# International Workshop on Lot-Sizing - IWLS'2022

**24th-26th August 2022, Oslo, Norway**

# Foreword

**Velkommen til Oslo!**

Dear colleagues and friends,

It is a real pleasure to welcome you in Oslo for the International Workshop on Lot-Sizing 2022. This is the first time IWLS takes place in Scandinavia. After this complicated period, we hope that you will enjoy the workshop that includes 24 in-person presentations. The program is rich and varied, with enough time for each presentation and between sessions to encourage fruitful exchanges between participants.

We also have planned three social events: The classical get together party on Tuesday, as in all the on-site editions of IWLS, a cruise tour in the Oslo fjord on Wednesday and, on Thursday, the gala dinner in Ekeberg restaurant preceded by a guided tour of the Ekeberg sculpture park for those who are interested. We believe you will appreciate these events, which are great opportunities to make new friends and start new collaborations.

We hope you will leave Oslo with some great souvenirs of the workshop and the city, and motivated to participate to the future editions of the International Workshop on Lot-Sizing.

Stéphane Dauzère-Pérès, Erna Engebrethsen, Mehdi Sharifyazdi, Karim Tamssaouet

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# Workshop Organization

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# Sustainability

A2-030 - Wednesday, 24/08 - 09:30-10:30

# Single-machine multi-product lot-sizing with on-site generation of renewable energy

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**Abstract**

Powering industrial processes (at least partially) with electricity generated on-site from renewable sources might help reducing the environmental impact of industrial production. The intermittence of such sources, together with the time-of-use pricing scheme widely used by electricity providers, lead to the need to solve an integrated energy supply and industrial production planning problem. We thus investigate a single-machine multi-product lot-sizing problem with on-site generation of renewable energy.

One modelling difficulty comes from the fact that the time discretization needed to manage product demand satisfaction, production planning and energy supply may significantly differ. We propose to deal with this difficulty by considering an extension of the Proportional Lot-Sizing Problem in which the same time slicing is used to handle the industrial production and the energy supply. We compare our model with the recently proposed extended General Lot-Sizing Problem which uses three levels of time slicing [1]. Our numerical results show that, in most cases, our model provides plans of the same quality, but with a reduced computational effort.

## 1 Introduction

Industrial companies are increasingly under pressure to mitigate the CO2 and pollution emissions linked to the manufacturing of industrial products. They are also confronted with a sharp rise of the price of conventional energy sources (gas, grid electricity...) so that the availability and affordability of energy is becoming a critical parameter in manufacturing. One way to deal with these two challenges consists in powering industrial processes with electricity generated on-site from renewable sources.

However, the intermittence of renewable energy sources (wind, sun...) makes it impossible to fully replace gas or grid electricity by on-site generated electricity to power an industrial process: both types of energy should thus be used in combination. Moreover, the time-of-use pricing scheme widely used by electricity providers means that it is necessary to accurately track the timing and quantity of grid electricity bought from these providers. Thus, an integrated energy supply and industrial production planning problem needs to be solved. This work is an attempt at modeling and solving such an integrated planning problem. We namely investigate a single-machine multi-product lot-sizing problem with on-site generation of renewable energy.

# 2 Problem description and modeling

We seek to plan production in an industrial plant producing several types of item to satisfy a time-varying external market demand. Backlogging is not is allowed and demand must be met on time. The plant comprises a single production resource. This resource may produce only one type of item at a time. Changing the type of item in production requires to carry out startup operations. These operations incur a fixed startup cost but the startup time is assumed to be negligible.

Energy is consumed during both startup operations and manufacturing. The amount of energy consumed during a startup depends on the product that the machine will be setup for. The energy consumed during manufacturing is proportional to the number of items produced. No energy is consumed when the machine is idle or when it is setup for a given product but not producing.

The energy supply system comprises three main elements: the onsite power generation devices (photovoltaic panels, wind turbines) converting a renewable energy source into electricity, the on-site energy storage system and the main electricity grid. The amount of electricity produced by the onsite power generation devices depends on the availability of the corresponding energy source and is given. The on-site energy storage system consists of batteries which can store electricity (either generated on-site or bought from the main grid). During peak-demand and/or peak-price periods, energy can be supplied to the production system by discharging the battery. Finally, electricity may be bought from the main grid at a given time-varying buying price. Figure 1 provides an overview of the studied integrated energy supply and industrial production system.

One modelling difficulty here comes from the fact that the time discretization needed to manage product demand satisfaction, production planning and energy supply may significantly differ. Indeed, following the terminology introduced by [2], we have to handle two exogenous time structures, i.e. two sets of points in time at which externally given events, that are defined by the data of the model, are considered.

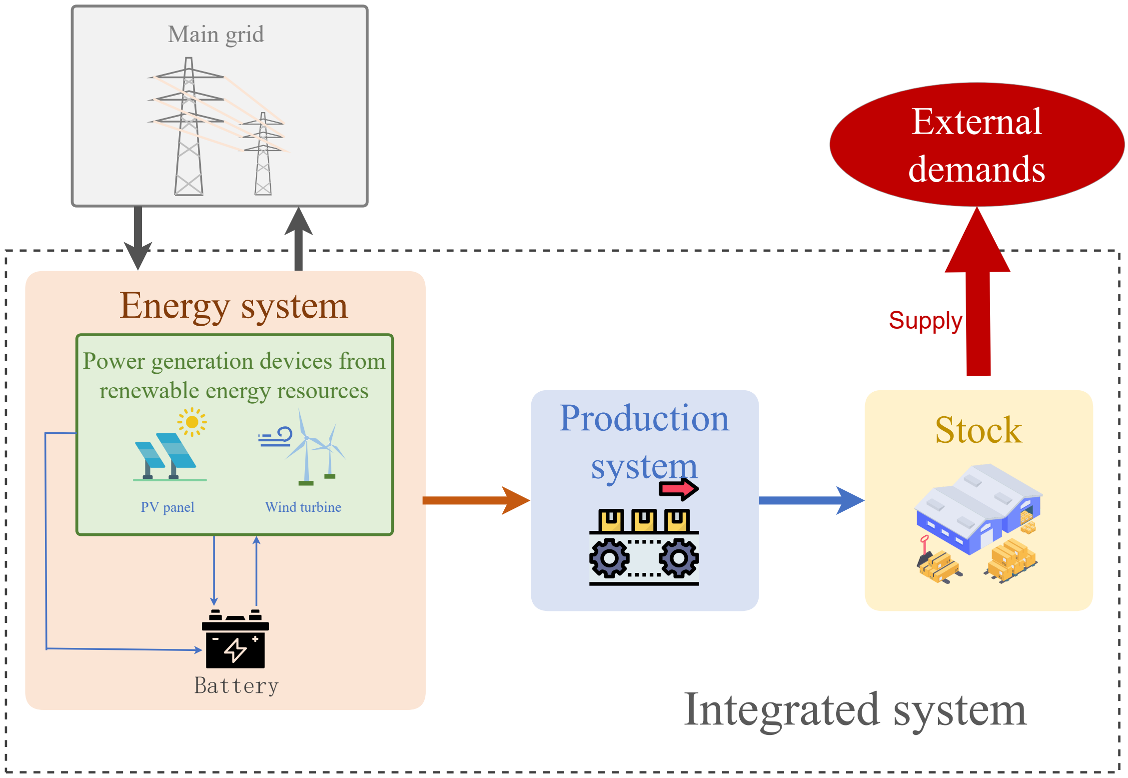


Figure 1: Studied integrated energy supply and industrial production system

The first time structure is imposed by the timing of the external demand and usually uses rather large time buckets (typically days or weeks). The second one is imposed by the discrete time grid used to track the availability of the on-site generated electricity and the varying price of grid electricity. This time grid usually uses much smaller time buckets (typically hours or 10-minutes intervals). In between these two exogenous time structures, to plan production, we have to handle a third endogenous time structure representing the points in time at which internal events are captured by decision variables.

To tackle this difficulty, Wichmann *et al.* [1] recently proposed a model based on an extension of the General Lot-Sizing Problem. This one is based on the use of three distinct time structures. However, it leads to the formulation of a large-size mixed-integer linear program involving many big-M type constraints, which results in significant computational difficulties to solve medium-size instances. In order to solve larger instances, we propose a new extension of the Proportional Lot-Sizing Problem in which the same time slicing is used to handle the industrial production and the energy supply. This amounts to considering that the endogenous time structure used to make planning decisions is the same as the exogenous one used to track the energy supply. The resulting model thus uses a set of macro-periods to follow the demand satisfaction and a set of micro-periods to track the energy supply and consumption. These energy-related micro-periods define the discrete external time grid used to plan production. As commonly done in PLSP models, we assume that at most two different types of products may be produced during a given energy-related micro-period.

## 3 Preliminary computational results

We carried out some numerical experiments to compare the quality of the energy supply and production plans obtained with both models. Numerical instances were randomly generated using data available in [1], [3] and [4]. Each instance was solved with CPLEX 12.8 using both the extended PLSP and the extended GLSP model. Our preliminary computational results first show that for small-size instances involving 3 products and 32 micro-periods, the problem can be solved to optimality much faster by using the PLSP model than by using the GLSP model. Moreover, in most cases, the cost of the production plan obtained with the PLSP model is the same as the one obtained in the the GLSP model. Moreover, for medium and large instances involving up to 10 products and 256 micro-periods, the solver can hardly find a feasible solution when applying the GLSP model, while it finds a relative good feasible solution when using the PLSP model.

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# A MILP heuristic for a production planning problem with returns and substitution

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**Abstract**

This work considers a production facility capable of making both new and used products. Returned products can be refurbished and sold as second hand items. If there is a shortage of such products, the company can satisfy their demand using new products at their original low price (downward substitution). If the returned products cannot be refurbished, they are dismantled and their key parts are remanufactured and made as good as new parts which will be used in the assembly of new finished products. The latter can also use new parts purchased from external suppliers at a higher cost compared to remanufactured parts. In this study, we solve a production planning problem associated with this configuration. The problem is solved using a Relax-and-Fix heuristic. Numerical experiments were carried out to assess the performance of the heuristic. We also present a detailed analysis of the value of integrating these processes (refurbishing, remanufacturing, and manufacturing) and the value of substitution.

# 1 Introduction

Reducing wastes is not just related to environmental issues. It can provide opportunities for large savings for companies. This can be achieved through the recovery of used items. Companies are moving from the traditional linear systems (take, make, dispose) to circular economy system where products are reused, shared, repaired, refurbished, remanufactured and recycled [2]. This emerging area of research attracts both academic and industry practitioners.

Management of electrical and electronic wastes has become a significant concern among manufacturers, and is one of the fastest growing areas of research due to shorter product life cycle and rapidly changing customer attitudes towards disposing used products. For example, 80% of mobile phones have been upgraded every 18-24 months [3]. Most of electrical and electronic products are also designed for recyclability, so they can be easily disassembled and recycled.

One of the main goals for introducing electrical and electronic returns is to transform the forward supply chain into the closed loop network (manufacturing and remanufacturing) and optimizing their cost and environmental impact simultaneously. Most studies relate to manufacturing and remanufacturing do not consider the refurbishing activities. However most of electrical and electronic returns have been lightly used or slightly damaged and quickly repairable for resale as second hand items. In this study we propose a hybrid manufacturing/remanufacturing model consisting of manufacturing, remanufacturing for making new products, and refurbishing for making used products. the demand of used products can satisfy their demand using new products at their original low price (downward substitution).

# 2 Problem Description

This work considers a production facility able to process new and returned products. It integrates three processes (paths). The first process repairs and refurbishes returns to satisfy demand of second hand products. Returns which cannot be refurbished are dismantled (in the second process) to extract from them key components which can be considered as good as new. The third process (path) uses the components disassembled or new purchased components to assembly a new products. We authorize downward substitution, where the new product may be used to meet the demand of second hand product at the the same price as the latter. The objective is to minimize the total cost combining fixed setup costs, and variable purchasing, processing and inventory holding costs. All demands must be satisfied (directly or through substitution) and constraints related to capacity and availability of products must be respected. The problem is formulated as a mixed integer linear programming model (MILP).

## 3 Solution approach: A Relax-and-Fix heuristic

It is easy to prove that this problem is NP-hard by showing that it is an extension of the One Warehouse multi retailer problem. Hence, to solve it, we propose a Relax and Fix (RF) heuristic ([4]). There exist several successful implementations of RF heuristics in production planning and scheduling (e.g. [1]). Computational experiments were carried out in order to evaluate the performance of the proposed heuristic, to analyse different integration levels between the three lines and to analyse the economic viability of substitution.

From a computational point of view, the RF heuristic is efficient and offers a good trade-off between solution quality and CPU times. This was verified through several experiments with different configurations obtained by adding or removing constraints related to production capacity and joint setups.

Through numerical experiments we also show that substitution and integration of of manufacturing, refurbishing and remanufacturing have considerable benefits from an economical point of view. We analyze these benefits vary as a function of different costs.

Figure 1 shows the relationship between quantities substituted, quantities collected, and quantities disassembled in the process as a function of the new production assembly (manufacturing) cost.

Figure 2 shows the value of substitution by comparing total costs of models with and without substitution. It indicates the range of values of manufacturing costs which make substitution economically interesting. If the manufacturing cost is too high (here starting at 5 monetary units), the two models have the same total cost and substitution is not a good option any more. This is confirmed by Figure 1 which shows that no substitution takes place starting at 5 monetary units.

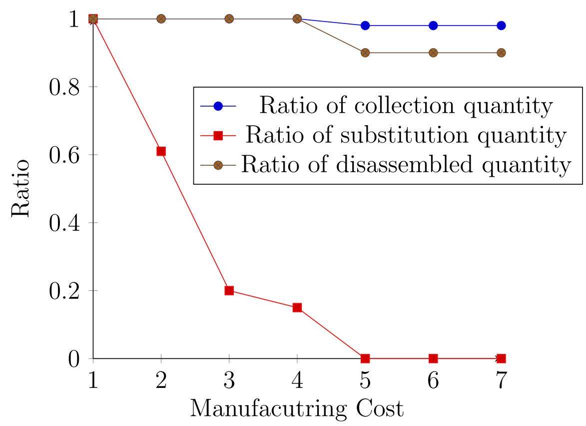


Figure 1: Ratio to the total demands for new and used products

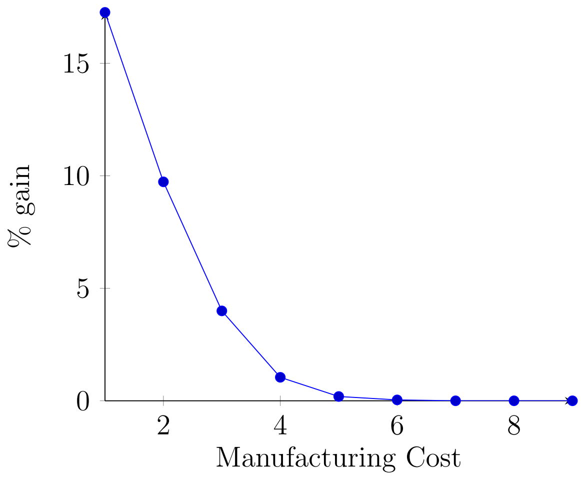


Figure 2: Value of substitution

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# Stochastic Lot Sizing – I

A2-030 - Wednesday, 24/08 - 11:00-12:30

# Dynamic lot sizing using forecast information

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**Abstract**

We study the stochastic lot-sizing problem where the item has non-stationary stochastic demand. Forecast information is updated as more demand realizations become available, and there exists a production lead time. The objective is to minimize costs of production, inventory and unsatisfied demand. We employ a *dynamic* planning strategy [1], rather than optimizing for a long planning horizon and only implementing part of the solution, as is the case in rolling horizon approaches that are commonly used to deal with new information and uncertainty [2]. We model the problem as a Markov Decision Process (MDP), and explicitly consider the forecast information in the state space to allow for the consideration of non-stationary demand processes. We solve the MDP in an approximate manner, using neural networks to represent policies, modifying a Deep Reinforcement Learning algorithm [3] to make it suitable in this highly stochastic and non-stationary environment. This method is compared to rolling horizon approaches, generic DRL methods and heuristics, in terms of costs, service level and planning stability.

# 1 Introduction & Literature

We consider a stochastic, single-item lot-sizing problem, where the item under consideration has non-stationary stochastic demand. The forecast for demand changes as more demand realizations become available, and there is a lead time after production. The difficulty of optimizing this forecast-based activity lies in two parts. The first question is how to forecast accurately, given the often seasonal and non-stationary demand patterns. The second challenge is how to optimize the lot-sizing problem, knowing that the forecast input will likely not reflect reality perfectly. A common approach is to take the forecast of the future as a given, and then solve the problem as if it is deterministic for a certain planning horizon [2]. The first part of the planning (the *frozen interval*) is executed. After the duration of the *re-planning periodicity*, a new planning is made for the full planning horizon. With this *rolling horizon* planning method, new or updated information can be considered at every re-planning moment.

Finding the best parameters for this rolling horizon method is not trivial. [2] show that the choice of the length of planning horizon, re-planning periodicity and frozen interval have a major impact on the effectiveness of the plan. Also the *end-effect* mitigation strategies are crucial in this methodology. Furthermore, one can imagine that only executing the first few steps of a plan, and then re-optimizing the plan may be sub-optimal. Instead, a *dynamic* planning strategy, where we only make decisions for the periods that are going to be executed could be advantageous. In cases where this strategy is used, the problem is typically formulated as a Markov Decision Process (MDP). Larger instances of MDP can be solved with Deep Reinforcement Learning (DRL) techniques, but studies show that stochastic environments can make the training process difficult and unstable. Additionally, the demand in these studies is typically assumed to be stationary, while in reality the demand patterns might change over time.

In several inventory control papers, the non-stationary demand process is studied, but these papers typically neglect setup costs and/or lead times. While [5] have proven that a state-dependent $(r, S)$ policy is optimal in the case of fixed order cost and a fluctuating demand environment, finding the parameters of this policy is difficult. The parameters could be determined by solving an MDP, where DRL could be used in large instances. This methodology is compared to heuristics from [6] and [7]. Additionally, existing literature studies mainly the case of backlogging in case demand exceeds available inventory, while in practice demand could be lost.

In this study, we have three research questions:

1. How does the performance of a dynamic planning strategy compare to alternative strategies (rolling horizon optimization, heuristics for $(r, S)$ parameters)?

2. How can we effectively use the demand / forecast information in the state representation of the MDP, considering this non-stationary and seasonal demand?

3. How can we learn in a stable and computationally efficient manner in this highly stochastic environment?

# 2 Problem formulation

In this section, we model the production system as an Integer Linear Program, which is used for the rolling horizon solution method. The demand forecast is updated every decision period. At the decision moment, we have a demand forecast $f_t$ for period $t$ ($t = 1, 2, \cdots, T + L,\ f_t \in \mathbb{N}$). The production step has a deterministic lead time ($L$). In this production system, costs are incurred for setting up production ($K$)[1] and holding an item on stock ($h$). At the same time, penalty costs ($p$) are incurred for the lost sales ($s_t$).

**Objective function** The objective is to minimize the costs over the model horizon ($T + L$). Since we can no longer impact decisions within the lead time of production ($t < L$), we only optimize the costs over the horizon which we can impact (1).

$$
\begin{aligned}
&\text{minimize} && \sum_{t=1+L}^{T+L} K\delta(q_t) + hI_t + ps_t && && (1)\\
&\text{subject to} && s_t \geq f_t - I_{t-1} - q_t, && t = 1+L, 2+L, \cdots, T+L && (2)\\
& && s_t \geq f_t - I_{t-1} - \hat{q}_t, && t = 1, \cdots, L && (3)\\
& && I_t = I_{t-1} + q_t - f_t + s_t, && t = 1+L, 2+L, \cdots, T+L && (4)\\
& && I_t = I_{t-1} + \hat{q}_t - f_t + s_t, && t = 1, \cdots, L && (5)\\
& && I_t, q_t, s_t \in \mathbb{N}, && t = 1, 2, \cdots, T+L && (6)
\end{aligned}
$$

Since we consider lost sales, we determine the lost sales variable $s_t$ in (2) as the part of the forecasted demand that cannot be satisfied by available inventory or incoming production. For the periods within the lead time, the incoming production ($\hat{q}_t$) is already in the pipeline, and is no longer a decision variable (3). The inventory transition function is given by (4) and (5). Note that in this sequential decision making process, $I_0$ is a result of previous production decisions and the actual demand realizations. Constraint (6) indicates that the inventory, production quantity and shortage are all non-negative integers.

