Lot sizing, preventive maintenance, and warranty decisions for imperfect production systems

Article *in* Journal of Quality in Maintenance Engineering · January 2006

DOI: 10.1108/13552510610654547

2 authors:


Mohamed Ben-Daya
American University of Sharjah


Syed Mubasher Noman
King Fahd University of Petroleum and Minerals

Journal of Quality in Maintenance Engineering

Lot sizing, preventive maintenance, and warranty decisions for imperfect production systems

M. Ben-Daya S.A. Noman

Lot sizing, preventive maintenance, and warranty decisions for imperfect production systems

M. Ben-Daya and S.A. Noman
Systems Engineering Department,
King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

Abstract

Purpose – Sets out to develop an integrated model that considers simultaneously inventory production decisions, PM schedule, and warranty policy for a deteriorating system that experiences shifts to an out of control state. The time to shift follows a general probability distribution with increasing hazard rate, so that time-based PM is effective in improving the system reliability.

Design/methodology/approach – A profit function is used to model the production system. Optimization techniques are used to generate optimal solutions for the problem. Although global optimality cannot be guaranteed, empirical results show that global optimal solutions are obtained.

Findings – The integrated model provides decisions on inventory levels, production run length, and PM schedule simultaneously. It is illustrated through numerical examples that investment in PM can lead to savings in warranty claims for repairable products. As a result, the overall profit per unit, in certain cases, is higher with PM than without PM.

Research limitations/implications – The production system is taken, numerical examples are presented and a sensitivity analysis is conducted to gain more insight into the developed model. In particular, the numerical analysis shows that a better PM program reduces warranty claims.

Practical implications – In addition to the joint optimization of production/inventory decisions and PM schedule, such models can be very useful in making resource allocation decisions between warranty and PM programs. It is clear from the numerical analysis that a better PM program reduces warranty claims.

Originality/value – The paper provides a joint optimization of production inventory decisions and the PM schedule for a system subject to a time to shift that follows a general probability distribution. Previous research considered only an exponential distribution and did not consider PM.

Keywords Preventive maintenance, Warranties, Lot size, Numerical analysis

Paper type Research paper

1. Introduction

The classical economic production quantity (EPQ) model (Nahmias, 1997; Silver *et al.*, 1998) assumes that the output of the production system is defect-free. Rosenblatt and Lee (1986) have found that when the production process is subject to a random process deterioration that shifts the system from an in-control state to an out-of-control state (and producing non-conforming items then), the resulting optimal EPQ is smaller than that of the classical model. Porteus (1986) has observed similar results. Lee and Rosenblatt (1989) also incorporated maintenance by inspection with restoration cost dependent on the detection delay. They assumed that the deterioration of the process is exponentially distributed. Their work was extended by Hariga and Ben-Daya (1998) and Lin *et al.* (1991), among others, for the case where the deterioration of the process

The authors would like to acknowledge the support of King Fahd University of Petroleum and Minerals.

Journal of Quality in Maintenance Engineering
Vol. 12 No. 1, 2006
pp. 68-80

1355-2511
DOI 10.1108/13552510610654547

follows a general distribution. For a further review of the determination of EPQ with imperfect production processes, the reader is referred to Ben-Daya and Rahim (2001).

In some cases, defects are detected after the product reaches the consumer. A common practice for many vendors is to offer a warranty period on their products. Warranties provide a protection to the user and, from a marketing point of view, induce him to buy the product. Djamaludin *et al.* (1994) studied the effects of product warranty policy on the optimal lot size for a production system in which the process goes out-of control with a given probability each time it produces an item and in which the inventory holding cost is assumed to be zero. The work of Djamaludin *et al.* (1994) was extended by Wang and Sheu (2003) to a discrete general shift distribution to provide an optimal lot size so that the long-run total cost of the set-up, the inventory holding and the warranty is minimized. Determination of the optimal production run length for a deteriorating production system in which products are sold with free minimal repair warranty was studied by Yeh *et al.* (2000). It is assumed that the time to shift from an in-control state to an out-of-control state is exponentially distributed, i.e. the process failure rate is a constant. Wang (2004) extended this work to consider processes subjected to random deterioration from an in-control state to an out-of-control state with a general shift distribution. Yeh and Chen (2005) revisited the problem of the joint determination of the optimal lot size and product inspection policy for a deteriorating production system when products are sold with free minimal repair warranty.

In this paper, we assume that the product is repairable and a free minimal repair warranty is offered to the user in case of failure of the item for a fixed period of time. We assume also that PM of the production process has an effect on the failure rate of the product sold under warranty while in use.

Production and inventory decisions, the preventive maintenance schedule and warranty policies are clearly interrelated problems. Traditionally, these three problems are dealt with separately. There is a need to develop models that capture the interdependence between them, leading to their joint optimization. Previous research looked only at the integration of warranty and lot sizing but did not consider PM as a means of improving the reliability of products. The purpose of this paper is to develop a model that links EPQ, maintenance requirements and warranty for a process with a general deterioration distribution. A solution procedure is proposed and numerical examples are presented for illustrative purposes. A sensitivity analysis with respect to key model parameters is conducted to provide further insights.

