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Optimizing production and imperfect preventive maintenance planning's integration in failure-prone manufacturing systems

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ABSTRACT

This paper investigates the issue of integrating production and maintenance planning in a failure-prone manufacturing system. It is assumed that the system's operating state is stochastically predictable, in terms of its operating age, and that it can accordingly be preventively maintained during preplanned periods. Preventive maintenance is assumed to be imperfect, that is when performed, it brings the manufacturing system to an operating state that lies between 'as bad as old' and 'as good as new'. Only an overhauling of the system brings it to a 'as good as new' operating state again. A practical integrated production and preventive maintenance planning model, that takes into account the system's manufacturing capacity and its operational reliability state, is developed. The model is naturally formulated as a mixed-integer non-linear optimization problem, for which an extended mixed-integer linear reformulation is proposed. This reformulation, while it solves the proposed integrated planning problem to optimality, remains quite demanding in terms of computational time. A fix-and-optimize procedure, that takes advantage of some properties of the original model, is then proposed. The reformulation and the fix-and-optimize procedure are tested on some test instances adapted from those available in the literature. The results show that the proposed fix-and-optimize procedure performs quite well and opens new research direction for future improvements.

1. Introduction

The current technological sophistication of the modern manufacturing systems creates a natural interdependence between production and maintenance activities on these systems. In automotive industry for example, some assembly lines are fully integrated and a failure of one of its machines or workstations may cause the full line to stop creating delays with considerable technical and economic consequences. In such environments, operating a manufacturing system that is not appropriately maintained, or even preventively maintained but at less opportune time periods, may result in significant productivity losses and additional costs which are among others caused by poor product quality and avoidable production overtimes. Thus, any system's deterioration in these highly automated and fully integrated production lines impacts not only the system itself but also the quality of the product, which may either require rework or is simply scrapped if it cannot be reworked. Consequently, the integration of production and preventive maintenance planning is essential to keep the manufacturing system operating optimally and at the desirable productivity level.

When a manufacturing system is operating, its various components are subject to degradations due either to age or usage. Some of these degradations can be modeled or approximated by some stochastic processes, which can be used to optimally integrate production and preventive maintenance plans. In practice however, production and maintenance are usually planned independently. Maintenance is scheduled and carried out based only on the system's reliability data, neither the forecasted production orders nor planned product mix are taken into account. Production planning is then carried out within the limits imposed by the system's maintenance schedule. This results often in production and maintenance plans which are not optimal with respect to the overall objective of minimizing the combined production and maintenance related costs.

This paper proposes an aggregate production planning model, which in addition to the relevant production data and parameters also takes the system's reliability statistics into account, to generate optimal tactical production plans that specify optimal periods for preventive maintenance as well. This planning integration is carried out at the

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tactical level to allow the planner to efficiently and effectively plan the required production and preventive maintenance tasks while taking advantage of the available information on expected product-mix orders and system's operating state. The actual scheduling of maintenance is then carried out, at the operational level, during those specified opportune periods. The model attempts to achieve an optimal trade-off between production related costs and maintenance related costs while fulfilling the aggregate production requirements. The model assumes that maintenance is imperfect and is first formulated, in its natural variables, as a mixed integer non-linear optimization problem. It is then shown that it can be reformulated as mixed-integer linear problem, which can be solved with the readily available solvers. However, the numbers of variables and constraints in this reformulation are too large. An approach to approximately solve the problem, while taking advantage of the linearity of the large reformulation, is also proposed and discussed.

The remainder of this paper is organized as follows: Section 2 briefly overviews relevant literature. Section 3 discusses in detail the development of an optimization model, and its properties, to generate optimal integrate production and imperfect preventive maintenance plans. Section 4 presents an extended mixed-integer linear reformulation of the problem. Section 5 presents a fix-and-optimize procedure to solve the problem using some well-constructed relaxed mixed-integer linear sub-problems, stemming from the reformulation discussed in Section 4. Then, the results of the reformulation and the fix-and-optimize procedure on some test instances are reported and discussed. Finally, some concluding remarks are presented in the conclusion.

2. A brief review of the literature

During its operation, a manufacturing system and its components are subject to random degradations which are inherent to either their age, usage or both. Maintenance must be regularly performed to assess the level of degradations and then, if necessary, attempt to bring the system and/or its critical components back to good operating conditions. As a result, researchers started investigating the issue of developing maintenance strategies for the stochastically degrading manufacturing systems. Maintenance of the system can be performed either preventively before or correctively after its failure or failure of one of its components. Preventive maintenance and replacement policies have been extensively investigated in the literature, starting with the preventive maintenance framework introduced by Barlow and Hunter in their seminal paper [1] (see also Barlow and Proshan [2]). Surveys summarizing the research status in this area at different epochs can be found in, among others, McCall [3], Pierskalla and Voelker [4], Sherif and Smith [5], Jardine and Buzacott [6], Valdez-Flores and Feldman [7], Cho and Parlar [8], Dekker [9], Pham and Wang [10], Van Der Duyn Schouten [11], Dekker et al. [12] and Lugtigheid et al. [13]. Recently Nakagawa and Mizutani, [14], extended the well-known results of periodic replacement with minimal repair policy as well as those of block replacement policy from infinite into finite time horizon setting. For more details about preventive maintenance policies made within finite time horizon see Nakagawa [15].

The major part of the available research literature, dealing with production and preventive maintenance integration, focuses on operations scheduling. The proposed integrated production and preventive maintenance scheduling models assume that the periods during which preventive maintenance is to take place are known in advance, and are mainly formulated as deterministic scheduling problem with availability constraints. These models are usually solved using adapted versions of the solution methods used for scheduling problems, see among others, Lee [16–18], Kubiak et al. [19], Lee and Chen [20] and Cassady and Kunatoglu [21].

