+02 −	**3.35E+02**	5.97E+02 −	5.78E+02 −	1.65E+03 −	**5.42E+02**
F25	4.14E+02 −	3.77E+02 ∼	4.65E+02 −	**3.66E+02**	6.82E+02 −	6.42E+02 −	1.11E+03 −	**5.94E+02**
F26	4.59E+02 −	5.15E+02 −	4.97E+02 −	**4.10E+02**	7.22E+02 −	7.49E+02 −	7.65E+02 −	**6.07E+02**
F27	2.05E+03 −	1.73E+03 −	2.38E+03 −	**1.55E+03**	4.23E+03 −	4.14E+03 −	6.27E+03 −	**3.54E+03**
F28	8.39E+03 −	1.24E+04 −	8.70E+03 −	**1.60E+03**	2.17E+04 ∼	3.23E+04 −	3.38E+04 −	**2.10E+04**
+/−/∼	4/18/6	10/11/7	0/23/5		4/13/11	2/19/7	0/22/6	
average rank	2.321	2.321	3.679	**1.679**	1.875	2.768	3.804	**1.554**

Experiment: Results (Summary)

+/−/∼ **in Wilcoxon test**
+ : ours underperforms
− : ours outperforms
∼ : cannot find significance

- **SADE-ATDSC has the scalability of performance to D**
  - SADE-ATDSC keeps deriving the best performance with the increase of D
  - The scalability to FEs is slightly weak (sometimes 2nd or 3rd rank at <500FEs)
  - The number of “-” is larger than that of “+” for almost all case in Wilcoxon test

