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An integrated production and preventive maintenance planning model

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Abstract

We are given a set of items that must be produced in lots on a capacitated production system throughout a specified finite planning horizon. We assume that the production system is subject to random failures, and that any maintenance action carried out on the system, in a period, reduces the system's available production capacity during that period. The objective is to find an integrated lot-sizing and preventive maintenance strategy of the system that satisfies the demand for all items over the entire horizon without backlogging, and which minimizes the expected sum of production and maintenance costs. We show how this problem can be formulated and solved as a multi-item capacitated lot-sizing problem on a system that is periodically renewed and minimally repaired at failure. We also provide an illustrative example that shows the steps to obtain an optimal integrated production and maintenance strategy.

Keywords: Production planning; Preventive maintenance; Integrated strategies

1. Introduction

There exists an extensive literature addressing the issue of production planning and an equally broad literature tackling maintenance planning questions. Production planning models seek typically to balance the costs of setting up the system with the costs of production and materials holding, while maintenance models attempt typically to balance the costs and benefits of sound maintenance plans in order to optimize the performance of the production system. In both domains, issues of production modeling and maintenance modeling have experienced an evident success both from theoretical and applied viewpoints. Paradoxically the issue of combining production and maintenance plans has received much less attention. The large part of the production planning models assumes that the system will function at its maximum performance during the planning horizon, and the large part of the maintenance planning

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models disregards the impact of maintenance on the production capacity and does not explicitly consider the production requirements. Actually, apart from the preventive maintenance actions that can be scheduled during down times, any unplanned maintenance action disturbs the production plan. It is therefore crucial that both production and maintenance aspects related to a production system are concurrently considered during the elaboration of optimal production and maintenance plans.

The purpose of this paper is to develop a combined production and maintenance model in a batch production system context. The main objective of the proposed model is to determine an integrated production and maintenance plan that minimizes the expected total production and maintenance costs over a finite planning horizon. The model takes into account the fact that the production system may fail randomly. A minimal repair is performed at failure and a periodic replacement is carried out periodically (Barlow and Hunter, 1960). According to this maintenance policy, the system failure rate remains undisturbed by any repair at failure between the periodic replacements (Barlow and Proschan, 1996). They are thus incorporated in the production–planning model through the definition of available production capacity in each period. In other words, the available production capacity in each period is a function of the system's effective capacity and the expected lost capacity due to preventive and corrective maintenance actions.

The remainder of this paper is organized as follows: in Section 2 a brief review of literature is presented. In Section 3 a mathematical model for integrated production and maintenance planning is developed. An algorithm to find the best integrated production and maintenance plan is presented in Section 4. Section 5 presents an illustrative example to show how the proposed algorithm works as well as the coherence of the obtained results from the mathematical model. Some possible extensions and remarks are discussed in the conclusion.

2. Brief literature review

The issue of unreliable production systems has been considered at different levels of production planning, and especially at the operational (scheduling) level. A larger part of scheduling problems discussed in the literature assumes that the maintenance periods are known in advance at the time when jobs are to be scheduled (Qi et al., 1999; Graves and Lee, 1999). It can be shown that these scheduling problems are reducible to scheduling problems with machine availability constraints, and that most of them can be proven to be very difficult (NP-Hard). Indeed, Adiri et al. (1989) showed that the single machine-scheduling problem with machine breakdowns is NP-hard even when the breakdowns are known in advance.

Lee (1996) studied single and parallel machines scheduling problems under a machine availability constraint, and with different performance measures (makespan, total weighted completion time, maximum lateness and number of tardy jobs). He provided polynomial algorithms for polynomial cases and proved NP-Hardness of others. Lee and Chen (2000) studied the problem of scheduling a set of jobs on a set of parallel machines where each machine must be maintained once during the planning horizon. The objective is to schedule jobs and maintenance activities so that the total weighted completion time is minimized. They studied two cases, in the first machines can be maintained simultaneously if necessary, and in the second only one machine can be maintained at any given time. They showed that, even when all jobs have the same weight, both cases of the problem are NP-hard. They also proposed a branch and bound algorithm based on the column generation approach for solving both cases of the problem.

