International Workshop on Lot-Sizing - IWLS'2019

21st-23rd August 2019, Paris, France

Foreword

Bienvenue à Paris!

Dear colleagues,

It is a real pleasure to welcome you to the 10th International Workshop on Lot Sizing, which comes back to France, at CNAM Paris, after its first edition in Gardanne ten years ago.

We carry on the tradition of the previous workshops that aim at discussing high quality research in a relaxed atmosphere. As in the previous editions, the workshop covers recent advances in lot sizing, including new approaches for classical problems, new relevant problems, the integration of lot sizing decisions in other problems and presentations of case studies. The goal of the workshop is also to favor exchanges between researchers and to enhance fruitful collaborations. Numerous joint works have been fostered by the nine previous editions of the workshop.

We would like to thank our sponsors for their support in organizing this workshop: CEDRIC, CNAM Paris for providing us the most suitable space and the access to the Musée des Arts et Métiers, Mines Saint-Etienne, EURO, EURO Working Group on Lot-Sizing, Labex Mathématique Hadamard, RFSI Ile de France, ROADEF for enabling free registrations to students.

We wish you a very pleasant and nice stay in Paris, and hope that you will find the workshop inspiring and productive.

Safia Kedad-Sidhoum, Céline Gicquel, Nabil Absi, Stéphane Dauzère-Pérès

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Integrated decisions – I

Modeling and solving an integrated lot-sizing and bin-packing problem with sequence-dependent setups

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Abstract

In this study, we are interested in an integration among lot-sizing and bin-packing decisions, considering the sequence-dependent setups. Due to dependency of the decisions and the addressed features, the problem is modeled as a mixed-integer nonlinear mathematical model, which considers two approaches to model the sequence-dependent setups. The mathematical models are linearized and solved by a mixed integer programming solver using separation cuts. In addition, symmetry breaking constraints are applied. A computational study using randomly generated data is conducted, in order to analyze the exact solution strategies to the integrated problem.

1 Introduction

In an industrial environment, several processes occur simultaneously at different stages of a production plant. In this paper, we study processes that consider lot-sizing and bin-packing decisions, more specifically, we are interested in treating these decisions in an integrated way.

The Lot-Sizing Problem considers the tradeoff between the setup and inventory holding costs to determine the minimal cost of a production plan for one (or several) machine(s) in order to meet the demand for each item. In the bin packing problem (Single Bin Size Bin Packing Problem (SBSBPP) according to [7]), there is the packing of a set of small items of different sizes into a number of large objects of identical size. In an analogy of cutting and packing problems, the bin-packing problem is characterized by a strongly heterogeneous assortment of small items, in contrast with the cutting stock problem that requires a weakly heterogeneous assortment of small items. The value, number, or total size of the necessary large objects has to be minimized, considering a weakly heterogeneous or strongly heterogeneous assortment of large objects.

The idea of integrating these decisions is to comprehend and take into account the interdependency, in the sense that an expanded view of the process contributes to a better decision-making, consequently to a global solution of the problem. Dealing with such integrated problem is challenging to researches, due to the fact that it might subsume more than one NP-hard problem. Therefore, this research fits into an updated, challenging and promising context of production planning by examining an integration among important problems of production processes.

2 Mathematical Models and Solution Methods

Considering the lot-sizing problem, the scheduling decisions manage to sequence the final products, whereas in the cutting stock problem, the scheduling decisions deal with the processes needed to perform the cutting operations for each object and are related to the variation in the number of pieces between two cut objects. A compact formulation to model the cutting stock problem is addressed. We consider a deterministic setting, with several time periods, so that the demand of final products can be satisfied either from production in the current time period and/or from inventory carried over from the previous period. The pieces required to assembly a final product in a determined time period can be cut in previous time periods, in this way, the synchronization between the problems considers a lead-time of one time period. Initial production inventory is considered only to pieces, due to the positive lead-time and stockouts are not accepted.

To treat the scheduling decisions in the integrated problem, we consider two different approaches from the literature, which are based on the *General Lot-Sizing and Scheduling Problem* (*GLSP*) [4] and on the *Asymmetric Traveling Salesman Problem*(*ATSP*) [3, 5]. In both strategies, we take into account sequence-dependent setup cost and setup time with the additional feature that setups may be carried over from one time period to the next and the setup can be preserved over idle time periods.

In the first mathematical model (*GLSP*), the idea is to split each time period (macro-period) into a fixed number of subperiods (micro-period) with flexible duration, in which only one setup is allowed in each micro-period, i.e., only one product or object can be produced or processed. The length of each micro-period is variable and it takes into account the setup time and production time of the product or process completed in that micro-period. Therefore, the micro-period decisions determine the number, size and sequence in which the final products and objects are performed in each time period.

In the second mathematical model, the scheduling decisions are modeled, in each problem involved considering the *ATSP* constraints, in which for each macro-period, several types of final products can be produced, as well as several objects can be cut. In order to treat the same time structure in both models, the *ATSP* based model aims to find a single sequence to the whole planning horizon for the final products, as well as for the objects in the cutting process. For this, the mathematical model should be able to save the final preparation state of the previous time period (macro-period) at the beginning of each time period (macro-period), to perform this, the setup carryover is modeled by considering the setup state at the beginning of each time period (macro-period) as a decision variable.

Exact solution approaches based on a mixed-integer programming and on a branch-and-cut procedure are used to find a good feasible solution to the integrated lot-sizing and bin-packing problem with sequence dependent setups. The *GLSP* based model is solved by an optimization package, whereas in the *ATSP* based model, two approaches are considered. Firstly, we address subtour elimination constraints using a polynomial set of linear constraints that are included a priori in the mathematical model for each pair of final products and cut objects in each time period. For this type of subtour elimination constraints we have tested the approaches proposed in [6], an improved version presented in [2] and the addiction case of clique constraints [1]. The resulting mathematical model, called *MTZ*, is solved by an optimization package. In the second approach, called *DFJ*, the subtour elimination constraints are dropped from the formulation and added in an interactive fashion, every time they are violated at the nodes of the branch-and-bound tree. These subtour elimination constraints are added in the model as separation cuts in the branch-and-cut, every time an integer solution is found at each node of the branch-and-bound tree to avoid generating too many cuts in the tree. We have tested three different ways to add the separation cuts.

3 Conclusions

The computational results of the approaches to the integrated problem are compared in terms of number of feasible instances, gap and computational time. Considering the mathematical models based on *ATSP* constraints (*MTZ* and *DFJ*), the *DFJ* approach (branch-and-cut) is able to find more feasible solution compared to the *MTZ*, with quite similar values for the computational time. An impact in the number of feasible solutions can be seen when the number of time periods increases.

Comparing the two mathematical models presented to the integrated problem, the model based on the *GLSP* strategy have a huge difficulty in finding feasible solutions, compared to the *DFJ* approach. The *DFJ* find 48% of the best results in terms of computational time compared to less than 5% to the *GSLP*. As a conclusion, we can say that the chosen approaches to model the scheduling decisions in the integrated problem are flexible in both environments and suitable for computing changeover setups.

Acknowledgments

This research was funded by the Conselho Nacional de Desenvolvimento Científico e Tecnológico, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior and the Fundação de Amparo a Pesquisa do Estado de São Paulo - FAPESP (process n 2016/01860-1, 2018/14895-3 and 2018/19893-9).

References

[1] T. Bektas and L. Gouveia. Requiem for the millertuckerzemlin subtour elimination constraints? European Journal of Operational Research, 236(3):820–832, 2014.

[2] M. Desrochers and G. Laporte. Improvements and extensions to the miller-tucker-zemlin subtour elimination constraints. Operations Research Letters, 10(1):27–36, 1991.

[3] B. Fleischmann. The discrete lot-sizing and scheduling problem with sequence-dependent setup costs. European Journal of Operational Research, 75(2):395–404, 1994.

[4] B. Fleischmann and H. Meyr. The general lotsizing and scheduling problem. OR Spectrum, 19(1):11–21, 1997.

[5] K. Haase. Capacitated lot-sizing with sequence dependent setup costs. OR Spectrum, 18(1):51–59, 1996.

[6] C. E. Miller, A. W. Tucker, and R. A. Zemlin. Integer programming formulation of traveling salesman problems. J. ACM, 7(4):326–329, 1960.

[7] G. Wäscher, H. Haußner, H. Schumann. An improved typology of cutting and packing problems. European Journal of Operational Research, 183(3):1109–1130, 2007.

A Branch-and-Cut Algorithm for an Assembly Routing Problem

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Abstract

We consider an integrated planning problem that combines production, inventory and inbound transportation decisions in a context where several suppliers each provide a subset of the components necessary for the production of a final product at a central plant. We provide a mixed integer programming formulation of the problem and propose several families of valid inequalities. The inequalities are used in a branch-and-cut (BC) algorithm. Computational experiments on a large set of generated test instances show that the inequalities significantly improve the performance of the BC algorithm.

1 Introduction

Few studies have considered the integration of production planning with inbound transportation for the collection of components from suppliers to assemble a final product. Hein and Almeder [1] introduce an integrated production and inbound routing problem with multiple components and products. Chitsaz et al. [2] study the case with multiple components and one final product but consider the inventory costs and storage capacity of the suppliers as well as a component storage area at the plant. They assume a one-to-one relationship between the suppliers and components.

We extend the model of Chitsaz et al. [2] to consider the case where each supplier may provide a subset of the components necessary for the final product and some components can be obtained from more than one supplier. We develop several new valid inequalities and through extensive computational experiments show that the inequalities notably enhance the performance of the BC algorithm.

2 Problem description

We consider a many-to-one assembly system with n suppliers ($N = \{1, ..., n\}$). The planning horizon includes l discrete time periods ($T = \{1, .., l\}$). To produce the final product (indicated by index zero), k distinct components ($K = \{1, ..., k\}$) are required. We assume that each supplier i may provide a subset of the components $K_i \subseteq K$. We define the problem on a complete undirected graph with the node set $N^+ = N \cup \{0\}$, where 0 represents the plant, and the edge set $E = \{(i,j) : i,j \in N^+, i < j\}$. We let $K^+ = K \cup \{0\}$. The suppliers as well as the central plant each have a global capacitated storage area for the components (L_i and L, respectively). Moreover, the central plant has a separate capacitated storage space for the final product (L_0). A fleet of m homogeneous vehicles, each with a capacity of Q, is available to transport the components from the suppliers to the plant. In each period t, an amount s_{ikt} of component k is made available at supplier i. The decisions to make include whether or not to produce the final product (y_t) and the quantity to be produced at the plant in each period (p_t), the inventory levels (I_{ikt}), supplier visit schedule (z_{it}) and order in each vehicle route (x_{ijt}), and the shipment quantities from the suppliers to the plant (q_{ijt}). A compact formulation for the Assembly Routing Problem (ARP) is presented as the following $\mathcal{M}_{ARP}$ model:

$$(\mathcal{M}_{ARP}) \;\; \min \sum_{t \in T} \Big(up_t + fy_t + \sum_{k \in K^+} h_{0k} I_{0kt} + \sum_{i \in N} \sum_{k \in K_i} h_{ik} I_{ikt} + \sum_{(i,j) \in E} c_{ij} x_{ijt} \Big) \tag{1}$$

s.t.

$$I_{00,t-1} + p_t = d_t + I_{00t} \quad \forall t \in T \tag{2}$$

$$I_{0k,t-1} + \sum_{i \in N_k} q_{ikt} = p_t + I_{0kt} \quad \forall k \in K, \forall t \in T \tag{3}$$

$$I_{ik,t-1} + s_{ikt} = q_{ikt} + I_{ikt} \quad \forall i \in N, \forall k \in K_i, \forall t \in T \tag{4}$$

$$p_t \leq C y_t \quad \forall t \in T \tag{5}$$

$$I_{00t} \leq L_0 \quad \forall t \in T \tag{6}$$

$$\sum_{k \in K} b_k I_{0kt} \leq L \quad \forall t \in T \tag{7}$$

$$\sum_{k \in K_i} b_k I_{ikt} \leq L_i \quad \forall i \in N, \forall t \in T \tag{8}$$

$$z_{0t} \leq m \quad \forall t \in T \tag{9}$$

$$\sum_{k \in K_i} b_k q_{ikt} \leq Q z_{it} \quad \forall i \in N, \forall t \in T \tag{10}$$

$$\sum_{(j,j') \in \delta(i)} x_{jj't} = 2 z_{it} \quad \forall i \in N^+, \forall t \in T \tag{11}$$

$$Q \sum_{(i,j) \in E(S)} x_{ijt} \leq \sum_{i \in S} \Big(Q z_{it} - \sum_{k \in K_i} b_k q_{ikt} \Big) \quad \forall S \subseteq N, |S| \geq 2, \forall t \in T \tag{12}$$

$$p_t \geq 0, y_t \in \{0,1\}, z_{0t} \in \mathbb{Z} \quad \forall t \in T \tag{13}$$
$$I_{0kt} \geq 0 \quad \forall k \in K^+, \forall t \in T \tag{14}$$
$$I_{ikt}, q_{ikt} \geq 0 \quad \forall i \in N, \forall k \in K_i, \forall t \in T \tag{15}$$
$$x_{ijt} \in \{0,1\} \quad \forall (i,j) \in E : i \neq 0, \forall t \in T \tag{16}$$
$$x_{0it} \in \{0,1,2\}, z_{it} \in \{0,1\} \quad \forall i \in N, \forall t \in T. \tag{17}$$

The objective function (1) minimizes the total production, setup, inventory, and transportation costs. The constraints (2)-(4) impose the product and components inventory flow. Constraints (5) impose the production capacity. Constraints (6)-(8) impose the storage capacity of the product and components. Constraints (9) impose the limit on the fleet size. Constraints (10) force a vehicle visit whenever components are shipped. Constraints (11) are the degree constraints. Constraints (12) are the subtour elimination constraints (SEC) and are referred to as generalized fractional subtour elimination constraints (GFSEC).

3 Inequalities and Algorithm

We present and discuss three classes of valid inequalities to improve the LP relaxation bound for the ARP. The first class contains inequalities of type *(l,S,WW)* [3]. Some of these inequalities take into account the known supply instead of the known demand. The second class concerns the cut-set type inequalities. We generalize and extend the cut-set type inequalities to provide integer lower bounds on the number of required production setups, number of vehicles dispatched, and supplier visits from period $e = 1$ to $t \in T$. The last class includes general inequalities for the ARP, such as the adaptation of the Dantzig-Fulkerson-Johnson (DFJ) SECs for multi-period vehicle routing problems .

The algorithm applies the valid inequalities at the root node and adds GFSECs and DFJs at each node of the search tree. To separate GFSECs, we use CVRPSEP [4] and we also propose two heuristic separation procedures. We adapted CCJ-DH, the unified matheuristic proposed in Chitsaz et al. [2], to obtain high quality feasible solutions as well as cutoff values. In our experiments we set a time limit of one hour both for the BC and for CCJ-DH.

4 Computational experiments

We generated three new classes of instances. The first class includes instances where each supplier provides a unique component type. The second class represents the case where each supplier provides a subset of components. The third class corresponds to the situation in which a unique component is offered by all suppliers. Each class includes instances with five different planning horizons ranging from 4 to 12 periods with a step of two. For each planning horizon we consider eight different numbers of suppliers, increasing by steps of 3. For each combination of the number of planning periods and suppliers we randomly generated five instances. Overall, 600 instances are generated for three classes, five planning horizons, eight numbers of suppliers, and five instances per category.

Table 1 reports a summary of the results on the performance of the BC when the default or the best-bound search strategies of CPLEX are employed, and either no inequality (None), only known inequalities or all inequalities (All) are applied. By comparing %UB and %BUB for each search strategy and each class, one observes the effect of applying CCJ-DH cutoffs within the BC. Overall, compared to the cases with no or only known inequalities, using all inequalities in BC with both search strategies notably increases the number of optimal solutions and significantly improves the %UB and %BUB for all classes. These results show that our new valid inequalities make a substantial difference in the success of the BC. We refer to Chitsaz et al. [5] for more explanations and detailed results.

Table 1: Summary of the results of the BC with the default and the best-bound search strategies, and with and without the valid inequalities on different instance classes*

Node	Valid	Class 1					Class 2					Class 3				
Selection	Ineq.	Size	#Opt	CPU	%UB	%BUB	Size	#Opt	CPU	%UB	%BUB	Size	#Opt	CPU	%UB	%BUB
Default	None	200	11	3157	69.6	96.7	200	5	3234	65.4	95.2	200	22	3045	79.6	95.9
	Known	200	51	2576	86.3	96.8	200	44	2729	83.9	95.2	200	107	1912	96.1	97.5
	All	200	103	1980	91.2	99	200	69	2420	85	97.9	200	155	1205	98.3	99.5
Best-Bound	None	200	8	3207	56.5	97.3	200	5	3260	36.9	96.3	200	14	3098	64.5	96.6
	Known	200	52	2578	57.3	97.3	200	64	2418	61.8	96.3	200	107	1872	89.8	98.1
	All	200	149	1422	84.7	99.4	200	119	1976	74.4	98.7	200	171	938	97.4	99.8

Size: Number of instances, #Opt: Number of optimal solutions
%UB: Average lower bound values as a percentage of the upper bound obtained by the BC without applying the CCJ-DH cutoffs
%BUB: Average lower bound values as a percentage of the BUB
* To calculate the BUB for each BC scenario, we considered the upper bounds obtained by either that BC scenario or CCJ-DH.

References

[1] Fanny Hein and Christian Almeder. Quantitative insights into the integrated supply vehicle routing and production planning problem. *International Journal of Production Economics*, 177:66–76, 2016.

[2] Masoud Chitsaz, Jean-François Cordeau, and Raf Jans. A unified decomposition matheuristic for assembly, production and inventory routing. *INFORMS Journal on Computing*, 31(1):134–152, 2019.

[3] Imre Barany, Tony Van Roy, and Laurence A Wolsey. Uncapacitated lot-sizing: The convex hull of solutions. In Bernhard Korte and Klaus Ritter, editors, *Mathe-*

matical programming at Oberwolfach II, volume 22 of *Mathematical Programming Studies*, pages 32–43. Springer, Berlin, Heidelberg, 1984.

[4] Jens Lysgaard, Adam N Letchford, and Richard W Eglese. A new branch-and-cut algorithm for the capacitated vehicle routing problem. *Mathematical Programming*, 100(2):423–445, 2004.

[5] Masoud Chitsaz, Jean-François Cordeau, and Raf Jans. A branch-and-cut algorithm for an assembly routing problem. Technical Report G-2018-49, GERAD, HEC Montral, Canada, 2018.

Integrated lot-sizing and maintenance planning

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Abstract

We consider a problem where lot-sizing and maintenance planning decisions are taken simultaneously. The computational complexity of several variants of the problem are studied depending on whether (i) Maintenance can be done at any point in time or not, (ii) There is a fixed aging component when starting production, and (iii) There is a minimum age before maintenance can be performed.

1 Introduction and problem motivation

In industry and in research, production planning and scheduling and maintenance planning and scheduling are most often performed independently. Usually, maintenance is first planned based on an estimated failure probability of machine components between two maintenance operations. Then, production is planned and scheduled given the maintenance plan. Hence, most papers on production scheduling consider that maintenance operations are performed at fixed times that are seen as machine unavailability periods in the scheduling horizon, see for instance [1] and [4]. In production planning, i.e. lot-sizing problems, maintenance is very rarely explicitly taken into account. Instead, the production capacity available in a period is reduced if a maintenance operation takes place. This is actually one of the motivations to consider time-varying production capacity in dynamic lot-sizing problems.

However, in many practical settings, maintenance is *flexible* and should be performed within a given time window, associated to the allowed risk on the machine. More precisely, the duration of each maintenance operation is provided, with its earliest and latest start times. On a scheduling horizon which is usually rather short, i.e. less than a day, considering a single or very few flexible maintenance operations is often enough. This is the case for instance in [2] and [5]. This flexibility can be very useful to maximize productivity by either postponing a maintenance operation to complete a production operation or starting early a maintenance operation when the machine is idle.

In practice, maintenance operations are not independent in the planning horizon, i.e., there is a maximum time between two consecutive maintenance operations. This implies that, if a maintenance operation is planned early in its time window, then the next maintenance time window will also be early. Moreover, it is often more realistic to consider a maximum number of products produced between two consecutive maintenance operations, i.e., the machine only "ages" when it processes products. Explicitly considering several related flexible maintenance operations in integrated lot-sizing and maintenance problems is important to determine realistic optimized plans. In addition to the classical trade-off between setup and inventory costs in lot-sizing, the complexity of planning related flexible maintenance operations and minimizing maintenance costs is added. To our knowledge, [3] are the only ones to explicitly include the planning of related flexible maintenance operations in a lot-sizing model for multiple items.

Based on the above motivation, we study the single-item integrated lot-sizing and maintenance problem with a maximum number of products to be processed between two maintenance operations. Our goal is to analyze the complexity of various cases, and to propose polynomial algorithms when possible. The main characteristics that we consider are: (1) Whether maintenance operations are only performed at the beginning of a period or at any point in time in a period, (2) Whether there is a minimum age before a maintenance operation can be performed or not and (3) Whether the age of the machine is only impacted by the number of processed products or also by setups.

2 Problem description & mathematical modeling

Consider the classical dynamic lot-sizing problem where the production periods and quantities of a single item have to be to decided on a finite planning horizon discretized in T periods, where deterministic demands have to be satisfied. The objective is to minimize the total cost, which combines the fixed setup costs and the variable holding and production costs.

Although production capacity is unlimited, we assume that preventive maintenance operations, each inducing a fixed maintenance cost, also have to be planned on the main machine processing the item. Following a concept often used in maintenance, we associate an "age" to the machine, which increases when producing the item and is reset to zero after a maintenance operation. A preventive maintenance policy has to be followed. More precisely, a maintenance operation cannot be planned before the machine has reached a minimum age and after the machine has reached a maximum age.

Note that, without loss of generality and to simplify the notations, we assume that the parameters and variables related to the age of the machine are expressed in number of items. The notations are detailed below.

Parameters:

d_t:	Demand in period t,
K_t:	Fixed setup cost in period t,
h_t:	Unit holding cost in period t,
c_t:	Unit production cost in period t,
$A^{\min}$:	Minimum age of the machine before maintenance is allowed,
$A^{\max}$:	Maximum allowed age of the machine,
F_t:	Fixed maintenance cost in period t,
A^f:	Fixed age setup, the age increase if the production of an item is setup in a period.

