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Integrated production planning and preventive maintenance in deteriorating production systems

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ABSTRACT

This paper discusses the issue of integrating production planning and preventive maintenance in manufacturing production systems. In particular, it tackles the problem of integrating production and preventive maintenance in a system composed of parallel failure-prone production lines. It is assumed that when a production line fails, a minimal repair is carried out to restore it to an 'as-bad-as-old' status. Preventive maintenance is carried out, periodically at the discretion of the decision maker, to restore the production line to an 'as-good-as-new' status. It is also assumed that any maintenance action, performed on a production line in a given period, reduces the available production capacity on the line during that period. The resulting integrated production and maintenance planning problem is modeled as a nonlinear mixed-integer program when each production line implements a cyclic preventive maintenance policy. When noncyclical preventive maintenance policies are allowed, the problem is modeled as a linear mixed-integer program. A Lagrangian-based heuristic procedure for the solution of the first planning model is proposed and discussed. Computational experiments are carried out to analyze the performance of the method for different failure rate distributions, and the obtained results are discussed in detail.

1. Introduction

To cope with the current tough competition many manufacturing companies have invested in highly automated production systems with sophisticated equipments. To be economically sustainable, these costly equipments should be exploited to the last instant of their maximum possible productive time. When an unplanned downtime, caused by a production line failure, occurs it often trims down the system's productivity and renders the current production plan obsolete. Revising the production plan in an emergency situation is usually very expensive and often causes increased variability in product quality and in service level. It is, therefore, essential that production planning and preventive maintenance activities be carried out in an integrated way to hedge against these often avoidable failures and re-planning occurrences. This paper proposes and discusses models to generate such integrated production and maintenance plans which aim at achieving an optimal trade-off between the various production and maintenance costs.

The issue of integrating production planning and preventive maintenance, in failure-prone manufacturing systems, is becoming an active area of research due to its importance in the current highly competitive environment. However, it is frequently tackled at the operational (the scheduling) level. In particular, scheduling problems on unreliable machines have received a considerable attention in the literature [1,6,11,12,17,18]. A large number of these scheduling problems assume that

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maintenance periods are known in advance, and thus reduce to scheduling problems with machine availability constraints [1,15,19]. A recent survey on scheduling problems with limited machine availability can be found in [20]. A general result is that these scheduling problems, with availability constraints, under different machine configurations and various objective functions are NP-hard [13,16]. Various heuristics as well as exact methods for their solution are proposed [3,8].

At the tactical level, there are only few papers discussing this issue. Wienstein and Chung [23] presented a three-part model to resolve the conflicting objectives of system reliability and profit maximization. An aggregate production plan is first generated, and then a master production schedule is developed to minimize the weighted deviations from the specified aggregate production goals. Finally, work center loading requirements, determined through rough cut capacity planning, are used to simulate equipment failures during the aggregate planning horizon. Several experiments are used to test the significance of various factors for maintenance policy selection. These factors include the category of maintenance activity, maintenance activity frequency, failure significance, maintenance activity cost, and aggregate production policy. Aghezzaf et al. [4] presented an integrated production and preventive maintenance planning model for a single-line production systems which can be minimal repaired at failure. They assumed that maintenance actions carried out on the production line reduce its capacity, and proposed a mathematical programming model to establish an optimal integrated production and maintenance plan for the single-line production systems.

In Section 2, a mathematical programming model for the integrated production and maintenance problem in production systems implementing cyclic preventive maintenance policies is proposed. In Section 3, a heuristic solution algorithm for the solution of the proposed model is proposed and discussed. Section 4 presents an integrated production and maintenance planning model for systems implementing general preventive maintenance policies. The last section presents some computational experiment to analyze the performances of the proposed models and their algorithms. Finally, some concluding remarks are presented in Section 6.

2. Integrated production planning and cyclic preventive maintenance

Consider a planning horizon H of length $T = N\tau$ covering N periods of fixed length τ, and a set of items $i \in P$ to be produced on a set of capacitated parallel production lines $j \in L$ during this planning horizon. Each production line $j \in L$ of the system produces each item $i \in P$ at a known production rate, expressed in product units per unit of time. During each period $t \in H$, a demand d_{it} of item $i \in P$ should be satisfied. It is assumed that each production line $j \in L$ has a known nominal capacity (given in time units) and is denoted by κ_j. Each preventive or corrective maintenance action performed on the production line consumes a certain percentage of this capacity. Thus, each preventive maintenance action and each corrective maintenance action on production line $j \in L$ consume, respectively $\theta_j^p = \alpha\kappa_j$ and $\theta_j^r = \beta\kappa_j$ capacity units (where $0 \leqslant \alpha \leqslant 1$ and $0 \leqslant \beta \leqslant 1$). It is also assumed that the failure distribution of each production line $j \in L$ is known. Let $f_j(t)$ and $F_j(t)$ denote its corresponding probability density and cumulative distribution functions, respectively. Let $r_j(t)$ denotes the failure rate function of the production line $j \in L$ at time t. It is well known that $r_j(t)$ is given by

$$r_j(t) = \frac{f_j(t)}{1 - F_j(t)}.$$

Finally, assume that preventive maintenance tasks are performed periodically on each production line $j \in L$, in the beginning of periods $t = 1, (n_j + 1), (2n_j + 1), (3n_j + 1), \ldots, N$, that corrective maintenance tasks are carried out on the production line when a failure occurs, and that any maintenance action is completed within the period in which it started. Fig. 1 shows an example of an integrated production and cyclic preventive maintenance plan and also the preventive maintenance cycle

