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A relax-and-fix heuristic approach for the capacitated dynamic lot sizing problem in integrated manufacturing/remanufacturing systems

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Abstract: In this paper, the capacitated dynamic lot sizing problem in integrated manufacturing/remanufacturing systems is addressed. These kinds of production systems are designed to satisfy the demands of different classes of single-level products not only by manufacturing raw materials, but also by remanufacturing returned products. A single machine with a limited capacity in each time period is used to perform both the manufacturing and remanufacturing operations. A mathematical programming formulation is proposed to optimally solve this problem. Since the problem is NP-hard (it is a generalized version of the classical capacitated dynamic lot sizing problem), a relax-and-fix heuristic is developed to solve the problem in a reasonable amount of time. To evaluate the efficiency of the proposed algorithm, some experimental instances are generated and solved. The obtained results show the effectiveness of the proposed algorithm.

Keywords: Remanufacturing; Production planning; Capacitated lot sizing; Relax and fix heuristic; Optimization.

1. INTRODUCTION

Today, due to the waste growth and land disposal restrictions, most of production companies try to incorporate sustainable elements in designing and managing their plants (Giglio et al., 2017). Remanufacturing systems are production plants that contribute to reduce the waste sent to the environment and they are also very effective from an economic point of view. In this class of systems, returned worn-out products are produced to new like conditions through a series of industrial processes (Naeem et al., 2013). The produced products can be sold with the same price of new products but they are less costly. For this reason, remanufacturing systems have gained lots of attentions in academic and industrial environments especially over the past two decades.

In the last five decades, many works in the scientific literature deal with dynamic lot sizing of manufacturing systems. Survey on such researches can be found in Drexl and Kimms (1997); Buschkhl et al. (2010). However, few studies on the dynamic lot sizing with product returns and remanufacturing (DLSPR) appeared in literature (Richter and Sombrutzki, 2000; Richter and Weber, 2001; Beltrán and Krass, 2002; Teunter et al., 2006, 2009; Schulz, 2011; Baki et al., 2014; Parsopoulos et al., 2015; Sifaleras et al., 2015; Zouadi et al., 2015). More specifically, one of the main extensions of DLSPR problem, the capacitated lot sizing problem with product returns (CLSPR), has gained little attention. In this class of problems, different items or products can be produced in each time period; the production of each item or product consumes a known amount of units of capacity, and only a limited production capacity is available in each period. Li et al. (2007) dealt with the capacitated dynamic lot sizing problem with return products under substitution; they assumed that there are three ways to fulfill the demands of two given products: manufacturing new products, remanufacturing the returned products, and outsourcing; in this connection, they proposed a mathematical formulation and a genetic algorithm to solve the problem. Pan et al. (2009) addressed the capacitated dynamic lot sizing problem with product returns and disposal option; by assuming that the capacities of production, disposal and remanufacturing are limited, and that backlogging is not allowed, they formulated the problem as a general model and proposed a dynamic-programming approach to solve it. Zhang et al. (2012) studied a variant of CLSPR in which a production system manufactures raw materials and remanufactures collected used products in order to fulfill the (separate) demands for them; they formulated the problem as a mixed-integer programming model and developed a Lagrangian relaxation-based solution approach for it. Sahling (2013) addressed a multi-product lot-sizing problem with

