Bivariate Normal Distribution:

$$f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\},$$

$-\infty < x < \infty, \quad -\infty < y < \infty.$

(a) the marginal distributions of X and Y are $\mathbf{N}\left(\mu_1, \sigma_1^2\right)$ and $\mathbf{N}\left(\mu_2, \sigma_2^2\right)$, respectively;

(b) the correlation coefficient of X and Y is $\rho_{XY} = \rho$, and X and Y are independent if and only if $\rho = 0$;

(c) the conditional distribution of Y, given $X = x$, is

$$\mathbf{N}\left(\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x-\mu_1), (1-\rho^2)\sigma_2^2\right);$$

(d) the conditional distribution of X, given $Y = y$, is

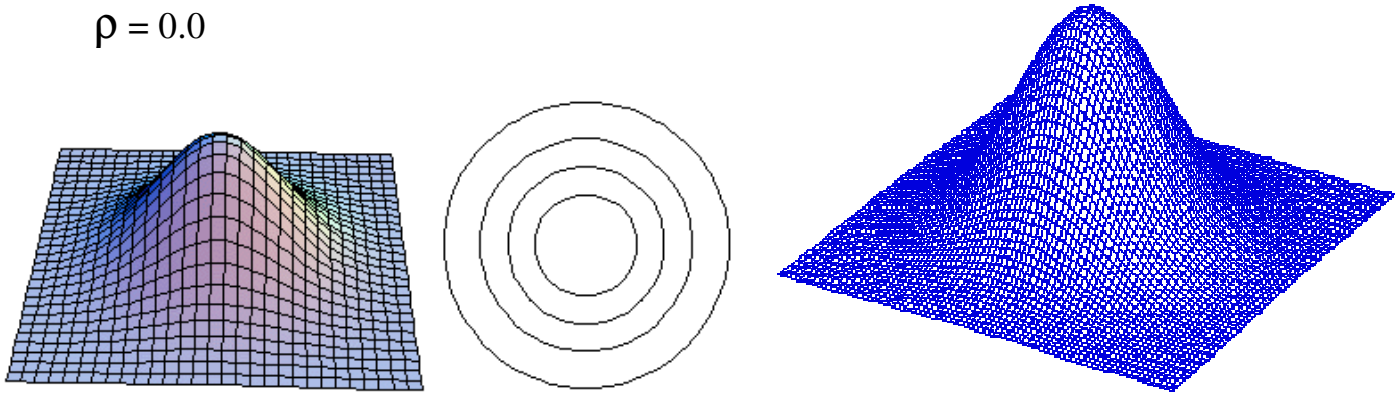
$$\mathbf{N}\left(\mu_1 + \rho\frac{\sigma_1}{\sigma_2}(y-\mu_2), (1-\rho^2)\sigma_1^2\right).$$

(e) $a\,X + b\,Y$ is normally distributed with

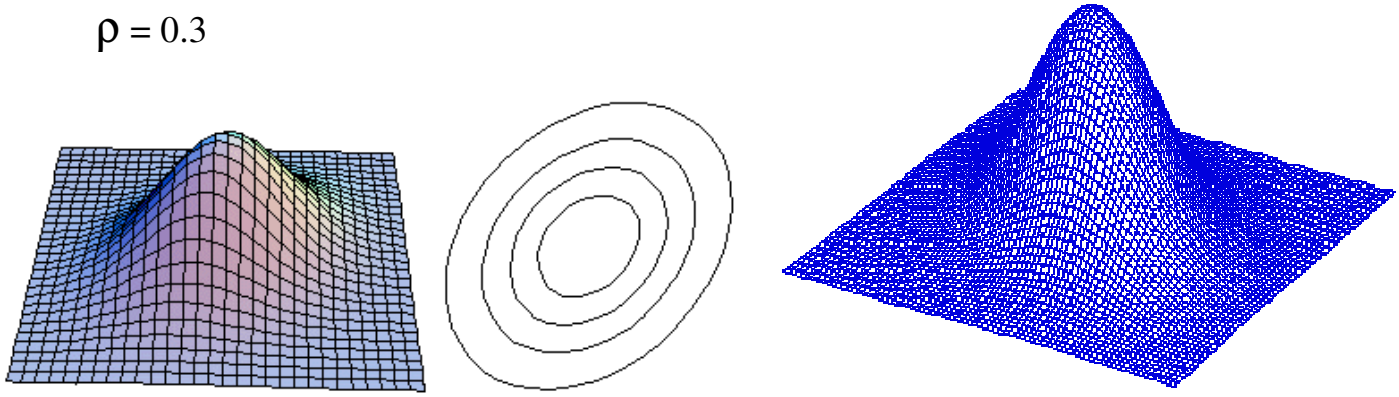
mean $\quad E(a\,X + b\,Y) = a\,\mu_1 + b\,\mu_2$ and

variance $\quad Var(a\,X + b\,Y) = a^2\sigma_1^2 + 2\,a\,b\,\rho\,\sigma_1\,\sigma_2 + b^2\sigma_2^2.$

