Notes from John C. Hull's famed textbook.

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1 Chapter 1: Introduction

A **derivative** can be defined as a financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables. The variables underlying a derivative are often the prices of traded assets. A stock option is a derivative whose price is dependent on the price of the stock. Note that derivatives can be dependant on almost any variable: from the price of hogs to the amount of snow in the Alpes.

1.1 Exchange-Traded Markets

A derivatives exchange is a market where individuals trade standardized contracts that have been defined by the exchange. Traditionally, derivative exchanges used the *open outcry system*; physically meeting up on the floor of the exchange, shouting and using a set of complicated hand signals. This is fortunately being replaced by electronic trading. The replacement has led to a growth in algorithmic trading, also known as blackbox trading, automated trading, high-frequency trading, or robo trading, or beep-boop-machine-make-money-out-of-thin-air trading. The last one is rarely used in the industry.

1.2 Over-the-counter Markets

This is an alternative to the exchange market and has become **much larger** than it in terms of the total volume of trading. Trades are done over the phone and usually between two fininsts or between a fininst and one of its clients (typically a corporate treasurer or a fund manager). Fininsts often act as **market makers** for the more commonly traded instruments; they are prepared to quote a bid price (to buy) and offer price (to sell). There is a credit risk associated with over-the-counter trade, while exchanges have organized themselves to eliminate virtually all credit risk.

1.3 Forward Contracts

A type of derivative that is an agreement to buy or sell an asset at a certain future time for a certain price (is a **forward contract**). In contrast, a **spot contract** is an agreement to buy or sell an asset today. These appear in over-the-counter trades. Each party involved in a forward contract can assume one of two positions:

1. the **long** position: agreeing to **buy** the asset at the specified date and price.
2. the **short** position: agreeing to **sell** the asset at the specified date and price.

The payoff from a **long** position in a forward contract on one unit of an asset is: $S_T - K$, where K is the delivery price (the one agreed to by both parties) and S_T is the spot price of the asset at maturity of the contract. The payoff in the **short** position is negative of that in the long position: $K - S_T$. As forward contracts cost nothing to enter, these payoffs are also the net gain or loss associated with the contract.
Forward contracts can be used in the landscape of stocks to benefit from expected changes in stock prices:

- If a stock is expected to drop ($\implies$ have a lower S_T in the near future), enter a forward contract to buy the stock in the future at $K <$ current price, having sold it now at the current price.

- If a stock is expected to rise (*implies* have a higher S_T in the near future), enter a forward contract to sell the stock in the future at $K >$ current price, having sold it now at the current price.

Both these points assume enough profit is made to justify any loans involved.

1.4 Futures Contracts

They are also agreements to buy/sell an asset at a certain time at a certain price, but are traded on an exchange. The commodities include pork bellies, live cattle, sugar, wool, lumber, copper, aluminum, gold, and tin. The financial assets include stock indices, currencies, and Treasury bonds. Future prices are regularly reported in the financial press.

1.5 Options

Options are traded both on exchanges and in the over-the-counter market. There are two types:

1. Call option: Gives the holder the right to buy the underlying asset by a certain date for a certain price.
2. Put option: Gives the holder the right to sell the underlying asset by a certain date for a certain price.

The specified price is called the **exercise price or strike price**, while the specified date is called the **expiration date or maturity**. Note that **American** options can be exercised at any date on or before the expiration, while **European** options can only be exercised on the expiration date.
The option gives the holder the right to exercise a trade; they don't have to actually make the trade. This freedom comes at a cost; precisely, the cost to acquiring an option.

1.5.1 Properties of Options

- The price of a call option decreases as the strike price increases, while the price of a put option increases as the strike price increases. (Is this because the prices is in general expected to drop, and hence buying at a higher price (call option with high strike price) is less preferred while selling at a higher price (put option with higher strike price) is more preferred?).
- However, both types of options become more valuable as their time to maturity increases.

Consider a case where an investor buys a call option with the asset as 100 shares of Google, the strike price as \$520 per share and the expiration date as 18/12/10, at a price of \$32 per share.