Lee (1997) extended his single and parallel machines study to the two-machine flow-shop scheduling problems with an availability constraint to minimize the makespan. He showed that the problem is NP-hard even when the unavailable time is known in advance. He also developed a pseudo-polynomial dynamic programming algorithm to solve the problem optimally. Kubiak et al. (2002) have extended these complexity results to the two-machine scheduling problem with an arbitrary number of non-availability periods. They proved that the problem of minimizing makespan in such a flow shop is NP-hard in the strong sense. More details on machine scheduling with an availability constraint can be found in Lee (2004).

At the aggregate planning level, the only work we are aware of is that of Wienstein and Chung (1999). They proposed a three-part model to evaluate an organization's maintenance policy. In their approach, an aggregate production plan is first generated, then a master production schedule is developed to minimize the weighted deviations from the goals specified at the aggregate level, and finally work center loading requirements are used to simulate equipment failures during the planning horizon. The authors have used several experiments to test the significance of various factors for maintenance policy selection. These factors include the category of maintenance activity, maintenance activity frequency, failure significance, maintenance activity cost, and aggregate production policy. The fundamental difference between Wienstein–Chung's approach and our's lies in the fact that our model takes, explicitly, into consideration the reliability parameters of the system at the early stage of the planning process. That is, when the aggregate plan is to be developed. As a consequence, the chances that our model performs better at the simulation phase are higher.

3. The mathematical model

We are given a planning horizon $H = N\tau$ including N periods of fixed length τ, and a set of products P to be produced during this planning horizon. For each product $i \in P$ a demand d_{it} is to be satisfied in each period $t \in H$. We assume that the production system has a known nominal capacity denoted by $C_{\max}$ and that each maintenance action consumes a certain percentage of this capacity. Thus, we assume that each planned preventive and unplanned maintenance action consumes, respectively, $L_p = aC_{\max}$ and $L_r = bC_{\max}$ capacity units (with $0 \leqslant a \leqslant b \leqslant 1$). Note that the assumption $a \leqslant b$ may be justified by the fact that more capacity resources may be consumed in case of a random failure since some offline activities for repair must in this case be accomplished online. Also, let ρ_i be the processing time for each unit of product $i \in P$.

Finally, it is assumed that the failure probability density function $f(t)$ and the cumulative distribution function $F(t)$ of the production system are known. We let $r(t)$ be the failure rate of a system at time t. It is well known that $r(t)$ is given by

$$r(t) = \frac{f(t)}{1 - F(t)}.$$

The maintenance policy suggests to replace the production system at predetermined instances $T = k\tau, 2k\tau, 3k\tau, \ldots$ and to carry out a minimal repair whenever an unplanned failure occurs. All maintenance actions are supposed to be perfectly performed.

Fig. 1 illustrates the combined production and maintenance strategy under consideration:

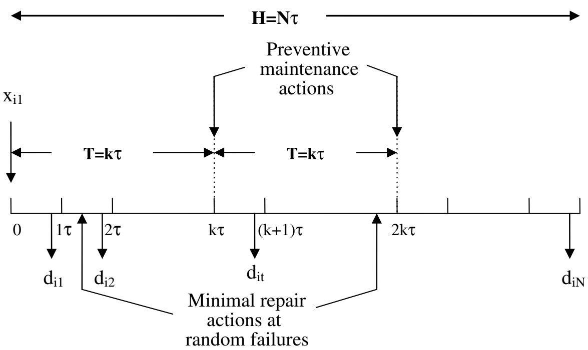

Fig. 1. Combined production and maintenance strategy.

