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An order-level lot size inventory model with inventory-level dependent demand and deterioration

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Abstract

Many inventory models have been developed for various deteriorating items with constant demand rate. It is a common experience that, for perishable consumer goods, the age of inventory has a negative impact on the demand due to the loss of consumer confidence on such product. Consumers tend to keep off perishable items which have reached closer to their expiry dates. This paper describes an inventory model in which the demand is considered as a composite function consisting of a constant component, and a variable component which is proportional to the inventory level in the periods when there is a positive inventory buildup. The rate of production is considered finite and the decay rate as exponential. The total cost function is composed of four cost components of all phases of the cycle (backorder replenishment, inventory buildup, inventory depletion and shortage). This cost function is later reduced to a two-variable function using boundary relations to determine iteratively the set of values of the variables that resulted in minimum cost per cycle. The optimum lot-size and order-level were then obtained with relations established previously. Results are demonstrated for an instance of the model.

Keywords: Lot size; Inventory-dependent demand with deterioration

1. Introduction

The traditional inventory model considers the ideal case in which depletion of inventory is caused by a constant demand rate. However, in real-life situations there is inventory loss by deterioration also. Again, the assumption of a constant demand rate may not be always appropriate for such consumer goods type of inventory (e.g. milk, meat, vegetables, radioactive materials, volatile liquids, etc.), as the age of inventory has a negative impact on demand due to the loss of consumer confidence on the quality of such products.

This paper tries to integrate a number of aspects on deteriorating inventory and inventory-dependent demand into a more general model. A model has been developed for such cases in which demand and deterioration rates are both affected by the elapsed time i.e. inventory level, since inventory level is a function of time [1]. Ghare and Schrader [2] qualified the phenomena of inventory deterioration and they categorized it into three types: direct spoilage, physical depletion and deterioration. Since this introduction a lot of work has been done on deteriorating inventory systems [3–11]. Misra developed a model with a finite replenishment rate but he did not consider backordering. Shah generalized Ghare and Schrader's model to allow

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for backordering, but replenishment rate was considered infinite. A very recent article in the field of deteriorating items with shortages was by Wee [12], which determined the lot-size for a declining market. Heng et al. [1] integrated Shah's [7] and Misra's [6] models to consider a lot-size, order-level inventory system with finite replenishment rate, constant demand rate and exponential decay. This model is an extension of the work done by Heng et al. [1] by incorporating the time-dependent demand into their model. This will give the model more flexibility and make it easily applicable to a large variety of inventory categories.

1.1. Problem definition

For perishable goods such as common food items (e.g. milk, meat, vegetables, etc.) it is observed that the age of inventory has a negative impact on consumer confidence for reasons such as (a) proximity to expiry dates (for applicable items), (b) detrimental effects on the quality of the product because of aging of inventory, and (c) general conception that an item lying unsold for a long time may be of inferior quality.

Hence, it is logical to conclude that during shortage periods there would be a constant demand, but when there is inventory buildup the demand would be a function of inventory level. In fact, the demand would still have the same constant component as before, only a fractional reduction (proportional to inventory level) would occur. This can be easily justified as the arrival of the normal set of customers but the declining of a fraction of them due to reasons stated above. This manifests itself as a portion of the on-hand stock remaining unused.

Incorporating the scenario of decreasing demand on the model developed by Heng et al. [1], a model is developed in this paper for typical perishable consumer goods with finite replenishment and inventory-level dependent demand and deterioration. A model of this nature, encompassing a large variety of perishable and non-perishable (by taking the deterioration rate as zero) items with fixed (by making the variable component zero) or variable demand patterns would be a very useful tool for inventory control in large retail setups and stores where the business and its success depends on the inventory control of a variety of items, with large variations in value, demand and deterioration patterns [13].

An expression for the total cost per cycle has been obtained by considering the various cost components in each phase of the cycle. The total cost is a function of four variables, which is further reduced to a two-variable function using boundary relations. An algorithm is used to search iteratively for the set of values of the variables which results in the minimization of total inventory cost per cycle. The optimum lot-size and order-level were then obtained with relations established previously.

