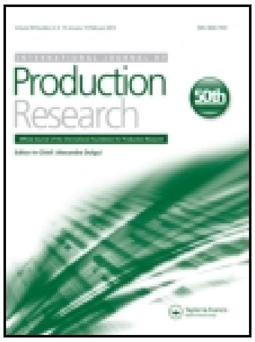

Taylor & Francis
Taylor & Francis Group

International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: https://www.tandfonline.com/loi/tprs20

Integrating run-based preventive maintenance into the capacitated lot sizing problem with reliability constraint

Zhiqiang Lu , Yuejun Zhang & Xiaole Han

To cite this article: Zhiqiang Lu , Yuejun Zhang & Xiaole Han (2013) Integrating run-based preventive maintenance into the capacitated lot sizing problem with reliability constraint, International Journal of Production Research, 51:5, 1379-1391, DOI: 10.1080/00207543.2012.693637

To link to this article: https://doi.org/10.1080/00207543.2012.693637

Published online: 22 Jun 2012.

International Journal of Production Research
Vol. 51, No. 5, 1 March 2013, 1379–1391

Integrating run-based preventive maintenance into the capacitated lot sizing problem with reliability constraint

Zhiqiang Lu†, Yuejun Zhang* and Xiaole Han

School of Mechanical Engineering, Shanghai Jiao Tong University, State Key Laboratory of Mechanical System and Vibration, Shanghai 200240, PR China

(*Received 27 September 2011; final version received 19 April 2012*)

A joint model for integrating run-based preventive maintenance (PM) into the capacitated lot sizing problem (CLSP) is proposed, in which the production system is subject to deterioration with usage and PM operations are implemented to restore the system. In this model, both production and PM operations are restricted by the system's maximum capacity, and the system reliability has to be maintained above a threshold value throughout the planning horizon. By linearisation of the reliability constraints, the problem is formulated as a mixed-integer linear programming. An explanatory example is given to illustrate the advantage of the joint model comparing with the interval-based PM policy in terms of system's overall cost. A three-stage heuristic is proposed to solve this integrated model, which includes a Lagrangian-based heuristic for the CLSP. The numerical experiments are conducted to evaluate the performance of the developed heuristics and the computational results show that the heuristics can provide good feasible solutions for the corresponding models. The discussion of the results is finally given in detail.

Keywords: capacitated lot sizing problem; run-based PM; reliability

1. Introduction

Production planning in the industrial environment normally comprises three ranges for decision making: long-term, medium-term and short-term. The medium-term planning tackles decisions on material requirements planning (MRP) and lot sizing over the planning period so as to minimise the overall costs while meeting demand requirements and satisfying existing capacity restrictions. The capacitated lot-sizing problem (CLSP) is one of the well-known models at the medium-term level. Since it takes into account the constraint of system capacity, this problem has been proved to be NP-hard even without setup times (Bitran and Yanasse 1982). Many meta-heuristics and mathematical programming based heuristics have been proposed to solve this problem (Jans and Degraeve 2007). Due to the importance in the industrial practice, the following extensions of CLSP are also widely studied: backorders, setup carry-over, sequencing and parallel systems (Quadt and Kuhn 2008).

On the other side, a production system is always subject to deterioration with usage, and thus maintenance operations have to be employed to retain the system in a reliable state or restore it after a breakdown. Corrective maintenance (CM) is performed right after a random breakdown of the system, whereas preventive maintenance (PM) is usually carried out before the system breakdown occurs. Since PM can be scheduled in advance to hedge against system uncertainties and guarantee system performance, various PM policies have been studied, e.g. age-dependent PM policy, periodic PM policy and failure limit policy for one-unit system (Wang 2002). An appropriate maintenance strategy can substantially improve the productivity and profitability, and the lack or ineffectiveness of maintenance planning can significantly restrict the competitive advantage of a company (Alsyouf 2007, 2009).

However, the role of maintenance is not highly recognised in practice, and the production and maintenance plans are usually treated in a separated way. This situation is partially caused by the fact that the maintenance and production departments are organisationally independent of each other, which results in the lack of communication regarding the requirement of each other's function. This dilemma can also be observed through the configuration of common ERP modules that the production and maintenance planning often belong to different modules using independently specific data from each service, and no coordination is applied to optimise the production and

*Corresponding author. Email: yuejun.tz@gmail.com
†Current address: School of Mechanical Engineering, Tongji University, Shanghai 201804, PR China.

ISSN 0020–7543 print/ISSN 1366–588X online

http://dx.doi.org/10.1080/00207543.2012.693637
http://www.tandfonline.com

maintenance concurrently (Najid *et al.* 2011). Another reason may be that the integration performance in literature is not good enough to motivate the practitioner to combine the decision process of production and maintenance. As a result, the integration method should be further explored to enhance the financial benefit, so that the current separation perspective can be improved in practice.

While integrating PM into the production plan at the tactical level, Weinstein and Chung (1999) have distinguished two different policies: interval-based PM is divided into classes based on intervals of time; run-based PM requires PM decision based on the cumulative run times of production system. The authors show that run-based PM is more effective than interval-based one in the environment of employing a chase aggregate strategy. For the CLSP model under a chase strategy, dynamic demands result in uneven system utilisations, which means system deterioration varies from one period to another, and thus run-based PM seems more profitable. Yet, the integrated model with interval-based PM policy has been highlighted (Aghezzaf *et al.* 2007, Aghezzaf and Najid 2008, Alaoui *et al.* 2010, Najid *et al.* 2011), and to our knowledge, only Fitouhi and Nourelfath (2012) have adopted the run-based PM, where the removal of periodicity constraint reduces more cost of production and maintenance.

