

# A PRODUCTION AND MAINTENANCE PLANNING MODEL FOR THE PROCESS INDUSTRY

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## Abstract

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*In this paper a model is developed to simultaneously plan preventive maintenance and production in a process industry environment, where maintenance planning is extremely important. The model schedules production jobs and preventive maintenance jobs, while minimizing costs associated with production, backorders, corrective maintenance and preventive maintenance. The formulation of the model is flexible, so that it can be adapted to several production situations. The performance of the model is discussed and alternate solution procedures are suggested.*

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## 1. Introduction

For most companies, maintenance represents a very significant function within the overall production environment. The developments in automation, and the resulting complexity of the systems involved, have made the reliability of the machines even more important. This is especially true in the process industry, characterized by expensive specialized equipment and stringent environmental considerations. Nowadays, with profit margins decreasing, the need for a good maintenance planning and control system is obvious. However, often maintenance is a secondary process in companies that have production as their core business. The result is that maintenance does not receive enough management attention. This was confirmed by a pilot survey (Ashayeri et al. (1994)) conducted at six chemical firms where maintenance was done in an ad hoc manner. Another reason for the lack of management attention is the belief that maintenance costs cannot be controlled. Management often looks at maintenance as a necessary evil, not as a means to reduce costs ( see also Paz and Leigh (1994)).

The model developed in this paper will show that a good maintenance plan, one that is integrated with the production plan, can result in considerable savings. This integration with production is crucial because production and maintenance have a direct relationship. Any breakdown in machine operation results in disruption of production and leads to additional costs due to downtime, loss of production, decrease in productivity and quality, and inefficient use of personnel, equipment and facilities.

In the next section, the special characteristics of the process industries are briefly reviewed. Section 3 discusses the maintenance problem in general, and maintenance in process industries in particular. Section 4 will then present a mathematical model for simultaneously planning preventive maintenance and production. The validation of the model is discussed in section 5, while its performance is reviewed in section 6. Section 7 presents the overall conclusions.

## **2. Characteristics of the Process Industry**

Process industries are defined in the APICS Dictionary (1987) as:

*"Process industries are businesses that add value to materials by mixing, separating, forming or chemical reactions. Processes may be either continuous or batch and usually require rigid process control and high capital investment"*

Typical examples of process industries include the chemical industry, petroleum industry, paper manufacturing and the food and beverages branch. The relative position of the process industries is put in perspective in the Product-Process matrix developed by Hayes and Wheelwright (1979), as is shown in figure 1.

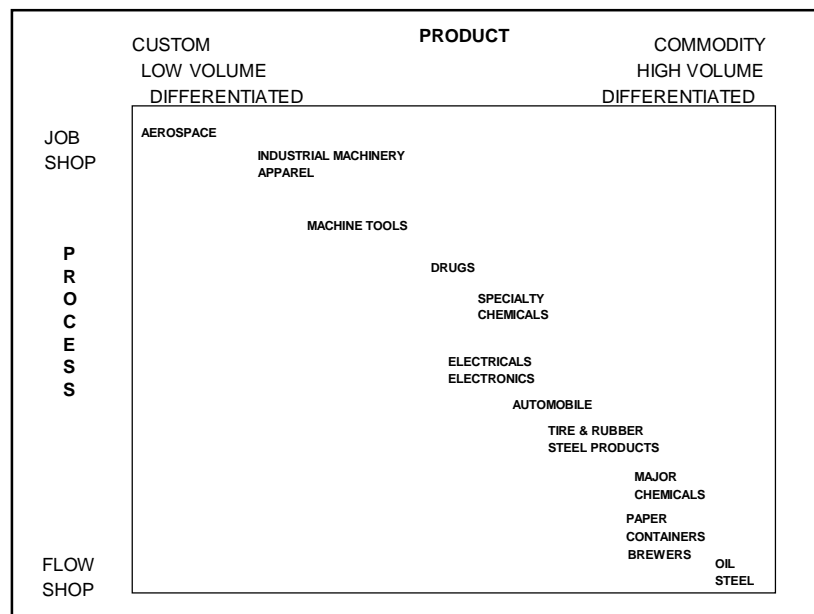


Figure 1: Product-Process matrix

Table 1 lists the major differences between the process industries and the discrete industry in the relation to the market, the production process, the quality of the products and processes, and the planning and control function (see Koene (1988), Taylor et. al. (1981), Vollman et. al. (1988)).

As to the difference in automation between discrete industries and process industries, one can observe that in the process industries the production process itself is highly automated, while in the discrete industry more emphasis is put on automating the planning and control system.

## **3. Maintenance Planning**

The importance of adequate planning in the maintenance function is emphasized by [Gits \(1994\)](#):

- Increased mechanization and automation require that a lot of maintenance has to be done. It has been found uneconomical to retain large maintenance staffs for emergencies that can be avoided through planning and systematized inspection.