## 2.1 Markov Decision Process

This lot-sizing problem can also be modeled as an MDP. In this MDP, we only consider actions relevant for the planning period ($q_{t+L}$) , rather than hypothetical actions for the future planning horizon ($q_{t+L}, q_{t+1+L}, ..., q_{t+T+L}$). At each time step in the MDP, the system is in some state $s_t$ (includes information about forecast, current inventory, production in pipeline), and the decision maker chooses action $a_t$ (a production

[1] $\delta(x)$ is the indicator function used to determine if setup costs are incurred.If $x > 0$, $\delta(x) = 1$, else $\delta(x) = 0$

quantity), that is feasible in that state. The set of feasible actions is denoted by $A_t$. At the next time step, the system transitions randomly into the next state $s_{t+1}$, giving a reward $R_{a_t}(s_t, s_{t+1})$ (the resulting costs of the action and observing demand). The probability of moving into the new state is given by the state transition function $P_{a_t}(s_t, s_{t+1})$, and is independent of all previous states and actions, satisfying the Markov property.

With the Bellman Optimality Equations [8] the optimal policy for the MDP could be found, but for larger problems such as the problem in this paper, it becomes very computationally expensive. We implement Deep Reinforcement Learning as an approximate method of solving the Bellman Optimality Equations.

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# Preventive and mitigative contingency planning for dynamic single-item stochastic lot-sizing

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**Abstract**

This paper formulates a stochastic model to assign a resource as buffer to the periods of a dyanmic single-item production (lot sizing) plan to minimize the expected total cost of deviation from the given plan and recovery actions. It also determines the optimal recovery actions to recover deviation at different periods of the plan. To find the optimal solution, a global optimization algorithm is designed based on stochastic dynamic programming and branch-and-bound. The algorithm has two phases. At the first phase, it generates a proper initial feasible solution and improves it by a neighborhood search procedure. A stochastic dynamic programming procedure is developed to give a lower bound for the optimal expected cost when the buffer resources for none or a number of periods are known. The first phase also gives a confidence interval for the optimal expected cost. The second phase finds the optimal solution by employing a branch-and-bound algorithm which uses the same method as the first phase to calculate lower bounds for the expected total cost.

**Keywords:** Mitigation plan, Buffer, Stochastic dynamic programming, Branch-and-bound, Lot-sizing

# 1 Introduction

This paper presents an optimization model and an algorithm to prevent and mitigate deviations from a given dynamic single-item production plan caused by uncertain factors such as the availability of resources and demand. The given plan includes production quantities and inventory values for each of the future periods in a sequence of periods. Any deviation from the given plan causes additional costs, such as penalties, lost sales, and extra inventory holding expenses. Decision-makers may cope with such deviations by actions either to prevent deviations before happening or to mitigate them after happening. In order to prevent deviations, limited resources, such as sub-contractors, can be assigned to future periods to act as a buffer. In case the planned production cannot meet demand because of realized uncertainties, the allocated resource can be used to produce more than planned. The allocation of some limited resources must be done at the beginning of the planning horizon. That is, these resources cannot be allocated to a period after the deviation is realized. However, there are mitigation actions, such as outsourcing or utilizing other resources, that may be used to recover from deviations. In contrast with the buffer resources, the decision on whether and which mitigation action to be taken will be made after the realization of the deviations. Nevertheless, both prevention and mitigation actions bear their own costs. Therefore, it is important to make a balance between prevention and mitigation costs on one hand and deviation costs on the other hand. The objective of the model presented in the paper is to minimize the expected total cost of deviations from the plan, as well as prevention and mitigation actions. The model defines a state for each period as the realized deviation from the planned inventory at the beginning of the period. In addition, one limited resource is formulated, which can be distributed among the periods beforehand in order to prevent potential deviations. Moreover, the model defines some mitigation actions to be taken at the beginning of each period, after the deviation is realized. The main decision variables of the model are (i) the buffer resource values given to the periods, and (ii) the mitigation actions to be taken at the beginning of the periods in case of each possible state. To take into account uncertainty, the model receives the probability distributions of state transitions from one period to the next, given buffer amounts and mitigation actions. These probability distributions depend on the probability distributions of demand at the corresponding periods. To solve the model, the paper develops a two-phase global optimization algorithm based on stochastic dynamic programming. The first phase tries to find a good feasible initial solution for the model and the second phase finds the optimal solution by a branch and bound algorithm.

# 2 Model formulation

Let *deviation* at a period be defined as the difference between the cumulative realized production and demand up to that period. Then, the paper formulates the aforementioned problem as follows:

$$MinZ = \sum_{i \in C} \sum_{d \in D} \overline{p}_{id} . (c'_{id} + c_i . k_{id}) \tag{1}$$

Subject to:

$$\overline{p}_{id} = \sum_{d \in D} \overline{p}_{(i-1)d} . p_{(i-1)dd'} \left( k_{(i-1)d}, b_{i-1} \right) \qquad \forall\ d' \in D\ ,\ i \in C\ ,\ i > 1 \tag{2}$$

$$\overline{p}_{1,0} = 1 \tag{3}$$

$$\overline{p}_{1,d} = 0 \qquad \forall\ d \in D\ ,\ d > 0 \tag{4}$$

$$\sum_{i \in C} b_i \leq \beta \tag{5}$$

$$b_i \in B \qquad \forall i \in C \tag{6}$$

$$k_{id} \in A \qquad \forall\ i \in C\ ,\ d \in D \tag{7}$$

Where sets, parameters and decision variables are defined as follows:

- Sets:
  - $C$: Set of periods, $C = \{1, 2, .., N\}$
  - $D$: Set of possible values of deviation. $D = \{0, 1, 2, \ldots, \delta\}$
  - $A$: Set of possible recovery actions. If $\alpha$ recovery actions are taken into account, $A = \{0, 1, \ldots, \alpha - 1\}$. In the example problem, we assumed that $A = \{0 \text{ (Do nothing) }, 1 \text{ (Increase capacity to a higher level)}\}$.
  - $B$: Set of possible values of buffer resource to be assigned to a period. $B = \{0, 1, 2, \ldots, \mu\}$
- Parameters:
  - $p_{idd'}(k, b)$: The probability of transition from deviation $d \in D$ in period $i \in C$ to delay $d' \in D$ in the next period, when recovery action $k \in A$ is chosen and buffer time $b \in B$ is assigned to period $i$. These probabilities are the conditional probabilities of deviation in a period, given the deviation, recovery action and buffer resource of the previous period. Therefore,

$$\sum_{d' \in D} p_{idd'}(k, b) = 1\ ,\ \forall i \in C, d \in D, k \in A, b \in B \tag{8}$$

  - $c_i(k)$: Cost of taking action $k \in A$ at period $i \in C$
  - $c'_{id}$: Cost of having deviation $d \in D$ at the beginning of period $i \in C$
- Decision variables:
  - $b_i$: Amount of buffer resource assigned to period $i \in C$ such that $b_i \in B$
  - $k_{id}$: The recovery action to be taken at period $i \in C$ when deviation $d \in D$ is observed, such that $k_{id} \in A$
  - $\overline{p}_{id}$: The probability of observing deviation $d \in D$ at the beginning of period $i \in C$. This variable is a function of the delay, recovery action and buffer resource in the previous period.

# 3 Solution algorithm

This paper proposes a two-phase solution method for the model as follows:
*Phase 1: Construction of a feasible solution and estimation of an upper bound for the objective function*
This phase consists of four steps. In step 1, a method is developed to give a lower bound for expected total cost from any period until the end of the planning horizon, when the deviation at the beginning of that period as well as the total buffer resource assigned to all the previous periods are known. In order to do so, an altered version of the problem is defined where the buffer resource is assigned to the periods at the same time as the recovery actions. That is, when the deviation is known, not beforehand. A stochastic dynamic programming model finds the optimal solution of the altered problem. Based on this solution, in step 2, a heuristic generates a feasible solution for the original model. For step 3, a method is developed based on stochastic dynamic programming to determine the expected total cost and optimal recovery actions associated with any given assignment of the buffer resource to the periods. Then, by using this method, the objective value of the solution generated in step 2 will be calculated. In step 4, by defining a neighborhood for each assignment of buffer resource, a local search optimization heuristic will be employed to improve the solution produced in step 2. Finally, the objective value of the final solution of this improving heuristic will be set as an upper bound for the second phase. The paper shows that this phase takes a polynomial time.
*Phase 2: Finding the optimal solution by a branch-and-bound algorithmn*
The branch-and-bound algorithm is represented by a tree graph. At each node of this graph, the values of the buffer resource assigned to a number of periods are known. The buffer resource value of a period can be known, only if the buffer resource values of all the previous periods are specified. Each node can be *branched* into some lower-level nodes if at least one period remains whose buffer resource value is not determined yet. A parent node at level $i$, where the buffer resource values of the periods 1 to $i$ are determined, is branched into children at level $i + 1$ by assigning all possible and feasible values to the buffer resource of period $i+1$. Hence, at each node, the minimum expected costs calculated by the dynamic programming model used in step 1 of phase 1, can be used as a lower bound for the minimum expected cost in the real world ($Z_L$). When a node is at the last level ($N$), it will be *fathomed* and the exact expected total cost will be compared to the upper bound of the objective function ($Z_U$) If it is lower, then $Z_U$ will be updated. Also, at any node before the last period, if $Z_L \leq Z_U$, then the node will be fathomed since it cannot generate better children. The algorithm terminates when all the nodes are fathomed. The computational complexity of phase 2 is exponential. However, the algorithm can be terminated at the end of phase 1, or any time during phase 2, with a feasible solution in hand and an estimate of its proximity to the optimal solution ($Z_U - Z_L$).

# A tree-search heuristic for a stochastic one-warehouse multi-retailer problem with multiple transportation modes

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**Abstract**

Large plants and distribution centres often deliver products to various retailers spread over a large territory, and can thus rely on multiple transportation modes such as air, road, rail or maritime. These different transportation modes have different costs but also different delivery lead times. We consider the problem faced by one plant or distribution centre and several retailers facing an uncertain demand: the Stochastic One-Warehouse Multi-Retailer Problem with Lead Times (SOWMRP-LT). The objective of the problem is to minimize the expected production, transportation, and storage costs. The multi-stage problem is solved approximately with a rolling horizon framework that relies on a two-stage representation of the problem. Heuristic acceleration techniques are developed to cope with the long width of the window considered in the rolling horizon, which is required by the long lead times taken into account.

## 1 The Stochastic One-Warehouse Multi-Retailer Problem with Lead Times

The present work considers an extension of the One-Warehouse Multi-Retailer Problem (OWMRP, [4]). The OWMRP consists of managing the inventory of a product at a production plant and at multiple retailers facing demands from customers. To do so effectively, production and transport decisions must be made to minimize manufacturing, holding, and shipping costs.

The advantages of a such system-wide approach were identified early on, including: cost reduction, increased availability of goods, and prevention of administrative duplication [6]. Nonetheless, the major disruptions of the supply chain that occurred during the past couple of years highlight the need to explicitly take uncertainty into account in order to reduce the costs and improve resilience [2].

Thus, we propose the Stochastic One-Warehouse Multi-Retailer Problem with Lead Times (SOWMRP-LT). It extends the OWMRP by concurrently using the services of multiples carriers, with different lead times and costs, and by considering uncertain demand at the retailers. Indeed, in general, a plant forwards its production to various retailers spread over a large territory, and can thus rely on multiple transportation modes, such as air, road, rail or maritime. These different transportation modes have different costs but also different delivery lead times. Consequently, many transportation modes with various prices and delivery times could be combined. In an uncertain context, a short delay allows for more flexibility to adjust production levels and react quickly, while slow carriers can economically transport a large amount of stock. In each period, the plant has to decide how much to produce, and how much to send to each of the retailers for each of the transportation modes.

Figure 1 depicts an example of an SOWMRP-LT instance with (from left to right) one plant, three carriers and two retailers. Figure 2 presents the time-expanded network on which the goods flow from the virtual source to the sink node ($s$ and $t$). Four periods are represented, but for the sake of clarity, the transportation arcs are drawn only for the first period. In addition, mind that, because of uncertain demand, we are also considering lost sales with a penalty factor (represented by the top red arc).

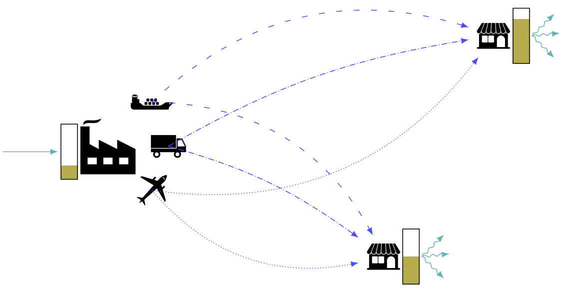

Figure 1: Production to sales setup

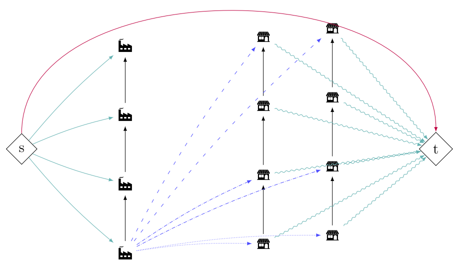


Figure 2: Time-expanded network of Figure 1

## 2 A tree search heuristic on a rolling horizon

The proposed SOWMRP-LT is a multi-stage problem. However, since it is intractable for realistic size instances, we developed a rolling horizon method that relies on a two-stage approximation. At each time period of the considered planning horizon, a scenario tree is first built to sample the possible outcomes in the periods to come with a Randomized Quasi Monte-Carlo procedure. Second, a restricted version of the SOWMRP-LT is solved on this scenario tree by imposing that the production and the transportation set-up must be the same for all scenarios at a given time period. The decisions made for the first period considered are then applied for the current period, before moving to the next once the demand is revealed, and repeating the process.

Despite the simplification, the restricted problem grows with the size of the scenario tree as well as the number of retailers, carriers and simulated periods. Thus, general-purpose solvers are impractical for this task. Instead, we chose to develop a tree search-based heuristic to manage the binary variables, i.e., production and transportation set-ups. Among the many existing methods [3], we decided to combine Limited Discrepancy Search (LDS, [1]) with Anytime Column Search (ACS, [5]).

The idea of LDS is to take a reference solution and to search among the solutions that lie within a restrained distance from it. In our case, it is desirable to start from the solution obtained during the previous iteration of the rolling horizon, i.e., for the previous time period. Indeed, the periods during which production will take place are not likely to change drastically, likewise for the transportation choices. Figure 3 presents an example of an LDS binary tree search on four decisions, with the reference solution on the left, allowing for $D_{lds} = 2$ differences.

ACS is a mechanism to prioritize the exploitation of promising branches of the tree to guarantee the identification of a good feasible solution early in the search. Such a strategy is essential, since, even with LDS, the number of possible combinations of production and transportation choices is too large to be completely explored. To achieve that, for each level of the tree search, only the best $D_{acs}$ nodes are expanded. But, to continue the search for the allowed time budget, the unexpanded nodes at each level are kept to repeat this procedure and expand the tree search width. Figure 4 represents the tree of Figure 3 explored using ACS with a beam of width $D_{acs} = 1$.

To choose the most promising nodes during the search with ACS, we developed a guide heuristic to evaluate solutions. An internal node of the search tree encodes a partial solution, as the binary decisions for later periods are not made yet. But, to evaluate these partial solutions, we complete them with the choices made in the reference solution used for the considered tree search. By definition of LDS, the remainder of the solution could not be significantly different, and, regardless, a guide function does not need to be exact. Having said that, the quantity of goods produced and sent needs to be adapted to the considered binary decisions. To do so, we solve a minimum cost bounded flow problem on the time-expanded network of the considered solution, like the one presented in Figure 2. This sub-problem consists of searching for the best improvement of the produced, stored, transported and missed quantity of goods such that the modifications are the same for all the scenarios. Thus, we use the simplex network algorithm to transport 0 units of flow from the sink to the source such that the upper (resp. lower) capacity on each arc is the maximum possible increase (resp. decrease) of the flow of goods on this arc among all the scenarios of the considered scenario tree. That is to say, we improve the solution for each scenario uniformly by solving a sub-problem whose size does not depend on the size of the scenario tree. Considering this sub-problem to guide the tree search significantly reduces the run-time, as the linear program necessary to tightly adjust the flow of goods to the chosen binary decisions and to the considered scenario tree needs to be solved only once at the end of the search, on the best solution found.

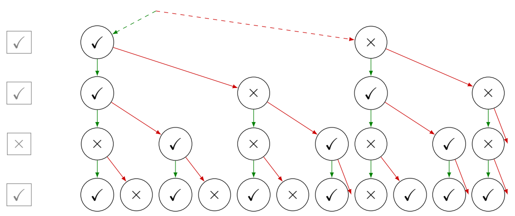

Figure 3: Example of an LDS with $D_{\text{lds}} = 2$

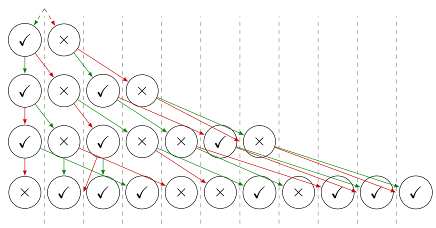

Figure 4: LDS from Figure 3 combined with an ACS with $D_{\text{acs}} = 1$

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# Integrated Decisions

A2-030 - Wednesday, 24/08 - 14:00-15:30

# The integrated production-transportation problem with process flexibility

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**Abstract**

We study an integrated multi-product production and distribution problem considering a network of multiple plants and customers, who are geographically dispersed, with direct shipment from the plants to the costumers. In addition to the decisions on production and distribution, a decision needs to be taken on the level of process flexibility in the network, i.e., which products can be produced in which plants. On one hand, a network with total flexibility allows for lower transportation costs, but requires large investments in flexibility and frequent setups. On the other hand, a network with a limited amount of flexibility, will increase the transportation costs, but requires a lower investment in flexibility. We propose mathematical models and a heuristic approach to solve the problem. Computational results are presented by varying key parameters and analyzing their impact on the value of flexibility, as well as the performance of the proposed approaches.

# 1 Introduction

In this research, the focus is on problems that appear in the context of industrial production and distribution planning. These problems involve the production of several products in multiple plants, and the distribution of these products to customers via direct shipments. These are complex tasks and need to be performed routinely. In general, production and distribution planning deals with decisions about the necessary production activities to transform the raw materials into finished products, and the transportation of these products to customers.

The planning of the production activities relates to the decisions about the quantity of products which must be produced. At the core, there is the lot sizing problem with production, setup, inventory and backlog costs. In addition, since we suppose that in the network the multiple plants and customers are geographically dispersed, transportation costs must also be taken into account. When the transportation between the plants and the customers is done using direct shipments, the problem is called the two-level production-transportation problem ([5]).

In addition to the decisions on production and distribution, we look at this problem in the context of a network of existing plants that can be flexible to make one or more different products. Nowadays, with the advancement of information technologies, aiming to be more competitive, companies' strategies give more importance to the benefits of flexibility. In line with this, researchers have recognized that flexibility concepts are important for building sustainable supply chains since they enable firms to be reactive, even in large-scale production, without sacrificing cost efficiency. The seminal paper of [4] analyzed the value of manufacturing process flexibility in a stochastic model with a single period. Since then, several authors have extended and analyzed the concept of limited flexibility configurations considering different stochastic environments. The main idea of the process flexibility studied by [4] is that a limited amount of flexibility, applied in the right way, can provide benefits close to the level offered by full flexibility. This is true even in a deterministic multi-period production planning environment, as studied by [3]. This new work extends the latter paper by considering the transportation decisions to the customers, thereby capturing a more complex and more realistic trade-off.