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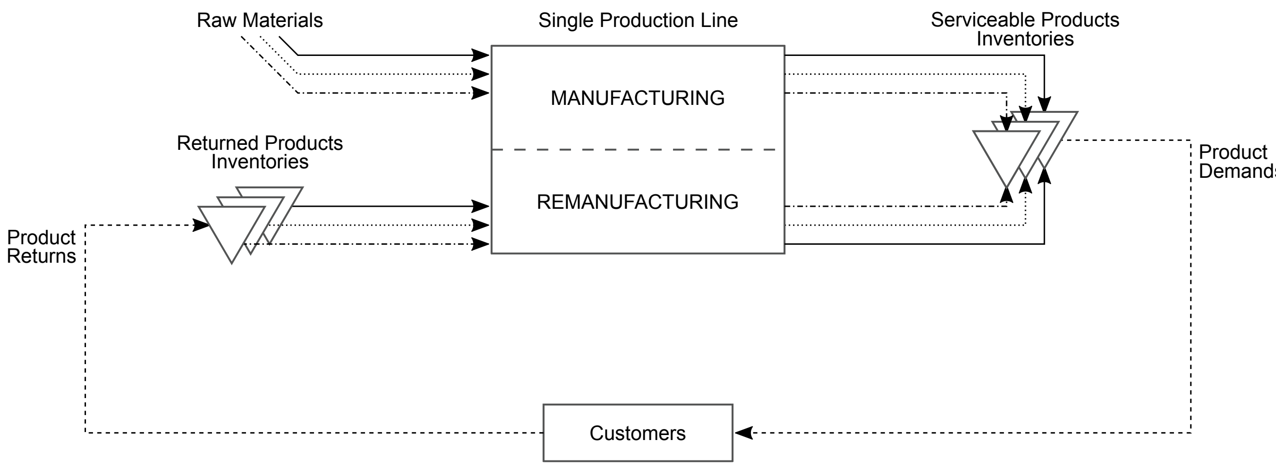

Fig. 1. The integrated manufacturing/remanufacturing system.

product returns and remanufacturing subject to a capacity constraint. He proposed a column-generation approach to determine lower bound bounds and transferred the obtained lower bound into a feasible solution by a truncated branch-and-bound approach using CPLEX. Sahling (2016) extended the problem and incorporated vendor selection into production and remanufacturing planning subject to emission constraints. A fix-and-optimize approach and a solution approach that combines column generation and a fix-and-relax heuristic are presented to solve the extended problem. Roshani et al. (2016) addressed the capacitated dynamic lot sizing problem in a closed remanufacturing system; they assumed that the demands of products are satisfied by only remanufacturing used products returned to the system and that a single machine with limited capacity is used to perform the remanufacturing operations; to solve the problem, they developed a simulated annealing algorithm utilizing an efficient neighborhood generation mechanism to always generate feasible neighbor solutions for the problem.

According to our best knowledge, there is no published study proposing a MIP-based solution approach for the capacitated lot sizing problem in integrated manufacturing/remanufacturing systems with joint setup (CLSP-MR-JS) in which different classes of single level products are produced using a single machine. In this paper, a mathematical formulation and a relax-and-fix heuristic (RFH) are proposed to address this research gap.

The remaining of the paper is as follows. In section 2, a mathematical programming formulation of the problem is described. The proposed RFH is presented in section 3 and computational studies are reported in section 4. Finally, concluding remarks are in section 5.

2. MATHEMATICAL PROGRAMMING FORMULATION

In this section, after the definition of the considered class of problems, a mathematical programming formulation is presented for CLSP-MR-JS. This formulation is a generalization of the mathematical formulation of the lot sizing problem with product returns proposed by Richter and Sombrutzki (2000) for pure remanufacturing systems.

2.1 Problem definition

We take into consideration the capacitated dynamic lot sizing problem in an integrated manufacturing/remanufacturing system designed to satisfy demands of different classes of single-level products by both manufacturing raw materials and remanufacturing returned products. In this problem, it is assumed that whenever a raw material is manufactured or a used product is remanufactured, they have the same qualities (so they are called serviceable products) and they can be used to satisfy the customer demands. The demands for each product and the number of used products returned to the system in each period (over a finite planning horizon T) are considered deterministic and known in advance. A simple sketch of the system is shown in Fig. 1.

The system utilizes a single machine to manufacture raw materials and remanufacture returned products. The capacity of the single machine is limited and the manufacturing and remanufacturing processes of each product, as well as the setup, consume known amounts of capacity in each time period. The problem is to determine the number of products manufactured and remanufactured in each period to satisfy the customer demands, so that total remanufacturing costs, return and serviceable products inventory costs, and setup costs are minimized.

