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Integrated optimal reliable design, production, and maintenance planning for multipurpose process plants

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Abstract

In multipurpose process plants, which are characterized by sharing different resources (equipment, manpower etc.) for production, unplanned equipment shutdown could affect the timely production of products and hence process profitability. Many approaches have been proposed to ensure high equipment availability by combining design, production, and scheduling frameworks with a maintenance optimization framework. In these approaches, the initial reliability characteristics of equipment, which also determine equipment availability, are considered fixed by problem definition. In this work, a combined design, production and maintenance planning formulation for multipurpose process plants is extended to incorporate the reliability allocation problem at the design stage. A simultaneous optimization framework is presented that addresses the problem of optimal allocation of reliability among equipment simultaneously with the selection of process configuration, production and maintenance planning for multipurpose process plants at the design stage. This framework provides the designer with the opportunity to select the initial reliability of equipment at the design stage by balancing the associated costs with its impact on the design and the availability in the operational stage. The overall problem is formulated as a mixed integer linear programming (MILP) model, and its applicability is demonstrated using a number of examples.

Keywords: Reliability allocation; Maintenance optimization; Multipurpose plants; Production planning; MILP

1. Introduction

Multipurpose process plants are used extensively to provide a flexible production platform for the production of many types of chemicals. Different products can be produced in these plants by sharing the available resources—equipment, raw materials, man-power, and utilities—over the time horizon of planning. The inherent operational flexibility offered by these plants, however, poses considerable complexity in the design and operation of these plants. For instance, the flexibility obtained by sharing equipment can be affected by unplanned equipment shutdowns due to equipment failures. Thus to achieve timely production of products at a minimum cost, it is critical to consider process availability during design and operation of multipurpose plants. Chemical process availability, also described as a system effectiveness criterion (Grievink, Smit, Dekker & Van Rijn, 1993; Pistikopoulos, Vassiliadis, Arvela & Papageorgiou, 2001), is determined by equipment and process reliability characteristics, and the implemented maintenance policy.

The initial reliability characteristics of a process are decided at the design stage where decisions about the process system configuration (e.g. redundancy, buffer storage) and the equipment initial reliabilities are made and can be improved by increasing the reliability of equipment and/or adding redundancy. However, im-

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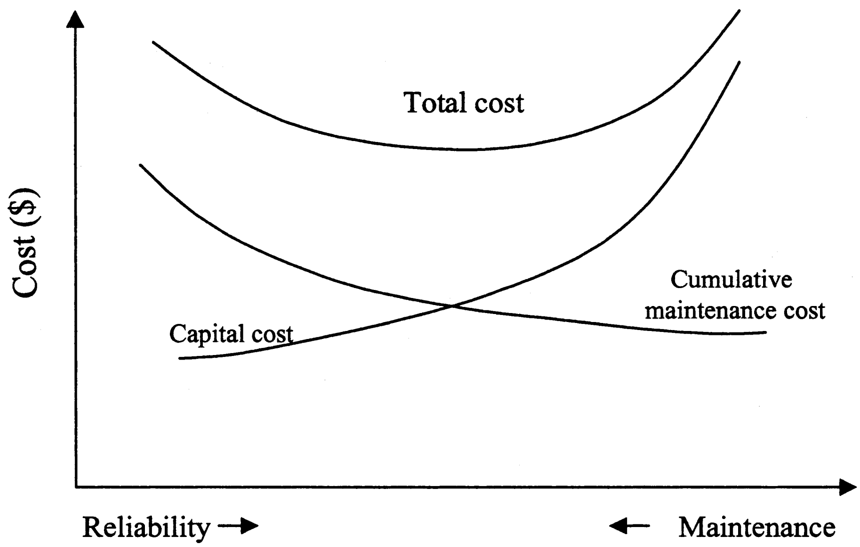

Fig. 1. Reliability vs. maintenance costs to achieve a high availability.

proving equipment initial reliability or adding parallel equipment requires additional capital investment at the design stage and, therefore, should be balanced against its benefits, i.e. higher profit due to higher availability. To obtain an optimal reliable design, the reliability optimization problem is solved at the design stage to maximize process reliability subject to budget constraints. A detailed discussion on various formulations and solution procedures can be found in Kuo, Prasad, Tillman and Hwang (2001) and Koolen (2001).

Once the design has been fixed, high process availability during the operational stage can be achieved by effective maintenance. The problem of maintenance optimization is to identify the maintenance policy that optimizes the balance between maintenance benefits and costs. Maintenance optimization methods have been studied extensively and are widely applied to the operation of process systems. This may be due to the cost driven optimization of the existing large base of older plants in refineries and petrochemical complexes.

In general, the problems of determining the optimal reliable design and maintenance policy are interdependent (see Fig. 1). Therefore, the challenge of achieving high process availability at the operational stage can be formulated to have the following key elements:

- a reliability model aimed at identifying the optimal process structure and reliability of process equipment units at the design stage;
- a maintenance model aimed at identifying the optimal maintenance policy to be implemented at the operational stage;
- appropriate linking variables that provide the mechanism for quantifying the interactions between process model, and maintenance and reliability models within an optimization framework.

In the past decade, several detailed formulations have been proposed for multipurpose plants to address the problem of achieving high process availability by introducing reliability and maintenance characteristics of units in the design, planning and scheduling formulations (Pistikopoulos et al., 2001; Sanmarti, Espuna & Puigjaner, 1997; Dedopoulos & Shah, 1995). In addition, for general process systems including multiproduct batch plant and continuous plants, a formulation has been proposed which also includes the uncertainties that are present at the design stage in the combined maintenance and design optimization problem (Vassiliadis & Pistikopoulos, 2001). This formulation, however, is computationally expensive. In most of the abovementioned approaches an important element is left out—the reliability optimization problem at the design level. Maintenance can restore degraded performance to initial levels but cannot significantly improve it. Significant improvement would require the selection of different and better equipment at the design stage. Therefore, the initial reliability of each process unit should be considered as another degree of freedom to the design problem.

