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Integrating noncyclical preventive maintenance scheduling and production planning for multi-state systems

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ABSTRACT

This paper integrates noncyclical preventive maintenance with tactical production planning in multi-state systems. The maintenance policy suggests noncyclical preventive replacements of components, and minimal repair on failed components. The model gives simultaneously the appropriate instants for preventive maintenance, and production planning decisions. It determines an integrated lot-sizing and preventive maintenance strategy of the system that will minimize the sum of preventive and corrective maintenance costs, setup costs, holding costs, backorder costs, and production costs, while satisfying the demand for all products over the entire horizon. The model is first solved by comparing the results of several multi-products capacitated lot-sizing problems. Then, for large-size problems, a simulated annealing algorithm is developed and illustrated through numerical experiments.

1. Introduction

Maintenance scheduling and production planning are two important activities which can significantly contribute to better business management in industry. These activities directly operate on the same resources and equipment. Due to the difference between maintenance and production purposes, their relationship has been considered as mutually in conflict, especially if the production and maintenance planning are done separately. According to Berrichi et al. [7], the conflicts may result in an unsatisfied demand in production, due to equipment unavailability if the production service does not respect the time needed for maintenance activities. Integration of maintenance and planning activities can avoid conflicts. In Aghezzaf et al. [2] and Chung et al. [13], the authors have shown the benefits of integrating maintenance and production planning. Communication and collaboration between the two departments are the main keys to doing successful planning in production systems.

Much research related to integrated production and maintenance planning can be found in the literature, especially during the last few years. In these integrated models, it is considered that the beginning times of preventive maintenance (PM) tasks are decision variables, as well as production jobs, and both (maintenance and production) are jointly scheduled [7]. Budai et al. [10] classified these problems into four categories: high level models, the economic manufacturing quantity models, models of production systems with buffers, and production/maintenance optimization models. In the last category, where our work is situated, many problems have been presented in the literature. Most of these models aim to optimize a combination of maintenance and/or production costs, production makespan or system availability (or unavailability). Berrichi et al. [7] suggested a model minimizing, simultaneously, the makespan for production and the system unavailability for systems with parallel machines. The model was solved by genetic algorithms. Berrichi et al. [8] improved the obtained results by using an ant colony algorithm. Ben Ali et al. [5] studied a job-shop scheduling problem under periodic unavailability periods for maintenance tasks. The problem was solved by developing an elitist multi-objective genetic algorithm minimizing makespan and total maintenance cost. Chung et al. [13] presented a model also optimizing the production makespan, with a reliability option based on the acceptability function for multi-factory networks. The maintenance strategy is suggested for both perfect and imperfect maintenance policies. A bi-objective optimization model minimizing simultaneously the production makespan and the system unavailability is considered by Moradi et al. [26], where production decisions assign the appropriate n jobs to m machines and maintenance decisions determine the instants of PM activities.

Pan et al. [31] suggested an integrated scheduling model incorporating both production scheduling and preventive maintenance planning for a single machine in order to minimize the maximum weighted tardiness. Cassady and Kutangolu [11] and

Abbreviations: PM, preventive maintenance; SA, simulated annealing; GA, genetic algorithm; TS, tabu search; MSS, multi-state system; PR, preventive replacement; MR, minimal repair; UGF, universal generating function; ES, exhaustive search

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Nomenclature

ΔE	loss of energy of the simulated annealing algorithm
δ_j	the reduction factor for the component j ($j=1,\dots,n$) when PM actions are performed at the beginning of production planning periods
AG	$n\times TS$ age matrix representing the effective age of each component j at the beginning of each maintenance planning period
A	$n\times T$ matrix representing the availability of each component j during each production planning period t ($t=1,\dots,T$)
CMR	$n\times n$ minimal repair cost diagonal matrix, where CMR_{jj} ($j=1,\dots,n$) is the minimal repair cost for the component j
CPR	$n\times n$ preventive replacement cost diagonal matrix, where CPR_{jj} ($j=1,\dots,n$) is the preventive replacement cost for the component j
M	$n\times TS$ matrix representing the expected number of failures of each component j during each maintenance planning period
$\mathbf{p_j^t}$	the probability distribution of the component j during the planning period t ($j=1,\dots,n$ and $t=1,\dots,T$)
Q	$TS\times T$ bloc diagonal scale reduction matrix
R	$n\times n$ diagonal matrix of cost reduction if PM actions are performed at the beginning of the production planning period
TMR	$n\times n$ minimal repair time diagonal matrix, where TMR_{jj} ($j=1,\dots,n$) is the minimal repair time for the component j
TPR	$n\times n$ preventive replacement time diagonal matrix, where TPR_{jj} ($j=1,\dots,n$) is the preventive replacement time for the component j
π_{pt}	cost of producing one unit of product p in period t
τ	length of the maintenance planning period
τ^{ts}	the s^{th} maintenance planning period of the production planning period t ($t=1,\dots,T$ and $s=1,\dots,S$)
A_j^{ts}	availability of the component j during the maintenance planning period τ^{ts} ($j=1,\dots,n$, $t=1,\dots,T$, $s=1,\dots,S$)
a_j^{ts}	age function of the component j at the end of the maintenance planning period τ^{ts} ($j=1,\dots,n$, $t=1,\dots,T$, $s=1,\dots,S$)
b_{pt}	backorder cost (lost opportunity and goodwill) per unit of product p by the end of period t
C	cooling constant of the simulated annealing algorithm
CM	total maintenance cost
CT	total maintenance and production costs
d_{pt}	demand of the product p to be satisfied at the end of period t
E	energy objective function of the simulated annealing algorithm
$f_j(\cdot)$	the lifetime function of the component j ($j=1,\dots,n$)
G_j	nominal production rate of component j
g_k	production rate for the state k, ($1\le k\le K$)
G_{MSS}^t	MSS available production capacity during the period t
H	planning horizon
h_{pt}	inventory holding cost per unit of product p by the end of period t
j	component index ($1\le j\le n$)
K	finite number of production rates
k	system state ($1\le k\le K$)
L	length of production planning periods t
$M_j(t)$	expected number of failures/repairs of the component j, in the time interval $[0,t]$
M_j^{ts}	the expected number of failures of the component j during the maintenance planning period τ^{ts} ($j=1,\dots,n$, $t=1,\dots,T$, $s=1,\dots,S$)
N	the number of possible combinations of the maintenance policy matrix **Z**
n	number of components
P	set of products
p	product, $p\in P$
$prob_k$	the steady-state probability of the state k, ($1\le k\le K$)
q_{ts}^i	binary variable equal to 1 if $t=i$ and 0 otherwise ($t=1,\dots,T$, $s=1,\dots,S$ and $i=1,\dots,T$)
$r_j(\cdot)$	the hazard function of the component j ($j=1,\dots,n$)
S	number of equal sub-periods of the interval L
s	maintenance planning period index ($1\le s\le S$)
Set_{pt}	fixed set-up cost of producing product p in period t
T	number of production planning periods
t	production planning period, ($1\le t\le T$)
T_e	temperature of the simulated annealing cooling process
T_{max}	maximal temperature of the simulated annealing algorithm
T_{min}	minimal temperature of the simulated annealing algorithm
w	random value from the interval [0, 1]
z_j^{ts}	binary variable equal to 1 if a PR is carried out on the component j at the beginning of the maintenance planning period τ^{ts}, and 0 otherwise

Decision variables

Z	binary matrix representing system preventive replacement policy
B_{pt}	backorder level of product p at the end of period t
I_{pt}	inventory level of product p at the end of period t
x_{pt}	quantity of product p to be produced in period t
y_{pt}	binary variable, which is equal to 1 if the set-up of product p occurs at the end of period t, and 0 otherwise

Sortrakul et al. [36] proposed an integrated maintenance planning and production scheduling model for a single machine minimizing the total weighted expected completion time to find the optimal PM actions and job sequence. Yu-Lan et al. [43] extended these researches where PM actions can be performed under flexible intervals (instead of equal intervals) which leads to more efficient solutions. Jin et al. [18] presented a model determining the optimal number of preventive maintenance activities in order to maximize the average profit under uncertain demand by using the financial “option” approach.

