Examples for 09/29/2015

1. Consider the population of high school graduates who were admitted to a particular university during a ten-year time period and who completed at least the first year of coursework after being admitted. We are interested in investigating how well Y, the first year grade point average (GPA), can be predicted by using the following quantities with $n = 20$ students:

X_1 = the score on the mathematics part of the SAT (SATmath)
X_2 = the score on the verbal part of the SAT (SATverbal)
X_3 = the grade point average of all high school mathematics courses (HSmath)
X_4 = the grade point average of all high school English courses (HSenglish)

Consider the model:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \varepsilon_i, \quad i = 1, 2, \ldots, 20,$$

where ε_i's are independent $N(0, \sigma^2)$ random variables.

```
fit <- lm(GPA ~ SATmath + SATverbal + HSmath + HSenglish, data = gpa)
summary(fit)
```

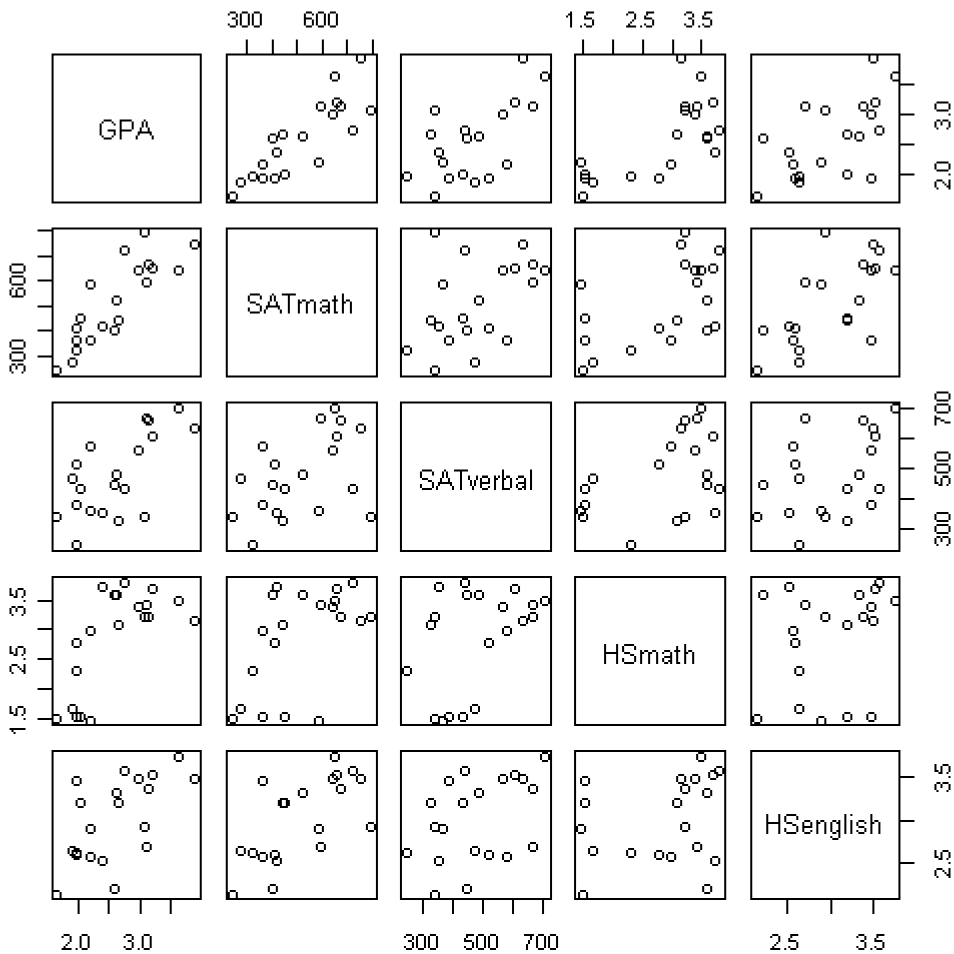
```
Call:
lm(formula = GPA ~ SATmath + SATverbal + HSmath + HSenglish)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.443283 -0.128374  0.002571  0.133996  0.538996 

Coefficients:
              Estimate  Std. Error  t value  Pr(>|t|)   
(Intercept)  0.1615496   0.4375321    0.369   0.71717   
SATmath      0.0020102   0.0005844    3.439   0.00365  **
SATverbal    0.0012522   0.0005515    2.270   0.03835  * 
HSmath       0.1894402   0.0918680    2.062   0.05697  . 
HSenglish    0.0875637   0.1764963    0.496   0.62700   
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2685 on 15 degrees of freedom
Multiple R-squared: 0.8528,     Adjusted R-squared: 0.8135
F-statistic: 21.72 on 4 and 15 DF,  p-value: 4.255e-06
```

```
pairs(GPA ~ SATmath + SATverbal + HSmath + HSenglish , data = gpa)
```