- New production systems, like JIT, with minimum stocks of finished products and work-in-process, have made interruptions to production costly.
- Failure to deliver on time, with the possible loss of future business, may result from interruptions to operations.
- Preventive maintenance or correction of defective conditions, not only decreases the cost of repairs but also maintains the quality and capacity of machinery.
- Utility and service expenses for steam, electricity, gas, water, and the like are reduced by a continuous maintenance program.
- Adequate planning of maintenance operations will insure that needed spare parts and materials are on hand.

	Process industries	Discrete industries
<b>Relationship with the market</b>		
Product type	Commodity	Custom
Product assortment	Narrow	Broad
Demand per product	High	Low
Cost per product	Low	High
Order winners	Price	Speed of delivery
	Delivery guarantee	Product features
Transporting costs	High	Low
New products	Few	Many
<b>The production process</b>		
Routings	Fixed	Variable
Lay-out	By product	By function
Flexibility	Low	High
Production equipment	Specialized	Universal
Labor intensity	Low	High
Capital intensity	High	Low
Changeover times	High	Low
Work in process	Low	High
Volumes	High	Low
<b>Quality</b>		
Environmental demands	High	Low
Danger	Sometimes	Hardly
Quality measurement	Sometimes long	Short
<b>Planning &amp; Control</b>		
Production	To stock	To order
Long term planning	Capacity	Product design
Short term planning	Utilization capacity	Utilization personnel
Starting point planning	Availability capacity	Availability material
Material flow	Divergent + convergent	Convergent
Yield variability	Sometimes high	Mostly low
'Explosion' via	Recipes	Bill Of Material
By and Coproducts	Sometimes	Not
Lot tracing	Mostly necessary	Mostly not necessary

Table 1: Differences between process industries and discrete industries

Special considerations are applicable to the planning and management of maintenance in plants which necessarily operate around the clock, like most chemical plants. Much of the maintenance work can only be done while the plant is shut down. Such a shutdown usually puts an expensive

sequence of equipment out of revenue-producing activity. When the plant has to be shut down because of an emergency maintenance, costs are even higher, given the fact that most in-process products are lost. Emergency shutdowns also put a great strain on personnel and can lead to unsafe methods of operation and danger to the environment, but not least, is the difficulty of hiring specialized maintenance personnel in case of an emergency.

Maintenance planning involves planning of the periods in which preventive maintenance is to be performed, but also determines the size of the maintenance crew, when and how many materials should be purchased, and the scheduling of the different maintenance jobs. Each of these functions will be elaborated on next.

### ***Preventive maintenance***

Preventive or planned maintenance is less costly than corrective maintenance. Preventive maintenance includes preplanned and scheduled adjustments, major overhauls, inspections and lubrications, to maintain equipment and facilities in such condition that breakdowns and the need for emergency repair are minimized. Corrective or breakdown maintenance on the other hand, involve the activities to repair, replace or otherwise restore equipment or facilities to operational status after a failure has occurred.

In order to set up an effective preventive maintenance plan, management should know the length of time the unit has been in operation and the length of time needed to repair the unit, subdivided into different phases of the maintenance job. Numerous quantitative models that determine when preventive maintenance should take place can be found in the literature (see e.g. Gertsbakh (1977) and Lyonnet (1991)).

### ***Determining the size of the maintenance crew***

The problem of determining the size of the maintenance crew can be found in many articles in the literature. Most authors use queuing theory or simulation to analyze the problem. The complexity of the models depends on whether single or multi-skilled craftsmen are present (for a summary of the literature on sizing of maintenance crews we refer to Mabini (1991)).

### ***Scheduling maintenance jobs***

The operational problem of scheduling different maintenance jobs consists of assigning certain jobs to particular operators for completion at a specified time, including sequencing and dispatching. While sequencing refers to determining the succession of jobs assigned to individual workmen, dispatching is concerned with putting the "sequenced" jobs into effect. PERT scheduling and related techniques may be used to address this problem (see e.g. French (1982), Selen and Heuts (1990)).

### ***Planning maintenance material***

Planning of maintenance material implies a good inventory and ordering strategy. This topic has been addressed extensively in the literature and will not be elaborated on. A summary of methodologies is given by Silver and Peterson (1985). The planning of maintenance material also involves replacement of equipment, which is an investment decision. Replacement problems have also been discussed extensively in the literature. For a summary, we refer to Mabini (1991). Most replacement models are based on a Markovian deterioration process, that is, the deterioration of a system to a future state depends only on the immediately preceding state.

### ***Computer aided maintenance***

Because of the vast quantity of technical and economical data involved in maintenance management, computer support is very desirable. Installing a computer-aided system brings these economies (e.g. see Pintelon (1990):

- reduced time spent on preventive maintenance, because of better planning
- less overtime worked
- less time given to corrective maintenance
- reduced loss of production due to breakdowns
- less time spent on the administrative tasks of the maintenance service
- longer lifetime of the equipment, because of better preventive maintenance
- reduced energy consumption

No need to say that, as with all other computer aided technologies, maintenance planning and control should be well structured. The maintenance planning software should be integrated with all other computer applications like production planning, inventory planning, and Computer Aided Design, as a better design influences the frequency and amount of maintenance required. Expert systems could also be useful in inspection and fault diagnosis.