A mathematical model for a single unit determining simultaneously the optimal value of lot size and the optimal preventive replacement interval with non-conformity constraints is suggested by Chelbi et al. [12]. Hajej et al. [17] investigated stochastic production planning and the maintenance scheduling problem for a single product and a single machine production system with subcontracting constraints. Ashayeri et al. [4] proposed a model optimizing total maintenance and production costs in discrete multi-machine environment with deterministic demand. Weinstein and Chung [42] worked on an integrated production and maintenance planning model in a hierarchical environment, where decisions about maintenance and production planning are made in aggregate and disaggregate levels. Coudert et al. [14] used a multi-agent paradigm and fuzzy logic for a cooperative production/maintenance scheduling to avoid conflict. A simulation-based approach is presented by Benmansour et al. [6] for joint production and preventive maintenance planning for a failure-prone machine in just-in-time context. Aghezzaf et al. [2] proposed a mixed non-linear periodic maintenance and production planning model, and a mixed linear model for a general preventive maintenance and production planning problem. Both models are solved with an approximate algorithm based on Lagrangian decomposition. Najid et al. [27] extended these researches and suggested an integrated model where preventive maintenance actions are planned in time windows and demand shortage is allowed when capacity is not sufficient to meet all demand. A hierarchical general (non-cyclic) model is developed by Sitompul and Aghezzaf [35]. The authors integrated preventive maintenance activities into the aggregate planning, while corrective maintenance and uncertainty due to machine breakdowns are tackled in the detailed level planning.

All the above mentioned papers assume that the production system may experience only two performance levels (perfect functioning or complete failure). In a recent contribution [30], we presented an integrated production and cyclical PM planning model for multi-state systems. Unlike the existing papers, this contribution assumes that the production system may experience a range of performance levels from perfect functioning to complete failure, which is more realistic. In Fitouhi and Nourelfath [16], we have extended our previous work (*i.e.* Nourelfath et al. [30]) to noncyclical PM for a single machine. In Nourelfath and Châtelet [28], the authors have dealt with the integration of production, inventory and maintenance planning for a parallel system with dependent components. The present paper proposes an integrated model for noncyclical PM and tactical production planning for multi-state systems. The model suggests a maintenance policy for each component where maintenance actions can be carried out at the beginning or inside any production planning period.

There are others existing papers dealing with joint optimization. In Levitin and Lisnianski [22], the authors formulated the joint redundancy and replacement schedule optimization problem generalized to multi-state system. In Levitin and Lisnianski [23], the authors considered a preventive maintenance optimization problem for multi-state systems, for which the reliability is defined as the ability to satisfy given production demand. The authors of Rosqvist et al. [33] presented a value-driven maintenance planning approach and applied it to approach to a production plant. In Vatn and Aven [40], the authors have shown the importance of linking maintenance and safety risks, and presented an approach to maintenance optimization where safety issues are important. The authors of Cowing et al. [15] presented a dynamic modeling of the trade-off between productivity and safety in critical engineering systems. In Vatn et al. [41], the authors presented an overall model for maintenance optimization. They developed an approach for identifying the optimal maintenance schedule for the components of a production system. Safety, health and environment objectives, maintenance costs and costs of lost production are all taken into account, and maintenance is thus optimized with respect to several objectives. Finally, in Nourelfath et al. [29], the authors have formulated a joint redundancy and imperfect preventive maintenance planning optimization model for seriesparallel multi-state degraded systems. A heuristic approach was also proposed to solve the formulated problem. This heuristic is based on a combination of space partitioning, genetic algorithms (GA) and tabu search (TS). After dividing the search space into a set of disjoint subsets, this approach uses GA to select the subspaces, and applies TS to each selected sub-space.

Although the above cited papers deal with joint optimization, to the best of our knowledge, the present paper is the first to develop an integrated model for noncyclical PM and tactical production planning for multi-state systems.

This new model will be first solved by comparing the results of several multi-product capacitated lot-sizing problems. Then, for large-size problems, a simulated annealing algorithm will be developed. Simulated Annealing (SA) has been applied to many production and maintenance planning problems. The SA performs in combinational optimization due to the use of analogous cooling operation for transforming a poor, unordered solution into an ordered and desirable solution, which can optimize the objective function [37]. SA can contribute to solve large scale problems with good quality in a reasonable computing time [32]. For the production planning research area, Teghem et al. [38] used SA algorithm in order to solve a mixed linear integer production planning for a book cover printing process. An adaptation of SA algorithms in project scheduling with limited resource was presented in Bouleimen and Lecocq [9]. Loukil et al. [25] proposed a SA algorithm to solve a production scheduling problem for flexible job-shop with batch production and process constraints. Shan et al. [34] improved the efficiency of solution research for a production assembly sequence by combining GA and SA approaches. Tang [37] applied the SA approach for a lot sizing problem to determine the optimal binary lot-sizing matrix decision. In maintenance planning, Leou [20] used the SA combined with GA for solving a maintenance scheduling problem optimizing reliability and operation cost. In Raza and Al-Turki [32], the authors have shown, through a large scale comparative study, the efficiency and the performance of SA approach for a maintenance scheduling problem minimizing total makespan. These successful implementations of SA algorithms for production and maintenance planning problems have motivated the proposed SA approach to solve the formulated optimization problem when the studied examples are large.

This paper is organized as follows. The next section presents the mathematical model, and its characteristics. Section 3 explains the maintenance policy, and methodology used to estimate the model parameters. An exhaustive search method and a simulated annealing approach are presented in Section 4 to solve the proposed integrated production and maintenance planning model. Numerical examples are presented in Section 5, which is solved with both solution methods. The proposed model is extended in Section 6. Finally, conclusions are given in Section 7.