The purpose of this paper is to establish a joint model for integrating run-based PM decision into the CLSP from the perspective of reliability. Reliability usually plays an important role in the performance improvement of the production system, and with a high reliability, the production process will become much more stable. Reliability decreases when the run time of the system increases, making it necessary to carry out PM operations during production to guarantee a given service. The objective of the integrated model is to minimise the total cost of production and PM over a given planning horizon. It is assumed that the production system is subject to deterioration with usage, which is measured by system's run time. The system also has to be maintained above a minimum reliability threshold through the planning horizon, i.e. the cumulative run times during each PM interval should be less than a maximum value. Meanwhile, we also consider the impact of PM on the production capacity that PM usually reduces system's available capacity for production. In sum, the relationship between the production and PM is established under the constraints of system capacity and reliability. The main difference between our work and Fitouhi and Nourelfath (2012) is that we add a reliability constraint in the joint model and stochastic breakdown is neglected. The explanatory example shows that PM intervals can be dynamically adjusted by the joint model, and thereby the system's total cost can be reduced especially when the system is at low levels of utilisation. In addition, we propose a three-stage heuristic to deal with our model and it provides a good performance in the computational experiments.

The remainder of this paper is organised as follows. In Section 2, the relative research in literature is surveyed. The mathematical model is proposed in Section 3, in which constraints of reliability are introduced into the CLSP and linearised. In Section 4, an explanatory example is given to examine the validation of the proposed model. A three-stage heuristic for the joint model and its computational experiments are presented in Sections 5 and 6, respectively. Finally, some conclusions are summarised in Section 7.

2. Literature survey

Production process is naturally coupled with maintenance operations, and the interactive planning and scheduling problems of production and maintenance activities have been addressed by many researchers in literature. These studies can be categorised into several streams.

The first stream deals with production scheduling problems emphasising the impacts of system breakdown and PM. Adiri *et al.* (1989) firstly propose a scheduling model with the production machine subject to a random breakdown during the task processing. This type of scheduling problem regarding the random breakdown is further studied by Lee and Lin (1992), Allahverdi and Mittenthal (1995) and Kasap *et al.* (2006). Liao and Chen (2003) and Low *et al.* (2010), on the other side, investigate the impact of periodic PM on the job scheduling of a single machine. Cassady and Kutanoglu (2003) present a different model to simultaneously optimise PM interval and job sequence, and their numerical results show an average reduction of 30% in expected total weighted tardiness compared to the solutions solved by the independent method. Ruiz *et al.* (2007) study two PM policies of maximising the availability and keeping a minimum level of reliability in a flow shop scheduling problem, which also demonstrates as well the significance of taking into consideration PM together with sequencing and the consequences of not doing so. Moradi *et al.* (2011) investigate integrated flexible job shop problem (FJSP) with PM activities under the multi-objective optimisation approaches.

The second stream attempts to derive the optimal lot size based on economic manufacturing quantity (EMQ) analysis while an unreliable system may undergo a random breakdown or PM operations. Groenevenlt *et al.* (1992) study the effects of machine breakdowns in the economic manufacturing context, and two production control policies (no-resumption and abort/resume) are proposed for coping with the stochastic interferences. They show that under both policies the optimal lot sizes will always be bigger than the ones in a corresponding deterministic case, and that the optimal lot size increases with the failure rate. Salameh and Jaber (1997) explore the effect of scheduled PM interruptions on the EMQ and the objective was achieved by trading off procurement cost, holding cost and maintenance and resetting cost per unit of time. Ben-Daya (2002) discusses the joint determination of economic production quantity and PM level for an imperfect process having a general deterioration distribution with increasing hazard rate. Chakraborty *et al.* (2008) present a generalised EMQ model for an unreliable production system in which the system might shift from an 'in-control' state to an 'out-of-control' state at any random time and might ultimately break down afterwards. El-Ferik (2008) proposes a model trying to jointly determine the EMQ and PM schedules under the realistic assumption that the production facility is subject to random failure and the maintenance is imperfect. Liao and Sheu (2011) apply the periodic PM to EMQ model for a randomly failing production process having a deteriorating production system with increasing hazard rate, and various special cases are considered, including the maintenance learning effect. Other continuous-run production models besides the EMQ are also studied, e.g. Salameh and Ghattas (2001), Zequeira *et al.* (2004, 2008) and Gharbi *et al.* (2007).

In the third stream, the integration of production and PM is studied at the tactical level and only several studies can be found. Weinstein and Chung (1999) firstly develop a hierarchical model to evaluate the significance of maintenance policy to aggregate production planning by varying maintenance factors like maintenance activity type, maintenance activity frequency, failure significance and maintenance activity cost. Aghezzaf *et al.* (2007) propose an integrated model to combine the periodic PM and CLSP, in which reliability parameters are considered at the early stage of planning process and better performance can be obtained at simulation phase. Aghezzaf and Najid (2008) further extend the above-mentioned model into the parallel systems and propose a Lagrangian-based algorithm to solve the problem. Najid *et al.* (2011) develop an integrated production and maintenance planning model with time windows and shortage cost. Alaoui *et al.* (2010) propose a joint production and maintenance planning model for a production system and solve it with a Langrangean heuristic. Fitouhi and Nourelfath (2012) deal with the problem of integrating non-cyclical preventive maintenance and CLSP for a single machine, and they find that the removal of periodicity constraint is directly affected by the demand fluctuation and can also reduce the total maintenance and production cost. Note that most integrated models in this stream are established under the interval-based PM policy, and a better result can be achieved while adopting run-based PM policy. Our work can be classified into this stream, which attempts to incorporate the run-based PM into CLSP, and the reliability requirement is additionally considered compared to other studies.

3. Model formulation

3.1 *Problem description*

Consider a single-unit system of which the production and PM plans over a finite horizon of T periods should be simultaneously determined. The deterministic demands for a set of items P in each period are given. The capacity constraint restricts the system's run time at each period. Setup is carried out while changing from one item to another, but setup time is neglected. Items can be hold across each period, but the associated inventory cost incurs. There is no initial inventory or backorder for any item.