Average rank

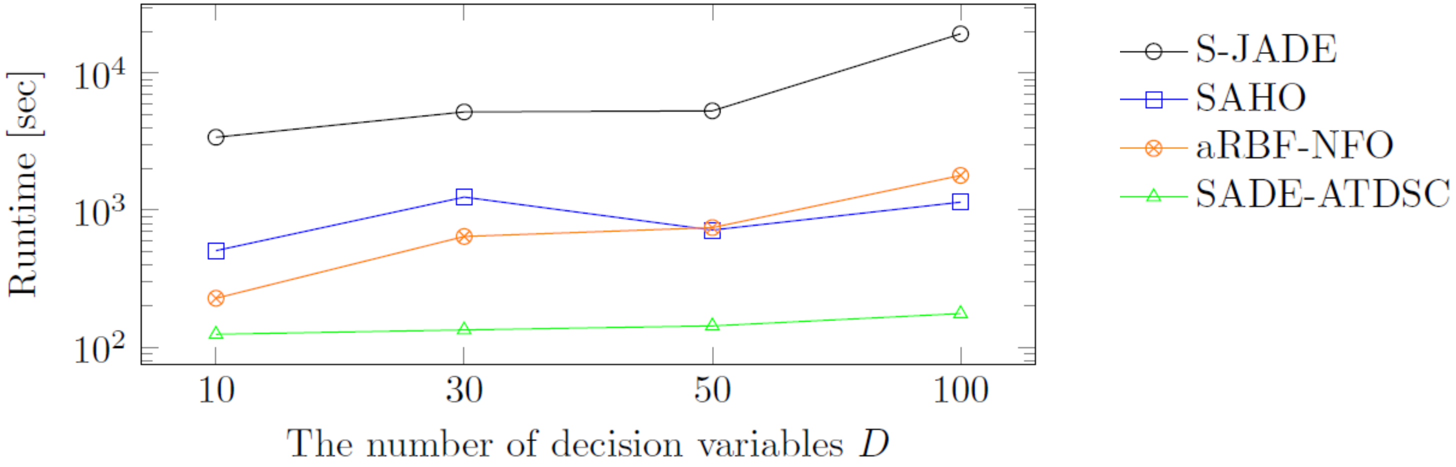


Wilcoxon signed-rank test

D	FE	vs S-JADE	vs SAHO	vs aRBF-NFO
10	300	4/16/ 8	7/ 8/13	1/22/ 5
	500	1/18/ 9	5/10/13	0/23/ 5
	1,000	2/16/10	4/14/10	1/22/ 5
30	300	6/11/11	5/12/11	0/20/ 8
	500	5/14/ 9	7/10/11	1/21/ 6
	1,000	3/15/10	10/12/ 6	1/21/ 6
50	300	11/10/ 7	10/ 8/10	0/21/ 7
	500	8/11/ 9	10/11/ 7	0/23/ 5
	1,000	4/18/ 6	10/11/ 7	0/23/ 5
100	300	6/ 9/13	13/ 6/ 9	0/21/ 7
	500	13/ 8/ 7	9/11/ 8	0/23/ 5
	1,000	4/13/11	2/19/ 7	0/22/ 6

Discussion 1/3: Computational Time

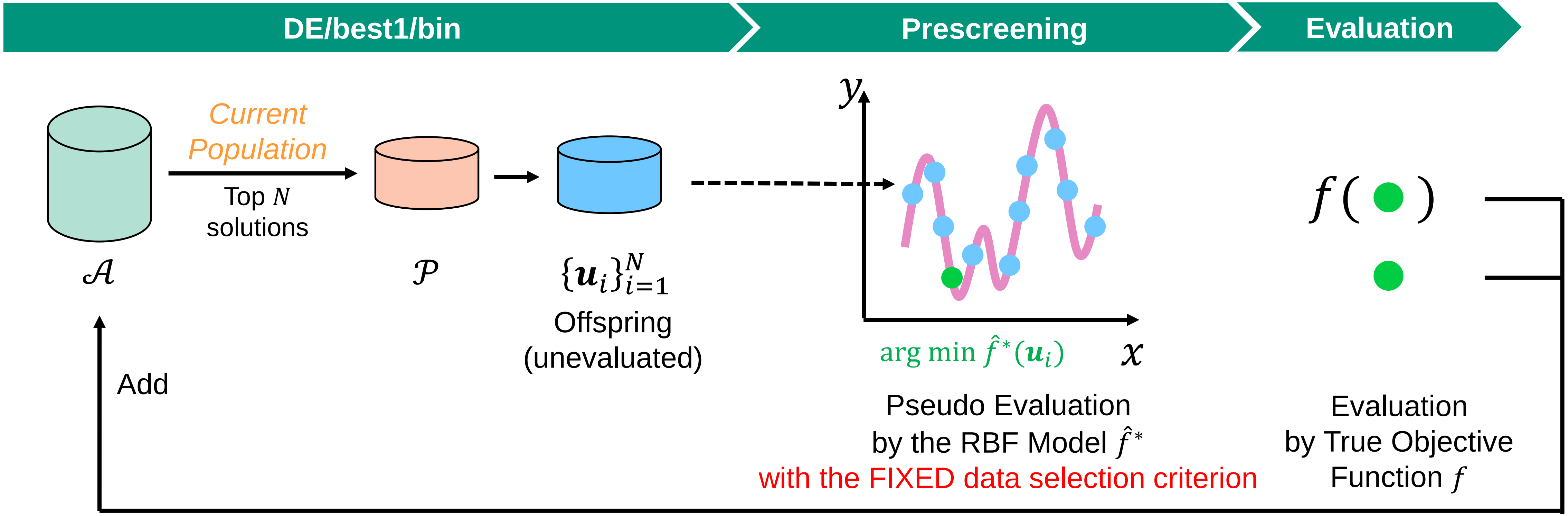
- **Comparison of computational time obtained the last experiment**
  - The averaged time is obtained at 1,000 FEs from 21 runs with all 28 functions
- **SADE-ATSDC is the fastest algorithm for all dimensions**
  - The computation time decreased by 50% to 99%



- S-JADE and SAHO perform iterated evolutionary searches on surrogate
- The number of candidates for adaptation is four and five for SADE-ATDSC and aRBF-NFO, respectively. The more model the algorithm builds, the much time is needed