During the last two decades, however, the research literature addressing the issue of integrating production and preventive maintenance, at the tactical planning level, witnessed a significant intensification. Starting with the paper of Wienstein and Chung [22], in which the authors presented a three-parts hierarchical production planning and scheduling model that also considers system's reliability. The first part in the model is an aggregated production planning model formulated as a linear program. The second part is a master production schedule with the objective of minimizing the weighted deviations with respect to the goals specified at the aggregate production planning level. The third part in the model deals with work center loading requirements, which are used to simulate equipment failures during the planning horizon. Several experiments are carried out to test the significant factors such as category of maintenance activity, maintenance activity frequency, failure significance, maintenance activity cost, and aggregate production policy for maintenance policy selection. In Aghezzaf et al. [23], the authors proposed an integrated production and preventive maintenance planning optimization model to generate optimal integrated production and preventive maintenance plans at the tactical level. The model assumes that the system is minimally repaired when it fails randomly during a production period. That is, the systems is returned to an operating state without altering its failure rate. When the system is preventively maintained it returns to an as good as new state. Its failure rate is the same as the that of a new system. It also assumes that any maintenance action, minimal repair or preventive maintenance, reduces the available production capacity of the system. This work is extended in [24] to deal with simultaneous optimization of production and preventive maintenance in multiple-lines stochastic degrading manufacturing system. Najid et al. [25] extended the model in [24] to include demand time windows and shortage cost. Nourelfath and Chatelet [26] discussed the same planning problem for a production system composed of a set of parallel components, in the presence of economic dependence and common cause failures. Zhao et al. [27] assume order-dependent-failure (ODF) and proposed an iterative method to solve the problem on a single-machine system. Fitouhi and Nourelfath [28] extended the model [23] to multi-state systems. Recently, Yalaoui et al. [29] proposed some very interesting exact and heuristic algorithms to efficiently solve moderate to larger instances of the model proposed in [24]. Liu et al. [30] studied the economic production quantity problem on a system, producing multiple items, and requiring preventive maintenance. Preventive maintenance is carried out during set-up times of some items. The authors analyzed the issue of jointly determining the optimal lot sizes and the preventive maintenance policy. Other versions of this latter model, including inventory, are discussed in the literature.

The model discussed in this paper extends the one proposed in [23] and its subsequent variations. It deals, though, with the non-periodic and imperfect preventive maintenance. When an imperfect preventive maintenance operation is performed it brings the system to an operating state that is between the two extreme operating states, namely the ‘as bad as old’ and the ‘as good as new’ states. Along the same lines as in [23] and [24], it is assumed that each imperfect preventive maintenance improves the available production capacity of the system and this according to the degree at which the maintenance is performed. However, the deterioration rate of the manufacturing system after each imperfect preventive maintenance increases and the production capacity decreases more rapidly due to more frequent minimal repair tasks. When the deterioration rate reaches an unacceptable level, the systems is overhauled and returns to an ‘as good as new’ state.

To model imperfect preventive maintenance, Malik [31] proposed an age reduction model based of the concept of system's ‘virtual age’ according to which a system becomes ‘younger’ whenever it undergoes an imperfect preventive maintenance. In other words, after an imperfect preventive maintenance, the age t of the system is reduced to $\alpha \times t$, where α is an age reduction coefficient $(0 \le \alpha \le 1)$. Consequently, the system becomes 'as good as new' if its age is reset to zero $(\alpha = 0)$ while it becomes 'as bad as old' if the age reduction coefficient $\alpha = 1$. In the latter case, the imperfect preventive maintenance corresponds to the minimal repair maintenance which consists in restoring the system to the state immediately preceding the failure. Nguyen and Murthy [32] investigated the issue of developing optimal preventive maintenance policies for repairable systems. They assumed that after a repair the system's age returns to age zero while the system's failure rate increases as the number of repairs carried out increases. Kijima [33], introduced two types of virtual age based preventive maintenance models. In Kijima's type I model, only the age part of the system between the last preventive maintenance and the current one is reduced. In Kijima's type II model, however, it is the whole virtual age of the system that is reduced after the preventive maintenance. Recently, Finkelstein [34] discussed the problem of finding optimal repair actions to the Kijima's type II imperfect repair model and characterized the conditions of existence of an optimal solution. Doyen and Gaudoin [35] proposed two classes of imperfect repair models in which repair effect is reflected in the change induced on the failure rate. Repair effect is expressed by a reduction of failure intensity in the first class of models, and by a reduction of the system's virtual age in the second class of models. Several special cases also were investigated. Many other models for imperfect preventive maintenance are discussed in Pham and Wang [10].

Nakagawa [36] (see also [14,15]) introduced the concept of hazard rate increase factor to model imperfect preventive maintenance. According to this model, the imperfect preventive maintenance could be seen as a replacement of the existing system with a new one but which is less reliable. In other words, after an imperfect preventive maintenance, the initial hazard rate $r_0(t)$ of the system is modified to become $\beta \times r_0(t)$, where β is such that $\beta \ge 1$ and is called the 'adjustment factor'. As a consequence, the hazard rate function increases with the number of imperfect preventive maintenance tasks carried out. Merging these imperfect preventive maintenance models, Lin et al. [37] proposed a hybrid model obtained by combining age reduction and hazard rate increase effects. The hybrid hazard rate model reduces the effective age to a certain lower value and tends to steepen the slope of the hazard rate function. If the hazard rate function of the system is $r_0(t)$ before performing an imperfect preventive maintenance task at time T, it becomes $\beta \times r_0(\alpha T + t)$ right after the preventive maintenance task, where $\beta \ge 1$, $0 \le \alpha \le 1$ and $t \ge 0$. When $\beta = 1$, the hybrid model reduces to an age reduction model, and it reduces to a hazard rate adjustment model when $\alpha = 0$. Lin et al. [38] extended this work to repairable systems with two categories of failure modes, namely the maintainable and non-maintainable failure modes. Only the system's failure rate corresponding to the maintainable failure mode is altered whenever an imperfect preventive maintenance task is performed. This hybrid model is also used as a modeling approach for imperfect preventive maintenance by [39]. Several extensions were also carried out including multi criteria approaches.

To conclude, this paper considers the issue of integrating production and imperfect preventive maintenance planning at the tactical level. Imperfect maintenance is modeled using a hybrid hazard rate function. The resulting optimization model underlying this integrated production and imperfect preventive maintenance planning is first formulated as a mixed-integer non-linear mathematical program. A linearized reformulation is then proposed together with fix-and optimize procedure which can be used to solve moderate to larger instances of the problem.

3. Integrated production and imperfect preventive maintenance planning

After some operating time and depending on the environment under which it operates, a production system will start experiencing performance deterioration and random failures. These failures and degradations usually result from wear of some components and equipment of the system. To keep the system operating at its optimum performance, it must be regularly maintained and overhauled when necessary. However, this regular maintenance must be carried out following some well-thought plan to avoid excessive degradation of the system's performance or failure of the system itself. Developing such a maintenance plan cannot be carried out without exploiting the right failure and degradation model of the system. The appropriate failure and degradation model is therefore the 'core' of any maintenance strategy. Nevertheless, the approach developed in the sections below constitutes a general framework in which these models can be considered as parameters to be adapted to each system and the environment in which it operates.In this section, the integrated aggregate production and maintenance planning problem, that investigated in this paper, is modeled and formulated as an optimization problem. It is assumed that the preventive maintenance is imperfect and modeled using an adapted hybrid hazard rate function described the subsequent subsection.