Meanwhile, a given reliability threshold for the system has to be maintained to guarantee the production stability. The reliability is affected by the system usage, which is measured by the actual run time. Thus, PM has to be performed to restore the system and satisfy the reliability constraint. Since the reliability threshold is often set with a high value, no breakdown is considered. Similar PM policy application can also be found in Ruiz *et al.* (2007). PM is only performed at the beginning of some production period, and certain units of capacity in the corresponding period are reduced due to the required time for a PM operation. In addition, PM is performed in the first period, and thus the system is started as good as new from the beginning of the planning horizon.

It can be seen that the relationship between the production and maintenance is based on the constraints of system capacity and reliability. The objective is to minimise the sum of setup cost, inventory holding cost, production cost and PM cost, while satisfying the demand for all products and reliability constraint over the entire horizon. The following parameters and decision variables are used in the mathematical model.

Model parameters

- d_{it} Demand of item i in period t.
- c_{it} Unit production cost of item i in period t.
- s_{it} Unit setup cost of item i in period t.
- h_{it} Unit inventory holding cost of item i in period t.
- c_p Cost of each PM operation.
- M A large number.
- C_t Available capacity in period t.
- R_s Reliability threshold value for the system.
- L The maximum length of the time interval between two consecutive PM operations.
- a_i Production processing time of item i.
- r Processing time of each PM operation.

Decision variables

- X_{it} Production volume of item i in period t.
- I_{it} Inventory volume of item i in period t.
- Y_{it} A binary variable that equals 1 if a setup is performed for item i in period t, or 0 otherwise.
- R_t Reliability value at the end of period t.
- Z_t The cumulative run times from the last PM operation to the end of period t.
- u_t A binary variable that equals 1 if PM is performed in period t, or 0 otherwise.

3.2 *The reliability constraint under run-based PM*

System's run time in period t can be denoted by $\sum_{i=1}^{P} a_i X_{it}$. If a PM operation is performed in period t, then the system is renewed and the cumulative run times from the last PM operation to the end of period t can be written as $Z_t = \sum_{i=1}^{P} a_i X_{it}$; otherwise, $Z_t = Z_{t-1} + \sum_{i=1}^{P} a_i X_{it}$. With the PM decision variable u_t, these two expressions can be combined as

$$Z_t = (1 - u_t)Z_{t-1} + \sum_{i=1}^{P} a_i X_{it}.$$

This equation expresses that if no PM is performed at the beginning of the corresponding period ($u_t = 0$), the cumulative run times of the system is the sum of its previous period run time and the current period run time. In contrast, when a PM is carried out (u_t=1), the system is renewed and the previous period run time is not counted.

Weibull distribution is frequently used in reliability/maintenance applications, which also has been used in other integrating models (Ruiz *et al*. 2007, Aghezzaf and Najid 2008, Fitouhi and Nourelfath 2012). We assume the system follows a Weibull lifetime distribution and let the shape and scale parameter be $\beta (\beta > 1)$ and η, respectively. According to the PM policy, PM should be performed before the reliability falls below the threshold, and the reliability constraint in period t can be constructed as

$$R_t = \exp\left[-\left(\frac{Z_t}{\eta}\right)^{\beta}\right] \geq R_s$$

We have denoted L as the maximum length of the time interval between two consecutive PM operations, thus $L = \eta[-\ln(R_s)]^{1/\beta}$, and then the reliability constraint can be converted into the constraint for the cumulative run times as:

$$Z_t \leq L.$$

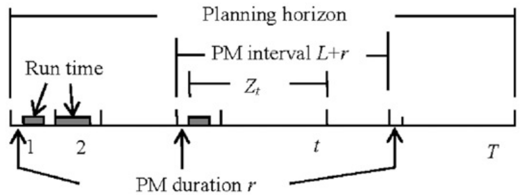

Figure 1. The relationship among L, r and Z_t.

Since the system reliability has to be kept above the given threshold R_s through the planning horizon, the above-mentioned constraint should be formulated for each period. The relationship among L, r and Z_t is illustrated in Figure 1.

In addition, time for PM operation will reduce the capacity of the production system, and thus the capacity constraint in the CLSP should be updated as

$$\sum_{i=1}^{P} a_i X_{it} + u_t r \leq C_t.$$

3.3 *The mathematical model*

The joint model for integrating run-based PM into the CLSP can be finally established as follows:

Minimise:

$$Z = \sum_{i=1}^{P}\sum_{t=1}^{T}(s_{it}Y_{it} + c_{it}X_{it} + h_{it}I_{it}) + \sum_{t=1}^{T} c_p u_t \tag{1}$$

Subject to:

$$X_{it} + I_{i,t-1} - I_{it} = d_{it} \quad \forall i \in \{1,\ldots,P\}, \quad \forall t \in \{1,\ldots,T\} \tag{2}$$

$$X_{it} \leq MY_{it} \quad \forall i \in \{1,\ldots,P\}, \quad \forall t \in \{1,\ldots,T\} \tag{3}$$

$$\sum_{i=1}^{P} a_i X_{it} + u_t r \leq C_t \quad \forall t \in \{1,\ldots,T\} \tag{4}$$

$$Z_t = (1-u_t)Z_{t-1} + \sum_{i=1}^{P} a_i X_{it} \quad \forall i \in \{1,\ldots,P\}, \quad \forall t \in \{1,\ldots,T\} \tag{5}$$

$$Z_t \leq L \quad \forall t \in \{1,\ldots,T\} \tag{6}$$

$$X_{it}, I_{it}, Z_t \geq 0 \quad \forall i \in \{1,\ldots,P\}, \quad \forall t \in \{1,\ldots,T\} \tag{7}$$