In this paper, we propose a new optimization problem that considers a network of customers and plants with specific transportation costs between each plant and customer. The decision on which product to make in which plant also has to take into account the trade-off with the transportation cost and hence the geographical dispersion of the demand.

# 2 Problem formulations and solution approach

We model a production planning problem with multiple items, plants and clients and transportation costs from plants to clients. The planning horizon is finite and subdivided into several periods. The plants have a predetermined production capacity and a limited amount of flexibility. In order to be able to produce a certain type of product, the plant needs to make a specific investment. The level of flexibility in each plant is a decision variable and a flexibility constraint is modeled by imposing a budget limit on the total amount invested in flexibility over all plants. The use of backlogging is allowed. In addition, to produce a given item in a specific period, a setup must be performed. The goal of the problem is to find a production plan that satisfies all constraints by minimizing the production, setup, inventory, backlog, and transportation costs. Figure 1 illustrates the integrated production-transportation problem considering 5 items, 3 production plants and 6 customers. Observe that for this example, there is a limited amount of flexibility in which production plant one can produce items one and four, production plant two can produce items three and four and production plant three can produce items two and five. Moreover, it is also important to note that all production plants can deliver their products to all customers with respective transportation costs.

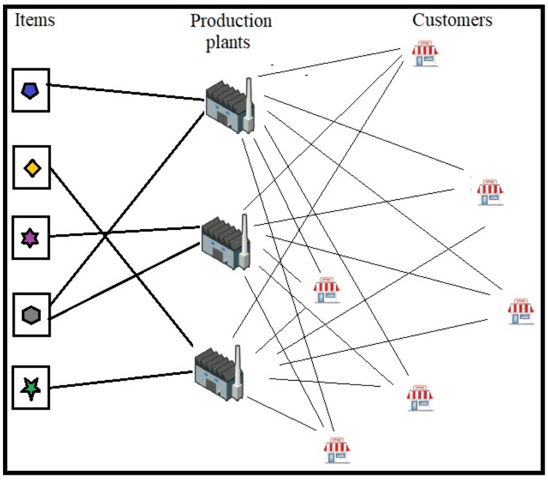


Figure 1: Graphical representation of the integrated production-transportation problem.

We propose two mathematical models for the analyzed problem, being one based on a classical formulation and the other a reformulation as a facility location problem. After analyzing the quality of the lower bounds, a third mathematical model is proposed which combines the first two formulations. To search for good feasible solutions, we propose a solution method denoted by KS, which consists in the hybridization of a MIP-based approach and a kernel search (KS) heuristic. The strategy is an adaption of solution methods proposed in [2]. While an MIP-based approach provides an initial solution, an intensification phase based on a kernel search heuristic [1] tries to improve the initial solution.

Our computational experiments show that, in terms of total costs, the proposed hybrid solution method presents on average better solutions with significantly lower computational times when compared with the results produced by a high-performance MIP software. Moreover, for several levels of capacity, the optimality gaps found by the proposed approach are significantly lower than those presented by the high-performance MIP software.

We present additional computational results aiming to analyze how different parameters impact the value of flexibility. Our analysis indicate that some of the main managerial insights derived for the case without transportation cost are no longer valid when we introduce (high) transportation cost. More specifically, with transportation costs, we find that flexibility adds benefits in the case of high capacity levels because flexibility allows to lower the transportation costs. Furthermore, we found that with high transportation cost and a high level of capacity, a limited amount of flexibility does not provide similar benefits as the case with full flexibility.

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# One-dimensional multi-period cutting stock problem with setup cost: a theoretical analysis


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**Abstract**

In this talk, we discuss the one-dimensional multi-period cutting stock problem with setup costs on cutting patterns. This problem is essentially an integration of two classical problems, namely the one-dimensional cutting stock problem and the lot-sizing problem. We present known and new formulations for the problem. Moreover, three extended reformulations are proposed in order to improve the lower bounds. An extensive theoretical analysis will be then presented in order to assess the strength of different formulations. A computational analysis complementary to the theoretical analysis will be also briefly presented, in order to provide further insights such computational challenges of various formulations and how theoretical differences vary in practice.

# 1 Introduction

In this talk, we discuss key results of the recent paper of [1], where we have carried out a theoretical analysis of the one-dimensional multi-period cutting stock problem with setups on cutting patterns (referred to as *MPCPSs* in the remainder of the paper). The reader is referred to the paper for full technical details (including problem characteristics and assumptions, mathematical formulations and theoretical results) as well as extensive computational results and discussion, as this short paper only aims to provide a brief exposure and summary.

In order to present the simplest possible formulation for the problem on-hand, we adapt the classical cutting stock formulation of [2] to this setting, with setups on cutting patterns. First, we define the variables of the formulation, as follows: $x_{jt}$ denotes the number of objects cut accoting to cutting pattern $j$ in period $t$, $y_{jt}$ is a binary variable indicating whether cutting pattern $j$ is used in period $t$ or not, and finally, $s_{it}$ indicates the amount of inventory of item $i$ at the end of period $t$.Then, the problem is as follows, to which we refer to as AGG (Adapted Gilmore and Gomory) formulation (as presented in [1]):

***AGG* model**

$$\textbf{Minimize} \sum_{t\in T}\sum_{i\in P} h_{it}s_{it} + \sum_{t\in T}\sum_{j\in J}(c_t y_{jt} + c x_{jt}) \tag{1}$$

subject to:

$$x_{jt} \le |M_t| y_{jt} \qquad \forall j,\ \forall t \tag{2}$$

$$s_{i,t-1} + \sum_{j\in J} \overline{a}_{ijt} x_{jt} = d_{it} + s_{it} \qquad \forall i,\ \forall t \tag{3}$$

$$x_{jt} \in \mathbb{Z}^+,\ \ y_{jt} \in \{0,1\} \qquad \forall j,\ \forall t \tag{4}$$

$$s_{it} \ge 0 \qquad \forall i,\ \forall t \tag{5}$$

Here, the objective function (1) minimizes the sum of holding costs of items, setup costs for the cutting patterns, and material costs of objects. Constraints (2) ensure that $y_{jt} = 1$ holds if objects are cut using the cutting pattern $j$ in period $t$, i.e., $x_{jt} > 0$. Inventory balance constraints are given by (3), and nonnegativity and integrality of variables are guaranteed by (4) and (5).

# 2 Formulations and Extended Formulations

In our work, we have considered and assessed numerous alternative formulations for *MPCPSs* in order to explore which formulation(s) would be most useful to researchers and practitioners. After an extensive theoretical analysis and computational experimentation, we eliminated many of these due to their limited use. Hence, in this work, we propose and evaluate only two more adapted formulations (in addition to AGG), which we refer to as Adapted Johnston and Sadinlija (AJS) and Adapted Reflect (ARE) formulations. These formulations partly stem from the earlier works of [3] and [4, 5], respectively.

Next, we have evaluated different strategies to improve lower bounds these basic formulations can generate. After considering various extended reformulations and valid inequalities, we have concluded to also include facility location extended reformulations of the 3 formulations on hand. It is important to note that this task is not as straightforward as is the case for classical lot-sizing problems, as the matching between variables in the extended space and variables in the original space is not direct in some cases. We will briefly discuss this in the talk.

# 3 Discussion and Future Perspectives

Following a thorough theoretical analysis, the main result we can establish is a ranking regarding the strengths of different formulations and extended formulations. Facility location reformulations of all 3 formulations are stronger than the 3 original formulations in this regard, while ARE and AGG provide strongest formulations (whether in case of original formulations or extended reformulations.)

Computational analysis focused on evaluating exact formulations and extended reformulations, as well as combining heuristics with these formulations and reformulations for more computationally efficient approaches, as exact approaches naturally suffer in case of more challenging and larger problems. Various observations from these computational experimentations will be discussed in the talk.

For future research, there are numerous avenues of interest. One rather obvious direction is to investigate the use of local search or other metaheuristics in the practical context. Another future direction is to expand the basic problem setting in this paper with various other practicalities, such as capacities, multiple machines or sequence dependent setups. Finally, integrating the decisions regarding process configuration in the problem will be also an attractive future direction, as various industries suffer from a large number of cut configurations.

# Acknowledgments

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# Integrated lot-sizing and scheduling problem in tire industry: tonnage constraint sensitivity analysis

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## 1 Introduction

Lot sizing problems have been studied widely in the literature over the last decades. The expected output of lot sizing is to give a complete picture over a planning horizon of how many parts to produce at each period and how many pieces to carry in inventory. It takes its origin in the well-known Economic Order Quantity (EOQ) model [1] under the assumption of single item, constant demand and infinite planning horizon. Since then, numerous researchers have built more realistic models to tackle real world problems ([5], [6], [7], [8], [11] and [12]). A review of the litterature allow us to position our problem as a multi-item, single-level, multi-resource lot sizing and scheduling problem with independent setups, backorder consideration, and client prioritization (See Figure 1).

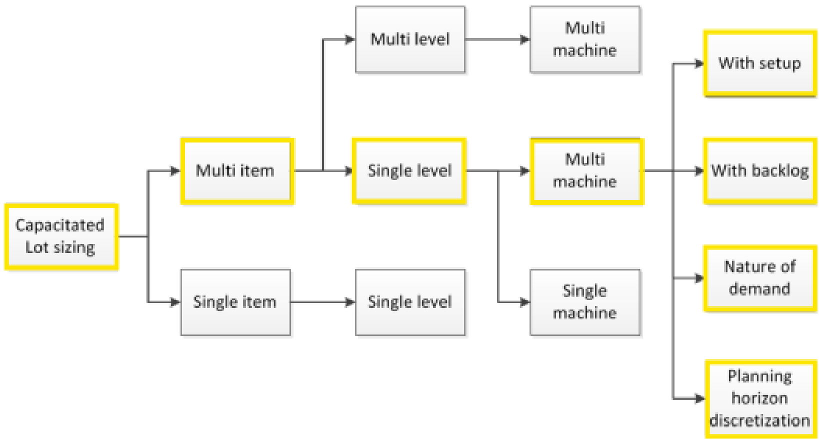


Figure 1: Problem positioning

# 2 Problem description and Contribution

The tire manufacturing process is divided into five major sub-processes : raw material mixing, semi-finite products manufacturing, tire assembling, tire curing, and finally verification and quality control. We focus on the curing sub-process production planning problem. It is the most important stage as it has been identified as the bottleneck by the company, requires consequent setup times and is highly restricted by tire - heater eligibility matrix. The plant management face a complex production planning problem with a wide portfolio of tires to be produced on unrelated parallel machines with numerous eligibility constraints. Furthermore market trends tend to dilute demand signal on more and more different references of tire. Thus the portfolio is getting wider and wider to match customer expectations and makes even more difficult the planning problem of the company. The production is based on a make-to-stock inventory policy, so that the inventory level stays between a minimum and a maximum level calculated to prevent shortage and keep Working Capital Requirement to a minimum.

During the curing process the green tire is put into a mold that provides a specific pattern for the tire. Each mold is tire-specific: it can be used for exactly one type of tire. For some tire references several molds are available, though for most tires there is only one mold. Every mold can be placed in several heaters, respecting the tire-heater eligibility matrix. Still each heater can contain at most one mold at a time. The curing time depends on the tire produced and the heater used. The heaters capacity therefore links together different tire references that compete for the same resource - available time of a given heater where the molds can be placed in. Except for the first and the last period of the production campaign, tires are produced in a continuous run and production is always done at full capacity. This type of production is often referred as "all-ornothing" production. Also, only one type of tire can be cured in a heater within one period. Thus, our problem is classified as a small-bucket lot sizing problem.

In [2] a MIP formulation of the different constraints of this problem is proposed. To continue this work, a client prioritization modelling is presented in [3]. A more in depth analysis of this problem is available in [4]. One particular set of constraints specific to the case-study considered are the tonnage constraints. Indeed, to control the costs and maximize throughput of the plant, the management team has set a production volume target for each day and week of the planning horizon, which is measured in tons of tires produced. The aim of this work is to propose a sensitivity analysis of this set of constraints, based on the key performance indicators described in [4].

# 3 Conclusion

The originality of the problem studied in this paper is the application of a simultaneous lot-sizing and scheduling problem in tire industry with specific constraints. This work is a continuation of the previous studies presented in [2], [3] and [4]. It allows the company to make a step further in his journey to industry 4.0 and an agile and digital manufacturing system.

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# Pricing

A2-030 - Wednesday, 24/08 - 16:00-17:00

# Joint dynamic pricing and lot-sizing problem with cross-price iso-elastic demand model

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**Abstract**

The presented work addresses a joint dynamic pricing and lot-sizing problem for a firm that uses its finite production capacity to manufacture a set of items. The demand for each item is nonlinear and integrates cross-price effect between products. The firm has to make decisions on production planning and the prices during several time periods to maximize the profit. Two variants of the simulated annealing are proposed. The solution's coding is based on setups and prices matrix. A set of moves are used to explore the neighborhood of the current solution. Numerical results show the efficiency of the proposed methods on a set of generated instances.

# 1 Introduction

Dynamic pricing is a pricing strategy where firms change dynamically the prices of products and services according to the demand at a different times Narahari et al.[5]. The first application of dynamic pricing are in service industries like airlines Smith et al.[6]. Pricing was considered initially by companies as a separate element to improve their profit. However, in the manufacturing industries the coordination of pricing decisions with the supply chain's decisions like production is critical (Chan et al.[2]). Researches studied a variety of problems in this topic with different assumptions on production systems, demand function, etc. Bajwa et al.[1] and Couzon et al.[4] are example of literature papers that coordinate pricing and lot-sizing decisions. The objective of our work is to propose a general model for joint pricing and lot-sizing decisions that incorporates the cross-price effects on demand between products. Two variants of the simulated annealing are proposed to solve the problem. The proposed methods are tested among some literature inspired instances. The obtained results are promising.

# 2 Problem description

The problem studied considers a firm that produces and sells a set of $J$ items over a finite and discrete horizon divided into $T$ periods. The firm has to set the production plan and the prices for each item to maximize the revenues. Five variables are associated for each tuple (item $j$, period $t$). $X_{jt}$, $I_{jt}$ and $Y_{jt}$ correspond to the production, inventory quantities and setup variables. The variable $Y_{jt} \in \{0,1\}$ and it's equal to 1 only if $X_{jt} > 0$. $p_{jt}$ and $d_{jt}$ correspond to prices and the demand for item $j$ during period $t$. For each period $t$, the firm has to fix $X_{jt}$ such that the total production doesn't exceed the available capacity in the same period. The inventory for each item at the beginning and at the end of the horizon is equal to 0. In addition, for each $j \in \{1, 2, .., J\}$ and $t \in \{2, 3, .., T-1\}$, the inventory balance defined by the equality $X_{jt} + I_{j,t-1}$ - $d_{jt} = I_{jt}$ must be hold.

The market demand $d_{jt}$ is assumed to be iso-elastic and it's equal to $d_{jt} = a_j \prod_{i=1}^{J} p_{it}^{b_{ji}}$. Coefficient $b_{ji}$ is cross elasticity between the demand of item $j$ and the price of item $i$. The objective function is equal to $\pi = \sum_{t=1}^{T} \sum_{j=1}^{J} \Big(p_{jt}d_{jt} - c_{jt}X_{jt} - h_{jt}I_{jt} - a_{jt}Y_{jt}\Big)$ and corresponds to the total profit for all the horizon to maximize. Parameters $c_{jt}$, $h_{jt}$ and $a_{jt}$ are unit production cost, unit inventory cost and setup cost for item $j$ during period $t$, respectively.

# 3 Resolution approach

Two variants of the simulated annealing denoted $SA_1$ and $SA_2$ are proposed to solve the problem. The simulated annealing is an iterative process inspired from the thermodynamics. At the beginning, an initial solution is generated and the temperature $T_0$ is initialized to a high value. At each iteration $i$, a new solution $s_v$ is generated from the neighborhood of the current one $s_i$. Then, $s_v$ is accepted according to a probability that depends on the objective values of $s_i$ and $s_v$ and on the current temperature $T_i$ which decreases from an iteration to another one according to a specific cold schemes. The process stops the search when the temperature reaches a value $T_f$ or when the iterations number reaches a predefined $Iter_{max}$.

The solution's coding is based on setups variables and prices variables for the $SA_1$ and $SA_2$, respectively. For the $SA_1$, the initial solution $s_0$ is generated by setting randomly the setups variables and then solving the generated non-linear programming $NLP$. Then, at each iteration $i$, three types of moves are used to explore the neighborhood of the current solution. The first and the second move swap the setup values of a selected product and period, respectively. The last move swaps the setup value of a randomly selected tuple (product,period). For the $SA_2$, the initial solution $s_0$ is generated by setting randomly the prices for each product at each period and then solving the generated multi-items lot-sizing problem. Then, at each iteration of $SA_2$, prices of a randomly selected product or period are changed to explore the neighborhood of the current solution.

# 4 Numerical results

The current section presents a preliminary results obtained by the $SA_1$ and $SA_2$. The conducted tests are based on a set of instances generated by considering ($J$=5, $T$=6), ($J$=5, $T$=12). $c_{jt}$, $h_{jt}$, $a_{jt}$ and $C_t$ are generated uniformly as in Chen et al. [3]. For $a_{jt}$ and $C_t$, the parameter $a$ is fixed to 100 and $b$ to 250. Then $a_{jt}$ and $C_t$ are distributed uniformly from $[0.5a, 1.5a]$ and $[0.5b, 1.5b]$, respectively. The values of $c_{jt}$ and $h_{jt}$ are generated uniformly from $[4, 6]$ and $[0.5, 1.5]$, respectively. The parameters $b_{jj}$ are distributed uniformly from $[-4, -2]$. The cross-price elasticity parameters $b_{ji}$ $(i \neq j)$ follow the uniform distribution from $[0, 1]$. The parameter $a_j$ is generated uniformly from the $[0.5C_t, 1.5C_t]$. Finally, for each tuple ($J$, $T$), 10 instances are generated.

$SA_1$ and $SA_2$ are implemented on Python3. Theirs parameters are set to ($T_0 =$ 50, $T_f = 1$, *cool* $= 0.96$). To compare theirs results, the *rpd* metric is defined by $SA_{i\_rpd} = \frac{SA_{i\_obj} - max(SA_{1\_obj}, SA_{2\_obj})}{max(SA_{1\_obj}, SA_{2\_obj})}$. The following Table shows the obtained results on each instance.

Table 1 shows that $SA_1$ reaches the best solution for all test instances. Analyzing the values of *rpd*, one can notice that the results of $SA_1$ and $SA_2$ are closed for $(J = 5, T = 6)$ (all the *rpd* values are less than 2%, except the instance 4 for which the *rpd* is equal to 10%). However, for $(J = 5, T = 12)$ the *rpd* increases to reach an average value of 8%.

| $(J=5,T=6)$ | $SA_{1\_obj}$ | $SA_{2\_obj}$ | $SA_{1\_rpd}$ | $SA_{2\_rpd}$ | $(J=5,T=12)$ | $SA_{1\_obj}$ | $SA_{2\_obj}$ | $SA_{1\_rpd}$ | $SA_{2\_rpd}$ |
|---|---|---|---|---|---|---|---|---|---|
| Instance 1 | **2927843** | 2900413 | 0,00% | -0,94% | Instance 1 | **721432** | 613010 | 0,00% | -15,03% |
| Instance 2 | **1273278** | 1253341 | 0,00% | -1,57% | Instance 2 | **1177205** | 1105920 | 0,00% | -6,06% |
| Instance 3 | **11479157** | 11100336 | 0,00% | -3,30% | Instance 3 | **969491** | 878055 | 0,00% | -9,43% |
| Instance 4 | **309189** | 277148 | 0,00% | -10,36% | Instance 4 | **362641** | 317141 | 0,00% | -12,55% |
| Instance 5 | **453996** | 453747 | 0,00% | -0,05% | Instance 5 | **487180** | 465603 | 0,00% | -4,43% |
| Instance 6 | **803892** | 780203 | 0,00% | -2,95% | Instance 6 | **978464** | 854801 | 0,00% | -12,64% |
| Instance 7 | **283939** | 279510 | 0,00% | -1,56% | Instance 7 | **437694** | 409072 | 0,00% | -6,54% |
| Instance 8 | **205925** | 204871 | 0,00% | -0,51% | Instance 8 | **646268** | 615004 | 0,00% | -4,84% |
| Instance 9 | **1735405** | 1710547 | 0,00% | -1,43% | Instance 9 | **1554607** | 1451843 | 0,00% | -6,61% |
| Instance 10 | **2045686** | 2019355 | 0,00% | -1,29% | Instance 10 | **330268** | 325902 | 0,00% | -1,32% |
| avg | - | - | 0,00% | -2,40% | avg | - | - | 0,00% | -7,94% |

Table 1: $SA_1$ and $SA_2$ results

# 5 Conclusion

The presented work considers a joint-pricing and lot-sizing problem with a demand that integrates cross-price effect. Two variants of the simulated annealing are presented. The preliminary results show the superiority of the $SA_1$. For deeply analysis of the two methods, the next step of our work is to test them on a large instances.

