Examples for 11/19/15, Part 3

1. Six samples of each of four types of cereal grain grown in a certain region were analyzed to determine thiamin content, resulting in the following data (μg/g):

Wheat	5.2	4.5	6.0	6.1	6.7	5.7
Barley	6.5	8.0	6.1	7.5	5.9	5.6
Maize	5.8	4.7	6.4	4.9	6.0	5.2
Oats	8.3	6.1	7.8	7.0	5.6	7.2

	n_i	$\bar{x}_i$	s_i	s_i^2
Wheat	6	5.7	0.7668	0.588
Barley	6	6.6	0.9508	0.904
Maize	6	5.5	0.6693	0.448
Oats	6	7.0	1.0139	1.028

Source	**SS**	**DF**	**MS**	**F**
Between	9.24	3	3.08	4.151
Within	14.84	20	0.742	
Total	24.08	23		

$F_{0.05}(3, 20) = 3.10$

$t_{0.025}(20) = 2.086$

A $100\times(1-\gamma)$-percent confidence interval the difference $\mu_i - \mu_j$, $i \neq j$, is given by

$$\overline{Y}_i - \overline{Y}_j \pm t_{\gamma/2}(N - J \text{ d.f.}) \cdot s_{pooled} \cdot \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

where $s_{pooled} = \sqrt{\text{MSW}}$.

a) Construct a 95% confidence interval for the difference between the average thiamin content for Oats and Maize.

Tukey's pairwise comparison:

With $100\times(1-\gamma)$-percent confidence *all* pairwise differences $\mu_i - \mu_j$ are bracketed by the bounds

$$\left(\overline{\mathrm{Y}}_i - \overline{\mathrm{Y}}_j\right) \pm \frac{q_{\gamma,J,N-J}}{\sqrt{2}} \cdot s_{pooled} \cdot \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

where $s_{pooled} = \sqrt{\mathrm{MSW}}$,

$q_{\gamma,J,N-J}$ = values from Studentized Range table.

b) Use a 95% confidence level and Tukey's pairwise comparison procedure to compare the average thiamin content for Oats with the average thiamin content for Maize.

Contrast in the means $\mu_1, \mu_2, \ldots, \mu_J$

$$c_1\mu_1 + c_2\mu_2 + \ldots + c_J\mu_J = \sum_{j=1}^{J} c_j \mu_j \qquad \text{where } \sum_{j=1}^{J} c_j = 0$$

Scheffé's multiple comparison:

With $100\times(1-\gamma)$-percent confidence *all* contrasts in the *J* population means of the form $\sum_{j=1}^{J} c_j \mu_j$ are bracketed by the bounds

$$\sum_{j=1}^{J} c_j \overline{\mathrm{Y}}_j \pm \sqrt{F_\gamma(J-1, N-J)} \cdot s_{pooled} \cdot \sqrt{(J-1)\cdot \sum_{j=1}^{J} \frac{c_j^2}{n_j}}$$

where $s_{pooled} = \sqrt{\mathrm{MSW}}$.

c) Use a 95% confidence level and Scheffé's multiple comparison procedure to compare …

(i) the average thiamin content for Oats with the average thiamin content for Maize;

(ii) the average thiamin content for Oats and Barley with the average thiamin content for Maize;

(iii) the average thiamin content for Oats and Barley with the average thiamin content for Maize and Wheat;