The remainder of this paper is organized as follows. The problem definition, assumptions and notation are presented in the next section. Section 3 deals with model development. The solution procedure and numerical results are discussed in section 4. Finally, section 5 contains a summary of the paper and some concluding remarks.

2. Model development

2.1 Statement of the problem

Consider a production system that can be in one of two states – in-control or out-of-control. It is assumed that the elapsed time, t, of the system in the in-control state follows a known general probability distribution. Once the system shifts to an out-of-control state, it stays there until the system is inspected to reveal its state and restored back to an in-control state. The restoration cost is a function of the detection delay (Rosenblatt and Lee, 1986).

PM activities are known to improve the reliability of equipment. In other words, preventively maintained equipment will experience fewer failures. One way to model this is to assume that, after PM, the failure rate of the system is somewhere between "as good as new" and "as bad as old". This concept is called "imperfect maintenance" and has been introduced by many authors (Nakagawa, 1980; Pham and Wang, 1996). It can be assumed that the failure rate of the equipment decreases after each PM. This amounts to a reduction in the effective age of the equipment. In this paper, the system undergoes preventive maintenance at regular intervals of length h_j ($j = 1, 2, \ldots, n$), with $\sum_{j=1}^{n} h_j$ being the total production time per cycle. The system's effective age is reduced by a factor b_j ($0 = b_0 \leq b_1 \leq b_2 \leq \ldots \leq b_n < 1$) after the jth PM.

Due to manufacturing variability, an item is nonconforming with probability π_1 when the process is in-control and π_2 when the process is out-of-control, where $\pi_1 < \pi_2$. Since non-conforming items can only be detected after a period of time in use, all the items produced are released for sale with a free minimal repair warranty. Failures that occur within the warranty period (W) result in valid warranty claim and are rectified by minimal repairs at no cost to the buyer. After minimal repair, the hazard rate of an item remains the same as that just before the failure. Every minimal repair incurs a cost of C_{m} to the manufacturer. However, it is assumed that due to a few obvious characteristics, the units produced in the out-of-control state can be differentiated from those produced in the in-control state, and can be sold at a lower price s.

For the production system described above, the expected total cost incurred includes the manufacturing cost, the inventory holding cost, the set-up cost, inspection and preventive maintenance costs, the restoration cost and the warranty cost. The problem is to find the optimal production run length t_n and inspection schedule $\{h_j; j = 1, 2, \ldots, n\}$, such that profit is maximized. First, we state the main assumptions under which the model is developed.

2.2 Model assumptions

- The process inspection is assumed to be error free.
- The probability of producing a nonconforming unit in the in-control state is less than that in the out-of-control state.
- The hazard rate for a non-conforming item is greater than that of a conforming item.
- The repair cost of both conforming and non-conforming items is the same.
- Age reduction after PM is dependent on the amount invested in PM.
- Units produced in the in-control state can be differentiated from those produced in the out-of-control state and can be sold at higher price.
- The process restoration cost is a function of the detection delay.
- The scheduled inspection, preventive maintenance and restoration durations are negligible.

2.3 Notations

The following are the notations used in this paper:

n	number of intervals in production run;
Q	production lot size;
d	demand rate(units/unit time);
p	production rate;

A	set-up including initial inspection cost;
ν	selling price per items produced during the in-control state;
s	selling price per items produced during the out-of-control state;
h	inventory carrying cost (\$/unit time,);
C_{p}	production cost excluding set-up;
π_1	probability of producing non-conforming item in the in-control state;
π_2	probability of producing non-conforming item in the out-of-control state;
y_j	age of the system before PM;
b_j	age reduction factor after the jth PM;
η	imperfectness factor;
w_j	$b_j y_j$ age of the system after PM;
h_j	production time interval at the jth PM;
t_{p}	$\sum_{j=1}^{n} h_j$, total production time;
$\mathrm{ETC}(n,\{h_j\})$	expected total cost per unit produced;
$\mathrm{ETP}(n,\{h_j\})$	expected total profit per unit produced;
$E(\mathrm{toc}_j)$	expected production time in out-of-control state in the jth interval;
N_j	expected number of defective items produced in the jth interval;
$E(N)$	expected number of items produced during the out-of-control period;
$E(NC)$	expected number of non-conforming items produced per cycle;
W	length of warranty period;
$r_1(t)$	hazard function for conforming items;
$r_2(t)$	hazard function for non-conforming items;
C_{m}	minimal repair cost during warranty period;
WC	warranty cost per item;
C_{r0}	fixed restoration cost;
C_{r1}	time-dependent restoration cost;
C_{pm}	inspection and preventive maintenance cost of the process; and
C_{pm}^{0}	maximum PM cost resulting in perfect system.