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# Solving profit-maximizing lot size problems with discrete prices

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**Abstract**

This work concerns a discrete-time dynamic lot size model with dynamic pricing, where demand in a period is a function of prices in the same period, as well as a number of preceding periods. Prices can be chosen from a discrete set, and a single price must be chosen in each period. Having non-zero production incurs a setup cost.

The problem consists of deciding, for each time period, whether one should produce, how much to produce, and which price to set. The objective is to maximize total profit, which is defined as total revenue minus total setup cost, inventory holding cost and production cost.

A dynamic programming algorithm is presented, which solves the problem to optimality. This solution method can also be used as a heuristic algorithm for a version of the problem with continuous prices.

# 1 Introduction

We consider a decision maker selling a product, whose goal is to decide prices throughout a discrete planning horizon in order to maximize his or her total profit. The profit function is a general function of the prices in some number of preceding time periods. Prices may be chosen from a predetermined list of options. These price options may be different in each time period, and the number of options in each time period may vary.

In the following section, we will present some models, some of which include setup decisions, while others do not. For the models that do not include setup decisions, profit functions may be left general. However, for the models with setup decisions included, profit functions are defined as the difference between total revenue, and the sum of production costs, setup costs and inventory holding costs. In order to present the models in an intuitive way, we will also define the revenue and cost functions, even for the models that only require general profit functions.

# 2 Problem variations

## 2.1 Model 1 - pure pricing, one lag period

We consider four versions of the above presented problem. Model 1 is the simplest model, in which one must decide on a price in each period, in order to maximize total profit, given as total revenue minus total production cost. Demand is a function of the price in the current period and the price in the previous period, and production is assumed to occur in every period.

$$\begin{aligned}
\Pi &= \text{total profit} \\
d_t &= \text{demand in period } t \\
p_t &= \text{price in period } t \\
c_t &= \text{marginal production cost in period } t \\
P_t &= \text{set of allowed prices in period } t
\end{aligned}$$

$$\max \Pi = \sum_{t=1}^{T} d_t(p_t - c_t) \tag{1}$$

subject to

$$\begin{aligned}
d_1 &= f_1(p_1) & (2) \\
d_t &= f_t(p_{t-1}, p_t),\ t = 2 \dots T & (3) \\
p_t &\in P_t,\ t = 1 \dots T & (4)
\end{aligned}$$

The objective function (1) represents the sum of revenue minus production costs for all time periods. Constraints (2)-(3) define demand in period 1 as a function of its price, and demand in other periods as a function of prices in the same period and the previous period. Constraint (4) enforces that prices are chosen from the predetermined set of allowed prices.

We model this problem as a longest path problem on a layered network, see figure 1. Nodes are divided into $T$ disjoint sets, not including the source and sink nodes. Each node represents a pricing decision, and travelling along an arc from one node to another rewards a nonnegative profit. Travelling to the sink node gives zero profit. The objective is to find the longest path from the source node $o$ to the sink node $s$.

Such a network problem has already been treated in the literature. See [1] for a backwards dynamic programming algorithm that solves a similar shortest path problem to optimality in $O(|A|)$ time, where $A$ is the set of arcs. We present a similar, forwards algorithm which also finds the optimal solution in $O(|A|)$ time.

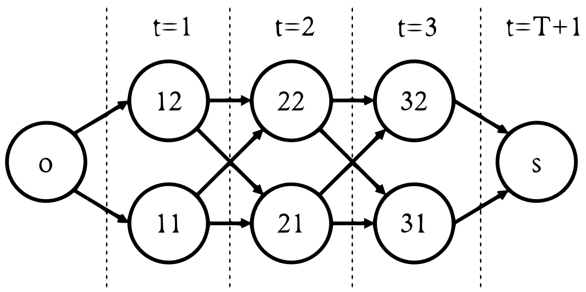


Figure 1: Graph representation of model 1.

### 2.2 Model 2 - pricing and setup, one lag period

In model 2, we add setup decisions to model 1. Production only occurs in periods in which setup occurs, and having setup incurs a given cost, regardless of amount produced. Product may be held in inventory between periods, also at a cost.

Under the assumption of Wagner-Whitin costs, we present a dynamic programming algorithm to solve this problem to optimality in $O(T^2K^3)$ time, where $K = \max(|P_t|)$, the highest number of prices in any period.

### 2.3 Model 3 - pure pricing, multiple lag periods

Model 3 is an extension of model 1 in that demand is a function of prices in any number of preceding prices, not only one. The objective functions in the two models are the same, see (1), as well as the constraint ensuring that prices are discrete, see (4). Constraints (2)-(3) are replaced by, respectively,

$$d_t = f_t(p_1, p_2, \ldots, p_{t-1}, p_t),\ t = 1 \ldots q \tag{5}$$

$$d_t = f_t(p_{t-q}, p_{t-q+1}, \ldots, p_{t-1}, p_t),\ t = q+1 \ldots T \tag{6}$$

where $q$ is the number of preceding periods, the prices of which influence demand in the current period.

Unfortunately, the number of combinations of preceding prices for the demand function is bounded by $K^q$, and even calculating every possible value of a demand function is exponential in complexity. This makes it difficult to avoid such complexity in an exact solution algorithm. However, by making assumptions on the nature of the demand function, it is possible to rule out some combinations that will not appear in an optimal solution. Without assumptions on the demand function, we present an algorithm that solves the problem to optimality in $O(TK^q)$ time.

## 2.4 Model 4 - pricing and setup, multiple lag periods

Model 4 is the most complex of the four, in which setup variables are included, and demand is a function of prices in multiple preceding periods. As this work is still unfinished, model 4 has not yet been made, although we believe that it will be possible to combine the algorithms for models 2 and 3 and to solve this problem.

# 3 Speed-up techniques

Some additional techniques may be employed, in order to speed up the execution of the aforementioned solution algorithms. These are based on some assumptions on the demand functions used, and assumptions of the available prices in the problem. For instances where these conditions are satisfied, one may disregard every price set at or below the unit production cost. Furthermore, as the solution algorithm progresses, one may also disregard every price in period $t$ that is smaller than the optimal price in period $t$ for the $1 \dots t$ sub-problem.

Additionally, if the horizon theorem (see [2] for details) turns out to be true for the models with setup variables, then it may also be used to speed up the solution algorithms for these problems, as not every potential final setup period needs to be checked.

# 4 Computational experiments

Some computational experiments will be performed, measuring the performance of the algorithm against that of commercial solvers.

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# Complexity and Exact Algorithms

A2-030 - Thursday, 25/08 - 09:00-10:30

# Lot sizing and maintenance planning problems: complexity analysis and exact solution approaches

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**Abstract**

In this talk, we study an integrated lot-sizing and maintenance planning problem on a single machine. Maintenance is performed based on the age of the machine that is related to production decisions. The goal is to decide, on a planning horizon of several periods, when and how much to produce, and when to maintain the machine. We study the complexity of the single-item and the multi-item versions. We show that the multi-item version is strongly NP-Hard and the single-item version is NP-Hard for some cases and polynomially solvable for other cases. More precisely, we study the following versions: (1) maintenance can be executed only when a minimum age of the machine is reached and before a maximum age, (2) production setups generate fixed age setups for the machines, and (3) maintenance can be performed at any point in time. For each version, we characterize some structural properties of dominant solutions to derive polynomial dynamic programming algorithms.

# 1 Introduction

In this talk, we study an integrated lot-sizing and maintenance planning problem on a single machine. Maintenance is performed based on the age of the machine that is related to production decisions. The goal is to decide, on a planning horizon of several periods, when and how much to produce, and when to maintain the machine. We study the complexity of the single-item (that we coin ULS-M) and the multi-item versions. We show that the multi-item version is strongly NP-Hard and the single-item version is NP-Hard for some cases and polynomially solvable for other cases. More precisely, we study the following versions: (1) maintenance can be executed only when a minimum age of the machine is reached and before a maximum age, (2) production setups generate fixed age setups for the machines, and (3) maintenance can be performed at any point in time. For each version, we characterize some structural properties of dominant solutions to derive polynomial dynamic programming algorithms. The detailed description of the problem as well as a mathematical modeling can be found in [1].

# 2 NP-Hard cases

Before proposing polynomial algorithms for various cases, we first showed that two general cases of the single-item problem are NP-hard: (1) With no fixed age setups, no fixed maintenance cost and time-varying maximum ages, and (2) With time variant fixed age setups. We also showed that multi-item version of the problem is strongly NP-Hard.

# 3 Polynomial cases

In this section we provide some obtained results where we show that some variants of the studied problem can be solved in polynomial time. The structural properties as well as the dynamic programming algorithms will be detailed during the presentation.

## 3.1 ULS-M without fixed age setups

We study the problem where a maintenance operation can occur only at the end of a period and without considering the fixed age setup. We show that this problem can be solved in polynomial time using a dynamic programming algorithm. In order to construct this algorithm, we make a decomposition into subplans, based on some structural properties. All these properties are based on the cycle free property of network flow problems with concave costs where extreme solutions are cycle free. Using these properties, we derive a polynomial time algorithm that runs in $O(T^7 \log T)$.

### 3.2 ULS-M with fixed age setups and stationary cost parameters

Here, we consider the problem where age setups induce a fixed deterioration. We suppose that a maintenance operation can only take place at the end of a period. In this section, we assume that costs are time-invariant. We show how to solve the problem in polynomial time by dynamic programming. In order to construct this algorithm, we make a decomposition into subplans, based on some structural properties. Because of the minimum machine age required before a maintenance operation, an 'artificial setup' might be required in an optimal solution to enforce a jump in the machine age. Note that this situation does not occur if fixed age setups are not considered. Because of these 'artificial setups' the cycle free property does not apply. Based on the proposed structural properties, we derive a polynomial dynamic programming that runs in $O(T^{10})$.

### 3.3 ULS-M with maintenance at any point in time and without fixed age setups

In this part, in contrast to the case with maintenance at the end of a time period, in an optimal solution, we can have a period where production first takes place, then there is a maintenance operation, and then production occurs again. We show that this problem can be solved in polynomial time using a dynamic programming algorithm. This algorithm is based on several structural properties. All these properties are based on the cycle free property of network flow problems with concave costs where extreme solutions are cycle free. Using these properties, we derive a polynomial time algorithm that runs in $O(T^8 \log T)$.

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# Continuous proportional lot sizing and scheduling problem

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**Abstract**

In this study, we tackle the problem of the multi-item, capacitated multi-resource lot sizing problem with start-up costs. The main motivation of this theoretical study comes from the glass container industry. While planning the production, one has to decide when and which quantity of different color glass melts to schedule on several furnaces, with the aim of minimizing the overall production and holding costs. In this setting, the customer's deterministic demand for bottles of different colors has to be satisfied. We propose a new variant of the multi-item lot sizing and scheduling problem to address the afore mentioned challenge. As it requires a full capacitated and continuous production process on each furnace over the whole planning horizon, we call this new variant the Continuous Proportional Lot sizing and Scheduling problem (CPLSP). CPLSP is closely related to two small-bucket models from the literature: the Discrete Lot sizing and Scheduling Problem (DLSP) and the Proportional Lot sizing and Scheduling Problem (PLSP). It is also related to the Changeover Scheduling Problems, where the aim is to minimize the number of changeovers while respecting the deadlines of the tasks. We establish some complexity results and we propose polynomial time algorithms for the case of two items.

# 1 Industrial motivation

We consider a production planning problem that appears in a glass container production system of the biggest European manufacturer, with plants in 6 countries. We focus on the problem of scheduling glass color campaigns in several capacitated furnaces. The objective is to decide for each period (=month), one glass color to be assigned to each furnace in order to satisfy the demand of the customers over several months. This problem can be positioned within tactical decision making in the glass container industry. See Figure 1 for a simple illustration of the system studied.

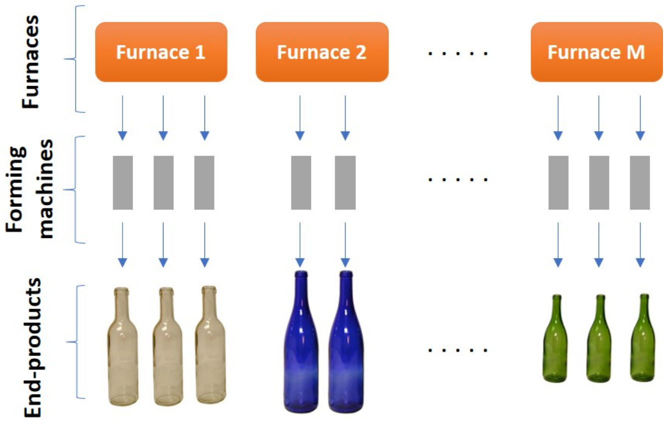


Figure 1: Parallel identical furnaces producing multi-color containers in the glass container manufacturing system under study.

In the real production system, the furnaces melt glass 7/24 hours with a full capacity, without any interruption, except for the color changes and maintenance activities. This color change process is very time consuming (up to a few days) which limits in practice the total number of colors assigned to a given furnace: typically, only two different colors are assigned to the same furnace. Instead of fixed setup cost, we consider start-up cost each time a color change takes place.

# 2 Continuous Proportional Lot-sizing and Scheduling Problem (CPLSP)

The problem considered is thus a multi-item (=colors), capacitated multi-resource (=furnaces) lot sizing problem with start-up costs. In the literature, Proportional Lot sizing and Scheduling Problem (PLSP) is found to be the closest one to our study. PLSP allows at most one color change per period, occuring at any time in this time period. Note that PLSP is a small-bucket model. In small bucket models, the time horizon is divided into small time intervals in which the machine can produce only a small number of different items (zero, one or two, depending on the models). Two classical constraints can be encounted in small bucket models: (a) Setup at the beginning of the time period; (b) All-or-nothing assumption: In each period, either the resource is idle or at full capacity. In the literature, we can identify three main small bucket models. The following figure compares them to our problem CPLSP.

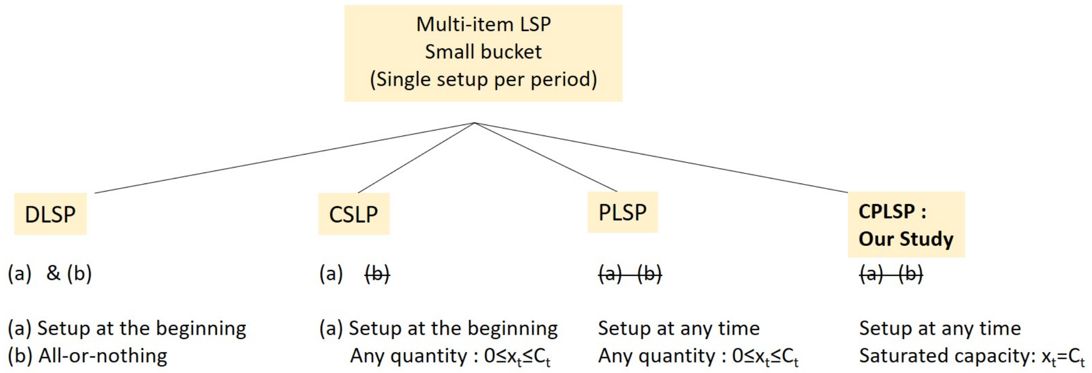


Figure 2: Illustration of small-bucket problems for multi-item LSP. (a) setup at the beginning (b) all-or-nothing policy. **DLSP** *(Discrete Lot-sizing and Scheduling Problem)*, **CSLP** *(Continuous Setup Lot-sizing Problem)*, **PLSP** *(Proportional Lot-sizing and Scheduling Problem)*.

Note that the main difference of our model CPLSP and the classical PLSP is that, in our model we do not authorize to produce nothing due to the high energy costs that would be triggered. In a given period the capacity is always saturated. For a literature survey on the integrated lot sizing and scheduling problems, see [3]. For complexity results on the small-bucket multi-item LSP with start-up times, see [4].

# 3 Link to the Changeover Scheduling Problem

Our problem can be formulated as a Changeover Scheduling Problem (CSP). See Bruno and Downey [1] for the polynomial and NP-hard cases of CSP. In CSP, a task $j$ corresponds to an order with a deadline (=order date), a processing time (=the quantity requested) and a family (=color). With $n$ tasks to schedule on several machines (=furnaces), the objective is to minimize the total cost of start-up and inventory holding to find the optimal scheduling. Cheng et Kovalyov [2] propose a dynamic programming algorithm in $O(n^F/F^{F-2})$ time for CSP, for a fixed number of family $F$. They also propose an algorithm called *TWO*, solving CSP in linear time with 2 families and unitary start-up costs. Notice that there is no holding cost in the CSP model, which differentiates it from the CPLSP model.

# 4 Mathematical formulation and results

We introduce the Continuous Proportional Lot-sizing and Scheduling Problem (CPLSP) where the furnaces produce glass at full capacity $C$. $N$ different items (colors of glass melt) have to be scheduled on parallel and identical furnaces. Each item $i$ has a demand $d_{it}$ to satisfy in period $t$ (=month) over $T$ periods. We consider an inventory holding cost $h_i$ for each unit stored and a start-up cost $g_i$ each time a color change takes place for the item $i$. The decision variables are: $x_{it}^m$ the quantity to produce of item $i$ in period $t$ on furnace $m$, $y_{it}^m$ the binary variable indicating if the furnace $m$ is configured for item $i$ in period $t$, $z_{it}^m$ the change-over binary variable and $s_{it}$ the stock level of item $i$ in $t$. Here the MILP formulation of CPLSP:

$$
\begin{array}{lll}
\min & \sum_{t=1}^{T} \sum_{i=1}^{N} h_i s_{it} + \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{m=1}^{M} g_i z_{it}^m & \\
s.t. & s_{it-1} + \sum_{m=1}^{M} x_{it}^m = s_{it} + d_{it} & \forall i \in \{1..N\}, t \in \{1..T\} \\
 & C = \sum_{i=1}^{N} x_{it}^m & \forall m \in \{1..M\}, t \in \{1..T\} \\
 & x_{it}^m \leq C(y_{it}^m + y_{it-1}^m) & \forall m \in \{1..M\}, i \in \{1..N\}, \forall t \in \{1..T\} \\
 & z_{it}^m \geq y_{it}^m - y_{it-1}^m & \forall m \in \{1..M\}, i \in \{1..N\}, \forall t \in \{1..T\} \\
 & \sum_{i=1}^{N} y_{it}^m = 1 & \forall m \in \{1..M\}, t \in \{1..T\} \\
 & x_{it}^m, s_{it} \geq 0, z_{it}^m, y_{it}^m \in \{0,1\} & \forall m \in \{1..M\}, i \in \{1..N\}, t \in \{1..T\}
\end{array}
$$

We propose a dynamic programming algorithm in $O(T^2)$ time to solve 2-item-CPLSP to the optimality. We generalize this result to solve 2-item-CPLSP with additional start-up times. We also propose an $O(M^3T^3)$ algorithm for 2-item-CPLSP on $M$ parallel machines.