Model parameters

f_{it}	fixed cost of producing item i in period t
p_{it}	variable cost of producing one unit of item i in period t
h_{it}	variable cost of holding one unit of item i by the end of period t
c_p	cost of each preventive replacement
c_r	cost to carry out a corrective maintenance action ($c_p \leqslant c_r$)
τ	duration of each basic planning period
ρ_i	processing time for each unit of product i

Model variables

x_{it}	quantity of item i produced in period t
I_{it}	inventory of item i at the end of period t
y_{it}	binary variable (y_{it} equal 1 if item i is produced in period t and 0 otherwise)
T	preventive maintenance cycle

Mathematical model PPM:

$$\text{Minimize} \quad \sum_{t\in H}\sum_{i\in P}(f_{it}y_{it} + p_{it}x_{it} + h_{it}I_{it}) + \frac{N\tau}{T}\left(c_p + c_r\int_0^T r(t)\mathrm{d}t\right),$$

$$\text{Subject to:} \quad x_{it} + I_{it-1} - I_{it} = d_{it} \quad \text{for } t \in H \text{ and } i \in P, \tag{1}$$

$$x_{it} \leqslant \left(\sum_{s\in H, s\geqslant t} d_{is}\right) y_{it} \quad \text{for } t \in H \text{ and } i \in P, \tag{2}$$

$$\sum_{i\in P} \rho_i x_{it} \leqslant C(t) \quad \text{for } t \in H, \tag{3}$$

$$x_{it}, I_{it}, T \geqslant 0;\ y_{it} \in \{0,1\} \quad \text{for } t \in H \text{ and } i \in P,$$

where the function $C(t)$ defines the available capacity in each period t.

This capacity $C(t)$ is given by

$$C(t) = C_{\max} - L_p - L_r \int_0^{\tau} r(u + (t-1)\tau)\mathrm{d}u,$$

if the preventive maintenance takes place in period t.

For all other periods t the capacity $C(t)$ is given by

$$C(t) = C_{\max} - L_r \int_0^{\tau} r(u + (t-1)\tau)\mathrm{d}u.$$

Clearly if we know the length of the preventive maintenance cycle, the values of the function $C(t)$ can be determined. The resulting problem is then the usual capacitated lot-sizing problem. In general, this problem is NP-Hard (Bitran and Yanasse, 1982). There are, however, some cases in which the problem can be proven to be polynomial. The case in which the failure rate $r(t)$ is a constant function and ($L_p = 0$) is an example. In this latter case the capacity function $C(t)$ is constant and the problem is polynomial (Florian and Klein, 1971).

4. A solution algorithm

To solve the above mathematical programming problem (PPM) we assume, without loss of generality, that the length of the planning horizon H as well as the length of the preventive maintenance cycle T are given in multiples of the basic planning period duration τ (i.e., $H = N\tau$ and $T = k\tau$).

Let $n_I = \lfloor N/k \rfloor$ if the ratio N/k is integer and $n_I = t\lfloor N/k \rfloor + 1$ otherwise (where $\lfloor N/k \rfloor$ is the highest integer smaller or equal than N/k).

The maintenance and planning model (PPM) can now be rewritten as follows:

PPM$_r$:

$$\text{Minimize} \quad Z(k) = \sum_{n=1}^{n_I} \left(c_p + \sum_{t=(n-1)k+1, t\leqslant N}^{nk} \left(c_r \int_0^{\tau} r(u + (t-(n-1)k-1)\tau)\mathrm{d}u + \sum_{i\in P} (f_{it}y_{it} + p_{it}x_{it} + h_{it}I_{it}) \right) \right)$$

Subject to : Constraints Eq. (1), (2), and

$$\sum_{i\in P} \rho_i x_{it} \leqslant C(t) = \begin{cases} C_{\max} - L_p - L_r \int_0^{\tau} r(u + (t-1)\tau)\,\mathrm{d}u & \text{if } t = (n-1)k+1 \\ C_{\max} - L_r \int_0^{\tau} r(u + (t-1)\tau)\mathrm{d}u & \text{if } (n-1)k+2 \leqslant t \leqslant nk \end{cases} \quad (3')$$

for $1 \leqslant n \leqslant n_I$ and $(n-1)k+1 \leqslant t \leqslant nk$ with $t \leqslant N$,

and $x_i, I_{it} \geqslant 0; k \in N; y_{it} \in \{0, 1\}$ for $1 \leqslant n \leqslant n_I$ and $(n-1)k+1 \leqslant t \leqslant nk$ and $i \in P$.