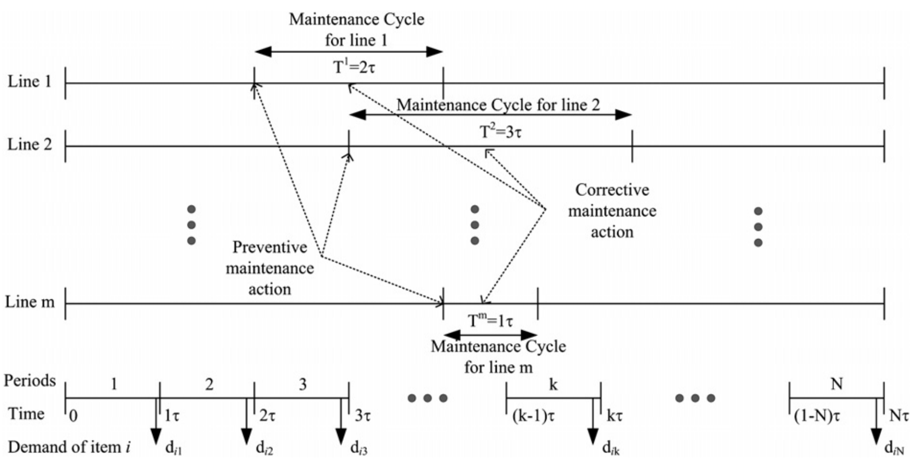

Fig. 1. An integrated production and cyclic preventive maintenance plan.

for each production line. The parameters T^1, T^2, and T^m shown in Fig. 1 are maintenance cycles of the corresponding lines.

2.1. A production and cyclic preventive maintenance planning model

The proposed mathematical program here models the problem of determining optimal integrated production and cyclic preventive maintenance plans in a multi-line production system. It assumes that when a production line fails, a minimal repair is carried out to restore it to an 'as-bad-as-old' status. When a preventive maintenance is carried out, the production line is restored to an 'as-good-as-new' status. It also assumes that the expected failures increase with elapsed time since the last preventive maintenance.

To define the parameters of the model, let c_j^p be the cost of preventively maintaining production line $j \in L$ in the beginning of each maintenance cycle, and c_j^r be the cost of performing a corrective maintenance action on this line when a failure occurs. Also, let f_{it}^j be the setup cost of producing item $i \in P$ in period $t \in H$ on production line $j \in L, p_{it}^j$ be the variable cost of producing item $i \in P$ in period $t \in H$ on production line $j \in L, h_{it}^j$ be the inventory holding cost of item $i \in P$ in period $t \in H$, and ρ_{ij} be the processing time of item $i \in P$ on production line $j \in L$ (expressed in time units per item). For the variables of the model, let x_{it}^j be the quantity of item $i \in P$ produced on line $j \in L$ during period $t \in H, I_{it}$ be the inventory of item $i \in P$ at the end of period $t \in H, T^j = n_j\tau$ be the length of the preventive maintenance cycle for production line $j \in L$, and y_{it}^j be a binary decision variable set to 1 if item $i \in P$ is produced on line $j \in L$ during period $t \in H$ and 0 otherwise.

2.1.1. The mathematical model

$$(\text{PCPM}) \quad \text{Minimize} \quad Z_{\text{PM}} = \sum_{t\in H}\sum_{i\in P}\left(\sum_{j\in L}(f_{it}^j y_{it}^j + p_{it}^j x_{it}^j) + h_{it}I_{it}\right) + \sum_{j\in L}\lceil T/T^j\rceil\left(c_j^p + c_j^r\int_0^{T^j} r_j(t)\mathrm{d}t\right)$$

$$\text{subject to} \quad \sum_{j\in L} x_{it}^j + I_{i,t-1} - I_{it} = d_{it} \quad \text{for all } i \in P,\ t \in H, \tag{1}$$

$$\sum_{i\in P}\rho_{ij}x_{it}^j \leqslant \kappa_j(t) \quad \text{for all } j \in L,\ t \in H, \tag{2}$$

$$x_{it}^j - d_{tN}^i y_{it}^j \leqslant 0 \quad \text{for all } i \in P,\ j \in L,\ t \in H, \tag{3}$$

$$\sum_{i\in P} y_{it}^j \leqslant 1 \quad \text{for all } j \in L,\ t \in H, \tag{4}$$

$$x_{it}^j, I_{it} \geqslant 0, \quad y_{it}^j \in \{0,1\}, \quad T^j \in \{\tau, 2\tau, \ldots, N\tau\} \quad \text{for all } i \in P,\ j \in L,\ t \in H,$$

where $d_{tN}^i = \sum_{s=t}^N d_{is}$, and the function $\kappa_j(t)$ defines the available capacity of the production line $j \in L$ in period $t \in H$ as is explained below.

Constraints (1) are the flow conservation constraints defined for any item $i \in P$ and any period $t \in H$. They guarantee that the available inventory of an item augmented with the quantity produced, on each production line, of that item is sufficient to satisfy its demand in period t. The remainder is stocked for subsequent periods. Constraints (2) are the capacity restrictions defined for each production line $j \in L$ in each period $t \in H$. They guarantee that the quantity produced on a production line does not exceed the available capacity of that line in period t. Constraints (3) force payments of the fixed costs, in any period $t \in H$, for any item $i \in P$ that is produced during that period. Constraints (4) ensure that at any moment no more than one product is scheduled on a production line.

It is time now to define exactly how the available capacity function of each production line in each period is determined. Consider a production line $j \in L$, and assume that its preventive maintenance cycle is given by $T^j = n_j\tau$. Assume also that a maintenance action on this line took place in the beginning of a period $s \in H$, and that it covers the periods $\{s, s+1, \ldots, (s+n_j-1)\}$. The available capacity of the production line $j \in L$ in period s is then given by equation:

$$\kappa_j(s) = \kappa_j - \theta_j^p - \theta_j^r\int_0^\tau r_j(u)\mathrm{d}u. \tag{5}$$

The capacity $\kappa_j(t)$ for the periods $t \in \{s+1, \ldots, (s+n_j-1)\}$, during which only the necessary corrective maintenance actions take place, is given by equation:

$$\kappa_j(t) = \kappa_j - \theta_j^r\int_0^\tau r_j(u + (t-s)\tau)\mathrm{d}u. \tag{6}$$

Notice that when preventive maintenance periods are known in advance, the available capacity of each production line, during each period, can be determined, and that the expected maintenance cost can be computed in advance. In this case, the problem (PCPM) reduces to the usual multi-item multi-line capacitated lot-sizing problem. Therefore, one can conclude that the problem (PCPM) belongs to the class of NP-hard problems since it contains, as a special case, the single-line capacitated lot-sizing problem known to be NP-hard [7].