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# Exact solution of capacitated dynamic lot sizing problems by branch-and-price algorithms using the SCIP framework

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**Abstract**

We consider the problem to solve the multi-level capacitated lot sizing problem (MLCLSP) via a branch-and-price algorithm based on a Dantzig-Wolfe decomposition by product types. A convex combination of Wagner-Within solutions [6] as in Manne's model [3] is known not to be sufficient to reproduce the entire solution space of the MLCLSP. In this paper, we therefore decouple the decisions on the setup patterns, to be made in the pricing problems, from those on the production quantities, to be made in the LP master problem. The resulting branch-and-price algorithm has been implemented using the SCIP framework. We present the underlying problem decomposition, the implementation and some first numerical results.

# 1 Introduction

The multi-level capacitated lot sizing problem (MLCLSP) deals with the problem to determine time-phased production quantities as well as inventory levels for multiple final and intermediate products such that the independent demand for those final as well as the dependent demand for intermediate products is met, capacity restrictions of the required production resources are respected and the resulting setup and holding costs are minimized. Direct compact formulations of that problem in terms for production quantities, inventory levels and binary setup decisions require a set of so-called "Big-M-constraints" to couple binary setup decisions to continuous production quantity decisions. If a branch&bound method is used to solve the problem, long computation times result from the weak lower bounds induced by those Big-M-constraints. Stronger bounds can be found using alternative formulations, e.g., the so-called "simple plant location formulation" or the "shortest-route formulation", see, e.g., [5]. However, this comes at the price of more complex models and additional variables and constraints.

## 2 Dantzig-Wolfe-Decomposition

A Dantzig-Wolfe-Decomposition can be used to obtain tighter bounds within the linear programming (LP) relaxation required within a branch&bound framework to solve the MLCLSP or its single-level counterpart, the CLSP. A direct application of this approach operating with a set of production schedules of the Wagner-Whitin type [6] embedded in a master problem as in Manne's formulation [3] leads to the problem that some possible solutions of the MLCLSP cannot be produced. For this reason, Degraeve and Jans [2] proposed a model variant overcoming this problem by operating with an extended set of setup schedules such that so-called non-dominant schedules can be considered in which there is a setup, but no production. While this may seem counter-intuitive at the first glance, it allows to build convex combinations of those schedules such that optimal solutions of the underlying MLCLSP can be created within a branch&price process. Unfortunately, the set of the non-dominant schedules required in this approach can be large.

It is for this reason that we propose an alternative approach. The idea is to let the branch&price approach to decide about certain time-phased limits of the production quantities. Those limits can be obtained relatively easily from the Wagner-Within solutions stemming from the pricing problems. This approach allows us to partially decouple production quantity decisions from setup decisions and to still use Wagner-Whitin solutions to determine tight lower bounds in the branch&bound process.

## 3 Implementation in SCIP and first numerical results

Implementing a branch&price algorithm is a complex task as the column generation process has to be aligned with a cleverly designed branching methodology. We used the SCIP framework [1] to implement the branch&price approach. This turned out to be both technically challenging but also fruitful in the sense that we could concentrate our work on the development of the pricer (to solve the sub-problems in the Dantzig-Wolfe decomposition resulting from our special model) and special branching schemes (to avoid unbalanced search trees as they are typically the result of branching on fractional binary variables of the compact model underlying the Dantzig-Wolfe decomposition). As this is currently work in process, we report some first and preliminary numerical results.

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# Stochastic Lot Sizing – II

A2-030 - Thursday, 25/08 - 11:00-12:30

# Multi Item Capacitated Lot Sizing Problem with Stochastic Demand Timing and Setup Times

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## 1 Introduction

Production planning under uncertainty has always been a challenging task for practitioners and a frequently visited research avenue for researchers. In order to draw attention to the growing trend on this subject and to indicate possible research directions, two surveys on stochastic lot sizing problems can be found in [1] and [4]. These reviews show that the uncertainty in demand quantity is the main focus of researchers. We believe that a major drawback of previous studies is that uncertain demands in different periods are uncorrelated and treated as independent random variables, which is often unrealistic in practical settings. To answer this criticism, [2] have recently proposed a novel way of modelling the uncertainty on demand in the single-item dynamic lot sizing problem. In their approach, the quantities of demands are deterministic but their timing might be stochastic, i.e. they might fully occur in a given window of multiple periods, with a given probability for each period. Since demand quantities are known but their occurrences are stochastic, demands are naturally correlated.

In our paper, we consider the multi-item capacitated case, to consider more practical settings than in [2], by relying on some of the contributions of [2] for the single-item uncapacitated case.

## 2 Problem statement and mathematical model

In this study, the multi-item capacitated lot sizing problem with stochastic demand timing is addressed. On a production horizon discretized in $T$ periods, the deterministic demands $D_{kt}$ of item $k$ in period $t$ are known, for which no backlog or lost sale is allowed. Moreover, stochastic demands $d_i$ of item $k(i)$ are also given, for which the timing is stochastic with probability $p_t^i \geq 0$ for period $t$ within a given time interval $[l^i, u^i] \subseteq [1, T]$. The demand quantity $d_i$ is not stochastic, and backlog is allowed until period $u^i$. The objective of the problem is to determine a production plan that minimizes the total expected cost and that for which both the deterministic and randomly occurring stochastic demands must be satisfied.

Let $[l^i, u^i] \subset [1, T]$ be an interval indexed by $i$, where the stochastic demand $d^i$ of item $k(i)$ fully occurs in a single period, with a probability of $p_t^i \geq 0$ for each period $t \in [l^i, u^i]$ and such that $\sum_{t=l^i}^{u^i}(p_t^i) = 1$.

The expected cost is calculated as follows:

$$EC^i(t) = \sum_{l=t}^{u^i} h_{k(i)l} \sum_{m=l+1}^{u^i} p_m^i + \sum_{l=l^i}^{t-1} b_{k(i)l} \sum_{m=l^i}^{l} p_m^i \tag{1}$$

where $h_{k(i)l}$ is the holding cost of item $k(i)$ in period $l$, and $b_{k(i)l}$ is the backlog cost of item $k$ in period $l$.

Since demand $d^i$ is stochastic, the inventory variable is also stochastic, which makes the modelling our problem challenging. To deal with it, [2] propose to model the problem with binary variables $z_{lt}$, which is the fraction of the deterministic demand $D_t$ produced in period $l \leq t$, and $z_l^i$, the fraction of the stochastic demand quantity $d_i$ produced in period $l \leq u^i$. Continuous variables $x_{k,t}$, resp. binary $y_{k,t}$, model the production quantity, resp. the setup state, of item $k$ in period $t$. The mathematical model is formalized below:

$$\begin{aligned} \min z = \sum_{k=1}^{K}\sum_{t=1}^{T} f_{kt}y_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{l=t}^{T}(c_{kt} + \sum_{m=t}^{l-1} h_{km})z_{ktl}D_{kl} \\ + \sum_{i\in I}\sum_{t=1}^{u^i}(c_{k(i)t} + EC^i(t))z_t^i d^i \end{aligned} \tag{2}$$

$$\sum_{l=1}^{t} z_{klt} = 1 \quad \forall t = 1, ..., T, k = 1, ..., K \tag{3}$$

$$\sum_{l=1}^{u^i} z_l^i = 1 \quad \forall i \in I \tag{4}$$

$$\sum_{k=1}^{K} a_{kt}(\sum_{l=t}^{T} z_{ktl}D_{kl}) + \sum_{i\in I;t\leq u^i} a_{k(i)t}z_t^i d^i + \sum_{k=1}^{K} su_{kt}y_{kt} \leq cap_t \quad \forall t = 1, ..., T \tag{5}$$

$$\sum_{l=t}^{T} z_{ktl}D_{kl} + \sum_{i\in I;t\leq u^i} z_t^i d^i \leq (\sum_{l=t}^{T} D_{kl} + \sum_{i\in I;t\leq u^i,k=k(i)} d^i)y_{k,t}$$

$$\forall t = 1, ..., T, \forall k = 1, ..., K \tag{6}$$

$$y_{kt} \in \{0, 1\} \quad \forall t = 1, ..., T, \forall k = 1, ..., K \tag{7}$$

$$0 \leq z_{ktl},\ z_t^i \leq 1 \quad \forall i \in I, \forall t = 1, ..., T, \forall l = t, ..., T, \forall k = 1, ..., K \tag{8}$$

The objective function (2) minimizes the total expected cost including set up, production, holding and backlog costs. Constraints (3) and (4) ensure that both deterministic and stochastic demands are satisfied, respectively. The limited production capacity is ensured in Constraints (5), where $a_{kt}$ is the capacity consumed by one unit of item $k$ in period $t$, $su_{kt}$ is the set up time of item $k$ in period $t$ and $cap_t$ is the capacity available in period $t$. Constraints (6) link the setup and production quantity variables, where $(\sum_{l=t}^{T} D_{kl} + \sum_{i\in I;t\leq u^i,k=k(i)} d^i)$ is the upper bound of $x_{k,t}$.

# 3 Numerical results

To validate and analyze the mathematical model, a set of instances defined from the instances generated by [3] are tailored according to the settings of our problem. In particular, the probability of occurrence of a stochastic demand in any period $t \subset [l^i, u^i]$ is normally distributed. The instances include 100 products and 24 periods, and each product has 10 stochastic demands. The standard solver IBM ILOG CPLEX 12.9 is used.

The preliminary results (Table 1) show that a feasible solution cannot be reached, displayed by *"Nf"* for *"No feasible"*, for most instances after 60 seconds. When limiting the computational time to 120 and 300 seconds, a feasible solution can be reached but the optimality gap is significant. These results motivate us to develop heuristic approaches to solve the problem for large instances.

# 4 Conclusions and perspectives

In this work, inspired from the work of [2], a mixed integer linear model is proposed for the multi-item capacitated lot sizing problem when the demand timing is stochastic. The model has been tested on various instances to show its limitations.

A Lagrangian relaxation heuristic is being developed that relies on the dynamic programs proposed in [2]. Constraint (5) in the model is relaxed to solve uncapaci-

Table 1: Numerical results obtained after 60, 120 and 300 seconds

| | Gap (%) | | | | Gap (%) | | |
|---|---|---|---|---|---|---|---|
| **Ins** | **60** | **120** | **300** | **Ins** | **60** | **120** | **300** |
| **1** | Nf | Nf | 0.85 | **12** | Nf | 2.29 | 1.45 |
| **2** | Nf | 5.44 | 4.59 | **13** | Nf | Nf | 4.37 |
| **3** | Nf | Nf | 1.09 | **14** | Nf | 2.37 | 0.84 |
| **4** | Nf | 2.51 | 1.28 | **15** | Nf | 37.87 | 37.87 |
| **5** | Nf | 8.24 | 3.72 | **16** | Nf | 1.18 | 0.83 |
| **6** | 54 | 54 | 54 | **17** | Nf | Nf | 2.26 |
| **7** | Nf | 2.01 | 1.87 | **18** | 38.75 | 2.41 | 2.16 |
| **8** | Nf | 3.37 | 1.19 | **19** | 7.12 | 3.07 | 0.66 |
| **9** | Nf | 3.63 | 1.23 | **20** | Nf | 2.40 | 1.80 |
| **10** | Nf | 4.26 | 0.69 | **21** | 9.86 | 2.55 | 2.55 |
| **11** | Nf | 3.71 | 3.66 | **22** | 8.54 | 1.86 | 1.25 |

tated single-item problems. In the workshop, we plan to present this approach and the corresponding numerical results.

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# Bi-objective stochastic lot-sizing with coordinated shipments

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**Abstract**

Inspired by sustainability goals, we consider the problem of coordinating shipments in a stochastic lot-sizing setting. There are multiple items, which are shipped periodically from a single supplier to satisfy customer demands. This demand is dynamic and stochastic, but we assume that demand distributions are known or can be estimated. Costs are associated with the amount of inventory of each item and with each order of an item. There is an opportunity to achieve environmental savings by combining orders implying fewer shipments. This leads to a bi-objective lot-sizing problem with coordinated shipments where both the amount of shipments as well as costs need to minimized, such that a service level constraint is satisfied. We study a static-dynamic version, where first the ordering periods are determined, and given these ordering periods the ordering plan per item should be obtained. The complexity of the problem lies in the fact that not each item may be ordered in a potential ordering period, as fixed ordering costs are incurred for each order placed. We propose several heuristic approaches for this problem based on dynamic programming and test the performance in a computational study.

## 1 Introduction

We consider the problem of determining when and how much to ship from suppliers to a warehouse. In our problem, there are multiple items (and one supplier for each of them), which are shipped periodically to a warehouse to satisfy the demand of customers. Demand of customers is dynamic and stochastic, but we assume that demand distributions are known for each item and each period. The cost are associated with the amount on inventory of each item and with each order of an item. However, based on a company case, there is also a second objective of minimizing the number of periods in which deliveries take place to achieve environmental savings.

In our version of the problem, the order periods for each item have to be set beforehand, but the quantity to be ordered can be determined when demand is known. So at the start of the planning period, we have to select the periods in which orders are placed. In the selected periods, we can then order the quantities necessary to bring our inventory levels to predetermined order-up-to amounts. This approach is known as the so-called static-dynamic one in the literature (see e.g. [1]). To ensure that a sufficient share of demand is fulfilled, a fill rate is set. We assume that a delivery can be made instantaneously and the order quantity is unlimited.

In fact, the problem under consideration is a stochastic bi-objective version of the economic lot-sizing problem, as demand is stochastic and there are the objectives of cost and the number of deliveries. In summary, the input of the problem is as follows:

$T$: number of periods,

$K$: number of items (product categories),

$d_{kt}$: stochastic demand for item $k$ in period $t$,

$s_{kt}$: set-up cost for item $k$ in period $t$,

$h_{kt}$: inventory holding cost per unit for item $k$ in period $t$.

# 2 The model

In order to model the problem, define $c^k_{ij}$ as the cost of having a set-up at time $i$ to cover demand of item $k$ for periods $i, \ldots, j$ with the next set-up at time $j+1$. These are the cost $s_{ki}$ of ordering item $k$ at time $i$ plus the expected costs of the ending inventories in the periods $i, i+1, \ldots, j$, given that an order quantity $Q$ is sufficient to satisfy $FR\%$ of demand between $i$ and $j$, that is

$$c^k_{ij} = s_{ki} + \sum_{t=i}^{j-1} h_{kt}\mathbb{E}(I_{kt})$$

with

$$\mathbb{E}(I_{kt}) = \int_0^\infty (Q-x)^+ f^k_{it}(x)dx = \int_0^Q (Q-x) f^k_{it}(x)dx,$$

and $Q$ such that

$$\int_0^Q x f^k_{ij}(x)dx = FR \cdot \sum_{t=i}^{j} \mathbb{E}(D_t), \tag{1}$$

where $FR$ is the specified fill rate in each order cycle and $f_{ij}(x)$ is the distribution function of having a total demand of $x$ in $[i, j]$.

Since we are taken a static-dynamic approach and the cost parameters $c_{ij}^k$ can be computed (analytically or numerically) upfront, we can model the problem as a MIP problem. In order to do that, we define the following decision variables:

$x_{ijk}$: binary variable equal to 1 iff the schedule for item $k$ contains order cycle $[i, j]$

$z_t$: binary variable equal to 1 iff there is a delivery in period $t$.

We use an $\epsilon$-constraint approach, meaning that we set a bound $\epsilon$ on the number of deliveries (i.e., $\sum_{i=1,\dots,T} z_t \leq \epsilon$) and solve a cost minimization problem for each $\epsilon \in \{1, \dots, T\}$ to get the Pareto frontier. For a given $\epsilon$, the MIP model is as follows:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{T} \sum_{j=i}^{T} c_{ijk} x_{ijk} & & (2)\\
\text{s.t.} \quad & \sum_{j=1}^{T} x_{1jk} = 1 & \forall k \qquad & (3)\\
& \sum_{i=1}^{j} x_{ijk} = \sum_{i=j+1}^{T} x_{j+1,i,k} & \forall j, k \qquad & (4)\\
& \sum_{j=i,\dots,T} x_{ijk} \leq z_i & \forall i, k \qquad & (5)\\
& \sum_{i=1,\dots,T} z_i \leq \epsilon & & (6)\\
& x_{ijk}, z_i \in \{0, 1\} & \forall i, j, k \qquad & (7)
\end{aligned}
$$

Similar models can be found in [1], except that we have an additional constraint to bound the number of shipments.

# 3 Heuristic solution approach

As larger instances cannot be solved in a reasonable amount of time by the MIP model, we propose a heuristic approach based on dynamic programming (DP). Note that there are two related problems that need to be solved. The overarching problem is to find a set of $m$ delivery periods $V$ in $\{1, \dots, T\}$ which minimizes total cost. Given the ordering periods in $V$, for each item $k$ we need to solve a subproblem in which the objective is to find the shortest path (note that not each ordering period has to be used for each item, as this will incur set-up cost).

In the DP approach we compute the optimal order schedule per item $k$ during the course of the algorithm. To formally describe this approach, we introduce the following notation:

- $v_j^m$: minimum cost up till period $j$ with at most $m$ shipments,
- $V_j^m$: set of 'optimal' order periods in interval $[1, j]$ when having at most $m$ order periods,
- $SP_k(V; j)$ optimal cost for item $k$ in $[1, j]$ when only orders in periods of $V$ are allowed.

Note that $SP_k(V; j)$ can be determined efficiently by a shortest path approach. The DP approach is summarized in Steps 1 and 2 below. When computing the cost $v_j^m$ in (8) of Step 2, we take the 'optimal' order schedules $V_{i-1}^{m-1}$ (i.e., a schedule with $m-1$ order periods for $[1, i-1]$), we add an order period in $i$, and compute for each item what the 'optimal' cost are in $[1, j]$ when using only order periods in $V_{i-1}^{m-1} \cup \{i\}$.

**Step 1:** Initialization for $j = 1, \ldots, T$
$v_j^1 = \sum_k c_{1jk}$
$V_j^1 = \{1\}$

**Step 2:** DP recursion
for $j = 2, \ldots, T$ and $m = 2, \ldots, j$ we have the recursion

$$v_j^m = \min \left\{ v_j^{m-1}, \min_{i=2,\ldots,j} \sum_k SP_k(V_{i-1}^{m-1} \cup \{i\}); j) \right\} \tag{8}$$

set $i^* = \arg\min_{i=2,\ldots,j} \sum_k SP_k(V_{i-1}^{m-1} \cup \{i\}); j)$
if $v_j^m < v_j^{m-1}$, then $V_j^m = V_{i^*-1}^{m-1} \cup \{i^*\}$, otherwise $V_j^m = V_j^{m-1}$

The total complexity of the heuristic is $O(qKT^3)$, as all shortest paths from node $i$ to all $j \geq i$ can be determined jointly. The DP approach can be further refined by not only keeping track of the best, but to keep track of the $q$ best solutions. Preliminary computational tests show that the heuristic approach performs well with optimality gaps of not more than 1% when $q = 10$, and with further improvements being attained when we increase $q$ to 20.

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# Data-driven robust optimization approach to supply planning using an approximated uncertainty set

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**Abstract**

This study addresses the problem of planning the supply operations of an assembly line when the later are subcontracted to an external service provider. This problem is motivated by a case study of the aircraft industry in which uncertainties in the delivery of raw material leads to inefficiency on the assembly line. We develop two classic and data-driven robust optimization models based on existing uncertainty set and we propose an approximation of the data-driven uncertainty set in order to improve the tractability of the resulting robust models. Experimental results show that robust optimization models can improve the efficiency of the assembly line by avoiding raw material unavailability. The data driven method provide the best solutions by taking into account correlations and asymmetries in the uncertainties but results in untractable models on large instances. We show that our approximation results in comparable solutions with a significant reduction of the computation time.