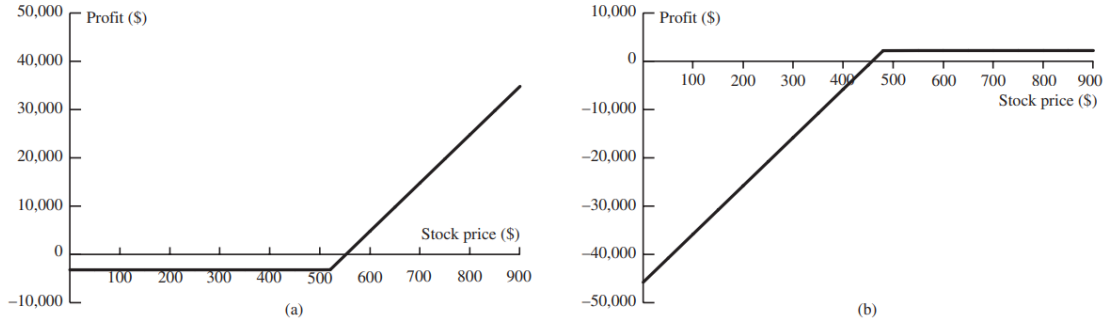
- The investor has spent \$3200 in acquiring the option.
- If Google's share price doesn't rise above \$520 by the expiration date, the investor has no reason to exercise the option (paying \$520 for a share while the bid price is much below that) and has thus lost \$3200.
- If before the expiration date, Google's share price rises above \$520, the investor can exercise the option; buy the shares at \$520, and sell them at the current price S_T, making a profit of $100(S_T - (520 + 32))$.
- Note that we're neglecting TVoM here.

The person who sells the put option is obliged to do the buying at the option's maturity (or before). Consider a case where an investor sells a put option with on 100 shares of Google with the strike price as \$480 per share and the expiration date as 18/09/10, at a price of \$22.2 per share.

- Immediately, the investor receives a cash inflow of $100 \times 22.2 = \$2220$
- Suppose Google's share price up to the expiration date remains above \$480, the buyer of the option has no incentive to sell his shares at \$480, and doesn't exercise the option.
- Suppose Google's share price S_T is below \$480 before the expiration date, the buyer of the option sells the shares back to the investor at \$480, and buys them at S_T making a profit of: $100 \times (480 - (S_T + 22.2))$.

- In this case, the investor is forced to buy shares worth $S_T < 480$ at 480, and suffers a net loss of: $100 \times (480 - (S_T + 22.2))$.
- The investor's gain can be defined as 100 times $S_T + 22.2 - 480$ when $S_T < 480$ and as 22.2 when $S_T > 480$.

The diagrams below summarize the two analyses with the assumption of European options.



Note that there are four types of participants in the options market with the choice of put/call option and buying or selling the option. Buyers are said to take **long positions** and sellers are said to take **short positions**. Selling an option is also known as **writing an option**.

1.6 Types of Traders

Derivatives markets have been successful mainly because of the variety of traders they have attracted and their great deal of liquidity. When an investor wants to take one side of a contract, there is usually no problem in finding someone who is prepared to take the other side. Traders can be broadly divided into three categories:

1. Hedgers: use derivatives to reduce the risk they face from potential future movements in a market variable.
2. Speculators: use derivatives to bet on the future direction of a market variable.
3. Arbitrageurs: take offsetting positions in two or more instruments to lock in a profit.

1.7 Hedgers

Hedgers can use future contracts to, for example, hedge the risk of foreign exchange. Companies that agree to pay a fixed amount in a currency, say C2, different from their operating currency, say C1, can buy futures of the amount in C2 at a fixed price in C1; hence avoiding any loss due to fluctuations in the exchange rate between C1 and C2. Conversely, a company set to receive a payment in C2 in the future can sell futures of C1 at the current exchange rate to hedge the foreign exchange risk. It should be noted that the companies may have been better off without using the future contracts, but they could have been worse off too.
"The purpose of hedging is to reduce risk. There is no guarantee that the outcome with hedging will be better than the one without hedging."

1.7.1 Hedging with Options

We use options to hedge against the possible decline in the price of shares held by us. One method is to buy put options at a high enough strike price K to lower-bound our possible loss by $CP - K$, where CP is the current price of the shares.

- If the price drops below K, we exercise our option to sell the shares at $\$K$ and buy them back at the lower price.
- If the price doesn't drop below K, we don't exercise our option, we have lost the amount we paid for the option, but we've been protected from the risk.

We note that options provide a lower bound for the loss incurred by price dips, while retaining the possibilty to earn from price rises. Futures, however, fix the price to protect against price dips but also keep us from benefitting from price rises.

1.8 Speculators

Speculators are either betting on the price of an asset going down or going up.