$$Y_{it}, u_t \in \{0,1\} \quad \forall i \in \{1,\ldots,P\}, \quad \forall t \in \{1,\ldots,T\} \tag{8}$$

$$L = \eta[-\ln(R_s)]^{1/\beta} \tag{9}$$

The objective function (1) is to minimise the total cost of setup, production, inventory and PM. Constraints (2) ensure the demand realised by inventory from previous periods or by the production in period t and the surplus items are preserved for the next period. Constraints (3) demonstrate that certain item can be produced in a period only if a related setup is performed. Constraints (4) concern the production capacity influenced by PM decision. In order to keep the reliability above the threshold value, Constraints (5) indicate the cumulative run times in a PM interval is affected by both production and PM decision. Constraints (6) force the cumulative run times in a PM interval should be no longer than the maximum PM length. Constraints (7) and (8) enforce non-negative and binary variables, respectively. Expression (9) gives the maximum PM interval length with reliability threshold and other parameters. Clearly, the relationship between the production and maintenance is based on the system capacity and reliability constraints, and both plans are optimised under a unified framework.

Furthermore, the non-linear Constraints (5) can be replaced by the following linearised constraints:

$$Z_t \geq \sum_{i=1}^{P} a_i X_{it} + Z_{t-1} - u_t M, \tag{9}$$

$$Z_t \geq \sum_{i=1}^{P} a_i X_{it}. \tag{10}$$

Note that Constraint (5) is equivalent to $u_t = 1,\ Z_t = \sum_{i=1}^{P} a_i X_{it}$ or $u_t = 0, Z_t = Z_{t-1} + \sum_{i=1}^{P} a_i X_{it}$. Constraints (9) and (10) are equivalent to $u_t = 1, Z_t \geq \sum_{i=1}^{P} a_i X_{it}$ or $u_t = 0, Z_t \geq \sum_{i=1}^{P} a_i X_{it} + Z_{t-1}$. Constraints (9) and (10) are relaxation of Constraint (5), but only with respect to dependent variable Z_t, the feasible region of key decision variables (X_{it}, u_t) remains unchanged, so do the optimal decisions.

4. An explanatory example

In this section, an explanatory example is presented to compare the performances of interval-based and run-based PM policies, and it is solved by CPLEX 12.1. Under the interval-based PM policy, PM plan is firstly decided with fixed time intervals and then the production plan is generated without considering reliability constraint, while under the run-based PM policy, both plans are simultaneously addressed by the proposed model.

Consider a planning horizon composed of 12 production periods, each with maximum capacity of 40. Two types of items are to be produced in lots to meet the demands. Table 1 presents the processing time and the unit cost of setup, production and holding for each item. Table 2 shows the specific demands for item A and B, which are uniformly distributed between [0, 20] and [0, 10], respectively. Meanwhile, let $c_p = 60$, $r = 8$, $\eta = 500$, $\beta = 1.63$, $R_s = 0.95$, and thus $L \approx 80$. Under interval-based PM policy, the production system has to be maintained every two periods, and the cost for production and PM is 360 and 877, respectively.

This optimal solution under the run-based PM policy is obtained by solving the joint model and the results are given in Table 2. Compared with interval-based PM policy, the most significant advantage of the run-based one is

Table 1. Items' data.

Item	a_i	s_{it}	c_{it}	h_{it}
A	1	10	2	1
B	2	15	5	3

Table 2. Demand data and computational results under run-based PM policy.

Period		1	2	3	4	5	6	7	8	9	10	11	12
d_{it}	A	11	15	11	14	10	15	16	16	5	9	2	13
	B	4	2	4	8	4	9	4	5	7	5	5	9
X_{it}	A	11	15	25	0	11	14	16	21	0	11	0	13
	B	6	0	4	12	0	13	0	5	7	5	5	9
u_t	0–1	1	0	0	1	0	0	1	0	0	1	0	0
PM cost			240										
Production cost			881										
Total cost			1121										

that two PM operations are cancelled, which means 120 units of PM cost are reduced, while production cost increases 4 units. Thus, the total cost decreases 116 units, which indicates that it is more profitable to choose the run-based PM when the system utilisation is low.

Figure 2 shows that the reliability curve with the run-based PM policy does not deteriorate smoothly as it depends on the run time at each period, whereas the system is assumed to be deteriorated periodically under the interval-based PM policy. It can also be found that, under run-based PM, all the PM operations are executed before the reliability value declines to the threshold, which implies that the optimal run time in each PM interval is smaller than the maximum PM interval length.

We further compare the integration effectiveness between run-based and interval-based PM policies at five levels of capacity utilisation, in which we add one unit of demand for item B at each period and thus the utilisation increases by 5% at each level. Figure 3 presents that the cost gap between run-based and interval-based PM converges as the capacity utilisation increases. It also indicates that the joint model can dynamically adjust PM intervals according to the usage of the system, and two PM policies have similar performance at the tight capacity utilisation.

5. The three-stage heuristic

It can be observed that the structure of the integrated model is generally based on the CLSP. Hence, we propose a three-stage heuristic to solve it. At the first stage, an initial interval-based PM plan is generated and thus the constraints for the reliability or cumulative run times can be removed from the joint model. The second stage mainly solves the independent CLSP by a Lagrangian-based heuristic, which is a sub-gradient optimisation method based on the Lagrangian relaxation of the capacity constraints. Note that Lagrangian-based heuristic for the CLSP has been proposed by many researchers with the standard primal and dual procedures but different infeasibility resolving procedures (Thizy and Van Wassenhove 1985, Trigeiro *et al*. 1989, Diaby *et al*. 1992, Hindi *et al*. 2003). At the final stage, a run-based PM plan is determined by sequentially re-inserting PM operations into the periods based on the system utilisation from the second stage, and this run-based PM decision process is generally inspired by Weinstein and Chung (1999). The detailed algorithm is stated as follows:

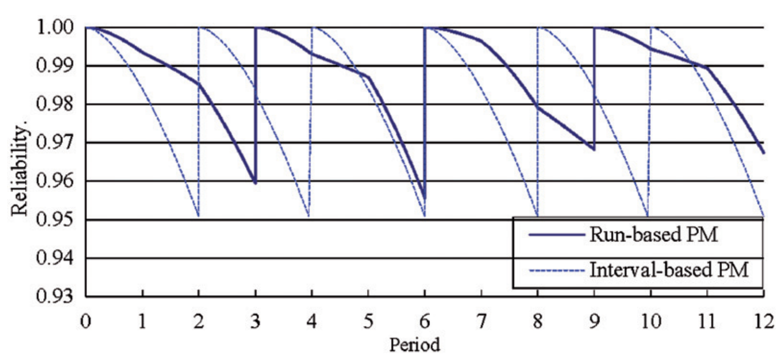

Figure 2. Reliability differences under two PM policies.

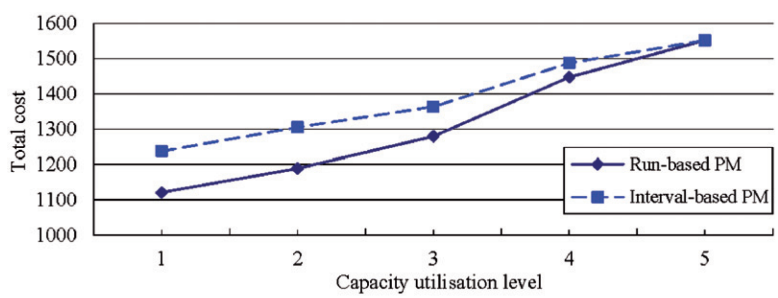

Figure 3. Cost trends of two PM policies at different levels of system utilisation.

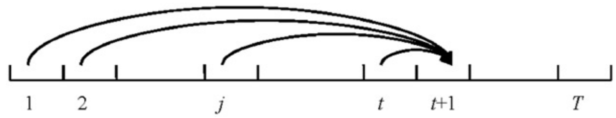

Figure 4. Forward shift certain production to period $t+1$.

Stage 1: Generate an initial interval-based PM plan. Calculate the maximum PM interval, $L = \eta[-\ln(R_s)]^{1/\beta}$, and the number of production period in each PM interval, $\lfloor L/C_t \rfloor$. Set the PM decision variables $\hat{u}_t = 0$, or 1, from the first period. Update the capacity constraint (4) for each period $\sum_{i=1}^{P} a_i X_{it} \le C_t - \hat{u}_t r$.

Since Constraints (5) and (6) are always satisfied under the initial interval-based PM plan, the joint model is reduced to a CLSP with updated capacities.

Stage 2: Solve the independent CLSP by the Lagrangian-based heuristic. Denote the Lagrangian multiplier as λ_t and dualise the capacity constraints for each period. The mathematical formulation of relaxed problem becomes

Minimise:

$$Z(\lambda_t) = \sum_{i=1}^{P} \sum_{t=1}^{T} [s_{it} Y_{it} + (c_{it} + \lambda_t a_i) X_{it} + h_{it} I_{it})] + \sum_{t=1}^{T} [c_p \hat{u}_t + \lambda_t (\hat{u}_t r - C_t)]$$

Subject to Constraints (2), (3), (7) and (8). And then at each iteration the procedure works as follows:

(1) Primal procedure. Calculate the updated production cost $c'_{it} = c_{it} + \lambda_t a_i$. Solve P number of single-item uncapacitated lot sizing problems through dynamic programming.
(2) Calculate the lower bound $Z(\lambda_t)$.
(3) Calculate the overtime or undertime for each period based on the results of step (1). The vector of these values is the subgradient, which is employed to calculate the direction vector.
(4) Smoothing procedure. If the production plan from step (1) is infeasible, then revise the plan by the smoothing procedure until no capacity constraint is exceeded and minimise the opportunity costs from step (2). The detailed steps are explained in the following.
(5) Dual procedure. Update the Lagrangian multipliers λ_t according to the subgradient method.

The smoothing procedure

The smoothing procedure is applied to eliminate the overtime at some periods by shifting certain production to other target periods. It is started from the solution of the Lagrangian relaxation, which is a lower bound for the optimal solution. The shifting method only includes a single forward pass compared to Trigeiro *et al.* (1989) and Hindi *et al.* (2003), but both forward and backward shifting of production are considered if overtime occurs. The forward shifting ensures that there is enough available capacity in the earlier periods for the overtime or reduces overtime by shifting production forward, while the backward shifting moves the overtime to the earlier periods with the smallest cost. The smoothed plan is held until a better one is found in the later iteration.

Assume overtime occurs in period t and item i is one of the items produced in this period. Since only a single forward pass is involved, we can assume there is no overtime, or the overtime has been eliminated in the earlier periods. Denote overtime in period t and undertime in period j $(1 \le j \le t-1)$ by $ot(t)$ and $ut(j)$, respectively, and the shifting procedure is stated as follows:

Direction selection

Check if $ot(t)$ is larger than the sum of the undertime from period 1 to t. If yes, go to the forward shifting phase, otherwise go to the backward shifting phase.

Forward shifting

(1) Select all the available production lots, from period 1 to t, with extra volume for the demands after t (Figure 4).
- If extra volume is produced in j, calculate the dual cost incurred per unit of undertime increased while shifting the extra volume to period $t+1$.

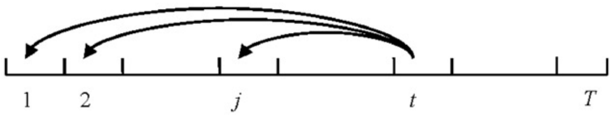

Figure 5. Backward shift the production to the earlier periods before t.