Variables:

x_t:	Production quantity in period t,
x_t^+:	Production quantity in period t before maintenance,
x_t^-:	Production quantity in period t after maintenance,
I_t:	Inventory level in period t,
y_t:	Setup variable in period t,
a_t:	Reduction in machine age after maintenance in period t,
A_t^-:	Machine age (in number of items) at the start of period t just before maintenance,
A_t^+:	machine age (in number of items) at the end period t just after maintenance,
z_t:	indicator if maintenance performed in period t.

Formulation of basic model:

Based on the network flow representation, the problem can be modelled as:

$$
\begin{array}{lllr}
\min & \sum_{t=1}^{T} (K_t y_t + p_t x_t + h_t I_t + F_t z_t) & & (1) \\
\text{s.t.} & I_{t-1} + x_t = d_t + I_t & t = 1, \dots, T, & (2) \\
& x_t \le M y_t & t = 1, \dots, T, & (3) \\
& x_t = x_t^- + x_t^+ & t = 1, \dots, T, & (4) \\
& A_t^- = A_{t-1}^+ + x_{t-1}^+ + x_t^- + A^f y_t & t = 1, \dots, T, & (5) \\
& A_t^- \le A^{\max} & t = 1, \dots, T, & (6) \\
& A_t^+ = A_t^- - a_t & t = 1, \dots, T, & (7) \\
& A^{\min} z_t \le a_t \le A^{\max} z_t & t = 1, \dots, T, & (8) \\
& x_t, x_t^-, x_t^+, I_t, A_t^-, A_t^+, a_t \ge 0 & t = 1, \dots, T, & (9) \\
& y_t, z_t \in \{0, 1\} & t = 1, \dots, T. & (10)
\end{array}
$$

Constraints (2) and (3) are the classical inventory balance and setup-forcing constraints. In Constraints (4), the production is split in production before and after maintenance in a period while, in Constraints (5), the machine age is updated. Note that we assume that the age setup is accounted for just before a (possible) maintenance operation starts. Constraints (6) ensure that the age of the machine does not exceed the maximum allowed age. Constraints (7) update the machine age after a maintenance, and Constraints (8) ensure that updating only occurs when the machine age is within the appropriate bounds.

3 Main results

Our main results can be summarized as follows. We show that the problem:

- Is a generalization of a capacitated lot-sizing problem in the single-item case,
- Can be solved in polynomial time by dynamic programming when certain parameters are time-invariant in the single-item case,
- Becomes strongly NP-hard with multiple items.

References

[1] Aggoune, R., 2004. Minimizing the makespan for the flow shop scheduling problem with availability constraints, European Journal of Operational Research 153 (3), 534–543.

[2] Azem, S., Aggoune, R., Dauzère-Pérès, S., 2007. Disjunctive and time-indexed formulations for non-preemptive job shop scheduling with resource availability constraints. In: 2007 IEEE international conference on industrial engineering and engineering management. IEEE, pp. 787–791.

[3] Géhan, M., Absi, N., Castanier, B., Dauzère-Pérès, S., Lemoine, D., 2013. Tactical production planning under system availability constraint. In: International Workshop on Lot Sizing (IWLS 2013).

[4] Tamssaouet, K., Dauzère-Pérès, S., Yugma, C., 2018. Metaheuristics for the job-shop scheduling problem with machine availability constraints, Computers & Industrial Engineering 125, 1–8

[5] Yang, S. L., Ma, Y., Xu, D. L., Yang, J. B., 2011. Minimizing total completion time on a single machine with a flexible maintenance activity. Computers & Operations Research 38 (4), 755–770.

Integrated decisions – II

Buyback contract terms integrated with batch ordering decisions

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Abstract

We study the single-item lot sizing problem with only full batch replenishment under three types of buyback contract for a system of a retailer replenishing from a supplier. In the first buyback contract type, the return periods are known in advance and no product return is possible outside those given periods. In the second type, the retailer can only return products to the supplier in the first j periods, with period j being fixed in the contract terms. Finally, in the third form, the retailer can return products only in replenishment periods. The quantity of returned products is limited with a coefficient called 'return rate', being either 100% (full return) or less than 100% (partial return). We consider the disposal concept to get rid of the products which cannot be stored nor returned to the supplier. We present a mixed integer programming formulation for each type, followed by dynamic programming algorithms for some polnomially solvable cases. We also show the equivalence of the problem for the second and third types of buyback contract with partial return to the resource constrained shortest path problem with double sided inequality constraints. Some results follow.

1 Problem description & Literature survey

We consider a retailer replenishing from a supplier one type of product over a finite horizon in a deterministic setting. Both sides sign a buyback contract, consisting in the possibility for the retailer to return back to the supplier the unused product units within the authorized periods. The buyback contract has very few parameters: the selling price (corresponding to the unit purchase price), the return revenue (what the retailer earns for each product returned), return dates and a return percentage to respect. Not all the unused units can be returned back to the supplier if the total quantity exceeds this initial percentage that is fixed in the terms. This is a protection for the supplier.

In this study, we integrate both Lot Sizing Problem (LSP) decisions and buyback contract terms' cost and constraints. The lot sizing problem is used to model the replenishment part: which quantity to replenish and when to replenish them are the main decisions made in LSP. Integrated with buyback, the overall decisions also include when and how much to return back to the supplier. Note that we consider batch replenishment for LSP part: the product units are loaded into the batches of a certain limited capacity V_t. Then, the replenishment decision becomes the number of batches to replenish in each period. We assume that only full batch replenishment is authorized. In what follows, this integrated problem is called LSP-BB for Lot Sizing Problem with Batch replenishment under Buyback contract terms.

The buyback contracts arise in several industrial settings. Products with the following properties are often replenished under this special type of contract (Pasternack (1985), Hou *et al.* (2010)) :

- Limited life time due to physical decay (dairy products, baked goods, pharmaceuticals, cosmetics).
- Risk of obsolescence (fashion apparel, computer hardware and software, greeting cards, magazines, newspapers).
- High carrying costs or products with rapidly saturated demands (books and recorded music).

Our problem is positioned in the intersection of various problems. We consider LSP with batch replenishment, which is quite well studied in the literature. We consider a special type of Capacity Reservation Contract (CRC) which is the buyback contract. To the best of our knowledge, it is the first time that the buyback contract is integrated into the lot sizing problem in a deterministic and dynamic setting. We also consider the disposal option in our problem. See Figure 1 for the position of our problem into the literature. Only the most relevant papers appearing in this figure are cited at the end of this document.

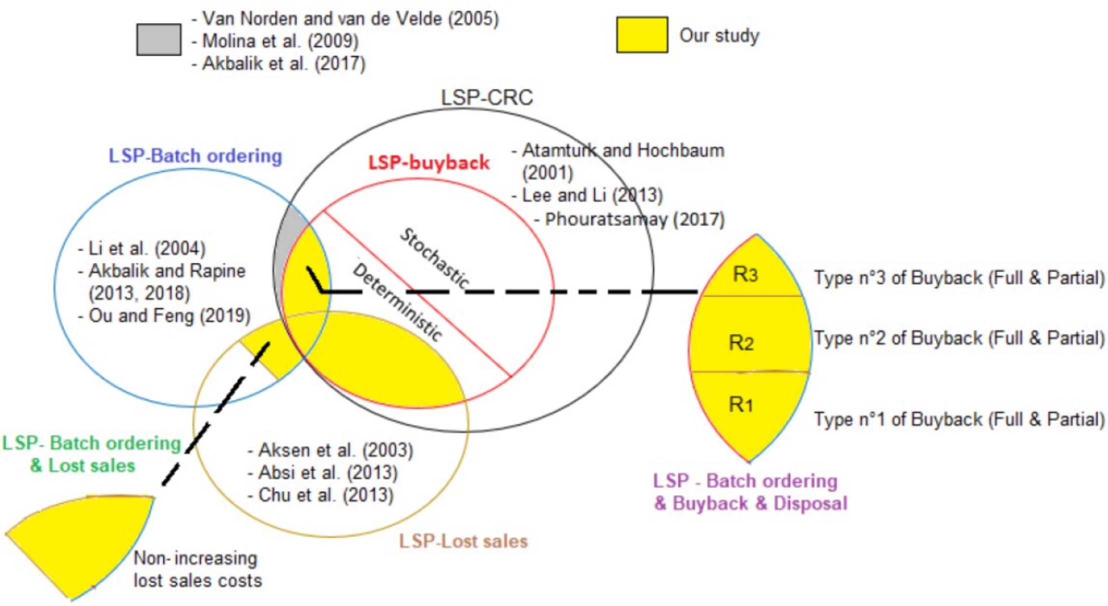

Figure 1: Our positioning into the literature, Farhat (2019)

Three types of buyback contract are considered in our study: R1, R2 and R3. In R1, the return periods are fixed and given. In R2, the products can be returned back only in the first j periods; and in R3, return possibility is only permitted in procurement periods. For more details, the reader is referred to the Ph.D manuscript of Farhat (2019).

2 Some results

We have proposed efficient polynomial time algorithms for different cases studied. Most of them are polynomial time dynamic programming algorithms. For the last cases concerning partial return policies for buyback contract types R2 and R3, we have shown the equivalence to the Resource Constrained Shortest Path Problem (RCSPP).

The overall complexity results are illustrated in Figure 2. Some notations are given as follows. ρ: return percentage, with $\rho = 100\%$ being full return and $\rho < 100\%$ partial return; w= return periods (cyclic or acyclic); OFB: only full batch; FTL: full truck load; T: number of periods in the overall planning horizon; $LSP - BR_i$: LSP with batch replenishment and buyback contract type R_i.

[1]: Lagrangian heuristic for RCSPP of Beasley and Christofides (1989); NI: Non-increasing; NN: Non-negative

Buyback forms	Return policies		Batch ordering: OFB	Batch ordering: FTL	Batch size: V	Batch size: V_t	Lost sales costs: NI	Lost sales costs: General	Disposal quantity: NN	Complexity
LSP-BR$_1$	$\rho = 1$	$w = 1$	×			×				$O(T)$
				×		×				$O(T)$
			×			×		×		$O(T)$
				×		×		×		$O(T)$
		$w > 1$	×		×					$O(Tw)$
				×	×					$O(Tw^3)$
			×		×		×			$O(Tw^3)$
				×	×		×			$O(Tw^3)$
		w_i	×		×				×	$O(T^4)$
	$\rho < 1$	w_i	×		×				×	$O(T^5)$
LSP-BR$_2$	$\rho = 1$		×		×				×	$O(T^5)$
	$\rho < 1$		×		×				×	[1]
LSP-BR$_3$	$\rho = 1$		×		×				×	$O(T^5)$
	$\rho < 1$		×		×				×	[1]

Figure 2: Our overall complexity results for LSP-BB, Farhat (2019)

References

[1] Akbalik, A., A. B. Hadj-Alouane, N. Sauer, H. Ghribi, NP-hard and polynomial cases for the single-item lot sizing problem with batch ordering under capacity reservation contract, European Journal of Operational Research 257(2) (2017) 483-493.

[2] Beasley, J.E., N. Christofides, An algorithm for the resource constrained shortest path problem, Networks, 19 (1989) 379-394.

[3] Farhat, M. *PhD manuscript: Batch replenishment planning under capacity reservation contract*, 2019. Université de Lorraine & Ecole Nationale d'Ingénieurs de Tunis.

[4] Hou, J., A.Z. Zeng, L. Zhao, Coordination with a backup supplier through buyback contract under supply disruption, Transportation Research Part E: Logistics and Transportation Review 46(6) (2010) 881-895.

[5] Li, C.L., V.N. Hsu, W.Q. Xiao, Dynamic Lot Sizing with Batch Ordering and Truckload Discounts, Operations Research 52(4) (2004) 639-654.

[6] Pasternack, B.A., Optimal pricing and returns policies for perishable commodities, Marketing Science 4(2) (1985) 166-176.

A Branch-and-Cut Algorithm for the Production-Routing Problem with Coordinated and Capacitated Lot-Sizes and Backorders

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Abstract

We present a branch-and-cut algorithm to tackle a production-routing problem consisting of multiple products and customers served by a heterogeneous fleet of vehicles. By considering these decisions together, companies achieve reduced inventory and transportation costs compared to when these decisions are made sequentially. To accelerate the performance of this algorithm, we employ a decomposition heuristic for determining tactical distribution routes. Customers are clustered through k-means++ and a location-allocation subproblem based on their contribution to overall demand, and these clusters remain fixed over the entire planning horizon. A routing subproblem dictates the order in which to visit customers in each period. Under four business scenarios, we vary the degree of flexibility in demand and transportation by considering split deliveries and backorders, two settings that are not commonly studied in the literature.

1 Introduction

The production-routing problem (PRP) combines the lot-sizing, distribution and routing decisions made by single a production facility carrying out all three operations. This is a typical arrangement for vendor-managed inventory (VMI), where a supplier controls the replenishment process for its customers. In this way, the vendor ensures that sufficient inventory is maintained at the customer level and can also coordinate its production and deliveries in such a way that manufacturing and vehicle capacities are efficiently used.

There are many industries to which PRP applies, for example, furniture [7], beverages [8, 1], and perishable foods [6]. However, few consider the production (or replenishment) of multiple items (e.g. [5, 7]), let alone multiple *families* of items. The extension to include an additional item or two is not trivial, especially if the production of these items is coordinated. Lot-sizing decisions would now be impacted by the fixed cost of setting up manufacturing for a family of items and not just the per unit production cost. This makes the lot-sizing portion of the PRP even more challenging to solve. Given that PRP is already NP-hard due to the embedded traveling salesman problem within the vehicle routing constraints, this added complexity poses challenges in developing appropriate solution methods that can both find near-optimal solutions and do so in a reasonable amount of time.

Based on our survey of the literature, there are a few PRP variants that have received little attention. These include the *coordinated* replenishment of multiple items, allowance for backorders, a heterogeneous fleet of vehicles, and split deliveries. Also, apart from [9], solution approaches tend to first solve for production and distribution quantities, and then determine vehicle routes. However, in practice, routes might remain fixed over a certain time horizon so that drivers may build rapport with their assigned retailers and thus provide better customer service, and so that workloads are equitable among drivers [3, 4].

We address these characteristics in our formulation of the multi-item PRP subject to capacitated and coordinated replenishments. We propose a Tactical Routes Heuristic (TRH) to fix customer routes over the planning horizon, reflecting current industry practices for equitable routing. Furthermore, we study the impact of flexible demand and transportation restrictions on vehicle utilization and cost using a branch-and-cut procedure that is warm-started with TRH solutions.

2 Solution Approach

We approach PRP by applying a Tactical Routes Heuristic (TRH) to determine an initial upper bound to PRP. In the first phase, a districting problem is solved to create customer clusters (or districts). Customers are clustered based on geographical proximity and total demands over the planning horizon. This districting problem is solved by designating certain customers as district "centers" and then allocating customers to them. To initialize district centers, the k-means++ algorithm assigns cluster centers based on a weighted probability. In this problem, total horizon demands are used as the weights.

In the second phase, a traveling salesman problem is solved for each cluster to determine the optimal customer visit sequence. Since each cluster remains unchanged over the time horizon, only production and shipments are determined in the third phase. This TRH solution is used to warm-start the branch-and-cut procedure.

3 Computational Results

TRH and PRP were coded in Python 2.7 with CPLEX 12.8, and run on a PC with an Intel Core i5 3.41GHz processor and 8GB RAM. We limit our study to the case of two product families ($L = 2$), though our formulation and solution approach can be extended to any value of L. No data sets for the multi-family PRP exist, so we use the 50-customer data instances developed by [2] (Set B50), modified to include multiple product families and a heterogeneous fleet of vehicles. Four business scenarios, [1A], [1B], [2A], and [2B], are studied by making slight adjustments to constraints and variables in the original PRP formulation (see Figure 1).

	Backorders Allowed	No Backorders Allowed
Split Deliveries Allowed	**[1A]**	**[1B]**
No Split Deliveries Allowed	**[2A]**	**[2B]**

Figure 1: Business Scenario Nomenclature

We solve all four scenarios with the branch-and-cut algorithm alone and compare the results to when branch-and-cut is warm-started with the TRH solution. While the branch-and-cut algorithm alone does fairly well on problems of small sizes (fewer customers, fewer items, and fewer time periods), the solution quality begins to degrade as problems become larger. Solution time increases rapidly, and many problems are not solved to optimality. The quality of the solution at the root node is also quite poor.

When the branch-and-cut algorithm is warm-started with the results of TRH, the overall solution time improves drastically. While larger instances ($T = 9$, $n \geq 30$) still do not solve to optimality, the duality gaps are much lower. The solution at the root node is much better, suggesting that even these interim solutions may be suitable to implement if available computational time is limited.

When comparing scenarios [1A] through [2B] to each other and to TRH solutions alone, we can better see the impact of backorders and split deliveries on vehicle utilization, solution time, and overall cost. TRH provides better vehicle utilization and computational times compared to all scenarios. Compared to scenarios [1A] and [2A], for certain large instances TRH produces items more frequently and incurs higher costs. However, compared to [1B] and [2B], TRH has the advantage by incurring less in inventory costs by backordering more frequently.

By examining differing scenarios, supply chain managers can better understand the cost and vehicle utilization impacts of shifting demand requirements or delaying shipments, and can be armed with meaningful information when negotiating terms of agreement with clients.

References

[1] Begen, M., Puterman, M.L. and Wu, H. Development of an operational and tactical decision support tool for a Canadian beverage firm: a case study. *European Journal of Industrial Engineering*, 13(2),245–263 (2019).

[2] Boudia, M., Louly, M.A.O. and Prins, C. Combined optimization of production and distribution. In *CD-ROM proceedings of the International Conference on Industrial Engineering and Systems Management (IESM '05)* (2005).

[3] Carlsson, J.G. and Delage, E. Robust partitioning for stochastic multivehicle routing. *Operations Research*, 61(3),727–744 (2013).

[4] Lei, H., Wang, R. and Laporte, G. Solving a multi-objective dynamic stochastic districting and routing problem with a co-evolutionary algorithm. *Computers & Operations Research*, 67, 12–24 (2016).

[5] Li, Y., Chu, F., Chu, C. and Zhu, Z. An efficient three-level heuristic for the large-scaled multi-product production routing problem with outsourcing. *European Journal of Operational Research*, 272(3), 914–927 (2019).

[6] Li, Y., Chu, F., Feng, C., Chu, C. and Zhou, M. Integrated production inventory routing planning for intelligent food logistics systems. *IEEE Transactions on Intelligent Transportation Systems*, 20(3), 867–878 (2019).

[7] Miranda, P.L., Cordeau, J-F., Ferreira, D., Jans, R. and Morabito, R. A decomposition heuristic for a rich production routing problem. *Computers & Operations Research*, 98, 211–230 (2018).

[8] Senoussi, A., Dauzère-Pérès, S., Brahimi, N., Penz, B. and Mouss, N.K. Heuristics based on genetic algorithms for the capacitated multi vehicle production distribution problem. *Computers & Operations Research*, 96, 108–119 (2018).

[9] Solyalı, O. and Süral, H. A multi-phase heuristic for the production routing problem. *Computers & Operations Research*, 87, 114–124 (2017).

An integrated multi level lot sizing problem with a single railway track transport problem

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Abstract

This paper presents a linear model that solves an integrated lot sizing and transport problem as part of a global supply chain system. The purpose of the model is to simultaneously propose a production plan, a stock plan and a transport plan in order to satisfy a known demand on a finite discrete horizon for a multi-level, multi-product production system. Assuming that local planning usually leads to sub-optimal solutions, this work is realized in the context of a global supply chain, which means that it reproduces the overall functioning of all the subsystems in order to find a solution that takes into account two types of constraints: local constraints that reflect the specificities of the different links in the chain. The second type are inter-process constraints, it expresses the interactions between the different sites and the propagation of the demand from downstream to upstream in the different subsystems. This planning was carried out as part of a mining supply chain and takes into account the different capacities of production, storage and transport for all the components of the logistics chain, but also the travelling time between the different sites.

1 Introduction

One of the major motivations for this research is to consider the planning problem of supply chain planning as a whole, where production, inventory, and transportation decisions are integrated. Traditional models usually address only one or two of these issues independently of the other(s). The most classical approach was to first plan production, and then establish distribution decision, which is equivalent to follow the decision-making process to the physical flow of the product. Though, knowing that 'local' planning usually leads to sub-optimal solutions, it goes without saying that integrating these decisions can lead to considerable increases in efficiency and effectiveness [1] [2] [3]. Also, addressing local problems while trying to improve supply chain performance might increase supply chain global cost instead of decreasing it [4]. IBM is a real-world example who improved its semiconductor supply chain efficiency by implementing a unified production, shipping and distribution plan. In fact, this global approach allowed IBM to improve its customer service by increasing its on-time deliveries by 15 percent, to increase its assets utilization by 2-4 percent of costs and to decrease its inventory by 25-30 percent [5]. A second example is from McKesson who is healthcare services company who distributes more than one-third of all pharmaceutical products in North America. This company started using, at 2010, an integrated decision support tool that optimizes their distribution network, supply flow, inventory, and transportation policies; this has allowed McKessons pharmaceutical division to reduce its committed capital by more than 1 billion dollars by the end of 2012 [6]. Furthermore, Kellogg company reduced its production, inventory, and distribution costs by an estimated 4.5 million dollars in 1995 due to its integrated operational planning system, and projects to save 35 to 40 million dollars per year by using their tactical integrated planning program [7]. Finally, due to its impact, several researchers published states of art in issue of the simultaneous planning of the different supply chain components [8] [9] [10].

2 Problem desciption and proposed MILP

Taking into account the advantages of a holistic approach instead of treating the supply chain problems separately or sequentially, this work deals with the integrating production, inventory and distribution planning which are three of the most classical problems in supply chain management. According to [11], the integration of these three problems gives birth to Integrated lot sizing with direct shipment problem. Thus, this work addresses simultaneously a production, inventory and transport problem. The considered transport is system is characterized by a single railway train track with delivery time windows. In other words, it integrates a Multi-Level Capacitated Lot Sizing (MLCLSP) problem and a train transport problem with delivery time windows. The integration of these specific problems in multi-level supply chain structure gives birth to an original problem that, to the best of our knowledge, has not been addressed in the literature. In order to solve this problem, we propose a linear optimization model that takes into account two type of constraints: local constraints of the different subsystems composing the global productive system, but also the global ones that translates the interactions between these subsystems and the spread of the demand along this supply chain. This model calculates simultaneously a production, storage and transport plan that satisfies a known demand while minimizing total production, storage and distribution costs and maximizing demand satisfaction rate.