In this paper we address this gap and present a mathematical formulation for the integrated optimal reliable design, production and maintenance planning for multipurpose process plants. The existing formulation of simultaneous design, production and maintenance planning of multipurpose plants (Pistikopoulos et al., 2001) is extended to include the decisions that must be made when selecting the initial reliability of equipment units at the design stage. The overall problem is formulated as a mixed integer linear programming (MILP) model. The resulting MILP problem employs the same rigorous maintenance and production planning models employed in earlier formulations with the

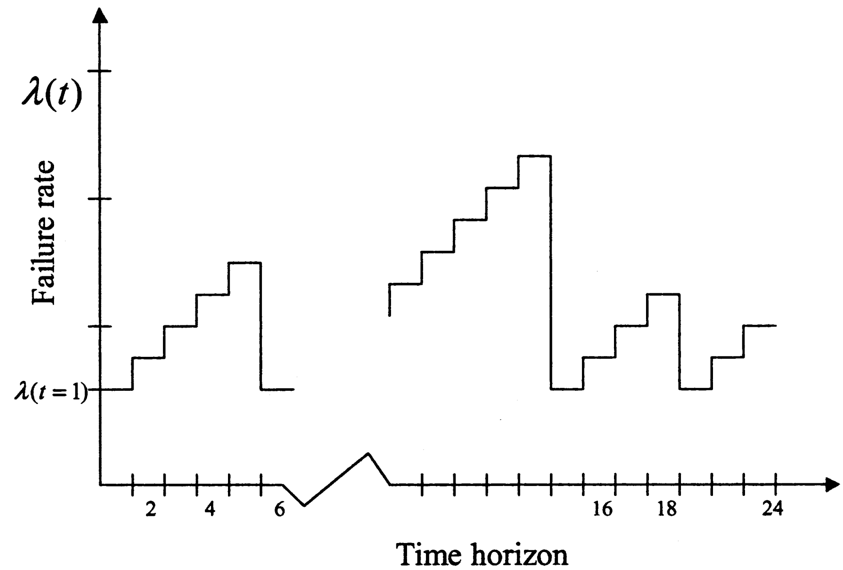

Fig. 2. Failure rate profile without considering reliability optimization at design stage.

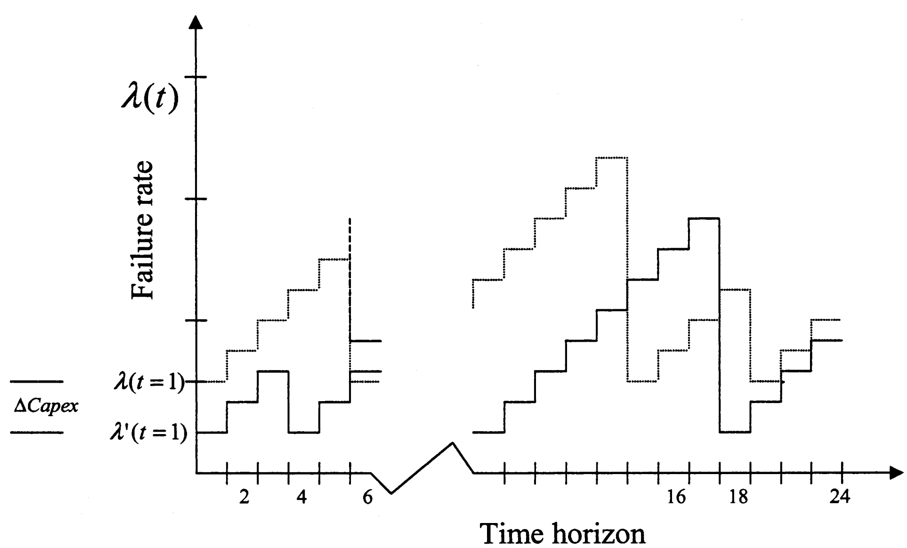

Fig. 3. Failure rate profile with reliability optimization at design stage.

addition of a reliability allocation model, which is the contribution of this work. It should be noted that the application of the proposed framework requires additional data about different reliability improvement options that are available at the design stage, with their associated additional costs. These data can usually be obtained from company in-house purchase and maintenance departments or from external equipment suppliers. In cases where the data for a particular unit is not readily available, the cost of additional resources required to generate these data may be included in the total additional costs.

The rest of the paper is structured as follows. The modeling framework is characterized in the next section. Then, a definition of the problem under consideration is stated. The mathematical formulation is described in Section 4. Subsequently, in Section 5, two illustrative examples are used to demonstrate the applicability of the proposed model. Finally, the conclusions are given in Section 6.

2. The modeling framework

Pistikopoulos et al. (2001) recently proposed a system effectiveness optimization framework for the simultaneous approach of design, production and maintenance planning of multipurpose process plants. In their model, they use a piecewise-constant increasing equipment failure rate for all equipment and develop an analytical preventive maintenance optimization model to reflect the effect of high failure rate on equipment uptime in each period, see Fig. 2. Pistikopoulos et al. (2001) propose an objective function, which balances additional maintenance costs with increased profit. The initial failure rate $\lambda(t=1)$ of each process unit, however, is considered fixed in their model, i.e. the initial reliability characteristic of equipment is considered fixed.

As outlined in Section 1, the availability of equipment is a function of its reliability and the implemented maintenance policy. Therefore, for fixed target availability, equipment with different initial failure rate $\lambda'(t=1)$ (see Fig. 3), the design decisions such as volume, and the implemented maintenance policy are expected to be different. We have, therefore, extended their formulation to include a reliability optimization model where the initial reliability of equipment is considered to be a decision variable and the initial reliability of equipment is identified at the design stage.

Different reliability optimization formulations exist in reliability engineering literature. Depending on the choice of decision variables, whether it is the equipment's reliability or the number of parallel units added to each piece of equipment or both, different reliability optimization formulations can be formulated. In this work, we consider a reliability allocation formulation where, for a specific system configuration, different levels of initial process reliability can be achieved by selecting equipment at different levels of cost and reliability. With proper mathematical modification the current framework can be extended to cover other formulations.

For reliability allocation problems, the relation between cost and reliability of equipment can be described either as a closed form exponential continuous function (Mettas, 2000) or as discrete cost–reliability data sets (Majety, Dawande & Rajgopal, 1999). The amount of information available at the design stage and the level of complexity of the problem considered, influence the choice between discrete and continuous representation. In this work, to avoid non-linearities in the final formulation, we consider discrete cost–reliability data sets for the equipment.