2. Model formulation

The problem considered has finite replenishment rate, backorder and inventory decay, and the production rate is assumed to be sufficiently large enough to meet the demand. Both decay and demand are functions of the inventory level (during positive inventory). To model the problem, the inventory cycle is divided into four major phases (based on major changes in parameters): backorder replenishment period (Phase 1), inventory buildup period (Phase 2), inventory depletion period (Phase 3) and shortage period (Phase 4). Time for Phase i is indicated by T_i and in each time phase, T_i ($i = 1, \ldots, 4$), the inventory level is correspondingly depicted by $I_i(t)$ at time, t. The change in inventory in the four phases have been described by equations, using the various known system parameters and unknown variables (the time spans of the phases). The inventory model is shown in Fig. 1.

From Fig. 1 it is apparent that during the time period T_1, there is a shortage and so there is a constant demand rate. In the subsequent time period, T_2, there is an inventory buildup and so the deterioration rate, α becomes effective. The production is stopped in the third time period and the demand decrease rate, β becomes effective. If there had been a continuous production without any break, then the effective decay rate would have become $\alpha - \beta$, but that is not the case here which makes the present model different from that of Heng et al. [1].

Assumptions. (1) Demand rate is dependent on the inventory level. (2) Production rate is known and constant. (3) Number of units is continuous and not

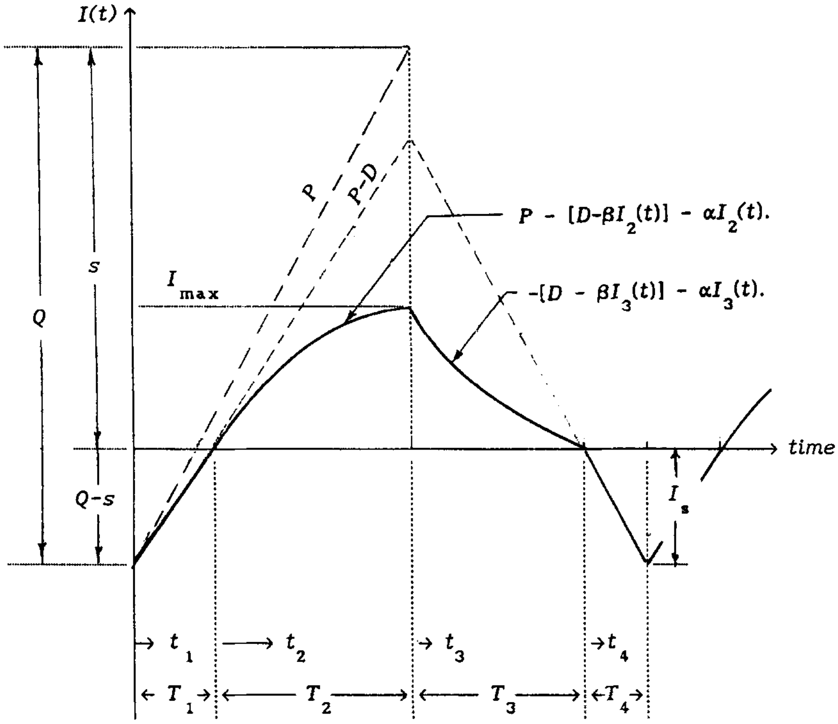

Fig. 1. Graphical representation of the inventory model.

discrete. (4) Production rate is greater than demand rate. (5) Deterioration rate is known and constant.

Notation

c = cost of a deteriorated unit, \$/unit
c_1 = inventory carrying cost, \$/unit/unit time
c_2 = back-ordering cost, \$/unit/unit time
c_3 = cost of a setup, \$/setup
D = demand rate, units/unit time
$I(t)$ = inventory level at time t
$I_i(t)$ = inventory level at time t during time period $T_i, i = 1,\dots,4$
I_{max} = maximum inventory level within a cycle
$I_\$$ = maximum inventory shortage or backorder
P = production rate, units/unit time
Q = production lot-size, units/batch
α = deterioration rate, units/unit time
β = the rate of decrease of demand
s = order level
T = cycle time
T_1 = production time when backorder is replenished
T_2 = production time when inventory builds up
T_3 = time period when there is no production and inventory depletes
T_4 = time period when there is no production and shortage occurs
T_i^o = optimal time length for phase i, for $i = 1,\dots,4$
TC = total cost, \$/unit time