# 1 Introduction

In the last decades, *Robust optimization (RO)* has become on of the most popular approach to deal with uncertainties in optimization problems. The idea of robust optimization is to restrict the possible values of uncertain parameters to an *uncertainty set* $\mathcal{U}$, then to optimize against the worst realisation within this set to obtain solutions that are robust to all scenarios in $\mathcal{U}$ [1]. The work of [2], [3], [4] and [5] have opened the way for RO and since then, a lot of efforts have been dedicated to develop different types of uncertainty set to obtain tractable robust models. Today, with the growing complexity of supply chains, RO appears like a promising approach to reduce the impact of uncertainties and to improve the reliability of industrial systems.

# 2 Problem description

We consider a production planning problem where an assembly line combines different components into a set of final products over a finite planning horizon. We assume that all component are available in a warehouse managed by a third party logistic provider (3PL). The 3PL is in charge of delivering components to the assembly line according to its orders as represented on Figure 1. For each picking operation, an operator

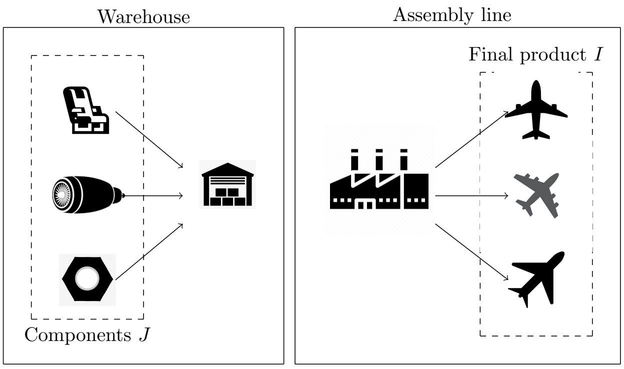


Figure 1: Diagram of the problem

collects a given quantity of a single type of component, bounded by a maximum batch size. Thus several picking operations of the same component $j \in J$ may be scheduled in the same period due to this limitation. We model the picking time of a particular batch of component $j$ as follows: 1. a fixed time $p_j$, that corresponds to the travel time between the shipping point and the zone where components $j$ are stored and 2. a picking time per unit denoted $\tau_j$. The overall picking time of a given period of the planning horizon is bounded by the maximum work capacity of the 3PL $C$. Any demand for product that is not satisfied immediately is backlogged and incurs a backlogging penalty cost in each period until the corresponding product is assembled in a subsequent period. Whenever a component is available on the assembly line but is not immediately used to manufacture an end product, it disturbs the production process by interfering with people and other goods moving nearby. We model this situation with a per-unit, per-period obstruction cost. The problem consists in planning the quantity of each component delivered to the assembly line by the TPL in each period such that the sum of the obstruction and backlogging costs is minimized.

The main source of uncertainty in this application comes from the fact that the assembly line has an incomplete or imprecise knowledge on the picking times at the 3PL. As a consequence, some combinations of orders may exceed the picking capacity of the 3PL provider, forcing the latter to postpone some operations to subsequent periods. We assume that we are given a set of historical setup times $\mathcal{D} = \{\mathbf{p}^{(1)}, \ldots, \mathbf{p}^{(N)}\}$ which can be used to derive uncertainty sets.

# 3 Data-driven robust optimization approach

In the literature, several attempt have been proposed to construct uncertainty set directly from historical data of optimization problems. By doing so, the produced uncertainty set can integrate information about the distributions of uncertain parameters such as correlations or asymmetries and then improve the quality of the robust solutions. In our work, we focus on the construction of *Data-driven uncertainty sets* that can be used to obtain tractable robust formulations of the problem described above. We present and compare the results obtained with three distinct uncertainty sets: 1. The classic Budget based uncertainty set such as described in [5] 2. The SVC based uncertainty set described in [6] 3. We propose an approximation of the SVC based uncertainty set in order to improve the tractability of the induced robust model Our approximation method aims at reducing the complexity of the SVC based uncertainty set while maintaining the same representation of uncertain parameters. By doing so, we are able to obtain robust solutions that are comparable to those obtained with the SVC based uncertainty set with a significant reduction of the computation time needed to solve the robust model.

# 4 Experimental results and conclusions

The different robust models have been evaluated on a set of instances generated to represent realistic cases inspired by our industrial partner. The different uncertainty sets are built with a data-set $\mathcal{D}$ of $N = 1000$ data-points and the obtained solutions are evaluated on a larger test data-set of $N' = 10000$ data points following the same distribution. Our experimental results can be summarized as:

1. The different robust models reduce the impact of picking time uncertainties on the production system.
2. By accurately modeling correlations and asymmetries in the distribution of picking times, the SVC based uncertainty set of [6] lead to a lower solution cost than the classic budget based uncertainty set.
3. The application of the SVC based uncertainty set to large instances leads to large robust models that are not tractable in practice.
4. Using our approximation of the SVC based uncertainty set lead to solutions that are comparable to those of the SVC based uncertainty set with a significant reduction of the computation time.

Future research directions are multiple and includes: 1. The extension of the model to a multi-stage model to represent more accurately the problem faced by our industrial partner. 2. Validate the models on industrial instances built from real historical data.

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# Heuristics

A2-030 - Thursday, 25/08 - 14:00-15:00

# A Flow Time Oriented Lot Sizing Model for a Serial Two-Stage Production-Inventory System: Analytical Approximation and Simulation-Based Optimization

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**Abstract**

Standard lot sizes that can be adapted to demand variations over time can be a reasonable alternative to dynamic lot sizing especially in discrete manufacturing. This provides the possibility to consider the impact of lot sizes on the relationship between flow time, work-in-process and output by means of steady-state queueing or simulation models.

Lot sizing models of this type have been developed mainly for single-stage production systems. We develop a model for a serial two-stage production-inventory system and analyze how the flow time effects of lot sizes propagate to the entire multi-stage system. Optimal lot sizes and inventory control parameters for both stages are derived by an analytical approximation and by simulation-based optimization.

We show that mainly by altering the inventory control parameters the flow time effects of lot sizes influence the mean total flow time. The results indicate that the structural insights known from single-stage models seem to pertain in a two-stage system. Insights further suggest adaptive lot sizing strategies when system utilization varies over time.

Keywords: Lot sizing, queueing, inventory control, flow times.

# 1 Introduction

Motivated by a real-world lot sizing problem faced by a metal processing company in Austria we analyze flow time oriented lot sizing for a serial two-stage production

- inventory system using an approximate analytical model and simulation-based optimization. The paper builds on four streams of literature: (1) Queueing-theoretical lot sizing models, (2) parameterization of (r, Q) policies, (3) lot streaming and (4) traditional lot sizing models, primarily the MLCLSP.

# 2 The Two-stage Lot Sizing Model

We consider a serial two-stage production system depicted in Figure 1 that consists of the single-server work centres MS1 and MS2 with their respective queues Q1 and Q2. The system produces $J$ products with identical setup and processing times, identical demand (constant demand rate) and identical lot sizes. The products are different from the beginning, that is, there is a 1:1 relation of the intermediate product produced by stage 1 and the final product produced by stage 2. For each product $j$ an intermediate inventory and a finished-goods inventory is held.

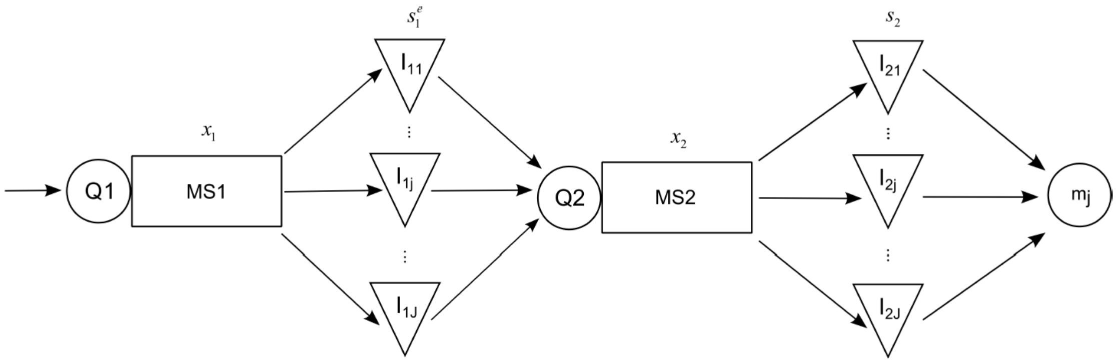


Figure 1: The serial two-stage production system

The production system faces a deterministic demand rate $m$ (product units per time unit) for each product $j$. Setup time per lot $r_n$, processing time per unit $a_n$ and lot size $x_n$ for the stages $n$ ($n$=1, 2) are identical for all products. E[$W_n$] denotes the mean number of units at stage $n$, E[$I_n$] is the mean inventory after stage $n$. E[$T_n$] and E[T] denote the mean flow time at stage $n$ and mean total flow time, respectively. $h_0, h_1, h_2$ are the holding cost for raw material, intermediate and final products, respectively, per unit and period, $R_n$ the direct setup costs per period at stage $n$. The lot sizes $x_n$ are the decision variables.

The two-stage lot sizing **Model LGR** can be formulated as follows:

$$Minimize\ Z = h_0E[W_1] + h_1E[I_1] + h_1E[W_2] + h_2E[I_2] + R_1 + R_2 \tag{1}$$

Subject to:

$$x_1 = kx_2 \qquad if\ x_2 \leq x_1 \tag{2}$$

$$x_1 = \frac{1}{k}x_2 \qquad if\ x_2 > x_1 \tag{3}$$

$$x_n > \frac{Jmr_n}{1 - Ja_n m} \qquad for\ n = 1, 2 \tag{4}$$

$$E[T_n] = g_n(x_n) \qquad for\ n = 1, 2 \tag{5}$$

$$x_n, k \in N \tag{6}$$

The WIP levels within the production stages and in the SKU inventories depend on the inventory control policy. We assume an independent reorder point system for each product at stage 2 with a reorder point $s_2$ that is identical for all products. For stage 1 we apply an echelon stock reorder point system with echelon stock reorder point $s_1^e$. This results in the task of minimizing (1) over the decision variables $x_1, x_2, s_1^e, s_2$.

# 3 Deriving solutions to Model LGR

The model can be solved by analytically approximating the inventory levels for a given solution vector and solving the resulting model, or by simulation-based optimization. The analytical approximation links the inventory control policy to the arrival process at the severs and applies an M/M/1 model to both servers. Parameter setting of the inventory control system is based on [2]. The simulation-based optimization is developed using FlexSim 16 and performed by OptQuest$^{TM}$ that uses elements from scatter search, taboo search and neural networks.

# 4 Results

**Theorem 1 (from analytical approximation)**
For given parameters of the inventory control system at both stages (stockout probability at stage 2 and $s_1^e$ ) the total flow time through the system E[T] (from entering the queue at stage 1 to outflow from FGI) is independent of the mean flow times at both stages. The mean flow times at the servers influence the mean total flow time only via the inventory control parameters.
**Theorem 2 (from analytical approximation)**
We assume a lot sizing policy that imposes the constraint $x_2 \leq x_1$, e.g., according to the MRP logic. Furthermore, there is no value added by stage 2, that is, $h_1 = h_2$. $x_1$ (and hence E[T$_1$] ) are exogenous parameters. In this case the optimal lot size at

stage 2 is equal to the lot size at stage 1.

**Corollary 1**
$x_1 = x_2$ means that in lot sizing both stages can be combined. This corresponds to the result in [1] and is realistic if we interpret stage 1 as raw parts manufacturing with high setup times and stage 2 as component manufacturing with highly automated technology.

**Theorem 3**
If minimization of mean total flow time is the objective: For fixed $x_1, x_1 = kx_2$ with $k \geq 1$ , integrality constraint on $k$ relaxed, the optimal $x_2$ is only weakly dependent of $x_1$ and at least close to the value of $x_2$ that minimizes $\mathrm{E}[\mathrm{T}_2]$ (in contrast to Theorem 2!). Note that the analytical model does not approximate the intermediate inventory accurately for non-integer $k$. For integer $k$ the optimal $x_2$ oscillates around this value. In the simulation-based optimization (that correctly models the intermediate inventory over time) the optimal values for $k$ were integer in all experiments (fixed $x_1$, $x_1 = kx_2$ with $k \geq 1$ ).

**Theorem 4**
As productive bottleneck utilization increases, the optimal solution can shift between the two lot sizing policies $x_1 \geq x_2$ and $x_1 < x_2$ .

*Observation:*
If the optimum is characterized by $x_1 = \frac{x_2}{k}$ with $k \geq 1$, the simulation-based optimization can lead to non-integer optimum values for $k$. Note the contrast to the result for the opposite case $x_1 \geq x_2$. Note, however, that in our case all lot sizes of a product at stage 1 are equal, which means that the (larger) lot at stage 2 does not define a complete cycle. The remaining intermediate inventory is outweighed by the reduced flow time at the servers.

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# New construction heuristic for capacitated lot sizing problems

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## 1 Introduction

Lot sizing is one of the core decisions to be made in production planning and inventory management. One of the basic concepts of lot sizing considering multiple items competing for the same production resource and a deterministic but dynamic demand is the capacitated lot sizing problem (CLSP). The CLSP is known to be NP-hard and finding optimal solutions for even small instances is time-consuming and usually out-of-range in a practical setting. Therefore, heuristic solution approaches are the preferred methods. Although, they have several disadvantages such as that they are tailored to a specific problem and small changes in the problem setting make them unusable, or that the solution quality is sometimes poor and not predictable. Simple and fast construction heuristics are rare, usually inflexible, and providing insufficient solution quality.

We propose a novel two-step construction heuristic (2-SCH) that overcomes these downsides and provides considerable more flexibility and high solution quality.

The CLSP is an extension of the classical Wagner-Within (WW) model, where multiple items are competing for the same limited resource. It is a single-level, multi-item big-bucket model and serves as one of the basic models in production planning. There are many variants and extensions of the model depending on the specific application used for.

We consider the CLSP with and without setup times. The mathematical formulation of CLSP with overtime and setup times is presented in (1) - (5).

$$\min Z = \sum_{i=1}^{P} \sum_{t=1}^{T} (sc_i \cdot Y_{it} + hc_i \cdot I_{it}) + \sum_{t=1}^{T} oc \cdot O_t \qquad (1)$$

subject to

$$I_{it} = I_{i(t-1)} + X_{it} - d_{it} \qquad \forall i, t \qquad (2)$$

$$\sum_{i=1}^{P} (X_{it} + st_i \cdot Y_{it}) \leq C_t + O_t \qquad \forall t \qquad (3)$$

$$X_{it} \leq (\sum_{\tau=1}^{T} d_{i\tau}) \cdot Y_{it} \qquad \forall i, t \qquad (4)$$

$$Y_{it} \in \{0, 1\},\ I_{it}, X_{it}, O_t \geq 0 \qquad \forall i, t \qquad (5)$$

The decision variables are lot sizes $X_{it}$ for item $i$ in period $t$, setups $Y_{it}$, inventory $I_{it}$, and overtime $O_t$. Objective function (1) minimizes the sum of setup, holding and overtime costs. It is subject to the inventory balance constraint (2), capacity constraint (3) and setup state constraint (4).

Two well-known construction heuristics for single-level CLSP without setup time are: Dixon-Silver-Heuristic [1] and ABC-Heuristic [2]. Recently a new extension of the Dixon-Silver-Heuristic has been proposed where the local decision criterion is optimized using genetic programming (GP) [3].

A simple heuristic was presented in [4] for the single-level CLSP with setup times. All mentioned methods are based on (modified) Silver-Meal criterion and create a production plan stepwise from the first to the last period. They are rather inflexible because they can hardly be adapted to other lot-sizing problems.

Recently, [5] proposed a simple construction heuristic embedded in a metaheuristic for the practical lot-sizing problem of a pharmaceutical company. This construction heuristic is a simple, rule-based method to add new demand to an existing production plan. The order in which demand is added to the plan is optimized by a genetic algorithm. We used that idea to develop a new 2-step construction heuristics.

# 2 2-step construction heuristic (2-SCH)

In the first step of the 2-SCH, demand information is sorted and in the second step a production plan is built up by including the previously sorted demand information step-by-step. Hence, our approach splits the general problem of constructing a production plan into two subproblems:

**Extension of partial production plan:** How to add a demand element to the current partial production plan?

**Sort demand:** How to sort the demand elements in the demand list?

To solve the first subproblem and add a demand element $d_{it}$ to a partial production plan we consider four cases in the given order:

**Case I: Use inventory.** The available inventory is used to satisfy $d_{it}$ and a new demand element $d_{i(t+1)}$ is created to be added immediately and ensure the feasibility of the plan.

**Case II: Enough capacity in $t$.** Three options are considered - add $d_{it}$ to $t$ ($NOW$), extend existing lot(s) in previous period(s) ($EXT$), create a new lot and extend existing lot(s) in previous period(s) ($NEW$) - and the cheapest one is executed.

**Case III: Not enough capacity in $t$.** Four options are considered - $EXT$, $NEW$, add possible amount to $t$ and apply Option $EXT$ for the remaining amount, add possible amount to $t$ and apply Option $NEW$ for the remaining amount - and the cheapest one is executed.

**Case IV: Overtime.** If the available capacity in periods up to $t$ is insufficient to produce the whole demand, we add as much production as possible within regular capacity in periods $1, ...t-1$ and add the remaining part to $t$, so that overtime $O_t$ is created.

We introduce the **shift of production** routine, which consists of a right-shift and a left-shift operation. It is included in the cost calculations of the above mentioned options and is performed every time when a new production lot is (potentially) created.

To solve the second subproblem we consider different sorting rules, based on period, cost or capacity utilization. Better results are observed if we allow to alter the sorting of demand list on the fly and use the ***postponement*** routine. If the relative difference between additional cost of option $EXT$ and additional cost of option $NEW$ is smaller than threshold $TH$, we will not add the considered demand element until there is a change in the plan for the considered item. As soon as another demand element for the considered item is added to the partial production plan, we try to add the postponed demand element again.

# 3 Computational experiments and conclusions

The preliminary tests on the CLSP without setup times allowed to identify the reduced set of sorting rules and threshold values that provide the good solution quality.

Results of the experiments on 1471 instances of CLSP without setup times report the average gap to MIP-solutions as 2.46%, whereas ABC-Heuristic leads to 3.71%

average gap and Dixon-Silver-Heuristic to 4.77%. 2-SCH performance is comparable to the best result of GP method (in both cases the reported average gap is 2.82% on 540 instances from the validation set).

Using the same parameters as for the CLSP without setup times, we obtained solutions without overtime for 720 instances out of 751. The average gap is 7.06% for 2-SCH and 8.91% for Trigeiro's simple heuristic (reported for 713 instances, for which feasible solutions were found by both methods). We were able to found feasible solution for all instances when we either applied 2-SCH with other sorting rules and threshold values than the ones used for CLSP without setup times or performed simple local search on the demand lists for some instances.

2-SCH has a number of advantages apart from delivering better results in terms of the average relative gap to the best MIP-solutions. First, this is a flexible method that can be applied to different problem extensions. Second, the problem representation as a sequence of demand elements with item and period indices, which is used by 2-SCH, is handy to use with metaheuristics and naturally allows the proposed method to be used as a part of a more complex algorithm. Finally, 2-SCH follows the rule-based concept that easily allows to create and adapt a production plan on the fly, which has known benefits in the practical setting. This also opens a perspective to use 2-SCH for stochastic problems as, for instance, a fast tool to compute many different scenarios.

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# Mathematical Formulations and Reformulations

A2-030 - Friday, 26/08 - 09:00-10:30

# Multi-item dynamic lot sizing with multiple transportation modes and item fragmentation

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## 1 Introduction

In this work, we consider a problem of ordering multiple items from a single source by a downstream supply chain member, such as a retailer in a distribution environment or a manufacturer in a production environment. The downstream supply chain member makes replenishment decisions for items with deterministic demand over a finite planning horizon. The costs include holding and ordering costs for each item, along with the transportation cost for shipping the items from the vendor. In each period, the lots of various items that are ordered are transported directly from the supplier to the customer using the selected transportation modes. The objective is to minimise the total cost while fully satisfying the customer demand from the current shipments and inventory.