The following identifies the key elements in our proposed framework:

- the simple product state-task network (STN) as described by Kondili, Pantelides and Sargent (1993) through which batch transformations are described;
- a combined aggregate production and maintenance planning model as proposed by Pistikopoulos et al. (2001), which describes the interactions between production and maintenance planning by associating the utilization of process assets and resources with the availability of equipment;
- a reliability allocation model that identifies the initial reliability characteristics of equipment at the design stage by optimizing the balance between the additional capital costs of the design and the impact of improved reliability on the equipment availability;
- an objective function, that provides a trade-off between increased revenues due to extra equipment availability and two additional costs (a) increased capital costs for improving unit's initial reliability and (b) increased operational costs for preventive actions.

3. Problem definition

The integrated optimal reliable design, production and maintenance planning problem for multipurpose process plants considered in this paper is stated as follows.

Given:

- production recipes (STN), i.e. the processing time for each task at the suitable units;
- potentially available processing units with available different sizes, cost and reliability;
- demand of products in each time period;
- the time horizon under consideration;
- reliability and maintenance characteristics of units;
- the operating and capital cost data involved in the plant/process installation and operation.

Determine:

- the units selected and size of each unit;
- the optimal initial reliability for each unit at the design stage;
- the optimal maintenance plan describing the number and the time of preventive maintenance actions in the operational stage;
- optimal production plan;
- the optimal value of an appropriate performance criterion, such as the overall expected profit.

4. Mathematical formulation

The multiperiod planning model adopted in this work, is based on a STN process representation proposed by Kondili et al. (1993). The STN is a directed graph with two types of distinctive nodes, the state nodes denoted by a circle and the task nodes denoted by a rectangle. The time horizon is discretized into a number of time periods, t, of equal duration, H. A notation list used in the formulation of the integrated optimal reliable design, production and planning model is provided below.

Indices

i	processing tasks
j	units
s	states of material
u	utilities
t	time periods
k	unit sizes
l	unit initial failure rate
θ	number of periods elapsed since unit j was last maintained

Sets

$S_i/\bar{S}_i$	sets of states consumed/produced by task i
$T_s/\bar{T}_s$	sets of tasks receiving/producing materials in state s
I_j	set of tasks for which unit j is suitable
ψ_j	set of unit sizes available for unit j
K_i	set of units suitable for task i
ζ_j	set of possible initial failure rates for unit j

Parameters

$\bar{V}_{jk}$	size k for unit j
$\bar{\lambda}_{jl}$	initial failure rate l for unit j
$\rho_{is}/\bar{\rho}_{is}$	proportion of input/output of task i from state $s \in S_i/\bar{S}_i$
p_i	set-up and processing time of task i
$\beta_{uij\omega}/\delta_{uij\omega}$	fixed/variable demand factor for utility u by task i in unit j at the time ω relative to the start of the task
$\phi_{ij}^{\min}/\phi_{ij}^{\max}$	minimum/maximum utilization factor
$A_{uf}^{\max}$	maximum availability level of utility u during time period t
$N_{ujt}^{\max}$	maximum number of batches when task i is performed in unit j during time period t
H	duration of each period
Δ_j^c	corrective maintenance (repair) duration of unit j
Δ_j^p	preventive maintenance duration of unit j
τ_j	maximum number of consecutive elapsed time periods without maintenance of unit j
$\gamma_{j\theta}$	failure rate value for unit j when the last maintenance action took place θ periods ago
K_j^0	fixed cost for unit j over considered time horizon of planning
K_j^1	variable size factor for unit j over considered time horizon of planning
K_{jl}^2	cost factor for unit j with failure rate l over considered time horizon of planning
η_{st}	unit price of state s during period t
C_{ut}	unit cost of utility u during period t
C_{jt}^p	preventive maintenance cost of unit j during period t
C_{jt}^c	corrective maintenance cost of unit j during period t
α_j	constant increment in failure rate

Variables

Binary variables

E_j	1 if unit j is chosen; 0 otherwise
E_{jk}	1 if size k is chosen for unit j; 0 otherwise
E_{jl}	1 if failure rate l is chosen for unit j; 0 otherwise
X_{jt}	1 if preventive maintenance is performed on unit j during period t; 0 otherwise
$Z_{jt\theta}$	1 during period t if unit j was maintained for the last time θ periods ago; 0 otherwise

Continuous variables

N_{ijt}	number of batches of task i processed in unit j over time period t
S_{st}	amount of material in state s in storage at the end of period t
D_{st}	amount of material delivered to external customers from state s over period t
V_j	size of unit j
B_{ijt}	amount of material undergoing task i in unit j during period t
U_{jt}	expected uptime of unit j during period t
λ_{jt}	failure rate of unit j during period t
$\gamma_{j\theta}$	failure rate value for unit j when the last maintenance action took place θ periods ago

The following are the basic constraints of the optimal reliable design, production, and maintenance planning model.

4.1. Design constraints

$$E_j = \sum_{k \in \psi_j} E_{jk} \qquad \forall j \tag{1}$$

$$V_j = \sum_{k \in \psi_j} \bar{V}_{jk} E_{jk} \qquad \forall j \tag{2}$$

The design constraints (1) and (2) determine the system structure by selecting units and their sizes out of a superstructure of units.

4.2. Resource utilization constraints

$$\sum_{i \in I_j} p_i N_{ijt} \le U_{jt} \qquad \forall j,\ t \tag{3}$$

The resource utilization constraints state that the total processing time on a unit cannot exceed the expected uptime of the unit.

4.3. Capacity constraints

$$\phi_{ijt}^{\min}\sum_{k\in\psi_j}\bar{V}_{jk}E_{jk}N_{ijt}\le B_{ijt}\le\phi_{ijt}^{\max}\sum_{k\in\psi_j}\bar{V}_{jk}E_{jk}N_{ijt} \quad (4)$$

$\forall i,\ j\in K_i,\ t$

Capacity constraints (4) suggest that batch sizes are allowed to vary between minimum and maximum values.