2.2 Problem assumptions

The following assumptions are made.

- Different single-level products are produced through both manufacturing of raw materials and remanufacturing of returned products.
- The demands of products can be satisfied by both manufacturing raw materials and remanufacturing returned products.
- Demands and returns are deterministic and may vary over time.
- Backlogging is not allowed.
- Shortage is not allowed.
- Remanufacturing and holding unitary costs, as well as setup costs, are know in advance.
- Lot streaming is not allowed.
- The single machine is always available.

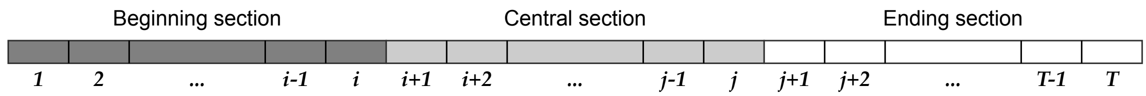

Fig. 2. The categorizing of periods

- Only one product can be processed by the machine at a time.

2.3 Mathematical model

Sets and parameters

- N: number of classes of products;
- T: number of time periods or buckets (optimization horizon);
- $\mathcal{P} = \{1, \ldots, j, \ldots, N\}$ set of classes of products;
- $\mathcal{T} = \{1, \ldots, t, \ldots, T\}$ set of time periods;
- D_{jt}: demand of products of class j at the end of period t;
- R_{jt}: number of returned products of class j at the beginning of period t;
- U^{M}_{jt}: unitary manufacturing cost (per each unit of products of class j and per each t);
- U^{R}_{jt}: unitary remanufacturing cost (per each unit of products of class j and per each t);
- S_{jt}: setup cost (per each lot of products of class j and per each t);
- H^{R}_{jt}: unitary holding cost in the return inventory (per each unit of products of class j and per each t);
- H^{S}_{jt}: unitary holding cost in the serviceable inventory (per each unit of products of class j and per each t);
- Γ^{M}_{j}: amount of capacity which is required to manufacture each product of class j;
- Γ^{R}_{j}: amount of capacity which is required to remanufacture each product of class j;
- Γ^{S}_{j}: amount of capacity consumed by setup relative to products of class j;
- C_t: maximum available capacity in period t;
- B_{jt}: big number, calculated as the sum of the demands of product j from period t to final period T;
- OC_t: unitary over-capacity cost, that is, cost per unit of additional capacity in period t.

Continuous decision variables

- x^{M}_{jt}: quantity (lot) of products of class j that are manufactured in periods t;
- x^{R}_{jt}: quantity (lot) of products of class j that are remanufactured in period t;
- i^{R}_{jt}: level of the return inventory relative to the products of class j in period t;
- i^{S}_{jt}: level of the serviceable inventory relative to the products of class j in period t;
- o_t: amount of additional capacity required by the single machine in period t.

Binary decision variables

- y_{jt}: binary variable which is equal to one if a non-null lot of products of class j is produced in period t, and zero otherwise.

Problem formulation

$$\min \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{P}} (U^{\mathrm{M}}_{jt}\, x^{\mathrm{M}}_{jt} + U^{\mathrm{R}}_{jt}\, x^{\mathrm{R}}_{jt} + S_{jt}\, y_{jt} + H^{\mathrm{R}}_{jt}\, i^{\mathrm{R}}_{jt} + H^{\mathrm{S}}_{jt}\, i^{\mathrm{S}}_{jt}) + \sum_{t\in\mathcal{T}} OC_t\, o_t \tag{1}$$

s.t.