3 Industrial application

The model is tested and validated on a real mining supply chain, it proposes an integrated solution applicable for simultaneously production, storage and distribution planning. Our industrial partner has provided us with a large amount of data regarding production costs, production and inventory capacities, shipping costs, and daily demand for a year-long planning horizon. We have used this information to evaluate our method under different production scenarios. The program has been solved in few minutes using Xpress solver. The model proposes for each period, product and site a production plan a storage plan and a transport plan, which identifies the quantity to be transported from each production site to the other. In conclusion, we have solved a real multi-plant, production planning and distribution problem in which production, inventory, demand and distribution decisions are optimised simultaneously. A mathematical formulation for a rich dynamic LSP problem with transport time windows is proposed. We are now working on comparisons of models and tests and analyzes of model robustness.

References

[1] D. M. Lambert and M. C. Cooper, Issues in supply chain management, Industrial marketing management, vol. 29, no. 1, pp. 6583, 2000.

[2] A. Kanda and S. G. Deshmukh, Supply chain coordination: perspectives, empirical studies and research directions, International journal of production Economics, vol. 115, no. 2, pp. 316335, 2008.

[3] K. Arshinder, A. Kanda, and S. G. Deshmukh, A review on supply chain coordination: coordination mechanisms, managing uncertainty and research directions, in Supply chain coordination under uncertainty, Springer, 2011, pp. 3982.

[4] Y. Adulyasak, J.-F. Cordeau, and R. Jans, Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems, INFORMS Journal on Computing, vol. 26, no. 1, pp. 103120, 2013.

[5] A. Degbotse, B. T. Denton, K. Fordyce, R. J. Milne, R. Orzell, and C.-T. Wang, IBM blends heuristics and optimization to plan its semiconductor supply chain, Interfaces, vol. 43, no. 2, pp. 130141, 2013.

[6] K. Katircioglu et al., Supply chain scenario modeler: A holistic executive decision support solution, Interfaces, vol. 44, no. 1, pp. 85104, 2014.

[7] G. Brown, J. Keegan, B. Vigus, and K. Wood, The Kellogg company optimizes production, inventory, and distribution, Interfaces, vol. 31, no. 6, pp. 115, 2001.

[8] S. Moons, K. Ramaekers, A. Caris, and Y. Arda, Integrating production scheduling and vehicle routing decisions at the operational decision level: A review and discussion, Computers Industrial Engineering, vol. 104, pp. 224245, Feb. 2017.

[9] M. Daz-Madroero, D. Peidro, and J. Mula, A review of tactical optimization models for integrated production and transport routing planning decisions, Computers Industrial Engineering, vol. 88, pp. 518535, 2015.

[10] B. Fahimnia, R. Z. Farahani, R. Marian, and L. Luong, A review and critique on integrated productiondistribution planning models and techniques, Journal of Manufacturing Systems, vol. 32, no. 1, pp. 119, 2013.

[11] Y. Adulyasak, J.-F. Cordeau, and R. Jans, Optimization-based adaptive large neighborhood search for the production routing problem, Transportation Science, vol. 48, no. 1, pp. 2045, 2014.

Sustainability

A Min-Max Approach for the Robust Two-level Lot Sizing Problem with Multiple Components and Remanufacturing

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Abstract

In this paper, we consider a robust lot sizing problem with two levels and the option of remanufacturing. The upper level consists of a single item under independent demand, and the lower level consists of multiple components under dependent demands. Such a setting often stems from practice, where returned products are processed by remanufacturing not a whole unit of a product but rather its components, while also manufacturing any further necessary components. Therefore, the problem considers a range of decisions, including manufacturing and remanufacturing to satisfy lower level demand, assembly of components to satisfy upper level demand, and inventory of returned items, components, and the final product. The uncertainty in this setting primarily stems from the number of items returned (as well as the quality of returned items,) and we address such uncertainty by defining uncertainty sets in a robust optimization framework. As an efficient solution method, we propose a decomposition approach with two subproblems, Decision Makers Problem (DMP) and Adversarial Problem (AP), which are handled in an iterative fashion. We present preliminary computational results with a number of observations and insights.

1 Introduction

There are a number of different ways to achieve product recovery, which enables the return of used products to the production cycle. The most effective way for product recovery is remanufacturing, which essentially processes a returned product to at least the same quality level of a newly manufactured product. Although remanufacturing has been receiving increasing attention from researchers in various disciplines due to its significant savings and environmental potentials, the lot-sizing research in this area is still in its early stages. We refer the interested reader to the recent studies of [4] and [5] for detailed reviews of accomplishments to date. Before discussing the role of uncertainty, we formally define the problem of interest in the deterministic setting, starting with the sets and parameters.

- Set of time periods: $t = 1, ..., \mathcal{T}$
- Set of components required to produce final-level item: $c = 1, ..., \mathcal{C}$
- Set of scenarios considered in DMP: $s = 1, ..., \mathcal{S}$
- m^0: cost of assembling demand-level (final) items
- h^0: holding cost for the final item
- h^c: holding cost for component c, where $c = 1, .., \mathcal{C}$
- K^c: setup cost for component c
- w^c: returns holding cost for component c
- m^c: manufacturing cost for component c
- r^c: remanufacturing cost for component c

We consider multiple components at the second level, and a single manufacturing decision (denoted as x_t^0) on the first level (demand level). Our choice of x_t^0 only incurs a linear production cost. Note that returns are on the component level (second level). We have the option of manufacturing (x_t^c) or remanufacturing (q_t^c) a component, where c takes on values from the set of components: $c = \{1, ..., \mathcal{C}\}$. For production to take place a joint setup has to be made, where our setup decision is indicated by the binary variable y_t^c. Once components are produced, they can be kept in components' inventory for a cost of h^c, where I_t^c indicates the number of components that are being kept in inventory at time period t. Next, we present the formulation for the problem as follows:

$$\min \sum_{t=1}^{\mathcal{T}} \Big(m^0 x_t^0 + h^0 I_t^0 + \sum_{c=1}^{\mathcal{C}} h^c I_t^c$$
$$+ \sum_{c=1}^{\mathcal{C}} w^c \big(\sum_{i=1}^{t} (R_i - q_i^c)\big) + m^c x_t^c + r^c q_t^c + K^c y_t^c \Big)$$
$$x_t^0 + I_{t-1}^0 = I_t^0 + D_t \qquad \forall t = 1, ..., \mathcal{T}, \ \forall c = 1, ..., \mathcal{C} \tag{1}$$
$$x_t^c + q_t^c + I_{t-1}^c = I_t^c + x_t^0 \qquad \forall t = 1, ..., \mathcal{T}, \ \forall c = 1, ..., \mathcal{C} \tag{2}$$
$$\sum_{i=1}^{t} (R_i - q_i^c) \geq 0 \qquad \forall t = 1, ..., \mathcal{T}, \ \forall c = 1, ..., \mathcal{C} \tag{3}$$
$$x_t^c + q_t^c \leq M_t y_t^c \qquad \forall t = 1, ..., \mathcal{T}, \ \forall c = 1, ..., \mathcal{C} \tag{4}$$

Constraint (1) is the flow balance constraint for the final-item level. Similarly, we use constraint (2) for the component level. Constraint (3) ensures that the returns inventory level is non-negative. Finally, constraint (4) is the setup constraint, where a joint setup cost of K^c is incurred if setup takes place for component c on a given time period.

2 Uncertainty and Robust Problem

As the lot-sizing problem attempts to make decisions for the future, there are naturally inherent uncertainties with regards to input parameters. Even when a make-to-order production system with known demand quantities is considered, the uncertainties involved in returned products (whether their quantities or qualities) will remain intact. In order to address such uncertainties, we consider the robust optimization framework, which does not require probability distributions but rather employs so-called "uncertainty sets" for parameters. Since the seminal work of [3] suitable for problems with discrete decision variables, there have been significant advances in the domain of robust optimization, we refer the interested reader to the extensive review of [2].

Here we introduce the case where returns belong to a budgeted polytope. We consider the following:

$$Z(\Gamma) := \{z \in [0,1]^T : \sum_{i=1}^{t} z_i \leq \Gamma_t, \ \forall t = 1, \ldots, \mathcal{T}\}$$

$$U(\Gamma) := \{R \in \mathbb{R}_+^T : R_t = \overline{R_t} + \hat{R}_t z_t, z \in Z(\Gamma)\}$$

Let the following define the extreme points in the set $U(\Gamma)$. Then, we are seeking a solution that remains feasible for the following set of extreme points:

$$U(\Gamma) := Conv(\{R^1, R^2, \ldots, R^J\})$$

We next present the following robust formulation of the problem:

$$\min \sum_{t=1}^{\mathcal{T}} \Big(m^0 x_t^0 + h^0 I_t^0 + \sum_{c=1}^{\mathcal{C}} (h^c I_t^c$$
$$+ w^c Q_t^c + m^c x_t^c + r^c q_t^c + K^c y_t^c) \Big)$$

$$x_t^0 + I_{t-1}^0 = I_t^0 + D_t \qquad \forall t = 1, ..., \mathcal{T},\ \forall c = 1, ..., \mathcal{C} \qquad (5)$$

$$x_t^c + q_t^c + I_{t-1}^c = I_t^c + x_t^0 \qquad \forall t = 1, ..., \mathcal{T},\ \forall c = 1, ..., \mathcal{C} \qquad (6)$$

$$Q_t^c \geq \sum_{i=1}^{t} (R_i^s - q_i^c) \qquad \forall t = 1, ..., \mathcal{T},\ \forall c = 1, ..., \mathcal{C},\ \forall s = 1, ..., J \qquad (7)$$

$$x_t^c + q_t^c \leq M_t y_t^c \qquad \forall t = 1, ..., \mathcal{T},\ \forall c = 1, ..., \mathcal{C} \qquad (8)$$

$$Q, x, q \geq 0, y \in \{0, 1\}^{\mathcal{T} \times \mathcal{C}} \qquad (9)$$

This characterisation of uncertainty sets using the convex hull of their extreme points allows us a decomposition approach such as of [1], where a restricted version of the robust problem with only a subset of extreme points, called "Decision Maker's Problem" (DMP), is solved iteratively with an "Adverserial Problem" (AP), which generates extreme points for DMP. We will present preliminary results and a number of observations and insights in this talk.

References

[1] Ö.N. Attila, A. Agra, K. Akartunalı, and A. Arulselvan, A decomposition algorithm for robust lot sizing problem with remanufacturing option, International Conference on Computational Science and Its Applications (ICCSA 2017), Lecture Notes in Computer Science (LNCS), Vol. 10405, Springer, pp.684-695 (2017)

[2] D. Bertsimas, D.B. Brown, and C. Caramanis, Theory and applications of robust optimization. SIAM Review, 53(3), 464-501 (2011)

[3] D. Bertsimas, and M. Sim. The price of robustness. Operations Research, 52(1), 35-53 (2004)

[4] M.J. Retel Helmrich, R. Jans, W. van den Heuvel, and A.P. Wagelmans, Economic lotsizing with remanufacturing: complexity and efficient formulations, IIE Transactions, 46(1), 67-86 (2014)

[5] S.A. Syed Ali, M. Doostmohammadi, K. Akartunalı, and R. van der Meer, A Theoretical and Computational Analysis of Lot-sizing in Remanufacturing with Separate Setups, International Journal of Production Economics, 203:276–285 (2018).

A bi-level single-item lot-sizing problem for industrial symbiosis

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Abstract

Motivated by the waste accumulation and the rarefaction of natural resources, legislation and habits are evolving in order to convert unavoidable production residues into useful and high added-value products. In this context, we propose and formalize a new single-item lot-sizing problem that considers the industrial symbiosis between two production units. During the production process of a main product, a production residue is generated by the first production unit. This residue can be considered either as a by-product to be used as raw materials by the second production unit, or as a waste to be disposed of. This residue can be either stored with a limited capacity or unstorable. The second production unit can use this by-product as raw materials or purchase raw materials from an external supplier. In this work, we prove that the studied problem is NP-Hard. We develop a solution method based on Lagrangian decomposition, jointly with a Lagrangian heuristic to provide an upper bound. We conducted several experiments to show the performance and the limits of our approach, and to derive managerial insights.

1 Introduction

Industrial symbiosis represents all forms of binding traditionally separate industrial entities in a joint production system, suitable for providing, sharing and reusing resources to create mutual added value [3]. One of the most common beneficial form of symbiotic industrial production is the process by which, by-products of one production unit become the raw materials for another, as illustrated in Figure 1.

Application of the industrial symbiosis allows production residues to be used in a more sustainable way and contributes to the development of a circular economy.

Joint production systems and their applications have been widely studied in the related literature: semi-conductor fabrication [1], float glass manufacturing [8], food industry [5], etc. As particular case of joint production systems, the industrial symbiosis in production planning is a hot topic of major importance for the transition towards a circular economy. To the best of our knowledge, only Sridhar et al. [7] studied a generic non-linear production problem dealing with by-products.

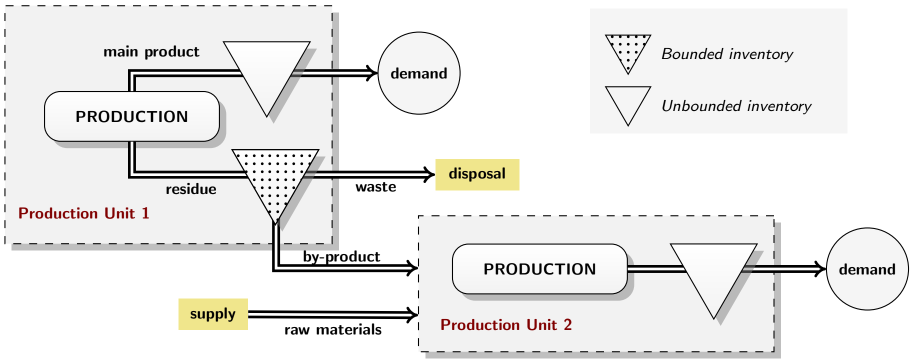

Figure 1: Process flow diagram of the ULS-IS-BI problem

Given the circular economy concerns and industrial needs, we investigate a single-item lot-sizing problem for the industrial symbiosis between two production units (PU1 and PU2) with disposal and purchasing options. This form of production can be related to the well-studied bi-level production planning problems, where a production entity has to determine its production plan at each level [2, 6, 9]. As shown in Figure 1, these two levels (i.e. production units) are linked by the by-product created by the first production unit and used as raw materials by the second one.

The rest of this paper is structured as follows. After the problem statement given in Section 2, a solution approach based on the Lagrangian decomposition is given in Section 3. Section 4 explains the instance generation and the methodology used to evaluate the competitiveness of the proposed approach and derive managerial insights.

2 Problem statement

Consider a bi-level single-item lot-sizing problem including the industrial symbiosis between two production units PU1 and PU2. These production units may produce two different products in order to meet their own deterministic demand. We coin our problem ULS-IS.

PU1 generates a by-product at the same time as the main product. By definition, by-products are lawful undesirable production outputs, whose further use is economically and environmentally sustainable. Accordingly, the generated by-product can be either disposed of, or sent to PU2 for a unitary cost lower than the unitary disposal cost.

At the second level, production unit PU2 can choose either to use the by-product generated by PU1 as raw materials or to order at an external supplier. The unitary cost of buying the by-product is lower than the purchasing cost of the raw materials.

In addition to the aforementioned costs, the classical costs related to the lot-sizing problems are included in the objective function for each production unit, namely: fixed setup costs, unitary production costs and unitary inventory holding cost of the main product. In the case where the by-product can be stored with a limited capacity, a unitary holding cost is added in the objective function. The objective function aims at minimizing the sum of all costs of both PU1 and PU2 occurring over the entire planning horizon.

By reduction from the capacitated lot-sizing problem [4], Theorem 1 holds.

Theorem 1. *The ULS-IS problem is NP-Hard regardless the storability of the by-product.*

3 Lagrangian decomposition algorithm

Given the complexity of the ULS-IS problem (see Theorem 1), we propose a Lagrangian decomposition algorithm to solve the problem under study. To do this, variables corresponding to the by-product flows are duplicated. This operation leads to the problem decomposition into two subproblems SP1 and SP2. In the case where the generated by-product cannot be store, these two subproblems can be solved by using dynamic programming algorithms running in $O(T \log T)$.

To construct feasible solutions, we propose the following Lagrangian heuristic, which operates in two phases:

- **Smoothing phase:** Given the optimal solutions of subproblems SP1 and SP2, this phase constructs a feasible solution in the following way: **(i)** if there is production in PU1 and PU2, then transportation takes place between PU1 and PU2, and the residual quantity is disposed of or purchased, otherwise **(ii)** the production process only in PU1 implies the disposal of the generated by-product and, respectively, the production process only in PU2 necessitates the raw materials purchasing from an external supplier.

- **Improvement phase:** This phase consists in moving production quantities from a period to another one with a view to reducing the disposal and purchasing quantities. The moves can be done forward or backward.

4 Experimental study

Numerical experiments are conducted on a large set of heterogeneous instances. To generate these instances, several critical parameters are identified, namely: **(i)** the ratio linking the inventory holding costs of PU1 and PU2, **(ii)** the setup cost-holding cost ratio, **(iii)** the demands of PU1 and PU2, **(iv)** the size of the planning horizon. Each of these parameters takes its values in a set of cardinality equal to 3. For each combination of these parameters and for each of 4 different sizes of the planning horizon, 10 instances are generated.

To analyze the competitiveness of the proposed solution method based on the Lagrangian decomposition to solve the ULS-IS problem, we carried out the comparison between:

- The straightforward and facility location models solved by using `IBM ILOG CPLEX` solver,
- Different settings of the Lagrangian decomposition algorithm, based on the Lagrangian heuristic given in Section 3 with and without a multi-start procedure.

In order to reveal the managerial and economic implications of the coordination between production units PU1 and PU2, the comparison between the centralized and decentralized versions of the ULS-IS problem has been also conducted.

References

[1] G. R. Bitran and S. M. Gilbert. Co-production processes with random yields in the semiconductor industry. *Operations Research*, 42(3):476–491, 1994.

[2] N. Brahimi, N. Absi, S. Dauzère-Pérès, and S. Kedad-Sidhoum. Models and lagrangian heuristics for a two-level lot-sizing problem with bounded inventory. *OR spectrum*, 37(4):983–1006, 2015.

[3] Marian R Chertow. Industrial symbiosis: literature and taxonomy. *Annual review of energy and the environment*, 25(1):313–337, 2000.

[4] M. Florian and M. Klein. Deterministic production planning with concave costs and capacity constraints. *Management Science*, 18(1):12–20, 1971.

[5] D. Lee and M. H. Tongarlak. Converting retail food waste into by-product. *European Journal of Operational Research*, 257(3):944–956, 2017.

[6] S-L Phouratsamay, S. Kedad-Sidhoum, and F. Pascual. Two-level lot-sizing with inventory bounds. *Discrete Optimization*, 2018.

[7] S. Sridhar, J. Linderoth, and J. Luedtke. Models and solution techniques for production planning problems with increasing byproducts. *Journal of Global Optimization*, 59(2-3):597–631, 2014.

[8] Z. C. Taşkın and A. T. Ünal. Tactical level planning in float glass manufacturing with co-production, random yields and substitutable products. *European Journal of Operational Research*, 199(1):252–261, 2009.

[9] M. Van Vyve, L. A. Wolsey, and H. Yaman. Relaxations for two-level multi-item lot-sizing problems. *Mathematical Programming*, 146(1-2):495–523, 2014.

Perishable items

Lot Sizing for a Perishable Item

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Abstract

Motivated by the inventory management for blood products, we address the uncapacitated lot sizing problem for a perishable item that has a deterministic and fixed life time. We first study the problem with deterministic demand and derive strong valid inequalities. Then we consider uncertain demand in a multistage stochastic setting. As the size of the model grows exponentially in the number of periods, we implement a scenario-wise decomposition method to obtain lower and upper bounds.

1 Introduction

This study is on extending the lot sizing problem to consider the perishability of products. Perishable inventory refers to products that lose their value and quality over time and that are disposed after a certain time. Items can be perishable because of the rapid changes and developments in the technology as in the case of smartphones. On the other hand, they might deteriorate like dairy products, fruits and vegetables [1]. Other examples of perishable items are newspapers, flowers, concert and game tickets.

Deteriorating inventory is also an important concept in health-care systems. Blood platelet is a highly perishable item with a very short lifetime and unpredictable supply and demand [2]. Vaccines are also perishable products. They are stored in vials and when a vial is opened vaccines can be used only in a safe-for-use time [3].

In the presence of deteriorating inventory minimizing wastage is an important aspect of the problem. On the other hand, satisfying the demand on time and minimizing the number of shortages are also crucial. In this study, we consider the lot sizing problem with perishable items and we take into account both the possibility of wastage and shortage.

2 Literature Review

There are many studies on perishable inventory. Nahmias [4] reviews ordering policies for perishable inventory problems. He examines problems for both single and multiple products with deterministic and stochastic demands. He categorizes the deteriorating inventory problem into two categories; fixed life perishability and random lifetime. Fixed life perishability refers to problems where products have fixed time of expiration, whereas random lifetime refers to problems in which items may decay at any time. In this study, we assume fixed life perishability.

Hsu [5] studies the uncapacitated lot sizing problem with perishable products where inventory decays with a deterioration rate at each period and the unit holding cost is age-dependent. The author represents the problem as a network flow problem with flow loss and comes up with a dynamic programming algorithm to solve it. He assumes that items are produced, not ordered, so they enter the inventory in the production period when they are one period old. In our study, we assume that products are purchased. So, they can be at any age when they enter the inventory.

Sargut and Işık [6] consider a similar problem as Hsu [5]. They extend the problem by adding capacity constraints. They explore properties of the optimal solutions and present a dynamic programming based heuristic for their problem.

Önal et al. [7] also study the problem of lot sizing with perishable items. They assume that items have deterministic deterioration rate and examine different mechanisms to allocate products to the customers. They show that the uncapacitated problem can be solved in polynomial time with all allocation mechanism whereas the capacitated problem is NP-Hard for some mechanism.