Introducing a continuous positive variable $\overline{EN}_{ijkt}$ can linearize the nonlinearities of the form, $N_{ijt}E_{jk}$ in equation 4 (Voudouris & Grossmann, 1992):

$$\overline{EN}_{ijkt}\equiv N_{ijt}E_{jk}\qquad\forall i,\ j\in K_i,\ \ k\in\psi_j,\ t \quad (5)$$

together with the following constraints:

$$\overline{EN}_{ijkt}\le N_{ij}^{\max}E_{jk}\qquad\forall i,\ j\in K_i,\ \ k\in\psi_j,\ t \quad (6)$$

$$N_{ijt}=\sum_{k\in\psi_j}\overline{EN}_{ijkt}\qquad\forall i,\ j\in K_i,\ t \quad (7)$$

where $N_{ij}^{\max}$ describe the maximum number of batches of task i that can be produced in unit j and is given by:

$$N_{ij}^{\max}=\frac{H\left(1-\Delta_j^c\max_{l\in\zeta_j}\{\bar{\lambda}_{jl}\}\right)}{p(i)}\qquad\forall i,\ j\in K_i \quad (8)$$

Substituting equation (5) into equation (4), the capacity constraints are now given as:

$$\phi_{ijt}^{\min}\sum_{k\in\psi_j}\bar{V}_{jk}\overline{EN}_{ijkt}\le B_{ijt}\le\phi_{ijt}^{\max}\sum_{k\in\psi_j}\bar{V}_{jk}\overline{EN}_{ijkt} \quad (9)$$

$\forall i,\ j\in K_i,\ t$

The relevant set of capacity constraints covers (6)–(9).

4.4. Material balance constraints

$$S_{st}=S_{s,t-1}+\sum_{i\in\bar{T}_s}\sum_{j\in K_i}\bar{\rho}_{is}B_{ijt}-\sum_{i\in T_s}\sum_{j\in K_i}\rho_{is}B_{ijt}-D_{st} \quad (10)$$

$\forall s,\ t$

The material balance constraints state that the amount of material in state s at the end of period t is the amount in storage at the end of the last period, plus the amount added by producer task, subtracting the amount consumed by consumer tasks and the amount delivered.

4.5. Demand constraints

$$D_{st}^{\min}\le D_{st}\le D_{st}^{\max}\qquad\forall s,\ t \quad (11)$$

The demand constraints (11) suggest that the demand of state s in each period t fluctuates between lower and upper bounds.

4.6. Utility constraints

$$\sum_i\sum_{j\in K_i}\sum_{\omega=0}^{p_i-1}(\beta_{uij\omega}N_{ijt}+\delta_{uij\omega}B_{ijt})\le A_{ut}^{\max}H \quad (12)$$

$\forall u,\ t$

The utility constraints (12) ensure that the utilization level of utilities such as steam, cooling water, manpower etc. does not exceed corresponding availability constraints.

4.7. Reliability allocation constraints

$$\gamma_{j1}=\sum_{l=1}^{\zeta_j}\bar{\lambda}_{jl}E_{jl}\qquad\forall j \quad (13)$$

$$E_j=\sum_{l\in\zeta_j}E_{jl}\qquad\forall j \quad (14)$$

$$\gamma_{j\theta}=\gamma_{j\theta-1}+\alpha_j,\qquad\forall j,\ 2\le\theta\le\tau_j \quad (15)$$

Reliability allocation constraints (13) determine the units' initial failure rate at the design stage. Constraints (14) ensure that only one kind of failure rate is selected at the design stage, while constraints (15) describe $\gamma_{j\theta}$ transition between periods, $2\le\theta\le\tau_j$.

4.8. Failure rate constraints

$$\lambda_{jt}=\sum_{\theta=1}^{\tau_j}\gamma_{j\theta}Z_{jt\theta}\qquad\forall j,\ t \quad (16)$$

$$X_{jt}\le E_j\qquad\forall j,\ t \quad (17)$$

$$Z_{jt\theta}\le X_{j,t-\theta}\qquad\forall j,\ t,\ \theta=1\dots\tau_j \quad (18)$$

$$\sum_{\theta=1}^{\tau_j}Z_{jt\theta}=E_j\qquad\forall j,\ t \quad (19)$$

Constraints (16)–(19) describe the failure rate of unit λ_{jt} as a function of maintenance policy implemented in the operational stage.

In constraint (16), the nonlinearities of the form $\gamma_{j\theta}Z_{jt\theta}$ can be linearized by introducing a continuous variable, $h_{jt\theta}$ (Floudas, 1995):

$$h_{jt\theta}\equiv\gamma_{j\theta}Z_{jt\theta} \quad (20)$$

together with the constraints:

$$\gamma_{j\theta}-\gamma_{j\theta}^{\max}(1-Z_{jt\theta})\le h_{jt\theta}\le\gamma_{j\theta}-\gamma_{j\theta}^{\min}(1-Z_{jt\theta})$$

$\forall j,\ t,\ \theta=1\dots\tau_j$

$$\gamma_{j\theta}^{\min}Z_{jt\theta}\le h_{jt\theta}\le\gamma_{j\theta}^{\max}Z_{jt\theta}\qquad\forall j,\ t,\ \theta=1\dots\tau_j \quad (21)$$

where, $\gamma_{j\theta}^{\min}$ and $\gamma_{j\theta}^{\max}$ are lower and upper limit on variable $\gamma_{j\theta}$, respectively, and defined by following expressions:

$$\gamma_{j\theta}^{\min}=\min_{l\in\zeta_j}\{\bar{\lambda}_{jl}\}\qquad\forall j \quad (22)$$

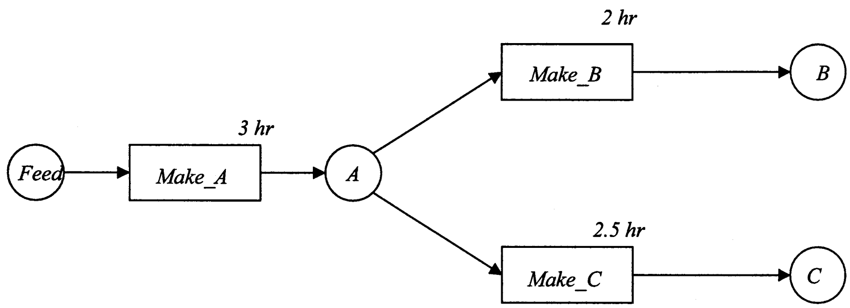

Fig. 4. STN for example 1.

