Proposition 1. Let $\kappa : \mathcal{X} \to \mathcal{Y}$ be a probability kernel and $\mu \in \mathcal{P}(\mathcal{X})$. Then $\kappa(B \mid x)\delta_x(A) = \kappa\delta_x(A \times B)$ for all $A \in \Sigma_{\mathcal{X}}$ and $B \in \Sigma_{\mathcal{Y}}$.

Proof. This is straight from definition. $\square$

Proposition 2. Let $f_1 : \mathcal{X}_1 \to \overline{\mathbb{R}}, f_2 : \mathcal{X}_2 \to \overline{\mathbb{R}}, \ldots$ are measurable. Then

$$f : \prod_i \mathcal{X}_i \to \overline{\mathbb{R}}$$
$$(x_1, x_2, \ldots) \mapsto \limsup_{n\to\infty} \sum_{i=1}^{n} f_i(x_i)$$

is measurable.

Proof. $\sum_{i=1}^{n} f_i \circ c_i$ is measurable since it is a composition of projections, the f_is and summation which are all measurable. $\square$