#### **4. A Production and Maintenance Planning Model**

Most of the articles on maintenance discuss the determination of the intervals between which preventive maintenance should be performed, using for instance the age-based model (see e.g. Zimmerman and Sovereign (1974)).

The simultaneous planning of production and preventive maintenance is a problem that has not received much attention in the literature. Joshi and Gupta (1986) make use of a production schedule and equipment failure history to plan maintenance with the aim of minimizing expected breakdown costs. A dissertation by Pintelon (1990) also develops models for simultaneous production and maintenance planning, but these determine the time between two preventive maintenance jobs without taking into account the scheduling of maintenance outside these intervals.

The model presented in this paper determines whether to schedule preventive maintenance whenever a new job is to be processed (scheduled). The model trades off the costs of preventive maintenance, and the sum of inventory holding, corrective maintenance (breakdown costs), backorder and setup costs. In other words, whenever a job's processing time is long, or its setup cost is high, or it is using an expensive material, the model examines whether it is beneficial to first perform preventive maintenance, or start the job and run the risk of a breakdown. In the latter case, a job will be delayed, resulting in a possible additional setup and lost material. This makes the maintenance model a Condition Based Maintenance System, where the conditions are determined by the length of the processing time, setup cost structure and material used.

In this model, the production environment may consist of several production lines. It is assumed that each production line has *one* bottleneck machine. When multiple lines are available, jobs which cannot be processed on their preferred production line due to preventive maintenance, may still be processed on the other production lines in case of high backorder costs.

The model developed in this paper is based on a production planning model by Bruvold and Evans (1985). They developed a mixed-integer program production planning model with significantly less binary variables as compared to other related research efforts, and which allows for different objective functions. This production planning model is modified and extended to take preventive maintenance decisions into account.

### Problem Formulation

As discussed earlier, the model deals with the scheduling of production jobs and preventive maintenance jobs on multiple production lines. Common production planning models use as principal decision variables the quantity to be produced in a certain period. The model proposed below only indicates whether or not to produce a certain product in a certain period on a certain production line. The assumption is therefore made that the production rate is constant throughout the planning period. This assumption is very reasonable in the chemical industry, where it is very hard if not impossible to change the production rate in the short term. The model minimizes total costs, including preventive maintenance costs, expected breakdown costs or expected corrective maintenance costs, inventory costs, backorder costs and setup costs. Production costs are not taken into account, as they are not relevant to the objective function.

The following additional assumptions are made in the model:

- equipment is as good as new after a preventive maintenance service
- when a breakdown occurs, the equipment has to be repaired or replaced immediately
- the expected breakdown costs increase strictly with time elapsed since the last repair
- only one product can be produced on a particular production line in a particular period
- maintenance takes the same time on all production lines
- setup costs are sequence-independent
- demand is deterministic

### The Model

The indices:

i: products,  $i=1,...,N$   
j: production line,  $j=1,...,M$   
t: time,  $t=0,...,T$

Period 0 is included as a dummy period to initialize some of the variables.

The decision variables:

$\delta_{ijt}$ : 1 if product  $i$  is produced on line  $j$  in period  $t$   
0 otherwise

$\phi_{ijt}$ : 1 if product  $i$  is produced on line  $j$  in period  $t$  but **not** in period  $t-1$   
0 otherwise

$m_{jt}$ : 1 if preventive maintenance is performed on line  $j$  in period  $t$   
0 otherwise

$z_{jt}$ : 1 if preventive maintenance is done on line  $j$  in period  $t$  but **not** in period  $t-1$   
0 otherwise

$y_{jmt}$ : 1 if in period  $t$  the last preventive maintenance job on line  $j$  ended in period  $m$   
0 otherwise

$\alpha_{ijmt}$ : 1 if  $y_{jmt}$  and  $\delta_{ijt}$  are 1

0 otherwise

$\Gamma_{it}^+$ : Net inventory of product  $i$  at the end of period  $t$  (in tons of product)

$\Gamma_{it}$ : Backorders of product  $i$  at the end of period  $t$  (in tons of product)

The variable  $y_{jmt}$  is needed, because when preventive maintenance is performed, the production line is assumed to be as good as new, so that the possibility of a breakdown solely depends on the last period in which maintenance was performed.

The parameters of the model are listed in table 2.