Table 1
Design alternatives: example 1

Unit type	Available unit sizes ($\bar{V}_{jk}$)				Available unit reliabilities ($\bar{\lambda}_{jl}$)		
					$l=1$	$l=2$	$l=3$
Unit 1	150	175	200	250	0.002	0.0015	0.001
Unit 2	50	80	150	200	0.004	0.003	0.002
Unit 3	60	100	125	200	0.002	0.0015	0.001

$$\gamma_{j0}^{\max} = \max_{l \in \zeta_j}\{\bar{\lambda}_{jl}\} + \tau_j \alpha_j \qquad \forall j \tag{23}$$

Combining equation (20) with (16), equation (16) can be replaced by:

$$\lambda_{jt} = \sum_{\theta=1}^{\tau_j} h_{jt\theta} \qquad \forall j,\ t \tag{24}$$

The relevant set of constraints for the failure rates comprises (17), (18), (19), (21)–(24).

4.9. Uptime definition constraints

$$U_{jt} = H(E_j - \Delta_j^c \lambda_{jt}) - \Delta_j^p X_{jt} \qquad \forall j,\ t \tag{25}$$

Assuming that equipment can fail both during minimal repair and preventive maintenance, the expected equipment uptime for unit j during period t is given by constraints (25) (Dedopoulos & Shah, 1995).

4.10. Objective function

$$\begin{aligned}\max \Phi = \sum_{st} \eta_{st} D_{st} - &\sum_{ut} C_{ut} \sum_{i} \sum_{j \in K_i} \sum_{\omega=0}^{p_i - 1} (\beta_{uij\omega} N_{ijt} + \delta_{uij\omega} B_{ijt}) \\ &- \sum_{jt} C_{jt}^p X_{jt} - \sum_{jt} C_{jt}^c (HE_j - U_{jt} - \Delta_j^p X_{jt})/\Delta_j^c \\ &- \sum_j \left(K_j^0 E_j + K_j^1 \sum_{k \in \psi_j} \bar{V}_{jk} E_{jk} + \sum_{l \in \zeta_j} K_{jl}^2 E_{jl} \right)\end{aligned} \tag{26}$$

In expression (26), the first term represents the profit generated by the delivered products, the second term denotes the total cost of utilities, and the third and fourth term correspond to preventive and corrective maintenance costs, respectively. Finally, the fifth term corresponds to design costs as a function of equipment initial failure rate.

Table 2
Cost data: example 1

Unit type	Fixed cost (K_j^0)	Size cost factor (K_j^1)	Failure rate cost factor (K_{jl}^2)		
			$l=1$	$l=2$	$l=3$
Unit 1	5000	100	0	2200	6000
Unit 2	20 000	300	0	2200	6000
Unit 3	20 000	350	0	2200	6000

Table 3
Failure rate and maintenance data: example 1

Unit type	α_j	Δ_j^c (h)	C_{jt}^c	Δ_j^p (h)	C_{jt}^p
Unit 1	0.001	24	50	6	1000
Unit 2	0.001	40	100	9	2000
Unit 3	0.001	30	75	7	2000

5. Numerical examples

Two illustrative examples are presented to show the applicability of the model. The problems are modeled and solved within the GAMS (Brooke, Kendrick & Meeraus, 1988) modeling environment using the CPLEX MILP optimizer. The computations were carried out on an AMD athlon processor.

5.1. Example 1

The first illustrative example is taken from Pistikopoulos et al. (2001) where a multipurpose plant (as described by the STN shown in Fig. 4) must be designed at a maximum expected profit to produce two main products B and C. Three potential units of different sizes and initial failure rates are considered to be available at the design stage. The available sizes and initial failure rates for different units are given in Table 1. Unlimited storage capacity is assumed for feedstock and final products B and C, while no storage facility is considered for intermediate A. Unit 1 is suitable for task Make_A, while units 2 and 3 can perform both tasks Make_B and Make_C.

This example assumes an operating horizon of 2 years, discretized in 24 1-month time periods. The demand for product B and C is assumed to range between 20 000 and 50 000 units for each month period with a unit price per period of $\eta_{st} = 0.5$ for both B and C. It is furthermore assumed that at least one preventive maintenance action must be performed for each chosen unit every six time periods, i.e. $\tau_j = 6$. The cost and maintenance data for all three units are given in Tables 2 and 3. Note that in Table 2, we introduced the additional cost data for improving the initial failure rate of a unit (K_{jl}^2). The assumed additional cost data reflects the commonly used exponential relationship between initial reliability and capital cost.

The resulting MILP problem involves 480 binary variables, 2042 continuous variables and 3247 constraints and was solved in 0.33 s CPU time with a relative gap of 0.035. The optimal equipment sizes and initial failure rates, obtained from the solution of the proposed model, are depicted in Table 4. The corresponding failure rate profiles for the three units are shown in Fig. 5. The corresponding maintenance policy is given in Fig. 6(a). A large equipment size is selected for unit 1, which is explained by the importance of unit 1, being the only unit that can be used for task Make_A. Furthermore, reduced initial failure rates of 0.001 and 0.002 have been allocated at the design stage for unit 1 and 2, respectively.

The solution obtained is then compared with the results obtained by Pistikopoulos et al. (2001), where the initial reliabilities of units were considered fixed. The optimal sizes of units obtained in their work are given in Table 4 with the corresponding maintenance policy schedule shown in Fig. 6(b). The design costs, maintenance costs, and deliveries costs, obtained from the present model and Pistikopoulos et al.'s (2001) model, are compared in Table 5. It is interesting to note the trade-off between various costs in Table 5. In this work, the increased design and preventive maintenance costs are balanced by a reduction in corrective maintenance costs and increased revenues. This leads to an overall increase in expected profit. It should be noted here that the results obtained are sensitive to the additional data assumed in this work for reliability improvement options and their associated costs. Nevertheless, the results presented in Table 5 adequately illustrate the underlying trade-off between capital investment and maintenance costs, which is the purpose of this work. It is furthermore interesting to note from Table 4 and Fig. 6 that considering the initial reliability of process units as a decision variable in a combined design, production

Table 4
Results: example 1

Unit type	This work		Pistikopoulos et al. (2001)	
	Optimal size (V_j)	Optimal initial failure rate (λ_{j1})	Optimal size (V_j)	Initial failure rate[a] (λ_{j1})
Unit 1	250	0.001	250	0.002
Unit 2	150	0.002	80	0.004
Unit 3	60	0.002	125	0.002

[a] Assumed fixed in their model.