$$i^{\mathrm{R}}_{j(t-1)} + R_{jt} - x^{\mathrm{R}}_{jt} = i^{\mathrm{R}}_{jt}\ , \quad \forall\, j \in \mathcal{P}, \forall\, t \in \mathcal{T} \tag{2}$$

$$i^{\mathrm{S}}_{j(t-1)} + x^{\mathrm{R}}_{jt} + x^{\mathrm{M}}_{jt} - i^{\mathrm{S}}_{jt} = D_{jt}\ , \quad \forall\, j \in \mathcal{P}, \forall\, t \in \mathcal{T} \tag{3}$$

$$x^{\mathrm{M}}_{jt} + x^{\mathrm{R}}_{jt} \le B_{jt}\, y_{jt}\ , \quad \forall\, j \in \mathcal{P}, \forall\, t \in \mathcal{T} \tag{4}$$

$$\sum_{j\in\mathcal{P}} (\Gamma^{\mathrm{R}}_{j}\, x^{\mathrm{R}}_{jt} + \Gamma^{\mathrm{M}}_{j}\, x^{\mathrm{M}}_{jt} + \Gamma^{\mathrm{S}}_{j}\, y_{jt}) \le C_t + o_t\ , \quad \forall\, t \in \mathcal{T} \tag{5}$$

$$x^{\mathrm{M}}_{jt} \ge 0\ ,\ x^{\mathrm{R}}_{jt} \ge 0\ ,\ i^{\mathrm{R}}_{jt} \ge 0\ ,\ i^{\mathrm{S}}_{jt} \ge 0\ , \quad \forall\, j \in \mathcal{P}, \forall\, t \in \mathcal{T} \tag{6}$$

$$y_{jt} \in \{0, 1\}\ , \quad \forall\, j \in \mathcal{P}, \forall\, t \in \mathcal{T} \tag{7}$$

The objective function (1) considers the minimization of the sum of total manufacturing and remanufacturing costs, total setup cost, and total holding cost. Constraints (2) and (3) provide the dynamics of return inventory and serviceable inventory, respectively, that is, they determine the values of the inventory levels as functions of the returned products, the demand, the quantity of manufactured and remanufactured products, and the previous inventory levels. Constraints (4) ensure that the joint setup variable y_{jt} is forced to one if at least one between x^{M}_{jt} and x^{R}_{jt} is greater than zero. Note that, B_{jt} is calculated as $\sum_{\tau=t}^{T} D_{j\tau}$ for all $j \in \mathcal{P}$ and $t \in \mathcal{T}$. Constraints (5) state that production quantities and setups must meet the capacity limit in each period. Finally, constraints (6) and (7) define the classes of decision variables.

3. PROPOSED SOLUTION APPROACH

The classic capacitated dynamic lot sizing problem (CLSP) is NP-hard (Karimi et al., 2003). Thus, CLSP-MR-JS, which is a generalized version of CLSP, is NP-hard too. For this reason, in this paper, a relax-and-fix heuristic is proposed to solve the problem. Note that, CLSP-MR-JS can be considered as a version of dynamic lot-sizing with remanufacturing and joint setup (DLSPRj) proposed by Teunter et al. (2006). The difference between this model and CLSP-MR addressed here is that the current model is a capacitated multi products system in which the cost of manufacturing and remanufacturing of products are time variant; in this connection, Retel Helmrich et al. (2014) proved that DLSPRj in which the cost of manufacturing and remanufacturing of products are time variant is NP-hard.

3.1 Relax-and-fix heuristic

In this section, a relax-and-fix heuristic (RFH) (Toledo et al., 2015) is proposed to solve CLSP-MR. This heuristic reduces the complexity of the problem by solving a series

Algorithm 1 The relax-and-fix algorithm

Step 1 Initialization:

Step 1.1 Read instance data (number of periods, number of products, demands of products, returned numbers of products, manufacturing and remanufacturing costs).

Step 1.2 Read the number of periods in central section θ, and rolling parameter γ.

Step 1.3 Set the counter of the first period of central section (α) to 1, the counter of the last period of it (β) to θ, and the iteration numbers ($iter$) to 1.

Step 2 While $iter \leq \left(\left\lceil \frac{T-\theta}{\gamma} \right\rceil +1 \right)$ do:

Step 2.1 Build the relax-and-fix (RF) model: for periods from α to β (central section) consider the original problem; for periods from $\beta+1$ to T (ending section) relax the binary variables according to the relaxing strategy.