When explicitly considering transportation capacities, one of the possible extensions of the inventory models highlighted in [1] is the integration of more operational decisions related to the container loading configurations. Such extension is common for other supply chain related decisions such as vehicle routing problems. In case of inventory models, this extension is relevant when dealing with several products having different configurations and container utilization rates. In this work, we assume that the items are palletized in standard euro-pallets without any geometrical or weight differences. The only attributes of the items determining the loading configuration are the lot sizes expressed as the number of items/pallets. However, the container loading is explicitly modeled to indirectly include the handling operations in the optimization scope. This integration is based on the assumption that avoiding the splitting of the ordered items among several containers, termed *fragmentation*, leads to simplification of loading and unloading operations, i.e., packing one product separately into two trucks is preferred over packing the product into three trucks.

# 2 Problem Motivations and Modelling

The ordering, transportation, and good receiving costs grow with the variety of products ([2]). Reducing the number of products loaded in a container positively impacts warehousing costs as this leads to less effort in the receiving, inventory update, and put away processes. As described in [3], some customers handle items separately for every truck. This means that, for every truck, the processing effort depends on the number of different items (but also on other factors, such as the number of pallets). Given a fixed number of items to transport, it is possible to reduce the number of items loaded on a container by reducing the number of fragmentations. However, limiting fragmentation possibility impacts the utilization of transportation resources negatively. Figure 1 illustrates the conflict between the efficiency of transportation operations and the efficiency of handling operations under the prism of fragmentation. Considering the example of a supplier shipping three products, each with an order of 20 pallets, Figure 1 compares the loading of the containers with a capacity of 30 pallets under the fragmentation-allowed policy and the fragmentation-forbidden policy.

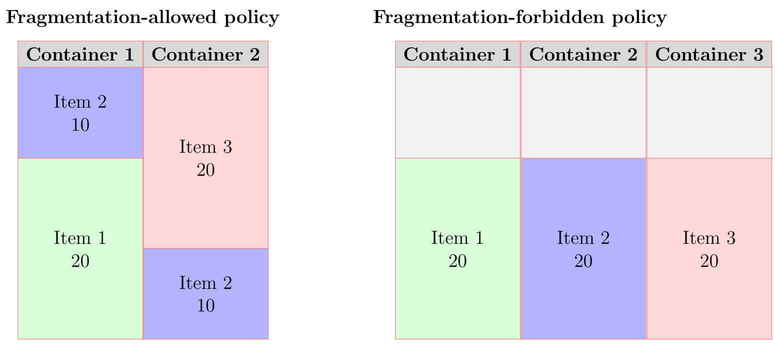


Figure 1: Best truck loading with fragmentation-allowed policy and fragmentation-forbidden policy

The primary motivation of our work is that taking both transport and handling operations into account in lot-sizing models can tackle the conflict highlighted above.

This argument is based on the insight that lot sizes play a critical role in the efficient usage of transportation resources and the benefit of fragmentation. To illustrate this point, let us consider a lot-sizing problem with three items and a planning horizon of two periods. Table 1 presents the demand for the three items and two feasible replenishment plans. Assuming the availability of one Full-truckload (FTL) mode with a capacity of 30 pallets, Figure 2 shows the best container loading with the fragmentation-forbidden policy. The quantities ordered in the second period fit in one container for both plans. The quantities ordered in the first period of Plan I correspond to the bin packing problem illustrated in Figure 1. As shown above, the ordered quantities induce either a single fragmentation or require one additional container, depending on the fragmentation-acceptance policy. However, the quantities ordered in the first period in Plan II can fit in two containers without any fragmentation. In other words, the conflict between the efficiency of transportation operations and the efficiency of handling operations can be avoided or mitigated by an appropriate choice of lot sizes.

| | Demand | | Plan I | | Plan II | |
|---|---|---|---|---|---|---|
| Item | $t = 1$ | $t = 2$ | $t = 1$ | $t = 2$ | $t = 1$ | $t = 2$ |
| 1 | 20 | 20 | 20 | 20 | 20 | 20 |
| 2 | 15 | 15 | 20 | 10 | 30 | 0 |
| 3 | 10 | 10 | 20 | 0 | 10 | 10 |

Table 1: Lot sizing problem and two optimal replenishment plans

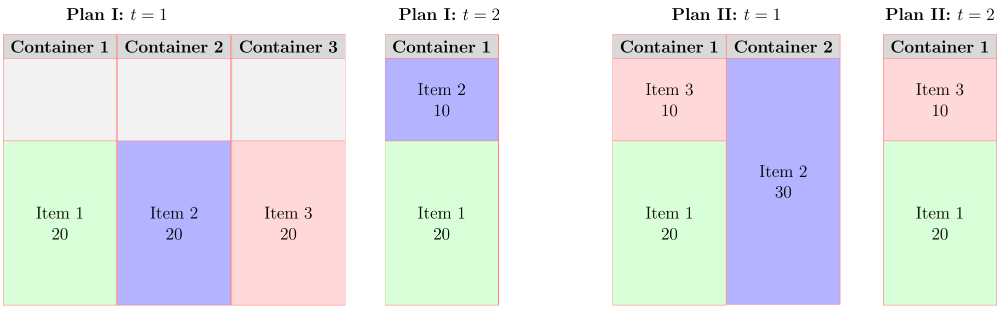


Figure 2: Plans I and II with fragmentation-forbidden policy

As it is difficult to estimate the cost related to fragmentation accurately, we consider a hard constraint on the maximum allowed number of possible fragmentations for all items in each period. The resulting optimization model is a dynamic multi-item lot-sizing model with multiple transportation modes with various capacities and fragmentation constraints.

Based on realistic data, this work analyzes the impact of various fragmentation strategies on the costs and computational times of mathematical models solved with a standard solver. Due to the complexity of the problem, several Mixed Integer Linear Programming models are tested and analyzed numerically. The different formulations of the problem are obtained by combining different formulations of the lot sizing sub-problem and the variable cost and size bin packing sub-problem.

# 3 Computational Results

Based on real data from a Scandinavian distribution company for fast-moving consumer goods, we generated problem instances and compared the different mathematical formulations in terms of computational time and number of solved instances using a standard solver. The numerical results showed that the best formulation relies on the model for the variable cost and size bin packing sub-problem proposed in [4]. We also analyzed the effect of varying some problem parameters and fragmentation constraints on the computational times.

Some managerial insights are also drawn. First, we are interested in identifying the cases where the integrated model may lead to significant cost savings compared to the sequential approach, where the decisions related to transportation are taken in a second stage using the lot sizes determined in the first stage, where the transportation costs and capacities are ignored. We are also interested in the impact on the total cost of the tightness of the constraint on the number of fragmentations. The results show that the costs of fully forbidding fragmentation can be significant, while a very small cost increase can be expected when allowing a small number of fragmentations per period.

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# Strong Valid Inequalities for Two-Period Relaxations of Big Bucket Lot-Sizing Problems

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**Abstract**

In this study, we investigate two-period relaxations for lot-sizing problems with big bucket capacities and zero setup times. More particularly, we identify an important mixed integer set representing relaxation of these subproblems and then present various families of strong valid inequalities for such relaxation. We then extend these inequalities in a novel fashion to the original space of two-period subproblems, and also propose a new family of valid inequalities in the original space. We then present and discuss the separation problems associated with these valid inequalities. Finally, we present the computational experiments indicating that the proposed inequalities can be indeed very effective improving lower bounds substantially.

# 1 Introduction

Production planning problems have been interesting for both researchers and practitioners for more than 50 years. The problem aims to determine a plan for how much to produce and stock in each time period during a time interval called planning horizon. It is an important challenge for manufacturing companies because it has a strong impact on their performance in terms of customer service quality and operating costs. In this study, we focus on multi-level, multi-item production planning problems with big bucket capacities, i.e., each resource is shared by multiple items and hence different items can be produced in a specific time period. These real-world problems remain challenging to solve to optimality as well as to obtain strong bounds.

Let $NT$, $NI$ and $NK$ be the number of periods, items, and machine types, respectively. We assume that each machine type operates only on one level, and each level can employ a number of machine types. The set *endp* indicates all end-items, i.e. items with external demand; the other items are assumed to have only internal demand. Let $x_t^i$, $y_t^i$, and $s_t^i$ represent production, setup, and inventory variables for item $i$ in period $t$, respectively. The setup and inventory cost coefficients are indicated by $f_t^i$ and $h_t^i$ for each period $t$ and item $i$. The parameter $\delta(i)$ represents the set of immediate successors of item $i$, and the parameter $r^{ij}$ represents the number of items required of $i$ to produce one unit of item $j$. The parameter $d_t^i$ denotes the demand for end-product $i$ in period $t$, and $d_{t,t'}^i$ is the total demand between $t$ and $t'$. The parameter $a_k^i$ represents the time necessary to produce one unit of $i$ on machine $k$, and $ST_k^i$ is the setup time for item $i$ on machine $k$, which has a capacity of $C_t^k$ in period $t$. Let $M_t^i$ represent the maximum number of item $i$ that can be produced in period $t$. Following the notation of [2], the multi-level, multi-item production planning problems with big bucket capacities can then be formulated:

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{NT}\sum_{i=1}^{NI} f_t^i y_t^i + \sum_{t=1}^{NT}\sum_{i=1}^{NI} h_t^i s_t^i && \\
s.t. \quad & s_{t-1}^i + x_t^i = s_t^i + d_t^i, && t \in [1, NT], i \in endp, && (1)\\
& s_{t-1}^i + x_t^i = s_t^i + \sum_{j\in\delta(i)} r^{ij} x_t^j, && t \in [1, NT], i \in [1, NI] \setminus endp, && (2)\\
& \sum_{i=1}^{NI} (a_k^i x_t^i + ST_k^i y_t^i) \le C_t^k, && t \in [1, NT], k \in [1, NK], && (3)\\
& x_t^i \le M_t^i y_t^i, && t \in [1, NT], i \in [1, NI], && (4)\\
& y \in \{0,1\}^{NT\times NI}, x \ge 0, s \ge 0. && && (5)
\end{aligned}
$$

Here, (1) and (2) are flow conservation constraints for end-items and intermediate items respectively. The constraints (3) are the big bucket capacity constraints, and (4) guarantee that the setup variable is equal to 1 if production occurs. Finally, (5) give the integrality and non-negativity constraints.

We note that uncapacitated relaxation and single-item relaxation have been studied previously by [5]. In addition, [4] introduced and studied the single-period relaxation with preceding inventory, where they also derived cover and reverse cover inequalities for this relaxation. Finally, we also remark the work of [3] on a single-period relaxation as a relevant study.

## 2 Two-Period Relaxation

Now, we present the feasible region of a two-period, single-machine relaxation of the multi-level, multi-item production planning problems with big bucket capacities, denoted by $X^{2PL}$ (see [1] for details).

$$
x_{t'}^i \le \widetilde{M}_{t'}^i y_{t'}^i, \qquad i \in \{1, \ldots, NI\}, t' = 1, 2,
$$

$$
\begin{aligned}
& x^i_{t'} \leq \widetilde{d}^i_{t'} y^i_{t'} + s^i, && i \in \{1,\ldots,NI\}, t' = 1,2, \\
& x^i_1 + x^i_2 \leq \widetilde{d}^i_1 y^i_1 + \widetilde{d}^i_2 y^i_2 + s^i, && i \in \{1,\ldots,NI\}, \\
& x^i_1 + x^i_2 \leq \widetilde{d}^i_1 + s^i, && i \in \{1,\ldots,NI\}, \\
& \sum_{i=1}^{NI} (a^i x^i_{t'} + ST^i y^i_{t'}) \leq \widetilde{C}_{t'}, && t' = 1,2, \\
& x \geq 0, s \geq 0, y \in \{0,1\}^{2\times NI}.
\end{aligned}
$$

Since we consider a single machine, we dropped the $k$ index from this formulation, however, all parameters are defined in the same lines as before. Observe that for a given time period $t$, the obvious choice for the "horizon" of this two-period relaxation would be $t+1$, i.e., $t' = 1,2$ relate to the periods of $t, t+\alpha$ with $\alpha \in \{1,\ldots,NT-t\}$. The parameters can be associated with the original problem parameters using the relations $\widetilde{M}^i_{t'} = M^i_{t+(t'-1)\alpha}$, $\widetilde{C}_{t'} = C^k_{t+(t'-1)\alpha}$, and $\widetilde{d}^i_{t'} = d^i_{t+(t'-1)\alpha,t+\alpha}$ for all $i$ and $t' = 1,2$.

Next, we remark the following polyhedral result for $X^{2PL}$ (see [1] for details).

**Proposition 2.1** *Assume that $\widetilde{M}^i_t > 0, \forall t \in \{1,\ldots,NT\}, \forall i \in \{1,\ldots,NI\}$ and $ST^i < \widetilde{C}_t, \forall t \in \{1,\ldots,NT\}, \forall i \in \{1,\ldots,NI\}$. Then conv($X^{2PL}$) is full-dimensional.*

In this study, we investigate the case of setup times $ST^i = 0, \forall i \in \{1,\ldots,NI\}$. Under this assumption, we establish a promising relaxation of $X^{2PL}$ and then study its polyhedral structure. We derive several families of strong valid inequalities for such relaxation and establish their facet-defining conditions. We then map and extend these valid inequalities to the original space of two-period subproblems. Septation problems associated with these valid inequalities are then presented and discussed. We finally conduct a computational experiment to measure the effectiveness of proposed inequalities in closing the integrality gap and present the results.

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# Modeling inventory capacity constraints in capacitated lot-sizing with demand and production rates

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## 1 Introduction

This paper relies on the work presented in [1]. We consider the so-called Capacitated Lot-Sizing Problem (CLSP) with setup times, where several items have to be produced over a discrete planning horizon subject to capacity restrictions, as introduced in [2]. In addition to these classical capacity constraints, inventory bounds are added that limit the products that can be kept in inventory ([3], [4] and [5]). These bounds are relevant in various industrial applications. Moreover, the increasing research considering stochastic demands emphasizes the need to have a minimum stock level on the inventory at each period. Some works consider production and demand rates to model the evolution of the production and the inventory in production planning ([6]). It is also considered in the Economic Lot-Scheduling literature ([7]). However, in the lot-sizing literature, inventory bounds are always set on discrete periods, not taking into account the dynamic nature of the inventory evolution within each period.

We discuss different CLSP models to capture the inventory evolution within periods. The paper is organized as follows. Section 2 introduces the problem formulation. Section 3 proposes a first model with a uniform production rate and a bounded demand, while Section 4 presents a second model where the production occurs at maximum rate and the demand is instantaneous. Numerical results will be presented and discussed in the workshop. Some conclusions and perspectives are given in Section 5.

## 2 Problem formulation

We consider the CLSP with minimum and maximum ending inventories introduced in [8], where $N$ items have to be produced over a planning horizon of $T$ periods. The quantity of item $i \in [\![1, N]\!]$ produced at period $t \in [\![1, T]\!]$ is given by variable $X_{it} \geq 0$. The binary variable $Y_{it}$ indicates whether a setup for item $i$ occurs at period $t$ or not. Variable $I_{it} \geq 0$ is the inventory variable for item $i$ at the end of period $t$. Finally, variable $L_{it} \geq 0$ defines the quantity of lost sales for item $i$ at the end of period $t$. We extend the definition of $I_{it}$ with $t = 0$ to describe the initial inventory of item $i$. The objective function consists in minimizing the total production, setup, inventory and lost sales costs of all items over the planning horizon. The inventory bounds are modeled as follows:

$$\underline{I_{it}} \leq I_{it} \leq \overline{I_{it}}, \quad \forall i \in 1, \ldots, N,\ \forall t \in 1, \ldots, T \tag{1}$$

Each item $i$ at period $t$ has a fixed setup time $s_{it} \geq 0$ and a demand $d_{it} \geq 0$. The available capacity for each period $t$ is denoted $c_t^{max} \geq 0$.

## 3 Uniform production and bounded demand

For sake of clarity, only the single-item case is analyzed in this section. The obtained constraints will then be translated to the multi-item CLSP. We assume in this first model that the production occurs at a uniform rate immediately after a setup time. We propose an approximation of the demand within a cone of uncertainty and new linear constraints to guarantee that the inventory satisfies lower and upper bounds at each period under this demand and production scenario. Each demand is approximated by two slopes and two offsets. The first slope corresponds to the demand at its earliest time, the second to the demand at its latest time. The offsets allow for these slopes to be adjusted as tightly as possible. We define four new parameters for each time period: $o_t^e$ (resp. $o_t^l$) is the offset before the start of the early (resp. late) demand, $r_t^e$ (resp. $r_t^l$) is the early (resp. late) demand rate.

To handle the minimum (resp. maximum) intermediate inventory levels, we only need to consider the slope for the approximation of the early (resp. late) demand. Based on the analysis of the inventory evolution within each period, we get the following set of additional constraints for each period $t$:

$$\text{if } s_t \le o_t^e + \frac{d_t}{r_t^e}: \quad I_{t-1} - d_t + L_t + \tfrac{X_t}{c_t^{max} - s_t}(o_t^e + \tfrac{d_t}{r_t^e} - s_t) \ge \underline{I_t} \tag{2}$$

$$\text{if } o_t^e \le s_t \le o_t^e + \frac{d_t}{r_t^e}: \quad I_{t-1} - \alpha_t(1 - \tfrac{L_t}{d_t})(s_t - r_t^e) \ge \underline{I_t} \tag{3}$$

$$\text{if } s_t > o_t^e + \frac{d_t}{r_t^e}: \quad I_{t-1} - d_t + L_t \ge \underline{I_t} \tag{4}$$

$$\text{if } s_t \le o_t^l: \quad I_{t-1} + \tfrac{X_t}{c_t^{max} - s_t}(o_t^l - s_t) \le \overline{I_t} \tag{5}$$

# 4 Production at maximum rate and instantaneous demand

In the model described in this section, demand for item $i$ at period $t$ is supposed to be instantaneous and is due at time $t_{it}^d \le c_t^{max}$. It is a reasonable assumption for long periods (for instance when periods are aggregated at the end of the time horizon). Production occurs at maximum rate $\frac{1}{b_{it}}$, where $b_{it} > 0$ is the unitary production time.

Each production decision variable $X_{it}$ is split into two decision variables $X_{it}^b$ and $X_{it}^a$ that respectively represents the production before and after the demand occurs. Assuming that production occurs either before or after the demand, we split each setup decision variable into two setup variables $Y_{it}^b$ and $Y_{it}^a$. Under these assumptions, the maximum inventory level is reached right before $t_{it}^d$ and $X_{it}^b$ units have been produced. The minimum inventory level is reached either at the beginning of the period or right after $t_{it}^d$. In this case, $X_{it}^b$ units were produced and $d_{it} - L_{it}$ units supplied. We get the following set of additional constraints for each period $t$ and for each item $i$:

$$\sum_{j,\ t_{jt}^d \le t_{it}^d} (b_{jt} X_{jt}^b + s_{jt} Y_{jt}^b) \le t_{it}^d, \quad \forall i \in 1, \ldots, N,\ \forall t \in 1, \ldots, T \tag{6}$$

$$\sum_{j,\ t_{jt}^d \ge t_{it}^d} (b_{jt} X_{jt}^a + s_{jt} Y_{jt}^a) \le c_t^{max} - t_{it}^d, \quad \forall i \in 1, \ldots, N,\ \forall t \in 1, \ldots, T \tag{7}$$

$$I_{i,t-1} + X_{it}^b \le \overline{I_{it}}, \quad \forall i \in 1, \ldots, N,\ \forall t \in 1, \ldots, T \tag{8}$$

$$I_{i,t-1} + X_{it}^b - d_{it} + L_{it} \ge \underline{I_{it}}, \quad \forall i \in 1, \ldots, N,\ \forall t \in 1, \ldots, T \tag{9}$$

# 5 Conclusions

We have introduced two ways to model intermediate inventory constraints under assumptions regarding demand and production in capacitated lot-sizing problems.

