Continuing from p. 38 bottom ... By Lemma 5.9 [Vershynin, 2010] the ψ_2-norm of Z_j satisfies

$$\left\|Z_j\right\|_{\psi_2} \leq CH_\xi V_{\max} \tag{1}$$

where $C > 0$ is an positive absolute constant and so is sub-gaussian. Thus by [Vershynin 2010, p. 11, Lemma 5.5]

$$\mathbb{E}\exp\left(cZ_j^2/\left\|Z_j\right\|_{\psi_2}^2\right) \leq e \tag{2}$$

so

$$\mathbb{E}\max_{j\in N_\delta} Z_j^2 = \frac{\max_{j\in[N_\delta]}\left\|Z_j\right\|_{\psi_2}^2}{c}\mathbb{E}\left(\max_{j\in[N_\delta]}\frac{cZ_j^2}{\max_{k\in[N_\delta]}\left\|Z_k\right\|_{\psi_2}}\right) \tag{3}$$

$$\leq \frac{C^2H_\xi^2V_{\max}^2}{c}\mathbb{E}\left(\max_{j\in N_\delta}\frac{cZ_j^2}{\left\|Z_j\right\|_{\psi_2}}\right) \tag{4}$$

$$\leq \frac{C^2H_\xi^2V_{\max}^2}{c}\log\left(\mathbb{E}\max_{j\in N_\delta}\exp\left(\frac{cZ_j^2}{\left\|Z_j\right\|_{\psi_2}}\right)\right) \tag{5}$$

$$\leq \frac{C^2H_\xi^2V_{\max}^2}{c}\log\left(\sum_{j\in[N_\delta]}\mathbb{E}\exp\left(\frac{cZ_j^2}{\left\|Z_j\right\|_{\psi_2}}\right)\right) \tag{6}$$

$$\leq \frac{C^2H_\xi^2V_{\max}^2}{c}\log\left(eN_\delta\right) \tag{7}$$