2.1. Formulation of the system

The change in inventory in the four phases (Fig. 1) can be described by the following equations:

Phase 1: In this phase, all units produced are immediately consumed at a constant (demand) rate. Hence, an inventory shortage is marked to diminish with time as the demand is met from the production immediately. The rate of change of inventory at time t in this phase, $I_1'(t)$, is expressed as:

$$I_1'(t) = P - D. \tag{1}$$

Phase 2: Inventory builds up to the maximum level in this phase of the cycle. For the rate of decrease of demand, β, the effective demand rate is $[D - \beta I_2(t)]$ because a negative effect on the constant demand rate which is time dependent is considered (hence, also the inventory level). Also, for the deterioration rate α, there is an exponential deterioration of inventory $\alpha I_2(t)$. The rate of change of inventory at time t in the phase, $I_2'(t)$, is given by

$$I_2'(t) = P - [D - \beta I_2(t)] - \alpha I_2(t). \tag{2}$$

Phase 3: Production is discontinued and the inventory depletes due to demand, $[D - \beta I_3(t)]$, and deterioration, $\alpha I_3(t)$ for reasons similar to those given in Phase 2. The rate of change of inventory at time t in the phase, $I_3'(t)$, is thus given by

$$I_3'(t) = -[D - \beta I_3(t)] - \alpha I_3(t). \tag{3}$$

Phase 4: Shortage increases to maximum level; since there is no inventory, the demand is not influenced by it, and is thus constant. All units are backordered. The rate of change of inventory at time t, $I_4'(t)$, is given by

$$I_4'(t) = -D. \tag{4}$$

2.2. Total cost function

The total cost constists of four cost components: carrying cost, backordering cost, deterioration cost, and replenishment cost. Based on the four basic phases of the inventory cycle, these costs are evaluated below.

2.2.1. Carrying cost

The carrying cost is incurred only in the phases in which there is a positive inventory (Phases 2 and 3). We, therefore, have to find the total inventories carried in these two phases to obtain the carrying cost per unit time. Solving Eq. (2) for $I_2(t)$ (see Appendix A) and Eq. (3) (see Appendix B) for $I_3(t)$, we get

$$I_2(t) = \frac{P-D}{\alpha-\beta}(1 - \exp(-(\alpha-\beta)t)). \tag{5}$$

and

$$I_3(t) = \frac{D}{(\alpha-\beta)}([\exp(\alpha-\beta)T_3 - \exp(\alpha-\beta)t_3]/\exp(\alpha-\beta)t_3), \tag{6}$$

respectively. The carrying cost per unit time, C_1, is given by

$$C_1 = \frac{c_1}{T_1+T_2+T_3+T_4} \times \left\{ \int_0^{T_2} I_2(t)\,\mathrm{d}t + \int_0^{T_3} I_3(t)\,\mathrm{d}t \right\}. \tag{7}$$

Substituting the expressions for $I_2(t)$ and $I_3(t)$ in Eq. (7) and simplification of it (see Appendix C) yields

$$C_1 = \frac{c_1}{(T_1+T_2+T_3+T_4)} \left(\frac{(P-D)T_2 - DT_3}{\alpha-\beta} \right). \tag{8}$$

2.2.2. Backordering cost

The backordering cost is incurred only when there is a shortage (Phases 1 and 4). The maximum inventory shortage is $I_z = (P-D)T_1 = DT_4$. The average shortage over T_1 is $(P-D)T_1/2$, and the average shortage over T_4 time period is $DT_4/2$. Therefore, the backordering cost per unit time, C_2, is given by

$$C_2 = \frac{c_2(P-D)T_1^2 + c_2DT_4^2}{2(T_1+T_2+T_3+T_4)}. \tag{9}$$

2.2.3. Replenishment cost

As in Heng et al. [1] for a setup cost, c_3, the replenishment cost per unit time is given by