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$cp_j$ :	preventive maintenance costs on production line $j$ (in dollars)
$cf_j$ :	corrective maintenance costs on production line $j$ (in dollars)
$h_i$ :	inventory costs of product $i$ per ton per period (in dollars)
$l_i$ :	backorder costs of product $i$ per ton per period (in dollars)
$s_{ij}$ :	setup costs of product $i$ on production line $j$ (in dollars)
$Cap_j$ :	capacity available on production line $j$ per period (in time units per period)
$p_{ij}$ :	processing time of product $i$ on production line $j$ (in time units per ton)
$R_{ij}$ :	production rate of product $i$ on production line $j$ (in tons per period)
$D_{it}$ :	demand of product $i$ in period $t$ (in tons)
$RT$ :	length of preventive and corrective maintenance (in periods)
$\lambda_{jt}$ :	probability of breakdown on production line $j$ in period $t$ , when the previous maintenance was done in period 0
$c_{jmt}$ :	expected breakdown cost in period $t$ on production line $j$ , when the previous preventive maintenance job ended in period $m$ <sup>1</sup>

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Table 2: Parameters Used in the Production and Maintenance Planning Model

### The constraints:

Although a large number of binary variables are defined, only  $\delta_{ijt}$  and  $m_{jt}$  need to be constrained to 0 or 1. The remaining variables may be uniquely defined 0 or 1 through the use of other constraints. For example,  $\phi_{ijt}$  is uniquely determined by the following constraint:

$$\phi_{ijt} \geq \delta_{ijt} - \delta_{ij,t-1} \quad i = 1, \dots, N, j = 1, \dots, M, t = 1, \dots, T \quad (1)$$

Note that, if  $\delta_{ijt}=1$  and  $\delta_{ij,t-1}=0$  (product  $i$  is produced in period  $t$  but not in period  $t-1$ ),  $\phi_{ijt}=1$ , as it will never be greater than 1 because of its positive objective coefficient. Similar arguments apply when  $\delta_{ijt}=0$  and  $\delta_{ij,t-1}=1$ . When both  $\delta_{ijt}$  and  $\delta_{ij,t-1}$  are zero or one, the positive objective coefficient of  $\phi_{ijt}$  will make sure that  $\phi_{ijt}$  is zero.

The variable  $z_{jt}$  denotes the period in which a preventive maintenance job starts, and defined as:  $z_{jt} = (1 - m_{j,t-1})m_{jt}$ . This is a nonlinear expression, but can be solved by introducing the following three constraints<sup>2</sup>:

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<sup>1</sup> Expected breakdown cost in period  $t$  on production line  $j$ , where the previous preventive maintenance job ended in period  $m$ , is calculated as the probability of a breakdown in period  $t$ ,  $\lambda_{jt}$ , multiplied by the corrective maintenance costs, or:  $c_{jmt} = cf_j P[X_j = t] = cf_j \lambda_{jt}$ , with  $X_j$  equalling the life of production line  $j$ .

$$z_{jt} \leq 1 - m_{j,t-1} \quad j = 1, \dots, M, t = 1, \dots, T \quad (2)$$

$$z_{jt} \leq m_{jt} \quad j = 1, \dots, M, t = 1, \dots, T \quad (3)$$

$$z_{jt} \geq (1 - m_{j,t-1}) + m_{jt} - 1 \quad j = 1, \dots, M, t = 1, \dots, T \quad (4)$$

The constraint sets (2), (3) and (4) also fix  $z_{jt}$  at either 0 or 1.

To make sure that no jobs can be planned **during** a preventive maintenance (which requires RT periods of time) we need the next constraints to make  $m_{jk}$  one for every  $k=t, \dots, t+RT-1$ :

$$z_{jt} \leq m_{jk} \quad j = 1, \dots, M, t = 1, \dots, T, k = t, \dots, t + RT - 1 \quad (5)$$

Besides constraints (1) through (5), some other key constraints are needed. A fundamental material balance equation for each period is stated as:

$$\text{Production} + \text{beginning inventory} - \text{ending inventory} = \text{demand}$$

Or mathematically:

$$\sum_{j=1}^M R_{ij} \delta_{ijt} + (I_{i,t-1}^+ - I_{i,t-1}^-) - (I_{it}^+ - I_{it}^-) = D_{it} \quad i = 1, \dots, N \quad t = 1, \dots, T \quad (6)$$

where inventory is defined as  $I_{it}^+ - I_{it}^-$ , or positive net inventory minus backorders.

$R_{ij}$  is defined as the capacity of production line  $j$  divided by the processing time of product  $i$  on production line  $j$ , or:  $R_{ij} = \text{Cap}_j / p_{ij}$ .

