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Resumo: Este artigo examina os modelos de dimensionamento de lotes e sequenciamento com ênfase nos casos de mono-estágio. O objetivo aqui é apresentar os diferentes aspectos de tais modelos na área de pesquisa operacional e os métodos mais usuais para solucioná-los. Os métodos metaheuristicos consolidam uma abordagem na qual estes problemas estão sendo tratados na maioria das literaturas pesquisadas. Neste artigo o leitor encontrará muitas referências, porém resalva-se que a velocidade das publicações na área de inteligência artificial e pesquisa operacional tem aumentado significativamente. Entre as metaheuristicas que poderão ser encontradas tem-se Busca Tabu; *Simulated Annealing*; Algoritmos Geneticos; Algoritmos Memeticos; *Threshold Acceptance*; e métodos híbridos de Busca Local com Programação Inteira Mista.

Palavras-chave: Metaheuristicas, Dimensionamento de Lotes, e Sequenciamento.

Abstract: This paper surveys Lot-Sizing and Scheduling Models emphasising single-stage cases. The objective here is to present different aspects of such models in the operational research area and notes the most common modern methods to solve them. Metaheuristic methods feature heavily in the research literature. In this work the reader will find many references, though it must be noted that the speed with which the publications in the artificial intelligence and operational research areas is increasing significantly. Among the metaheuristics discussant are Taboo Search; Simulated Annealing; Genetic Algorithms; Memetic Algorithms; Threshold Acceptance and hybrid Local Search-Mixed Integer Programming methods.

Keywords: Lot-sizing, Scheduling, Mixed Integer Programming, Metaheuristics.

Introduction

This paper first presents definitions of Lot-Sizing and Scheduling and then presents a range of models later includes the problems. The reader will find many features of lot-sizing and scheduling problems and the manner in which they are treated by operational researchers and computer scientists.

There are many models, involving different features; e.g. the presence of single or multiple-machines. In the latter case these can be parallel machines (in a single stage), machines in sequence (i.e. multistage) or even parallel multistage machines. The formulation can involve set-up costs and set-up times that can be fixed, or vary by product or be sequence-dependent. Another feature is the demand, which can be constant, or vary over regular periods, or vary over irregular periods.

1 Basic Lot-Sizing and Scheduling Models

The trivial problem for the lot-sizing cases can be formulated for a single stage with infinite production capacity and a single product to be planned over T periods of time:

$$\text{Minimise } \sum_{t=1}^{T} hI_t \tag{1}$$

Subject to

$$I_{t-1} + x_t - I_t = d_t \quad \forall t = 1,\dots,T \tag{2}$$

$$I_t \geq 0 \text{ and } x_t \geq 0 \quad \forall t = 1,\dots,T \tag{3}$$

where h represents the inventory holding cost of the product from one period to the next and d_t represents the product demand at the end of period *t*. The decision variables x_t represent the quantities to be produced in each period *t* and I_t represents the inventory of the product at the end of period *t*.

Expression (1), which represents the objective function, minimum total holding cost of the inventory resulting from the quantities produced. Equation (2) relates inventory at the start and end of period *t* to the production and demand in that period.

This very simple model has a trivial solution, namely $x_t = d_t \quad \forall \quad t = 1,\dots,T$, i.e., it is possible to produce exactly the amount of demand and therefore all I_t will be zero.

Introducing five different aspects into the model can develop this trivial model:

1.1 Capacity Constraint

Include capacity constraints as follows:

$$x_t \leq C_t \quad \forall t \tag{4}$$

where C_t represents the production capacity for each period *t*. If $d_t > C_t$ then it is not trivial to find an optimal solution.

1.2 Presence of Backlogs

If backlogs are allowed in the production system then equations (1), (2) and (3) are modified as follows:

$$\text{Minimise } \sum_{t} (hI_t^+ + gI_t^-) \tag{5}$$

Subject to:

$$I_{t-1}^{+} - I_{t-1}^{-} + x_t - I_t^{+} + I_t^{-} = d_t \quad \forall t \tag{6}$$

$$I_t^{+} \geq 0\,,\; I_t^{-} \geq 0 \text{ and } x_t \geq 0 \quad \forall t \tag{7}$$

and the capacity constraint (4).