3.1. Modeling imperfect preventive maintenance with hybrid failure rates

In reliability engineering, information on the probability of failure of an operating system is of critical importance. Failure can be defined in different ways, and might mean mechanical breakdown, deterioration beyond some critical level of the system's performance, or appearance of some particular defects in the system's output. Failure rate is a good measure which describes how a system improves or deteriorates with its usage and age. The instant failure rate function $r(t)$, called simply the failure rate, is defined as

$$r(t) = \frac{g(t)}{1 - G(t)} \tag{1}$$

where X, $g(t)$ and $G(t)$ denote the non-negative random variable defining the failure time of the system, the failure time density and failure distribution functions respectively. This means physically that $r(t)\Delta t \approx Pr\{t < X \le t + \Delta t X > t\}$ represents the probability that a system with age t will fail in an interval $[t, t + \Delta t[$ for small $\Delta t > 0$.

As mentioned above, in this paper we consider imperfect preventive maintenance with a hybrid failure rate function. If after the kth preventive maintenance the failure rate function remains below some threshold function, $r_{k_{max}}(t)$, the system can again be preventively maintained. However, if it reach or exceeds this threshold level, it is overhauled and will return to an 'as good as new' state. We adopt a hybrid failure rate model defined along the same line as in Lin et al. [37]. We consider a system whose lifetime is randomly distributed and for which the corresponding initial hazard rate function is given by the function $r_0(t)$. If the kth preventive maintenance takes place $T_k \tau$ units of time after an overhaul, that is in the beginning of period T_k having fixed length τ, the hazard rate function $r_k(t)$ of the system is then defined as

$$r_k(t) = \beta_k r_0(t + \alpha_k T^k_{AOT}), \quad t \in [0, (T_{k+1} - T_k)\tau[,$$
$$\text{for all } k, 1 \le k \le k_{max} \text{ and integer} \tag{2}$$

where T^k_{AOT} is the actual operating time of the system since the beginning of the planning horizon until the beginning of period T_k, the period during which the kth preventive maintenance is taking place. It is the timespan during which the system was in actual fact producing and not undergoing maintenance, neither preventive nor corrective. The parameters α_k and β_k stand, respectively, for the age reduction coefficient and the hazard rate increase coefficient (adjustment factor) such that $0=\alpha_1<\alpha_2<\cdots<\alpha_{k_{max}}=1$ and $1=\beta_1\le\beta_2\le\cdots\le\beta_{k_{max}}$. Note again that for $t\in[0,T_1[$, $r_0(t)$ is the hazard rate of the system, which is initially assumed to be 'as good as new', that is after an overhauling. The baseline failure rate function $r_0(t)$ is assumed to be a monotonically increasing function throughout the text.

3.2. Modeling integrated production and imperfect preventive maintenance

This subsection presents a mixed integer linear program to determine optimal integrated production and preventive maintenance plans on a production system subject to random failures. The model assumes that the preventive maintenance is imperfect, that any failure of equipment is immediately repaired, and that the expected number of failures increase with elapsed time since the last preventive maintenance action. Without loss of generality we assume that during the planning horizon the failure rate function never exceeds threshold function $r_{k_{max}}(t)$. This is not difficult to add to the model.

We consider a production system, that is subject to random failures, on which a set of items $j\in P=\{1,\ldots,N\}$ is to be produced during a planning $H=\{1,\ldots,T\}$. We assume that each period $t\in H$ has a fixed length denoted by τ and that each item $j\in P$ has a deterministic demand d_{jt} which must be satisfied in that period. The quantity $d^j_{tT}=\sum_{t'=t}^{T}d^j_{t'}$ is the cumulative demand of item $j\in P$ from t to T. Each item $j\in P$ is produced at a known production rate ρ_j expressed in product units per unit of time. The production system has a known periodic constant production capacity (given in time units) and is denoted by κ_{max}. We assume that each maintenance action performed on the system consumes some given amount of system's capacity.

To define the variables of the model, we let Q_{jt} denote the quantity of item $j\in P$ produced during period $t\in H$, I_{jt} the inventory of item $j\in P$ at the end of period $t\in H$, x_{jt} the binary decision variable set to 1 if the production of item $j\in P$ is launched in period $t\in H$ and 0 otherwise, finally we let y_t be a binary decision variable set to 1 if the production system is set up to production in period $t\in H$ and 0 otherwise and $\mathbf{y}=(y_t)_{t\in H}$ denote the setup decisions vector. We also let z^k_{st}, with $k\le s\le t$, denote a binary decision variable set to 1 if the last preventive maintenance of the system before the time period t is the k^{th} one and has taken place during the time period s, and 0 otherwise. By convention we assume that the manufacturing system is preventively maintained in the beginning of period 1, that is $z^1_{11}=1$.

To further identify the parameters of the model we let θ^k_{PM} and c^k_{PM} be respectively the expected time and cost of the k^{th} preventive maintenance of the production system, and θ^k_{CM} and c^k_{CM} be respectively the expected time and cost of performing a corrective maintenance on the system when a failure has occurred, after the k^{th} preventive maintenance. We also let f_{jt} be the fixed cost incurred if the system is operating during period $t\in H$ to produce item $j\in P$, p_{jt} be the variable production cost of producing item $j\in P$ in period $t\in H$, and h_{jt} be the inventory holding cost of item $j\in P$ in period $t\in H$. Finally, we let $c^k_{st}(\boldsymbol{y})$ and $\kappa^k_{st}(\boldsymbol{y})$ be respectively expected maintenance cost and expected loss in production capacity of the system during period t, when the k^{th} and last preventive maintenance action before time period t has taken place in the time period s, with $k\le s\le t$. These parameters depend on the system's setup vector $\boldsymbol{y}$ and are given by

$$c^k_{st}(\boldsymbol{y})=\begin{cases} c^k_{PM}+c^k_{CM}\int_0^\tau\beta_k r_0\left(u+\alpha_k\left[\sum_{t'=1}^{s-1}y_{t'}\right]\tau\right)du & \text{if } t=s, k\le s,\\ c^k_{CM}\int_0^\tau\beta_k r_0\left(u+\alpha_k\left[\sum_{t'=1}^{s-1}y_{t'}\right]\tau+\left[\sum_{t'=s}^{t-1}y_{t'}\right]\tau\right)du & \text{if } s<t\le T.\end{cases}\quad(3)$$

and

$$\kappa^k_{st}(\boldsymbol{y})=\begin{cases} \theta^k_{PM}+\theta^k_{CM}\int_0^\tau\beta_k r_0\left(u+\alpha_k\left[\sum_{t'=1}^{s-1}y_{t'}\right]\tau\right)du & \text{if } t=s, k\le s,\\ \theta^k_{CM}\int_0^\tau\beta_k r_0\left(u+\alpha_k\left[\sum_{t'=1}^{s-1}y_{t'}\right]\tau+\left[\sum_{t'=s}^{t-1}y_{t'}\right]\tau\right)du & \text{if } s<t\le T.\end{cases}\quad(4)$$

Note that we start with $k=1$, meaning that first maintenance (i.e. overhauling) of the system took place in the beginning of the first period at $t=1$ of the planning horizon H.