$$C_3 = \frac{c_3}{T_1+T_2+T_3+T_4}. \tag{10}$$

2.2.4. Deterioration cost

Deterioration cost is incurred during periods of positive inventory (Phases 2 and 3). The number of units deteriorated during T_2 and T_3 are

$$\alpha \left[\int_0^{T_2} I_2(t)\,\mathrm{d}t + \int_0^{T_3} I_3(t)\,\mathrm{d}t \right]$$

and

$$I_{\max} - \int_0^{T_3} [D - \beta I_3(t)]\,\mathrm{d}t_3,$$

respectively. The deterioration cost per unit time is, therefore, given by

$$C = \left[c \left(\int_0^{T_2} \{(P-D) + \beta I_2(t)\}\,dt - I_{\max} + I_{\max} - \int_0^{T_3} \{D - \beta I_3(t)\}\,dt \right) \right] \Big/ (T_1 + T_2 + T_3 + T_4). \tag{11}$$

Substituting the expressions for $I_2(t)$ and $I_3(t)$ in Eq. (11) and subsequent simplification of it (see Appendix D) yields

$$C = c\alpha \left[\frac{(P-D)T_2 - DT_3}{(\alpha - \beta)} \right] \Big/ (T_1 + T_2 + T_3 + T_4). \tag{12}$$

2.2.5. Total cost

The total cost per unit time, TC, is given by

$$\begin{aligned}
\mathrm{TC} &= C + C_1 + C_2 + C_3 \\
&= c\alpha \left[\frac{(P-D)T_2 - DT_3}{(\alpha - \beta)} \right] \Big/ (T_1 + T_2 + T_3 + T_4) \\
&\quad + \frac{\alpha c_1}{(T_1 + T_2 + T_3 + T_4)} \left(\frac{(P-D)T_2 - DT_3}{\alpha - \beta} \right) \\
&\quad + \frac{c_2(P-D)T_1^2 + c_2 D T_4^2}{2(T_1 + T_2 + T_3 + T_4)} \\
&\quad + \frac{c_3}{(T_1 + T_2 + T_3 + T_4)} \\
&= \{2[(P-D)T_2 - DT_3][c + \alpha c_1] \\
&\quad + (\alpha - \beta)[c_2 T_1^2 (P-D) + c_2 D T_4^2 + 2c_3]\} / \\
&\quad [2(T_1 + T_2 + T_3 + T_4)(\alpha - \beta)].
\end{aligned} \tag{13}$$

3. The solution method

Eq. (13) shows that the total cost, TC, is a function of T_1, T_2, T_3 and T_4. We know from boundary conditions (refer Fig. 1) that

$$I_z = (P-D)T_1 = DT_4 \tag{14}$$

$$T_4 = T_1 x \tag{15}$$

where, $x = (P/D - 1)$. The maximum inventory level at $t_3 = 0$ is

$$I_{\max} = I_3(t)|_{t_3=0} = \frac{D}{(\alpha - \beta)}(\exp(\alpha - \beta)T_3 - 1).$$

Also, at $t_2 = T_2$, i.e. at $t_3 = 0$, we equate the inventory level, $I_2(t)$, obtained from Eq. (5) with the maximum inventory level $I_{\max}$:

$$\frac{P-D}{\alpha - \beta}(1 - \exp[-(\alpha - \beta)T_2]) = \frac{D}{\alpha - \beta}(\exp[(\alpha - \beta)T_3] - 1). \tag{16}$$

Expanding the exponential terms in Eq. (16) and simplifying it as in Appendices C and D, we get

$$\frac{(\alpha - \beta)}{2} T_3^2 + T_3 - x \left\{ T_2 - \frac{(\alpha - \beta)}{2} T_2^2 \right\} = 0, \tag{17}$$

which is a quadratic equation in T_3. Solving for positive roots (as the time phases cannot have negative values), we obtain

$$T_3 = \frac{-1 + (1 + 2kx[T_2 - kT_2^2/2])^{1/2}}{k} \tag{18}$$

where, $k = \alpha - \beta$ and $x = (P/D - 1)$. Heng et al. [1, p. 192, Eq. (23)] deviated from the *exact* derivation of T_3 in terms of T_2; they have neglected the higher-order term in T_2. This paper did not neglect the higher-order terms, and hence, the results are relatively closer to the optimum.