This is also a linear programming. The objective function (5) minimises the holding inventory and backlog costs over the time. The parameter g represents the penalty the cost of one carrying over a backorder of one unit of product from one period to the next.

In equation (5) I_t^{+} and I_t^{-} represent, respectively, the inventory and backlog quantities at the end of period t and are decision variables in the system. There is no optimal solution where $I_t^{+} > 0$ and $I_t^{-} > 0$ simultaneously, since the objective function can be improved by decreasing both I_t^{+} and I_t^{-} at the same rate until one becomes zero.

1.3 Presence of Set-up Costs

In this case, the simplest lot-sizing problem was formulated by Wagner and Whitin [44]. Capacity limits are not considered. It is a single-product single-machine problem. A set-up cost is charge in period t of the item is produced in that period. This it may be more economical to pay some inventory costs by producing in some periods only rather than incur a set-up cost in every period. The objective function (8):

$$\text{Minimise } \sum_{t=1}^{T} sy_t + hI_t \tag{8}$$

minimise the sum of set-up and holding costs. It is necessary to introduce a binary decision variable, y_t which value 1 if the set-up of product occurs at the end of the period t and 0 otherwise. The following constraint forces $x_t = 0$ if $y_t = 0$ and frees $x_t \geq 0$ if $y_t = 1$.

$$x_t \leq M_t y_t \quad \forall t \tag{9}$$

where M_t is an upper bound x_t.

The model is no longer a Linear Programme, but rather a Mixed Integer Programme (MIP). Further aspects of set-up modelling are considered later in this paper.

1.4 Multistage Problems

The lot-sizing problem arises as part of MRP (Material Requirement Planning) logic ([3], [41] and [43]). In traditional MRP systems the problem is addressed level by level. In the single-level version of the problem we are faced with a set of net requirements which are produced by the MRP explosion and netting steps, and we must choose a set of lot sizes. The costs involved are fixed costs and holding costs. The multistage is referenced [3] so this lot-sizing problem encompasses the entire product structure. At each level the problem resembles the single level, but with the additional property that the lot sizes at each level, which form the solution, also cause part or all of the demand at the next level down the product structure. The problem is to simultaneously find a set of lot sizes at each level, combined together, will minimise the total fixed and holding cost in the system.

The terms "multistage" and "multilevel" can be taken as having the same meaning and therefore in this paper we will use the multistage term.

The multistage capacitated lot-sizing (MSCLS) problem is concerned with the determination of production lot sizes in resource-constrained multistage MRP systems so as to minimise the sum of production, set-up and inventory costs. The MSCLS problem, in [18] is shown to be NP-Complete.

This is important because it implies that it is unlikely that any algorithm can optimally solve large problems.

Readers must not confuse multistage with multiple products (sometimes called multiple-item). Multiple products can be multistage if the production of one item is dependent on another. Otherwise the production can be called single-stage, or single-level (see [32] and [33]).

1.5 Multiple Products

We can develop the model represented by constraints (4) to (7) by including the multiple products:

Minimise $\sum_i \sum_t h_{it} I_{it}^+ + g_{it} I_{it}^-$ (10)

Subject to

$I_{i,t-1}^+ - I_{i,t-1}^- + x_{it} - I_{it}^+ + I_{it}^- = d_{it} \quad \forall i,t$ (11)

In this case the variables I_{it}^+ and I_{it}^- represent respectively the inventory and the backlog of product *i* at the end of period *t* and x_{it} represents the quantity of product *i* produced in period *t*. The data d_{it} are the demand of product *i* at the end of period *t* and h_{it} the unit invent any holding cost of product *i* at the end of period *t*. To turn it into a capacitated problem, is necessary to add the constraints such as:

$\sum_i u_i x_{it} \le C_t \quad \forall t$ (12)

where u_i is the capacity need for one unit of product *i* and C_t is the total capacity of machine at period *t*.

