
Problem 3.3 (a)

Let M_k denote the binominally distributed random variable counting the number of requests sent to memory unit k , and let N_k denote the number of requests answered by memory unit k .

Then the support of N_k is $\{0, 1, 2\}$ and

$$N_k = 0 \iff M_k = 0$$

$$N_k = 1 \iff M_k = 1$$

$$N_k = 2 \iff M_k \geq 2$$

And hence

$$\begin{aligned} EN_k &= 0 \cdot P(M_k = 0) + 1 \cdot P(M_k = 1) + 2 \cdot P(M_k \geq 2) \\ &= P(M_k = 1) + 2(1 - P(M_k = 0) - P(M_k = 1)) \\ &= \frac{1}{n} \left(\frac{n-1}{n} \right)^{n-1} n + 2 \left(1 - \left(\frac{n-1}{n} \right)^n - \frac{1}{n} \left(\frac{n-1}{n} \right)^{n-1} n \right) \\ &= \left(\frac{n-1}{n} \right)^{n-1} + 2 \left(1 - \left(\frac{n-1}{n} \right)^n - \left(\frac{n-1}{n} \right)^{n-1} \right) \\ &\approx 2 - 3e^{-1} \end{aligned}$$

Hence if N denotes the total number of requests answered, then

$$N = \sum_{k=1}^n N_k$$

and hence

$$EN = \sum_{k=1}^n EN_k = n(2 - 3e^{-1})$$

This analysis holds only when $n > 2$, but for $n \geq 2$ all requests are clearly answered regardless of the outcomes of the requests so in this cases the expected number of requests answered is n .