Step 2.2 Solve the RF model.

Step 2.3 Fix the value of binary variables in the central period to the optimal values found in step 2.1.

Step 2.4 Set β to $\beta+\gamma$, α to $\alpha+\gamma$, $iter$ to $iter+1$.

Step 3 While-End.

Step 4 Report the solution of the last iteration as the solution of the problem.

of partially relaxed mixed-integer programming models whose number of binary variables are small enough to be solved by conventional branch-and-bound methods (Mohammadi et al., 2010). In this way, the proposed heuristic categorizes the planning periods into three different sections, namely, beginning, central and ending ones. In the beginning section, the value of some or all of the integer and binary variables are frozen to their optimal values in the previous iteration(s). In the central section, the original problem is considered; that is, we keep the binary variables. In the ending section, all or a set of binary variables are relaxed so that they can take any value between zero and one. Fig. 2 illustrates the categorizing scheme for the proposed algorithm.

3.2 Iterative approach

RFH starts to solve the problem by iteratively partitioning the periods, fixing and relaxing subsets of variables according to the above described procedure, and solving the simplified models in a rolling horizon manner. In the first iteration ($iter = 1$) the beginning section does not exist, the central section contains the first θ periods and the ending section contains the remaining periods. This first iteration finds the optimal solution for the original model in which the binary variables for periods from $\theta+1$ to T are continuously relaxed. When the optimal solution for such an iteration is found, the values of the binary variables in periods from α to β are fixed to their optimal value; afterwards α and β increase by γ and the iteration counter ($iter$) increases by one. Note that θ is the constant length of the central section, α and β are respectively the first and last period of the central section and γ ($\leq \theta$) is the incremental parameter. In the second iteration ($iter = 2$), the beginning section contains the first γ periods; periods from $\alpha = \gamma$ to $\beta = 2 \cdot \gamma$ constitute the central section and the remaining periods are in the ending section. In this iteration the model is solved with the adopted simplifications (relaxations and fixings) and the optimal solution is found. Afterwards α and β are increased by γ and a new iteration is started. This procedure is repeated while $iter$ is less than or equal to $\left\lceil \frac{T-\theta}{\gamma} \right\rceil +1$ (where $\lceil x \rceil$ is the least integer greater than or equal to x). In the last iteration there are only the beginning section, from periods 1 to $\alpha - 1$, and the central one, from periods α to T; if the optimal solution at this step is found, a certainly feasible for the whole original problem and it can be reported as the solution of the algorithm. It is worth noting that the solution of RFH is in general an upper bound of the optimal solution of the considered problem. The procedure for the proposed RFH is reported in algorithm 1. Note that the formula in step 2 of the algorithm calculates the number of iterations of the algorithm needed to solve the problem. As an example, suppose that T = 5, $\theta = 2$ and $\gamma = 1$, then the number of iteration of the algorithm is $\left\lceil \frac{5-2}{1} \right\rceil + 1$ which is equal to 4.

3.3 Relaxing and fixing strategies

The relaxing and fixing strategies used in the proposed RFH are now introduced. As stated before, the main goal of using relaxing and fixing strategies is to reduce the computational burden required to solve the problem at each iteration. Apparently, if the number of relaxed variables increases, the difficulty of solving the problems is reduced, but the quality of the generated solutions decreases too. For this reason, the definition of suitable relaxing and fixing strategies is very important for designing an effective heuristic. In the proposed RFH the following relaxing and fixing strategies is used:

- Fix the value of the setup variables in the beginning section. For periods of the ending section, let all binary variables to be continuous and take values between or equal to zero and one.