Some variations on the problem

This model is called the Capacitated Lot-sizing and Scheduling Problem (CLSP), by [13], for a single-stage (i.e. single-machine) multiple product system. To minimise the total of set-up and holding costs, the model in [13] is:

Minimise $\sum_{i=1}^{P} \sum_{t=1}^{T} (s_i y_{it} + h_i I_{it})$ (13)

Subject to

$I_{it} = I_{i,t-1} + x_{it} - d_{it} \qquad i = 1,\dots,P; t = 1,\dots,T$ (14)

$u_i x_{it} \le C_t y_{it} \qquad i = 1,\dots,P; t = 1,\dots,T$ (15)

$\sum_{i=1}^{P} u_i x_{it} \le C_t \qquad t = 1,\dots,T$ (16)

$y_{it} \in \{0,1\} \qquad i = 1,\dots,P; t = 1,\dots,T$ (17)

$I_{it}, x_{it} \ge 0 \qquad i = 1,\dots,P; t = 1,\dots,T$ (18)

where s_i the set-up cost for item *i* and h_i is a holding costs of product *i*.

Note that C_t/u_i is used as an upper bound on x_{it} in expression (15).

A unit production cost c_i can be also be inserted in the objective function, as follows:

Minimise $\sum_{i=1}^{P} \sum_{t=1}^{T} (s_i y_{it} + c_i x_{it} + h_i I_{it})$ (19)

2 Modelling of Time Periods

Two terms very much used in Lot-sizing Problems, are *small* and *big (or large) bucket*, as defined in [5] which makes a distinction:

"between "Big Bucket" models having long time periods in which several items can be set up and produced and "Small Bucket" models have short time periods in order to be able to model start-ups, switch-offs and/or changeovers. The "Small Bucket" models are then split further into those in which only one item can be set up per period, and those with possibly two set-ups per period".

The CLSP is called a large bucket (or big bucket) problem ([13], [5]) because several items may be produced per period.

The case where the (macro-) periods are subdivided in several micro-periods leads to the Discrete Lot-sizing and Scheduling Problem (DLSP), called a small bucket problem, in [13] and [5], because at most one item can be produced per period. The DLSP has the same objective function as the CLSP, but constraints (15) and (16) need to be replaced by:

$$u_i x_{it} = C_t s_{it} \quad i = 1,\ldots,P; t = 1,\ldots,T \tag{20}$$

and following the constraints:

$$\sum_{i=1}^{P} s_{it} \le 1 \quad t = 1,\ldots,T \tag{21}$$

$$y_{it} \ge s_{it} - s_{i,t-1} \tag{22}$$

where $s_{it} \in \{0,1\}$ indicates whether the machine is configured for item *i* in period *t* ($s_{it} = 1$) or not ($s_{it} = 0$) and $y_{it} \in \{0,1\}$ now indicates the start-up of a lot of item *i* in period *t*.

The 'all-or-nothing' assumption of the DLSP comes in via equation (20), where, in contrast to CLSP, the left- and right-hand sides must be equal. Constraint (21) makes sure that at most one item can be produced per period and that with (20), capacity limits are taken into account. The start-up of a new lot is spotted by the inequality (22).

In the Continuous Set-up Lot-sizing Problem (CSLP) constraint (20) of the DLSP is replaced by

$$u_i x_{it} \le C_t s_{it} \quad i = 1,\ldots,P; t = 1,\ldots,T \tag{23}$$

Constraint (20) of the DLSP forces the production system to produce to its full capacity, whereas constraint (23) of the CLSP allows the system to produce under its full capacity.

The Proportional Lot-sizing and Scheduling Problem (PLSP) occurs when the CSLP model does not use the full capacity of a period, in [13]:

"Roughly speaking, the basic idea of PLSP is to use remaining capacity for scheduling a second item in the particular period".