2.1.2. An exact algorithm for problem (PCPM)

The variables $(T^j)_{j\in L}$ defining the preventive maintenance cycles, for the production lines of the system, make the objective function of the problem (PCPM) to become a nonlinear function. If these preventive maintenance cycles are known in advanced the problem reduces to a multi-item capacitated multi-line lot-sizing problem. This latter problem can be solved with any usual linear mixed integer solver like Cplex. Alternatively, many branch and bound, branch and cut, and branch and price exact methods as well as many heuristic methods were proposed to solve the single-line version of this problem [2,5,9,10,14,22]. A recent survey on the multi-item capacitated lot-sizing problem can be found in [24]. The following steps describe an algorithm to solve the problem (PCPM):

- For each vector of preventive maintenance cycles $(n_1,\dots,n_j,\dots,n_{|L|})$ of the production lines $j\in L$, with $n_j\in\{1,2,\dots,N\}$, perform the following two steps:
  1. Compute the expected maintenance cost corresponding to the selected vector of maintenance cycles, and determine for each production line $j\in L$ the available capacity $\kappa_j(t)$ in each period $t\in H$.
  2. Solve the resulting pure production planning problem, with the above computed available capacities $(\kappa_j(t)_{t\in H})_{j\in L}$, and add together the resulting expected maintenance and production costs.
- Rank these total costs and select the plan resulting in the least cost, this is the optimal integrated production and cyclical preventive maintenance plan.

The number of possible combinations of the preventive maintenance cycle sizes is exponential of order $N^{|L|}$, where N is the number of periods in the planning horizon as defined above. Now, observe that when the number of production lines is very small (2 or 3), then the complexity of the above procedure depends actually on the complexity of the algorithm used to solve the pure capacitated lot-sizing problems.

The following section discusses a heuristic solution method, based on Lagrangian relaxation, for the problem (PCPM). Since the single-line production and cyclic preventive maintenance planning problem plays a central role in this procedure, a detailed procedure for solving this latter problem efficiently is given below.

2.2. Single-line integrated production and cyclic preventive maintenance planning model

Consider the problem of planning production and cyclic preventive maintenance on a single production line $j\in L$, denoted by $(\text{PCPM}^j_{\text{SL}})$. Assume that the demand fractions of items $i\in P$, which are to be produced on the production line j during each period $t\in H$, are known in advanced and are denoted by δ^j_{it}. Let I^j_{it} be new variables defining the proportion of inventory of item $i\in P$ at the end of period $t\in H$ resulting from the production on line $j\in L$. Assume as before that $T^j=n_j\tau$ denotes the length of the preventive maintenance cycle for the production line $j\in L$. The problem $(\text{PCPM}^j_{\text{SL}})$ can now be rewritten as follows:

$$\text{PCPM}^j_{\text{SL}}\quad \text{Minimize}\quad Z^j_{\text{PM}}=\sum_{t\in H}\sum_{i\in P}(f^j_{it}y^j_{it}+p^j_{it}x^j_{it}+h_{it}I^j_{it})+\Phi^j_{\text{PM}}(n_j)$$

$$\text{subject to}\quad x^j_{it}+I^j_{i,t-1}-I^j_{it}=\delta^j_{it}\quad \text{for all } i\in P,\ j\in L,\ t\in H, \tag{7}$$

$$\sum_{i\in P}\rho_{ij}x^j_{it}\leqslant\kappa_j(t),\quad \text{for all } j\in L,\ t\in H, \tag{8}$$

$$x^j_{it}-d^i_{tN}y^j_{it}\leqslant 0\quad \text{for all } i\in P,\ j\in L,\ t\in H, \tag{9}$$

$$\sum_{i\in P}y^j_{it}\leqslant 1\quad \text{for all } j\in L,\ t\in H, \tag{10}$$

$$x^j_{it},I^j_{it}\geqslant 0,\ y^j_{it}\in\{0,1\},\ n_j\in\{1,2,\dots,N\}\quad \text{for all } i\in P,\ t\in H,$$

where the available capacities are given by $\kappa_j(t)=\kappa_j-\theta^p_j-\theta^r_j\int_0^\tau r_j(u)\mathrm{d}u$ if $t=(n-1)n_j+1$, and $\kappa_j(t)=\kappa_j-\theta^r_j\int_0^\tau r_j(u+(t-((n-1)n_j+1))\tau)\mathrm{d}u$ if $(n-1)n_j+1\leqslant t\leqslant nn_j$, for $1\leqslant n\leqslant\lceil N/n_j\rceil$ and $(n-1)n_j+1\leqslant t\leqslant nn_j$ with $t\leqslant N$. The maintenance cost $\Phi^j_{\text{PM}}(n_j)$ is given by

$$\Phi^j_{\text{PM}}(n_j)=\sum_{n=1}^{\lceil N/n_j\rceil}\left(c^p_j+\sum_{t=(n-1)n_j+1,t\leqslant N}^{nn_j}\left(c^r_j\int_0^\tau r_j(u+(t-(n-1)n_j-1)\tau)\mathrm{d}u\right)\right).$$

It must be clear by now that the decision variables in this last model are the usual production variables x^j_{it}, y^j_{it}, I^j_{it}, and the variable n_j which defines the optimal size of the preventive maintenance cycle. To determine the optimal values of production plan and the size of the maintenance period, one may proceed as follows.

2.2.1. A single production line algorithm

For each preventive maintenance cycle n_j = 1 to N, corresponding respectively to the cases of one preventive maintenance in the beginning of each production period, and to the case of one preventive maintenance in beginning of the whole planning horizon.