Computational experiments, that will presented in the workshop, show that these models allow for production plans that better respect inventory constraints to be reconstructed.

A first perspective would be to design solution methods that are specific to the proposed models as well as a detailed complexity analysis for these new problems. We also believe that other assumptions could be considered to derive other inventory constraints than the ones derived in this work, or other original problems.

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# Stochastic Inventory Management

A2-030 - Friday, 26/08 - 11:00-12:30

# Economic Order Quantity with Supply Uncertainty

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**Abstract**

The economic order quantity (EOQ) depends on supply uncertainties that may be generated by random transportation losses or imperfect quality. Assuming that increasing the order size provides a first-order stochastically dominant shift in the distribution of on-spec items received, we derive the corresponding stochastic EOQ in terms of elasticities and characterize its dependence on all problem parameters, which may not be monotone as it would be in the standard deterministic model. In addition, we show that uncertainty in supply may lead to larger or smaller optimal order quantities, and that while the optimal cost may increase or decrease following small improvements of the supply distribution, it can never be less than the optimal deterministic cost. We discuss various applications, including increasing resilience and diversification of supplies, delivery targets, supply-quality updating, as well as quality control.

## 1 Introduction

A key reason for holding inventory is to spread a fixed ordering cost over time. This is accomplished optimally only when the weekly expense of keeping an extra item in stock is equal to the additional capital savings from allocating the ordering cost to a longer interval. This logic was formulated by Harris [1] resulting in the "economic order quantity" (EOQ) as solution to the deterministic problem of minimizing the average cost of serving a constant flow of demand. The underlying dynamic optimization problem can be solved in a quasi-static manner because at the end of an order cycle, when all the initial inventory is used up, the system returns to its initial state. The renewal logic carries over to situations with (stationary) uncertainty. Indeed, shipments from suppliers may get lost resulting in a complete write-off; alternatively, they may be partially spoiled or contain defective parts (or both), so that in practice supply uncertainty may be substantial. This paper deals with such supply uncertainty under the basic premise that the distribution of on-spec deliveries "increases" (in the sense of first-order stochastic dominance) when a larger order quantity is chosen. We first introduce the "stochastic EOQ problem" aimed at determining an order quantity which minimizes long-run average cost in the presence of supply uncertainty. The solution to this problem is described using three types of elasticities, related to the sensitivities of write-offs, average on-spec deliveries, and the dispersion of these deliveries, to changes in the ordered amounts. Second, we show that the stochastic economic order quantity may well be smaller than the deterministic EOQ solution whenever average on-spec deliveries are sufficiently insensitive to changes in the order quantity, as would be the case for a supplier with capacity constraints. Third, we characterize the comparative statics for all standard parameters of the stochastic EOQ problem, including ordering cost, holding cost, input cost, and demand, and show that for all but one of them the standard monotonicity of the solution can be reversed, in addition to a generic dependence of the solution on the input cost, much unlike the deterministic EOQ solution which does not depend on the price of the ordered parts. We then turn our attention to comparing the absolute cost and show that a first-order stochastically dominant improvement of supply uncertainty may have an ambiguous impact on cost. Yet, the latter is (under mild conditions) minorized by the optimal cost in the presence of a loss-free perfect-quality supply. Finally, we discuss various applications of the model, including resilience, diversification, delivery targets, learning about performance, as well as quality testing.

# 2 Main Results

With the pioneering study by Silver [3] lying dormant, Salameh and Jaber [2] started an avalanche of research contributions when introducing a natural extension of the EOQ model to a situation where supply is in fact uncertain. Our model, which requires only supply distributions that are stochastically ordered in the chosen lot size, should be viewed in this vein. It provides a fairly simple solution to a more realistic version of the underlying dynamic inventory control problem, going beyond models of (random) proportional loss while also allowing for write-off events. This solution is conveniently described using three distinct elasticities of supply, related to the variation of write-off probabilities, expected deliveries, and the coefficient of variation, respectively, in the order quantity. The comparative statics of the stochastic EOQ solution, that is, the change of the optimal order quantity in the problem parameters such as per-unit input cost ($c$), demand rate ($D$), per-unit inventory holding cost ($h$), and ordering cost ($K$), can be characterized precisely in terms of the aforementioned supply elasticities. The changes compared to the standard EOQ solution are striking. For example, the stochastic EOQ goes up in $h$ if ordering more can strongly reduce supply uncertainty, or equivalently, when the delivery-dispersion elasticity is sufficiently negative. It is also possible that the stochastic EOQ decreases in demand, provided that the deliveries are very inelastic (so the sum of write-off elasticity and fulfilment elasticity are less than the fraction of input cost in the overall procurement cost). At that point a higher demand leads to an incentive to turn inventories faster so as to save on input cost given that ordering less has only a small impact on deliveries. Finally, the stochastic EOQ increases in per-unit input cost if and only if delivery is elastic, whereas the standard deterministic lot size does not depend on $c$ at all. It comes therefore no longer as a surprise that the stochastic EOQ can well be smaller than the classical deterministic EOQ solution, notably for suppliers with capacity constraints, while the optimized expected cost must still increase when supply is uncertain relative to the deterministic base case.

# 3 Conclusion

The findings have immediate practical consequences, as it is now possible to use general (semi-)parametric families of distributions to characterize supply behavior, e.g., in terms of resilience or supply diversification. Further applications include delivery targets, the updating of supply distributions within the family of beta-distributions, as well as quality control using a (possibly correlated) binomial distribution of inspection outcomes.

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# Stochastic Dynamic Programming Formulation for the $(R, s, S)$ Policy Parameters Computation

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**Abstract**

The $(R, s, S)$ is a stochastic inventory control policy widely used by practitioners. In an inventory system managed according to this policy, the inventory is reviewed at instant $R$; if the inventory is lower than the reorder level $s$ an order is placed. The order's quantity is set to raise the inventory level to the order-up-to-level $S$. This paper introduces a new stochastic dynamic program (SDP) algorithm to compute the $(R, s, S)$ policy parameters for the non-stationary stochastic lot-sizing problem. In recent work, [1] present an approach to compute optimal policy parameters under such assumptions. We present the first formulation of the $(R, s, S)$ problem as a functional equation of an SDP model. This model is an extension of Scarf's $(s, S)$. A simple implementation of the model requires a prohibitive computational effort to compute the parameters. However, we can speed up the computations by using K-convexity property and memoisation techniques. The resulting algorithm is considerably faster than the state-of-the-art, extending its adoptability by practitioners.

# 1 Problem description

This work considers the single-item, single-stocking location, stochastic inventory control problem over a $T$-period planning horizon. The demand's stochasticity and non-stationarity of period $t$ are modelled through the random variable $d_t$. Cumulative demand of periods $t$ to the beginning of period $j$ takes the form of $d_{t,j}$ with $j > t$. If the demand in a given period exceeds the on-hand inventory, the excess is backlogged and carried to the next period. Under these assumptions, the $(R, s, S)$ policy takes the vectorial form form $(\mathbf{R}, \mathbf{s}, \mathbf{S})$, with $\mathbf{R} = (R_1, \ldots, R_T)$; where $R_t$ , $s_t$ and $S_t$ denote respectively the length, the reorder-level and order-up-to-level associated with the $t$-th inventory review.

Policies are compared based on their expected cost. Stocktaking has a fixed cost of $W$. We denote by $Q_t$ the quantity of the order placed in period $t$. Ordering costs are represented by a fixed value $K$ and a linear cost, but we shall assume that the variable cost is zero without loss of generality. At the end of each period, a holding cost $h$ is charged for every unit carried from one period to the next. In case of a stockout, a penalty cost $b$ is charged for each item and period. We denote with $I_t$ the closing inventory level for period $t$, making $I_0$ the initial inventory. The order quantity $Q_t$ is fixed at every review moment before the demand realisation to raise the inventory level to $S_t$. The order is placed only if $t$ is a review period and the open inventory is below the order level $s_t$.

We consider the problem of computing the optimal $(\mathbf{R}, \mathbf{s}, \mathbf{S})$ can be formulated as follow:

$$C_1(I_0) \triangleq \min_{(\mathbf{R},\mathbf{s},\mathbf{S})} f_1(I_0, Q_1, R_1) + E[C_{1+R_1}(I_0 + Q_1 - d_{1,1+R_1})] \quad (1)$$

Where $C_1(I_0)$ is the expected cost of the optimal policy parameters starting at period 1 with the initial inventory $I_0$. In general, $C_t(I_{t-1})$ represent the expected inventory cost of starting at period $t$ with open inventory $I_{t-1}$. While, $f_t(I_{t-1}, Q_t, R_t)$ is the expected cost of a review cycle starting in period $t$ and ending up in period $t+R_t$; it comprises review, ordering, holding and penalty cost for the review cycle. $C_t(I_{t-1})$ values can be computed recursively when all the policy parameters are computed using the following formula:

$$C_t(I_{t-1}) \triangleq f_t(I_{t-1}, Q_t, R_t) + E[C_{t+R_t}(I_{t-1} + Q_t - d_{t,t+R_t})]) \quad (2)$$

with $C_{T+1}(I_T) \triangleq 0$. For a given $(\mathbf{R}, \mathbf{s}, \mathbf{S})$ parameters set, this formulation allows to compute the expected policy cost. However, the number of combinations of parameters is exponential, making this approach unusable for the computation of optimal ones.

# 2 Heuristic technique

The heuristic introduced in this work aims to compute locally optimal $R_t$ values to produce a near-optimal $(\mathbf{R}, \mathbf{s}, \mathbf{S})$ policy. The main idea is to move the assignment of the decision variable $R_t$ at period $t$ and do not fix all of them at the beginning of the time horizon. This can be done by transforming the recursive Equation 2 into:

$$\widehat{C}_t(I_{t-1}) = \min_{R_t} f_t(I_{t-1}, Q_t, R_t) + E[C_{t+R_t}(I_{t-1} + Q_t - d_{t,t+R_t})]) \quad (3)$$

Solving this recursion could lead to different optimal $R_t$ for different opening inventory levels $I_{t-1}$.

Our heuristics consists of choosing a locally optimal $R_t$ assuming that an order is placed in period $t$ and the possibility of placing a negative order. We define these locally optimal replenishment cycles as $R_t^a$. Knowing the expected cost of future periods $\widehat{C}_j$ with $j > t$, it is possible to compute the optimal $s_t$ and $S_t$ for that specific replenishment cycle $R_t$ using SDP. The best $S_t$ is the value that minimizes $\widehat{C}_t(S_t)$, since we place an order to reach the point with the lowest future expected cost.

$$S_t = \arg\min_{I_{t-1}} \widehat{C}_t(I_{t-1}) \tag{4}$$

So, assuming that an order is placed, the best replenishment cycle is the one that has the lowest cost after the inventory level is topped up to $S_t$:

$$R_t^a \triangleq \arg\min_{R_t} \widehat{C}_t(S_t) \tag{5}$$

The computation of $\widehat{C}_t$ requires the expected costs of future periods $\widehat{C}_j$ with $j > t$, which are dependent on the optimal $R_j$. We relaxed the cost function by defining $C_t^a$ as the expected cost of using local optimal $R_j^a$ for all periods $j$ after $t$. Given $C_{T+1}^a(I_T) = 0$, it is possible to compute the relaxed cost function in a backward way using the following approximate SDP functional equation:

$$C_t^a(I_{t-1}) \triangleq f_t(I_{t-1}, Q_t, R_t^a) + E[C_{t+R_t^a}^a(I_{t-1} + Q_t - d_{t,t+R_t^a})] \tag{6}$$

This formula computes a near-optimal replenishment schedule $\mathbf{R}^a$, and the set of order and order-up-to levels optimal for that given schedule. Due to the relaxation, $\mathbf{R}^a$ can differ from the optimal $\mathbf{R}$; however this event is rare.

The resulting approximate SDP formulation is more complex than the $(s, S)$ one, making the computational effort required to solve it prohibitive. This is mainly due to the computation of the expected cycle cost; its computation involves three variables in each period: current inventory, order size and length of the replenishment cycle. This computational effort can be considerably reduced applying the K-convexity property. The deployment of search reduction and memoisation techniques further improve the performances, and it has a crucial impact on the applicability of this model.

# 3 Experimental Results

We aim to evaluate the policies computed by the heuristic and the computational effort required. We assess the computational effort required to compute a policy and under an increasing time horizon. We used the same testbed presented in [1].

For the experiments, we use as a comparison the branch-and-bound (BnB) technique presented in [1]. This is the only $(R, s, S)$ solver for this problem configuration available in the literature. The solvers used are:**BnB-Guided** the branch-and-bound approach presented in [1], **SDP** the basic implementation of the SDP heuristic model, and **SDP-Opt**, the heuristic implementation deployed using the K-convexity property and the immediate cost memoisation.

Figure 1 shows the logarithm of the average computational timee. The simple implementation of the heuristic can barely solve tiny instances before the time limit,

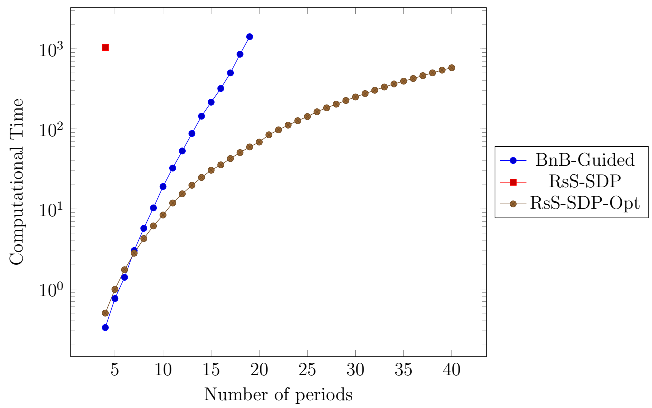


Figure 1: Computational time of the $(R, s, S)$ SDP over the number of periods.

making it useless for every practical use. The reduction of computational effort provided by K-convexity and memoisation is massive. The guided BnB slightly outperforms the optimised SDP for small instances up to 8 periods, then the gap between the two strongly increases, making it able to solve instances more than twice as big in the same amount of time. The memoisation offers a great speed up in the computational times, which is more significant in bigger instances. For bigger instances, the physical memory needed grows to require the usage of memory swap and a slow down in performances.

In this testbed, the heuristic always computes the optimal replenishment plan.

# 4 Conclusions

This paper presented a heuristic for the non-stationary stochastic lot-sizing problem with ordering, review, holding and penalty cost, a well-known and widely used inventory control problem. Computing $(R, s, S)$ policy parameters is computationally hard due to the three sets of parameters that must be jointly optimised. We presented the first pure SDP formulation for such a problem. The algorithm introduced solves to optimality a relaxation of the original problem, in which review cycles are considered independently, and items can be returned/discarded at no additional cost.

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# Optimal policies for inventory systems with limited number of replenishments

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## 1 Introduction

In this paper, we study the problem of managing inventories where a firm must match supply to non-stationary and stochastic demands over a finite planning horizon, subject to a limit on the number of replenishment orders that can be placed. The firm must decide in each period whether and how much to order while considering the effect of this decision on what can be ordered in future periods. The problem stands apart from classical inventory control problems in how economies of scale in replenishment is embedded into the inventory system. In traditional inventory systems, economies of scale is due to fixed ordering costs. These are imposed on each replenishment independently. In the system under consideration, economies of scale is due to the upper bound on the number of replenishments. This necessitates allocating replenishments dynamically over time which creates an interdependency among replenishment periods.

We are motivated by practical settings where firms face limits on the number of replenishments over some compliance period due to contractual agreements. In particular, our study emanates from a contractual agreement between a university hospital and its supplier. The hospital establishes a procurement contract with a supplier using a tendering procedure and the supplier who wins the tender supplies the hospital by contract for a fixed period of time. The contract places all logistics costs on the supplier and specifies the number of replenishment orders that can be placed over the contract period. The hospital is then faced with the challenge of matching supply to demand, while rationing the specified number of contract-based replenishments over time. To do this optimally, one needs to employ a dynamic policy where replenishment decisions are made considering the number of remaining contract-based replenishments and the time until the end of the contract period. This approach may lead to significant cost savings as compared to rudimentary approaches such as issuing replenishment orders over equally-sized intervals within the contract period—which is indeed the hospital's practice.

Contractual agreements with fixed number of replenishments are also relevant in industries where products have short selling seasons. This is particularly the case in the apparel industry. The majority of apparel companies make their procurements ahead of the selling season as apparel suppliers favor producing in large quantities. However, it is well-known that in-season orders significantly reduce inventory costs. This provides an incentive to establish agreements that allow for a few replenishment orders across the selling season as a middle ground (see e.g. Li et al. 2009, Chen et al. 2016). The problem also reflects on systems with self-imposed limits on the number of replenishments, rather than contractual agreements. This is particularly relevant in the context of managing carbon emissions. It is well-known that a considerable share of carbon emissions is due to transportation, wherein the amount of emissions is largely determined by the frequency of shipments (Marklund and Berling 2017). Therefore, firms can effectively reduce their emissions by imposing limits on the number of replenishments which is a less-costly operational adjustment in comparison to conventional capital intensive initiatives (Tang et al. 2015).

# 2 Related literature

The literature on inventory systems with limitations on the number of replenishments is fairly limited and mainly concentrates on products with short selling seasons. The majority of the contributions in this line of research consider the case where the buyer has two procurement opportunities: an initial order before and another one within the selling season. The associated problem is to decide upon the replenishment quantities for these two procurement opportunities. This can be regarded as an extension of the classical newsvendor model where the buyer can capitalize the information that becomes available after the initial order through a second order opportunity. Different variants of the problem have been addressed in the literature. These involve models where the timing of the second order is known in advance (see e.g. Eppen and Iyer 1997, Donohue 2000, Jones et al. 2001, Fisher et al. 2001) and determined dynamically within the selling season (see e.g. Milner and Kouvelis 2002, 2005, Li et al. 2009). The main focus of these studies is the effect of information updates and supplier flexibility on the system performance. Li et al. (2009) is an exception in this regard as they also provide conditions under which the optimal ordering policy is well-behaved. Chen et al. (2016) considered a model which allows for multiple shipments over the selling season and developed mechanisms to facilitate supply chain coordination. However, they assume that replenishment periods and associated replenishment quantities are all fixed in advance of the selling season. In this study, we extend the aforementioned models by allowing multiple replenishment opportunities that can be exercised any time over a planning horizon, yet we focus on the characterization of the optimal inventory policy and do not focus on industry-specific considerations such as information updates and supply chain coordination.

# 3 Contributions

The contributions of our study can be summarized as follows. We model the problem as a two-dimensional stochastic dynamic program analyze its structural properties. Because the functional properties often used in the inventory control literature are not sufficient to capture the behaviour of the cost function associated with this stochastic dynamic program, we introduce a new concept which we refer to as "convex dominance". In contrast to existing functional properties, convex dominance defines a relationship between two real-valued functions, and, when applied to the same function, generalizes well-known properties such as convexity and $K$-convexity. While convex dominance is tailored for the purposes of the current study, it is a valuable concept for its own sake and has the potential to be useful in establishing structural results on a larger set of problems. We make use of convex dominance to establish a structural relationship between cost functions associated with different numbers of replenishments. This allows us to characterize the optimal replenishment decision for any period, provided that cost functions admit convex dominance. Then, we inductively prove that convex dominance is preserved over the periods in the planning horizon. Our analysis reveals that the optimal inventory policy is specified by dynamic re-order and order-up-to levels that depend on the remaining number of replenishments. We also provide bounds on optimal re-order and order-up-to levels, building on two specific cases of the problem.

Even though we fully characterize the structure of the optimal policy, finding the optimal policy parameters remains a computational challenge. To that end, we develop a simple method to approximate the cost functions associated with different numbers of replenishments. The approximate cost function immediately translates into an efficient computational approach by which policy parameters can be computed heuristically. We assess the performance of the heuristic against the optimal policy and show that it performs close to optimal.

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