2. The mathematical model

2.1. Problem description

Let us consider a production system containing a set of machines (called components) arranged according to a given configuration. We use a generic multi-state system (MSS) model that may represent any configuration, *e.g.* series, parallel, series-parallel [42,30,16,37,32], networks, etc. We assume that the states of the components are binary (*i.e.*, either good or failed). Each two-state component j $(1 \le j \le n)$ is characterized by its own nominal performance rate G_j. The production system is consequently considered as an MSS with a finite number K of production rates g_k $(k = 1, 2, \ldots, K)$, where g_k is defined as the number of products per time unit and each type of product requires the same effort. To each production rate corresponds a state k with a steady-state probability $prob_k$. The system may produce (at different production rates) a set of products P during a given planning horizon H including T periods. Each period t $(t=1,\ldots,T)$ has a fixed length L. For each product $p\in P$ a demand d_{pt} is to be satisfied at the end of period t. Planned preventive maintenance and unplanned corrective maintenance can be performed on each component of the MSS. The maintenance policy suggests preventive replacements (PR) of components (cyclic or not cyclic). Furthermore, a minimal repair (MR) is carried out whenever an unplanned component failure occurs. For each component j is associated preventive and corrective maintenance times and costs, in addition of its expected number of failures/repairs in the time interval [0, t], denoted by $M_j(t)$. The expected maintenance cost (denoted by CM) during the planning horizon is the sum of preventive and corrective maintenance costs. The available production capacity of the MSS during a period t (denoted by G^t_{MSS}) is measured by its average global production rate during this period (*i.e.*, number of produced items per time unit). In this problem we assume that:

- Components are economically, stochastically and structurally independent, which means respectively that the cost of join maintenance of a group of components is equal to the total cost of individual maintenance of these components, the condition of components does not influence the lifetime distribution of other components and each component forms an entity that is not further subdivided for a reliability study.
- Preventive maintenance is performed according to the “as good as new” policy (perfect maintenance).
- Minimal repair does not affect the components age.
- All products need the same amount of work according to their performance rate.
- There is no initial stock or backorder.

2.2. The model

Minimize

$$CT=\sum_{p\in P}\sum_{t=1}^{T}(h_{pt}I_{pt}+b_{pt}B_{pt}+\pi_{pt}x_{pt}+Set_{pt}y_{pt})+CM(\mathbf{Z}) \tag{1}$$

Subject to

$$x_{pt}-I_{pt}+I_{p(t-1)}+B_{pt}-B_{p(t-1)}=d_{pt},\quad p\in P \quad \text{and} \quad t=1,\ldots,T, \tag{2}$$

$$x_{pt}\le\left(\sum_{q\ge t}d_{pq}\right)y_{pt},\quad p\in P \quad \text{and} \quad t=1,\ldots,T, \tag{3}$$

$$\sum_{p\in P}x_{pt}\le G^t_{MSS}L,\quad t=1,\ldots,T, \tag{4}$$

$$x_{pt}, I_{pt} \text{ and } B_{pt} \text{ integers},\ p\in P \text{ and } \quad t=1,\ldots,T, \tag{5}$$

$$y_{pt} \text{ binary } (p\in P \text{ and } t=1,\ldots,T), \text{ and } \mathbf{Z} \text{ is binary matrix}. \tag{6}$$

$$B_{p0}=0 \text{ and } I_{p0}=0,\ p\in P. \tag{7}$$

The objective function to be minimized is a non-linear cost equation as represented by Eq. (1). It consists of a total holding cost, a backorder cost (backlogs are allowed), a total production cost, a total set-up cost, and a total maintenance cost $CM(\mathbf{Z})$. According to this objective function, the decision variables are I_{pt}, B_{pt}, x_{pt}, y_{pt} and $\mathbf{Z}$.

The first constraint (2) relates inventory or backorder at the start and end of period t to the production and demand in that period. There is no optimal solution where $I_{pt}>0$ and $B_{pt}>0$ simultaneously, since the objective function can be improved by decreasing both I_{pt} and B_{pt} until one becomes zero. Eq. (2) ensures simply that the sum of inventory (or backorder) of product p at the end of period t is equal to its inventory (or backorder) in the previous period plus the total production of that product in that period, minus the product demand for that period. The second constraint (3) forces $x_{pt}=0$ if $y_{pt}=0$ and frees $x_{pt}\ge 0$ if $y_{pt}=1$. In Eq. (3), the quantity $(\sum_{q\ge t}d_{pq})$ is an upper bound of x_{pt}. Eq. (4) corresponds to the available production capacity constraint. Finally, the constraint (5) and (6) show the non-negativity and the binary aspect of the decision variables respectively. Constraint (7) determines the initial inventory and backorder conditions.

As in the above model (given by Eqs. (1)–(7)), the values of $CM(\mathbf{Z})$ and $G^t_{MSS}(\mathbf{Z})$ have to be calculated, a method to estimate these values will be presented in the next section.

3. Evaluation of $CM(\mathbf{Z})$ and $G^t_{MSS}(\mathbf{Z})$

The evaluation of the total maintenance cost and the MSS capacity is related to the maintenance policy applied to the production system. Each production planning period t $(t=1,\ldots,T)$ has a fixed length L, which is divided into S equal sub-periods τ^{ts} $(s=1,\ldots,S)$ called maintenance planning period where

$$\tau^{ts}=\frac{L}{S}=\tau,\ t=1,\ldots,T,\ \text{and } s=1,\ldots,S. \tag{8}$$

For each component j, a preventive replacement can be carried out at the beginning of each maintenance period τ^{ts} $(t=1,\ldots,T$ and $s=1,\ldots,S)$ according to “as good as new” policy. For unplanned failure, a corrective maintenance is performed according to the minimal repair policy as shown in Fig. 1.

In order to represent the maintenance policy, we define the maintenance policy matrix $\mathbf{Z}=(z_j^{ts})_{n\times TS}$ where z_j^{ts} is a binary variable equal to 1 if a PR is carried out on the component j at the beginning of the maintenance planning period τ^{ts}, and 0 otherwise. For each component j we have $T\times S$ decision variables z_j^{ts} corresponding to all possible PR actions that can be carried out at the beginning of each maintenance planning period τ^{ts} $(t=1,\ldots,T$ and $s=1,\ldots,S)$ as shown in Fig. 2.

Fig. 3 presents an example of the maintenance policy matrix $\mathbf{Z}$ for a system with 4 components, 5 production planning periods and 2 maintenance planning sub-periods. For component 4 for example, a PR action is performed at the beginning of the first and the fifth maintenance planning periods (first sub-period of the first production planning period and the second sub-period of the third production planning period).

We define the age matrix $\mathbf{AG}(\mathbf{Z})=(a_j^{ts})_{n\times TS}$ obtained from the maintenance policy matrix $\mathbf{Z}$. The age matrix represents the effective age of each component j at the beginning of each maintenance planning period. The effective component age takes into consideration the instant of the previous PR and the length of the maintenance planning period according to the following

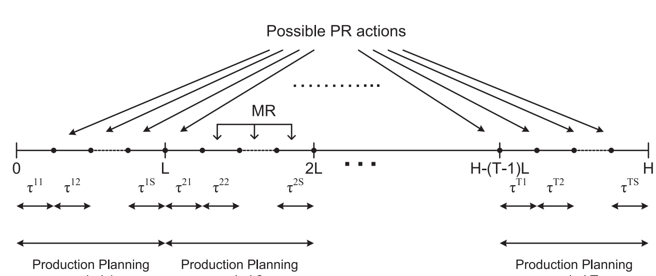

Fig. 1. Maintenance policy for a component j of the MSS.