2.4 Model formulation

The objective is to maximize profit, which is defined as the difference between revenues and costs. The expected revenue per unit produced is given by:

$$\text{REV} = \frac{\nu[pt_{\text{p}} - E(n)] + sE(N)}{pt_{\text{p}}} = \nu - (\nu - s)\frac{E(N)}{pt_{\text{p}}}. \tag{1}$$

In addition to the set-up cost A, the production cost per item C_{p} and the inspection and PM cost C_{pm}, the following costs are incurred per cycle:

- the inventory carrying cost;
- the restoration cost; and
- the warranty cost

These costs are derived next. The inventory carrying cost is given by:

$$E(HC) = h\frac{(p - d)t_{\text{p}}}{2d}. \tag{2}$$

Before deriving the remaining costs, let us explain how the effect of PM on the system is modeled. After each PM, the age of the system is somewhere between "as good as new" and "as bad as old", depending on the level of PM activities. The reduction in the age of the equipment is a function of the cost of preventive maintenance. Let $y_k(w_k)$ denote the effective age of the equipment right before (after) the kth PM. Then, a linear relationship is considered between age reduction and PM level as follows:

$$w_k = \left(1 - \eta^{k-1}\frac{C_{\text{pm}}}{C^0_{\text{pm}}}\right)y_k, \tag{3}$$

where $0 < \eta \leq 1$. Non-linear relationships between age reduction and PM cost may be considered as well (Ben-Daya and Makhdoum, 1997).

Note that the effective age of the equipment at time t_j is given by:

$$y_1 = h_1, \tag{4}$$

$$y_j = w_{j-1} + h_j, \quad j = 2, \ldots, m. \tag{5}$$

Two important remarks are in order:

(1) There is a finite number of PM levels dictated by the system maintenance requirements. Hence the variable C_{pm} is not continuous but can only assume a finite number of possible values. The above equations simply provide the underlying function relating age reduction to PM cost, and do not in any way imply that their relationship is continuous.

(2) The parameter η is an imperfectness factor which implies that there is a degradation in the effect of PM on the age of the system. A full PM brings the system farther from the "as good as new condition" as more PMs are performed.

We are now ready to derive the restoration and warranty costs. The process is restored to the in-control state whenever it is found to be out-of-control during an inspection. Therefore the expected restoration cost during the interval of length h_j is given by:

$$E(RC_j) = \frac{\int_{w_{j-1}}^{y_j}[C_{\text{r0}} + C_{\text{r1}}(y_j - t)]f(t)\,\mathrm{d}t}{\bar{F}(w_{j-1})}.$$

Hence the expected restoration cost per item for a cycle is given by:

$$E(RC) = \sum_{j=1}^{n-1} \frac{C_{\mathrm{r}0} \int_{w_{j-1}}^{y_j} f(t)\,\mathrm{d}t}{pt_{\mathrm{p}} \bar{F}(w_{j-1})} + \sum_{j=1}^{n-1} \frac{E(\mathrm{toc}_j)}{pt_{\mathrm{p}}}$$

$$= \frac{C_{\mathrm{r}0}}{pt_{\mathrm{p}}} \left[n - 1 - \sum_{j=1}^{n-1} \frac{F(\bar{y}_j)}{F(w_j - 1)} \right]$$

$$+ \frac{C_{\mathrm{r}1}}{pt_{\mathrm{p}}} \sum_{j=1}^{n-1} \frac{\left\{ y_j [F(y_j) - F(w_j - 1)] - \int_{w_{j-1}}^{y_j} tf(t)\,\mathrm{d}t \right\}}{\bar{F}(w_{j-1})}. \quad (6)$$

The expected number of items produced during the out-of-control period of the jth interval of length h_j is given by:

$$E(N_j) = pE(\mathrm{toc}_j),$$

where $E(\mathrm{toc}_j)$ is the time the machine is in the out-of-control state before it is inspected and restored back to the in-control state, and is given by:

$$E(\mathrm{toc}_j) = \frac{\int_{w_{j-1}}^{y_j} (y_j - t) f(t)\,\mathrm{d}t}{\bar{F}(w_{j-1})} = \frac{\left\{ y_j [F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)\,\mathrm{d}t \right\}}{\bar{F}(w_{j-1})}. \quad (7)$$

Therefore, expected production in the out-of-control state is given by:

$$E(N_j) = \frac{p\left\{ y_j [F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)\,\mathrm{d}t \right\}}{\bar{F}(w_{j-1})}. \quad (8)$$

The total expected number of items produced during the out-of-control periods during a production cycle is given by:

$$E(N) = \sum_{j=1}^{n} E(N_j) = \sum_{j=1}^{n} \frac{p\left\{ y_j [F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)\,\mathrm{d}t \right\}}{\bar{F}(w_{j-1})}. \quad (9)$$

Since items produced during in-control periods will be non-conforming with probability π_1 and items produced during out-of-control periods will be non-conforming with probability π_2, the total expected number of non-conforming items per cycle is given by:

$$E(NC) = \pi_1 [pt_{\mathrm{p}} - E(N)] + \pi_2 E(N) = (\pi_2 - \pi_1) E(N) + pt_{\mathrm{p}} \pi_1. \quad (10)$$