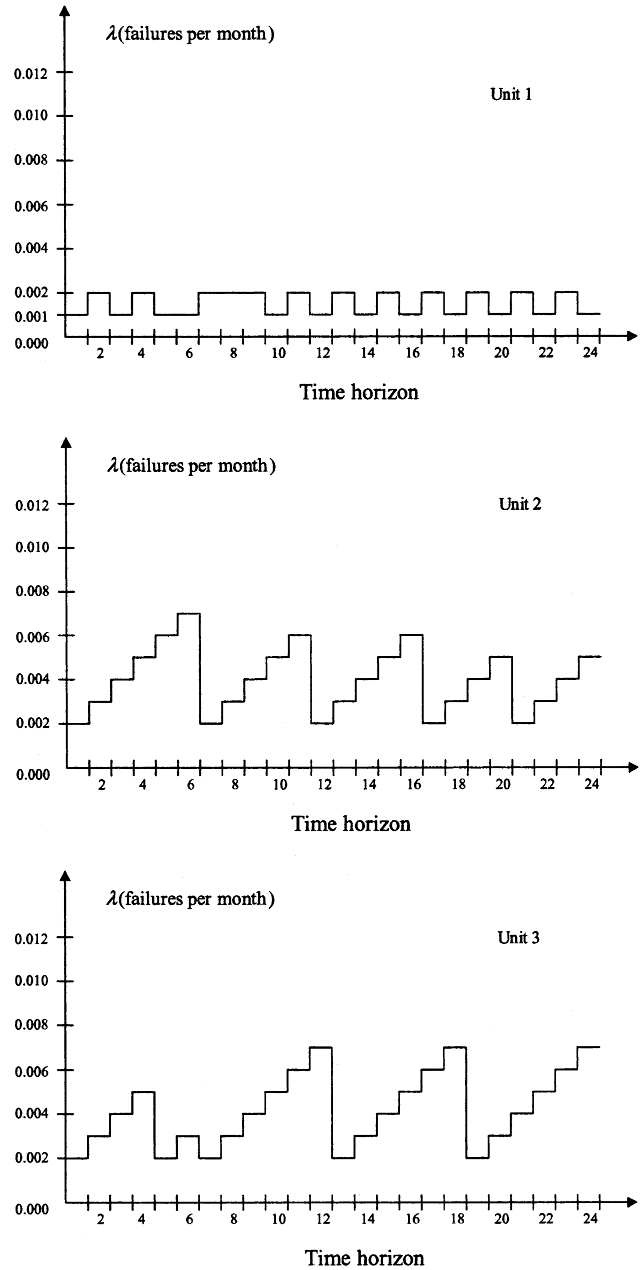

Fig. 5. Optimal failure rate profiles: example 1.

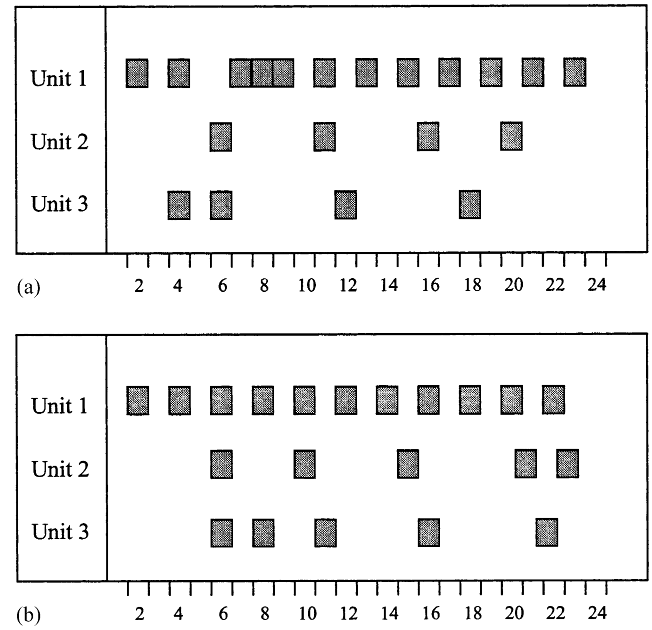

Fig. 6. Optimal preventive maintenance schedule: example 1 (a) with reliability optimization at design stage; (b) without reliability optimization at design stage (Pistikoploulos et al., 2001).

and maintenance planning model leads to a different optimal design and maintenance plan.

5.2. Example 2

The second example is a large-scale industrial example taken from Dedopoulos and Shah (1995). The plant concerned involves both batch and semi-continuous operations to produce ten different products. The process involves five different steps: blending, reaction, conveying, drying and packaging. Blending is a batch operation of 3 h creating the initial feedstock to be fed to the reactors. There are two batch reaction types of duration 5 and 6 h, respectively, produce two different intermediates (R1 and R2). In the conveying step, the reactor products are transferred continuously to an intermediate storage via a bucket conveyor. The materials are dried semicontinuously in the drying step. Dried products are fed to packaging lines to produce end products semicontinuously. The process is described by the STN in Fig. 7. The details of available processing and storage resources are given in Tables 6 and 7, respectively. The available sizes and initial failure rates for different units are given in Table 8.

An operating horizon of 4 years, discretized into 24 2-month time periods, is considered in this example. The demand and unit price per period for all products are given in Table 9. The minimum and maximum capacity utilization factors are assumed to be $\phi_{ij}^{\min} = 0.25$ and $\phi_{ij}^{\max} = 1$. The cost and maintenance data for all units are given in Tables 10 and 11, respectively. It is assumed that at least one preventive maintenance action must be performed to each chosen unit every nine time periods, i.e. $\tau_j = 9$.