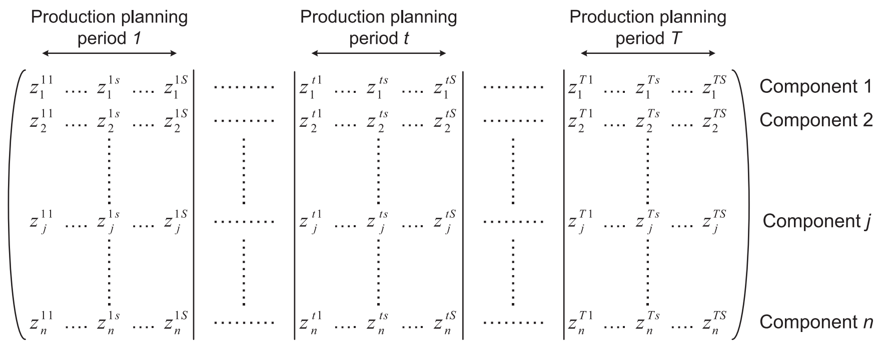

Fig. 2. Structure of the Maintenance Policy matrix.

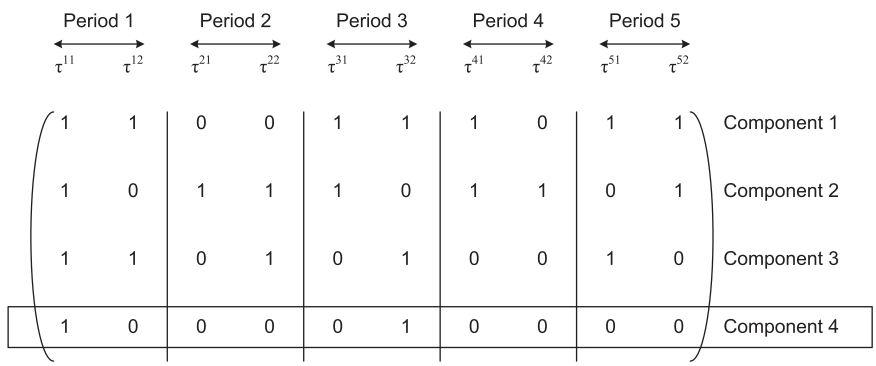

Fig. 3. Example of the maintenance policy matrix for $n=4$, $T=5$ and $S=2$.

equation:

$$a_j^{ts} = \begin{cases} (1-z_j^{ts}) & \text{if } j=1,\dots,n,\ t=1,\ s=1, \\ (1-z_j^{ts})(a_j^{t(s-1)}+\tau) & \text{for } j=1,\dots,n,\ t=1,\dots,T,\ s=2,\dots,S, \\ (1-z_j^{ts})(a_j^{(t-1)S}+\tau) & \text{for } j=1,\dots,n,\ t=2,\dots,T,\ s=S. \end{cases} \tag{9}$$

Through the age matrix $\mathbf{AG}(\mathbf{Z})_{n\times TS}$ we introduce the matrix $\mathbf{M}(\mathbf{AG}(\mathbf{Z}))=(M_j^{ts})_{n\times TS}$, which represents the expected number of failures. It is denoted $\mathbf{M}(\mathbf{Z})=(M_j^{ts})_{n\times TS}$, and defined according to the non-homogeneous Poisson process by

$$M_j^{ts} = \int_{a_j^{ts}}^{a_j^{ts}+\tau} r_j(x)\,dx, \text{ for } j=1,\dots,n,\ t=1,\dots,T \text{ and } s=1,\dots,S, \tag{10}$$

where $r_j(\cdot)$ is the hazard function of the component j computed from the lifetime distribution $f_j(\cdot)$. The matrix $\mathbf{M}$ gives the average number of renewals for each component during all the maintenance planning periods. Therefore, total maintenance cost $CM(\mathbf{Z})$ is the summation, for all components, of PR costs given by the matrix **CPR** and the maintenance policy matrix **Z**, with the MR costs given by the MR cost matrix **CMR** and the renewal matrix $\mathbf{M}(\mathbf{Z})$. The total maintenance cost is

$$\begin{aligned} CM(\mathbf{Z}) &= \|\mathbf{CPR}\times\mathbf{Z}+\mathbf{CMR}\times\mathbf{M}(\mathbf{Z})\| \\ &= \sum_{j=1}^{n}\sum_{t=1}^{T}\sum_{s=1}^{S}(CPR_{jj}z_j^{ts}+CMR_{jj}M_j^{ts}), \end{aligned} \tag{11}$$

where $\|\cdot\|$ is the matrix norm defined by the summation of the absolute value of all its components.

Based on the same principle, for each component j, we can obtain the availability A_j^{ts} during each maintenance planning period τ^{ts} ($t=1,\dots,T$ and $s=1,\dots,S$) according to the equation

$$A_j^{ts} = \frac{\tau^{ts}-z_j^{ts}TPR_{jj}-M_j^{ts}TMR_{jj}}{\tau^{ts}},\ 1\le j\le n,\ 1\le t\le T \text{ and } 1\le s\le S. \tag{12}$$

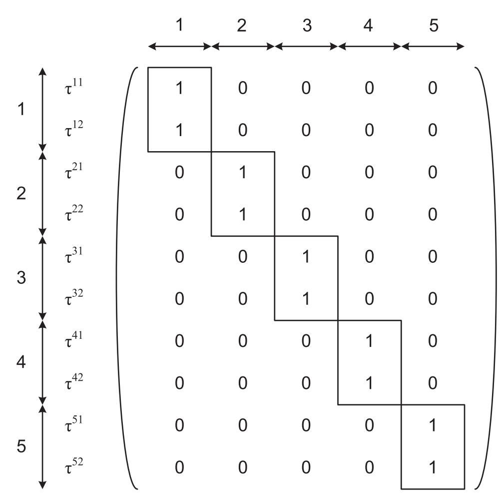

Fig. 4. Scale reduction matrix **Q** for $n=4$, $T=5$ and $S=2$.

However, to estimate the capacity G_{MSS}^t, we need to evaluate the availability of each component j ($j=1,\dots,n$) during the production planning period (and not the maintenance planning period). In order to reduce the planning scale from the maintenance planning to the production planning scale (change matrix dimension from TS to T), we introduce the bloc diagonal scale reduction matrix $\mathbf{Q}=(q_{ts}^i)_{TS\times T}$ where q_{ts}^i ($t=1,\dots,T$, $s=1,\dots,S$ and $i=1,\dots,T$) is a binary variable equal to 1 if $t=i$ and 0 otherwise as illustrated in the example in Fig. 4. Consequently, the availability matrix $\mathbf{A}=(A_j^t)_{n\times T}$ can be defined by using the matrix **Q**, the time matrix **TPR** and **TMR**, the maintenance policy matrix **Z** and the renewal matrix $\mathbf{M}$ according to the equation

$$(A_j^t)_{n\times T}=\frac{[L(\mathbf{1})_{n\times T}-\mathbf{TPR}_{n\times n}\mathbf{Z}_{n\times ST}\mathbf{Q}_{ST\times T}-\mathbf{TMR}_{n\times n}\mathbf{M}(\mathbf{Z})_{n\times ST}\mathbf{Q}_{ST\times T}]}{L}, \tag{13}$$

where

$$A_j^t=\frac{L-\sum_{s=1}^{S}(z_j^{ts}TPR_{jj}+M_j^{ts}TMR_{jj})}{L},\ j=1,\ldots,n \text{ and } t=1,\ldots,T, \tag{14}$$

and $(\mathbf{1})_{n\times T}$ is the $n\times T$ matrix where all its elements are equal to 1.