The fraction of non-conforming items per cycle is as follows:

$$q = \frac{E(NC)}{pt_{\mathrm{p}}} = \sum_{j=1}^{n} \frac{p\left\{ y_j [F(y_j) - F(w_{j-1})] - \int_{w_{j-1}}^{y_j} tf(t)\,\mathrm{d}t \right\}}{\bar{F}(w_{j-1})} \frac{[\pi_2 - \pi_1]}{pt_{\mathrm{p}}} + \pi_1. \quad (11)$$

The hazard function of the conforming items is $r_1(x)$, and that of the non-conforming items is $r_2(x)$. Hence, the expected warranty cost is given by:

$$E(WC) = C_{\mathrm{m}}\left[(1-q)\int_0^w r_1(x)\,\mathrm{d}x + q\int_0^w r_2(x)\,\mathrm{d}x\right]. \tag{12}$$

The expected total cost per item is made up of the manufacturing cost, the inspection and PM cost, the inventory carrying cost, the restoration cost and the warranty cost. Thus:'

$$\mathrm{ETC}(n,\{h_j\}) = C_{\mathrm{p}} + \frac{A+(n-1)C_{\mathrm{pm}}}{pt_{\mathrm{p}}} + E(HC) + E(RC) + E(WC), \tag{13}$$

where $E(HC)$, $E(RC)$ and $E(WC)$ are given by equations (2), (6) and (12), respectively. Therefore, the expected total profit per unit produced can be obtained as:

$$\mathrm{ETP}(n,\{h_j\}) = \nu - (\nu - s)\frac{E(N)}{pt_{\mathrm{p}}} - \mathrm{ETC}, \tag{14}$$

$$\begin{aligned}\mathrm{ETP}(n,\{h_j\}) = {} & \nu - \frac{(\nu-s)E(N)}{pt_{\mathrm{p}}} - C_{\mathrm{p}} - \frac{A+(n-1)C_{\mathrm{pm}}}{pt_{\mathrm{p}}} - \frac{(p-d)t_{\mathrm{p}}h}{2d}\\ & - \frac{C_{\mathrm{r0}}}{pt_{\mathrm{p}}}\left[n-1-\sum_{j=1}^{n-1}\frac{\bar{F}(y_j)}{\bar{F}(w_{j-1})}\right] - \frac{C_{\mathrm{r1}}}{pt_{\mathrm{p}}}\sum_{j=1}^{n-1}E(\mathrm{toc}_j)\\ & - C_{\mathrm{m}}\left\{\frac{[E(N)(\pi_2-\pi_1)+pt_{\mathrm{p}}\pi_1](R_2-R_1)}{pt_{\mathrm{p}}} + R_1\right\},\end{aligned}$$

which can be simplified to:

$$\begin{aligned}\mathrm{ETP}(n,\{h_j\}) = {} & \nu - C_{\mathrm{p}} - \frac{A+(n-1)C_{\mathrm{pm}}}{pt_{\mathrm{p}}} - \frac{(p-d)t_{\mathrm{p}}h}{2d} - \frac{C_{\mathrm{r0}}}{pt_{\mathrm{p}}}\left[n-1-\sum_{j=1}^{n-1}\frac{\bar{F}(y_j)}{\bar{F}(w_{j-1})}\right]\\ & - \frac{[\nu-s+C_{\mathrm{m}}(\pi_2-\pi_1)(R_2-R_1)]}{t_{\mathrm{p}}}\sum_{j=1}^{n}\frac{y_j[F(y_j)-F(w_{j-1})]-\int_{w_{j-1}}^{y_j}tf(t)\,\mathrm{d}t}{\bar{F}(w_{j-1})}\\ & - \frac{C_{\mathrm{r1}}}{pt_{\mathrm{p}}}\sum_{j=1}^{n-1}\frac{\left\{y_j[F(y_j)-F(w_{j-1})]-\int_{w_{j-1}}^{y_j}tf(t)\,\mathrm{d}t\right\}}{\bar{F}(w_{j-1})} - Cm[\pi_1(R_2-R_1)+R_1],\end{aligned} \tag{15}$$

where $R_1 = \int_0^w r_1(x)\,\mathrm{d}x$ and $R_2 = \int_0^w r_2(x)\,\mathrm{d}x$.

3. Solution procedure and numerical results

First, we discuss the problem of solving the above model to obtain the optimal values for the decision variables. Also, we will discuss the way by which the inspection frequency should be regulated and the optimization procedure used to determine the optimal solution.

The problem is to determine simultaneously the optimal lengths of the inspection intervals, namely $h_1, h_2, \ldots, h_m$, and the number of inspections and PMs, n.