Önal [8] considers lot sizing with perishable items in a two-level supply chain. He assumes that customers buy the product with the longest shelf-life and formulate algorithms to solve the problem.

For further studies, we refer the reader to the literature review of Janssen et al. [9] as well as the book of Nahmias [10] on perishable inventory systems.

3 Contributions of the present study

We propose two formulations for the deterministic problem. The first formulation is a natural formulation based on production, stock and setup variables whereas the second formulation is a facility location formulation. Unlike the classical lot sizing problem, when items have a fixed shelf life, the facility location formulation contains fewer variables than the natural formulation. Indeed, instances that cannot be solved to optimality in half an hour with the natural formulation can be solved within a few seconds using this facility location formulation. However, this formulation cannot be extended to include age-related constraints, which are rather common in practice. For this reason, we strengthen the natural formulation using valid inequalities and use these inequalities in solving the variant of the problem with additional constraints on the composition of orders in terms of age. Computational experiments show that the proposed valid inequalities are very effective in solving the problem and its variant.

In the second part of our study, we consider a multi stage stochastic setting with uncertain demand. We first test the effectiveness of our valid inequalities in solving the stochastic problem. Then for larger sizes, we decompose the problem based on a grouping of scenarios and use the group subproblems to obtain lower and upper bounds. We report the results of our computational experiments where we test the effectiveness of different grouping techniques on the quality of the bounds and the time to compute them.

References

[1] S. K. Goyal and B. C. Giri, Recent trends in modeling of deteriorating inventory, European Journal of Operational Research, 134, 1-16, (2001)

[2] R. Haijema, J. van der Wal, N. M. van Dijk, Blood platelet production: Optimization by dynamic programming and simulation, Computers & Operations Research, 34, 760-779, (2007)

[3] A. Dhamodharan and R. A. Proano, Determining the optimal vaccine vial size in developing countries: a monte carlo simulation approach, Health Care Management Science, 15, 188-196, (2012)

[4] S. Nahmias, Perishable inventory theory: A review, Mathematical Programming, 4, 601-796, (1982)

[5] V. N. Hsu, Dynamic economic lot size model with perishable inventory, Management Science, 46, 1159-1169, (2000)

[6] F. Z. Sargut and G. Işık, Dynamic economic lot size model with perishable inventory and capacity constraints, Applied Mathematical Modelling, 48, 806-820, (2017)

[7] M. Önal, H. Romeijn, A. Sapra, and W. van den Heuvel, The economic lot sizing problem with perishable items and consumption order preference, European Journal of Operational Research, 244, 881-891, (2015)

[8] M. Önal, The two-level economic lot sizing problem with perishable items, Operations Research Letters, 44, 403-408, (2016)

[9] L. Janssen, T. Claus, and J. Sauer, Literature review of deteriorating inventory models by key topics from 2012 to 2015, International Journal of Production Economics, 182, 86-112, (2016)

[10] S. Nahmias, Perishable Inventory Systems. Springer US, (2011)

A new inventory issuing rule based on the expected remaining lifespan for a randomly perishable product

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Abstract

A perishable product such as many fresh produces has a limited lifespan. In this research, it is assumed that each product unit can perish after a random amount of time independently of the other units. The product units are checked at the end of every period and the perished units are disposed off before the start of the next period. The objective is to minimize the expected total cost due to setups, production, inventory holding, shortage and the disposal of perished units. In this environment, it is possible that a more recently-produced unit perishes sooner than an older unit. By formulating the problem as a stochastic dynamic program, some numerical examples show that the FIFO (First-In-First-Out) rule does not always represent the optimal inventory issuing policy. Hence, a new inventory issuing rule is proposed in this research based on the expected remaining lifespan of the units. This is the optimal policy in all of the test instances and dominates the FIFO rule under the given circumstances.

1 Introduction

When a product is perishable, it has a limited lifespan which can be either a deterministic value or can be a bounded random variable. It is argued in the literature that due to various perturbations in the production and handling conditions such as changes in the temperature, humidity, packaging, and contact to air and sunlight, the lifespan of each product unit can be regarded as a random variable [1].

An operations manager categorizes the product units in the warehouse based on their ages, while tracing the age categories can result in a more efficient utilization of inventories and consequently in a better cost management, especially by avoiding inventory perishability as much as possible. In this environment, the operations manager uses certain rules to decide simultaneously about (i) the lot sizes in each planning period and (ii) the order of the age categories that the inventories should be dispatched to the customers. The latter decision rule is known in the literature as "inventory issuing rule". The main inventory issuing rules in the literature are: FIFO (First-In, First-Out) and LIFO (Last-In, First-Out).

There are two assumptions among many scholars and practitioners in the perishable inventory society. First, it is assumed that those units which are produced earlier perish sooner ([5] page 888). Second, it is assumed that the percent of the perished units in each period (i.e., the probability of the perishability in a period) increases as the units get older such that in a given period, those units which were produced earlier are supposed to perish faster (because of having a higher probability of perishability) ([2] page 1161, [5] page 882). Based on this viewpoint on perishability, FIFO is commonly supposed to be the optimal inventory issuing rule for minimizing the total costs of a producer, especially when the lifespan is a fixed value or when a fraction of units are perished over periods. For example, Janssen et al. (2016) briefly discuss in their literature review about the recent papers that are related to inventory issuing rules ([3] page 95). They claim that FIFO rule is always optimal by citing to the early literature review Nahmias (1982) [4].

In this paper, it is shown that both of the above assumptions are not necessarily correct for the case of independent perishability of the units in a periodic inventory control system, where the lifespan of a unit is a bounded random variable which follows a general (discrete) probability distribution function. And consequently, it can be shown (numerically) that FIFO is non-optimal when minimizing the total costs.

In more details, the problem is formulated in the framework of a stochastic dynamic program. Then, a new inventory issuing rule is proposed, which is based on the Expected Remaining Lifespan (ERL) of a perishable product with a general (discrete) random lifespan. Then, computational experiments are provided to assess the superiority of the proposed issuing rule over FIFO rule.

References

[1] Amorim, P., Meyr, H., Almeder, C., Almada-Lobo, B., Managing perishability in production-distribution planning: A discussion and review, Flexible Services and Manufacturing Journal, 25, 389-413 (2013)

[2] Hsu, V.N., Dynamic economic lot size model with perishable inventory. Management Science, 46(8), 1159-1169 (2000)

[3] Janssen, L., Claus, T., Sauer, J., Literature review of deteriorating inventory models by key topics from 2012 to 2015. International Journal of Production Economics, 182, 86-112 (2016)

[4] Nahmias, S., Perishable inventory theory: A review. Operations Research, 30(4), 680-708 (1982)

[5] Önal, M., Romeijn, H.E., Sapra, A., van den Heuvel, W., The economic lot-sizing problem with perishable items and consumption order preference. European Journal of Operational Research, 244, 881-891 (2015)

Decomposition approaches

Benders Decomposition for a Stochastic Three-level Lot Sizing and Replenishment Problem with a Distribution Structure

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Abstract

We address a stochastic and uncapacitated three-level lot sizing and replenishment problem with a distribution structure (3LSPD). We consider one production plant that produces one type of item over a discrete and finite planning horizon. The items produced are transported to warehouses and then to retailers using direct shipments. The objective is to minimize the operational costs. We use a Benders decomposition approach and develop a Benders-based branch-and-cut algorithm to solve the problem. We take advantage of the substructures identified in the decomposition and design efficient procedures to solve the subproblems obtained. We propose computational enhancements. In particular, we develop an algorithm to derive Pareto-optimal cuts without solving an auxiliary problem. We perform computational experiments to assess the performance of our decomposition approach and see the impact of our enhancements. The Benders-based branch-and-cut algorithm we propose clearly outperforms CPLEX.

1 Introduction

Supply chain planning often takes as a starting point forecasts of the future demand, yielding deterministic values for the demand to satisfy. However, if these forecasts are misleading, it would result in costly decisions for the companies. Taking uncertainty into account can be very beneficial but also increases the difficulty of the operational problems to be solved.

We address here an integrated three-level lot sizing and replenishment problem with a distribution structure, in a two-stage decision process (2S-3LSPD). We consider the supply chain of a general manufacturing company which comprises one production plant, several warehouses and multiple retailers. All retailers are linked to a unique warehouse and each warehouse is linked to the production plant, resulting in a distribution structure. The flow of goods ordered by the retailers is entirely fixed: the product goes from the production plant (where it is produced), to a warehouse (where it is stored) and finally to a retailer (where it is sold). The objective of the problem is to determine, for each time period, the production quantities at the plant and the flow of goods between the facilities to minimize the operational costs of the whole system.

The retailers face a stochastic and dynamic demand for a unique item. The distribution of the demand for each retailer is assumed to be known, and uncertainty is taken into account through the use of demand scenarios. In our two-stage decision process, the demands of each retailer for the entire time horizon are revealed once the first stage decisions are made. These first stage decisions correspond to the production and ordering setup decisions for each facility and each time period. The second stage involves production, replenishment and inventory decisions, which are made once the demands are revealed. This separation between the first and second stage decisions is exactly the static-dynamic uncertainty strategy first proposed by Bookinder and Tan [3] for the stochastic single item LSP.

The motivation to use Benders decomposition on the 2S-3LSPD is to efficiently solve this problem by exploiting the different substructures that appear in the multi-commodity formulation presented in Section 2.

2 Benders decomposition

To apply Benders decomposition, we start from the multi-commodity formulation MC, adapted from Melo and Wolsey [5]. We denote by δ_{kt} the Kronecker delta that takes the value 1 if $k = t$ and 0 otherwise. Let Ω be the finite set of demand scenarios and let p_ω be the probability of scenario $\omega \in \Omega$. We denote by $d_{rt\omega}$ the demand of retailer r in period t under scenario ω. Let R, T and F be the set of retailers, time periods and facilities, respectively. Let $\delta_w(r)$ be the warehouse linked to retailer r. Let y_{it} be a boolean setup variable taking value 1 if and only if there is production or an order placed by facility i in period t. If we denote by $x^{lr}_{kt\omega}$ the quantities produced/ordered in level l in period k to satisfy $d_{rt\omega}$ and by $\sigma^{lr}_{kt\omega}$ the stock in level l at the end of period k to satisfy $d_{rt\omega}$, the MC formulation is as follows:

$$\text{Min} \sum_{t\in T} \left(\sum_{i\in F} sc_{it} y_{it} + \sum_{\omega\in\Omega} p_\omega \sum_{r\in R} \sum_{k\leq t} \left(hc_{pk}\sigma^{0r}_{kt\omega} + hc_{\delta_w(r)k}\sigma^{1r}_{kt\omega} + hc_{rk}\sigma^{2r}_{kt\omega} \right) \right) \quad (1)$$

$$\sigma^{0r}_{k-1,t,\omega} + x^{0r}_{kt\omega} = x^{1r}_{kt\omega} + \sigma^{0r}_{kt\omega} \quad \forall\, t \in T, k \leq t \in T, r \in R, \omega \in \Omega \quad (2)$$

$$\sigma^{1r}_{k-1,t,\omega} + x^{1r}_{kt\omega} = x^{2r}_{kt\omega} + \sigma^{1r}_{kt\omega} \quad \forall\, t \in T, k \leq t \in T, r \in R, \omega \in \Omega \quad (3)$$

$$\sigma^{2r}_{k-1,t,\omega} + x^{2r}_{kt\omega} = \delta_{kt} d_{rt\omega} + (1-\delta_{kt})\sigma^{2r}_{kt\omega} \quad \forall\, t \in T, k \leq t \in T, r \in R, \omega \in \Omega \quad (4)$$

$$x^{0r}_{kt\omega} \leq d_{rt\omega} y_{pk} \quad \forall\, t \in T, k \leq t \in T, r \in R, \omega \in \Omega \quad (5)$$

$$x^{1r}_{kt\omega} \leq d_{rt\omega} y_{\delta_w(r)k} \quad \forall\, t \in T, k \leq t \in T, r \in R, \omega \in \Omega \quad (6)$$

$$x^{2r}_{kt\omega} \leq d_{rt\omega} y_{rk} \quad \forall\, t \in T, k \leq t \in T, r \in R, \omega \in \Omega \quad (7)$$

$$x^{0r}_{kt\omega}, x^{1r}_{kt\omega}, x^{2r}_{kt\omega}, \sigma^{0r}_{kt\omega}, \sigma^{1r}_{kt\omega}, \sigma^{2r}_{kt\omega} \geq 0 \quad \forall\, t \in T, k \leq t \in T, r \in R, \omega \in \Omega \quad (8)$$

$$y_{it} \in \{0,1\} \quad \forall t \in T, i \in F. \quad (9)$$

The objective function (1) minimizes the sum of the setup costs and of the expected inventory holding costs at each facility for each time period, sc_{it} and hc_{it}, respectively. Constraints (2)-(4) represent the inventory balance equations for each commodity $d_{rt\omega}$ at the different facilities. Constraints (5)-(7) are the setup forcing constraints for the different facilities.

When the binary setup decisions are fixed, we obtain a continuous linear problem which can be solved efficiently. This framework is well suited for the use of Benders decomposition. The original idea of Benders decomposition is to partition the complete problem into two smaller problems, namely the master problem and the subproblem. The master problem is a simplified version of the original problem where only some variables have been kept, along with the constraints in which they are the only ones to appear. The master problem also contains an artificial variable representing a lower bound on the cost of the subproblem. The subproblem is exactly the original problem without the constraints that have been kept in the master problem. In this primal subproblem, the variables present in the master problem are fixed to given values. In our case, we keep the binary setup variables y_{it} in the master problem. The production and inventory variables x and σ are present in the primal subproblem, along with constraints (2)-(8). The primal subproblem can be decomposed into $|R||T||\Omega|$ shortest path subproblems, one for each commodity $d_{rt\omega}$. We solve all these shortest paths problems with Dijkstra's algorithm and derive the dual solution of each subproblem by solving additional shortest paths problems. Indeed, the solution of the dual subproblem of each commodity leads to the generation of a so-called Benders cut which is in our case an optimality cut since the dual subproblems are feasible.

The optimality cuts can be generated from any solution and not only from an optimal integer solution to the master problem. Therefore, we solve the 2S-3LSPD in a standard branch-and-cut (B&C) framework with the use of callbacks. At each node of the B&C tree for the Benders reformulation, the dual subproblems are solved, thus generating optimality cuts. Indeed, each dual subproblem acts as a separation problem to generate cuts.

Even when implemented using callbacks, several features slow down the Benders-based B&C. We implemented several ideas to speed up the solution process. The first three ideas aim to improve the lower bound during the search process. First, we add lower bound lifting inequalities. These inequalities give a better approximation of the cost of the primal subproblem, given a set of binary setup values. Here, we compute the minimal holding costs that will be incurred, given a feasible integer solution to the master problem. Second, we add optimality cuts at the root node, based on fractional solution. In that case, the primal subproblems to be solved are minimum cost flow problems. Finally, we take advantage of the integrality requirements on the master variables when deriving Benders cuts and develop valid inequalities which lead to better LP relaxation values for the master problem. These inequalities come from the observation of Bodur and Luedtke [2]. The fourth idea deals with the choice of a good optimality cut. To tackle this issue, it is possible to solve an auxiliary problem which returns, among the optimal solutions to the dual subproblem, the best one in terms of dominance of the cut generated. This cut is called the Pareto-optimal cut. We developed a specialized algorithm, based on the idea of Magnanti et al. [4] to derive such cuts without the use of a general purpose solver. Finally, the fifth idea explores the different ways of aggregating cuts from the different subproblems.

The numerical experiments performed on numerous instances with these enhancements show the superiority of our approach compared to CPLEX.

References

[1] Y. Adulyasak, J.-F. Cordeau and R. Jans, Benders Decomposition for Production Routing Under Demand Uncertainty, Operations Research, 63(4), 851-867 (2015)

[2] M. Bodur and J. R. Luedtke, Mixed-Integer Rounding Enhanced Benders Decomposition for Multiclass Service-System Staffing and Scheduling with Arrival Rate Uncertainty, Management Science, 63(7), 2073-2091 (2016)

[3] J. H. Bookbinder and J.-Y. Tan, Strategies for the Probabilistic Lot Sizing Problem with Service-Level Constraints, Management Science, 34(9), 1096-1108 (1988)

[4] T. L. Magnanti, P. Mireault, and R. T. Wong, Tailoring Benders Decomposition for Uncapacitated Network Design, Mathematical Programming Study 26(10), 112-154 (1986)

[5] R. A. Melo and L. A. Wolsey, Uncapacitated two-level lot sizing, Operations Research Letters, 38(4), 241-245 (2010)

Parallelized approaches to solve the capacitated lot-sizing problem with lost sales and setup times

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Abstract

Multi-item lot-sizing problems faced in industrial contexts are often too large to be directly solved by standard optimization solvers. To this purpose, parallelization approaches are very useful to transform the single resolution of a complex problem into the resolution of smaller independent sub-problems, leading to much smaller computational times. We first motivate and propose new practically relevant instances for the Capacitated Lot-Sizing Problem (CLSP) with lost sales, setup times and target ending inventory. Two ways of parallelizing the problem are then introduced. A first approach is based on a Lagrangian relaxation of the item binding capacity constraints. We show that the problem linked to each item can be solved in polynomial time. A second approach uses a parallelized version of Relax-And-Fix and Fix-and-Optimize heuristics where, instead of optimizing the time intervals in chronological order at each iteration, different time intervals are optimized in parallel. The best interval is picked according to various strategies and is fixed for the following iterations.

1 Industrial context and motivations

A classical dynamic lot-sizing problem consists in planning quantities to be produced in each period of a finite horizon, discretized in periods, to satisfy demands, while optimizing the trade-off between setup and holding costs. The well-known Capacitated Lot-Sizing Problem (CLSP) considers multiple products and capacity constraints. For most instances of the literature, solutions of the CLSP have no ending inventory, because there are no incentives to store products at the end of the planning horizon. However, as shown for instance in the survey of [5], real-life planning processes are often conducted in a rolling horizon. Hence, to be practically relevant, plans should keep enough inventory of some products at the end of the planning horizon, and should have inventories of some products at the beginning of the planning horizon.

In this work, we focus on a specific and well-know lot-sizing problem: The CLSP with setup times. Trigeiro et al. [6] created a set of instances for this problem, that are commonly used as benchmarks or to create benchmarks to validate solution approaches ([1], [2], [3], [4]). Yet, as discussed earlier, these instances are not realistic because they consider neither initial nor ending inventories, which leads to start-of-horizon and end-of-horizon effects (see Figure 1). We are also allowing lost sales as the satisfaction of all demands cannot be guaranteed in many industrial contexts.

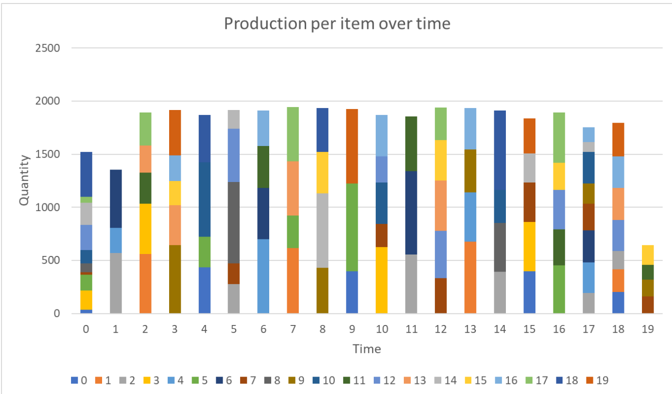

Figure 1: Optimal solution for an instance of Trigeiro et al. [6]

2 Creation of new instances

Following the characteristics of the instances of Trigeiro et al. [6], our assumption is that the demand variability over time and between products is small. We propose new instances where initial inventories and maximum ending inventories are added for each product, as well as a common target ending inventory for all products. The objective is to create instances that make sense in a rolling horizon setting. Indeed, it is unrealistic to define a target ending inventory for each single product in a given period. Using the information on cost components and the capacity, a MIP is solved to create new consistent data to complement and modify the instances of Trigeiro et al. [6]. The MIP still guarantees that there exists a feasible plan without lost sales.

3 Parallelization approaches for the CLSP with setup times, lost sales and target ending inventory

In this section, we introduce parallelization approaches for the CLSP with setup times, lost sales and target ending inventory. The latter characteristic is the main difference with previous research on related problems. First, a Lagrangian relaxation heuristic (used for instance in [3]) is considered, which can "naturally" be parallelized. Then, different ways to parallelize Relax-and-Fix and the Fix-and-Optimize heuristics (used for instance in [1]) are proposed.

3.1 Lagrangian relaxation heuristic

As in [6] and [3], the capacity constraints are relaxed using Lagrangian multipliers. When the maximum ending inventory is the same for each product, we show that the multi-item uncapacitated lot-sizing problem with target ending inventory and lost sales is polynomially solvable by decomposing into several uncapacitated single-item lot-sizing problems with ending inventory. At each iteration of the Lagrangian heuristic, a reconstruction method, following the same principle than in [6], is used to build a feasible solution. A first production plan is thus determined, that can be improved by various methods (for instance a parallelized Fix-and-Optimize heuristic).

3.2 Parallelized Relax-and-Fix and Fix-and-Optimize heuristics

Our approach works in two main steps. An initial solution is first obtained using a Relax-and-Fix heuristic. This solution is then improved with a Fix-And-Optimize heuristic.

In a classical Relax-and-Fix heuristic, sub-problems are optimized chronologically for each time interval. Instead, in our parallelized approach, sub-problems are optimized in parallel in order to select the best time interval to fix in the following iterations. The setups are then fixed for the "best" sub-problem at each iteration in an iterative process (see illustration in Figure 2).

To evaluate the quality of each partial solution and define the best interval, several score functions are defined. One of the score functions consists in reconstructing a feasible solution from the partially optimized solution. We also consider different ways to define the time intervals, leading to different resolutions strategies. The same approach can be applied to the Fix-and-Optimize heuristic, where the Boolean variables outside of the optimized time interval are fixed to a given value rather than relaxed.