Although the equations, $\partial(\mathrm{TC})/\partial T_i = 0$ for $i = 1, \ldots, 4$, are quadratic in nature and seemingly solvable, a closed-form solutions for T_1, T_2, T_3 and T_4, and the test of sufficient conditions for optimality, are certainly not a trivial exercise. Therefore, an iterative solution procedure is thus presented in Algorithm 1 to obtain an optimal production lot size, Q_o, and an order level, s_o. The production lot size, Q_o, is actually produced during the time $T_1 + T_2$.

Hence,

$$Q_o = (T_1^o + T_2^o)P \tag{19}$$

and from Eq. (14) the maximum inventory shortage or backorder during time T_4 is $I_s = DT_4$ from which the optimal order level, s_o is given by

$$s_o = Q_o - DT_4^o. \tag{20}$$

Algorithm 1. Finding optimal lot size and order level

Step 1: Assume $0 \leqslant T_1^o \leqslant \mathrm{UB}_1$, $0 \leqslant T_2^o \leqslant \mathrm{UB}_2$ where UB_1 and UB_2 are known upper bounds. Set $T_1^o = 0$, $T_2^o = 0$, $\mathrm{TC}_{\mathrm{new}} = 0$ and fix step length Δ_1 and Δ_2 for T_1 and T_2, respectively.

Step 2: Compute T_3, T_4 and TC using Eqs. (18), (15) and (13), respectively.

Step 3: $\mathrm{TC}_{\mathrm{old}} \leftarrow \mathrm{TC}_{\mathrm{new}}$ and $\mathrm{TC}_{\mathrm{new}} \leftarrow \mathrm{TC}$.

Step 4: If $\mathrm{TC}_{\mathrm{new}} < \mathrm{TC}_{\mathrm{old}}$, store $T_1^o = T_1$, $T_2^o = T_2$, $T_3^o = T_3$, $T_4^o = T_4$ and $\mathrm{TC}^o = \mathrm{TC}_{\mathrm{new}}$.

Step 5: If $T_2 < \mathrm{UB}_2$, then $T_2 \leftarrow T_2 + \Delta_2$, go to *Step 2*.
else *Step 6*.

Step 6: If $T_1 < \mathrm{UB}_1$, then $T_1 \leftarrow T_1 + \Delta_1$, go to Step 2.

Step 7: Compute $Q_o = (T_1^o + T_2^o)P$ and $s_o = Q_o - DT_4^o$.

Algorithm 1 basically computes the optimal values for T_1, T_2, T_3 and T_4 to minimize the total cost. The algorithm uses an iterative search procedure assuming that the total cost function is convex. Computational results for an instance of the problem are provided using a numerical example.

Example 1. Heng et al. [1] data is used here to compare the present method. The rate of decrease of demand, β has been additionally considered in this example. Heng et al. [1] has considered $\alpha = 0.05$, but in this paper α is assigned a value of 0.07 to consider higher deterioration rate and β is assigned 0.02 to consider a decrease in demand. This assumption will generate $\alpha - \beta = 0.05$. So practically, α in Heng et al. [1] model is replaced by $\alpha - \beta$ with the assumption $\beta = 0$. The input parameters are as follows:

$P = 75$ gal/wk,
$D = 50$ gal/wk,
$c = \$10.00$/gal,
$c_1 = \$4.00$/gal/wk,
$c_2 = \$2.00$/gal/wk,
$c_3 = \$100.00$/production run,
$\beta = 0.02$,
$\alpha = 0.07$ (0.05 in Heng's problem).