4. COMPUTATIONAL EXPERIMENTS

To evaluate the performance of the presented algorithm, a set of experimental instances of the capacitated dynamic lot sizing problem with product returns have been generated randomly. In particular, 16 test instances with four different levels of product classes and four different numbers of periods have been considered. In order to find the optimal solutions of these instances, the mathematical model presented in section 2 has been coded and solved with IBM CPLEX 12.6 MIP solver. Moreover, the proposed heuristic has been coded in JAVA language and the partially-relaxed models were solved using IBM ILOG CPLEX 12.6 Concert Technology. The experiments have been carried out on a PC with a Intel core(TM)i7-6700HQ CPU@2.60 GHz processor and 8.00 GB RAM. All the parameters of the algorithm have been fixed after some tuning experiments. For all levels of products, when the number of periods is 10, 20, 30 and 40 the values of θ are fixed to 6, 12, 15 and 20, respectively, and the values of γ are set to 2, 2, 5 and 5. The *timelimit* for solving the partially-relaxed models is set to 100 seconds.

Table 1 summarizes the results of computational experiments and shows the comparison between the solution provided by CPLEX and the one obtained with the proposed RFH approach. The instance index (Problem), the number of products (N) and the number of periods (T) are specified in the first three columns of the table. The next four columns in Table 1 report the results found by the MIP solver for the original model, showing the lower bound (LB) and the upper bound (UB) of the objective function (which is the value of objective function of the best found feasible solution), the optimality gap (Gap) between such two bounds (computed as $100 \cdot \frac{UB-LB}{UB}$), as well as the Cpu time (in seconds). Note that, an overall maximum time limit equal to the cpu time needed for the proposed RFH to solve the same instance was imposed for these tests. The next four columns report the results found by the MIP solver for the original model in which an overall maximum time limit of 3600 s was imposed for these tests. In the last five columns, Table 1 includes the results found by the proposed RFH; in particular, the value of objective function ($Cost$), its relative gap (Gap) with respect to the lower bound of the MIP solution (computed as $100 \cdot \frac{Cost-LB}{Cost}$), and the Cpu time; *Dev-1* and *Dev-2* are the percentage deviations of the best cost produced by RFH from the solution found by the MIP solver (i.e., UB) when the overall maximum time limit is fixed to the cpu time needed for the proposed RFH to solve the same instance (CPLEX-1) and 3600 seconds (CPLEX-2). Note that the positive values of *Dev* shows that the proposed RFH found a worse solution, while the negative values indicates RFH generates a better solution.

A review of the results in Table 1 and the comparisons between the results of RFH with the results of CPLEX-1 show that RFH was able to generate better solutions for 10 out of 16 instances, the same results were found for one instance and slightly worse solutions were produced for five instances with the avarage deviation of -9.47%. The comparisons between the results of RFH with the results of CPLEX-2 show that RFH was capable to attain the optimal solution for five out of the eight instances that the MIP solver was able to optimally solve in less than one hour. Besides, when considering the instances not optimally solved by the MIP solver within one hour, the proposed RFH generated better results for two instances, the same results were found for one instance and worse solutions were produced for six instances, with the minimum and maximum *Dev* equal to -74.1% and 11.34%, respectively. On the basis of such considerations, and taking into account the average computational times of both the MIP solver and RFH (which are 1931.47 and 184.58 seconds, respectively), it can be said that RFH succeeded in producing acceptable results for the test instances within reasonably short computation times.

One of the advantages of proposed RFH is that this algorithm is able to found good upper bounds for the optimal solution of the CLSP-MR-JS in a reasonable amount of time. This algorithm decomposes the problem into several sub problems that can be solved easily in a short processing time. Thus, for those problems that cannot be solved in a reasonable amount of time, it is recommended to apply the proposed RFH to solve them; otherwise, it is more convenient to use optimization software to solve the original model. Besides, the efficiency of the proposed RFH is affected by the value of γ and θ parameters. Thus, it is recommended to tune the parameters of the algorithm properly before applying to solve the problems.