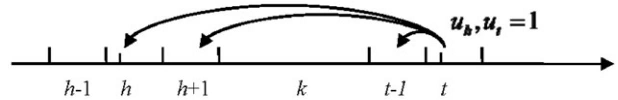

Figure 6. Backward shift the production to the earlier periods in the corresponding PM interval.

- If extra volume is produced in t, calculate the dual cost incurred per unit of overtime decreased while shifting the extra volume to period $t+1$.

(2) Select the shifting with minimal dual cost incurred per unit of undertime increased or overtime decreased.
(3) If the overtime in t has been eliminated, then stop the shifting; otherwise go to the direction selection phase.

Backward shifting

Calculate the dual cost incurred per unit of overtime decreased while shifting certain volume of item i in period t to the earlier available period j (Figure 5). Select the shifting with minimal dual cost incurred per unit of overtime decreased. The volume for shifting is given as follows:

- if $a_i X_{it} \le ot(t)$, the volume for shifting is $\min(\lfloor ut(j)/a_i \rfloor, X_{it})$;
- if $a_i X_{it} > ot(t)$, the volume for shifting is $\min(\lfloor ut(j)/a_i \rfloor, \lfloor ot(t)/a_i \rfloor)$.

Clearly, backward shifting will not violate the capacity constraints of the earlier periods.

Stage 3: Determine the run-based PM plan.

(1) Remove the interval-based PM plan in Stage 1 and update the capacity for each period.
(2) Insert PM into the period t sequentially until Z_{t-1} is just less than L and set the corresponding variable $u_t = 1$, otherwise $u_t = 0$.
(3) If overtime occurs under the new PM plan, shift certain production to the target period k ($h \le k \le t-1$, where h is the latest period performing the PM operation) (Figure 6). Only backward shifting is executed in this situation and the shifting criterion is similar to the backward shifting procedure in Stage 2.

6. Computational results

Two computational experiments are conducted to test the solution quality of the Lagrangian-based heuristic for the CLSP and the three-stage heuristic for the integrated model. Thus we can generally distinguish whether the solution gap of the three-stage heuristic is caused by the production or PM plan. Both heuristics are programmed in C# and run on a PC with a Core 2 CPU 2.10 GHz and 2 GB RAM. The number of iteration parameter for Lagrangian-based heuristic is set to 50 for all the problems. The exact solutions are obtained through CPLEX 12.1 solving the CLSP and the linearised model, respectively. Note that since the optimal solutions by CPLEX may take several hours, or even more, as the number of items and periods increases, we only compare the heuristic results and the upper bounds of CPLEX at these cases. For each combination of items and periods, 5 test problems are generated, and the gap is computed as follows:

$$Gap\ (\%) = \frac{Z_H - Z_{CPLEX}}{Z_H + Z_{CPLEX}} \times 200,$$

where Z_H is the near optimal solution by the heuristic and Z_{CPLEX} is the optimal solution or upper bound by CPLEX at 1800s.

6.1 *Results for the Lagrangian-based heuristic in Stage 2*

The Lagrangian-based heuristic in Stage 2 for the independent CLSP is tested. Problem dimension is represented by the number of items $P \in \{10, 20\}$ and the number of periods $T \in \{12, 24, 48\}$. The production system has the same

Table 3. Solution gaps of the Lagrangian-based heuristic for CLSP.

		Capacity utilisation								
		0.95			0.85			0.75		
Items	Periods	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean
10	12	1.57	0.55	1.02	0.71	0.00	0.37	0.10	0.00	0.03
10	24	1.71	1.17	1.55	0.89	0.10	0.35	0.09	0.00	0.03
10	48	2.21*	1.41*	1.88*	0.60	0.17	0.33	0.90	0.01	0.05
20	12	0.20	0.08	0.49	0.10	0.00	0.04	0.01	0.00	0.00
20	24	0.85	0.40	0.58	0.30	0.00	0.09	0.02	0.00	0.00
20	48	1.09*	0.51*	0.74*	0.13	0.00	0.06	0.01	0.00	0.00

Note: *The Lagrangian-based heuristic is compared with the upper bound of CPLEX at 1800s.

capacity at each period, and the average capacity utilisation $\mu \in \{0.95, 0.85, 0.75\}$. The following parameters are integers uniformly generated from an interval $[a, b]$:

- Unit production cost of item (c_{it}): $U[5, 10]$;
- Unit inventory holding cost of item (h_{it}): $U[1, 5]$;
- Unit setup cost of item (s_{it}): $U[50, 200]$;
- Processing time (a_i): $U[1, 20]$;
- Demand (d_{it}): $U[10, 150]$.

Capacity is the last parameter to be determined. The total run time of the system is firstly calculated by the demand and processing time of each item, and then the capacity C_t is obtained through dividing the total run time by average capacity utilisation μ and T. The expression can be denoted by

$$C_t = \left(\sum_{i=1}^{P} \sum_{t=1}^{T} a_i d_{it} \right) \Big/ (\mu T).$$

Demand is continuously produced until

$$\sum_{i=1}^{P} \sum_{t=1}^{t} a_i d_{it} \leq tC_t \quad (\forall t \in \{1, \ldots, T\}),$$

otherwise no feasible solution can be found. Instances are generated until the required number of problems is solved for each combination of parameters.

Table 3 presents the solution gaps of the Lagrangian-based heuristic in Stage 2 for the independent CLSP. Note that the solution quality has a substantial improvement as the capacity utilisation decreases, or as the number of items increases. We can also observe that the solution gap has a growing trend as the number of periods increases at capacity utilisation 0.95, but this increasing trend is not distinct at capacity utilisation 0.85 and 0.75. Such a situation also takes place in the three-stage heuristic for the joint model. This can be explained by the fact that the soothing procedure tends to move production tasks backward to eliminate the overtime and the inventory cost has an evident increase at the tight capacity utilisation with a long planning horizon. In general, the results obtained are fairly good and robust in terms of optimality gaps.