For a Markovian model, a uniform inspection scheme provides a constant integrated hazard over each interval. Banerjee and Rahim (1988) extended this fact to non-Markovian models by choosing the length of inspection intervals such that the integrated hazard over each interval is the same for all intervals, that is:

$$\int_{w_{j-1}}^{y_j} \frac{f(t)}{\bar{F}(t)} \mathrm{d}t = \int_0^{h_1} \frac{f(t)}{\bar{F}(t)} \mathrm{d}t. \tag{16}$$

If the time that the process remains in the in control state follows a Weibull distribution, that is, its probability density function is given by:

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \mathrm{e}^{-(\frac{t}{\theta})^{\beta}}, \quad t > 0, \quad \beta \geq 1, \quad \theta > 0,$$

then using equation (16), the length of the inspection intervals h_j ($j = 2, \ldots, m$) can be determined recursively as follows:

$$h_j = [(w_{j-1})^{\beta} + h_1^{\beta}]^{1/\beta} - w_{j-1}, \quad j = 2, \ldots, m. \tag{17}$$

This relationship reduces the number of decision variables from $n + 1$ to only two, namely n and h_1. The following simple solution procedure is proposed to minimize the profit function ETP.

3.1 Solution algorithm

- Step 1 – set $n = 1$.
- Step 2 – calculate the optimal h_1 which maximizes profit $\mathrm{ETP}(n, h_1^*)$.
- Step 3 – if n = 1, set $\mathrm{ETP}^* = \mathrm{ETP}(n, h_1^*)$.
- Step 4 – If $\mathrm{ETP}(n, h_1^*) \geq \mathrm{ETP}^*$ then $n = n + 1$, go to step 2 else.
- Step 5 – $n^* = n - 1$ and $h_1^* = h_1$ corresponding to n^*.

Since it is difficult to find a closed form solution, the Golden section method is used to determine the optimal value of h_1 for each given value of n, following the above algorithm. The computer code is written in Fortran, while IMSL subroutines of GAMIC are used to evaluate the gamma function. The program is run on a Pentium III computer with 256 MB RAM.

Note that for Weibull distribution, we have:

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \mathrm{e}^{-(\frac{t}{\theta})^{\beta}},$$

$$F(t) = 1 - \mathrm{e}^{-(\frac{t}{\theta})^{\beta}},$$

$$\bar{F}(t) = \mathrm{e}^{-(\frac{t}{\theta})^{\beta}}.$$

Using equation (16), it can also be shown that $\bar{F}(y_j) = \bar{F}(w_{j-1})\bar{F}(h_1)$. Hence:

$$\begin{aligned} \bar{F}(y_j) &= \bar{F}(w_{j-1})\bar{F}(h_1), \\ \mathrm{e}^{-\left(\frac{y_j}{\theta}\right)^\beta} &= \mathrm{e}^{-\left(\frac{w_{j-1}}{\theta}\right)^\beta}\,\mathrm{e}^{-\left(\frac{h1}{\theta}\right)^\beta}, \\ h_j &= (w_{j-1}^\beta + h_1^\beta)^{1/\beta} - w_{j-1}, \end{aligned} \tag{18}$$

$$\int_{w_{j-1}}^{y_j} tf(t)\,\mathrm{d}t = \int_{w_{j-1}}^{y_j} t\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}\mathrm{e}^{-\left(\frac{t}{\theta}\right)^\beta}\,\mathrm{d}t = \theta\int_{(w_{j-1}/\theta)^\beta} U^{1/\beta}\,\mathrm{e}^{-U}\,\mathrm{d}U.$$

This is similar to an incomplete gamma function. An incomplete gamma function $\gamma(a,x) = \int_0^x t^{a-1}\,\mathrm{e}^{-t}\,\mathrm{d}t$ is bounded by a complete gamma function $\Gamma(a) = \int_0^\infty t^x\,\mathrm{e}^{-t}\,\mathrm{d}t$ and can be calculated from numerical tables or by subroutines for numerical integration. Therefore, the above expression can be written as:

$$\theta\gamma\left[1+1/\beta, \left(\frac{y_j}{\theta}\right)^\beta\right] - \theta\gamma\left[1+1/\beta, \left(\frac{w_{j-1}}{\theta}\right)^\beta\right]. \tag{19}$$

The profit function for a Weibull distribution is given by:

$$\begin{aligned} \mathrm{ETP}(n, \{h_j\}) &= \nu - C_\mathrm{p} - \frac{A+(n-1)C_\mathrm{pm}}{pt_\mathrm{p}} - \frac{(p-d)t_\mathrm{p}h}{2d} \\ &- \frac{C_{\mathrm{r}0}}{pt_\mathrm{p}}\left(n-1-\sum_{j=1}^{n-1}\frac{\mathrm{e}^{-\left(\frac{y_j}{\theta}\right)^\beta}}{\mathrm{e}^{-\left(\frac{w_{j-1}}{\theta}\right)^\beta}}\right) \\ &- \frac{[\nu - s + C_\mathrm{m}(\pi_2-\pi_1)(R_2-R_1)]}{t_\mathrm{p}}\sum_{j=1}^{n}\frac{y_j\left(\mathrm{e}^{-\left(\frac{w_{j-1}}{\theta}\right)^\beta} - \mathrm{e}^{-\left(\frac{y_j}{\theta}\right)^\beta}\right) - \theta\Psi_j}{\mathrm{e}^{-\left(\frac{w_{j-1}}{\theta}\right)^\beta}} \\ &- \frac{C_{\mathrm{r}1}}{pt_\mathrm{p}}\sum_{j=1}^{n}\frac{y_j\left(\mathrm{e}^{-\left(\frac{w_{j-1}}{\theta}\right)^\beta} - \mathrm{e}^{-\left(\frac{y_j}{\theta}\right)^\beta}\right) - \theta\Psi j}{\mathrm{e}^{-\left(\frac{wj-1}{\theta}\right)^\beta}} - C_\mathrm{m}[\pi_1(R_2-R_1)+R_1]. \end{aligned} \tag{20}$$

where:

$$\Psi_j = \gamma\left[1+1/\beta, \left(\frac{y_j}{\theta}\right)^\beta\right] - \gamma\left[1+1/\beta, \left(\frac{w_{j-1}}{\theta}\right)^\beta\right].$$

Now, let us consider the following numerical example. Suppose that the life-time distribution of both conforming and non conforming items are Weibull with hazard rate functions $r_1(t) = \lambda_1^{\beta_1}\beta_1 t^{\beta_1-1}$ $r_2(t) = \lambda_2^{\beta_2}\beta_2 t^{\beta_2-1}$, respectively. Assume that the shape parameters are:

$$\beta_1 = \beta_2 = 1.5 + 0.5\left(1 - \frac{C_{\mathrm{pm}}}{C^0_{\mathrm{pm}}}\right),$$

and the scale parameters are $\lambda_1 = 1/6$ and $\lambda_2 = 1/2$, and the warranty period $W = 6$. The remaining parameters are $p = 600,\ d = 400,\ A = 1,000,\ C_{\mathrm{pm}} = 80,\ C^0_{\mathrm{pm}} = 100,\ \nu = 20,\ s = 15,\ \pi_1 = 0.15,\ \pi_2 = 0.65,\ C_{\mathrm{m}} = 1,\ C_{\mathrm{r0}} = 20,\ C_{\mathrm{r1}} = 1,\ C_{\mathrm{p}} = 10,\ h = 1,\ b_j = [1 - \eta^{j-1}(C_{\mathrm{pm}}/C^0_{\mathrm{pm}})],\ \beta = 2$ and $\theta = 1$.

Table I shows the effect of θ and thereby the mean of the production process shift to the out-of-control state on various costs (QC = lost revenue due to selling out-of-control production at lower price, RC = restoration cost, WC = warranty cost), profit, h_1, n and total production time per cycle, t_{p}. It can be seen that for lower values of θ, where t_{p} is much higher than mean, profits with PM are much higher than those without PM. This difference between profits with and without PM continues to decrease with increasing mean. For a mean higher than t_{p}, the profit difference with PM and without

θ	Mean	h_1	n	t_{p}	QC	RC	WC	Profit	
*0.3	*0.265	0.149	19	2.200	0.457	0.059	1.939	5.145	With PM
		0.157	16	0.626	0.739	0.191	2.791	2.662	Without PM
*0.5	*0.443	0.206	14	2.289	0.316	0.029	1.871	5.724	With PM
		0.209	31	1.170	0.458	0.138	2.566	4.258	Without PM
*0.8	*0.709	0.272	11	2.418	0.218	0.015	1.824	6.097	With PM
		0.288	27	1.490	0.325	0.070	2.460	5.074	Without PM
*1.0	*0.886	0.322	9	2.364	0.194	0.011	1.812	6.235	With PM
		0.331	24	1.620	0.272	0.049	2.417	5.357	Without PM
*1.3	*1.152	0.372	8	2.449	0.154	0.007	1.793	6.371	With PM
		0.390	21	1.790	0.218	0.032	2.374	5.622	Without PM
*1.5	*1.329	0.416	7	2.412	0.144	0.006	1.788	6.435	With PM
		0.426	19	1.860	0.193	0.025	2.354	5.741	Without PM
*2.0	*1.772	0.491	6	2.465	0.113	0.003	1.773	6.546	With PM
		0.503	16	2.010	0.149	0.015	2.319	5.937	Without PM
*3.0	*2.658	0.688	4	2.363	0.096	0.002	1.766	6.669	With PM
		0.632	12	2.190	0.101	0.007	2.281	6.134	Without PM

Table I. Results for varying mean time to shift by changing θ

C_{m}	h_1	n	t_{p}	QC	RC	WC	Profit	
*1.0	0.322	9	2.364	0.194	0.011	1.812	6.235	With PM
	0.331	24	1.623	0.272	0.049	2.417	5.357	Without PM
*1.5	0.299	10	2.428	0.169	0.010	2.701	5.331	With PM
	0.312	26	1.592	0.240	0.048	3.588	4.155	Without PM
*2.0	0.291	10	2.364	0.160	0.010	3.593	4.432	With PM
	0.299	27	1.553	0.219	0.047	4.750	2.963	Without PM
*2.5	0.274	11	2.434	0.143	0.009	4.471	3.534	With PM
	0.287	28	1.517	0.201	0.046	5.901	1.779	Without PM
*3.0	0.268	11	2.383	0.137	0.009	5.357	2.641	With PM
	0.277	28	1.466	0.187	0.045	7.048	0.602	Without PM

Table II. Results for varying cost of minimal repair C_{m}

PM is less pronounced due to a higher restoration cost caused by more frequent shifts to the out-of-control state.