Discussion 2/3: Selection Tendency in Total 1/2

- **Additional experiment and Comparison to adaptation results**
  - Four fixed algorithms with different data selection criteria are run
    - We check how similar the selection tendency as the adaptation results and the results in the additional experiment obtained by four fixed algorithms are



Discussion 2/3: Selection Tendency in Total 2/2

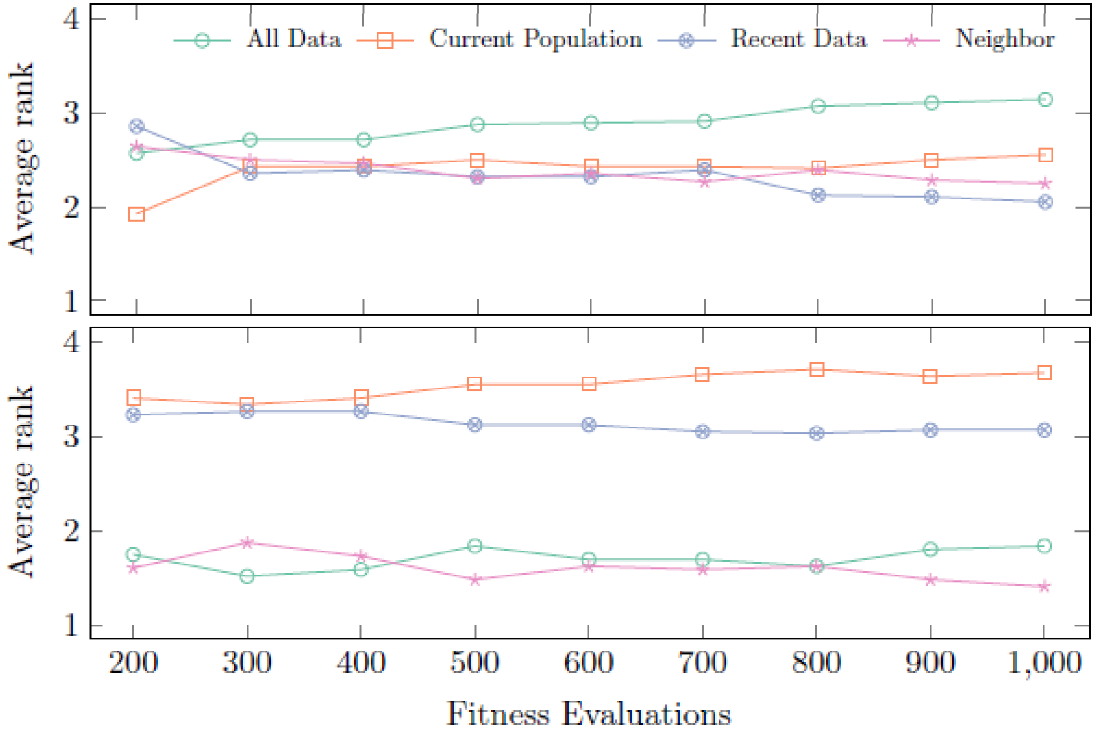
- **The adaptation result and the additional experiment result are alike**
  - The superior criterion in additional experiment tends to be selected most in SADE-ATSDC in main experiment