Table 1
Comparison with results of Heng et al. [1]

Description	Notation	Heng et al.	Present method
Phase-1 time	T_1^o	1.343 wk	1.500 wk
Phase-2 time	T_2^o	0.646 wk	0.300 wk
Phase-3 time	T_3^o	0.320 wk	0.148 wk
Phase-4 time	T_4^o	0.671 wk	0.750 wk
Opt. cycle time	T_o	2.980 wk	2.698 wk
Opt. order level	s_o	115.589 gal	97.499 gal
Opt. lot size	Q_o	149.145 gal	134.999 gal
Minimum cost	TC_o	\$67.11/wk	\$74.66/wk

The results are presented in Table 1. In the example, it is noticed that the total cost is slightly higher than that in Heng et al. Higher deterioration rate, α, results in the increase of total deterioration cost, and decrease in demand, β, results in the increase of total inventory carrying cost. Consequently, the total cost in the present model is amplified in comparison with that in Heng et al. model.

4. Sensitivity analysis

A sensitivity analysis is performed to study the effect of deterioration rate, α and demand decrease rate, β on the total cost. It is clear from Eq. (13) that the total cost TC is a convex function is β, the demand decrease rate; but the total cost is insignificant for small variation in β. Thus the α-curves in Fig. 2 appear to be linear though it is not the case. It can be also observed in Fig. 2 that for very low values of β, and high values of α, the total cost, TC is highly sensitive. The sensitivity of the total cost is determined by the proximity of the curves. As the value of β increases and the values of α decreases, the variation in the total cost is less pronounced, which is shown by the total cost curves that are not closely packed.

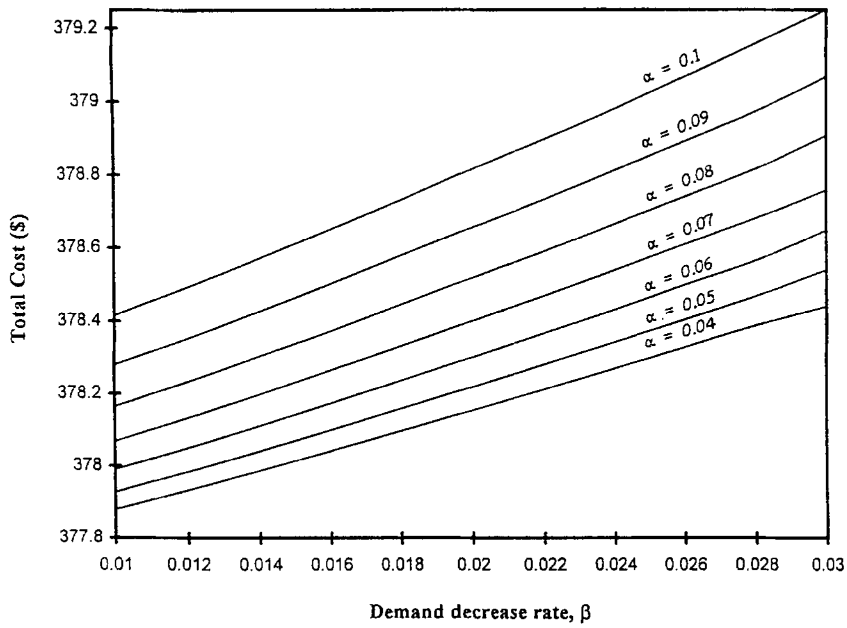

Fig. 2. Effects of α and β on total cost.

5. Conclusion

A generalized inventory model for the lot-size order-level system with finite production rate and inventory-level dependent demand and deterioration is derived. This model which incorporates flexibility of varying demand may be applied to a wide variety of items. All combinations of deterioration or no deterioration and varying or constant demands may be solved by this approach.

Earlier research considered various pattern of deterioration with constant demand. However, a deteriorating inventory will always be subject to decrease in demand. Since this study assumed a decrease in demand with the deterioration rate, this model is a closer representation of a real-life situation.

It is not appropriate to keep a constant rate of replenishment when the inventory deteriorates and the demand decreases in an inventory system because that will result in a replenishment which will ultimately waste due to deterioration or unnecessary, inventory carrying. To decrease deterioration and holding costs, the replenishment rate can also be considered as a function of inventory level as a further improvement of this model. In that case however the rate of decrease would also be a decision variable. Further, the results of the sensitivity analysis prove the fact that the parameters α and β do influence the total cost to an appreciable extent and variations in α and β have an impact on the total cost.