1. For each preventive maintenance cycle of size n_j, determine the corresponding maintenance cost function $\Phi^j_{\mathrm{PM}}(n_j)$, then determine for each period $t \in H$ the available capacity $\kappa^{n_j}_j(t)$.
2. Solve the resulting pure production planning problem $(\mathrm{PCPM}^j_{\mathrm{SL}}(n_j))$, for this fixed cycle of size n_j, to obtain the optimal value $\widehat{Z}^j_{\mathrm{PM}}(n_j)$ using any efficient specialized algorithm, or a usual linear mixed integer solver.
3. Compare the resulting values $\widehat{Z}^j_{\mathrm{PM}}(n_j)$ for $n_j = 1$ to N. The value of $\hat{n}_j$ that results in the least value of $\widehat{Z}^j_{\mathrm{PM}}(n_j)$ is chosen as the optimal preventive maintenance cycle size for the production line, and the optimal production plan resulting from the solution of $(\mathrm{PCPM}^j_{\mathrm{SL}}(\hat{n}_j))$, is chosen as the optimal production plan for the line.

To take advantage of this single-line production and cyclic preventive maintenance algorithm, a heuristic solution method for the problem (PCPM), based Lagrangian relaxation decomposition, is proposed in the following section.

3. A Lagrangian relaxation based approximation procedure for (PCPM)

The mathematical program (PCPM) models the integrated production and preventive maintenance planning problem on parallel production lines implementing cyclic preventive maintenance policies. To take advantage of the properties of the single-line version of the problem, the problem (PCPM) is reformulated to make these single-line versions appear as sub-problems. Two new sets of variables δ^j_{it} and η^j_{it} are introduced, both defining the fraction of demand of item $i \in P$ produced on production line $j \in L$ in period $t \in H$, (i.e. $\delta^j_{it} = \eta^j_{it}$). The new variables I^j_{it}, defining the proportion of inventory of item $i \in P$ at the end of period $t \in H$ resulting from production on line $j \in L$, are also introduced. The resulting model becomes then.

3.1. A reformulated mathematical model (PCPM_R)

$$(\mathrm{PCPM}_R) \quad \text{Minimize} \quad Z_{\mathrm{PM}} = \sum_{j\in L}\left(\sum_{t\in H}\sum_{i\in P}(f^j_{it}y^j_{it} + p^j_{it}x^j_{it} + h_{it}I^j_{it}) + \lceil N/n_j \rceil \left(c^p_j + c^r_j \int_0^{n_j\tau} r_j(t)\mathrm{d}t\right)\right)$$

$$\text{subject to} \quad x^j_{it} + I^j_{i,t-1} - I^j_{it} = \delta^j_{it} \quad \text{for all } i \in P, j \in L, t \in H, \tag{11}$$

$$\sum_{i\in P}\rho_{ij}x^j_{it} \leqslant \kappa_j(t) \quad \text{for all } j \in L,\ t \in H, \tag{12}$$

$$x^j_{it} - d^i_{tN}y^j_{it} \leqslant 0 \quad \text{for all } i \in P,\ j \in L,\ t \in H, \tag{13}$$

$$\sum_{i\in P} y^j_{it} \leqslant 1 \quad \text{for all } j \in L,\ t \in H, \tag{14}$$

$$\delta^j_{it} - \eta^j_{it} = 0 \quad \text{for all } i \in P,\ j \in L,\ t \in H, \tag{15}$$

$$\sum_{j\in L}\eta^j_{it} = d_{it} \quad \text{for all } i \in P,\ t \in H, \tag{16}$$

$$x^j_{it}, I^j_{it}, \delta^j_{it}, \eta^j_{it} \geqslant 0,\ \ y^j_{it} \in \{0,1\},\ \ n_j \in \{1,2,\ldots,N\} \quad \text{for all } i \in P,\ j \in L,\ t \in H.$$

Constraints (11) are the new flow conservation constraints defined for each production line $j \in L$. Constraints (15) and (16) guarantee, for each item $i \in P$ and each period $t \in H$, that the sum of the quantities shipped from inventories accumulated by the production lines $j \in L$ are sufficient to cover the demand of item $i \in P$ in period $t \in H$.

Examining carefully the problem (PCPM_R), one can observe that if constraints (15) are relaxed, the problem splits up into $(|L| + 1)$ sub-problems. The first $|L|$ sub-problems consist of planning production and cyclical preventive maintenance on each single production line $j \in L$ for a given demand vector δ^j. The last sub-problem is a simple linear program, which assigns the demands to be produced to the production lines with the objective of minimizing the marginal production costs of each production line.

Now, let λ^j_{it} be the Lagrangian multipliers associated with constraints (15). The resulting sub-problem for each production line $j \in L$ is given by

$$\mathrm{PCPM}^j_{\mathrm{LR}}(\lambda): \quad \text{Minimize} \quad V^j_{\mathrm{PM}}(\lambda) = Z^j_{\mathrm{PM}} - \sum_{t\in H}\sum_{i\in P}\lambda^j_{it}\delta^j_{it}$$

$$\text{subject to} \quad x^j_{it} + I^j_{i,t-1} - I^j_{it} = \delta^j_{it} \quad \text{for all } i \in P,\ j \in L,\ t \in H, \tag{17}$$

$$\sum_{i\in P}\rho_{ij}x^j_{it} \leqslant \kappa_j(t) \quad \text{for all } j \in L,\ t \in H, \tag{18}$$

$$x^j_{it} - d^i_{tN}y^j_{it} \leqslant 0 \quad \text{for all } i \in P,\ j \in L,\ t \in H, \tag{19}$$

$$\sum_{i\in P} y^j_{it} \leqslant 1 \quad \text{for all } j \in L,\ t \in H, \tag{20}$$

$$x^j_{it}, I^j_{it}, \delta^j_{it} \geqslant 0,\ \ y^j_{it} \in \{0,1\},\ \ n_j \in \{1,2,\ldots,N\} \quad \text{for all } i \in P,\ \ j \in L,\ \ t \in H,$$

where $\kappa_j(t)$ is the available capacity of the production line $j \in L$ in period $t \in H$, defined by Eqs. (5) and (6), and Z^j_{PM} given by

$$Z^j_{\mathrm{PM}} = \sum_{t\in H}\sum_{i\in P}(f^j_{it}y^j_{it} + p^j_{it}x^j_{it} + h_{it}I^j_{it}) + \lceil N/n_j \rceil \left(c^p_j + c^r_j \int_0^{n_j\tau} r_j(t)\mathrm{d}t\right).$$