Observe that the actual operating time of the system T^k_{AOT} is approximated here by $\left[\sum_{t'=1}^{s-1}y_{t'}\right]\tau$. This is acceptable if the total time devoted to preventive and corrective maintenance is significantly smaller than the system's productive total time, since the beginning of the planning horizon. If however this preventive and corrective maintenance total time is significantly large and cannot be ignored when evaluating the system's age before a preventive maintenance, T^k_{AOT} becomes far more complex to compute. This complication results from the fact that T^k_{AOT} depends on the previous actual operating times until period T_{k-1} during which the $(k-1)^{\text{th}}$ preventive maintenance has taken place. The relationship linking T^k_{AOT} and T^{k-1}_{AOT} is of the form:

$$T^k_{AOT}=T^{k-1}_{AOT}+\left[\sum_{t'=T_{k-1}}^{T_k-1}y_{t'}\right]\tau-\theta^{k-1}_{PM}-\theta^{k-1}_{CM}\int_0^{(T_k-T_{k-1})\tau}\beta_{k-1}r_0\left(u+\alpha_{k-1}T^{k-1}_{AOT}\right)du\quad(5)$$

This relationship can be written in form of constraints and incorporated in the integrated production and imperfect preventive maintenance planning model (*IPImPMPP*) proposed below. Of course the resulting optimization problem becomes much more complex. In the remainder of the paper we work with the assumption that $T^k_{AOT}\approx\left[\sum_{t'=1}^{s-1}y_{t'}\right]\tau$.

To illustrate how these parameters can be computed, consider a production system with a maximal capacity κ_{max}. Assume that the system is preventively maintained twice, as shown in Fig. 1. The arrows indicate the periods during which the system is setup to production.

Assume that the production system is subject to random failures occurring according to a Gamma distribution, $\Gamma(m=2,\nu=2)$, with a shape parameter $m=2$ and a rate parameter $\nu=2$. Also, assume that ($\theta^k_{PM}=1$, $c^k_{PM}=28$) and ($\theta^k_{CM}=5$, $c^k_{CM}=35$) be the expected time and cost of the k^{th} preventive and corrective maintenance of the production system respectively. The coefficients α_k and β_k are given by $\alpha_1=0$ and $\alpha_k=0.15+10(k-1)$ for $k=2,\ldots,8$, and $\beta_k=1$ for $k=1,\ldots,8$. Table 1 shows, respectively, the preventive maintenance plan, the production setups plan, age after each preventive maintenance with the corresponding failure rates and finally the resulting expected capacity loss:

Recall that the density function of a Gamma distribution $\Gamma(m,\nu)$ with a shape parameter m and a rate parameter ν is given by

$$r_0(t)=\begin{cases}\dfrac{\nu^m t^{m-1}exp^{(-\nu t)}}{\Gamma(m)} & \text{if} \quad t,\nu\ge0, m>0,\\ 0 & \text{if} \quad t<0.\end{cases}\quad(6)$$

Also, notice that after an overhauling, if no preventive maintenance carried out and if the system is setup to production in each period, the expected number of failures as a function of the production system's age, is given by the Table 2 below as in [23] and [24].

In Section 4, we show how the expected capacity loss can be obtained as a function of the system's age in matrix forms. This allows us to reformulate the integrated production and imperfect

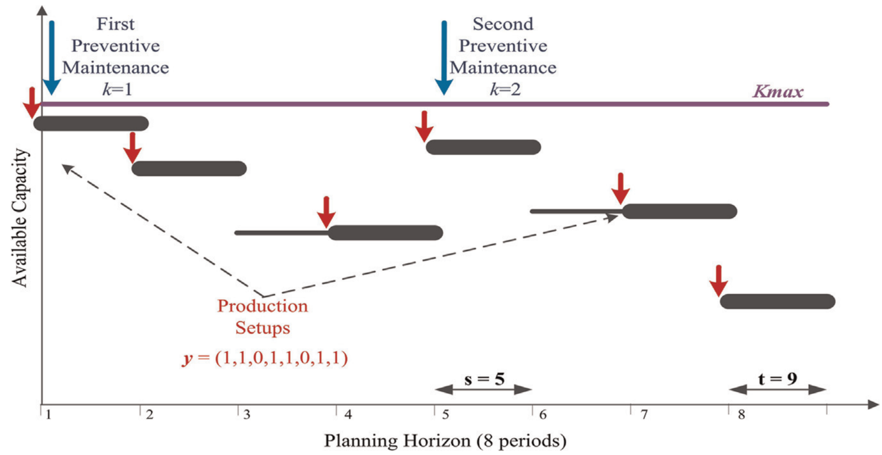

Fig. 1. An example of imperfect preventive maintenance plan with production setups.

Table 1
Expected number of failures given the preventive maintenance plan and the indicated production setups.

Period	Actual age	Preventive maintenance plan	Production setups	Age after preventive maintenance	Expected number of failures	Expected capacity loss
1	0.00τ	$k=1$	1	0.00τ	0.901	5.507
2	1.00τ		1	1.00τ	1.489	7.446
3	2.00τ		0	2.00τ	1.664	8.318
4	3.00τ		1	2.00τ	1.664	8.318
5	4.00τ	$k=2$	1	0.75τ	1.412	8.061
6	5.00τ		0	1.75τ	1.632	8.161
7	6.00τ		1	1.75τ	1.632	8.161
8	7.00τ		1	2.75τ	1.731	8.659

Table 2
Expected number of failures given the production is setup in each period and no preventive maintenance is carried out.

Actual age	0.00τ	1.00τ	2.00τ	3.00τ	4.00τ	5.00τ	6.00τ	7.00τ
Expected number of failures	0.901	1.489	1.664	1.749	1.799	1.833	1.857	1.875

preventive maintenance planning problem as mixed-integer linear program.