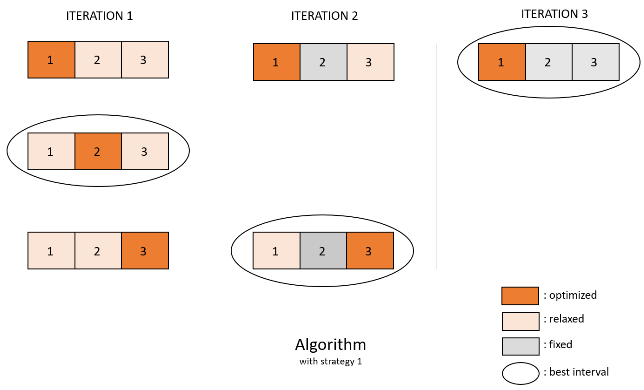

Figure 2: Illustration of parallelized Relax-and-Fix heuristic

3.3 Numerical results

Computational experiments were performed on the modified instances of Trigeiro et al. [6]. In the workshop, we will present and discuss the results of our parallelized approaches compared to the standard solver IBM ILOG CPLEX and the "classical" Relax-And-Fix and Lagrangian relaxation heuristics. The different strategies of our Parallelized Relax-and-Fix and Fix-and-Optimize heuristics will also be compared and analyzed.

References

[1] Absi, N., Kedad-Sidhoum, S. *MIP-based heuristics for multi-item capacitated lot-sizing problem with setup times and shortage costs.* RAIRO Operations Research, 41, 2007, 171192.

[2] Absi, N., Kedad-Sidhoum, S. *The multi-item capacitated lot-sizing problem with setup times and shortage costs.* European Journal of Operational Research, 185 (3), 2008, 1351-1374.

[3] Absi, N., Detienne, B., Dauzère-Pérès, S. *Heuristics for the multi-item capacitated lot-sizing problem with lost sales.* Computers & Operations Research 40 (1), 2013, 264-272.

[4] De Araujo, S.A., De Reyck, B., Degraeve, Z., Fragkos, I., Jans, R. *Period Decompositions for the Capacitated Lot Sizing Problem with Setup Times.* INFORMS Journal on Computing, 27(3), 2015, 431448.

[5] Jans, R., Degraeve, Z. *Modeling industrial lot sizing problems: a review.* International Journal of Production Research, 46(6), 2008, 1619-1643.

[6] Trigeiro, W.W., Thomas, L.J., McClain, J.O. *Capacitated lot-sizing with setup times.* Management science, 35(3), 1989, 353-366.

Stochastic lot sizing

Multi-stage stochastic capacitated lot sizing with different backlog control mechanisms

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Abstract

This work focuses on the single-level, multi-product dynamic lot sizing problem subject to capacity constraints with stochastic demand, where backlogs are controlled by different service constraint mechanisms. A customizable multi-stage production planning approach in rolling horizons is introduced, where decisions for each period are taken in two steps: The setup pattern is determined some periods in advance based on distribution parameters of demand. The production quantities, however, are still adjustable, factoring in previous demand observations in the final decision. The cost function considers additional costs for short-termed changes of the setup pattern. Adaptations are only made if the cost reduction justifies additional system nervousness. Model extensions allow risk averse optimization of the conditional value at risk and robust optimization for problem instances with limited overtime or late demand realization.

1 Introduction and problem description

The presented planning approach aims at dealing with the problem of finding optimal decisions on both the setup pattern and the production quantities for the single level, multi period, multi product lot sizing problem with a single production resource and scarce production capacity. Backlogs are allowed and controlled by customer oriented service constraints limiting the mean waiting time of demand fulfilment. Setups on short notice are more expensive than those scheduled long-termed.

A multi-stage approach is introduced, which takes prior demand realizations into account when deciding on the production quantities for subsequent periods. With this multi-stage approach meeting a given service level can be guaranteed, which cannot be achieved with static approaches. It even allows stochastic lot-sizing without the need to allow backlogs, which makes this approach especially valuable for industries where backlogs cannot be accepted, such as in supply networks with just-in-time production. First results show that the adaptation to demand realizations also results in lower total costs.

2 The responsive multi-stage algorithm

Within the proposed multi-stage planning algorithm production plans are determined in rolling horizons. Figure 1 shows an examplary production plan determined with this approach in the current period T2. For each period, decisions are taken in two steps. As a first step, an initial setup pattern for the respective period is determined some periods in advance based on distribution parameters of demand, before demand realizations for that period are known. This ensures a certain level of predictability and stability of the production plans.

At a later stage the production plan is reoptimized to determine production quantities. The plan is adapted to demand realizations, factoring in all available demand information and updated demand forecasts. Already fixed variables are retained. However, the updated plan can imply changes in the fixed setup pattern, due to cancelled setups or additional setups scheduled on short notice. Those short-termed decisions are considered particularly expensive and therefore short-termed adjustments are only applied to react to unexpected demand realizations if unavoidable or economically reasonable.

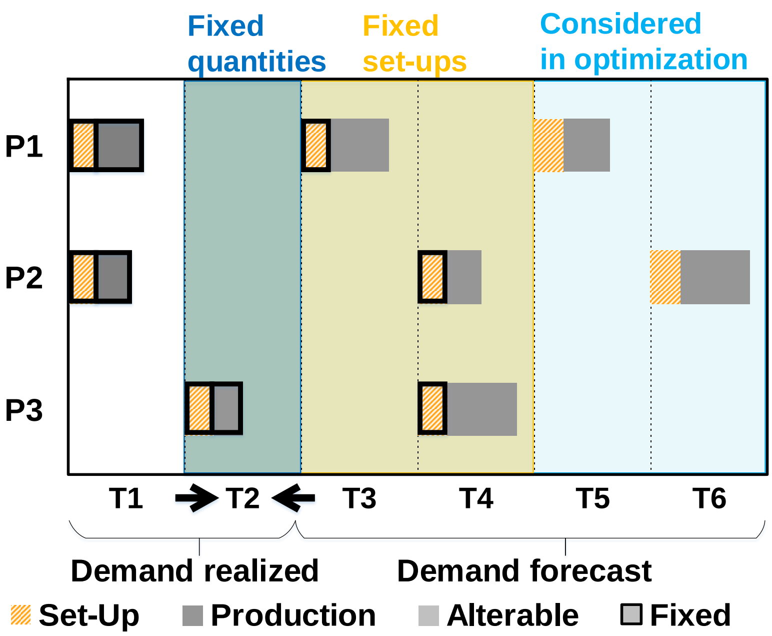

Figure 1: Exemplary plan determined with the proposed multi-stage algorithm

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3 Backlog control mechanisms

The multi-stage algorithm allows the guaranteed attainment of the target service level. However, this might lead to short-termed additional setups resulting in both system nervousness and additional costs. Therefore different mechanisms to control backlogs are implemented.

Controlling expected service levels
This approach controls backlogs by limiting the *expected* service level only. If particularly high demand realizations arise, deviations between the realized and the target service level are accepted. As a consequence, in the next reoptimization production plan adaptations are made, such that the expected service level reaches the target area by the end of the next (rolling) planning horizon. By accepting temporary violations of the target service level this approach results in the least total costs.

Controlling realized service levels
This approach controls backlogs by limiting the *realized* service level. If particularly high demand realizations arise, additional setups are scheduled to still be able to achieve the target service level by additional production. With this procedure the target service level can be attained at any time. It can also be applied for stochastic lot sizing without any realized backlogs. However, unavoidable additional production might lead to an increase in total costs. The objective function can be extended to take into account expected costs for additional production as a function of production quantities and demand information. Those costs for additional production can consist of costs for both additional setups and additional overcapacity. The extension incentivises dynamic implicit safety stocks, whose levels are chosen endogenously depending on current utilization. Those safety stocks reduce the realized total costs and increase stability of the production plans.

Cost oriented production plan selection
The approaches mentioned above limit backlogs by setting service level constraints. However, each solution, that satisfies this constraint, is feasible. Therefore there is no incentive to overachieve the service constraint. Hence the allowed amount of backlogs is exploited systematically to reduce expected holding costs. In many cases a production plan with less backlogs would only lead to a slight increase in total costs. In consequence, the cost oriented production plan selection algorithm determines solutions with different levels of service. Figure 2 shows an overview of the algorithm. If the increase in total costs for a production plan with higher service levels does not exceed a chosen threshold, this plan will be implemented.

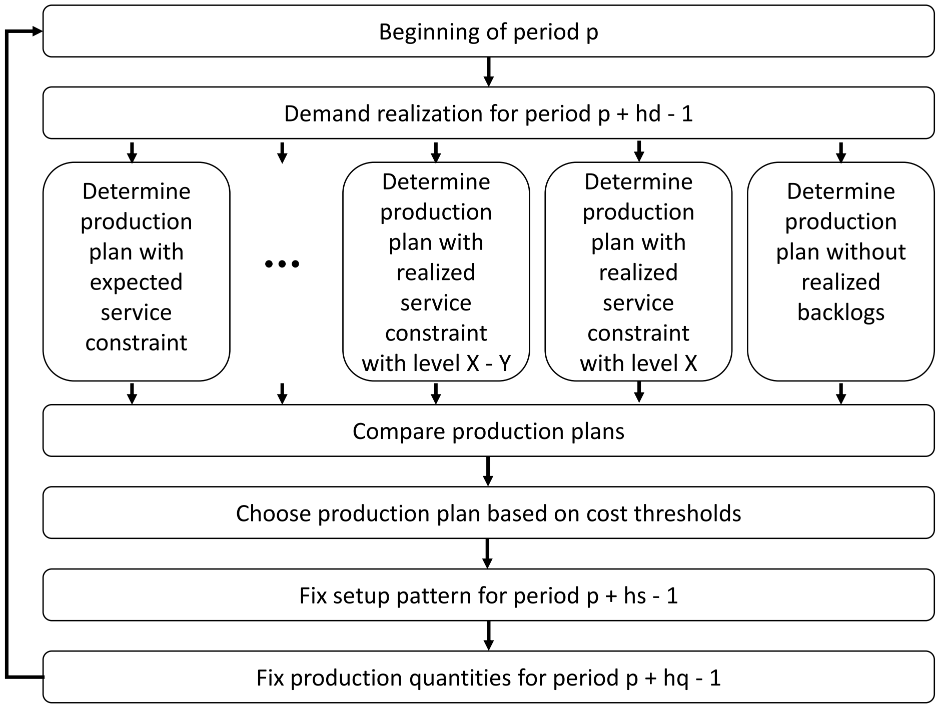

Figure 2: Algorithm for the cost oriented production plan selection

4 Risk averse optimization of the conditional value at risk

Optimization of the (approximated) mean of total costs can lead to solutions with high variance of the distribution of total costs as a function of uncertain demand realizations. This effect is particularly strong, if unavoidable additional production might become necessary. In optimal solutions often only low safety stocks are produced, such that additional setups become necessary if high demand realizations arise.

Optimization of the *conditional value at risk* (CVaR) on the contrary regards risk aversion in stochastic optimization by taking into account only a given percentage of scenarios with the highest total costs. By optimizing the CVaR in the proposed lot sizing approach, higher amounts of safety stocks are produced, which on the one hand might lead to a higher mean of total costs, but on the other hand prevent additional setups in scenarios with high demand realization and therefore are able to reduce the variance of the distribution of the objective function value by improving the robustness to high demand realizations.

Optimal (R,s,S) policy for single-item inventory lot sizing problem with stochastic non-stationary demand

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Abstract

We propose an efficient branch-and-bound approach for computing the optimal (R,s,S) policy parameters under non-stationary stochastic demand. In this policy, the inventory position is raised to an order-up-to level S at a review instant when the inventory position is at or below reorder level s. To the best of our knowledge, no method for computing the optimal policy parameters has been reported in the literature.

Our solution is based on the stochastic dynamic programming approach for the (s,S) policy. We use a search tree to compute the optimal replenishment cycles. To speed up the computations we applied a branch-and-bound technique with dynamic programming bounds. These bounds allow the branch and bound to prune of up to 99.8% of the search tree without compromising the optimality. Numerical experiments show that this technique can solve instances of realistic size in reasonable time.

1 Introduction

Production/inventory theory provides policies for managing and control inventories under different constraints and environments. An interesting class of inventory control problems is the one that considers the single-location, single-product case under non-stationary stochastic demand with complete backordering and penalty cost. This problem has been extensively studied because of its key role in real-world applications. Many inventory control policies can be adopted for this problem [1].

We focus on order-up-to-S strategies, wherein at each order the inventory level is raised to the order-up-to-level S, and the order size is decided when the order is placed and not at the start of the planning horizon. Three policies are presented in the literature for this particular problem: (s,S), (R,S) and (R,s,S). In the (s,S) policy the inventory level is raised up to S if it is lower than s. In the (R,S) policy an order is placed every R periods to raise the inventory level to the order-up-to-level. In the (R,s,S) policy the inventory level is checked every R periods, and if it is less than or equal to s an order is placed. Under the non-stationary demand assumption, the policies take the form (s^n, S^n, R^n) where subscript i denotes the i^{th} period.

If the costs considered are *ordering*, *holding* and *penalty* costs then the (s,S) policy is proved to be optimal [2]. However, in many practical applications (R,S) is more appealing as it allows orders to be scheduled in advance. Moreover, the addition of a *review* cost compromises the optimality of the (s,S) policy. The review cost is charged when we need to assess the inventory level to decide whether to place an order or not. This has a strong practical value because it allows us to model inventory assessment cost, order cancellation penalty, order preparation cost, etc. These costs would have been charged for every period in the (s,S) and added to the order cost in the (R,S). The (R,s,S) policy is the most flexible, and is the only optimal policy in the presence of non-zero review cost. However, computing (R^n, s^n, S^n) policies under stochastic non-stationary demand is a computationally hard task. To the best of our knowledge, no optimal solution is available in the literature.

2 Method

The problem has n time periods, each with a demand represented by a random variable with known probability distribution function. The solution must compute the values of δ, $\mathbf{s}$ and $\mathbf{S}$ that minimize the expected total cost, where: δ_t is a binary variable taking value 1 iff t is a review period; in period t an order will take place if t is a review period and the stock level is lower than s_t; and if an order occurs in period t then the inventory will be replenished up to S_t.

Given a fixed assignment of δ the problem is reduced to the (s,S) policy which can be solved as a stochastic dynamic program. The algorithm is widely known in literature and it is the most efficient method to compute the optimal (s,S) policy parameters. Let $C_t(I_{t-1})$ represents the expected total cost of an optimal policy over periods $t, \dots n$. Each C_t represents a single stage of the dynamic programming algorithm. The solution is computed backwards, so C_t depends only on C_{t+1}. Finally, $C_1(I_0)$, where I_0 is the initial inventory, contains the expected cost for the optimal (s,S) policy associated to the δ assignment.

No complete method is available to compute the optimal (R^n,s^n,S^n)-policy, so we shall use a brute force solution as baseline: fix the review moments and compute the optimal solution for the resulting (s,S) problem with the stochastic dynamic program; repeat for every possible assignment of δ and report the best result.

2.1 Memoization through binary tree

The cost function depends only on the following periods, so the last period can assume just two different sets of C_n, depending on whether or not it is a review period. The penultimate period can be computed in four different ways, two for each possible last period. Iterating this process we can restructure the computations in a complete binary tree. If we visit it in pre-order traversal it is possible to avoid recomputing the same stages multiple times.

Each node represents the computations of a single stage of the dynamic programming. When we compute the stage represented by a leaf we obtain the (s,S) policy associated to a fixed δ assignment represented by the path from the leaf to the root. So each leaf represents a (R^n,s^n,S^n)-policy.

2.2 Branch and bound technique

The binary tree structure of the solution allows the deployment of branch and bound techniques. If we can prove that all the solutions present in the subtree rooted in a node are not optimal we can prune the tree without compromising the optimality.

This can be done by comparing the minimum cost of a stage with the cost of the optimal solution so far. We can prune the subtree if the first is greater than or equal to the second, as the overall cost increases monotonically as we descend in the tree.

This pruning can be made more effective adding pre-computed bounds on the minimal cost from a tree level to the leaves. With these bounds we can prune approximately 75% of the search tree for a 10-period instance and 95% for a 20-period one.

If we visit the tree in a random order, we are more likely to find a sub-optimal solution earlier. This reduced the number of unpruned nodes by approximately one third.

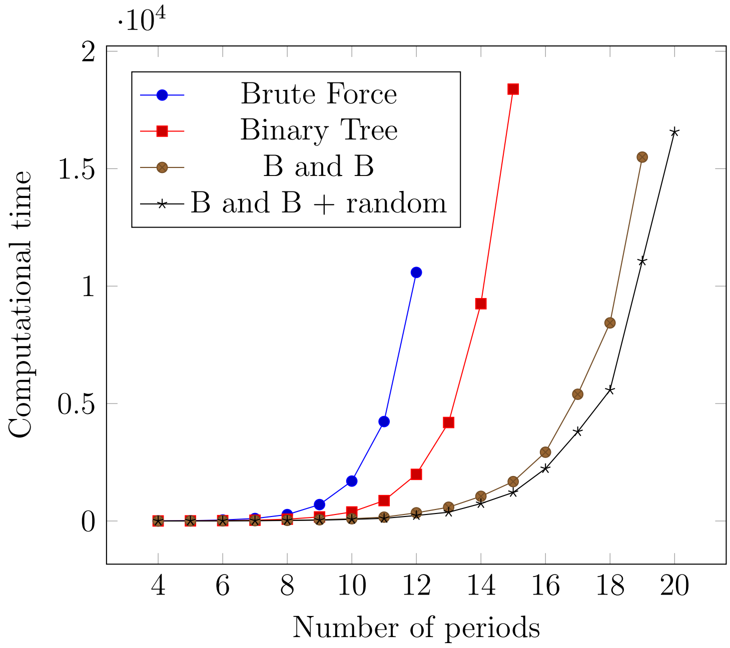

Figure 1: Computational time over the number of periods for the 4 different solvers.

3 Conclusion

We presented herein the first optimal solution for the (R,s,S)-policy problem with non-stationary stochastic demand. This technique allows considering the review cost, usually neglected in the literature, that can be used to model many real-world costs.

Figure 1 shows the computational time required to solve instances of increasing size. The technique presented herein can solve in reasonable time instances almost twice as big as the baseline, making it applicable to practical problems.

This work can be used to develop (R,s,S) heuristics and to evaluate their quality by computing their optimality gap.

References

[1] E. A. Silver, D. F. Pyke, R. Peterson. Inventory Management and Production Planning and Scheduling. Wiley, 1998.

[2] H. Scarf. The Optimality of (5,5) Policies in the Dynamic Inventory Problem. Mathematical Models in Social Sciences, Stanford University Press, 1959.

Single-item dynamic lot sizing with stochastic demand timing

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Abstract

A novel relevant way of modeling uncertainty on demand in the single-item dynamic lot-sizing problem is proposed and studied. The uncertainty is not related to the demand quantity, but to the demand timing, i.e., the demand fully occurs in a period of a given time interval with a given probability. Polynomial dynamic programs are proposed to solve the problem for the general case of multiple stochastic demands, and for several special cases.

1 Introduction

This paper tackles a single-item dynamic lot-sizing problem, i.e. quantities to be produced or replenished on a finite planning horizon discretized in periods must be determined to satisfy time-varying demands. The total cost, combining the fixed setup costs and the variable inventory and production costs, must be minimized. Because uncertainty is considered, backlog costs associated to delaying the satisfaction of some demands in a period are also included in the total cost.

Most of the literature in lot sizing is studying deterministic problems. Recent surveys on stochastic lot sizing can be found in [3] . In their survey, [1] show that the vast majority of the research literature in single-item stochastic dynamic lot sizing considers stochastic demand quantities. Stochastic costs and yield have also been studied, also combined with stochastic demands, but stochastic lead times have been very rarely considered, see [2] for a single-item lot-sizing problem with stochastic lead times.

Our problem setting significantly differs from previous studies because we consider that the demands are deterministic in terms of volumes but that their timing might be stochastic. More precisely, a given demand quantity might occur in multiple periods with a given probability to fully occur in each period. This is particularly true in operational or tactical production and inventory planning on several weeks with periods of one day, where demands and orders are well established.

2 Problem modeling

We consider the single-item uncapacitated dynamic lot-sizing problem with a planning horizon of T periods in the classical deterministic sense, as follows:

$$\begin{aligned}
\min\ & \sum_{t=1}^{T} f_t y_t + \sum_{t=1}^{T} h_t s_t + \sum_{t=1}^{T} c_t x_t && && (1)\\
\text{s.t. } & x_t + s_{t-1} - s_t = D_t && t = 1, \ldots, T && (2)\\
& x_t \leq M_t y_t && t = 1, \ldots, T && (3)\\
& y_t \in \{0,1\}; x_t \geq 0; s_t \geq 0 && t = 1, \ldots, T && (4)
\end{aligned}$$

For any period t, the variables x_t and s_t represent production and inventory quantities, respectively, and binary y_t variables indicate whether a production setup takes place or not. The objective (1) is to find a minimum cost production plan, where the total cost consists of fixed setup costs f_t (charged only if production is strictly positive, i.e., $y_t = 1$), per unit inventory holding costs h_t, and per unit production costs c_t, respectively, for all periods in the horizon. We also assume all cost parameters to be strictly positive, i.e., no "free lunch", and no speculative production costs, i.e., $c_t + \sum_{\ell=t}^{t'-1} h_\ell \geq c_{t'}$, $\forall t, t' \in [1, T]$ such that $t < t'$. The flow balance constraints (2) ensure on-time satisfaction of demand D_t, whereas the relationship between production and setup variables is set by (3), where M_t is an upper bound on x_t, e.g. $M_t = \sum_{\ell=t}^{T} D_\ell$. Constraints (4) are the integrality and non-negativity constraints.

In addition to the deterministic demands D_t, $\forall t \in [1, T]$, that need to be satisfied on time, we simultaneously consider stochastic demands, as follows. Let $[l_i, u_i] \subset [1, T]$ be an interval, indexed by i, where it is certain that a demand of d^i will occur at once in one period, with a probability of $p_t^i \geq 0$ for each period $t \in [l_i, u_i]$ and such that $\sum_{t=l_i}^{u_i} p_t^i = 1$. Note that $p_t^i = 0$ for $t \leq l_i - 1$ and $t \geq u_i + 1$. Let I be the set of such intervals with stochastic demand in the planning horizon and, for ease of notation, let $|I| = n$.