As already mentioned, observe that if the vector δ^j is given, then the quantity $\sum_{t\in H}\sum_{i\in P}\lambda^j_{it}\delta^j_{it}$ in the objective function of $(\mathrm{PCPM}^j_R(\lambda))$ becomes a constant. The resulting problem reduces then to the problem of planning production and cyclical preventive maintenance on a single production line, in which δ^j_{it} are the demands for each item $i \in P$ in each period $t \in H$.

The last sub-problem $(\mathrm{DASP}(\lambda))$ determines, for each item $i \in P$ in each period $t \in H$, the best assignment of the demand d_{it} to the production lines $j \in L$, and is given bt

$$\begin{aligned} \mathrm{DASP}(\lambda):\quad \text{Minimize}\quad & V^0_{\mathrm{PM}}(\lambda) = \sum_{j\in L}\sum_{t\in H}\sum_{i\in P}\lambda^j_{it}\eta^j_{it} \\ \text{subject to}\quad & \sum_{j\in L}\eta^j_{it} = d_{it} \quad \text{for all } i \in P,\ \ t \in H, \\ & \eta^j_{it} \geqslant 0 \quad \text{for all } i \in P,\ \ j \in L,\ \ t \in H. \end{aligned} \tag{21}$$

If $\widehat{V}^j_{\mathrm{PM}}(\lambda)$ and $\widehat{V}^0_{\mathrm{PM}}(\lambda)$ are, respectively, the optimal values of the problems $(\mathrm{PCPM}^j_{\mathrm{LR}}(\lambda))$ and $(\mathrm{DASP}(\lambda))$ for a given $\lambda \in \mathbf{R}$, then the Lagrangian dual associated with the above relaxed problem is given by

$$W_{LD} = \underset{\lambda\in\mathbf{R}}{\text{Maximize}}\left(\sum_{j\in L}\widehat{V}^j_{\mathrm{PM}}(\lambda) + \widehat{V}^0_{\mathrm{PM}}(\lambda)\right).$$

In the following sections, a heuristic procedure to solve the problem (PCPM) using the sub-problems $(\mathrm{PCPM}^j_{\mathrm{LR}}(\lambda))$ and $(\mathrm{DASP}(\lambda))$ is presented and discussed. The value obtained with this heuristic procedure is then compared to a very good lower bound on the optimal solution of the problem to evaluate and analyze the optimality gap.

3.2. A Lagrangian-based solution procedure for the problem (PCPM)

To launch the solution process explained below, one needs to start with a first assignment of demands to the production lines. One may start with an assignment of demands to the production lines in proportions of their processing rates (fair share):

$$\delta^j_{it} = \frac{\rho_{ij}}{\sum_{j'\in L}\rho_{ij'}}d_{it} \quad \text{for all } i \in I,\ \ t \in H,\ \ j \in L.$$

Step 1: Solve the linear programming relaxations of the problems $(\mathrm{PCPM}^j_{\mathrm{LR}})$ obtained from the problems $(\mathrm{PCPM}^j_{\mathrm{LR}}(\lambda))$ by letting $\lambda = 0$ and δ^j as defined above. Use the single production line algorithm as described above but where the pure production planning problems $(\mathrm{PCPM}^j_{\mathrm{SL}}(n_j))$, for fixed values of n_j, are solved as linear programs (LP-relaxation).

Step 2: Get the shadow prices λ^j_{it} associated with the constraints (17) for the optimal problem $(\mathrm{PCPM}^j_{\mathrm{SL}})$, that is, the optimal $(\mathrm{PCPM}^j_{\mathrm{SL}}(\hat{n}_j))$. To assure convergence, after a given fixed number of iterations, multipliers obtained by the usual subgradient procedure are added to these shadow prices, component by component.

Step 3: Using these shadow prices λ^j_{it} solve the problem $(\mathrm{DASP}(\lambda))$ and get the new values for δ^j. These values are used in step 1 to get the new shadow prices as explained in step 2, and the process is continued until the values of the problems $(\mathrm{PCPM}^j_{\mathrm{LR}})$ are not changing.

Step 4: Once the final values of δ^j are obtained, then for each line one may use the single production line algorithm described above, but now the problems $(\mathrm{PCPM}^j_{\mathrm{SL}})$ are solved as integer linear programs. The collection of optimal plans for each production line $j \in L$ is taken as the best approximate to the optimal solution for the problem PCPM.

3.3. An illustrative example

Consider a production system composed of two identical production lines each with a maximal capacity $\kappa_1 = \kappa_2 = 15$. Two items are to be produced in lots on these two lines during a planning horizon of eight periods. The setup, production and holding costs on each line of each item in each period are respectively 25, 5, and 2. It is also assumed that $\rho_{ij} = 1$ for all $i \in P$ and $j \in L$. Table 1 shows the periodic demands of each item in each period.