As mentioned previously, each component j is considered as a 2-state system functioning in perfect state with a production rate G_j or in complete failure state. For each period t, the probabilities associated to the 2 states $\{0, G_j\}$ for each component j can be defined by

$$\mathbf{p_j^t}=\{(1-A_j^t),A_j^t)\},\ j=1,\ldots,n \text{ and } t=1,\ldots,T. \tag{15}$$

By using the probability distribution $\mathbf{p_j^t}$ and the 2 states $\{0, G_j\}$ for each component j, the MSS capacity G_{MSS}^t for each period $t=1,\ldots,T$ can be estimated depending on the system configuration (series, parallel, series-parallel, network, etc.). Many methods used to evaluate the MSS capacity have been developed in the literature [24]. Since we have binary-state components, the MSS capacity can be evaluated with the structure function approach as used in Nourelfath et al. [30]. The authors show a clear example of the method application for a 2-parallel binary component system and applied it to generate a cyclical integrated maintenance and production planning for multi-state systems. However, the structure function is only efficient for small-size systems due to state enumeration used by the approach. In fact, if we consider, for example, a 20 independent parallel component will have 2^{20} possible state. In this case, the Universal Generating Function (UGF) introduced by Ushakov [39] can be used to evaluate the MSS capacity. The approach suggests algebraic procedures and operators using performance distributions of each component. The UGF method is a recursive approach which presumes obtaining the UGF of subsystems containing several basic components then treating the subsystem as a single component with the obtained UGF when computing the UGF of a higher level subsystem [21].

4. Solution method

Model (1)–(7) are considered as an integer non-linear model. The decision variables are the maintenance policy matrix $\mathbf{Z}$ on one hand and, for each period t $(t=1,\ldots,T)$ and each product $p\in P$, the production, inventory backorder and set up variables x_{pt}, I_{pt}, B_{pt} and y_{pt}, respectively, on the other hand. However, for each maintenance policy matrix $\mathbf{Z}$, through the estimation of model parameters $CM(\mathbf{Z})$ and G_{MSS}^t $(t=1,\ldots,T)$ according to the methodology used in the previous section, model (1)–(7) can be considered as an integer linear lot-sizing problem. The variables x_{pt}, I_{pt}, B_{pt} and y_{pt} related to the lot-sizing problem are simultaneously determined when solving the optimization model by any method.

Consequently, two solution methods will be developed: the Exhaustive Search (ES) method and the Simulated Annealing (SA) algorithm. Both the methods are based on the same principle, which is fixing the maintenance policy matrix $\mathbf{Z}$, computing the parameters $CM(\mathbf{Z})$ and G_{MSS}^t $(t=1,\ldots,T)$ and solving the integer linear lot-sizing problem by using the MATLAB solver. Nevertheless, the difference between these two methods consists in the choice of the maintenance policy matrix $\mathbf{Z}$.

4.1. Exhaustive search method

This method is similar to the solution method used in Fitouhi and Nourelfath [16]. The ER method consists of enumerating all possible combinations of the maintenance policy matrix $\mathbf{Z}$, then computing their relative parameters $CM(\mathbf{Z})$ and G_{MSS}^t $(t=1,\ldots,T)$ and finally solving all possible lot sizing problems in order to select the best solution. The number of lot-sizing problems to solve corresponds to the number of possible combinations N of the maintenance policy matrix $\mathbf{Z}$ which depends on the number of periods T, the sub-period S and the number of components n according to the equation

$$N=2^{n(ST-1)}. \tag{16}$$

The guarantee of obtaining the optimal solution is the main advantage of this method. However, it can only be applied to small problems. In fact, the number of possible combinations of the maintenance policy matrix $\mathbf{Z}$ increases exponentially if n, S or T increases according to the previous equation. For example, for a production system with 5 components, 4 periods and 2 sub-periods, the number of possible combinations of the maintenance policy matrix $\mathbf{Z}$ is around 10^{10} possibility which, using a 2.33 GHz processor, should take more than eight years to be solved. Consequently, in order to solve that kind of a problem, the simulated annealing heuristic is developed in the next section.

4.2. Simulated Annealing algorithm

The Simulated Annealing (SA) is an optimization technique for combinatorial problems introduced by Kirkpatrick et al. [19] and based on principles of thermodynamics annealing in solid. Starting from an initial feasible solution, the SA algorithm defines a move set that allows the choice of another possible solution from the neighbourhood, under a cooling process of the temperature T_e. In general, the moving process is a random process. The value of the objective function E' is then evaluated and compared to the objective function E obtained by the previous solution. If the movement improves the objective function, the new solution is taken, else the loss of energy $\Delta E=\exp(-(E-E')/T_e)$ is compared to w which is a random value from the interval [0, 1]. If $w<\Delta E$ the solution is selected else it is rejected. The temperature T_e decreases from a maximal temperature T_{max} to a minimal temperature T_{min} by following a cooling schedule. Many criteria can be chosen to stop the SA algorithm such as reaching the minimal temperature, maximal number of iterations, maximal number of unchanged solutions, etc.

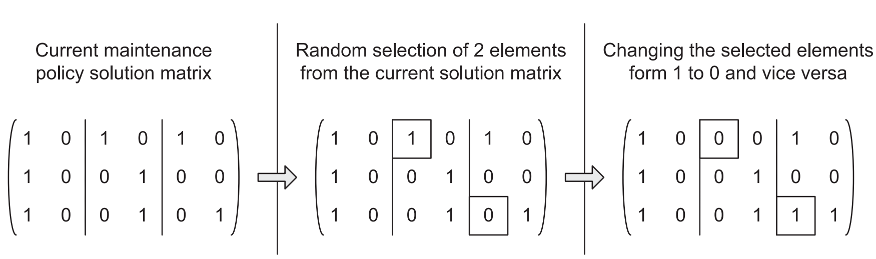

Fig. 5. Example of neighbourhood move for a production system with $n=3$, $T=3$ and $S=2$.

```
Choose initial solution maintenance policy matrix Z_0
Compute CM and G^t_MSS relative to Z_0
Solve the integer linear lot-sizing problem relative to Z_0 and determine
the initial solution (x_pt)_0, (I_pt)_0, (B_pt)_0 and (y_pt)_0
Compute the total production and maintenance cost CT_0
CT ← CT_0
Z ← Z_0
x_pt ← (x_pt)_0, I_pt ← (I_pt)_0, B_pt ← (B_pt)_0 and y_pt ← (y_pt)_0
T_e ← T_max
while T_e ≥ T_min do
    Move Z to Z' according to the neighbourhood movement protocol
    Compute CM and G^t_MSS relative to Z'
    Solve the integer linear lot-sizing problem relative to Z' and
    determine solutions (x_pt)', (I_pt)', (B_pt)' and (y_pt)'
    Compute the total production and maintenance cost CT' relative to
    Z'
    if CT' < CT then
        Z ← Z'
        x_pt ← (x_pt)', I_pt ← (I_pt)', B_pt ← (B_pt)' and y_pt ← (y_pt)'
        CT ← CT'
    else
        Choose randomly w ∈ [0, 1]
        if w < exp(-(CT-CT')/T_e) then
            Z ← Z'
            x_pt ← (x_pt)', I_pt ← (I_pt)', B_pt ← (B_pt)' and y_pt ← (y_pt)'
            CT ← CT'
        else
            Keep current solution Z, x_pt, I_pt, B_pt and y_pt
        end
    end
    T_e ← T_e C
end
Print Z, x_pt, I_pt, B_pt, y_pt and CT
```

Fig. 6. Simulated annealing pseudo-code.