Table II shows the effect of minimal repair cost C_m on various costs, profit, t_1, n and t_p. It can be seen that as C_m increases the profit decreases, as expected. This is mainly due to an increase in the warranty cost, as it is directly related to C_m. To reduce the effect of a higher warranty cost, production runs are shorter and the number of PMs is increased to benefit from their effect on the chances of going out-of-control. Note the percentage difference in profit between the case with PM and that without PM. For

Table III.
Results for varying warranty period W

W	t_1	n	t_p	QC	RC	WC	Profit	
*3.0	0.508	5	2.204	0.445	0.014	0.638	6.839	With PM
	0.369	22	1.728	0.341	0.051	0.618	7.188	Without PM
*4.0	0.498	5	2.161	0.429	0.013	1.002	6.472	With PM
	0.356	23	1.707	0.317	0.051	1.090	6.709	Without PM
*5.0	0.488	5	2.116	0.412	0.013	1.421	6.048	With PM
	0.343	24	1.682	0.293	0.050	1.690	6.098	Without PM
*6.0	0.477	5	2.068	0.395	0.013	1.889	5.573	With PM
	0.331	24	1.623	0.272	0.049	2.417	5.357	Without PM
*7.0	0.466	5	2.021	0.378	0.013	2.401	5.052	With PM
	0.318	25	1.589	0.249	0.048	3.265	4.487	Without PM
*8.0	0.455	5	1.975	0.362	0.012	2.955	4.488	With PM
	0.305	26	1.555	0.228	0.047	4.236	3.492	Without PM
*9.0	0.425	6	2.194	0.321	0.012	3.524	3.883	With PM
	0.293	27	1.522	0.210	0.046	5.327	2.370	Without PM
*10.0	0.416	6	2.146	0.308	0.012	4.150	3.244	With PM
	0.281	28	1.489	0.193	0.046	6.540	1.124	Without PM

Table IV.
Effect of PM cost

C_{pm}	t_1	n	S + PM	QC	RC	WC	Profit
20	0.244	28	1.073	0.136	0.021	2.155	6.015
40	0.275	17	1.135	0.164	0.016	2.035	6.046
60	0.299	12	1.158	0.183	0.013	1.920	6.127
80	0.322	9	1.155	0.194	0.011	1.812	6.235
100	0.344	7	1.130	0.195	0.009	1.711	6.364

Table V.
Results for varying maximum investment in PM

C^0_{pm}	C_{pm}	t_1	n	QC	RC	WC	Profit
100	80	0.322	9	0.194	0.0111	1.812	6.235
200	160	0.399	7	0.291	0.0128	1.859	5.847
300	240	0.476	5	0.401	0.0135	1.912	5.549
400	320	0.532	4	0.486	0.0136	1.953	5.301
500	400	0.590	3	0.578	0.0128	1.997	5.090
Without PM		0.331	24	0.272	0.049	2.417	5.357

higher values of C_m the percentage difference is more than for lower values. This implies that PM is very effective in such cases.

Table III shows the effect of the warranty period W on various costs, profit, t_1, n and t_p for $C_{pm}^0 = 300$ and $C_{pm} = 250$. It can be seen that profit goes down with increasing W, as expected. By increasing the number of PMs and shortening the length of the intervals between PMs, the system tries to compensate for the increase in warranty cost by reducing quality and restoration costs. Here again, notice the profit difference with and without PM: it increases with increasing W, implying that PM is highly effective in reducing the warranty cost, and is recommended in such cases.

Table IV shows the effect of investment in PM on various costs, profit, t_1 and n. It can be seen that for higher ratios of C_{pm} to C_{pm}^0, profit increases. For very little investment in PM the improvement factor is not much to compensate for the investment. The savings are mostly from warranty and restoration costs. For higher investment in PM the system tries to increase the span of production, thereby reducing the set-up and PM costs per unit.

Table V shows the effect of perfect PM cost when 80 percent of it is invested in PM. It can be seen that for lower costs of perfect PM, C_{pm}^0, savings can be seen for all involved costs. As the cost of perfect PM increases, the savings are seen mainly in restoration and warranty costs. It can also be seen that it is profitable to perform PM when $C_{pm}^0 < 400$; above this value the PM cost is higher than the benefits it offers.

