Monads and other abstractions

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4 Jul 2014

Mathematics

Meaning

There isn't any.

Abstraction

Structures, and relationships between structures.

Sets

“Stuff”

A set is a collection of elements.

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```
type Set a = a -> Bool
```

Extensional

Can be defined by stating its elements.

$\{$ *True*, *False* $\}$

Intensional

Or by describing them.

$$\{\ x \mid x \in \mathbb{N}, \mathit{even}(x)\ \}$$

Distinction

Values can be extensionally equal, but intensionally distinct.

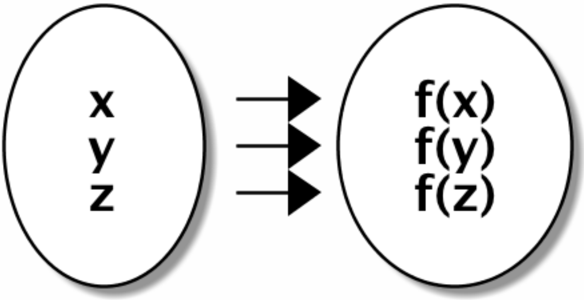
$n \mapsto 2(n+5)$
$n \mapsto 2n+10$

Deceptively simple

With just a definition of sets, and seven axioms (and we've already seen two!), you can generate a good part of mathematics.

Functions

As maps



Higher-order functions

$$id\ x = x$$

$$(f \circ g)\ x = f(g(x))$$

Properties of functions

$$f : cod \to dom$$

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$$f : cod \to dom$$

Definition (Idempotent)

$f \circ f = f$

Properties of functions

$$f : cod \to dom$$

Definition (Idempotent)

$f \circ f = f$

Definition (Involutive)

$f \circ f = id$

Homomorphism

"Structure preserving."

Isomorphism

An isomorphism is a pair of functions satisfying two equations:

$$f \circ g = id_{dom(f)}$$

$$g \circ f = id_{dom(g)}$$

Isomorphism

In terms of the types involved:

$$A \cong B$$

$$g : A \to B$$

$$f : B \to A$$

Laws

Algebras

Algebraic Structures

Magmas

Semigroups

Monoids

Groups

Type Algebras

Equational Reasoning

Quantification

Existential

$$\exists p, P(p)$$

Universal

$$\forall p, P(p)$$

Universal

True?

$\forall$ x, $\exists$ y $\rightarrow$ x = y

Universal

True?

$\forall$ x, $\exists$ y $\rightarrow$ x $\neq$ y

Parametricity

Curry-Howard Isomorphism

Free objects

Category Theory

Functors

Applicatives

Monads

Free Monads