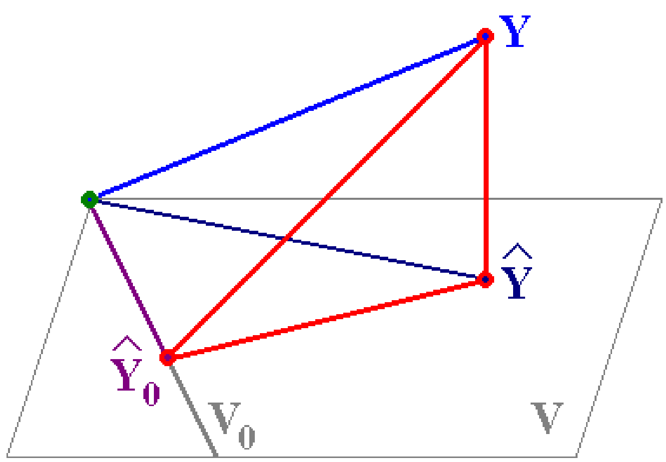
GPA vs. SATmath, GPA vs. SATverbal, and GPA vs. HSenglish suggest a linear relationship. GPA vs. HSmath does not look linear. Either additional higher-order terms in HSmath (for example, the second-order term) are needed, or the values of one or more variables should be transformed before analysis.

```
fit2 <- lm(GPA ~ SATmath + SATverbal, data = gpa)
fit3 <- lm(GPA ~ SATmath + HSmath, data = gpa)
sum(fit$residuals^2)
[1] 1.081499
sum(fit2$residuals^2)
[1] 1.388384
sum(fit3$residuals^2)
[1] 1.528179
```

Suppose we wish to test the claim that SATverbal and HSenglish do not affect the first year GPA. That is, we wish to test $H_0 : \beta_2 = \beta_4 = 0$ vs. H_a : at least one of β_2 and β_4 is significantly different from 0. Perform the test at a 10% level of significance.

Full Model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \varepsilon_i$.

Null Model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_3 X_{i3} + \varepsilon_i$.



$$V = \{ a_0 \mathbf{1} + a_1 \boldsymbol{x_1} + a_2 \boldsymbol{x_2} + a_3 \boldsymbol{x_3} + a_4 \boldsymbol{x_4}, \quad a_0, a_1, a_2, a_3, a_4 \in \mathbf{R} \},$$

$$\dim(V) = 5.$$

$$V_0 = \{ a_0 \mathbf{1} + a_1 \boldsymbol{x_1} + a_3 \boldsymbol{x_3}, \quad a_0, a_1, a_3 \in \mathbf{R} \}, \qquad \dim(V_0) = 3.$$

Numerator d.f. $= \dim(V) - \dim(V_0) = 5 - 3 = \mathbf{2}$.

Denominator d.f. $= n - \dim(V) = 20 - 5 = \mathbf{15}$.