The resulting MILP problem involves 1680 binary variables, 8012 continuous variables and 13 377 constraints and was solved in 0.95 s CPU time with a relative gap of 0.004. The optimal equipment sizes and initial failure rates, obtained from the solution of the proposed model, are depicted in Table 12. The optimal preventive maintenance policy obtained for this example

Table 5
Design, deliveries, and maintenance costs: example 1

	Value of deliveries	Total preventive maintenance cost	Total corrective maintenance cost	Design cost	Expected profit
This work	690 905	28 000	13 536	148 000	501 370
Pistikopoulos et al. (2001)	674 050	24 000	17 208	137 750	495 092

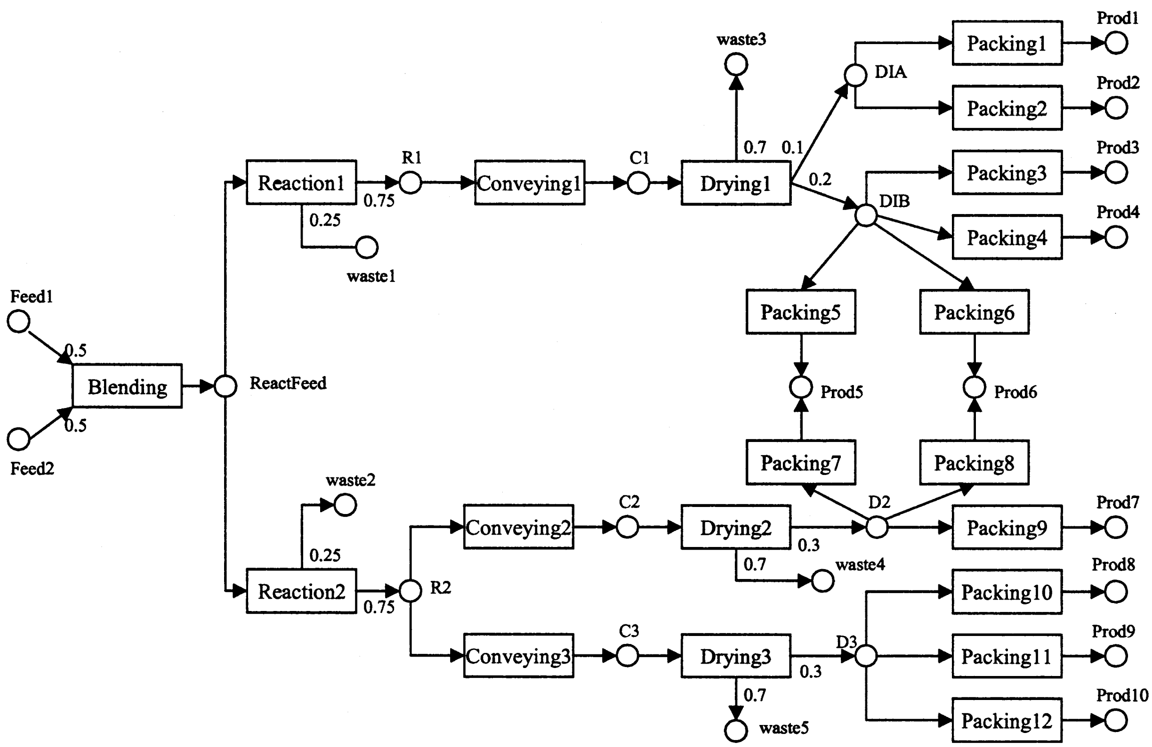

Fig. 7. STN for example 2.

Table 6
Details of processing resources: example 2

Unit type	Suitable tasks
Blender	Blending
Reactor	Reaction
Conveyor	Conveying
Dryer_A	Drying 1, drying 2
Dryer_B	Drying 3
PackLine1	Packing 1, Packing 2, Packing 3, Packing 4
PackLine2	Packing 5, Packing 6, Packing 7, Packing 8
PackLine3	Packing 9, Packing 10, Packing 11, Packing 12

is shown in Fig. 8. Note that for the conveyor equipment the biggest possible size and most reliable option available is selected. This can be explained by its importance to perform conveying one to three tasks. In addition, better initial reliability is allocated to reactor and packline 1–3. This particular example can be explained by the marginal cost of increasing the size of equipment and its initial reliability. The solution obtained is also compared with the results obtained with

Table 7
Details of storage resources: example 2

Storage unit	Capacity	Suitable tasks
Tank1	250	ReactFeed
Tank2	100	C1
Tank3	100	C2
Tank4	100	C3
Warehouse	25 000	Prod 1–prod 10

the model formulation of Pistikopoulos et al. (2001) in Tables 12 and 13. In Table 12, it is interesting to note the selection of sizes of pakline 2 and 3 in both optimal solutions. Table 13 describes the trade-offs between various costs terms used in the objective function for both formulations.

6. Conclusions

In this work, we have presented a new mathematical formulation for the integrated optimal reliable design,

Table 8
Design alternatives: example 2

	Available unit sizes ($\bar{V}_{jk}$)			Available unit reliabilities ($\bar{\lambda}_{jl}$)		
	$k=1$	$k=2$	$k=3$	$l=1$	$l=2$	$l=3$
Blender	32.0	36.0	40.0	0.0005	0.0004	0.00035
Reactor	40.0	49.0	56.0	0.0005	0.0004	0.00035
Conveyor	4.5	5.25	6.0	0.0020	0.0019	0.0018
Dryer_A	5.0	5.5	6.0	0.0020	0.0019	0.0018
Dryer_B	2.0	2.4	3.0	0.0025	0.0024	0.0023
PackLine 1	0.5	0.8	1.2	0.0025	0.0024	0.0023
PackLine 2	0.25	0.45	0.65	0.0025	0.0024	0.0023
PackLine 3	0.5	0.8	1.2	0.0025	0.0024	0.0023

Table 9
Product demand and price data: example 2

Product	dmin	dmax	Price
Product 1	150	300	500
Product 2	120	360	500
Product 3	150	450	500
Product 4	225	600	500
Product 5	225	510	500
Product 6	165	390	500
Product 7	90	240	500
Product 8	60	180	500
Product 9	75	240	500
Product 10	105	300	500

production and maintenance planning for multipurpose process plants. A reliability allocation model is coupled with the existing design, production, and maintenance optimization framework to identify the optimal size and initial reliability for each unit of equipment at the design stage. An explicit objective function is proposed, which balances the additional design and maintenance costs with the benefits obtained due to increased process availability.

In contrast to earlier approaches, which focus mainly on deriving an effective maintenance policy at the operational stage, the proposed integrated approach also provides a designer with an opportunity to improve the operational availability at the design stage by selecting better equipment.