Maintenance and production cannot be scheduled in the same period on a certain production line. Constraint (7) prevents this from happening:

$$\sum_{i=1}^N \delta_{ijt} + m_{jt} \leq 1 \quad j = 1, \dots, M \quad t = 0, \dots, T \quad (7)$$

The variable  $y_{jmt}$  must equal one when the last preventive maintenance job on production line  $j$  occurred in period  $m$  and we are now in period  $t$ . To ensure this, we define  $y_{jmt}$  as:

$$y_{jmt} = (1 - m_{jt})(1 - m_{j,t-1}) \dots (1 - m_{j,m+1}) m_{jm}$$

This is again nonlinear, so we use the same technique as applied in constraints (2) through (4):

$$y_{jmt} \leq 1 - m_{jk} \quad j = 1, \dots, M, t = 1, \dots, T, k = m + 1, \dots, t \quad (8)$$

$$y_{jmt} \leq m_{jm} \quad j = 1, \dots, M, t = 1, \dots, T, m = 0, \dots, t - 1 \quad (9)$$

$$y_{jmt} \geq \sum_{k=m+1}^t (1 - m_{jk}) + m_{jm} - (t - m) \quad j = 1, \dots, M, t = 1, \dots, T, m = 0, \dots, t - 1 \quad (10)$$

where  $y_{jmt}$  is greater than or equal to 0.

To initialize some of the variables, we introduce a dummy period 0, where the previous preventive maintenance was performed and where inventory was zero.

$$m_{j,0} = 1 \quad j = 1, \dots, M \quad (11)$$

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<sup>2</sup> The general rule for linearizing expressions like  $x_1 \dots x_k$  is as follows: replace  $x_1 \dots x_k$  with  $y$  and add the next three constraints:

- 1)  $y \leq x_j \quad 1 \leq j \leq k$
- 2)  $y \geq \sum_{j=1}^k x_j - (k-1)$
- 3)  $y \geq 0$



$$I_{i,0}^+ = I_{i,0} = 0 \quad i = 1, \dots, N \quad (12)$$

Constraint (11), in combination with (7) initialized the  $\delta_{ijt}$ 's to 0 in period 0.

As mentioned earlier, the objective function models the minimization of total costs, consisting of preventive maintenance costs, expected breakdown costs, inventory and backorder costs, and setup costs.

Preventive maintenance costs are one-time costs incurred when a preventive maintenance job is started:

$$\sum_{j=1}^M \sum_{t=1}^T z_{jt} c p_j \quad (13)$$

Expected breakdown costs are incurred each period that a job is busy. The  $y_{jmt}$  in the following expression makes sure that the correct probability of a breakdown **during** that period is selected, by only setting  $y_{jmt}$  to 1 if the previous maintenance was performed in period  $m$ :

$$\sum_{i=1}^N \sum_{j=1}^M \sum_{t=1}^T \delta_{ijt} \sum_{m=0}^{t-1} y_{jmt} c_{jmt} \quad (14)$$

This expression is non-linear, but can be linearized by replacing  $\delta_{ijt} y_{jmt}$  with the extra variable  $\alpha_{ijmt}$ . Then (14) is transformed to:

$$\sum_{i=1}^N \sum_{j=1}^M \sum_{t=1}^T \sum_{m=0}^{t-1} \alpha_{ijmt} c_{jmt} \quad (15)$$

The linearization takes two extra constraints:

$$\alpha_{ijmt} \leq \frac{1}{2} (\delta_{ijt} + y_{jmt}) \quad i = 1, \dots, N, j = 1, \dots, M, t = 1, \dots, T, m = 0, \dots, t-1 \quad (16)$$

$$\alpha_{ijmt} \geq \delta_{ijt} + y_{jmt} - 1 \quad i = 1, \dots, N, j = 1, \dots, M, t = 1, \dots, T, m = 0, \dots, t-1 \quad (17)$$

Again, the positive objective coefficients will make sure that  $\alpha_{ijmt}$  is always restricted to either 0 or 1.

Inventory costs, back-order costs and setup costs are stated respectively as:

$$\sum_{i=1}^N \sum_{t=1}^T I_{it}^+ h_i \quad (18)$$

$$\sum_{i=1}^N \sum_{t=1}^T I_{it}^- l_i \quad (19)$$

$$\sum_{i=1}^N \sum_{j=1}^M \sum_{t=1}^T \phi_{ijt} s_{ij} \quad (20)$$

## 5. Validation of the Model

To demonstrate how the model works, four maintenance scenarios are discussed. The first is a basic real-life case, with five products scheduled on two identical production lines with the same speed. The only difference being that production line 1 is old and a inhibits larger probability of a breakdown, which is reflected in higher expected breakdown costs per period. Demand for the five products is given in table 3. The planning horizon consists of ten periods of two weeks each . Round the clock production makes the capacities of both production lines equal to 336 hours per period. This capacity will be fully utilized in the basic example. The production rates (capacity divided by processing times) of the five products are given in table 4. Preventive maintenance costs are 67,500 dollars for each production line. These are one-time costs, including labor costs. material costs, opportunity costs, and the like. Costs of corrective maintenance are much higher, and amount to 250,000 dollars for each production line. Expected breakdown costs were calculated, using the probabilities of a breakdown as listed in table 5. Inventory and backorder costs are the same for each product: 10 dollars and 100 dollars per ton of product per time period, respectively. Setup costs are fixed at 1,000 dollars for each product on both lines.