Thus in the PLSP can be observed by:

$$u_i x_{it} \le C_t (s_{i,t-1} + s_{it}) \quad i = 1,\ldots,P \quad and \quad t = 1,\ldots,T \tag{24}$$

replaces (20) and also includes

$$\sum_{i=1}^{P} u_i x_{it} \le C_t \quad t = 1,\ldots,T \tag{25}$$

In equation (24) we see that in order to produce a certain item in a certain period of time, it is necessary to have the machine already configured either at the beginning or at the end of that period. Once it has been configured the total capacity requirement per period has its limit in equation (25).

In [14] the PLSP with set-up times (PLSPST) is presented where the objective function is expression (13). The capacity constraint now include is set-up time w_{it}.

$$\sum_{i=1}^{P}(u_i x_{it} + w_{it}) \le C_t ; t = 1,...,T \tag{26}$$

Reference [14] for full details of an interesting formulation.

The General Lot-Sizing and Scheduling Problem (GLSP), proposed by [13] and [17], features multiple products, single-machine sequence-dependent set-up costs, small bucket time, but with no set-up times nor backlogging.

The macro-periods *t* are each divided into a fixed number of non-overlapping micro-periods with variable length, where S_t denotes the set of micro-periods *s* belonging to the macro-period *t* and all micro-periods are ordered in the sequence *s*=1,…, *S*.

Two models are provided in [17], the first being GLSP-CS (Conservation of Set-up State) and the second GLSP-LS (Loss of Set-up State). The mathematical model for GLSP-CS is as follows:

$$\text{Min } \sum_{i,t} h_i I_{it} + \sum_{ijs} s_{ij} z_{ijs} \tag{27}$$

Subject to

$$I_{it} = I_{i,t-1} + \sum_{s \in S_t} x_{is} - d_{it} \tag{28}$$

$$\sum_{i,s \in S_t} a_i x_{is} \le K_t \tag{29}$$

$$a_i x_{is} < K_t y_{is} \tag{30}$$

$$x_{si} \ge m_i (y_{is} - y_{i,s-1}) \tag{31}$$

$$\sum_i y_{in} = 1 \tag{32}$$

$$z_{ijs} \ge y_{i,s-1} + y_{j,s-1} - 1 \tag{33}$$

Using the notation of [17], the variables are: $I_{it} \ge 0$ is the inventory of product *i* at the end of macro-period *t*, x_{is} is the production of quantity of item *i* in micro period *s*; $y_{is} = 1$ if the machine is set-up for product *i* in the micro-period *s* and 0 otherwise; z_{ijs} = 1 if the changeover of the product *i* to product *j* take a place in the beginning of micro-period *s* and 0 otherwise.

The parameters are: K_t is the capacity (time) available in macro-period *t*, a_i is a capacity consumption (time) needed to produce one unit of product *i*, m_i is the minimum lot size of product *i* , s_{ij} is the cost of a changeover from product *i* to product *j*, I_{i0} is the initial inventory of product *i* at the beginning of the planning horizon. y_{i0} = 1 if the machine is already set-up for product *i* at the beginning of period 1 and 0 otherwise.

Constraint (31) enforces minimum lot sizes in order to avoid set-up changes without product changes, avoiding an incorrect calculation of set-up costs/times in an optimal solution if set-up costs/times do not satisfy the triangle inequality ($s_{ik} + s_{kj} \ge s_{ij} \quad \forall i, j, k$) as can occur in the chemical industry.

In [35], constraint (29) is modified to include set-up times:

$$\sum_{i,s \in S_t} a_i x_{is} + \sum_{i,j,s \in S_t} st_{ij} z_{ijs} \le K_t \tag{34}$$

where st_{ij} is the set-up time of the changeover from product *i* to product *j*.

Models and applications of the Continuous Time Lot-sizing and Scheduling Problem (CTLSP), including the Batching and Scheduling problem (BSP), can be found in [5], [13], [14], [15] and [37].

3 Multiple-Machine Problems

The next section in this paper covers the multiple-machine problem but at the same time preserving the single-stage feature, i.e., machines in parallel.