The resulting optimization problem corresponds to an MILP formulation, which requires a modest computational effort. The applicability of the proposed model has been demonstrated in two numerical examples. The examples clearly show that the method proposed in this work for including reliability allocation in the design stage leads to a significantly different design (unit sizes, expected profit...) and accordingly different mainte-

Table 10
Cost data: example 2

Unit type	Fixed cost (K_j^0)	Size cost factor (K_j^1)	Failure rate cost factor (K_{jl}^2)		
			$l=1$	$l=2$	$l=3$
Blender	230 000	11 000	0	12 000	35 000
Reactor	230 000	11 000	0	12 000	35 000
Conveyor	200 000	45 250	0	12 000	35 000
Dryer_A	230 000	45 250	0	12 000	35 000
Dryer_B	235 000	65 500	0	12 000	35 000
PackLine 1	235 000	100 500	0	12 000	35 000
PackLine 2	235 000	100 750	0	12 000	35 000
PackLine 3	235 000	100 750	0	12 000	35 000

Table 11
Maintenance data: example 2

Unit type	α_j	Δ_j^c (h)	C_{jt}^c	Δ_j^p (h)	C_{jt}^p
Blender	0.0001	48	2000	32	15 000
Reactor	0.0001	48	2000	32	20 000
Conveyor	0.0005	40	2200	40	30 000
Dryer_A	0.0005	40	2200	32	12 000
Dryer_B	0.0005	40	2200	32	12 000
PackLine 1–3	0.0001	40	2200	40	30 000

Table 12
Results: example 2

Unit type	This work		Pistikopoulos et al. (2001)	
	Optimal size (V_j)	Optimal initial failure rate (λ_{j1})	Optimal size (V_j)	Initial failure rate[a] (λ_{j1})
Blender	32	0.0005	32	0.0005
Reactor	40	0.0004	40	0.0005
Conveyor	6	0.0018	6	0.0020
Dryer_A	5	0.0020	5	0.0020
Dryer_B	2	0.0025	2	0.0025
PackLine 1	0.8	0.0024	0.8	0.0025
PackLine 2	0.65	0.0024	0.45	0.0025
PackLine 3	0.5	0.0024	0.8	0.0025

[a] Assumed fixed in their model.

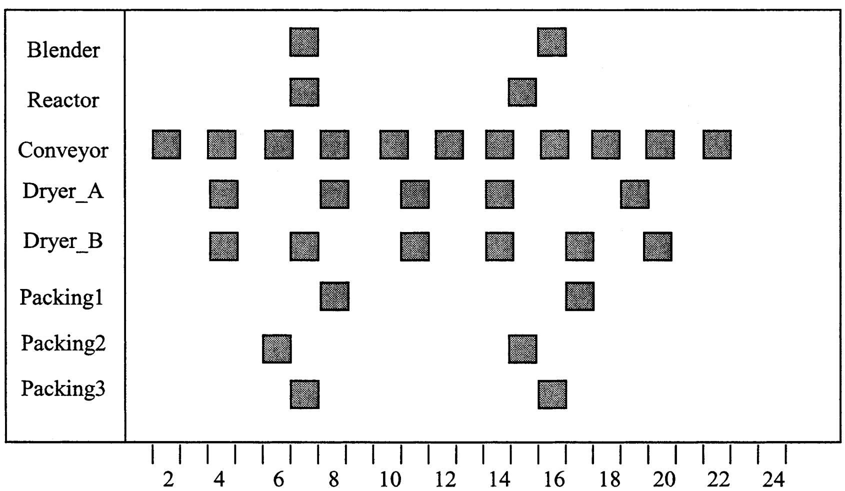

Fig. 8. Optimal preventive maintenance schedule: example 2.

Table 13
Design, deliveries, and maintenance costs: example 2

	Value of deliveries ($\times 10^3$)	Total preventive maintenance costs ($\times 10^3$)	Total corrective maintenance cost ($\times 10^3$)	Design cost ($\times 10^3$)	Expected profit ($\times 10^3$)
This work	28 157	712	1344	3530	22 571
Pistikopoulos et al. (2001)	27 908	700	1394	3512	22 302

nance policy in comparison with the existing approaches for combining design, production and maintenance planning.

References

Brooke, A., Kendrick, D., & Meeraus, A. (1988). *GAMS: a user guide*. California: Scientific Press.

Dedopoulos, I. T., & Shah, N. (1995). Long-term maintenance policy optimization in multipurpose process plants. *Chemical Engineering Research and Design 74*, 307–320.

Floudas, C. A. (1995). *Nonlinear and mixed-integer optimization*. New York: Oxford University Press.

Grievink, J., Smit, K., Dekker, R., & Van Rijn, C. F. H. (1993). Managing reliability and maintenance in the process industry. *Conference on foundation of computer aided operations, FOCAP-O* (pp. 133–157). Colorado, USA.

Kondili, E., Pantelides, C. C., & Sargent, R. W. H. (1993). A general algorithm for short-term scheduling of batch operations-I. MILP formulation. *Computers and Chemical Engineering 17*, 211–227.

Koolen, J. L. A. (2001). *Design of simple and robust process plants*. Weinheim, Germany: Wiley-VCH GmbH.

Kuo, W., Prasad, V. R., Tillman, F. A., & Hwang, C. (2001). *Optimal reliability design*. Cambridge University Press.

Majety, S. R. V., Dawande, M., & Rajgopal, J. (1999). Optimal reliability allocation with discrete cost–reliability data for components. *Operations Research 47*, 899–906.

Mettas, A. (2000). Reliability allocation and optimization for complex systems. *Proceedings annual reliability and maintainability symposium* (pp. 216–221). Los Angeles, USA.

Pistikopoulos, E. N., Vassiliadis, C. G., Arvela, J., & Papageorgiou, L. G. (2001). Interactions of maintenance and production planning for multipurpose process plants—a system effectiveness approach. *Industrial Engineering and Chemical Research 40*, 3195–3207.

Sanmarti, E., Espuna, A., & Puigjaner, L. (1997). Batch production and preventive maintenance scheduling under equipment failure uncertainty. *Computers and Chemical Engineering 21*, 1157–1168.

Vassiliadis, C. G., & Pistikopoulos, E. N. (2001). Maintenance scheduling and process optimization under uncertainty. *Computers and Chemical Engineering 25*, 217–236.

Voudouris, V. T., & Grossmann, I. E. (1992). Mixed integer linear programming reformulations for batch process design with discrete equipment sizes. *Industrial Engineering and Chemical Research 31*, 1315–1325.