<b>Demand <math>D_{it}</math></b>	Product 1	Product 2	Product 3	Product 4	Product 5
Period 1	42	0	0	84	42
Period 2	42	56	42	42	0
Period 3	21	28	84	21	42
Period 4	21	0	84	42	42
Period 5	21	56	42	21	21
Period 6	21	0	42	0	21
Period 7	0	56	42	21	42
Period 8	42	28	0	0	84
Period 9	84	0	168	0	21
Period 10	42	0	0	21	21

Table 3: Demand Table

<b>Production rate <math>R_{ij}</math></b>	Product 1	Product 2	Product 3	Product 4	Product 5
Line 1	84	56	168	84	84
Line 2	84	56	168	84	84

Table 4: Production rate

<b>probability <math>\lambda_{jt}</math></b>	period 1	period 2	period 3	period 4	period 5	period 6	period 7	period 8	period 9	period 10
line 1	0	0	0.02	0.03	0.04	0.07	0.11	0.17	0.24	0.32
line 2	0	0	0.01	0.02	0.03	0.06	0.10	0.16	0.23	0.31

Table 5: Probability of a breakdown

Figure 1 illustrates the relationship between cumulative demand and cumulative capacity, indicating that the factory loaded almost up to its maximum capacity.

This basic model was solved, using the optimization package OMP (Beyers & Partners (1993)) and resulted in the *optimal* solution listed in figure 2, with objective function value of 277,620 dollars. Figure 3 shows the inventories and backorders of the optimal solution.

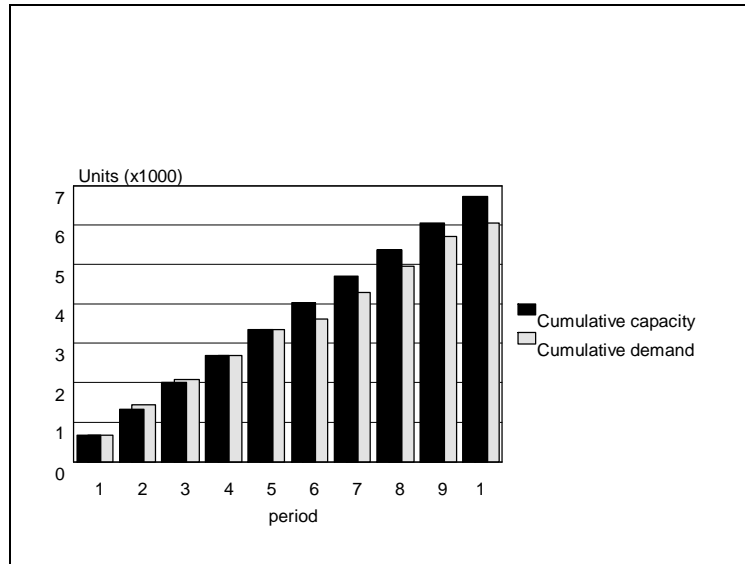


Figure 1: Cumulative demand versus cumulative capacity

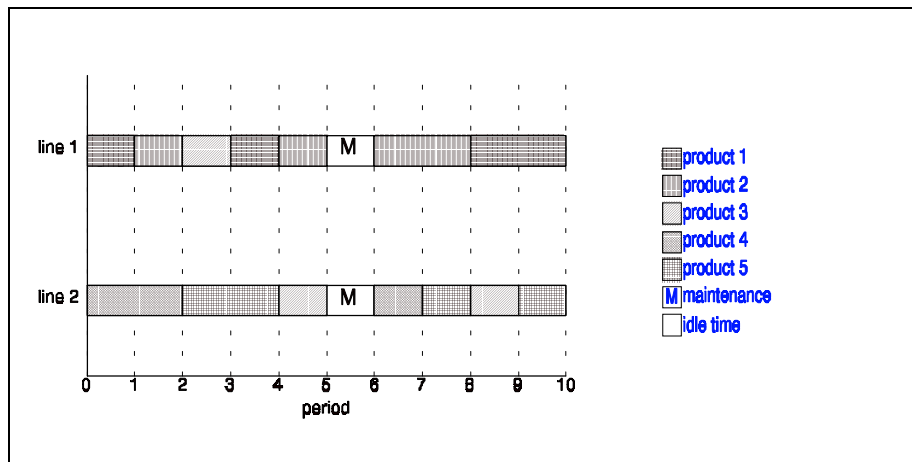


Figure 2: Optimal schedule for basic scenario

The three other scenarios referred to earlier, are summarized table 6.

In the first scenario, the setup costs of product 1 were increased to 3,000 dollars. As expected, the original optimal solution remained unchanged, because the increase in setup costs does not exceed the backorder costs of products 2 and 3 that would be rescheduled to save one setup of product 1. These results further validate our model.

The second scenario reduces the capacity of the second (cheaper) production line by 50%, resulting in a drastic increase in the number of backorders. Again, this result is to be expected, and is listed in table 7. Note that the model chose to give priority to the product with the highest total demand. The objective function value rose to 443,960 dollars. The third scenario then consisted of increasing the backorder costs of product 1. One would expect that in this case the model would try to avoid shortages of this product, which resulted. The total costs amounted to 334,270 dollars.