	SS	*DF*	*MS*	*F*
Diff.	$\text{SSResid}_{\text{null}} - \text{SSResid}_{\text{full}}$	$\dim(V) - \dim(V_0)$	…	…
Full	$\text{SSResid}_{\text{full}}$	$n - \dim(V)$	…	
Null	$\text{SSResid}_{\text{null}}$	$n - \dim(V_0)$		

	SS	*DF*	*MS*	*F*	
Diff.	0.44668	2	0.22334	**3.098**	← Test Statistic
Full	1.08150	15	0.07210		
Null	1.52818	17			

Critical Value: $F_{0.10}(2, 15) = \mathbf{2.70}$.

Decision: **Reject H_0**.

```
anova(fit3,fit)
Analysis of Variance Table

Model 1: GPA ~ SATmath + HSmath
Model 2: GPA ~ SATmath + SATverbal + HSmath + HSenglish
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     17 1.5282
2     15 1.0815  2   0.44668 3.0976 0.0748 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Suppose we wish to test the claim that high school performance does not affect the first year GPA. That is, we wish to test $H_0: \beta_3 = \beta_4 = 0$ vs. H_a: at least one of β_3 and β_4 is significantly different from 0. Perform the test at a 10% level of significance.

Full Model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \varepsilon_i$.

Null Model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i$.

$$V = \{ a_0 \mathbf{1} + a_1 \boldsymbol{x_1} + a_2 \boldsymbol{x_2} + a_3 \boldsymbol{x_3} + a_4 \boldsymbol{x_4}, \quad a_0, a_1, a_2, a_3, a_4 \in \mathbf{R} \},$$

$$\dim(V) = 5.$$

$$V_0 = \{ a_0 \mathbf{1} + a_1 \boldsymbol{x_1} + a_2 \boldsymbol{x_2}, \quad a_0, a_1, a_2 \in \mathbf{R} \}, \qquad \dim(V_0) = 3.$$

Numerator d.f. $= \dim(V) - \dim(V_0) = 5 - 3 = \mathbf{2}$.

Denominator d.f. $= n - \dim(V) = 20 - 5 = \mathbf{15}$.

	SS	*DF*	*MS*	*F*	
Diff.	0.30688	2	0.15344	**2.128**	← Test Statistic
Full	1.08150	15	0.07210		
Null	1.38838	17			

Critical Value: $F_{0.10}(2, 15) = \mathbf{2.70}$.

Decision: **Do NOT Reject H_0**.

```
anova(fit2,fit)
Analysis of Variance Table

Model 1: GPA ~ SATmath + SATverbal
Model 2: GPA ~ SATmath + SATverbal + HSmath + HSenglish
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     17 1.3884
2     15 1.0815  2   0.30689 2.1282 0.1536
```

Test $H_0 : \beta_2 = \beta_3 = \beta_4 = 0$ vs. H_a : at least one of β_2, β_3, and β_4 is significantly different from 0. Use a 5% level of significance.

Full Model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \varepsilon_i$.

Null Model: $Y_i = \beta_0 + \beta_1 X_{i1} + \qquad\qquad\qquad\qquad + \varepsilon_i$.

