STAT 420 Fall 2015

Answers for 10/08/2015

2. The following data are indexed prices of gold and copper over a 10-year period. Assume that the indexed values constitute a random sample from a bivariate normal distribution.

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(x-\bar{x})\cdot(y-\bar{y})$	$(y-\bar{y})^2$
76	80	16	12	256	192	144
62	68	2	0	4	0	0
70	73	10	5	100	50	25
59	60	−1	−8	1	8	64
53	64	−7	−4	49	28	16
54	68	−6	0	36	0	0
55	65	−5	−3	25	15	9
58	62	−2	−6	4	12	36
57	67	−3	−1	9	3	1
56	73	−4	5	16	−20	25
600	680	0	0	500	288	320

a) Test for the existence of linear relationship between the indexed prices of the two metals. That is, test $\mathrm{H}_0: \rho = 0$ vs. $\mathrm{H}_1: \rho \neq 0$. Use a 5% level of significance.

$\bar{x} = \dfrac{600}{10} = 60.$ $\qquad \bar{y} = \dfrac{680}{10} = 68.$ $\qquad r = \dfrac{288}{\sqrt{500}\,\sqrt{320}} = \mathbf{0.72}.$

Test Statistic: $\qquad t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \dfrac{0.72\cdot\sqrt{10-2}}{\sqrt{1-0.72^2}} = \mathbf{2.9345}.$

Rejection Region: Rejects H_0 if $t < -t_{0.025}(8 \text{ df})$ or $t > t_{0.025}(8 \text{ df})$.

$\pm t_{0.025}(8 \text{ df}) = \pm 2.306.$ $\qquad$ **Reject $\mathbf{H_0}$.**

Since $t_{0.01}(8 \text{ df}) = 2.896 < 2.9345 < 3.355 = t_{0.005}(8 \text{ df})$,

$2 \times 0.005 = 0.01 < \text{p-value} < 0.02 = 2 \times 0.01.$ $\qquad$ (p-value ≈ 0.01887)

OR

$$W = \frac{1}{2}\ln\frac{1+r}{1-r} = \frac{1}{2}\cdot\ln\left(\frac{1+0.72}{1-0.72}\right) = 0.907645.$$

Under H_0, $$\mu_W = \frac{1}{2}\ln\frac{1+\rho_0}{1-\rho_0} = \frac{1}{2}\cdot\ln\left(\frac{1+0}{1-0}\right) = 0,$$

$$\sigma_W^2 = \frac{1}{n-3} = \frac{1}{7}.$$

Test Statistic: $$z = \frac{W-\mu_W}{\sigma_W} = \frac{0.907645-0}{\sqrt{1/7}} = \mathbf{2.40}.$$

Rejection Region: Rejects H_0 if $z < -z_{0.025}$ or $z > z_{0.025}$.

$\pm z_{0.025} = \pm 1.960.$ **Reject H_0.**

P-value = $2 \times P(Z > 2.40) = 2 \times 0.0082 = 0.0164$.

b) Is there enough evidence to conclude $\rho > 0.40$. That is, test $H_0: \rho = 0.40$ vs. $H_1: \rho > 0.40$. Use a 5% level of significance. What is the p-value of this test?

$$W = \frac{1}{2}\ln\frac{1+r}{1-r} = \frac{1}{2}\cdot\ln\left(\frac{1+0.72}{1-0.72}\right) = 0.907645.$$

Under H_0, $$\mu_W = \frac{1}{2}\ln\frac{1+\rho_0}{1-\rho_0} = \frac{1}{2}\cdot\ln\left(\frac{1+0.40}{1-0.40}\right) = 0.423649,$$

$$\sigma_W^2 = \frac{1}{n-3} = \frac{1}{7}.$$

Test Statistic: $$z = \frac{W-\mu_W}{\sigma_W} = \frac{0.907645-0.423649}{\sqrt{1/7}} = \mathbf{1.2805}.$$

Rejection Region: Rejects H_0 if $z > z_{0.05}$.

$z_{0.05} = 1.645.$ **Do NOT Reject H_0.**

P-value = right tail = $P(Z > 1.2805) = \mathbf{0.10}$.