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Appendix A. Evaluating inventory level in the inventory buildup phase

We now derive Eq. (5) from Eq. (2).

$$I_2'(t) = P - [D - \beta I_2(t)] - \alpha I_2(t)$$

or

$$I_2'(t) = P - D - (\alpha - \beta) I_2(t)$$

or

$$I_2'(t) + (\alpha - \beta)I_2(t) = P - D. \tag{A.1}$$

Multiplying both sides of Eq. (A.1) by $\exp[\int(\alpha - \beta)\,dt]$, we get,

$$I_2'(t)\exp\left[\int(\alpha-\beta)\,dt\right] + (\alpha-\beta)I_2(t)\exp\left[\int(\alpha-\beta)\,dt\right] = (P-D)\exp\left[\int(\alpha-\beta)\,dt\right]$$

or

$$\frac{d}{dt}\left(I_2(t)\exp\left[\int(\alpha-\beta)\,dt\right]\right) = (P-D)\exp\left[\int(\alpha-\beta)\,dt\right]$$

or

$$I_2(t)\exp\left[\int_0^{t_2}(\alpha-\beta)\,dt\right] = \int_0^{t_2}(P-D)\exp\left[\int(\alpha-\beta)\,dt\right]\,dt + k_1$$

or

$$I_2(t) = \frac{\int_0^{t_2}(P-D)\exp\left[\int(\alpha-\beta)\,dt\right]\,dt + k_1}{\exp\left[\int_0^{t_2}(\alpha-\beta)\,dt\right]},$$

where, $t_2 = t - T_1$. At $t_2 = 0$, $I_2 = 0$, hence $k_1 = 0$. Therefore,

$$I_2(t) = \frac{\int_0^{t_2}(P-D)\exp\left[\int(\alpha-\beta)\,dt\right]\,dt}{\exp\left[\int_0^{t_2}(\alpha-\beta)\,dt\right]},$$

$$I_2(t) = \frac{(P-D)\{\exp[(\alpha-\beta)t_2 - 1]\}}{\exp[(\alpha-\beta)\,t_2]}$$

$$= \frac{(P-D)}{(\alpha-\beta)}(1 - \exp[-(\alpha-\beta)t_2]). \tag{A.2}$$

Appendix B. Evaluating inventory level during depletion period

We derive Eq. (6) from Eq. (3).

$$I_3'(t) = -D - (\alpha-\beta)I_3(t)$$

or

$$I_3'(t) + (\alpha-\beta)I_3(t) = -D. \tag{B.1}$$

Multiplying both sides by $\exp[\int(\alpha-\beta)\,dt]$, we get,

$$I_3'(t)\exp\left[\int(\alpha-\beta)\,dt\right] + (\alpha-\beta)I_3(t)\exp\left[\int(\alpha-\beta)\,dt\right] = -D\exp\left[\int(\alpha-\beta)\,dt\right]$$

or

$$\frac{d}{dt}\left(I_3(t)\exp\left[\int(\alpha-\beta)\,dt\right]\right) = (-D)\exp\left[\int(\alpha-\beta)\,dt\right]$$

or

$$I_3(t) = \frac{-D\int_0^{t_3}\exp\left[\int(\alpha-\beta)\,dt\right]\,dt + k_2}{\exp\left[\int_0^{t_3}(\alpha-\beta)\,dt\right]}$$

where, $t_2 = t - T_1 - T_2$, or

$$I_3(t) = \frac{-D\{\exp[(\alpha-\beta)t_3 - 1]\} + k_2}{(\alpha-\beta)\exp[(\alpha-\beta)t_3]}$$

$$= \frac{-D}{(\alpha-\beta)}(1 - \exp(-(\alpha-\beta)t_3)) + k_2.$$

At $t_3 = 0$, $I_3 = I_{\max}$ and at $t_3 = T_3$, $I_3 = 0$. Hence, $k_2 = I_{\max}$ and

$$I_{\max} = \frac{D}{(\alpha-\beta)}(\exp(\alpha-\beta)T_3 - 1).$$

Therefore,

$$I_3(t) = [D\{1 - \exp(\alpha - \beta)t_3 + \exp(\alpha - \beta)T_3 - 1\}/(\alpha - \beta)]/(\exp[-(\alpha - \beta)t_3])$$
$$= \frac{D}{(\alpha - \beta)}([\exp(\alpha - \beta)T_3 - \exp(\alpha - \beta)t_3]/\exp(\alpha - \beta)t_3). \quad \text{(B.2)}$$