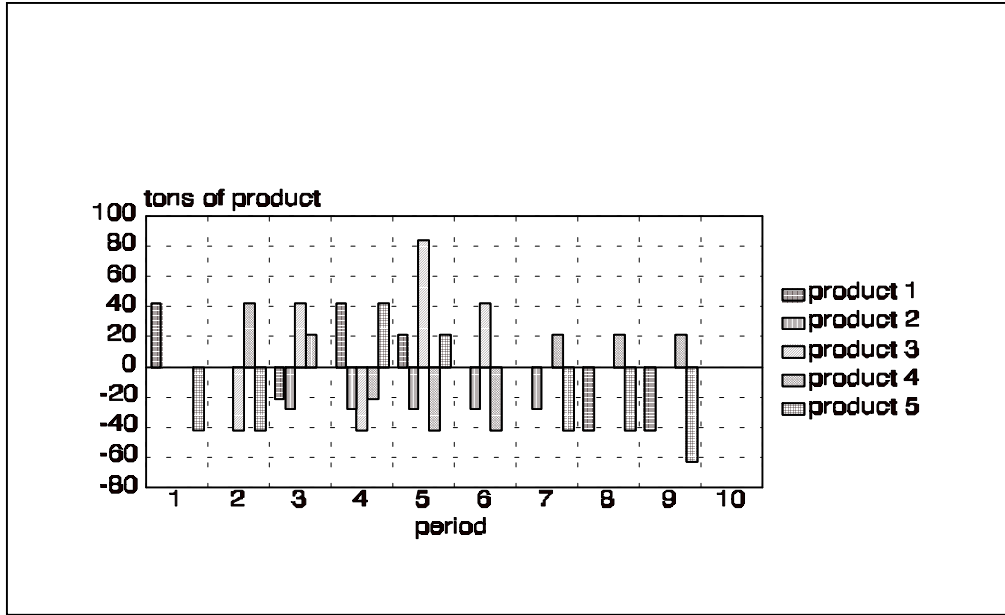


Figure 3: Inventories and backorders for the basic scenario

	Basic scenario	Scenario 1	Scenario 2	Scenario 3
Setup costs	$s_{ij}=1,000$ for all $i,j$	$s_{ij}=3,000$ for all $j$ $s_{ij}=1,000$ for $i=2,\dots,5$ , $j=1,2$	$s_{ij}=1,000$ for all $i,j$	$s_{ij}=1,000$ for all $i,j$
Capacity	$Cap_j=336$ for all $j$	$Cap_j=336$ for all $j$	$Cap_1=336$ $Cap_2=168$	$Cap_j=336$ for all $j$
Backorder costs	$l_i=1,000$ for all $i$	$l_i=1,000$ for all $i$	$l_i=1,000$ for all $i$	$l_1=3,000$ $l_i=1,000$ for $i=2,\dots,5$

Table 6: Summary of the four scenarios

	Job sequence	Objective value (\$)
Scenario 1	Line 1: 1 2 3 1 2 M 2 2 1 1 Line 2: 4 4 5 5 3 M 4 5 3 5	285,620
Scenario 2	Line 1: 4 4 5 1 M 5 2 1 1 4 Line 2: 4 3 3 1 3 3 M 5 4 5	443,960
Scenario 3	Line 1: 1 2 3 5 M 2 2 1 1 5 Line 2: 4 4 1 5 2 M 3 4 3 5	334,270

Table 7: Results of three scenarios

All three scenarios validated the working of the model. It is important to note that, while the basic scenario was solved optimally, the other three scenarios were not. Instead, good feasible solutions were obtained using a special branching strategy in the Branch and Bound procedure, because of the computation time involved. This branching method and the important issue of computational efficiency, is elaborated on next.

## 6. Branching Strategy and Performance of the Model

The optimal solution, listed in figure 1, took 24 hours of computing time on an Olivetti M6 460 with a clock speed of 66 MHz. This is of course too long for any practical implementation, and another procedure for generating good, feasible solution in a reasonable amount of time had to be found. A special branching method was used, and will be discussed below. This method was used to generate the results of the three scenarios, and produced good feasible solutions in about two minutes on a HP9000. Although the HP9000 is about three times as fast as the Olivetti M6 460, the computation time was brought down to a manageable level. However, the notion of achieving an absolute optimal solution was abandoned.

The branching method consisted of:

- branching on 0.2
- priority on binary variable  $m_{jt}$
- SOS branching

“Branching on 0.2” means that, when the LP-relaxation solution for a particular binary variable  $x$  is for instance  $x=0.25$ , the branch that sets  $x$  at 1 will be processed first, because the cut-off point is 0.2. In general, the LP relaxation solution shows which cut-off point is best used. In this model, quite a lot of variables had an LP solution of about 0.25.