SA algorithm is applied in this section to solve the model (1)–(7). The initial solution **Z** used in our case will be the no preventive maintenance policy ($z_j^{ts}=0,\ \ j=1,\dots,n,\ \ t=1,\dots,T,$ $s=1,\dots,S$ except for $z_j^{11}=1$). The move set is defined by a random change from the current solution where one or many elements will be randomly selected from the maintenance policy matrix **Z** and will be changed from 0 to 1 or vice versa as shown in the example in Fig. 5.

The model parameters $CM(\mathbf{Z})$ and G_{MSS}^t $(t=1,\dots,T)$ are computed and the integer linear lot-sizing problem relative to the new maintenance policy matrix **Z** is solved. The new value of the objective function (the total maintenance and production costs) is consequently determined and compared to the last selected objective function value according to the loss of energy criteria described previously. The temperature will decrease from T_{max} to T_{min} by following the cooling schedule $T_{m+1}=T_mC$, where C is a constant. The SA algorithm runs until the cooling schedule reaches T_{min}. The complete SA algorithm is presented in Fig. 6.

5. Numerical examples

5.1. Problem data

Let us consider a multi-state system containing 3 binary-state components in a series-parallel configuration as shown in Fig. 7. The characteristics and the lifetime distribution of each component are given in Table 1.

The planning horizon H is 4 months where the production planning period is determined monthly ($T=4$). Each production period is divided into 2 sub-periods ($S=2$) considered as a maintenance planning period. Consequently, for each component, a PR action can be performed every 2 weeks. For each product, the monthly demands are presented in Table 2.

Table 3 gives the holding, backorder, set-up and production costs for each product. These costs are the same for all periods.

According to Section 3, the multi-state system capacity will be estimated by using the system structure function. In fact, the system is a combination of two parallel components (1 and 2), connected in series with component 3 as shown in Fig. 7. If we consider that each component j is featured by the two-state (capacity) and their probability $(\{0,G_j\},\{(1-A_j^t),A_j^t\})$, for $t=1,\dots,4$ and $j=1,\dots,3$, the multi-state system capacity is obtained by the following equation:

$$G_{MSS}^t=\mathrm{Min}\{(G_1A_1^t+G_2A_2^t),(G_3A_3^t)\},\ \text{for } t=1,\dots,4. \tag{17}$$

5.2. Results and discussion

This section presents the problem results obtained by the Exhaustive Search (ES) method and the Simulated Annealing (SA) algorithm. All the algorithms presented in this paper were coded with MATLAB solver on an i7 3.64 GHz processor.

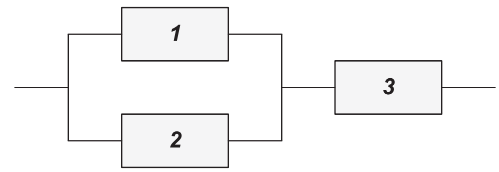

Fig. 7. Three binary-state components series-parallel system.

Table 1
Characteristics of the production system components.

Comp. j	G_j (itm/month)	CPR_j ($)	CMR_j ($)	TPR_j (week)	TMR_j (week)	Lifetime distribution
1	105	5000	200	0.09	0.05	Weibull (3,3)
2	110	4000	300	0.08	0.04	Weibull (2,2)
3	205	3000	250	0.09	0.02	Weibull (2,2)

Table 2
Demands of products.

Period t	Demand of product 1 d_{1t} (items)	Demand of product 2 d_{2t} (items)
1	95	95
2	95	95
3	95	90
4	90	90

Table 3
Cost data of products.

Product p	Holding cost h_{pt} ($)	Backorder cost b_{pt} ($)	Set-up cost s_{pt} ($)	Production cost π_{pt} ($)
1	40	150	1000	100
2	40	150	1000	100

Table 4
Maintenance policy matrix optimizing maintenance and total costs and their capacities.

Comp. j	Maintenance policy optimizing total cost Production planning period								Maintenance policy optimizing maintenance cost Production planning period							
	T_1		T_2		T_3		T_4		T_1		T_2		T_3		T_4	
1	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0
2	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
3	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
G_{MSS}^t	193.52		187.52		188.85		182.88		193.52		171.22		193.52		171.22	

Table 5
Production plan optimizing the maintenance cost.

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	95	0	0	1	98	3	0	1
2	79	0	16	1	92	0	0	1
3	111	0	0	1	82	0	8	1
4	73	0	17	1	98	0	0	1

Table 6
Production plan optimizing the total cost.

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	98	3	0	1	95	0	0	1
2	92	0	0	1	95	0	0	1
3	95	0	0	1	90	0	0	1
4	90	0	0	1	90	0	0	1

5.2.1. Solution with the exhaustive search method

The solution with the exhaustive search method took 2 h and 30 min. The number of the maintenance policy matrix **Z** obtained by Eq. (16) is $N=2^{21}=2\,097\,152$ combinations. The solution optimizing the maintenance cost *CM*, and the total production and maintenance costs are compared and presented in Table 4, with their respective maintenance policy matrixes **Z** and their capacities.

The maintenance policy optimizing the maintenance cost presents a periodic policy for all components, and its total cost is $136 255. However, the maintenance policy optimizing the total cost presents an acyclical policy for the component 1. The relative total cost of this policy is $132 894 which means that the gain obtained by the integration of the preventive maintenance and the production planning is around 2.5%. The production plans corresponding to the optimization of the maintenance cost and the total costs are presented in Tables 5 and 6, respectively. The 2.5% gain can be explained by the weak capacity given by the maintenance policy optimizing the maintenance cost according to Table 4, which generates a large quantity of backorders as shown in Table 5. The integration of maintenance and production planning increases the MSS capacity by modifying the maintenance policy carried out for component 1, which allows us to obtain a better concordance between the demand and the MSS capacity.

5.2.2. Solution with the simulated annealing algorithm

For the numerical example, the SA is applied according to the algorithm defined in Section 4.2. The cooling process will get down from $T_{max}=100$ to $T_{min}=0.1$ according to the rule $T_{m+1}=T_m C$ where $C=0.999$. The temperature limit is the only stopping criterion used in this algorithm. From a current maintenance policy matrix **Z**, the move set is defined by changing the value of 2 elements chosen randomly from the maintenance policy matrix. The numerical example was executed 100 times and found the optimal solution in and around 86% of cases. For the 100 trial, the standard deviation is 269.15 and the coefficient of variation is 0.20%; which shows that, for our numerical example, the solution method is robust and the given non-optimal solutions are close to the optimum. The SA algorithm took an average time of 25.5 s to find the optimal solution. The advantage of using the proposed SA algorithm lies in its ability to find the optimal solution for this numerical example much quicker than the ES method used as a reference here. We recall that the latter took 2 h and 30 min to find this optimal solution for the same problem.