We assume that no backlog is allowed for deterministic demands and that, accordingly, no backlog is allowed for any stochastic demand d^i after period u_i. Note that, however, stochastic demand d^i may be satisfied with inventory carried from before l_i, while backlogging is allowed within the interval $[l_i, u_i]$ with a variable backlog cost b_t. As it is usually the case, we assume that backlog is more costly than inventory, i.e., $b_t > h_t\, \forall t$.

$$EC_i(t) = \sum_{l=t}^{u_i} h_l \sum_{k=l+1}^{u_i} p_k^i + \sum_{l=l_i}^{t-1} b_l \sum_{k=l_i}^{l} p_k^i \qquad (5)$$

Note that the first and second terms of (5) correspond to the expected holding and backlogging costs, respectively. Also, note that the first term is equal to 0 for $t = u_i$, and the second term is equal to 0 for $t \leq l_i$.

It is possible to show that $\arg\min EC_i(t) \in [l_i, u_i]$.

3 Summary of the main results

The various cases that we studied and the main associated results are summarized below.

3.1 Stochastic Demand Timing with a Single Interval

In this case, we assume that there is a single interval i with stochastic demand d^i throughout the planning horizon. Because backlog on d^i is only allowed before u_i, d^i is either produced before or at l_i, i.e., no backlog cost is incurred, or between $l_i + 1$ and u_i, i.e., both inventory and backlog costs are incurred.

We first proved that there is an optimal solution in which d^i is not produced in multiple periods, and that if stochastic demand is produced, it is not produced in isolation from deterministic demand, thus limiting the number of states in the dynamic program. It is then possible to derive a dynamic program with a complexity of $O(TlogT)$.

3.2 General Case of Stochastic Demand Timing

Some dominance properties can be defined to differentiate different cases of overlapping intervals. In this section, we look into the general case with multiple intervals of stochastic demand timing, where there is no dominance relationship between the overlapping intervals.

For the general case, a dynamic program is proposed whose time complexity may be exponential. However, in the relevant practical case when the ratio between the unit inventory and backlog costs in each period is time independent, i.e., $h_t = \alpha_t h$ and $b_t = \alpha_t b$ with $\alpha_t > 0\ \forall t$ (or, equivalently, $h_t/b_t = h/b$, $\forall t$), the dynamic program has a polynomial time complexity.

3.3 Special Cases of Stochastic Demand Timing

We study two relevant special cases of stochastic demand timing, which enable us to show that the dynamic program for the general case has a polynomial time complexity due to the significant reduction of valid states.

In the first case, none of the intervals with stochastic demand timing overlap, i.e., $\forall i, j \in I$ either $l_i \geq u_j + 1$ or $l_j \geq u_i + 1$ holds. In the second case with dominant overlapping intervals, we assume that, for any pair of stochastic demands d_i and d_j in I, either d_i dominates d_j or the opposite.

4 Conclusions and Perspectives

Various results and the dynamic programs will be presented in the workshop. The dynamic programs can be extended to the case with backlog costs on deterministic demands and on stochastic demands after the last period in the interval.

Various extensions of this work are being investigated, including the case where $\sum_{t=l_i}^{u_i} p_t^i < 1$, i.e., there is a probability that demand d_i may not occur at all. This implies that some production quantity might end up in inventory and thus be used to satisfy other demand in the planning horizon.

References

[1] Brahimi, N., Absi, N., Dauzère-Pérès, S., Nordli, A., 2017. Single-item dynamic lot-sizing problems: An updated survey. European Journal of Operational Research 263 (3), 838–863.

[2] Huang, K., Küçükyavuz, S., 2008. On stochastic lot-sizing problems with random lead times. Operations Research Letters 36 (3), 303–308.

[3] Tempelmeier, H., 2013. Stochastic lot sizing problems. In: Handbook of Stochastic Models and Analysis of Manufacturing System Operations. Springer, 313-344.

[4] Wagner, H., Whitin, T., 1958. Dynamic version of the economic lot size model. Management Science 5 (1), 89–96.

A dynamic programming based decomposition approach for the stochastic uncapacitated single-item lot-sizing problem

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1 Introduction

We investigate an extension of the uncapacitated single-item lot-sizing problem (ULS) in which the problem parameters, demand and costs, are subject to uncertainty. We consider a multi-stage decision process corresponding to the case where the value of the uncertain parameters unfolds little by little following a discrete-time stochastic process and the production decisions can be made progressively as more and more information on the demand and cost realizations is collected. In order to address this problem, we rely on a multi-stage stochastic integer programming approach and assume that the underlying stochastic input process has a finite probability space so that the information on the evolution of the uncertain parameters can be represented by a discrete scenario tree. Halman et al. [4] showed that a special case of the stochastic ULS in which the setup costs are set to zero and the uncertain demand can take only two possible values in each period is NP-Hard.

Several research were devoted to the polyhedral study of the mixed-integer linear program obtained when formulating the stochastic ULS on a scenario tree. Guan et al. [3] extended the (l, S) valid inequalities to a general facet-defining class called (Q, S_Q) for the stochastic variant and proved them suffice to describe the convex hull for the two period case. Later, Di Summa and Wolsey extended the work of Guan et al. [3] by showing that the (Q, S_Q) valid inequalities are dominated by a set of mixing inequalities and provided some particular cases where these mixing inequalities suffice to fully describe the convex hull. More recently, Guan, Ahmed, and Nemhauser [2] proposed a general method for generating cutting planes for multi-stage stochastic integer programs based on combining valid inequalities for individual scenarios and they provided a new set of valid inequalities for the uncapacitated and capacited stochastic lot-sizing problem.

Unfortunately, implicit enumeration methods, such as branch-and-cut algorithms, do not scale up well with the size of the scenario tree. Decomposition methods, such as Benders' decomposition, are thus an attractive alternative to tackle instances with large-size scenario trees. In particular, the Stochastic Dual Dynamic Programming (SDDP) approach proposed by Pereira and Pinto [5] has been widely used to solve large-size multi-stage stochastic linear programs. This approach relies on a dynamic programming formulation of the stochastic problem. In this formulation, the overall problem is decomposed into a series of single-node sub-problems in which the future costs of the decision made at node n are represented by an expected cost-to-go function. In a linear setting, the expected cost-to-go functions are piecewise linear convex and can thus be under-approximated through a set of supporting hyperplanes. Recently, Zou, Ahmed, and Sun proposed a new extension called Stochastic Dual Dynamic integer Programming (SDDiP) of this method in order to solve multi-stage stochastic integer programs with binary state decision variables and non-convex expected cost-to-go functions. One of their main contributions was to introduce a new class of cutting planes, called Lagrangian cuts, which satisfies the validity, tightness and finiteness conditions ensuring the convergence of the algorithm.

We propose to develop a stochastic dual dynamic integer programming approach to solve the stochastic ULS on large scenario trees. We first investigate a stochastic dynamic programming formulation of the stochastic ULS based on continuous state variables. As proposed by Zou, Ahmed, and Sun, we reformulate the obtained nodal sub-problems using a binary approximation of the inventory decision variables in order to obtain binary state variables. This allows us to use the SDDiP algorithm proposed by Zou, Ahmed, and Sun to solve the problem. Second, we propose an improved version of the SDDiP algorithm of Zou, Ahmed, and Sun [6] in which a cutting-plane generation phase based on continuous state variables is carried out to build a first approximation of the expected cost-to-go functions before actually running the SDDiP algorithm.

2 Mathematical formulations

We aim at planning production of a single type of item on a single resource over a planning horizon of T periods under uncertain demand and costs. We consider a multi-stage decision process and assume a stochastic input process with finite probability space.

The resulting information structure can be represented as a scenario tree $(\mathcal{V}, \mathcal{E})$ with T levels or stages. Each node $n \in \mathcal{V}$ corresponds to a single stage t^n. Let $\mathcal{V}^t$ be the set of nodes belonging to stage t. Each node n has a unique predecessor node denoted a^n belonging to stage $t^n - 1$ and represents the state of the system that can be distinguished by the information unfolded up to period t^n. At any non-terminal node of the tree, there are one or several branches to indicate future possible outcomes of the random variables from the current node. Let $\mathcal{C}(n)$ be the set of children of node n. The probability associated with the state represented by the node n is denoted by ρ^n and the transition probability from node n to its child node m is denoted by ρ^{nm}. A scenario is defined as a path in the tree from the root node to a leaf node and represents a possible outcome of the stochastic input parameters over the whole planning horizon.

The stochastic input parameters are defined as follows for each node $n \in \mathcal{V}$: d^n represents the discrete demand, f^n the setup cost, h^n the unit inventory holding cost and g^n the unit production cost. Moreover, we assume that at each stage, the realization of the random parameters happens before we have to make a decision for this stage, i.e. we assume that the values of d^n, f^n, h^n and g^n are known before we have to decide on the production plan at node $n \in \mathcal{V}$.

Extensive MILP formulation

Based on the uncertainty representation described above, the stochastic ULS can be reformulated as a deterministic equivalent problem in the form of a mixed-integer linear program (MILP). We introduce the following decision variables at each node $n \in \mathcal{V}$: x^n defines the quantity produced,

$y^n \in \{0,1\}$ is the setup variable and s^n represents the inventory level. This leads to the following MILP formulation:

$$\min \sum_{n\in\mathcal{V}} \rho^n (f^n y^n + h^n s^n + g^n x^n) \tag{1}$$

$$\text{s.t.} \quad x^n \le M^n y^n \qquad \forall n \in \mathcal{V} \tag{2}$$

$$s^n + d^n = x^n + s^{a^n} \qquad \forall n \in \mathcal{V} \tag{3}$$

$$x^n, s^n \ge 0 \qquad \forall n \in \mathcal{V} \tag{4}$$

$$y^n \in \{0,1\} \qquad \forall n \in \mathcal{V} \tag{5}$$

The objective function (1) aims at minimizing the expected total cost, over all nodes of the scenario tree. This cost is the sum of the expected setup, inventory holding and production costs. Constraints (2) link the production quantity variables to the setup variables. Note that the value of the M^n constant can be set by using an upper bound on the quantity that can be processed at node n, usually defined as the maximum future demand as seen from node n. Constraints (3) are the inventory balance constraints. Constraints (4)-(5) provide the decision variables domain.

Dynamic programming formulation

An alternative to the extensive formulation of the stochastic ULS discussed above is a dynamic programming formulation involving nested expected cost-to-go functions. This approach decomposes the original problem into a series of single-node sub-problems which are linked together by dynamic programming equations.

More precisely, the sub-problem related to node n focuses on defining the production plan for node n based on the entering stock level, s^{a^n}, imposed by its parent node a^n in the scenario tree. Its objective value comprises two terms: a term related to the setup, production and inventory holding costs incurred at node n and a term called the expected cost-to-go function which represents the expected future costs, over all $m \in \mathcal{C}(n)$, incurred by the production decisions made at node n.

For each node $n \in \mathcal{V} \setminus \{0\}$, the sub-problem is formulated as:

$$Q^n(s^{a^n}) := \min(f^n y^n + h^n s^n + g^n x^n) + \sum_{m\in\mathcal{C}(n)} \rho^{nm} Q^m(s^n) \tag{6}$$

$$\text{s.t.} \quad x^n \le M^n y^n \tag{7}$$

$$s^n + d^n = x^n + s^{a^n} \tag{8}$$

$$x^n, s^n \ge 0 \tag{9}$$

$$y^n \in \{0,1\} \tag{10}$$

Here $Q^n(\cdot)$ represents the optimal objective value at node n as a function of the entering stock level s^{a^n}. The expected cost-to-go function at node n is defined as $\mathcal{Q}^n(\cdot) := \sum_{m\in\mathcal{C}(n)} \rho^{nm} Q^m(\cdot)$. Note that for all leaf nodes, i.e. for all $n \in \mathcal{V}^T$, $\mathcal{Q}^n(\cdot) \equiv 0$.

3 Stochastic dual dynamic integer programming algorithm

The main idea of this algorithm is to solve the stochastic ULS by solving a sequence of single-node sub-problems in which the expected cost-to-go function $\mathcal{Q}^n(\cdot)$ is approximated by a piece-wise linear function. Each iteration of the SDDiP algorithm comprises a sampling step, a forward step and a backward step. In the sampling step, a subset of scenarios is sampled from the scenario tree. In the forward step, the algorithm then proceeds stage-wise from $t = 1$ to T by solving, at each node of the sampled scenarios, a dynamic programming equation with an approximate expected cost-to-go function. At the end of this step, the state decision variables are stored and a statistical upper-bound of the problem is computed as the weighted average over all sampled scenarios. In the backward step, we proceed stage-wise from the last stage T to the root node and solve at each node a suitable relaxation of the forward problem. The algorithm then adds supporting hyperplanes to the approximate cost-to-go functions of the previous stage. Finally, the nodal problem solved at the root node provides a lower bound of the overall problem. The algorithm stops when the upper and lower bound are close enough, according to a convergence criteria.

In the stochastic ULS, the state variables are the inventory variables, s^n, which are defined as continuous decision variables. Hence, in order to be able to apply the SDDiP algorithm to this problem, we resort to a binary approximation of the state variables. This binarization is obtained by replacing the continuous variable s^n by a set of binary variables $u^{n,\lambda}$ such that $s^n = \sum_{\lambda \in \mathcal{B}} 2^\lambda u^{n,\lambda}$. Here $u^{n,\lambda} = 1$ if coefficient 2^λ is used to compute the value of s^n, 0 otherwise. Moreover, in order to generate the cuts during the backward step of the algorithm, we introduce local copies of the binary state variables. More precisely, $z^{n,\lambda}$ is an auxiliary decision variable representing the value of the state variable at the parent node of n, i.e. it is a local copy at node n of the state variable $u^{a^n,\lambda}$. This leads to the following reformulation of the nodal sub-problem for node $n \in \mathcal{V}$:

$$Q^n(u^{a^n}) := \min(f^n y^n + h^n s^n + g^n x^n) + \sum_{m \in \mathcal{C}(n)} \rho^{nm} Q^m(u^n) \tag{11}$$

$$\text{s.t.} \quad x^n \leq M^n y^n \tag{12}$$

$$\sum\nolimits_{\lambda \in \mathcal{B}} 2^\lambda u^{n,\lambda} + d^n = x^n + \sum\nolimits_{\lambda \in \mathcal{B}} 2^\lambda z^{n,\lambda} \tag{13}$$

$$z^{n,\lambda} = u^{a^n,\lambda} \qquad \forall \lambda \tag{14}$$

$$x^n, s^n, z^n \geq 0; y_n \in \{0,1\} \tag{15}$$

$$u^{n,\lambda} \in \{0,1\} \qquad \forall \lambda \tag{16}$$

where u^n denotes the vector of binary variables $u^n = (u^{n,0}, ..., u^{n,\lambda}, ..., u^{n,B})$.

We propose an extension of the SDDiP algorithm that consists in carrying out an initial phase before actually running the SDDiP algorithm. This leads to a two-phase algorithm.

In the first phase (PHASE I), we build a first approximation of the expected cost-to-go functions by generating cuts based on formulation (6)-(10) which uses continuous inventory state variables rather than reformulation (11)-(16) which uses a binarization of the inventory state variables. More precisely, the nodal sub-problem at node n is reformulated by introducing an auxiliary variable σ^n representing the value of the inventory variable at the parent node s^{a^n}. This results in the following sub-problem:

$$Q^n(s^{a^n}) := \min(f^n y^n + h^n s^n + g^n x^n) + \mathcal{Q}^n(s^n)$$

$$\text{s.t.} \quad (7), (9), (10)$$

$$s^n + d^n = x^n + \sigma^n \tag{17}$$

$$\sigma^n = s^{a^n} \tag{18}$$

Similarly to the SDDiP algorithm, the expected cost-to-function $\mathcal{Q}^n(\cdot)$ is under-approximated by a set of cuts:

$$\tilde{\psi}_i^t(s^n) := \min\{\theta^t : \theta^t \geq \sum_{m \in \mathcal{C}(n)} \rho^{nm}(\tilde{v}_l^m + \tilde{\pi}_l^m s^n) \qquad \forall l = \{1, ..., i-1\}\} \tag{19}$$

where $\tilde{v}_l^m$ and $\tilde{\pi}_l^m$ are the coefficients of the cuts generated at iteration $l < i$.

This leads to the following approximated sub-problem $\tilde{P}_i^n(s^{a^n}, \tilde{\psi}_i^t)$:

$$\underline{\tilde{Q}}_i^n(s_i^{a^n}) := \min(f^n y^n + h^n s^n + g^n x^n) + \tilde{\psi}_i^t(s^n) \tag{20}$$

subject to (7), (9), (10), (17), (18).

In PHASE I, we generate only strengthened Benders' cuts. In these cuts, the value of $\tilde{\pi}_i^m$ is set to the dual value of the copy constraint (18) in the linear relaxation of $\tilde{P}_i^n(s^{a^n}, \tilde{\psi}_i^t)$. The value of $\tilde{v}_i^m$ is set to the optimal value of the Lagrangian relaxation of $\tilde{P}_i^n(s^{a^n}, \tilde{\psi}_i^t)$ in which the Lagrangian multiplier of the dualized copy constraint $\sigma^n = s^{a^n}$ is set to $\tilde{\pi}_i^m$ in the objective function.

In the second phase (PHASE II), we reformulate (6)-(10) by making a binary approximation of the continuous state variables. Note that any valid cut generated in PHASE I for the formulation (6)-(10) provides a valid cut for the reformulation (11)-(16) by setting $\pi_i^{m,\lambda} = \tilde{\pi}_i^m, \forall \lambda \in \mathcal{B}$ and $v_i^m = \tilde{v}_i^m$. We then further improve the under approximation of the expected cost-to-go functions by generating integer optimality, Lagrangian and strengthened Benders' cuts as done in Zou, Ahmed, and Sun [6].

We will present some numerical results that show that the initial phase significantly improves the quality of the solution found by the algorithm proposed in Zou, Ahmed, and Sun [6].

References

[1] Di Summa, M., and Wolsey, L. A. 2008. Lot-sizing on a tree. *Operations Research Letters* 36(1):7–13.

[2] Guan, Y., Ahmed, S., and Nemhauser, G. L. 2009. Cutting planes for multistage stochastic integer programs. *Operations research* 57(2):287–298.

[3] Guan, Y., Ahmed, S., Nemhauser, G. L., and Miller, A. J. 2006. A branch-and-cut algorithm for the stochastic uncapacitated lot-sizing problem. *Mathematical Programming* 105(1):55–84.

[4] Halman, N., Klabjan, D., Mostagir, M., Orlin, J., and Simchi-Levi, D. 2009. A fully polynomial-time approximation scheme for single-item stochastic inventory control with discrete demand. *Mathematics of Operations Research* 34(3):674–685.

[5] Pereira, M. V., and Pinto, L. M. 1991. Multi-stage stochastic optimization applied to energy planning. *Mathematical programming* 52(1-3):359–375.

[6] Zou, J., Ahmed, S., and Sun, X. A. 2017. Stochastic dual dynamic integer programming. *Mathematical Programming* 1–42.

Extensions on the demand

Capacitated single-item production planning with lost sales

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Abstract

In this paper, the capacitated single-item production planning with lost sales and no setup cost is studied. Although the problem can be solved using linear programming, fast algorithms are provided for several cases, in particular when lost sale costs are "dominant", i.e. when not accepting demands is only due to limited capacity.

1 Introduction

In this paper, we study a single-item production planning problem with limited production capacity and authorized lost sales but with no setup costs. This is a particular lot-sizing problem with linear production costs. There have been a large number of publications on single-item lot-sizing problems since the seminal paper of [5]. A recent literature survey can be found in [2] which, in particular, underlines the importance of three different variants of the Wagner-Whitin model, namely lost sales, production capacity limitations, and linear production costs (no setup costs).

In this work, we propose polynomial time algorithms to solve problems with these three characteristics. We present some characteristics that allow us to develop fast algorithms to solve the problems. More particularly, we show that the algorithms of [1] and [3] can be adapted to solve some cases in $O(T \log T)$, where T is the planning horizon.

In order to distinguish this problem from single-item lot sizing problems of which one important characteristic is the setup cost, we use the term "production planning" instead of "lot sizing" in the sequel.

A production plan needs to be established for a single item on a time horizon of T periods ($t = 1, \ldots, T$) where not all demands have to be satisfied, i.e. lost sales are allowed. Lost sales in a period are incurred either because production and inventory costs to satisfy the lost demand are larger than the lost sale cost, or because of limited capacity. In the remainder, we call the problem with "dominant" lost sale costs when lost sales are only due to limited capacity, i.e. lost sale costs are large enough.

When all unitary costs are linear, the problem can be solved as a linear program. In this paper, fast algorithms are proposed to solve the case with dominant lost sale costs, and some cases with non-dominant lost sale costs.

Single-item production planning problems with linear production costs can be solved in polynomial time even if there are capacity restrictions. The problem can be formulated as a linear programming model which can be solved, in the worst case, using a general polynomial time algorithm for LPs. However faster dedicated algorithms were proposed for the case with general production capacities and no lost sales. [4] consider the lot-sizing problem with linear production cost and bounded storage capacity. [3] solve a similar problem where capacity restrictions are imposed on production instead of storage. Finally, [1] study several extensions of the problem where backordering is allowed and upper bounds are imposed both on production capacities and backordered quantities. All of these three papers solved the problems in $O(T \log T)$. However, [4] is the first to solve the particular problem with bounded inventory. [1] generalize the problem of [4] and solve it without worsening the running time. [3] present a simple algorithm that does not require any data transformation compared to [1].

There are two main reasons for lost sales to be chosen as a planning option: i) When there is not enough capacity and ii) When the marginal profit of the demand is too low to be justified by investment in capacity extensions or high production and inventory holding costs. The second situation can be translated by relatively lower penalty costs. Hence, we distinguish between *dominant* (high) and *non-dominant* lost sale costs.

In our work, lost sales are allowed and different cases are discussed depending on whether the lost sale costs are dominant or not dominant, and whether they are constant or time dependent.

2 Problem modeling

To formalize the problem, the notations are introduced below.
Parameters:

d_t: Demand in period t,

c_t, h_t, p_t: Production, inventory holding, and lost sale costs per unit of product in period t,

u_t: Production capacity in period t,
$U_t = \sum_{j=1}^{t} u_j$: Cumulative production capacity up to period t,
$D_t = \sum_{j=1}^{t} u_j$: Cumulative production capacity up to period t.

Decision variables:

X_t, I_t, L_t: Production, inventory, and lost sales levels in period t,
$q_t = d_t - L_t$: Quantity of demand d_t that is accepted.