Assume that the cost of any preventive maintenance action and the costs of any corrective maintenance action on any of the two production lines are given, respectively by $c^p_1 = c^p_2 = 40$ and $c^r_1 = c^r_2 = 35$. Also, assume that the two production lines

Table 1
Products' periodic demands

Periods	Demand of product 1	Demand of product 2
1	7	6
2	6	4
3	7	4
4	6	4
5	4	6
6	6	4
7	7	4
8	6	4

Table 2
Expected number of failures

Age	Expected number of failures
$[0, 1\tau[$	0.901
$[1\tau, 2\tau[$	1.489
$[2\tau, 3\tau[$	1.664
$[3\tau, 4\tau[$	1.749
$[4\tau, 5\tau[$	1.799
$[5\tau, 6\tau[$	1.833
$[6\tau, 7\tau[$	1.857
$[7\tau, 8\tau[$	1.875

are subject to random failures according to a Gamma distribution $\Gamma(m = 2, v = 2)$ with a shape parameter $m = 2$ and a rate parameter $v = 2$. Table 2 shows the expected number of failures as a function of the production line's age: Recall that the density function of a Gamma distribution $\Gamma(m, v)$ with a shape parameter m and a rate parameter v is given by

$$f(t) = \begin{cases} \frac{v^m t^{m-1} \exp^{(-vt)}}{\Gamma(m)} & \text{if } t, v \geqslant 0,\ \ m > 0, \\ 0 & \text{if } t < 0. \end{cases} \tag{22}$$

Finally, let $\theta_j^p = 1$ and $\theta_j^r = 5$ be respectively the amounts of capacity lost when a preventive maintenance action and an unplanned corrective maintenance action take place, respectively. Table 3 shows the available capacities in each period given as a function of the size of the preventive maintenance cycle $T = k\tau$.

The expected maintenance cost for each production line can now be obtained as indicated in Table 4, if the size of the preventive maintenance cycle is given.

To get the optimal solution of this problem 64 pure production planning problems are solved with a linear mixed integer solver. The problem is then solved with the Lagrangian relaxation based heuristic procedure as described above. The following are the summary results. The optimal value of the problem (PCPM) is given by $\widehat{Z}_{\mathrm{PM}} = 1735.89$ where the first production line is maintained after each three periods and the second production line maintained after each four periods. The Lagrangian-based

Table 3
Available capacities in each period as a function of the size of the preventive maintenance cycle $T = k\tau$

Periods	Available capacities as a function of $T = k\tau$							
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$
1	9.49	9.49	9.49	9.49	9.49	9.49	9.49	9.49
2	9.49	7.55	7.55	7.55	7.55	7.55	7.55	7.55
3	9.49	9.49	6.68	6.68	6.68	6.68	6.68	6.68
4	9.49	7.55	9.49	6.26	6.26	6.26	6.26	6.26
5	9.49	9.49	7.55	9.49	6.00	6.00	6.00	6.00
6	9.49	7.55	6.68	7.55	9.49	5.84	5.84	5.84
7	9.49	9.49	9.49	6.68	7.55	9.49	5.72	5.72
8	9.49	7.55	7.55	6.26	6.68	7.55	9.49	5.63

Table 4
Expected maintenance costs as a function of the size of the preventive maintenance cycle $T = k\tau$

Period	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$
Cost	572.39	494.68	487.46	486.19	487.97	493.90	506.77	500.84

approximation procedure provided the value $\widehat{Z}_{\mathrm{LR}} = 1770.09$ where the first production line is maintained after each four periods and the second line maintained after each three periods.

4. Production and general preventive maintenance planning

In many situations additional constraints, such as limited size and number of maintenance crews combined with the productivity requirement, can make it difficult to implement cyclical preventive maintenance strategies for all production lines of the systems. In this section, a model is presented that is more general in the sense that it relaxes the cyclic restriction, and can handle additional constraints. Before developing the model, some necessary additional parameters and variables should be defined.

Let $\kappa_s^j(t)$ be the expected loss in production capacity of line j, in period t, when the last preventive maintenance action before time t on the line j has taken place in period s, $s \leqslant t$. Thus, this expected loss is then given by

$$\kappa_s^j(t) = \begin{cases} \theta_p + \theta_r \int_0^\tau r_j(u)\mathrm{d}u & \text{if } t = s, \\ \theta_r \int_0^\tau r_j(u + (t-s)\tau)\mathrm{d}u & \text{if } t > s. \end{cases} \tag{23}$$

Let $c_s^j(t)$ be the expected maintenance cost of line j, in period t, when the last preventive maintenance action before time t on the line j has taken place in period $s, s \leqslant t$. Thus, this expected cost is then given by

$$c_s^j(t) = \begin{cases} c^p + c_r \int_0^\tau r_j(u)\mathrm{d}u & \text{if } t = s, \\ c_r \int_0^\tau r_j(u + (t-s)\tau)\mathrm{d}u & \text{if } t > s. \end{cases} \tag{24}$$

Finally, let z_{st}^j be a binary decision variable set to 1 if the preventive maintenance cycle on line j covering period t has started at period s, and 0 otherwise. The general integrated production and maintenance planning model (PGMP) is given by

$$\text{PGMP}: \quad \text{Minimize} \quad Z_{\mathrm{PM}}^G = \sum_{t\in H}\sum_{i\in I}\left(\sum_{j\in L}(f_{it}^j y_{it}^j + p_{it}^j x_{it}^j) + h_{it} I_{it}\right) + \sum_{t\in H}\sum_{s\in H, s\leqslant t} c_s^j(t) z_{st}^j$$

$$\text{subject to} \quad \sum_{j\in L} x_{it}^j + I_{i,t-1} - I_{it} = d_{it} \quad \text{for all } i \in P,\ t \in T, \tag{25}$$

$$\sum_{i\in I}\rho_{ij}x_{it}^j + \sum_{s\in H, s\leqslant t}\kappa_s^j(t) z_{st}^j \leqslant \kappa_j \quad \text{for all } j \in L,\ t \in T, \tag{26}$$

$$x_{it}^j - d_{tN}^i y_{it}^j \leqslant 0 \quad \text{for all } i \in P,\ j \in L,\ t \in T, \tag{27}$$

$$\sum_{i\in I} y_{it}^j \leqslant 1 \quad \text{for all } j \in L,\ t \in T \tag{28}$$

$$\sum_{s\in H, s\leqslant t} z_{st}^j = 1 \quad \text{for all } j \in L, t \in T, \tag{29}$$

$$z_{st}^j - z_{st-1}^j \leqslant 0 \quad \text{for all } j \in L,\ s \in T,\ t \in T,\ s < t, \tag{30}$$

$$x_{ij}^t, I_{it} \geqslant 0,\ y_{ij}^t,\ z_{st}^j \in \{0, 1\} \quad \text{for all } i \in P,\ j \in L, \text{ and } s, t \in H.$$

Constraints (26) are the new capacity constraints. These constraints define for each production line $j \in L$ the available capacity during each period. Constraints (29) determine the preventive maintenance periods for each production line. Constraints (30) guarantee, that if a period s is the last preventive maintenance period before $t > s$, then s is also the last preventive maintenance period before $t - 1$.

