Ejercicio 1

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{
        \cfrac{}{x:B\to B\to B, y:B\to B, z:B \vdash x:B\to B\to B}\,(T-VAR) \quad
        \cfrac{}{x:B\to B\to B, y:B\to B, z:B \vdash z:B}\,(T-VAR)
      }{x:B\to B\to B, y:B\to B, z:B \vdash (x\,z):B\to B}\,(T-APP) \quad
      \cfrac{
        \cfrac{}{x:B\to B\to B, y:B\to B, z:B \vdash y:B\to B}\,(T-VAR) \quad
        \cfrac{}{x:B\to B\to B, y:B\to B, z:B \vdash z:B}\,(T-VAR)
      }{x:B\to B\to B, y:B\to B, z:B \vdash (y\,z):B}\,(T-APP)
    }{x:B\to B\to B, y:B\to B, z:B \vdash ((x\,z)\,(y\,z)):B}\,(T-APP)
  }{x:B\to B\to B, y:B\to B \vdash (\lambda z:B.(x\,z)\,(y\,z)\,):B\to B}\,(T-ABS)
  }{x:B\to B\to B \vdash (\lambda y:B\to B.\lambda z:B.(x\,z)\,(y\,z)\,):(B\to B)\to B\to B}\,(T-ABS)
}{\vdash (\ \lambda x:B\to B\to B.\lambda y:B\to B.\lambda z:B.(x\,z)\,(y\,z)\,):(B\to B\to B)\to (B\to B)\to B\to B}\,(T-ABS)
$$

Ejercicio 5

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{
        \cfrac{}{x:B\vdash x:B}\,T-VAR
      }{\vdash (\lambda x:\ B.x):B\to B}\,T-ABS
    }{\vdash (\lambda x:\ B.x)\ as\ B\to B:B\to B}\,T-ASCRIBE \quad
    \cfrac{}{z:B\to B\vdash z:B\to B}\,T-VAR
  }{\vdash let\ z=((\lambda x:\ B.x)\ as\ B\to B)\ in\ z:B\to B}\,T-LET
}{\vdash (let\ z=((\lambda x:\ B.x)\ as\ B\to B)\ in\ z)\ as\ B\to B:B\to B}\,T-ASCRIBE
$$

Ejercicio 9

$$
\cfrac{
  \cfrac{
    \cfrac{}{\vdash unit:Unit}\,T-UNIT
  }{\vdash unit\ as\ Unit:Unit}\,T-ASCRIBE \quad
  \cfrac{
    \cfrac{
      \cfrac{
        \cfrac{}{x:(B,B)\vdash x:(B,B)}\,T-VAR
      }{x:(B,B)\vdash snd\ x:B}\,T-SND
    }{\vdash \lambda x:(B,B)\ .snd\ x:(B,B)\to B}\,T-ABS
  }{}
}{
\cfrac{\vdash (unit\ as\ Unit,\ \lambda x:(B,B)\ .snd\ x):(Unit,(B,B)\to B)}{\vdash fst(unit\ as\ Unit,\ \lambda x:(B,B)\ .snd\ x):Unit}\,T-FST}\,T-PAIR
$$