Selection ratio in adaptation obtained by SADE-ATSDC in main experiment

$D = 10$

$D = 100$

Average rank among four fixed algorithms in additional experiment

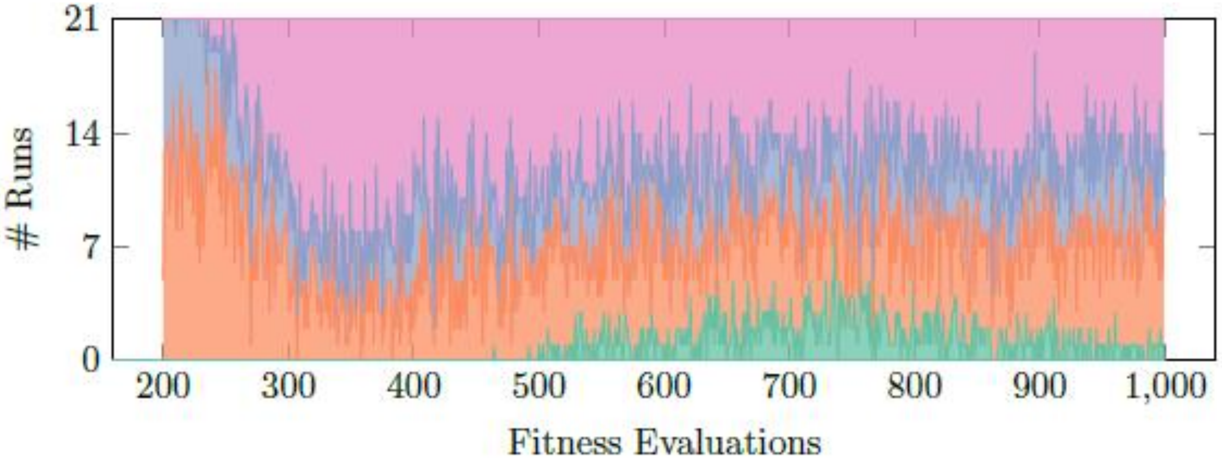


Discussion 3/3: Selection Tendency 2/2

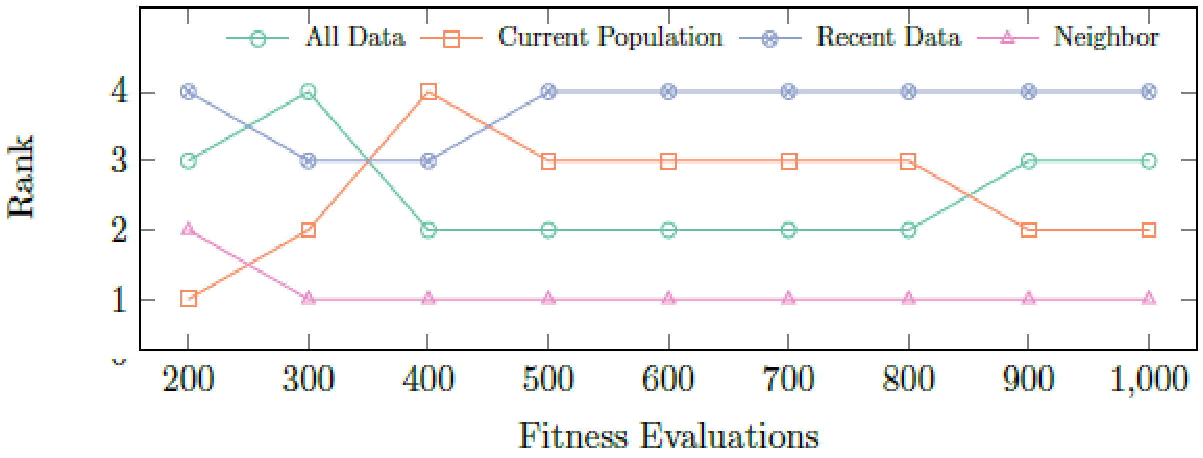
- **Selection Tendency by Adaptation in a Certain Problem**
  - We also check whether the adaptation result tracks the criterion from which the fixed algorithms derived the best result in terms of FEs
  - The adaptation result almost tracks the result of the additional experiment again

F27 ($D = 50$)

Selection ratio in adaptation obtained by SADE-ATSDC in the main experiment



Average rank among four fixed algorithms in the additional experiment



Conclusion

- **SAEA with Adaptation of Training Data Selection Criterion**
  - **SADE-ATDSC**: Surrogate-assisted DE with Adaptation of Training Data Selection Criterion
    - Multiple RBF models are constructed with different criteria
    - The one model with best accuracy is selected
  - SADE-ATDSC outperforms state-of-the-art SAEAs
    - The problem dimension is up to 100
    - SADE-ATDSC derives much smaller computational time than that of compared algorithms

- **Future Work**
  - Tuning of adaptation frequency
  - Adaptation of both data selection criterion and model type/settings

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