The advantage of this problem is that the objective function in not any more nonlinear, one can then solve it using any solver for linear mixed integer programs. In the following section, this model will be used to generate lower bounds for the test problems (PCPM) to assess the optimality gap for the heuristic procedure.

5. Design of experiment and computational results' analysis

To assess the quality of the solutions of the problem (PCPM) obtained with the Lagrangian relaxation based heuristic, a design of experiment based on some critical planning parameters which are thought likely to have a significant impact on these heuristic solutions is developed. Seven parameters are considered for the analysis: number of products, number of production lines, number of periods in the planning horizon, cost structure, demand structure, capacity tightness, and failure distribution of each production line. The design of experiment is structured in the same way as in [21].

The number of product $|P|$ is selected from $\{10, 25\}$, the number of periods N is selected from $\{12, 24\}$, and the number of production lines $|L|$ is selected randomly from $\{2, 4\}$. Failure distribution of each production line $j \in L$ is selected from Gamma distributions with a shape parameter 2 and rate parameters randomly selected from the set $\{1, 2\}$, and Weibull distributions with a shape parameter 2 and scale parameters randomly selected from the set $\{3, 4\}$.

The cost structure is assumed to have the following properties: the setup cost f_{it}^{j} for each product is selected randomly from two different intervals $[10, 50]$ for low setup cost problem and $[75, 100]$ for high setup cost problems, the production cost p_{it}^{j} is selected randomly from $[5, 10]$, and the holding cost is then defined as $h_{it} = \alpha\%\max_j(p_{it}^{j})$, where α is uniformly distributed between 5 and 20. The demand for each product $i \in P$ is assumed to have a stationary mean $\bar{d}_i$. The mean demands are randomly sampled from the interval $[75, 100]$, and the period-by-period demands for each product $i \in P$ are generated from a truncated normal distribution having mean $\bar{d}_i$ and a standard deviation randomly sampled from the interval $[0.25\bar{d}_i, 0.5\bar{d}_i]$.

The capacity is the last parameter set for each production line in each test problem. A target average utilization of capacity factor β is first introduced to define the capacity tightness. The factor β is set to 0.75, 0.85, and 0.95 corresponding respectively to situations with loose, moderately loose, and tight capacity constraints. The capacity required, in each period, is then computed as if a lot-for-lot solutions were implemented. These values are averaged over all periods and the result is divided by the number of production lines. The capacity of each production line in each period is then obtained by dividing the later result by the target average utilization of capacity factor β.

For each combination of number of products, number of production lines, number of periods, setup cost structure, and capacity tightness 15 test problems are generated. These 15 test problems are divided in three groups. A first group of five test problems is generated with Gamma failure distributions of all production lines, then a second group of five other test problems is generated with Weibull failure distributions of all production lines are Weibull distributions, and finally, a last group of five test problems is generated with failure distributions selected randomly from Gamma or a Weibull distributions (mixed). In this last group, test problems in which the failure distributions are all the same are discarded. This makes a total of 720 test problems. Each test problem is solved with the proposed Lagrangian-based solution procedure to obtain a very good upper bound $Z_{\mathrm{PM}}^{\mathrm{UB}}$ on the optimal value the problem (PCPM). The model (PGMP) is then solved for the same instance to get a lower bound on the optimal value the problem (PCPM). Finally, the gap is computed as follows:

$$\mathrm{Gap} = 100\frac{Z_{\mathrm{PM}}^{\mathrm{UB}} - Z_{\mathrm{PM}}^{G}}{Z_{\mathrm{PM}}^{\mathrm{UB}}}\%.$$

Table 5 shows the summary of the results of the averages of above ratios computer for each test group, and Table 6 presents the computational times required for the Lagrangian-based solution procedure. Notice that $G\text{-}^{*}$, $W\text{-}^{*}$, and $M\text{-}^{*}$ in the two tables mean, respectively, that the line has a Gamma failure distribution, a Weibull, or mixed (some have Gamma failure distributions and some others Weibull distribution).

The average solution gap ratios shown in Table 5 indicate that the tightness of the capacity constraint has a significant effect on the solution gap. Test problems with tight capacity constraints have much larger average gaps than those with loose

Table 5
Summary of solution gaps (mean)

Lines	Items	Periods	Capacity tightness					
			Loose Setup costs		Moderately loose		Tight	
			Low	High	Low	High	Low	High
G-2	10	12	2.034	6.842	3.840	13.273	15.609	16.653
G-2	10	24	2.132	7.171	4.025	13.910	16.358	17.452
G-2	25	12	2.294	7.719	4.332	14.973	17.608	18.786
G-2	25	24	1.553	5.226	2.933	10.137	11.921	12.718
G-4	10	12	2.250	7.570	4.249	14.685	17.270	18.425
G-4	10	24	1.590	5.348	3.002	10.375	12.201	13.017
G-4	25	12	1.658	5.577	3.130	10.818	12.722	13.573
G-4	25	24	1.924	6.474	3.634	12.558	14.768	15.756
W-2	10	12	1.272	4.279	2.402	8.301	9.762	10.415
W-2	10	24	1.752	5.893	3.308	11.432	13.444	14.343
W-2	25	12	1.215	4.089	2.295	7.932	9.328	9.952
W-2	25	24	1.561	5.250	2.947	10.185	11.977	12.779
W-4	10	12	1.933	6.502	3.650	12.613	14.833	15.825
W-4	10	24	1.701	5.721	3.211	11.098	13.050	13.923
W-4	25	12	1.663	5.593	3.140	10.851	12.760	13.614
W-4	25	24	1.368	4.603	2.583	8.929	10.500	11.202
M-2	10	12	2.039	6.858	3.850	13.304	15.646	16.692
M-2	10	24	1.681	5.656	3.175	10.972	12.903	13.766
M-2	25	12	1.506	5.066	2.844	9.828	11.557	12.330
M-2	25	24	1.331	4.476	2.512	8.683	10.211	10.894
M-4	10	12	1.862	6.263	3.515	12.149	14.287	15.243
M-4	10	24	1.773	5.965	3.348	11.571	13.607	14.518
M-4	25	12	1.781	5.991	3.363	11.621	13.666	14.581
M-4	25	24	1.425	4.793	2.690	9.298	10.934	11.665