Setting priority on  $m_{jt}$  means the model first decides on the values of these variables, before proceeding with the optimization. This measure proves very effective in situations where a clear priority rule exists, like in a transportation model where the mode of transportation (trucks, train, and the like) and the number of transportation vehicles to be purchased or leased, are to be decided. In such a case, priority is clearly set on the mode of transportation.

The last measure taken was to let the optimization package perform SOS branching, that is, branching on constraints. Say, for example, that the following constraint exists:

$$\sum_{k=1}^K x_k = I$$

Branching will then be done in two directions, corresponding with two parts of the constraint:

$$\begin{aligned} \sum_{k \in S} x_k &= I \\ \sum_{k \in S'} x_k &= 0 \end{aligned}$$

This leads to more efficient branching and less branches. The branching method reduced the computing time significantly. To show how close the feasible solution came to the optimal, we ran the basic scenario model of the previous chapter, with the above mentioned method. The program produced a feasible solution in about six minutes with an objective function value of 278,950 dollars. Comparing this to the optimal solution of 277,620 dollars, we may conclude that the branching heuristic performed very well.

To get an idea about the performance of this method for larger problems, a fifth scenario was generated using the basic model for a 30 period planning horizon. The model complexity is illustrated in table 8, the third column relating to the fifth scenario. The relaxed problem (LP) solution (lower bound) was found in 18 minutes and 11 seconds, the first good solution (less than double of the lower bound value) being produced in 48 minutes, all computations being performed on the HP9000.

As can be seen in table 8 the number of variables, but especially the number of constraints, increases dramatically with the number of time periods. The number of binary variables is a linear function in  $i, j$  and  $t$ :  $f(i, j, t) = j(i+1)(t+1)$ , but the total number of variables and the number of constraints have a more complicated structure in  $i, j$  and  $t$ ; defined as:

$$\text{total number of variables} = j(i+1)t! + (ij+2i+j)t + 2i$$

$$\text{number of constraints} = j(t!)^2 + 2jt! + (ij+i+5j)t + i + 2j$$

	N=5 M=2 T=10	N=5 M=2 T=20	N=5 M=2 T=30	N=10 M=2 T=10	N=20 M=2 T=10	N=5 M=3 T=10	N=5 M=5 T=10
Binary variables	132	252	372	242	462	198	330
Continuous variables	890	2970	6250	1650	3170	1280	2060
Total variables	1022	3222	6622	1892	3632	1478	2390
Constraints	2024	8634	22264	2184	3604	2181	3595

Table 8: Model Complexity (RT expressed in periods)

Fortunately, in the chemical industry, there is hardly ever a need for a planning horizon of over 20 periods, except perhaps in the pharmaceutical branch, where smaller time buckets could be useful because of the smaller batches and larger variety of products. Even then the model could still be used with a rolling horizon. Ten products on three production lines is also the maximum to be scheduled in most chemical companies. If more products are manufactured, grouping could be used.

For the sake of completeness, it is worth pointing out that the model can also be solved using one of the standard algorithms for solving mixed binary linear programming problems. For a review of some of these algorithms we refer to Shapiro (1979). Other computationally efficient commercial software include IBM Mathematical Programming system Extended/370 Program Reference Manual (1979), or IBM Optimization Subroutine Library (OSL) (1990), which have already been extensively applied in a study of large scale binary linear programming problems (Crowder et al. (1983)). MPSARX, developed by Van Roy and Wolsey (1987), is a state-of-the-art Mathematical Programming system (MPS) that can be implemented for solving our model. For additional references, see also Van Roy (1983, 1989), Van Roy and Wolsey (1983), Mikhalevich et al. (1983), Jackson and O'Neil (1983), Côté and Laughton (1984), Glover (1984), and Jeroslow (1984). Finally, for a review of the performance evaluation literature of mixed binary programming algorithms, we refer to von Randow (1985, pp. 198 and 199).

Obviously, for large scale problems the model may be hard to solve to optimality with a simple branching policy or standard software. In such cases, one may be forced to stop the procedure early, or to develop an appropriate heuristic. Solution techniques such as Simulated Annealing or Tabu search may be used as well. A direction for further research therefore involves improving the computational efficiency for large scale problems, which the authors are currently pursuing.

## **7. Conclusions**

Increased implementation of Computer Integrated Manufacturing concepts in the process industry raises issues in the planning of production and maintenance within an integrated process manufacturing system. The interactions of different production and preventive maintenance decisions impact on the proper use of available capacity and company profits. In order to tackle these interactions, a model was developed which minimizes several production and maintenance related cost factors during long- or medium-term planning horizons, taking into account the probability of break-downs.

Sequence dependent setups would make the formulation even tighter and result in a shorter computation time. Although the model was developed for the chemical industry, it may have useful application possibilities for the discrete manufacturing industries as well, particularly in flexible assembly systems where a bottleneck machine (cell) exists and the production is performed on a Just-in-Time basis.

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