```
fit4 <- lm(GPA ~ SATmath, data = gpa)
sum(fit4$residuals^2)
[1] 2.044274
```

$$V = \{ a_0 \mathbf{1} + a_1 \boldsymbol{x}_1 + a_2 \boldsymbol{x}_2 + a_3 \boldsymbol{x}_3 + a_4 \boldsymbol{x}_4, \quad a_0, a_1, a_2, a_3, a_4 \in \mathbf{R} \},$$

$\dim(V) = 5.$

$$V_0 = \{ a_0 \mathbf{1} + a_1 \boldsymbol{x}_1, \quad a_0, a_1 \in \mathbf{R} \}, \qquad \dim(V_0) = 2.$$

Numerator d.f. $= \dim(V) - \dim(V_0) = 5 - 2 = \mathbf{3}$.

Denominator d.f. $= n - \dim(V) = 20 - 5 = \mathbf{15}$.

	SS	*DF*	*MS*	*F*
Diff.	0.96277	3	0.3209233	**4.451**
Full	1.08150	15	0.07210	
Null	2.04427	18		

← Test Statistic

Critical Value: $F_{0.05}(3, 15) = \mathbf{3.29}$.

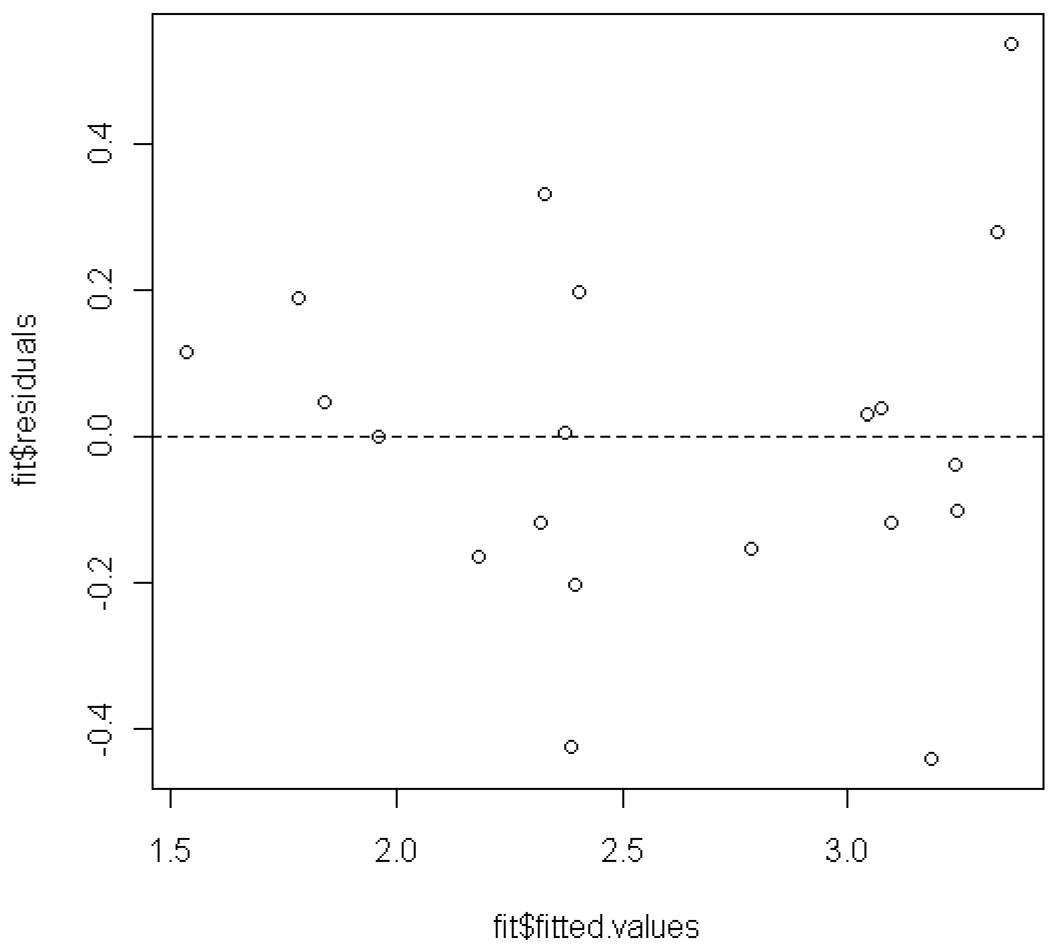
Decision: **Reject H_0**.

```
anova(fit4,fit)
Analysis of Variance Table

Model 1: GPA ~ SATmath
Model 2: GPA ~ SATmath + SATverbal + HSmath + HSenglish
  Res.Df    RSS Df Sum of Sq      F  Pr(>F)
1     18 2.0443
2     15 1.0815  3   0.96277 4.4511 0.01994 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