Table 6
Summary of the heuristic computational times (min)

Lines	Items	Periods	Capacity tightness					
			Loose Setup costs		Moderately loose		Tight	
			Low	High	Low	High	Low	High
G-2	10	12	1.29	1.62	1.76	2.05	2.04	2.49
G-2	10	24	1.30	1.58	1.61	1.98	2.32	2.23
G-2	25	12	3.13	3.45	3.73	4.12	4.52	4.66
G-2	25	24	2.60	3.17	3.31	4.35	4.47	4.79
G-4	10	12	4.03	4.80	4.86	5.83	5.90	7.48
G-4	10	24	4.47	4.64	4.90	5.77	6.34	7.29
G-4	25	12	15.69	19.10	20.56	25.64	24.42	30.84
G-4	25	24	17.17	18.99	19.23	24.44	26.75	29.46
W-2	10	12	1.36	1.74	1.83	1.90	2.15	2.39
W-2	10	24	1.44	1.54	1.85	1.88	2.12	2.52
W-2	25	12	2.71	3.26	3.44	3.95	4.40	4.90
W-2	25	24	3.02	3.44	3.63	4.21	4.03	4.75
W-4	10	12	3.99	5.09	5.76	6.43	5.74	7.03
W-4	10	24	4.32	5.20	4.89	6.41	5.97	7.79
W-4	25	12	15.96	20.68	21.34	24.92	27.13	27.32
W-4	25	24	15.48	19.35	22.76	25.33	24.66	27.90
M-2	10	12	1.37	1.59	1.89	2.18	1.99	2.26
M-2	10	24	1.49	1.56	1.69	1.90	2.11	2.61
M-2	25	12	2.95	3.47	3.69	4.32	4.36	5.03
M-2	25	24	2.81	3.26	3.71	4.37	4.32	4.94
M-4	10	12	3.86	4.51	4.88	6.35	6.39	7.81
M-4	10	24	3.82	5.07	5.20	5.97	6.34	6.90
M-4	25	12	16.37	19.99	22.21	23.55	23.18	27.09
M-4	25	24	16.44	19.78	19.43	22.61	23.17	26.95

capacity. This gap varies from 2% for loose capacity problems up to 16% when capacity is tight. Actually, this result was expected since, for test problems with loose capacity, the two independently obtained optimal production and maintenance plans usually produce feasible near-optimal integrated plans for the problem (PCPM). These ratios show also that the effect of setup cost is significant. When comparing the low and high setup cost columns in Table 5, one can see that the differences vary between 4% and 8%. However, when capacity is tight these differences are not important. This is probably due to the fact that when the capacity is tight, production takes place more frequently making the impact of the setup cost less significant. Table 5 shows also that when Weibull distributions with scale parameters of 3 or 4 are used, the solution gaps become smaller than when Gamma distributions are used. This is also expected to some extent, since the expected capacity loss, given in function of the time elapsed since the last preventive maintenance, is smaller in case of a Weibull distribution with scale parameters of 3 or 4. Overall, the results obtained are fairly good in terms of optimality gaps.

Table 6 shows the computational times of the heuristic procedure. These times vary from 2 to 30 min for hard and large cases. This can be explained by the fact that Cplex is used a certain number of times during each iteration to solve pure production problems. Most probably, a more careful implementation of the procedure would reduce significantly these computational times.

Finally, notice that in the proposed Lagrangian-based solution procedure, the assignment of demands to the production lines is carried out on the basis of marginal costs obtained through the solutions of LP-relaxations of the single-line linear mixed integer sub-problems and corrected with subgradient-based multipliers. When no improvement is realized the last assignment of demands is kept and the sub-problem is solved as a mixed integer problem. The test problems indicate that a large part of the gaps resulted from non-optimal assignments of the demands to the lines. To improve this gap the Lagrangian dual of the problem (PCPM_R) in which the variables η are eliminated and constraints (16) are relaxed is currently investigated.

6. Conclusion

This paper investigated the problem of integrating production planning and preventive maintenance in multi-line production systems subject to failures. It is assumed that the capacities of these production lines progressively decline as they produce due deterioration. Two mathematical programming models for the problem are proposed. The first model assumes that each production line of the system implements a cyclic preventive maintenance policy with a fixed cycle size. The second model relaxes this last restriction and establishes for each production line specific maintenance periods, which do not necessarily fall at equally distant epochs. The first model is an NP-hard nonlinear mixed integer program, for which a heuristic method based on Lagrangian relaxation is developed. The sub-problems, used to iteratively construct an approximate optimal solution, are single-line versions of the problem, one for each production line, and then a last sub-problem is a linear program used to assign demands to the production lines. The LP-relaxations of these single-line sub-problems are solved to provide marginal production costs on each line, and these marginal costs combined with subgradient multipliers are used to assign the demands to the production lines. A series of test problems were generated and solved with the Lagrangian-based solution procedure. The optimality gaps were assessed using lower bounds obtained with the second model, which is a linear mixed integer program. These results show that procedure performs fairly well in terms of solution gap. Computational times are still high for some cases. Improving the implementation of the procedure to reduce these times is now carried out.

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