Table 4 presents the summery of the mean computational times by the Lagrangian-based heuristic and CPLEX. The mean times for the heuristic are generally below 3 seconds, but it only shows a distinct advantage at tight capacity utilisation. Notice that at capacity utilisation 0.95, all the instances cannot be solved within 1800 seconds by CPLEX, but the upper bounds are very close to the optimal solutions (mostly less than 1%). This implies that the complexity of the CLSP is closely related to the tightness of the capacity utilisation.

6.2 *Results for the three-stage heuristic*

PM normally does not take a high proportion of capacity in practice, but it consumes a large amount of expense (Alsyouf 2007). Thus, based on the parameter structure in Section 6.1, we set the PM related parameters $L = 4 * C_t$, $r = 0.05 * C_t$ and $c_p = 200 * P$ for the joint model.

Table 4. Mean computational times of the Lagrangian-based heuristic and CPLEX.

		Capacity utilisation					
		0.95		0.85		0.75	
Items	Periods	Heuristic	CPLEX	Heuristic	CPLEX	Heuristic	CPLEX
10	12	0.13	0.78	0.12	0.21	0.11	0.08
10	24	0.32	70.93	0.25	0.60	0.26	0.22
10	48	1.35	1800*	1.17	3.00	1.10	0.45
20	12	0.18	0.97	0.16	0.30	0.14	0.14
20	24	0.59	183.84	0.44	0.99	0.46	0.35
20	48	2.74	1800*	2.27	3.29	2.12	0.57

Note: *CPLEX terminates at 1800s.

Table 5. Solution gaps of the three-stage heuristic for the joint model.

		Capacity utilisation								
		0.95			0.85			0.75		
Items	Periods	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean
10	12	1.29	0.34	0.88	0.75	0.10	0.33	0.22	0.00	0.07
10	24	1.62	0.60	1.13	0.75	0.08	0.49	0.22	0.01	0.08
10	48	1.89*	0.68*	1.25*	2.75*	2.16*	2.32*	0.05*	0.01*	0.03*
20	12	0.79	0.16	0.52	0.17	0.09	0.12	0.03	0.00	0.01
20	24	1.24	0.42	0.89	0.13	0.01	0.07	0.01	0.00	0.00
20	48	1.42*	0.78*	1.09*	2.00*	1.76*	1.91*	0.03*	0.00*	0.01*

Note: *The three-stage heuristic is compared with the upper bound of CPLEX at 1800s.

Table 6. Mean computational times of the three-stage heuristic and CPLEX.

		Capacity utilisation					
		0.95		0.85		0.75	
Items	Periods	Heuristic	CPLEX	Heuristic	CPLEX	Heuristic	CPLEX
10	12	0.14	1.27	0.12	0.73	0.11	0.51
10	24	0.36	635.41	0.27	91.64	0.25	1124.18
10	48	1.37	1800*	1.21	1800*	1.23	1800*
20	12	0.19	1.64	0.16	71.44	0.16	428.72
20	24	0.61	1056.82	0.47	391.17	0.46	1286.15
20	48	2.74	1800*	2.38	1800*	2.31	1800*

Note: *CPLEX terminates at 1800s.

Table 5 presents the solution gaps of the three-stage heuristic, and something interesting can be found compared with Table 3. First, the gap size decreases in most instances in Table 5. This is because that PM cost is additionally considered in the objective function of the joint model, and thus the impact from the Lagrangian-based heuristic is reduced. Also, the increasing trend of the solution gaps at columns with capacity utilisation 0.95 can still be found as the planning horizon increases. In addition, compared to Table 3, the solution gap has an abnormal growth at capacity utilisation 0.85 with 48 periods. This is probably due to the fact that, according to our PM insertion method, PM decision error is more likely to occur at moderately loose capacity utilisation through a long planning horizon. A more careful implementation of decision process will enhance the run-based PM quality.

Table 6 presents the mean computational times of the three-stage heuristic and CPLEX. The computational times by the heuristic have grown slightly compared to Table 4, while times by CPLEX increase significantly at the most instances. Namely, the joint model is more complicated than the independent CLSP and it is more difficult to find the optimal solution.

7. Conclusions

The contribution of our work is twofold. First, we propose a joint model to integrate run-based PM into the CLSP. The relationship between the production and maintenance is based on the constraints of system capacity and reliability, and they are concurrently optimised under the objective of minimising the sum of setup cost, inventory holding cost, production cost and PM cost. The explanatory example shows the advantage of the run-based PM policy. Second, since the joint model is difficult to be solved by CPLEX at a large scale, a three-stage heuristic is presented to solve this model with a good feasible solution in a reasonable execution time.

Future work from this study might follow several directions. One direction is to further consider the random breakdowns during the model formulation, which is usually an important decision factor in the PM planning. Another direction is to take into account the sequence of production lots, which will provide a more flexible timing for the PM operations. Each of the directions is a natural extension in the future work.

Acknowledgement

This research is supported by National Natural Science Foundation of China (No. 71171130, 50905115).

References

Adiri, I., *et al.*, 1989. Single machine flow-time scheduling with a single breakdown. *Acta Informatica*, 26 (7), 679–696.

Aghezzaf, E.H., Jamali, M.A., and Ait-Kadi, D., 2007. An integrated production and preventive maintenance planning model. *European Journal of Operational Research*, 181 (2), 676–685.

Aghezzaf, E.H. and Najid, M.N., 2008. Integrated production and preventive maintenance in production systems subject to random failures. *Information Sciences*, 178 (17), 3382–3392.

Alaoui, M., Najid, M.N., and Mohafid, A., 2010. Lagrangean Based heuristic for integrated planning production and maintenance problem with windows and shortage cost. [online]. Available from: http://www.enim.fr/mosim2010/articles/74.pdf [Accessed 29 May 2012].