The integrated production and imperfect preventive maintenance planning problem (*IPImPMPP*) can now be modeled as a linear mixed-integer program as described below

(*IPImPMPP*): Minimize

$$Z_{PPM}^{IP} = \sum_{t=1}^{T}\sum_{j=1}^{N}\left(f_{jt}x_{jt} + p_{jt}Q_{jt} + h_{jt}I_{jt}\right) + \sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s} c_{st}^{k}(\mathbf{y})y_t z_{st}^{k}$$

subject to

$$Q_{jt} + I_{j,t-1} - I_{jt} = d_{jt}, \quad \forall j \in P, \ \ t \in H \tag{7}$$

$$Q_{jt} - min\left\{d_{tT}^{j}, \kappa_{max}\right\}x_{jt} \le 0, \quad \forall j \in P, \ \ t \in H \tag{8}$$

$$x_{jt} - y_t \le 0, \quad \forall j \in P, \ \ t \in H \tag{9}$$

$$\sum_{j=1}^{N}\rho_j Q_{jt} + \sum_{s=1}^{t}\sum_{k=1}^{s}\kappa_{st}^{k}(\mathbf{y})y_t z_{st}^{k} \le \kappa_{max}, \quad \forall t \in H \tag{10}$$

$$\sum_{s=1}^{t}\sum_{k=1}^{s} z_{st}^{k} = 1, \quad \forall t \in H \tag{11}$$

$$z_{st}^{k} - z_{s,t+1}^{k} \ge 0, \quad \forall k, s, t \in H, k \le s \le t \le T-1 \tag{12}$$

$$\sum_{s=k-1}^{t-1} z_{s,t-1}^{k-1} - z_{tt}^{k} \ge 0, \quad \forall k, t \in H, 2 \le k \le T, k \le t \tag{13}$$

$$Q_{jt}, I_{jt} \ge 0, x_{jt}, y_t, z_{st}^{k} \in \{0,1\} \quad \forall j \in P, s, t \in H, k \le s \le t.$$

Constraints (7) are the flow conservation constraints in each period $t \in H$. They guarantee that the available inventory augmented with the quantity produced in a period t is sufficient to satisfy the demand d_{jt} of item j in that period. The remainder is stocked for the subsequent periods. Constraints (8) make certain that, when the production of an item is scheduled in a period, the system is setup accordingly to produce that item in that period. They also force disbursement of the fixed costs of the items produced during the period. Constraints (9) indicate whether the system is operating or not in each period, which is the case if it is setup to produce some product. Constraints (10) are capacity restrictions defined for each period $t \in H$. They guarantee that the quantity produced in period t does not exceed the available capacity of the system given its status in terms of the expected capacity loss during that period. Constraints (11) determine the periods during which preventive maintenance activities take place. To assure consistency, constraints (12) guarantee that if the last preventive maintenance action, before time period $t+1$, takes place in period $s < t$ and it is the k^{th} one, then this preventive maintenance action must also be the k^{th} and last one before period t. Again for consistency, constraints (13) assure that the k^{th} preventive maintenance takes place in some period $t \ge k$, only if the $(k-1)$th preventive maintenance took place in some period before t.

In this proposed integrated production and maintenance planning model, it is assumed that the durations of the preventive as well as the corrective maintenance actions, θ_{PM}^{k} and θ_{CM}^{k} respectively, are smaller than the planning's period duration τ. In cases where the durations of the preventive maintenance actions, for example, are higher than the length of the planning's period τ, an additional constraint must be added to the model. This constraint states that if a preventive maintenance action takes place in a period s, production in that period and the next ones up until $(s+\theta_{PM}^k-1)$ cannot take place. This can be formulated as

$$\sum_{t=s}^{s+\theta_{PM}^k-1} y_t \le \theta_{PM}^k(1-z_{ss}^k), \quad \forall s \in H, 1 \le k \le s$$

The same can be done for the corrective maintenance actions. In this case, however, a new binary variable must be introduced to indicate whether the system failed in a period or not. Of course in these cases T_{AOT}^k cannot be approximated as above anymore and the resulting optimization problem gets far more complex.

If one examines carefully the optimization problem (*IPImPMPP*), one can observe that it is a nonlinear mixed-integer optimization problem and that this non-linearity stems from the component $\sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s} c_{st}^k(\boldsymbol{y})z_{st}^k$ in the objective function and from the component $\sum_{s=1}^{t}\sum_{k=1}^{s} \kappa_{st}^k(\boldsymbol{y})z_{st}^k$ in the constraints (10). However, when the periods during which the system is setup to production are known in advance, the expected maintenance cost components and the expected available production capacity of the system during each period can directly be determined. The problem becomes linear optimization problem integrating a multi-item capacitated lot-sizing problem with preventive maintenance. In the following subsection we propose a reformulation of this model as mixed-integer linear program, which can then be solved using the readily available MILP solvers such as CPLEX and Gurobi.

4. Reformulation of the problem (IPImPMPP)

As is shown above the natural formulation of the problem is nonlinear, which however can reformulated and modeled as a mixed-integer linear program. To define the variables of the new model, we let $u_{st}(p,q)$, with $p \le s \le t$ and $0 \le q \le t-s+1$, to be a binary variable assuming value 1 if the system is setup to production p times during the planning horizon $\{1,\dots,s-1\}$ and q times during the periods $\{s,\dots,t-1\}$. We also define the variable $v_{st}^k(p,q)$, with $p \le s \le t$, $k \le s$ and $q \le t-s+1$, to be a binary variable assuming value 1 if the system is setup to production p time during the horizon $\{1,\dots,s-1\}$ and q time during the period $\{s,\dots,t\}$, and the k^{th} maintenance takes place in period s.

Finally, we let $c_{st}^k(p,q)$ and $\kappa_{st}^k(p,q)$ be respectively the expected maintenance cost and expected loss in production capacity of the system during period t, when the last preventive maintenance action before time period t has taken place in the beginning of period s, $s \le t$. These parameters are given by

$$c_{st}^k(p,q) = \begin{cases} c_{PM}^k + c_{CM}^k \int_0^\tau \beta_k r_0(u+\alpha_k p\tau)du & \text{if } t=s, \\ c_{CM}^k \int_0^\tau \beta_k r_0(u+\alpha_k p\tau+q\tau)du & \text{if } s<t\le T. \end{cases} \quad (14)$$

and

$$\kappa_{st}^k(p,q) = \begin{cases} \theta_{PM}^k + \theta_{CM}^k \int_0^\tau \beta_k r_0(u+\alpha_k p\tau)du & \text{if } t=s, \\ \theta_{CM}^k \int_0^\tau \beta_k r_0(u+\alpha_k p\tau+q\tau)du & \text{if } s<t\le T. \end{cases} \quad (15)$$

The integrated production and imperfect preventive maintenance planning problem (*IPImPMPP*) can now be reformulated and modeled with the linear mixed integer program described below