4. Conclusion

In this paper, we developed an integrated model that considers simultaneously inventory production decisions, PM schedule and warranty policy for a deteriorating system that experiences shifts to an out-of-control state. The time to shift follows a general probability distribution with increasing hazard rate so that time-based PM is effective in improving system reliability. Numerical examples are presented and a sensitivity analysis is conducted to gain more insights into the developed model. This model can be easily extended to also optimize the PM level and warranty period. Such models can be very useful in making resource allocation decisions between warranty and the PM program. As is clear from the numerical analysis, a better PM program reduces warranty claims.

References

Banerjee, P.K. and Rahim, M.A. (1988), "Economic design of $\bar{x}$-chart under Weibull shock models", *Technometrics*, Vol. 30, pp. 407-14.

Ben-Daya, M. and Makhdoum, M. (1997), *Integrated Production and Quality Model Under Various Preventive Maintenance Policies*, Systems Engineering Department, King Fahd University of Petroleum & Minerals, Dhahran.

Ben-Daya, M. and Rahim, M.A. (2001), "Integrated production, quality and maintenance models: an overview", in Rahim, M.A. and Ben-Daya, M. (Eds), *Integrated Modeling in Production Planning, Inventory, Quality and Maintenance*, Kluwer, New York, NY.

Djamaludin, I., Murthy, D.N.P. and Wilson, R.J. (1994), "Quality control through lot sizing for items sold with warranty", *International Journal of Production Economics*, Vol. 33, pp. 97-107.

Hariga, M. and Ben-Daya, M. (1998), "The economic manufacturing lot-sizing problem with imperfect manufacturing processes: bounds and optimal solutions", *Naval Research Logistics*, Vol. 45, pp. 423-33.

Lee, H.L. and Rosenblatt, M.J. (1989), "A production and maintenance planning model with restoration cost dependent on detection delay", *IIE Transactions*, Vol. 21 No. 4, pp. 368-75.

Lin, T.M., Tseng, S.T. and Liou, M.J. (1991), "Optimal inspection schedule in the imperfect production system under general shift distribution", *Journal of the Chinese Institute of Industrial Engineers*, Vol. 8 No. 2, pp. 73-81.

Nahmias, S. (1997), *Production and Operations Analysis*, McGraw-Hill, Singapore.

Nakagawa, T. (1980), "A summary of imperfect preventive maintenance policies with minimal repair", *Operations Research*, Vol. 14 No. 3, pp. 249-55.

Pham, H. and Wang, H. (1996), "Imperfect maintenance", *European Journal of Operational Research*, Vol. 94, pp. 425-38.

Porteus, E.L. (1986), "Optimal lot sizing, process quality improvement and setup cost reduction", *Operations Research*, Vol. 34, pp. 137-44.

Rosenblatt, M.J. and Lee, H.L. (1986), "Economic production cycles with imperfect production process", *IIE Transactions*, Vol. 18, pp. 48-55.

Silver, E.A., Pyke, F.D. and Petrson, R. (1998), *Inventory Management and Production Planning and Scheduling*, Wiley, New York, NY.

Wang, C.H. (2004), "The impact of a free-repair warranty policy on EMQ model for imperfect production systems", *Computers & Operations Research*, Vol. 31, pp. 2021-35.

Wang, C.H. and Sheu, S.H. (2003), "Optimal lot sizing for products sold under free-repair warranty", *European Journal of Operational Research*, Vol. 149, pp. 131-41.

Yeh, R.H. and Chen, T.H. (2005), "Optimal lot size and inspection policy for products sold with warranty", *European Journal of Operational Research*, forthcoming.

Yeh, R.H., Ho, W.T. and Tseng, S.T. (2000), "Optimal production run length for products sold with warranty", *European Journal of Operational Research*, Vol. 120, pp. 575-82.

Corresponding author
M. Ben-Daya can be contacted at: bendaya@ccse.kfupm.edu.sa

This article has been cited by:

1. Prashant M. Ambad, Makarand S. Kulkarni. 2015. An attractiveness index based approach for warranty optimization. *International Journal of Quality & Reliability Management* **32**:4, 415-431. [Abstract] [Full Text] [PDF]
2. Da Wen, Pan Ershun, Wang Ying, Liao Wenzhu. 2014. An economic production quantity model for a deteriorating system integrated with predictive maintenance strategy. *Journal of Intelligent Manufacturing* . [CrossRef]
3. Y.-C. Chou, P.-H. Huang. 2013. Integrating machine scheduling and self-healing maintenance by job-mix pull control. *International Journal of Production Research* **51**, 6194-6208. [CrossRef]
4. Mahmood Shafiee, Stefanka Chukova. 2013. Maintenance models in warranty: A literature review. *European Journal of Operational Research* **229**, 561-572. [CrossRef]
5. Prashant M. Ambad, Makarand S. Kulkarni. 2013. A methodology for design for warranty with focus on reliability and warranty policies. *Journal of Advances in Management Research* **10**:1, 139-155. [Abstract] [Full Text] [PDF]