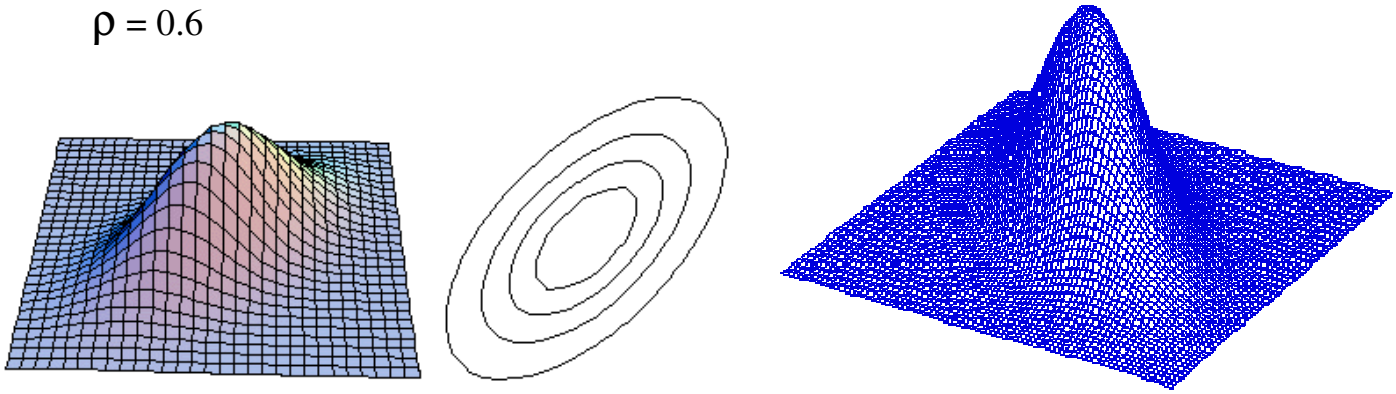
$\rho = 0.0$



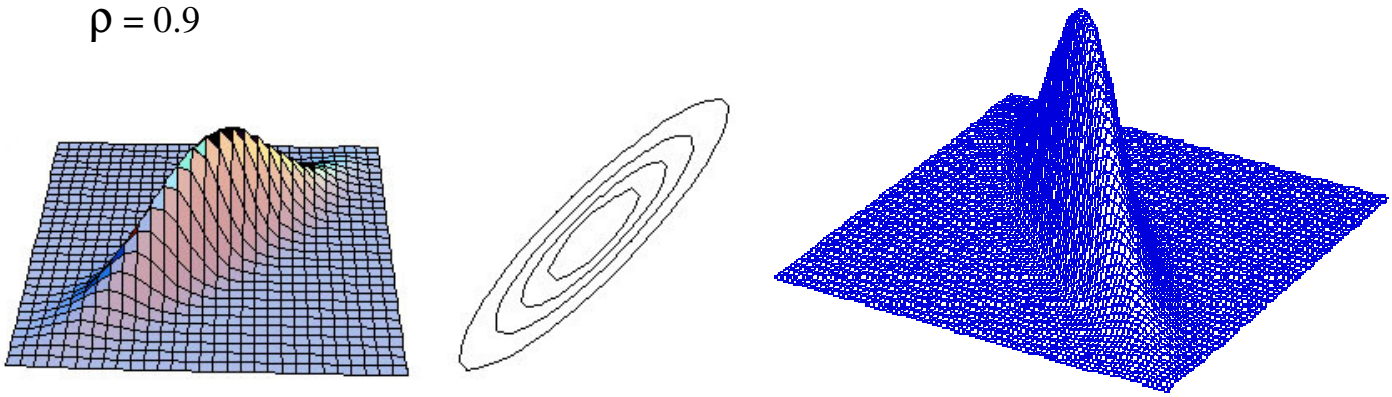
$\rho = 0.3$



$\rho = 0.6$



$\rho = 0.9$



1. A large class took two exams. Suppose the exam scores X (Exam 1) and Y (Exam 2) follow a bivariate normal distribution with

$$\mu_1 = 70, \qquad \sigma_1 = 10,$$
$$\mu_2 = 60, \qquad \sigma_2 = 15, \qquad \rho = 0.6.$$

a) A students is selected at random. What is the probability that his/her score on Exam 2 is over 75?

b) Suppose you're told that a student got a 80 on Exam 1. What is the probability that his/her score on Exam 2 is over 75?

c) Suppose you're told that a student got a 66 on Exam 1. What is the probability that his/her score on Exam 2 is over 75?

d) Suppose you're told that a student got a 70 on Exam 2. What is the probability that his/her score on Exam 1 is over 80?

e) A students is selected at random. What is the probability that the sum of his/her Exam 1 and Exam 2 scores is over 150?

f) What proportion of students did better on Exam 1 than on Exam 2?

g) Find $P(2X + 3Y > 350)$.

h) Find $P(5X + 3Y < 570)$.

i) Find $P(5X - 4Y > 150)$.