The linear programming formulation of the production planning problem with lost sales and linear costs is formalized below:

$$\text{Min} \quad \sum_{t=1}^{T} (c_t X_t + h_t I_t + p_t L_t) \tag{1}$$

$$I_{t-1} + X_t = d_t - L_t + I_t \qquad \forall\, t \tag{2}$$

$$X_t \leq u_t \qquad \forall\, t \tag{3}$$

$$X_t, I_t, L_t \geq 0 \qquad \forall\, t \tag{4}$$

The objective (1) minimizes the total cost, i.e. the sum of the production cost, the inventory holding cost, and the lost sale cost. The first set of constraints (2) are the inventory balance equations. Constraints (3) set an upper bound on production capacity. Constraints (4) are non-negativity constraints.

When lost sale costs (p_t) are dominant, i.e. large enough, lost sales only occur because of limited capacity. Mathematically, lost sale cost are dominant if the production and cumulative inventory holding costs are always smaller than the lost sale costs, that is, $\forall t, j \leq t, p_t > c_j + \sum_{k=j}^{t-1} h_j$. We set $p_t = p \forall t$, when lost sale costs are time independent.

3 Summary of the main results

3.1 Dominant lost sale costs

The algorithms to solve this problem calculates the cumulative demands ($D_t = \sum_{j=1}^{t} d_t$) and cumulative capacities ($U_t = \sum_{j=1}^{t} u_t$) in each period t starting at $t = 1$. In the first period such that $D_t - U_t = B > 0$, one has to take the decision to reject (part of) demands in periods $j = 1, \ldots, t$. Where the lost sale occurs depends on the variability of the lost sale costs. Two cases are considered: With constant lost sale costs and with time dependent lost sale costs.

A simple algorithm is proposed to solve the case with constant lost sale costs. For the case with time dependent lost sale costs, a two-phase procedure is proposed. In the first phase, the demands are smoothed by deciding where and how much demand to reject. The second phase is a scheduling phase based on the algorithm of [3].

3.2 Non-dominant lost sale costs

In this section, we assume that the unit production cost is constant, i.e. $c_t = c, \forall t$. We also consider that lost sale costs are relatively small, although several times higher than inventory holding costs which means that it is more reasonable to store the product over a certain number of periods rather than losing the sale. However, if storage capacity is limited this assumption is no longer valid.

Again, a simple algorithm is proposed to solve the case with constant lost sale costs. The case with time dependent lost sale costs is still being studied.

4 Future research directions

This study can be extended in different ways. An interesting research direction consists in explicitly integrating production planning decisions with sales. In fact, lost sales are supposed to be "controlled" by the sales department. Actually, they correspond to customer orders which are usually fully accepted or rejected. We believe that there is a need to bridge the gap between studies on lot sizing/production planning with lost sales and the literature on order acceptance.

References

[1] Ahuja, R .K., Hochbaum D. S., 2008. Solving linear cost dynamic lot-sizing problems in O(n log n) time. Operations research 56 (1), 255–261.

[2] Brahimi, N., Absi, N., Dauzère-Pérès, S., Nordli, A., 2017. Single-item dynamic lot-sizing problems: An updated survey. European Journal of Operational Research 263 (3), 838–863.

[3] Kovalyov, M., Pesch, E., 2014. An O(n log n) algorithm for a single-item capacitated lot-sizing problem with linear costs and no backlogging. International Journal of Production Research 52 (13), 3758–3761.

[4] Sedeño-Noda, A., Gutiérrez, J., Abdul-Jalbar, B., Sicilia, J., 2004. An O(T log T) algorithm for the dynamic lot size problem with limited storage and linear costs. Computational Optimization and Applications, 28 (3), 311–323.

[5] Wagner, H., Whitin, T., 1958. Dynamic version of the economic lot size model. Management Science 5 (1), 89–96.

The Profit-Maximizing Lot Size Problem with Pricing Lag Effects

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Abstract

In 1970, Joseph Thomas extended the well-known dynamic lot size model of Wagner and Whitin to include variable prices, as well as variable production costs. This work extends Thomas' model to include something called the lag effect.

Formally, this work models a disaggregated, finite-horizon, profit-maximizing dynamic lot size problem with pricing lag effects in demand. This effect models the tendency for customers to stock up on a product when it is offered for cheap, causing demand to increase in the given time period, while it decreases in the next. In effect, lowering the price causes demand to shift backwards in time, in addition to increasing demand as in most demand functions.

The problem is to decide in which time periods to produce, how much to produce in each period, and which price to set in each period. The objective is to maximize the total profit, defined as total revenue minus total setup cost, production cost and inventory holding cost.

Three theorems which restrict the amount of possible optimal solutions are proved. A solution is then provided to the production horizon problem, a sub-problem of the main problem. The production horizon problem is then solved with two different models: with and without the lag effect. The optimal solutions of these models are then compared for varying values of some key parameters.

1 Introduction

Most mathematical models describing purchasing behaviour consider demand as a function of price in a single time period. A notable exception is Ahn et al. [2]. Our model instead deals with demand as a function of the price in the same period, in addition the price in the previous period. We introduce this demand function in order to capture the behaviour where customers stock up on a product when it is offered for cheap. We call this effect the *lag effect*. In this formulation, setting a low price will cause some customers to buy for consumption not only in the same period, but also for the next period. In other words, some of the demand realized in a period originated from the next. At the same time, some of the current period's demand was already realised in the previous period. The periods may give demand to the previous and take from the next.

Our mathematical model is based on that of Thomas [1]. Ours is a disaggregated formulation, but the only essential difference is the structure of the demand function. Thomas proved the five theorems of Wagner and Whitin [3] for his model and provided a dynamic programming algorithm similar to the one developed by Wagner and Whitin. This work aims to take steps towards doing the same for our model, hopefully culminating in a polynomial time exact solution algorithm.

2 Problem description

The model seeks to maximize the total profit, defined as total revenue minus total setup cost, production cost and inventory holding cost. This is done by setting the right price p_t in each time period t in the time horizon T, and producing the right amount d_{it} in period i to cover demand in period t. Setup δ_t must be in effect in order to produce in each period. The demand function d_t in each period is the sum of all production happening the same period or before, to cover demand in period t. It is described as a sum of two parts: the lag-independent demand function ϕ_t and the lag function l_t, which is the demand transferred to period t from period $t+1$ due to the lag effect.

Mathematical model

$$\max \Pi = \sum_{t=1}^{T} (d_t p_t - s_t \delta_t - \sum_{i=t}^{T} h_{ti} d_{ti}) \quad (1)$$

$$d_t = \phi_t(p_t) + l_t(p_t) - l_{t-1}(p_{t-1}) \quad t = 1, ..., T \quad (2)$$

$$\sum_{i=1}^{t} d_{it} = d_t \quad t = 1, ..., T, \quad (3)$$

$$\begin{aligned}
\sum_{i=t}^{T} d_{ti} &\leq M_t \delta_t \quad t = 1, ..., T, && (4)\\
l_0(p_0) &= l_0 && (5)\\
l_T(p_T) &= l_T, && (6)\\
p_t, d_{it} &\geq 0 \quad i = 1, ..., T,\ t = i, ..., T, && (7)\\
\delta_t &\in \{0, 1\} \quad t = 1, ..., T. && (8)
\end{aligned}$$

The objective function (1) maximizes the total profit consisting of total revenue minus total setup cost, production cost and inventory holding cost. Constraint (2) defines the demand function, and constraint (3) links the demand function and the production-demand variables. Constraint (4) ensures no production without setup, and constraints (5) and (6) are initial and final conditions on the lag function. Finally, constraint (7) ensures non-negativity of price and production-demand variables, and (8) ensures that the setup variable is binary.

3 Analytical results

In this work we prove theorems 1, 2 and 3 of Thomas [1] for our lag model. In short, these theorems say that there exists an optimal solution to the problem, where there is no incoming inventory in any period with setup, and that each production amount is a sum of the demands of consecutive periods.

This means that there is an optimal solution which can be described as a chain of production horizons with no inventory going from one horizon to the next. A production horizon is a period with setup followed by some number of periods without setup. This number can be zero, in which case the setup period is followed by another setup period, and the production horizon is of length one.

In addition to proving these theorems, we also solve the production horizon problem to optimality. This procedure involves solving a linear equation system.

4 Numerical experiments

We did numerical experiments in order to compare our lag model to the lag-less model of Thomas [1]. In these tests, we compared the profit of the lag model to the Thomas model while varying three key parameters. We used linear demand functions and lag functions, with a particular lag function in mind. We modelled the lag function as the demand function of the next period, evaluated at the price of the current period, all multiplied by a factor f. Mathematically: $l_t = f\phi_{t+1}(p_t)$. The f factor represents the fraction of customers that are willing and able to stock up on the product. This formulation means that a customer will stock up if he is willing to stock up, and if the price is acceptable.

The three parameters we varied in the tests were as follows: The constant in the linear demand function, α, the stockup factor, f, and the time horizon, T. We compared the two models for the same parameter values, looking at absolute deviation and percentage deviation.

We found that the lag effect has a non-negligible impact on profit, as the lag model had 10-20% higher profit in many of the instances. Interestingly, the lag model never had worse profit than the lag-less model in any of our tests. It is unknown whether this is true for all possible demand and lag functions and parameter values.

The lag effect seems to have a greater impact on profit when sales volume is high, as marginal contribution to profit for α showed to increase as α increases. A similar result was found for the stockup factor f.

Lastly, the experiments showed that production horizons seem to yield diminishing returns on profit as they get longer. This is likely due to the high marginal cost of producing for a late period.

References

[1] Thomas, J. Price-Production Decisions with Deterministic Demand. Management Science, 16(11):747-750, 1970.

[2] Ahn, H; Gümüs, M; Kaminsky, P. Pricing and Manufacturing Decisions When Demand Is a Function of Prices in Multiple Periods. Operations Research, 55(6):1039-1057, 2007.

[3] Wagner, H and Whitin, T. Dynamic Version of the Economic Lot Size Model. Management Science 5(1):89-96, 1958.

Energy

Energy storage management with energy curtailing incentives in a telecommunications context

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1 Introduction

The electrical energy market has been subject to intensive research specially with the emergence of smart-grids providing more complex energy grids with multiple power sources, storage systems and local eco-friendly energy production [3, 7].

The use of batteries as backup in case of power outages is frequent in telecommunications companies that provide critical services to keep their network equipment always active [6]. In this context, for each equipment in the network there exists at least one battery for backup use and security rules on the battery usage must be considered. Firstly, it has to be immediately recharged to its full capacity B_{max} after each use. Furthermore, to increase the batteries' lifespan, they must be recharged with a constant boost power P_B.

However, those batteries could also be used for other purposes, such as participating in the energy market when they are not being used for backup. Since the energy price is not constant on time, batteries can be used in the periods when the energy costs more, also called *peak-time* periods, and recharged when the energy costs less as a strategy to reduce the electricity bill [4, 5, 10, 8]. However, there exists a limited amount U_{max} of energy that can be bought at each time period due to the grid capacity. With this bound and the battery capacity, trivial strategies such as buying all the energy demand over the planning horizon at the cheapest period are not possible. Such an energy market is known as *Retail Market*.

A second way is to use the batteries to participate to the *Curtailing Market*, introduced for the first time by Lee et al. [9]. In this context, a company can be called to reduce its energy consumption by receiving a reward. Considering a typical energy production and distribution system usually composed by generators, transmission and distribution operators and clients, the transmission operator (TO) is the agent responsible for the energy transmission and for the grid stability. When the consumption demand in a system is larger than the energy production, the TO has either to use its electrical energy reserves (e.g. call nuclear plants to produce more) or to call the customers that have a huge energy demand to cut down their consumption for a period (performing a *curtailing*) giving them a reward [1, 9]. Usually, the reward depends on the amount of energy that is cut down during a curtailing and rules to participate in this market are priorly contractualized [2].

2 Problem Definition

The problem treated in our study can be formally described as follows. Let us consider a customer with an electrical energy demand W_t in each period t over a planning horizon of T discrete time periods. The unitary cost C_t used to compute the electricity bill at each period t given.

Each curtailing has a minimal (resp. maximal) duration D_{min} (resp. D_{max}) that must be respected. In addition, during a curtailing, a minimum amount of power must be cut down at each period of time. In other words, for each period t of a curtailing, there exists a maximal amount of energy U_t that can be bought from the supplier. The way such an amount is computed is imposed by the TO depending on the country. Our study is based on the french context where this amount is defined as $W - P_C$, $\overline{W}$ being the mean demand forecast over the curtailing. P_C is a contractualized power that must be cut down.

Furthermore, a minimum amount of energy B_{min} must remain in the battery and the battery must be fully charged at the beginning and at the end of the time horizon for network safety purposes.

Managing batteries while respecting both usage and market rules is a key aspect to keep the network safe at optimal cost. Our paper addresses this aspect in a single battery setting. To the best of our knowledge, this is the first study where batteries are used for backup as well as to participate in the curtailing market.

	Rules	Stock management setting
Battery usage requirements	• Maximal capacity B_{max}	• Inventory with a limited capacity
	• Safety energy level B_{min}	• Safety stock
	• Immediate recharge	• If the inventory is not being used, it must be replenished
	• Constant recharge rate P_B	• The inventory is replenished at constant rate
Curtailing Market requirements	• Minimal curtailing duration D_{min}	• Minimum number of consecutive periods for each stock usage
	• Maximal curtailing duration D_{max}	• The stock can not be used more than a maximum number of periods
	• Minimal amount of energy that must be cut down during a curtailing P_C	• A minimum number of items must be supplied from the inventory at any period of time when the stock is used
Spot Market requirements	• Maximal amount of energy that can be bought U_{max}	• A production capacity on each period is imposed

Table 1: Relation between the specific rules of the considered problem and the stock management setting

In this context, the battery can be viewed as a power stock and its management treated as a particular production planning problem with specific rules on the i nventory. The battery is ready for use and no setup or installation costs are considered, and a new curtailing can start only if the battery is fully charged. Schneider et al. [11] studied a single-period electrical energy storage system from an inventory model point of view and proposed a general translation of technical requirements for energy storage systems into requirements for inventory models. In a similar vein, Table 1 presents the relation between the specific rules of the considered problem and the stock management setting.

3 Contributions

First part of our study aims at modeling the problem providing the battery usage rules at optimal cost as a mixed integer program. However, due to the size of the real instances, the model becomes hard to solve. In this context, based at the structure of the problem, a polynomial time algorithm is also proposed providing over the planning horizon the battery usage rules at optimal cost.

The derived idea is the enumeration of a subset of all possible curtailings that can be performed, and the reduction to a longest path problem in a directed acyclic graph (DAG), created from the enumerated subset of curtailings.

Formally, a curtailing c can be represented by its start and end times denoted (f_c, l_c) and by the amount of energy X_c that is cut down over its duration, also called *Depth of Discharge*. A curtailing can be then defined by the triple (f_c, l_c, X_c). Since curtailing duration is bounded by D_{min} and D_{max}, all possible pairs (f_c, l_c) can be enumerated in $O(T^2)$ time. Originally, X is a continuous variable and can not be extensively enumerated. However, we have proved that there exists an optimal solution of the problem for which the value of X_c in each curtailing of the solution is either a multiple of P_B or a multiple of U_t. Consequently, for any pair (f_c, l_c) there exists at most T values of X_c that needs to be enumerated.

Since a curtailing is defined by $c = (f_c, l_c, X_c)$, we are able to compute its potential gain g_c due to the sequences at the battery use and recharge. Firstly, the sequence of recharge of the battery is imposed by the immediate recharge rule. Secondly, since we know X_c for each curtailing c, and the cost of purchasing energy at each time period, we can easily determine how much energy should be consumed from the battery during each period of the curtailing, in order to minimize the cost.

Finally, a graph $G = (V, A)$ can be created where each enumerated curtailing is represented by a vertex in V. Two dummy vertices s and t are also added in V. The set of arcs A is defined as follows:

- for any $v_1, v_2 \in V - \{s,t\}, v_1 \neq v_2$, an arc from v_1 to v_2 of weight g_{v_1} is added if v_2 can be performed after v_1 with respect to the start/end times and the last charging period.

- for all $v_i \in V, v_i \neq s$, an arc from s to v_i of weight 0 is added.

- for all $v_i \in V, v_i \neq s, v_i \neq t$, an arc from v_i to t of weight g_{v_i} is added.

By construction, the graph G is a DAG and the longest path from s to t can be computed in polynomial time [12]. Let $p*$ be a longest path from s to t, the set of vertices in $p*$ gives us directly the set of curtailings to be performed at optimal cost $c*$, which is the value of $p*$.

The proposed algorithm works for any variant of the problem such that the computation of U_t is not influenced by other curtailings. However, in some cases, the computation of U_t could be influenced by the Depth of Discharge of the curtailing previously performed. In this context, our algorithm can not be applied.

The complexity of the algorithm is $O(V + A)$, where $|V|$ is bounded by T^3 and $|A|$ by T^6.

References

[1] Baldick, R., Kolos, S., & Tompaidis, S. , Interruptible Electricity Contracts from an Electricity Retailer's Point of View: Valuation and Optimal Interruption, Operations Research, 54(4), 627-642 (2006)

[2] Chrysikou, V., Alamaniotis, M., & Tsoukalas, L. H., A Review of Incentive based Demand Response Methods in Smart Electricity Grids, International Journal of Monitoring and Surveillance Technologies Research (IJMSTR), 3(4), 62-73, (2015).

[3] Dang, T., The Energy Web: Concept and challenges to overcome to make large scale renewable and distributed energy resources a true reality, In 2009 7th IEEE International Conference on Industrial Informatics (pp. 384-389),(2009).

[4] Daryanian, B., Bohn, R. E., & Tabors, R. D., Optimal demand-side response to electricity spot prices for storage-type customers, IEEE Transactions on Power Systems, 4(3), 897-903, (1989).

[5] Johnson, M. P., Bar-Noy, A., Liu, O., & Feng,, Y., Energy peak shaving with local storage. Sustainable Computing: Informatics and Systems, 1(3), 177-188 (2011).

[6] Kiehne, H. A., & Krakowski, H., Batteries in Power Supplies for the New Telecommunication Services, In INTELEC'84-International Telecommunications Energy Conference (pp. 34-40), IEEE,(1984).

[7] Koutsopoulos I., Hatzi V., & Tassiulas, Optimal energy storage control policies for the smart power grid, In 2011 IEEE International Conference on Smart Grid Communications (SmartGridComm), (pp. 475-480), L. (2011).

[8] Labidi, W., Smart grid-aware radio engineering in 5G mobile networks (Doctoral dissertation, Université Paris-Saclay),3(2),52-63,(2019).

[9] Lee, Maurice W., Hydroelectric power in the Columbia basin, J. Bus. Univ. Chicago, 26, 173–189, (1953).

[10] Mishra, A., Irwin, D., Shenoy, P., Kurose, J., & Zhu, T., Smartcharge: Cutting the electricity bill in smart homes with energy storage, In Proceedings of the 3rd International Conference on Future Energy Systems: Where Energy, Computing and Communication Meet (p. 29), ACM (2012).

[11] Schneider, M., Biel, K., Pfaller, S., Schaede, H., Rinderknecht, S., & Glock, C. H. , Optimal sizing of electrical energy storage systems using inventory models, Energy Procedia, 73, 48-58,(2015).

[12] Sedgewick, R., & Wayne, K. D., Algorithms (4th ed.), Addison-Wesley Professional, pp. 661–666,(2011).

Renewable versus grid energy for a mid term production and capacity adjustment planning

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Abstract

In this work, we study a tactical production, inventory and capacity adjustment planning for a firm using two energy supplies for its production activities: an on-site generated renewable energy source and the grid electricity. While the renewable energy can be considered as free of use for the firm, the amount of energy available typically depends on weather conditions for solar and wind powers, and thus varies from period to period. Conversely, the grid energy is virtualy unlimited, but is to be purchased from an external provider. We consider the single-item lot sizing problem integrated with these two sources of energy. The production system we consider has a stationary nominal capacity, which can be expanded by installing some temporary capacities. Temporary capacities are a multiple of a base value, and a fixed cost is incurred for each period an extra capacity is installed. This corresponds typically to change the working shift pattern. We have to determine in each period of the time horizon the temporary capacity adjustment to set-up, the amount of energy to buy from the grid and the quantity to produce, such that the demand is satisfied at a minimum cost. We establish that this problem is NP-hard, even with a single installable temporary capacity. We also identify several special cases where an optimal solution can be efficiently computed in polynomial time.

1 Introduction

More and more industrial companies are considering the use of on-site renewable energy sources to partly or entirely supply their energy, in order to reduce their cost and their dependency on external energy providers. This switch in energy supply is also a way for industry to reduce the greenhouse gas emissions related to production activities. Concrete industrial examples can be found in Wilde (2018) and Merchant Wind Power (2016), related to companies installing solar panels and wind turbines in their production sites to provide a part or the totality of the electricity required to power their plant. However, one difficulty in using on-site renewable energy sources is their dependency to weather conditions (sun or wind) : the amount of energy supplied can be extremely time-varying, hence, they cannot fully replace grid energy. Both sources should be used in coordination in order to find the best trade-off between production and energy costs. This means that the renewable energy generation should be integrated into the production planning, see Keller *et al.* (2016), to take advantage of its benefits. More references on energy considerations for production planning and scheduling can be found in Biel and Glock (2016) and Gahm *et al.* (2016).

In this study, we focus on a mid-term capacity, production and inventory planning problem taking into account two sources of energy (green and grid) to power the production activities. The energy required by production activities can be supplied either by a free-of-use renewable green source, with a time-varying capacity; or by the grid source, available at any required level but incurring a unit cost per kWh purchased. We consider a deterministic and dynamic demand, known in each period over a finite horizon. In addition to a nominal production capacity, we assume that additional temporary capacities can be installed in each period, incurring a fixed cost per extra capacity installed. Since we search the trade-off between production, inventory and capacity acquisition costs, this problem can easily be modeled as a variant of the single-item lot sizing problem.

