## Ex.3.10

Since $H_n < \ln(n) + 1$ we can use Chebyshevs inequality to deduce

$$\begin{aligned} P(X > \beta n \ln(n)) &\leq P(X > \beta n(H_n - 1) \\ &= P(\frac{X - nH_n}{nH_n} > \beta - \beta H_n^{-1} - 1) \\ &\leq \frac{n^2 \sum_{i=1}^n i^{-2} - nH_n}{n^2 H_n^2 (\beta - \beta H_n^{-1} - 1)^2} \\ &\leq \frac{1}{(\beta - 1)^2} \end{aligned}$$

When $\beta \geq \frac{1}{1 - H_n^{-1}}$ (not a very strict requirement). We have used that $\sum_{i=1}^n i^{-2} \leq H_n^2$ (easily seen).

## Pr.3.1

If $N$ is the random variable of the number of empty bins then $N = \sum_{i=1}^n N_i$ where $N_i$ is one if bin number $i$ is empty and zero otherwise. Thus by linearity of expectation

$$EN = \sum_{i=1}^n EN_i = n \cdot \left( \frac{n-1}{n} \right)^m$$

since every independent $m$ balls have to land in the $n-1$ other bins If $m = n$ this is $\approx n/e$.

## Problem 3.3 (a)

Let $M_k$ denote the binominally distributed random variable counting the number of requests sent to memory unit $k$, and let $N_k$ denote the number of requests answered by memory unit $k$.
Then the support of $N_k$ is $\{0, 1, 2\}$ and

$$\begin{aligned} N_k = 0 &\iff M_k = 0 \\ N_k = 1 &\iff M_k = 1 \\ N_k = 2 &\iff M_k \geq 2 \end{aligned}$$

And hence

$$
\begin{aligned}
EN_k &= 0 \cdot P(M_k = 0) + 1 \cdot P(M_k = 1) + 2 \cdot P(M_k \geq 2) \\
&= P(M_k = 1) + 2(1 - P(M_k = 0) - P(M_k = 1)) \\
&= \frac{1}{n}(\frac{n-1}{n})^{n-1} n + 2(1 - \left(\frac{n-1}{n}\right)^n - \frac{1}{n}\left(\frac{n-1}{n}\right)^{n-1} n) \\
&= \left(\frac{n-1}{n}\right)^{n-1} + 2\left(1 - \left(\frac{n-1}{n}\right)^n - \left(\frac{n-1}{n}\right)^{n-1}\right) \\
&\approx 2 - 3e^{-1}
\end{aligned}
$$

Hence if $N$ denotes the total number of requests answered, then

$$N = \sum_{k=1}^{n} N_k$$

and hence

$$EN = \sum_{k=1}^{n} EN_k = n\left(2 - 3e^{-1}\right)$$

This analysis holds only when $n > 2$, but for $n \leq 2$ all requests are clearly answered regardless of the outcomes of the requests so in these cases the expected number of requests answered is $n$.