($Re-IPImPMPP$): Minimize

$$Z_{PPM}^{IP} = \sum_{t=1}^{T}\sum_{j=1}^{N}\left(f_{jt}x_{jt}+p_{jt}Q_{jt}+h_{jt}I_{jt}\right) + \sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s}\sum_{p=0}^{s-1}\sum_{q=0}^{t-s} c_{st}^k(p,q)v_{st}^k(p,q)$$

subject to

$$Q_{jt}+I_{j,t-1}-I_{jt} = d_{jt}, \quad \forall j \in P, t \in H \quad (7)$$

$$Q_{jt}-min\left\{d_{tT}^j, \kappa_{max}\right\}x_{jt} \le 0, \quad \forall j \in P, t \in H \quad (8)$$

$$x_{jt}-y_t \le 0, \quad \forall j \in P, t \in H \quad (9)$$

$$\sum_{j=1}^{N}\rho_j Q_{jt} + \sum_{s=1}^{t}\sum_{k=1}^{s}\sum_{p=0}^{s-1}\sum_{q=0}^{t-s}\kappa_{st}^k(p,q)v_{st}^k(p,q) \le \kappa_{max}, \quad \forall t \in H \quad (10\text{revisited})$$

$$\sum_{s=1}^{t}\sum_{k=1}^{s} z_{st}^k = 1, \quad \forall t \in H \quad (11)$$

$$z_{st}^k - z_{s,t+1}^k \ge 0, \quad \forall k,s,t \in H, k \le s \le t \le T-1, \quad (12)$$

$$\sum_{s=k-1}^{t-1} z_{s,t-1}^{k-1} - z_{tt}^k \ge 0, \quad \forall k,t \in H, 2 \le k \le T, k \le t \quad (13)$$

$$\sum_{p=0}^{s-1}\sum_{q=0}^{t-s} p \cdot u_{st}(p,q) - \sum_{s'=1}^{s-1} y_{s'} = 0, \quad \forall s,t \in H, s \le t \quad (16)$$

$$\sum_{p=0}^{s-1}\sum_{q=0}^{t-s} q \cdot u_{st}(p,q) - \sum_{s'=s}^{t-1} y_{s'} = 0, \quad \forall s,t \in H, t, s \le t \quad (17)$$

$$\sum_{p=0}^{s-1}\sum_{q=0}^{t-s} u_{st}(p,q) = 1, \quad \forall s,t \in H, s \le t \quad (18)$$

$$y_t + z_{st}^k + u_{st}(p,q) - v_{st}^k(p,q) \le 2, \quad \forall k,s,t,p,q,\ p \le s-1, q \le t-s \quad (19)$$

$$\sum_{s=1}^{t}\sum_{k=1}^{s}\sum_{p=0}^{s-1}\sum_{q=0}^{t-s} v_{st}^k(p,q) - y_t \le 0, \quad \forall t \quad (20)$$

$$Q_{jt}, I_{jt} \ge 0, x_{jt}, y_t, z_{st}^k, u_{st}(p,q), v_{st}^k(p,q) \in \{0,1\} \forall j \in P, s,t \in H, k \le s \le t.$$

Constraints (7)–(13) remain unchanged, however constraints (16), (17) and (18) determine the values of the variables u and v. The last constraints (19) and (20) connect the variables y and z with u and v meaning that if the k^{th} and last preventive maintenance before period t takes place in period $s \le t$ and if the system is setup to production p times during the horizon $\{1,\dots,s-1\}$ and q times during the periods $\{s,\dots,t-1\}$ then $v_{st}^k(p,q)=1$.

Observe now that the model ($Re-IPImPMPP$) is a mixed-integer linear program which solves the integrated production and imperfect preventive maintenance planning problem to optimality. However, the price paid for this linearization is an explosion in the numbers of variables and constraints. As a consequence, with this model we will be able to solve only small to medium instances of the problem (*IPImPMPP*) within a reasonable computational time. However, it still can be used to evaluate the performance of any heuristic method proposed for this problem as it provides optimal solution values or lower bounds.

In the following section we propose a fix-and-optimize procedure which still uses a mixed-integer linear model but with far more less variables and constraints than those of the model ($Re-IPImPMPP$), and still provides an optimal integrated production and imperfect preventive maintenance plan, if carried out until its completion.

5. An iterative MILP-based solution heuristic

The mixed-integer linear programming model ($R-IPImPMPP$) solves the integrated production and imperfect preventive maintenance planning problem to optimality. However, when solved with the usual MILP solvers such as Cplex or Gurobi some instances of medium to large size require a very large amount of computational time. This is typically due the additional variables and constraints necessary to obtain valid mixed-integer linear formulation for the problem.

Now notice that if one knows in advance the periods during which the system is setup to production, i.e., if the variables $(y_t)_{t\in H}$ are fixed, we can then compute the coefficient $c^k_{st}(\boldsymbol{y})$ and $\kappa^k_{st}(\boldsymbol{y})$ become constant, so the model (*IPImPMPP*) becomes linear. The procedure proposed in this section takes advantage of this fact and provides a more efficient solution procedure for large instances of the problem (*IPImPMPP*).

5.1. Fix-and-optimize solution procedure to (Re-IPImPMPP)

For a given fixed vector $\boldsymbol{y}$ of production setups, we define the vector $(\overline{p},\overline{q})$ having as components these given by $\overline{p}_{st}=\sum_{s'=1}^{s-1}\overline{y}_{s'}$ if $s\geq 2$, and 0 otherwise; and $\overline{q}_{st}=\sum_{s'=s}^{t-1}\overline{y}_{s'}$ if $t\geq s+1$, and 0 otherwise, for each pair of period $s,t\in H$, such that $s\leq t$. We consider the problem ($Re-IPImPMPP_{FSV}(\overline{p},\overline{q})$) obtained from the problem ($Re-IPImPMPP$) by fixing the vector of production setups $\boldsymbol{y}$ and relaxing constraints (9)

($Re-IPImPMPP_{FSV}(\overline{p},\overline{q})$:

$$Z^{PM}_{FSV}(\overline{p},\overline{q})=Minimize\sum_{t=1}^{T}\sum_{j=1}^{N}\left(f_{jt}x_{jt}+p_{jt}Q_{jt}+p^E_{jt}Q^E_{jt}+h_{jt}I_{jt}\right)+\sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s}c^k_{st}(\overline{p}_{st},\overline{q}_{st})z^k_{st}$$

subject to:

$$Q_{jt}+Q^E_{jt}+I_{j,t-1}-I_{jt}=d_{jt},\quad \forall j\in P,\ t\in H \tag{7}$$

$$Q_{jt}-\left(\sum_{t'=t}^{T}d^j_{t'}\right)x_{jt}\leq 0,\quad \forall j\in P,\ t\in H \tag{8}$$

$$\sum_{j=1}^{N}\rho_jQ_{jt}+\sum_{s=1}^{t}\sum_{k=1}^{s}\kappa^k_{st}(\overline{p}_{st},\overline{q}_{st})z^k_{st}\leq\kappa_{max},\quad \forall t\in H \tag{10revisited}$$