The balance equation (11) now becomes:

$$I_{i,t-1} + \sum_{m}^{M} x_{imt} - I_{it} = d_{it} \forall i,t \tag{35}$$

where x_{imt} is the production of item *i* on machine *m* in period *t*.

The capacity constraint (12) becomes:

$$\sum_{i} u_{im} x_{imt} \leq C_{mt} \forall m,t \tag{36}$$

where u_{im} is the capacity needed for one unit of product *i* on machine *m* and C_{mt} is the capacity available on machine *m* in period *t*.

References [1] and [4] distinguish three special cases for single-stage in parallel machines: identical parallel machines, uniform parallel machines and unrelated parallel machines, a feature often omitted by the researchers.

Reference [36], develops the following features: multiple-products, multiple-machines heterogeneous, capacitated, sequence-dependent set-up times, without backlogging and the objective function has a particularity including the holding, sequence-dependent costs and production costs, in a single-stage scheme.

Reference [36] develops the model of [17] to the multiple machine case denoted General Lot-Sizing and Scheduling Problem Parallel Production Line (GLSPPL).

A different modelling approach is a adapted in [9], which presents two formulations to Lot-Sizing when set-up times are sequence-dependent in the context of rolling-horizon planning and scheduling for parallel machines. The first formulation is an exact model while the second is an approximation for rolling-horizon use.

Instead of using macro and micro periods, the models in [9] permit multiple set-ups within each planning period. As in [9], the n-*th* set-up can occur at different times on each machine. The variable $y_{ijmt}^{n} = 1$ if the n-*th* set-up in period *t* on machine *m* is from product *i* to product *j* and 0 otherwise. The result MIP is huge, but solved set-up on a rolling horizon basis, using continuous variable approximations for future set-ups. Research is continuing on such models.

4 Methods

Many approaches have been used to solve Lot-sizing and Scheduling Problems, as presented in Table 1.

Methods/approach	**References Single-Stage Problems**	**References Multiple-Stage Problems**
Mathematical Programming	[5],[10],[14],[21],[24],[40],[45]	[5],[6],[18],[28],[29],[30],[34]
Simulated Annealing	[8],[11],[35], [39]	[4],[26],[28],[29],[36],[39]
Evolutionary Algorithms	[11],[22], [25], [42]	[12],[22],[23],[26],[27],[31],[38]
Taboo Search	[11], [17], [35], [39]	[18],[29],[30],[31],[36],[39]
Simpler Heuristics	[2],[8],[10],[11],[14],[15],[16],[17],[20], [24],[32],[33],[35]	[6],[7],[34]

Table 1: List of publications in Lot-sizing and Scheduling Problems

The table is by no means complete, but note that a wide range of mathematical programming, heuristic and metaheuristic methods have been applied to solving lot-sizing and scheduling problems.

There seen to be proportionally more application of simpler heuristics to single-stage problems and proportionally more application of metaheuristics to the multiple-stage problems.

In the case of Evolutionary Algorithms just one reference used Memetic Algorithm to solve the problem, [38], for the multistage case.

Observe that several researchers ([5], [22] and [39]) applied the same method to both single and multiple stage methods. Conversely many papers ([6], [8], [10], [11], [14], [17], [24], [26], [29], [34], [35], [39] applied two or more methods to the same problem, permitting better understanding of these methods and the difficulties of the problem.

5 Conclusions

This paper presented a review of Lot-sizing and Scheduling models with an emphasis in single-stage problems. It covered several important aspects, such as, capacity constraints, backlogs, set-up costs and times, multiple machines and size of planning periods. The paper started with a very simple problem gradually including these aspects and variations of the problem in a structured manner to show the major features found in Lot-sizing and Scheduling Problems.

Finally, a classified and referenced listing of solution methods was presented with very general observations to serve as a starting point for the authors' research into effective solution methods for the multiple-product, multiple-machine lot-sizing and sequencing problem with sequence-dependent set-up times discussed in [9] and [10].

Future research will analyse the approaches referenced in this paper and apply it to the models in [9] and [10], particularly with respect to Memetic Algorithms.

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