Examples for 09/17/2015

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \dots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \dots & \dots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_n \end{bmatrix} \qquad \mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \qquad \mathrm{E}(\hat{\boldsymbol{\beta}}) = \boldsymbol{\beta}, \qquad \mathrm{Var}(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}.$$

```
> x = c(  2,  6,  8,  8, 12, 16, 20, 20, 22, 26)
> y = c( 58,105, 88,118,117,137,157,169,149,202)
> N = length(x)
>
> Xmat = cbind(rep(1,N), x)
> Xmat
         x
 [1,] 1  2
 [2,] 1  6
 [3,] 1  8
 [4,] 1  8
 [5,] 1 12
 [6,] 1 16
 [7,] 1 20
 [8,] 1 20
 [9,] 1 22
[10,] 1 26
>
> XX = t(Xmat) %*% Xmat
> XX
         x
   10  140
x 140 2528
>
> XXinv = solve(XX)
> XXinv
                             x
   0.44507042 -0.024647887
x -0.02464789  0.001760563
>
> XY = t(Xmat) %*% y
```

```
> XY
   [,1]
   1300
x 21040
> betahat = XXinv %*% XY
> betahat
  [,1]
    60
x    5

> # joint confidence region for the parameters
>
> fit = lm(y ~ x)
>
> library(ellipse)
> plot(ellipse(fit,c(1,2),level=0.95),type="l")
> # plot(ellipse(fit),type="l")
> title("95% joint confidence region for beta0 and beta1")
> points(fit$coeff[1],fit$coeff[2])
```