The decision variables remain, for each product i and each period t, x_{it}, I_{it}, y_{it} together with the variable k which defines the optimal length of the preventive maintenance cycle T ($T = k\tau$).

To determine the optimal values of production plan and the length T of the maintenance cycle, the following procedure has been used.

For $k = 1, 2, \ldots, N$, corresponding respectively to the cases of one preventive maintenance in each basic planning period and the one preventive maintenance in the horizon.

- *Step 1:* Based on the value of k, determine n_I and the corresponding maintenance cost function terms, then determine the available capacities $C^k(t)$ in period t.
- *Step 2:* Solve the resulting pure production planning problem (PPM$_r$ for fixed values of k) using any selected algorithm. In our case we solved the production planning problem using the mixed integer solver of CPLEX.
- *Step 3:* Compare the resulting values $Z(k)$, and select for the optimal preventive maintenance period size the value k^* such that $Z(k^*) = \min_k\{Z(k)\}$.

The production plan associated with $Z(k^*)$ is selected as the final production plan.

Notice that our algorithm solves the classical capacitated lot-sizing problem N times to determine the optimal combined production and maintenance plans. Thus, its complexity depends on the complexity of the capacitated lot-sizing problems in step 2. In other words when the capacitated lot-sizing problems solved in step 2 are polynomial, our whole method is polynomial. In case the lot-sizing problems are NP-Hard, which is the case when the failure rate function is strictly increasing in time, then the size of the problem that can be solved with this exact approach is very small. It is therefore advisable to solve the resulting capacitated lot-sizing problems in step 2 using one of the available heuristics.

5. An illustrative example

Let us consider the following planning horizon composed of 8 production periods, each with an available maximal capacity of $C_{\max} = 15$. Two products are to be produced in lots so that the demands are satisfied. Tables 1 and 2 show the setup, production and holding costs for each product and the periodic demands of each product respectively.

Table 3 shows the optimal plan for the two products without taking into account the capacity lost in maintenance (assuming that the system will not fail and does not require any preventive maintenance). The total cost for the optimal production plan is equal to 417.

Now, if we consider the preventive maintenance model with minimal repair at failure as the selected maintenance strategy of the production system with the following parameters: the cost of a preventive maintenance action is set to $c_p = 28$, and the cost of minimal repair action at failure is given by $c_r = 35$. We assume that the system lifetime is distributed according to Gamma distribution with the parameters $G(\alpha = 2, \lambda = 1)$. Table 4 shows the expected number of failures as a function of the system's age.

Fig. 2 shows the expected total cost of maintenance strategy as a function of the preventive replacement cycle T. From Fig. 2 the optimal value of T is equal to $3.941 \approx 4$. This strategy if selected will result in a total production and maintenance cost of 660.18 (see Table 5). As we shall see later this cost is not the optimal.

Table 1
Products' cost data

Product	Setup cost	Production cost	Holding cost
1	25	5	2
2	25	5	2

Table 2
Products' periodic demands

Period	Demand of product 1	Demand of product 2
1	2	3
2	3	2
3	2	3
4	3	2
5	2	3
6	3	2
7	2	3
8	3	2

Table 4
Expected number of failures for $\tau = 1$

Age	Expected number of failures
$[0, 1\tau[$	0.307
$[1\tau, 2\tau[$	0.595
$[2\tau, 3\tau[$	0.712
$[3\tau, 4\tau[$	0.777
$[4\tau, 5\tau[$	0.818
$[5\tau, 6\tau[$	0.846
$[6\tau, 7\tau[$	0.866
$[7\tau, 8\tau[$	0.882

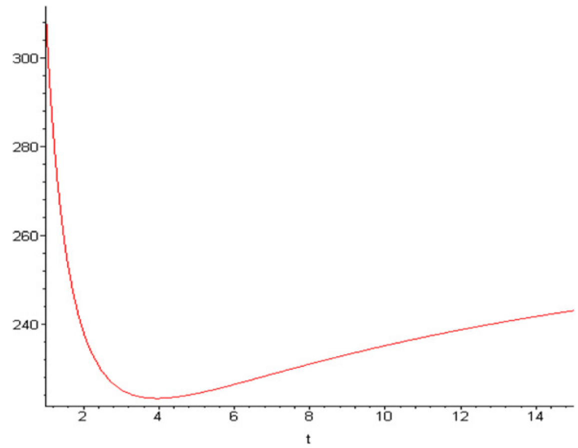

Fig. 2. The average maintenance cost of the system.