The contributions of this study are (*i*) Proposition of a production planning model in which capacity adjustment, production, inventory and energy supply decisions are simultaneously optimized ; (*ii*) Complexity classification of the related problems in terms of NP-hardness and polynomially solvable cases ; (*iii*) Proposition of new polynomial time algorithms for some particular cases

2 Problem description

The aim is to satisfy without shortage a deterministic demand d_t in each period t over a time horizon of length T, minimizing total production, capacity installation and energy costs. The production system has a nominal stationary capacity C. In each period t, up to m_t temporary capacities can be installed to extend this nominal capacity, each temporary capacity having the same size A and incurring the same fixed cost f_t. The amount of energy available in periods t from the on-site renewable source translates into a quantity B_t of products can be produced. This capacity is a soft limit, which can be extended by purchasing electricity from the grid, at a cost of π^+ per unit produced. Notice that we consider a stationary price for the electricity bought on the grid. Each unit produced, regardless the source of energy used, incurs a unit cost c_t of production, and each unit carried in stock from period t to period $t+1$ incurs a holding cost h_t. We assume non-speculative motives for the unit production and holding costs. Since these variable costs are linear, this is equivalent to consider null holding costs and non-increasing unit production costs.

MILP formulation

We consider the following decision variables:

x_t : production quantity in period t

y_t : the number of installed temporary capacities in period t

e_t^+ : the amount of units produced using grid energy in period t

The problem can be formulated as follows:

$$\begin{array}{llll}
\min & \sum_{t=1}^{T}(c_t x_t + f_t y_t + \pi^+ e_t^+) & & \\
s.t. & x_t \leq C + y_t A & \forall t \in \{1..T\} & (1) \\
 & y_t \leq m_t & \forall t \in \{1..T\} & (2) \\
 & x_t \leq B_t + e_t^+ & \forall t \in \{1..T\} & (3) \\
 & \sum_{i=1}^{t} x_i \geq \sum_{i=1}^{t} d_i & \forall t \in \{1..T\} & (4) \\
 & x_t, e_t^+ \geq 0, y_t \in \mathbb{N} & \forall t \in \{1..T\} & (5)
\end{array}$$

The objective function minimizes the total cost. Constraints (1) enforce the quantity produced to respect the effective capacity of each period. Constraints (2) guarantee that the number of installed temporary capacities does not exceed the maximal number authorized in a period. Constraints (3) limit the quantity produced in a period relatively to the amount of energy available from the green source and the amount of energy bought from the grid. Constraints (4) are classical lot sizing constraints for demand satisfaction without backlogging nor lost sales. Constraints (5) define the feasibility domain of each decision variable.

3 Complexity results & Polynomial cases

We have proven the problem to be NP-hard, even under restrictive conditions, see the following theorem :

Theorem 1. *The problem is NP-hard even in the case of a single temporary capacity. It remains NP-hard for instances with*

- *Non-Increasing green capacities B_t and stationary fixed cost f*
- *Non-Decreasing green capacities B_t (and time-varying fixed cost f_t)*

We have also established that the problem is polynomially solvable for several cases:

• **If the green source is sufficient to entirely supply the energy to produce at full capacity** in every period. It corresponds to assume that $B_t \geq C + m_t A$ $\forall t$. We can disregard the energy constraint, since no electricity is purchased from the grid in an optimal solution. The problem can be reduced to a lot-sizing problem with batch delivery, each batch representing the installation of a temporary capacity. It is solvable in time complexity $O(T^3 \log T)$

• **The green source is not sufficient to entirely supply the energy to produce at nominal capacity** in every period. It corresponds to assume that $B_t \leq C$ $\forall t$. Structural properties of the solution allow to reduce the problem to a discrete lot-sizing problem. It is solvable in time complexity $O(T^4 \log T)$

• **The amount of energy supplied by the green source is non-decreasing with time** and the fixed cost for installing a temporary capacity is stationary. Notice that relaxing one of these conditions render the problem NP-hard. We develop a greedy-based algorithm to optimally solve the problem in time complexity $O(T^4)$ in the case of a single temporary capacity.

References

[1] Biel, K. and Glock, C.H. Systematic literature review of decision support models for energy-efficient production planning. Computers & Industrial Engineering, 101:243-259, 2016.

[2] Gahm, C., Denz, F., Dirr, M., Tuma, A. Energy-efficient scheduling in manufacturing companies: A review and research framework. European Journal of Operational Research, 248(3):744-757, 2016.

[3] Keller, F., Braunreuther, S., Reinhara, G. Integration of on-site energy generation into production planning systems. In Procedia of the 23rd CIRP Conference on Life Cycle Engineering, pages 254-258, 2016.

[4] Merchant Wind Power. Dundee, Scotland: State-of-the-art green energy. http://www.merchantwindpower.co.uk/case-studies/dundee- scotland, 2016.

[5] Wilde, D. Bentley adds U.K.'s largest solar farm to its crewe factory. http://www.merchantwindpower.co.uk/case-studies/dundee- scotland, 2018

Applications

The Multi-Period Cutting Stock Problem with Diameter Conversion in the Construction Industry

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Abstract

The one-dimensional cutting stock problem is widely used for reinforcement steel bar in the construction industry. Diameter sizes of rebars are determined by the structural designer to provide tensile strength to the structure, and can be changed if cross-section area per concrete area stays constant. The decision maker can decide to convert diameter size to generate better cutting patterns for an effective usage of the resources. Besides, the decision maker deal with the lot-sizing problem by assuming the multi-period structure of rebar demand. We study the cutting stock problem in which decision maker decides diameter size, cutting time, and patterns to minimize the usage of raw material and holding cost. Note that it differs from the classical cutting stock problem as it is not known how many pieces should be cut until diameter sizes are chosen. We propose a pseudo-polynomial formulation and a genetic algorithm to solve the problem. Computational results are provided.

1 Introduction

One of the major cost items in the construction industry is reinforcement steel rebar which is widely used to provide necessary tensile strength to the structures. It is crucial to use rebars efficiently to decrease the cost of structures. Each part of the structures requires different sizes of these rebars in terms of diameter and length, and they can be obtained by cutting large rebars into smaller pieces. Diameter sizes and the number of rebars required are determined in a way to obtain necessary tensile strength. The chosen diameter size and the number of needed rebar of each structural element can be converted into different sizes if conversion rules are followed. Since the diameter sizes are convertible, choosing diameter sizes and generating cutting patterns together will yield efficient results to avoid trim loss. The problem is to determine cutting patterns, diameter sizes, and the number of required rebars to minimize the usage of reinforcement steel rebars. Since the diameter is a different dimension of the problem in addition to the length dimension, the problem differs from classical one-dimensional cutting stock problem (1-D CSP). On the other hand, sizes of the diameter dimension are uncertain and convertible to different sizes. Therefore, determining the appropriate diameter size is a part of the decision making process. Note that diameter sizes are decision variables of the problem and the problem differs from the classical two-dimensional cutting stock problem (2-D CSP). The problem is defined as a 1.5-dimensional cutting stock problem (1.5-D CSP) in the literature. Some construction projects may include more than one building such as multiple unit housing projects, holiday sites, public building complexes, etc. The need for rebars may occur in different time periods during the construction phase of these projects. Besides, some larger building projects may need different diameter rebars in different time periods according to the business plan. As a result, the decision maker of the construction project should decide the times of procurement and amount of rebars before rebars are needed. The aim of the decision maker is to determine the time period and cutting patterns of the items, so that trim loss and inventory holding cost are minimized. This problem is called the 1.5-dimensional multi-period cutting stock problem (1.5-D MPCSP).

2 Solution Approaches

We consider three formulations: Kantorovich [1], pseudo-polynomial arc-flow formulation of Carvalho [3], and reflect formulation of Delorme and Iori [2] to solve our problems. Since Kantorovich formulation is weak, we implement Branch&Price method by applying Dantzig-Wolfe decomposition. Arc-flow formulation of Carvalho [3] is pseudo-polynomial, and it becomes weak when stock capacity increases. Delorme and Iori [2] developed the reflect formulation which uses half of the length capacity in order to overcome this weakness. We modified these formulations to attack 1.5-D CSP and the 1.5-D MPCSP. Although Reflect formulation is very powerful for 1.5-D CSP, it is hard to solve moderate size instances if multi-period structure of the problem is considered. Therefore, we propose a genetic algorithm approach to solve large size instances of the 1.5-D MPCSP.

3 Experimental Results

We tested our methods on real data from different application areas such as hospitals, apartments, business center constructions, etc. According to our experimental results, reflect formulation outperforms Kantorovich and arc-flow formulations. However, it has difficulties on solving moderate and large size instances of 1.5-D MPCSP. On the other hand, our genetic algorithm reaches near optimal solution (within 5%) in a reasonable amount of time. As a result, we propose using reflect formulation for 1.5-D CSP in all instance sizes, and for 1.5-D MPCSP in small size instances. On the other hand, genetic algorithm is more succesful in solving moderate and large size instances.

References

[1] Leonid V Kantorovich, Mathematical methods of organizing and planning production, Management science, vol. 6, pp. 849–859 (1961)

[2] Maxence Delorme and Manuel Iori, Enhanced pseudo-polynomial formulations for bin packing and cutting stock problems, Technical Report, DEI "Guglielmo Marconi" Alma Mater Studiorum Università di Bologna, Italy (2017)

[3] Valerio De Carvalho, Exact solution of bin-packing problems using column generation and branch-and-bound, Annals of Operations Research, vol. 86,pp. 629–659 (1999)

Dynamic lot sizing for LNG inventories

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Abstract

We consider a lot-sizing problem motivated by Liquefied Natural Gas (LNG). LNG is an alternative transportation fuel. It is supplied from sources with different quality and price, at different costs. LNG is subject to quantity and quality decay. It is possible to upgrade or downgrade the quality of LNG by mixing an existing lot with a new one. LNG is provided to customers via special-purpose stations. We address the lot sizing problem in such a station. The problem entails finding a minimum-cost replenishment plan satisfying demands over a finite planning horizon, while meeting a minimum quality level. We initially formulate the problem as a mixed integer non-linear program and an approximate mixed integer linear program. These models are appealing as they can be directly fed into commercial solvers. But they are computationally expensive. Then, we focus on classes of solutions which can be obtained in polynomial-time and develop several heuristics with varying levels of complexity. We numerically illustrate that our heuristics provide high-quality solutions in short computational times.

1 Introduction

The vast majority of the inventory management literature on deteriorating products addresses problems where product quality decreases over time following a decay process. The product is discarded or used for an alternative purpose when the quality eventually drops below an acceptable level. In this paper, we consider a system where it is possible to interrupt the decay process and upgrade or downgrade quality by mixing different loads of the same product—with different levels of quality. This is common for many fuels and chemicals. When these substances are mixed, the quality of the mixture becomes the weighted average of the quality of the loads involved. The inventory policies used in systems involving such substances can exploit this property to manage product quality effectively, thereby reducing costs.

Our study is mainly motivated by Liquefied Natural Gas (LNG) inventories. LNG is a sustainable fuel used for road and maritime transportation. In order to keep it in its liquid state, LNG needs to be stored and transported at $-163°$ using specialized insulation. However, as there is no perfect insulation LNG is exposed to heat loss. This leads to constant boil-off, and, therefore, quantity and quality decay. The loss of quality reduces the efficiency of the LNG as a fuel. Besides, the use of low-quality LNG may even lead to permanent engine damage. Therefore, it is essential that LNG inventories meet a minimum quality level. LNG can be supplied from different markets. These differ in terms of quality, price, and ordering costs. It is therefore possible to dynamically manage the quality over time by mixing procurement lots from different suppliers. LNG is provided to customers via special-purpose stations with limited storage capacity. We address the lot sizing problem in such a station. The problem entails finding a minimum-cost replenishment plan satisfying deterministic demands over a finite planning horizon, while meeting a minimum quality level at all times.

2 Background

We consider a multi-supplier lot-sizing problem with quantity and quality decay. There is a variety of studies on inventory systems with deterioration. The majority of these studies are devoted to the analysis of systems with different deterioration processes. For an extensive overview, we refer the reader to Goyal and Giri (2001) and Bakker et al. (2012). There is also an extensive literature on inventory systems with multiple suppliers, where suppliers are differentiated by means of characteristics such as price, ordering cost, capacity, availability, lead time, and quality. The reader is referred to Minner (2003) for a review. Our study is closely related to these lines of research. However, we significantly deviate from the literature, as in our system it is possible to interrupt the deterioration process and upgrade and downgrade quality by mixing procurement lots from different suppliers. This requires bringing together concepts from inventory systems with deterioration and multiple suppliers. To the best of our knowledge, the current study is the first in the literature in this respect.

Our study also has strong ties with the so-called pooling problem (Audet et al., 2004; Dey and Gupte, 2015). This is a well-known problem which frequently appears in the petrochemical industry where crude oils characterized by different levels of quality attributes are mixed together in intermediate pools which are subsequently used to blend end-products with pre-specified quality requirements. The lot-sizing problem we address in the current study can be regarded as a generalized pooling problem, where the inventories carried over from one period to the next correspond to material flows in between intermediate pools. This analogy makes it possible to adapt and extend the well-established models and methods of the pooling problem to approach the lot-sizing problem on hand. However, the variants of the pooling problem where flows between intermediate pools is considered are known to be much more computationally challenging as compared to the standard pooling problem (Kolodziej et al., 2013).

3 Overview and Results

The problem we consider in this study is a combinatorial optimization problem with two almost exclusive components: a lot-sizing component that can be expressed as a mixed integer program and a quality-related component that entails non-linear and non-convex constraints.

We use a variety of methods to approach this problem. We initially formulate the problem as a mixed integer non-linear program. This formulation can be solved by one of the standard mixed integer non-linear programming solvers. However, our computational study with Baron—one of the most prominent commercial solvers—shows that the time-efficiency of such a formulation is very poor. This rules out any successful implementation. We then use a linearization approach for quality-related constraints, motivated by the successful applications in the context of the pooling problem. In particular, we make use of the radix-based method which is known to be very competitive among existing methods in the literature. This results in a mixed integer program which can also be fed into available solvers. While the linearization approach leads to significant improvements with respect computational efficiency, the model quickly becomes intractable as the length of the planning horizon increases—in line with what has been reported in the literature on the pooling problem.

These findings motivate us towards devising heuristics, rather than approaching the problem with exact or approximate mathematical models. To that end, we focus on classes of solutions which can be obtained in polynomial-time and develop several heuristics with varying levels of complexity. Our heuristics are based on the idea of decomposing the overall problem into replenishment cycles. This approach leads to structural properties which allows us to solve the associated sub-problems efficiently. We numerically illustrate that our heuristics provide high-quality solutions in very short computational times.

References

Audet, C., J. Brimberg, P. Hansen, S. L. Digabel, and N. Mladenović (2004). Pooling problem: Alternate formulations and solution methods. *Management science 50*(6), 761–776.

Bakker, M., J. Riezebos, and R. H. Teunter (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research 221*(2), 275–284.

Dey, S. S. and A. Gupte (2015). Analysis of milp techniques for the pooling problem. *Operations Research 63*(2), 412–427.

Goyal, S. K. and B. C. Giri (2001). Recent trends in modeling of deteriorating inventory. *European Journal of operational research 134*(1), 1–16.

Kolodziej, S. P., I. E. Grossmann, K. C. Furman, and N. W. Sawaya (2013). A discretization-based approach for the optimization of the multiperiod blend scheduling problem. *Computers & Chemical Engineering 53*, 122–142.

Minner, S. (2003). Multiple-supplier inventory models in supply chain management: A review. *International Journal of Production Economics 81*, 265–279.

A column generation approach for production planning in semiconductor manufacturing

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1 Introduction

In front-end semiconductor manufacturing (also called wafer manufacturing), production planning is very complex due to the system characteristics such as re-entrant flows, hundreds of operations to perform for each product, many heterogeneous machines of different types, etc. Efficient production planning is even more crucial in high-mix facilities, i.e. with many products and varying demands, which corresponds to most of European semiconductor manufacturing facilities.

In production planning, quantities of products to be released at each period of the planning horizon must be determined to meet demands at lowest cost while satisfying capacity constraints. In semi-conductor manufacturing several operations have to be performed in order to produce a given product. In detailed planning, production capacity is allocated at the operation level.

A large part of the literature in semiconductor manufacturing focused on modeling congestion and its effects on cycle times. The easiest way to consider congestion is to use fixed Lead Times (LT), i.e. a fixed delay in number of periods between the arrival of products at an operation and their completion time. However, fixed lead times do not take into account that the actual lead times depend on the workload of resources.

In 1996, Hung and Leachman [4] tackle this issue by proposing an approach that iterates between an optimization model and a simulation model. The optimization model optimizes the production plan given fixed lead times, and the simulation model determines the lead times given the production plan. A more recent way to model congestion is the use of Clearing Functions (see e.g. [2] and [1]), i.e. non-linear functions that determine the output of a resource according to the workload.

In this work, we study a model that considers flexible lead-times. A column generation approach is used to solve the studied problem.

2 Problem definition

In this section, we define the studied problem. First we introduce the problem with fixed lead-times and then we replace fixed lead times by flexible lead times.

We aim at planning the production of P products over a discrete time horizon that has two timescales. The time horizon is decomposed into T days and S weeks. Demands are expressed per product and per week. Each product needs a sequence of operations $\mathcal{L}_p$ to be processed on a set of workshops. Each workshop can process a finite set of operations and has a finite capacity. The goal is to decide quantities to be released per product p, per operation l and per period t (day). The set of operations for each product and their resource consumption provide the timing of operations. In order to trace production flows, an inventory variable that represents the work in process per product, per operation and per period (day) is introduced. A unitary work in progress cost is associated with each product and each operation. We also introduce a unitary inventory cost and a unitary backlogging cost for each product and each period (week). We also assume that for each operation, a product need to wait LT_{pl} periods before being processed. This is called a *Fixed Lead Time.* The goal is to satisfy demands while minimizing inventory, backlogging and work in process costs. The resulting model is a Linear Program.

The main constraints are: flow conservation constraints that ensure the link between the output of an operation and the input of the next operation, flow conservation constraints of the work in progress, flow conservation constraints that guarantee that fixed lead times are satisfied, flow conservation constraints for final products, ensuring the satisfaction of demands, and capacity constraints that limit, in each workshop, the quantities that can be processed. All decision variables are continuous and non-negative.

The flaws of this model mostly reside in Fixed Lead Time constraints that were already discuss in [3]. Authors also presented a more flexible way of modeling lead times called WIP penetration constraints (introduced by Hwang and Chang in 2003 [5]). WIP penetration constraints allow more freedom in production runs in order to obtain more balanced workloads, but introduces much more decision variables and constraints and constraints that limit the set of operations that can be performed during the same period. In the following section, we propose a new mathematical model based on timed production routes as an alternative to the classical straightforward formulation (as described above).

3 A new formulation using timed production routes

In this section, we propose a new mathematical formulation based on timed production routes. A timed production route is associated with a given product and provides the assignment of its operations to periods. The flows of WIPs are embedded within the set of all feasible production routes. Given the set of all feasible timed production routes, we derive a model that allocates production quantities to timed production routes in order to meet demands of final products while satisfying production capacity constraints and flow balance constraints for final products. To each timed production route and each period, a binary parameter is defined to indicate whether an operation is processed or not. A unitary cost is calculated for each timed production route. It provides the unitary cost of managing the WIP. We also introduce a new decision variable that decides the quantity to release on each timed production route. The objective function is to minimize the total inventory, backlogging and work in process costs. The resulting model is a Linear Program with a huge number of timed production routes. In order to solve this model, we use a column generation approach to generate useful timed production routes.

4 Column generation approach

In order to improve the tractability of the formulation based on timed production routes, we use a column generation approach. Rather than solving the model that considers all timed production routes, we define a reduced master problem based on a set of limited number of timed production routes (columns) that is enriched with new timed production routes in an iterative scheme. New timed production routes are obtained by solving a sub-problem for each product. The goal of each sub-problem is to provide new timed production routes (columns) with negative reduced costs in order to improve the objective function of the master problem. The column generation stops when the optimal solution of sub-problems does not provide a timed production route (column) with a negative reduced cost.

A sub-problem is defined for each product. It consists in solving an assignment problem that assigns each operation to the period where it is processed. This assignment satisfy all internal flows constraints. For the problem with fixed lead times, the assignment of the first operation provides the timing of all other operations. On the other hand, for the problem with WIP penetration constraints, more freedom is given, so the assignment of operations to periods should satisfy WIP penetration constraints and is driven by dual costs associated with capacity constraints and flow conservation constraints of final products.

To solve the resulting sub-problems a dynamic programming algorithm is developed. This dynamic program is a labeling algorithm. The first label is generated by the assignment of the first operation to a given period. Each label is extended to the next period by assigning between 0 and the maximum number of operations that can be executed in the same period. So each label can generate up to a maximum number of new labels per period and per operation. Dominance rules are used in order to reduce the number of generated labels. Note that the evaluation of labels considers dual costs.

5 Conclusion

In this work, we introduce a new method to deal with semiconductor flow complexity in production planning. Numerical experiment on a literature and industrial data sets will validate this approach. Several improvements to the timed production routes generation should be discussed and tested: what is the best way to tune column generation, how to accelerate the current implementation, which heuristic could be used to speed up the column generation, which realistic rules should be imposed to timed routes and what are their impacts on the solution quality.

Acknowledgment

This work is funded by Productive 4.0. The project receives grants from the European H2020 research and innovation program, ECSEL Joint Undertaking, and National Funding Authorities from 19 involved countries under grant agreement No 737459.

References

[1] Erinç Albey, Ümit Bilge, and Reha Uzsoy. An exploratory study of disaggregated clearing functions for production systems with multiple products. *International Journal of Production Research*, 52(18):5301–5322, 2014.

[2] Jakob Asmundsson, Ronald L Rardin, and Reha Uzsoy. Tractable nonlinear production planning models for semiconductor wafer fabrication facilities. *IEEE Transactions on Semiconductor Manufacturing*, 19(1):95–111, 2006.

[3] Sébastien Beraudy, Nabil Absi, and Stéphane Dauzère-Pérès. Detailed production planning models withflexible lead time constraints. In *9th Internatinal Workshop on Lot-Sizing, IWLS'2018*, pages 24–27, Ubatuba, Brazil, 2018.

[4] Yi-Feng Hung and Robert C Leachman. A production planning methodology for semiconductor manufacturing based on iterative simulation and linear programming calculations. *IEEE Transactions on Semiconductor manufacturing*, 9(2):257–269, 1996.

[5] Ting-Kai Hwang and Shi-Chung Chang. Design of a lagrangian relaxation-based hierarchical production scheduling environment for semiconductor wafer fabrication. *IEEE Transactions on Robotics and Automation*, 19(4):566–578, 2003.

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