Table 7
Estimated ES solution time for different problems.

n	T	S	N	Estimated time
3	4	2	$2^{21}=2\,097\,152$	2 h 30 min
3	5	2	$2^{27}=134\,217\,728$	9 days
4	4	2	$2^{28}=268\,435\,456$	14 days
3	4	3	$2^{33}\approx 8.6\times 10^{9}$	More than 1 year

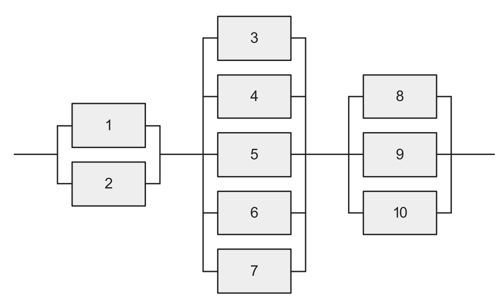

Fig. 8. 10-components MSS series-parallel.

5.3. Larger problems

According to Eq. (16), the number N of possible combinations of maintenance policy matrix **Z** increases exponentially if n, T or S increases. Taking into consideration that the previous numerical example took around 2 h and 30 min for $2^{21}=2\,097\,152$ mixed capacited lot-sizing problem to solve, in Table 7 we tried to estimate the processing time for ES if n, T or S increases. Table 7 shows that it is difficult to obtain an optimal solution for any MSS configuration bigger than the previous example ($n=3$, $T=4$ and $S=2$).

In order to obtain a feasible solution (optimal or not) in reasonable processing time for large-sized problems, the SA algorithm developed in Section 4.2 will be used as an alternative solution approach in the next section.

5.3.1. Example 1

In this section we consider a 10-components system $n=10$ with three subsystems (1, 2, and 3) connected in series. For subsystem 1, three components are in parallel. Subsystem 2 contains five parallel components. Subsystem 3 contains two parallel

Table 8
Characteristics of the components (n=10, T=5 and S=1).

Comp. j	G_k (items/month)	CPR_j ($)	CMR_j ($)	TPR_j (month)	TMR_j (month)	Lifetime distribution
1	100	1500	1000	0.03	0.06	Weibull (2,2)
2	50	1500	1200	0.03	0.08	Weibull (3,3)
3	100	2000	1400	0.04	0.1	Weibull (2,2)
4	50	2000	1600	0.02	0.1	Weibull (3,3)
5	50	3000	1800	0.05	0.12	Weibull (2,2)
6	50	2500	2000	0.06	0.14	Weibull (3,3)
7	50	3000	2200	0.05	0.15	Weibull (2,2)
8	50	3500	2400	0.04	0.13	Weibull (3,3)
9	150	4000	2600	0.03	0.1	Weibull (2,2)
10	100	4500	2800	0.01	0.09	Weibull (3,3)

Table 9
Demands of products (n=10, T=5 and S=1).

Period t	Demand of product 1 d_{1t} (items)	Demand of product 2 d_{2t} (items)
1	50	50
2	10	10
3	50	50
4	70	70
5	150	150

Table 10
Cost data of products (n=10, T=5 and S=1).

Product p	Holding cost h_{pt} ($)	Backorder cost b_{pt} ($)	Set-up cost s_{pt} ($)	Production cost π_{pt} ($)
1	40	250	500	100
2	40	250	500	100

Table 11
Maintenance policy for the best solution obtained by SA for MSS (n=10, T=5 and S=1).

Comp. j	T_1	T_2	T_3	T_4	T_5
1	1	0	0	1	0
2	1	0	0	1	0
3	1	0	0	1	0
4	1	0	0	1	0
5	1	0	0	1	0
6	1	0	0	1	0
7	1	0	0	1	0
8	1	0	0	1	0
9	1	0	0	1	0
10	1	0	0	1	0

components as shown in Fig. 8. The planning horizon H is 5 months where the production planning period and the maintenance planning period are determined monthly (T=5 and S=1). The characteristics, and the lifetime distribution of each component are given in Table 8. Costs and periodic demands are given in Tables 9 and 10, respectively.

The SA algorithm was applied to solve this example. The cooling process was down from $T_{max}=100$ to $T_{min}=0.1$ according to the rule $T_{m+1}=T_mC$ where C=0.999. The move set is defined by changing the value of 2 elements chosen randomly from the maintenance policy matrix. The numerical example was executed 100 times and the same best solution was found in 94% of the cases. The integrated maintenance and production planning generating the best obtained solution suggests a PR at the beginning of the fourth maintenance (and production) planning period for all system components as shown in Table 11. The total cost of the best solution is $170 612 with a standard deviation around 31 and the coefficient of variation is less than 0.02%; and was obtained in the average time of around 34 s while it is impossible to obtain any solution with ES method with the current technology. The production plan corresponding to the best obtained solution is presented in Table 12. Despite the fact that there is no guarantee that the best solution obtained by the SA approach is the optimal solution, the SA approach has consistently generated a feasible solution in short processing time with a very low standard deviation.

5.3.2. Example 2

We consider the same 10-components MSS described in the previous section (n=10) and shown in Fig. 8. The planning horizon H is 5 months where the production planning period is determined monthly (T=5), however, PR can occur every two weeks (S=2). The characteristics, the lifetime distribution of each component, the costs, and the periodic demands are the same as example 1 and are respectively given in Tables 8, 9 and 10.

The SA algorithm was applied for this example as well. The cooling process was down from $T_{max}=100$ to $T_{min}=0.1$ according to the rule $T_{m+1}=T_mC$. However, due to size of the problem and the huge number of maintenance policy alternatives, the cooling constant was increased to C=0.9997 in order to slow down the cooling process and to allow the algorithm to look into more possibilities. The move set is defined by changing the value of 2 elements chosen randomly from the maintenance policy matrix. The numerical example was executed 100 times and the same best solution was found in 96% of the cases.

The integrated maintenance and production planning generating the best obtained solution suggests a PR at the beginning of the sixth maintenance planning period for all system components, which corresponds to the second maintenance planning period of the third production planning period as shown in Table 13. The total cost relative to the best solution is $168 379 with a standard deviation around 57 and the coefficient of variation is less than 0.03%. The best solution was obtained in average time around 112 s. The production plan corresponding to the best obtained solution is presented in Table 14. Similarly to the previous example, the SA approach has consistently generated a feasible solution in short processing time with a low standard deviation.

On one hand, examples 1 and 2 show the efficiency of SA to obtain a feasible solution in a reasonable execution time. On the other hand, both examples use the same data except for the maintenance planning period (S=1 for example 1 and S=2 for example 2). The feasible solution generated by example 2 suggests a PR replacement at the middle of the third production planning period. This policy is economically better than the feasible solution obtained in example 1 and suggesting a PR replacement at the beginning of the fourth production planning period. This example shows that decreasing the maintenance planning period from a month to 2 weeks can reduce the total maintenance and production cost by giving more flexibility to the system to adjust the capacity according to the demand. However, without the optimal solution, this cost reduction can also be due to improvement of the quality of the feasible solution found by the SA algorithm. These assumptions will be validated in further works.

6. Extended model

The model presented by Eqs. (1)–(7) suggests a maintenance policy which allows PM actions at the beginning or inside the production planning period. Since the presented model is an extension to [16] research, the authors assumed that preventive maintenance can occur only at the beginning of production

Table 12
Best production plan obtained by the SA algorithm ($n = 10$, $T = 5$ and $S = 1$).