Appendix C. Carrying cost per unit time, C_1

We now derive Eq. (8) from Eq. (7). We have

$$C_1 = \frac{c_1}{T_1 + T_2 + T_3 + T_4}\left\{\int_0^{T_2} I_2(t)\,\mathrm{d}t + \int_0^{T_3} I_3(t)\,\mathrm{d}t\right\}. \quad \text{(C.1)}$$

Expanding the term $\int_0^{T_2} I_2(t)\,\mathrm{d}t_2$, we get,

$$\int_0^{T_2} I_2(t)\,\mathrm{d}t_2 = \int_0^{T_2} \frac{P - D}{\alpha - \beta}\{1 - \exp[-(\alpha - \beta)t_2]\}\mathrm{d}t_2$$
$$= \left(\frac{P - D}{\alpha - \beta}\right)T_2 - \left(\frac{P - D}{\alpha - \beta}\right) \times (1 - \exp[-(\alpha - \beta)T_2]/(\alpha - \beta))$$
$$= \left(\frac{P - D}{\alpha - \beta}\right)T_2 - I_{\max}/(\alpha - \beta).$$

Expanding the term $\int_0^{T_3} I_3(t)\,\mathrm{d}t$, we get,

$$\int_0^{T_3} I_3(t)\,\mathrm{d}t = \int_0^{T_3} \frac{D}{(\alpha - \beta)}\{[\exp(\alpha - \beta)T_3 - \exp(\alpha - \beta)t_3]/\exp(\alpha - \beta)t_3\}\mathrm{d}t$$
$$= -DT_3/(\alpha - \beta) + I_{\max}/(\alpha - \beta)$$

Hence,

$$C_1 = \frac{c_1}{(T_1 + T_2 + T_3 + T_4)}\left(\frac{(P - D)T_2 - DT_3}{\alpha - \beta}\right). \quad \text{(C.2)}$$

Appendix D. Deteriorating cost per unit time

We derive Eq. (12) from Eq. (11).

$$C = \left[c\left(\int_0^{T_2} \{(P - D) + \beta I_2(t)\}\,\mathrm{d}t - I_{\max} + I_{\max} - \int_0^{T_3} \{D - \beta I_3(t)\}\,\mathrm{d}t\right)\right] \Big/ (T_1 + T_2 + T_3 + T_4). \quad \text{(D.1)}$$

Now,

$$\int_0^{T_2} \{(P - D) + \beta I_2(t)\}\,\mathrm{d}t$$
$$= \int_0^{T_2} (P - D) + \frac{\beta(P - D)}{\alpha - \beta} \times \{1 - \exp[-(\alpha - \beta)t_2]\}\,\mathrm{d}t$$
$$= \left|(P - D)t_2 + \frac{\beta(P - D)}{\alpha - \beta} \times \{t_2 + \{\exp[-(\alpha - \beta)]t_2\}/(\alpha - \beta)\}\right|_0^{T_2}$$
$$= (P - D)T_2 + \frac{\beta(P - D)}{\alpha - \beta} \times \{T_2 + [\{\exp[-(\alpha - \beta)T_2\}/(\alpha - \beta)] - 1/(\alpha - \beta)\}$$
$$= (P - D)T_2 + \frac{\beta(P - D)}{\alpha - \beta}\{T_2 + [1/(\alpha - \beta)] \times [\{\exp(-(\alpha - \beta)T_2)\} - 1]\}.$$

Using the expressions that are yielded in Appendix C, the expression for C can be written as

Hence,

$$C = c\alpha\left[\frac{(P - D)T_2 - DT_3}{(\alpha - \beta)}\right] \Big/ (T_1 + T_2 + T_3 + T_4). \quad \text{(D.2)}$$

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