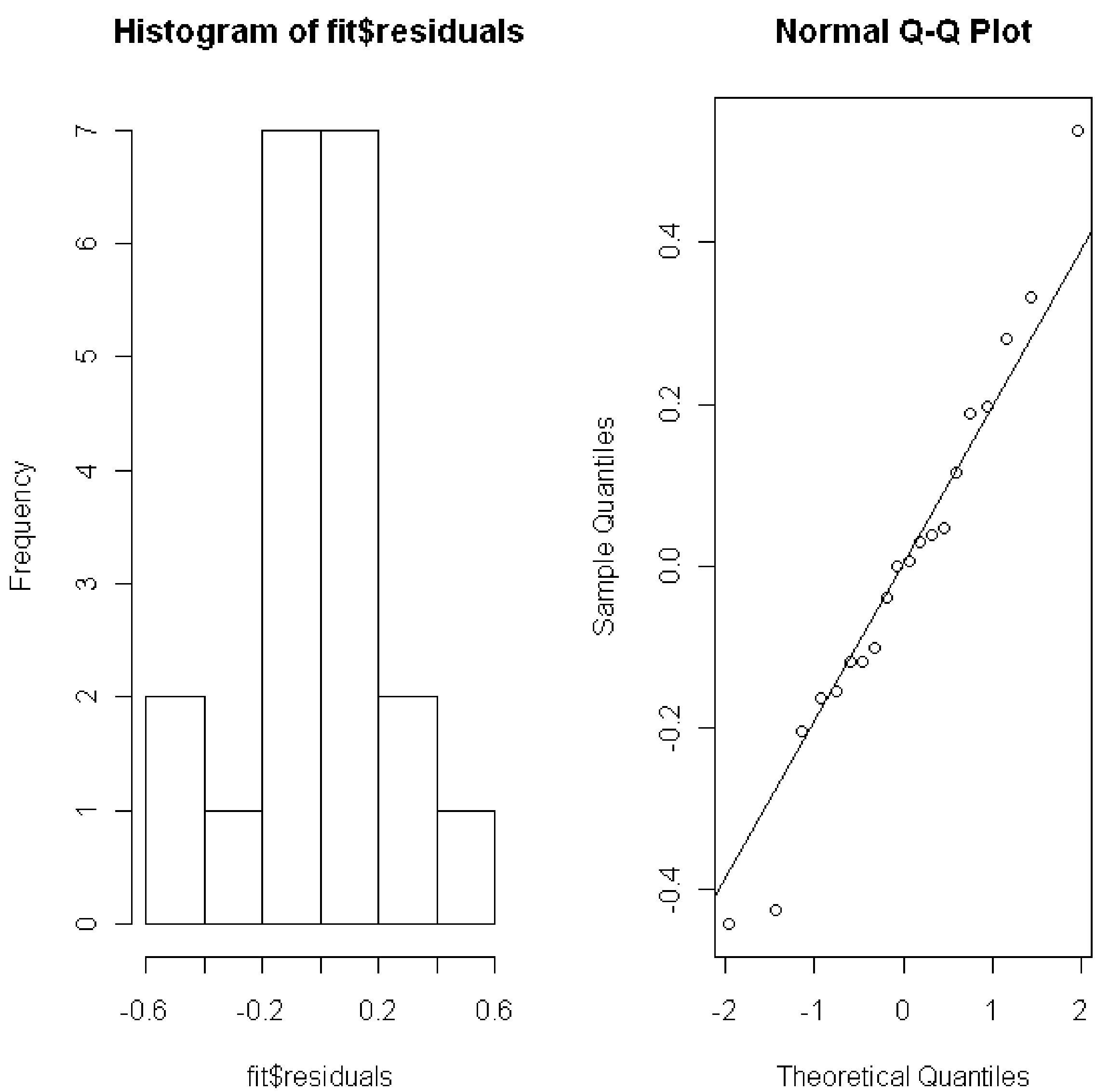
–

```
plot(fit$fitted.values,fit$residuals)
abline(h=0,lty=2)
```



The residuals versus the fitted values plot suggests that the variance σ^2 is not constant.

```
par(mfrow=c(1,2))
hist(fit$residuals)
qqnorm(fit$residuals)
qqline(fit$residuals)
```



```
par(mfrow=c(2,2))
plot(fit)
```