5. CONCLUSIONS

In this paper, we considered the capacitated dynamic lot sizing problem in manufacturing/remanufacturing systems. A mixed-integer linear programming formulation has been proposed for the problem. This model can be used to solve small-sized instances. Moreover, since the capacitated lot sizing problem with product returns is NP-hard, a relax-and-fix algorithm has been developed. This heuristic reduces the complexity of the problem by solving a series of partially-relaxed mixed-integer programming models whose number of binary variables are small enough to be solved by conventional branch-and-bound methods. With the aim of examining the efficiency of the proposed approach, a set of randomly generated instances has been produced. Then, the results obtained by the CPLEX MIP solver using the mathematical formulation and the ones yielded by the RFH algorithm have been compared. Such a comparison shows that the proposed RFH can generate good solutions for the experimental problems in reasonable amount of time.

About future research directions, more powerful solution methods such as Lagrangian relaxation may be proposed to solve this complex problem. Besides, sequence-dependent setups and changeover times and costs between two different manufacturing or remanufacturing product lots and between two same product lots produced through manufacturing or remanufacturing, may be also considered.

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Table 1. Comparison results

Problem	N	T	CPLEX-1 *LB*	CPLEX-1 *UB*	CPLEX-1 *Gap*	CPLEX-1 *Cpu*(s)	CPLEX-2 *LB*	CPLEX-2 *UB*	CPLEX-2 *Gap*	CPLEX-2 *Cpu*(s)	RFH *Cost*	RFH *Gap*	RFH *Cpu*(s)	RFH *Dev-1*	RFH *Dev-2*
1	5	10	115490.9	145305.5	20.52	0.30	145305.5	145305.5	0.00	1.01	145305.5	0.00	0.30	0.00	0.00
2		20	393641.2	413825.7	4.88	6.50	413672.3	413672.3	0.00	18.80	413683.7	0.00	6.50	-0.03	0.07
3		30	409487.8	433519.5	5.54	10.60	415406.5	415406.5	0.00	140.11	415406.5	0.00	10.60	-4.36	0.00
4		40	58891.9	59021.0	0.22	28.13	59010.0	59010.0	0.00	333.05	59010	0.00	28.13	-0.02	0.00
5	10	10	1289469.9	1332509.5	3.23	16.80	1312862.0	1312862.0	0.00	83.10	1332859	1.50	16.80	0.03	1.50
6		20	28721.7	48284.5	40.5	41.90	29031.0	29031.0	0.00	370.60	29031	0.00	41.90	-66.32	0.00
7		30	131070.8	131601.5	0.4	41.45	131510.0	131510.0	0.00	1105.80	131510	0.00	41.45	-0.07	0.00
8		40	364894.1	386019.5	5.47	314.50	367549.8	368429.5	0.24	3600.00	384580.5	4.42	314.50	-0.37	4.20
9	20	10	1128206.7	1144056.5	1.39	22.73	1144056.5	1144056.4	0.00	51.06	1204312	5.00	22.73	5.00	5.00
10		20	52292.8	92045.5	43.19	282.85	52381.4	91814	42.9	3600.00	52735	0.67	282.85	-74.54	-74.10
11		30	74633.4	74861.5	0.3	391.5	74781.8	74833.5	0.07	3600.00	74835	0.07	391.5	-0.03	0.00
12		40	450727.1	513837.0	12.28	500.5	458361.0	513509.5	10.74	3600.00	516014	11.17	500.58	0.42	0.48
13	30	10	3639611.7	3677045.0	1.02	234.3	3646161.2	3677015.5	0.85	3600.00	3737412	2.44	234.30	1.61	1.61
14		20	136130.6	175039.0	22.23	300.5	137771.8	156405.5	11.91	3600.00	156514.5	11.97	300.50	-11.83	0.07
15		30	253768.2	332489.4	23.67	360.50	279330.6	308739.0	9.53	3600.00	348255.5	19.79	360.50	4.52	11.34
16		40	420084.9	485514.5	13.48	500.62	422326.5	463752.0	8.93	3600.00	459893.5	8.16	500.62	-5.57	-0.84
Average			559195.2	590310.9	12.39	184.58	568094.9	581584.5	5.32	1931.47	591352.6	4.08	184.58	-9.47	1.65

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