$$\sum_{s=1}^{t}\sum_{k=1}^{s}z^k_{st}=1,\ \forall t\in H \tag{11}$$

$$z^k_{st}-z^k_{s,t+1}\geq 0,\quad \forall k,s,\ t\in H,\ k\leq s\leq t\leq T-1, \tag{12}$$

$$\sum_{s=k-1}^{t-1}z^{k-1}_{s,t-1}-z^k_{tt}\geq 0,\quad \forall k,\ t\in H,\ 2\leq k\leq T, k\leq t \tag{13}$$

$$Q_{jt},Q^E_{jt},I_{jt}\geq 0, x_{jt},z^k_{st}\in\{0,1\}\quad \forall j\in P,s,\ t\in H,\ k\leq s\leq t.$$

where $p^E_{jt}=\max_{t'\in H}\left\{f_{jt'}+p_{jt'}+\sum_{s'=1}^{T}h_{js'}+\sum_{t''=1}^{T}\sum_{s'=1}^{t''}\sum_{k=1}^{s'}c^k_{s't''}(\overline{p}_{s't''},\overline{q}_{s't'})\right\}$.

Now observe that adding the variables Q^E_{jt} for all $j\in P$ and $t\in H$, solves the infeasibility issue which might rise for some values of the vector $(\overline{p},\overline{q})$. The cost parameter p^E_{jt} is chosen so that only if the problem is infeasible that the variable Q^E_{jt} assumes a positive value. The problem ($Re-IPImPMPP_{FSV}(\overline{p},\overline{q})$ can be solved as a mixed-integer linear program using available MILP solvers such as Cplex or Gurobi. The numbers of variables and constraints, compared to those of the model ($Re-IPImPMPP$), are significantly lower.

The proposed fix-and-optimize procedure uses these problems to find an optimal or close to optimal solutions for the integrated production and imperfect preventive maintenance planning problem (*IPImPMPP*). We let $\left(Q^{PM}_{FSV}(\overline{p},\overline{q}),x^{PM}_{FSV}(\overline{p},\overline{q}),z^{PM}_{FSV}(\overline{p},\overline{q})\right)$ and $Z^{PM}_{FSV}(\overline{p},\overline{q})$ be the optimal solution for the problem ($Re-IPImPMPP_{FSV}(\overline{p},\overline{q})$) and its corresponding value, respectively. We also let (Q^{OPT}, x^{OPT},z^{OPT}) and $Z_{OPT}{}^{PM}$ be respectively the best solution found so far and value and its corresponding value

$$Z^{PM}_{OPT}=\sum_{t=1}^{T}\sum_{j=1}^{N}\left(f_{jt}x^{OPT}_{jt}+p_{jt}Q^{OPT}_{jt}+h_{jt}I^{OPT}_{jt}\right)+\sum_{t=1}^{T}\sum_{s=1}^{t}\sum_{k=1}^{s}c^k_{st}\Big(\sum_{s'=1}^{s-1}\max_{j\in P}\left\{x^{OPT}_{js'}\right\},\sum_{s'=s}^{t-1}\max_{j\in P}\left\{x^{OPT}_{js'}\right\}\Big)z^{k,OPT}_{st}$$

Two important remarks are ought to be mentioned, first in most of the cases when the procedure stops the resulting solution is an optimal feasible solution. This is mainly because if the number of production setups in the optimal solution to the problem ($Re-IPImPMPP$) is m^*, the 'While loop' improves this solution to determine for each pair (s,t) with $s\leq t$, the optimal values. Second, the procedure can also be used as a heuristic by stopping it when some predefined target gap is reached.

Algorithm 1. Fix-and-Optimize Heuristic.

for $m=1$ **to** T **do**
  for $l=1$ **to** m **do**
    for $t=1$ **to** T **do**
      for $s=1$ **to** t **do**
        let $\overline{p}_{st}=\min\{l,s-1\}$ and $\overline{q}_{st}=\min\{m,t-s\}$
      end for
    end for
    Solve the problem ($Re-IPImPMPP_{FSV}(\overline{p},\overline{q})$)
    let $Z^{PM}_{opt}:=Z^{PM}_{FSV}(\overline{p},\overline{q})$, and
    let $(Q^{OPT},x^{OPT},z^{OPT}):=\left(Q^{PM}_{FSV}(\overline{p},\overline{q}),x^{PM}_{FSV}(\overline{p},\overline{q}),z^{PM}_{FSV}(\overline{p},\overline{q})\right)$.
    while *Optimal* $\neq$ *Yes* **do**
      for $t=1$ **to** T **do**
        for $s=1$ **to** t **do**
          let $\overline{p}^*_{st}=\sum_{s'=1}^{s-1}\max_{j\in P}\left\{x^{OPT}_{js'}\right\}$ and
          let $\overline{q}^*_{st}=\sum_{s'=s}^{t-1}\max_{j\in P}\left\{x^{OPT}_{js'}\right\}$
        end for
      end for
      Re-solve the problem ($Re-IPImPMPP_{FSV}(\overline{p}^*,\overline{q}^*)$)
      if $Z^{PM}_{FSV}(\overline{p}^*,\overline{q}^*)<Z^{PM}_{opt}$ **then**
        let $Z^{PM}_{opt}:=Z^{PM}_{FSV}(\overline{p}^*,\overline{q}^*)$, and
        let $(Q_{OPT},x_{OPT},z_{OPT}):=\left(Q^{PM}_{FSV}(\overline{p}^*,\overline{q}^*),x^{PM}_{FSV}(\overline{p}^*,\overline{q}^*),z^{PM}_{FSV}(\overline{p}^*,\overline{q}^*)\right)$.
        let *Optimal* $\neq$ *Yes*
      else $\{Z^{PM}_{FSV}(\overline{p}^*,\overline{q}^*)\geq Z^{PM}_{opt}\}$
        let *Optimal* = *Yes*
      end if
    end while
  end for
end for

5.2. Analysis of some computation experiments

In this sub-section we present and discuss the results of the computational experiments carried out on a selected set of cases, available in the literature. In particular, a collection of test instances from the LOTSIZELIB [40] is used to evaluate the

Table 3
Maintenance data for instance A2007.

	Expected number of failures after k^{th} imperfect preventive maintenance							
	1	2	3	4	5	6	7	8
[0, 1t]	0.901							
[1t, 2t]	1.489	1.104						
[2t, 3t]	1.664	1.641	1.480					
[3t, 4t]	1.749	1.814	1.859	1.898				
[4t, 5t]	1.799	1.901	2.007	2.127	2.289			
[5t, 6t]	1.833	1.954	2.087	2.238	2.423	2.653		
[6t, 7t]	1.857	1.989	2.137	2.304	2.500	2.732	3.004	
[7t, 8t]	1.875	2.014	2.171	2.348	2.550	2.784	3.052	3.356

Table 4
Summary of the experimental results.