Table 3
Optimal production plan with maximum capacity

Periods	Production		Inventory		Setups	
	Product 1	Product 2	Product 1	Product 2	Product 1	Product 2
1	5	10	3	7	1	1
2	0	0	0	5	0	0
3	7	0	5	2	1	0
4	0	0	2	0	0	0
5	0	10	0	7	0	1
6	8	0	5	5	1	0
7	0	0	3	2	0	0
8	0	0	0	0	0	0

Table 5
Optimal production, maintenance and total costs for each maintenance scenario

Periods	Available capacities in each period as a function of the length of preventive maintenance period $T=k\tau$; $\tau=1$							
	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$
1	9.49	9.49	9.49	9.49	9.49	9.49	9.49	9.49
2	9.49	7.55	7.55	7.55	7.55	7.55	7.55	7.55
3	9.49	9.49	6.68	6.68	6.68	6.68	6.68	6.68
4	9.49	7.55	9.49	6.26	6.26	6.26	6.26	6.26
5	9.49	9.49	7.55	9.49	6.00	6.00	6.00	6.00
6	9.49	7.55	6.68	7.55	9.49	5.84	5.84	5.84
7	9.49	9.49	9.49	6.68	7.55	9.49	5.72	5.72
8	9.49	7.55	7.55	6.26	6.68	7.55	9.49	5.63
Optimal maintenance cost	309.92	238.19	228.51	**223.34**	224.77	229.44	238.96	231.10
Optimal production cost	**431.06**	433.76	443.48	436.84	434.28	447.28	447.28	444.28
Optimal total cost	740.98	671.95	671.99	660.18	**659.05**	676.72	686.24	678.38

In the following the integration of the maintenance and production plans to achieve the best production and maintenance costs is investigated. Let $L_p=1$ and $L_r=5$ (i.e., $a=0.333$ and $b=0.067$) the parameters defining the capacity lost when a preventive maintenance action and unplanned minimal repair actions are taken into account.

Table 5 shows the available capacities in each period given as a function of the size of preventive maintenance period $T=k\tau$, as well as the optimal production, maintenance and total costs. If we examine the pure production costs, we observe that the minimal production cost is obtained for $k=1$. As mentioned above the minimal maintenance cost is obtained for $k=4$. The total cost associated with each of these plans when both production and maintenance costs are considered are, respectively, 740.98 for $k=1$ and 660.18 for $k=4$. Actually, the total cost of an optimal combined production and maintenance plan is 659.06 and is obtained for $k=5$. In other words the optimal combined production and maintenance plan suggests that a preventive maintenance be carried out each 5τ units of time, and the system be minimally repaired at failure.

6. Conclusion

A joint production and maintenance planning model for a production system subject to random failures has been proposed. This model takes, explicitly, into account the reliability parameters of the system and its capacity in the development of the optimal production plan. At failure, a minimal repair is carried out to restore the system into the operating state without changing its failure rate function. The system is also replaced preventively at predetermined instants, regardless of its state and its age. To solve the problem an iterative solution framework has been developed. The illustrative example shows that the model is consistent and that the optimal production and preventive maintenance planning are coherent. The numerical procedure has been validated using a set of arbitrary selected data. The generic model can be applied for any set of data and any lifetime distribution of the system. The optimal planning is obtained using an iterative procedure and the mixed integer solver of CPLEX. An extension of the proposed model to a production system consisting of multiple machines is also investigated by the authors. The issues of maintenance strategies as well as integration of our model with operations scheduling are being also investigated.

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