Allahverdi, A. and Mittenthal, J., 1995. Scheduling on a two-machine flowshop subject to random breakdowns with a makespan objective function. *European Journal of Operational Research*, 81(2), 376–387.

Alsyouf, I., 2007. The role of maintenance in improving companies productivity and profitability. *International Journal of Production Economics*, 105 (1), 70–78.

Alsyouf, I., 2009. Maintenance practices in Swedish industries survey results. *International Journal of Production Economics*, 121 (1), 212–223.

Ben-Daya, M., 2002. The economic production lot-sizing problem with imperfect production processes and imperfect maintenance. *International Journal of Production Economics*, 76 (3), 257–264.

Bitran, G.R. and Yanasse, H.H., 1982. Computational complexity of the capacitated lot size problem. *Management Science*, 28 (10), 1174–1186.

Cassady, R.C. and Kutanoglu, E., 2003. Minimizing Job Tardiness Using Integrated Preventive Maintenance Planning and Production Scheduling. *IIE Transactions*, 35(6), 503–513.

Chakraborty, T., Giri, B.C., and Chaudhuri, K.S., 2008. Production lot sizing with process deterioration and machine breakdown. *European Journal of Operational Research*, 185 (2), 606–618.

Diaby, M., *et al.*, 1992. A Lagrangean relaxation approach for very-large-scale capacitated lot-sizing. *Management Science*, 38 (9), 1329–1340.

El-Ferik, S., 2008. Economic production lot-sizing for an unreliable machine under imperfect age-based maintenance policy. *European Journal of Operational Research*, 186 (1), 150–163.

Fitouhi, M.C. and Nourelfath, M., 2012. Integrating noncyclical preventive maintenance scheduling and production planning for a single machine. *International Journal of Production Economics*, 136 (2), 344–351.

Gharbi, A., Kenne, J.P., and Beit, M., 2007. Optimal safety stocks and preventive maintenance periods in unreliable manufacturing systems. *International Journal of Production Economics*, 107 (2), 422–434.

Groenevenlt, H., Pintelon, L., and Seidmann, A., 1992. Production lot sizing with machine breakdowns. *Management Science*, 38 (1), 104–123.

Jans, R. and Degraeve, Z., 2007. Meta-heuristics for dynamic lot sizing: a review and comparison of solution approaches. *European Journal of Operational Research*, 177 (3), 1855–1875.

Hindi, K.S., Fleszar, K., and Charalambous, C., 2003. An effective heuristic for the CLSP with set-up times. *The Journal of the Operational Research Society*, 54 (5), 490–498.

Kasap, N., Aytug, H., and Paul, A., 2006. Minimizing makespan on a single machine subject to random breakdowns. *Operations Research Letters*, 34 (1), 29–36.

Lee, C.Y. and Liman, S.D., 1992. Single machine flow-time scheduling with scheduled maintenance. *Acta Informatica*, 29 (4), 375–382.

Liao, C.J. and Chen, W.J., 2003. Single-machine scheduling with periodic maintenance and nonresumable jobs. *Computers & Operations Research*, 30 (9), 1335–1347.

Liao, G.L. and Sheu, S.H., 2011. Economic production quantity model for randomly failing production process with minimal repair and imperfect maintenance. *International Journal of Production Economics*, 130 (1), 118–124.

Low, C., *et al*., 2010. Minimizing the makespan in a single machine scheduling problems with flexible and periodic maintenance. *Applied Mathematical Modelling*, 34 (2), 334–342.

Moradi, E., Fatemi Ghomi, S.M.T., and Zandie, M., 2011. Bi-objective optimisation research on integrated fixed time interval preventive maintenance and production for scheduling flexible job-shop problem. *Expert Systems with Applications*, 38 (6), 7169–7178.

Najid, N.M., Alaoui-Selsouli, M., and Mohafid, A., 2011. An integrated production and maintenance planning model with time windows and shortage cost. *International Journal of Production Research*, 49 (8), 2265–2283.

Quadt, D. and Kuhn, H., 2008. Capacitated lot-sizing with extensions: a review. *4OR: A Quarterly Journal of Operations Research*, 6 (1), 61–83.

Ruiz, R., García-Díaz, J.C., and Maroto, C., 2007. Considering scheduling and preventive maintenance in the flowshop sequencing problem. *Computers & Operations Research*, 34 (11), 3314–3330.

Salameh, M.K. and Ghattas, R.E., 2001. Optimal just-in-time buffer inventory for regular preventive maintenance. *International Journal of Production Economics*, 74 (1–3), 157–161.

Salameh, M.K. and Jaber, M.Y., 1997. Optimal lot sizing with regular maintenance interruptions. *Applied Mathematical Modelling*, 21 (2), 85–90.

Thizy, J.M. and Van Wassenhove, L.N., 1985. Lagrangean relaxation for the multi-item capacitated lot-sizing problem: a heuristic implementation. *IIE Transactions*, 17 (4), 308–313.

Trigeiro, W.W., Thomas, L.J., and McClain, J.O., 1989. Capacitated lot sizing with setup times. *Management Science*, 35 (3), 353–366.

Wang, H., 2002. A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, 139 (3), 469–489.

Weinstein, L. and Chung, C.H., 1999. Integrating maintenance and production decisions in a hierarchical production planning environment. *Computers & Operations Research*, 26 (10–11), 1059–1074.

Zequeira, R.I., Prida, B., and Valdes, J.E., 2004. Optimal buffer inventory and preventive maintenance for an imperfect production process. *International Journal of Production Research*, 42 (5), 959–974.

Zequeira, R.I., Valdes, J.E., and Berenguer, C., 2008. Optimal buffer inventory and opportunistic preventive maintenance under random production capacity availability. *International Journal of Production Economics*, 111 (2), 686–696.