Instance	Values of the model ($Re-IPImPMPP$)		Values of fix-and-optimize algorithm		Gap
	Value	CPU in sec	Value	CPU in sec	(%)
A2007	815.435	1.59	815.435	3.19	0.000
tr6_15	337,355.000	4606.00	337,355.000	4339.00	0.000
tr6_30	677,888.000	20,533.36	677,948.000	2125.16	0.009
tr12_15	1,245,873.000	2810.00	1,245,920.000	2710.00	0.004
tr12_30	4,296,869.000	19,652.90	4,398,920.000	2088.91	2.375
tr24_15	2,502,277.000	1720.00	2,502,640.000	2920.00	0.015
tr24_30	8,305,836.000	28,842.40	8,306,310.000	2031.02	0.006
set1ch	107,532.000	260.00	107,532.000	130.00	0.000

performance of the model ($Re-IPImPMPP$) and the proposed fix-and-optimize algorithm for the integrated planning of production and imperfect preventive maintenance problem. Of course, the instances from the LOTSIZELIB were extended and adapted to the integrate maintenance optimization aspect. Specifically, data related to the reliability and maintenance of the manufacturing system is created and added to production data. The instance *A2007* is the test case used in [24]. The instances from LOTSIZELIB are denoted by *trN_T*, where $N=6$, 12, 24 is the number of items and $T=15$, 30 is the number of periods, and the instance *set1ch* is from [41]. The MILP models are coded in AMPL and are solved using the solver CPLEX 12.6. The computational tests were carried out on an Intel(R) Core(TM) i7-3770 CPU @ 3.40 GHz, 3401 MHz, 4 Core(s), 8 Logical with 32 GB RAM running under windows 7. The CPU times are given in seconds.

For reliability and maintenance data, we assume that the manufacturing system is subject to a random deterioration and that the failure rate is governed by a Gamma distribution $\Gamma(m=2, \nu=2)$ with a shape parameter $m=2$ and a rate parameter $\nu=2$ as in [24]. Recall that the density function of a Gamma distribution $\Gamma(m,\nu)$ with a shape parameter m and a rate parameter ν is given by

$$f(t)=\begin{cases}\dfrac{\nu^m t^{m-1}\exp^{(-\nu t)}}{\Gamma(m)} & \text{if } t,\ \nu\geq 0,\ m>0,\\ 0 & \text{if } t<0.\end{cases} \tag{21}$$

As an illustration, consider the instance A2007 where the production is to be planned on an 8 periods horizon, Table 3 below shows the expected number of failures as a function of the manufacturing system's age, in case of imperfect preventive maintenance with $\alpha_k=k/(3k+7)$ and $\beta_k=(12k+1)/(11k+1)$ for all k:

The following Table 4 summarizes the results of the experiments carried out. The first column of the table identifies the solved instances. The second column reports the optimal value of each instance obtained by solving the model ($Re-IPImPMPP$), assuming imperfect preventive maintenance with $\alpha_k=k/(3k+7)$ and $\beta_k=(12k+1)/(11k+1)$ for all k. Each instance is solved using AMPL/CPLEX 12.6, and the third column reports the resulting CPU running time. The fourth column reports the value of each instance obtained by the proposed Fix-and-Optimize algorithm. This algorithm is also coded in AMPL and the sub-problems ($Re-IPImPMPP_{FSV}(\overline{p},\overline{q})$) are solved for each instance using CPLEX 12.6. The fifth column reports the resulting CPU running time obtained using the AMPL/CPLEX 12.6 this coded Fix-and-Optimize algorithm.

When comparing results of the proposed Fix-and-Optimize algorithm to the results of the model ($Re-IPImPMPP$), clearly the proposed algorithm performs quite well and provides solutions which are close to optimal. In fact, Table 4 reveals that the highest gap does not exceed 2.4%. The Gap, in this case, is defined as the ratio of the difference between the value of the fix-and-optimize procedure and the value of the model ($Re-IPImPMPP$) to the value of the model ($Re-IPImPMPP$), and is expressed in percentages. In terms of CPU time, even though the Fix-and-Optimize algorithm may also last up to an hour and half for some instances, it clearly beats the model ($Re-IPImPMPP$) which requires up to 8 h to reach optimality for the same instance.

We are currently working on accelerating this Fix-and-Optimize method. One possible direction, that is currently investigated, is to not solve some sub-problems ($Re-IPImPMPP_{FSV}(\overline{p},\overline{q})$) to optimality, but stop when the solution value reaches some threshold. The initial experiments with this approach showed that the gap may increase up to 15%, but then the instances which required more than an 1 h and 20 min were solved in 15–20 min.

6. Conclusion

This paper tackles the problem of integrating production planning and imperfect preventive maintenance, at the tactical level, in failure prone manufacturing system. To model the imperfect preventive maintenance, a hybrid failure rate model is assumed. First, an optimization model for the problem is developed. This model is naturally formulated as a mixed-integer nonlinear optimization program. Then, an extended formulation for the problem is proposed, which solves it as a mixed-integer linear program. However, the numbers of variables as well as constraints get quickly very large for medium to large instances of the problem. This limits the usage of this extended formulation. Nevertheless, it can be used to provides lower bound to measure the performance of any heuristic that might later be developed for the problem.

Next, a fix-and-optimize iterative procedure is proposed, which also solves a set of mixed-integer linear optimization programs, but with significantly less variables and constraints than the proposed reformulation of the problem. These optimization problems are well-constructed relaxed mixed-integer linear sub-problems, also stemming from the reformulation of the problem discussed in

Section 4. The reformulated model and the procedure are tested on a set of instances adopted from the instances that are used for the capacitated lot-sizing problem. The results show that the procedure performs quite well, even though the CPU time is still relatively large. This iterative procedure, however, can also be used as a heuristic in two ways, either by stopping the process when some predefined acceptable performance level is reached or by replacing the exact solution of the relaxed linear mixed-integer sub-problems by some heuristic solutions.

To conclude, this paper proposes thus a first practical optimization model for solving the integrated production and imperfect preventive maintenance planning problem, at the tactical level. It also opens some venues for improvements of the proposed optimization model itself and also for the development of new heuristic approaches to this complex optimization problem. The proposed model and its reformulation provide also a first instrument to measure the performance of the to be developed heuristic approaches, before they are incorporating in some extended integrated planning module of the future ERP frameworks.

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