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	60	10	0	1	60	10	0	1
2	0	0	0	0	0	0	0	0
3	50	0	0	1	50	0	0	1
4	70	0	0	1	135	65	0	1
5	150	0	0	1	85	0	0	1

Table 13
Maintenance policy for the best solution obtained by SA for MSS ($n = 10$, $T = 5$ and $S = 2$).

Comp. j	T_1		T_2		T_3		T_4		T_5	
1	1	0	0	0	0	1	0	0	0	0
2	1	0	0	0	0	1	0	0	0	0
3	1	0	0	0	0	1	0	0	0	0
4	1	0	0	0	0	1	0	0	0	0
5	1	0	0	0	0	1	0	0	0	0
6	1	0	0	0	0	1	0	0	0	0
7	1	0	0	0	0	1	0	0	0	0
8	1	0	0	0	0	1	0	0	0	0
9	1	0	0	0	0	1	0	0	0	0
10	1	0	0	0	0	1	0	0	0	0

planning periods for practical motivations. This section will explain these motivations and present the modified model with some numerical examples.

6.1. Motivations

Much research related to preventive maintenance and production planning used the assumption that PM action can only be performed at the beginning of production planning, such as [2,1,3,30]. Many reasons can explain the use of this assumption. First, most modern equipment is highly automated and reliable, consequently the Mean Time Between Failure is high enough to use the production planning period as the measure unit for preventive maintenance planning. Second, the main principle of the integration of maintenance and production planning is to find the best deal between demand, capacity and production [16]. Decisions related to PM actions have to take into consideration the production information such as inventory, backorder and produced items. This information is generally available only at the end of production planning period, which gives a better opportunity to make a PM decision. Finally, performing PM actions at the beginning of production planning periods offers the opportunity to the same crew to perform set-up and PM actions, which can reduce costs and avoid some human resource management issues.

However, the model presented in this paper gives more flexibility by allowing PM actions at the beginning or at some instants inside the production planning period. In Section 3, we considered that each production planning period can be divided into S sub-period called preventive maintenance period where PM action can be performed at the beginning of these periods. This generalization can be useful when equipment needs some adjustment or PM intervention more frequently. Many companies generate their production planning monthly but would like to schedule their PM plan every week for example. Moreover, for a company having specialized maintenance services, there is no need to merge PM actions and production set-up. Finally, our model can generate PM actions "on request" which provides a better deal between maintenance and production planning. Notice that, for $S = 1$ the presented model will allow PM actions only at the beginning of production planning periods.

6.2. The modified model

The model presented by Eqs. (1)–(7) can be adjusted in order to fit with some real and practical opportunities. In fact, based on arguments presented in the previous sub-section, we consider, for each component j ($j = 1, \ldots, n$), that performing PM action at the beginning of production planning period can reduce the preventive maintenance cost CPR_{jj} by the percentage δ_j. Consequently, we define the $n \times n$ diagonal matrix $\mathbf{R} = (\delta_j)_n$ where δ_j ($j = 1, \ldots, n$) is the reduction factor for the component j when PM actions are performed at the beginning of production planning periods.

Based on the preventive maintenance matrix $\mathbf{Z}$, we define the matrix $\tilde{\mathbf{Z}} = (\tilde{z}_j^k)$ ($j = 1, \ldots, n$, $t = 1, \ldots, T$ and $s = 1, \ldots, S$) by

$$\tilde{z}_j^k = \begin{cases} z_j^{ts} & \text{if } k = (tS+1) \text{ for } j = 1, \ldots, n \text{ and } t = 0, \ldots, (T-1), \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

The introduced matrix $\tilde{\mathbf{Z}}$ is extracted from the maintenance policy matrix $\mathbf{Z}$ which considers only decision variables at the beginning of production planning period. The use of that matrix avoids increasing the number of model decisions variables. The maintenance cost is adjusted according to equation

$$\begin{aligned} CM(\mathbf{Z}) &= \|\mathbf{CPR} \times \mathbf{Z} - \mathbf{R} \times \mathbf{CPR} \times \tilde{\mathbf{Z}} + \mathbf{CMR} \times \mathbf{M}(\mathbf{Z})\| \\ &= \sum_{j=1}^{n} \sum_{t=1}^{T} \sum_{s=1}^{S} (CPR_{jj} z_j^{ts} + CMR_{jj} M_j^{ts}) - \sum_{j=1}^{n} \sum_{t=0}^{T-1} (\delta_j CPR_{jj} z_j^{tS+1}). \end{aligned} \quad (19)$$

By using Eq. (19), the total maintenance cost CM can be computed and the model presented by Eqs. (1)–(7) can be solved using the same Exhaustive Search method or the Simulated Annealing algorithm presented in Section 4.

6.3. Numerical example

We consider the same example given in Section 5 with 3 components, 4 production planning periods and 2 maintenance planning periods ($S = 2$). Based on the new assumption, we consider that, for each component j ($j = 1, 2, 3$), performing PM actions at the beginning of production planning period can reduce the preventive planning cost CPR_{jj} by the percentage δ ($\delta_1 = \delta_2 = \delta_3 = \delta$). By using the same solution methods presented in this paper, Table 15 shows the maintenance policy optimizing the integrated preventive maintenance and production problem for values of δ ranging from 10% to 50%.

The different maintenance policy matrix $\mathbf{Z}$ presented in Table 15 shows that starting from 30% of preventive maintenance cost reduction, the optimal maintenance policy avoids PM actions inside the production planning period. The compromise between maintenance, system capacity and demand promotes PM actions

Table 14
Best production plan obtained by the SA algorithm ($n=10$, $T=5$ and $S=2$).

Period	Product A				Product B			
	Production	Inventory	Backorder	Set-up	Production	Inventory	Backorder	Set-up
1	60	10	0	1	60	10	0	1
2	0	0	0	0	0	0	0	0
3	50	0	0	1	50	0	0	1
4	70	0	0	1	142	72	0	1
5	150	0	0	1	78	0	0	1

Table 15
Optimal maintenance policies for the different values of the reduction percentage δ.

δ (%)	Component 1 Production planning period								Component 2 Production planning period								Component 3 Production planning period							
	T_1		T_2		T_3		T_4		T_1		T_2		T_3		T_4		T_1		T_2		T_3		T_4	
10	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
20	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
30	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
40	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
50	1	0	1	0	1	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	1	0

at the beginning of production planning periods in order to reduce the total maintenance and production cost.

7. Conclusion

In this paper, we presented an integrated model for production and general preventive maintenance planning for multi-state systems. For the production side, the model generates, for each product and each production planning period, the quantity of inventory, backorder, items to produce and also the instant of set-up. For the maintenance side, for each component, we proposed the instant of each preventive maintenance action which can be carried out during the production planning period. A Matrix based methodology was used in order to estimate model parameters such as system availability and the general capacity. The proposed model was solved by the ES method and SA. The exhaustive search method gives the optimal solution however, due to computing time and the high number of combinations, it can only be used for small production systems. The SA method reduces the solution time. It was also shown that the integration of acyclical maintenance and production planning improves the total production and maintenance costs. The paper discussed the merit of allowing preventive maintenance actions only at the beginning or inside the production planning period through a modified model and a numerical example. Future work will extend the model presented in this paper to deal with the case of systems containing dependent components.

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