1.8.1 With Futures

Suppose a speculator believes that C2 will be increasingly more valuable than C1 over the next few months. The speculator can:

- Purchase, say 250,000 of C2 right away and sell it when it becomes more valuable.
- Enter the long position on a future implying the purchase of 250,000 of C2 at a rate K lower than the expected exchange rate in the future. Then, on the expiration date, we purchase 250,000 of C2 at this lowered rate and immediately sell it for a profit given by: $250,000 \times (S_T - K)$

Note that if the futures's promised price is lower than the current exchange rate, the profit obtained is even larger than that obtained by the first option, while the loss is reduced in magnitude too. The futures market allows the speculator to take a large speculative position with a relatively small initial outlay.

1.8.2 With Options

Options allow for more profit (and equivalently, more loss, albeit capped at the capital) to be made with the same investment capital, than would be made by purchasing shares rightaway. This is because options themselves are much cheaper than shares. The only reason the profit/loss is magnified is because you can buy more options with the same capital right now than you can buy shares. Summary: Good outcomes are magnified, while with bad outcomes, the entire initial investment is lost.

1.8.3 Comparison

It's important to note that with futures, both the loss and gain are unbounded but with options, the loss is bounded below by the amount paid for the options.

1.9 Arbitrageurs

Arbitrage involves locking in a riskless profit by simultaneously entering into transactions in two or more markets. A simple example of arbitrage involves selling and buying stocks listed in different markets in different currencies and profiting from the lag from the exchange rate. Small investors are left with neglegible profits in this scheme in light of transaction costs, but large investors make significant profits. Arbitrage opportunities don't last very long and such lags are temporary and fleeting.

1.10 Dangers

Sometimes, traders who have a mandate to be hedgers or arbitrageurs, become (by choice or ignorance) speculators; this can lead to disastrous effects. Risk management is an important part of trading firms and should be carried out without any negligence.

2 Chapter 2: Mechanics of Futures Markets

The (current) price of future contracts of a particular asset are determined by the laws of demand and supply:

- more buyers $\implies$ price goes up $\implies$ more sellers enter the market.
- more sellers $\implies$ price goes down $\implies$ more buyers enter the market.

Closing Out Positions

Most futures contracts do not lead to delivery because most traders choose to close out their positions prior to the delivery period specified in the contract. Closing out refers to entering into futures contracts that exactly counter your current long or short position on an asset, by having **the same amount of the asset in the opposite position** with **the same delivery period**. The loss/gain arises from the fact that the countering future is purchased at a much later date (which is probably close to the delivery period) and hence the price specified in the future is different from those entered previously. The loss/gain is determined by the change in the futures price between the first contract and the counter contract. We still review the delivery arrangements in futures contracts because the possibility of a final delivery ties the futures price to the spot price.

2.1 Specs of a Futures Contract

The futures contract must specify:

- The asset
- The contract size (how much of the asset is being traded)
- Where the delivery will be made
- When the delivery will be made

Additionally, some alternatives are specified for the grade of the asset that will be delivered or for the delivery locations. When the party with the short position is ready to deliver, it files a *notice to deliver* with the exchange, specifying the selections it has made with respect to said alternatives.

The Asset

Exchanges (the institutes organizing trades) put down the criterion for acceptable grades and respective prices of assets that are commodities. For ex., corn bushels can be described as as No.1 Yellow, No. 2 Yellow and No. 3 Yellow; each with no further than 1.5 cents from its centre color category. The financial assets in futures contracts are generally well-defined. For ex. the Japanese Yen. Treasury bonds and notes on the CBoT have some interesting features here. [1]

Contract Size

It specifies the amount of the asset to be delivered under one contract. A very large size is unusable by investors who wish to hedge relatively small exposures or take relatively small speculation positions while a very small size may render the contract expensive to trade because of costs associated with each trade; contract sizes are an important decision for the exchange. The correct size of a contract depends on the likely user, and exchanges have introduced minicontracts to attract small investors: mini-Nasdaq.

Delivery Arrangements

The exchange must specify the delivery spot; it's particularly important for commodities with large transportaion costs, like giant vipers. When alternative delivery locations are specified, the price received by the short position party is adjusted base on the location chosen by the party: price tends to be higher for delivery locations that are relatively far from the main sources of the commodity.

[1] The underlying asset in the Treasury bond contract is any long-term US Treasury bond that has a maturity of greater than 15 years and is not callable within 15 years. In the Treasury note futures contract, the underlying asset is any long-term Treasury note with a maturity of no less than 6.5 years and no more than 10 years from the date of delivery. In both cases, the exchange has a formula for adjusting the price received according to the coupon and maturity date of the bond delivered.

Delivery Months

A futures contract is referred to by its delivery month, but the contract must specify the precise period of delivery. Though for many futures contracts, the period is the entier month. The delivery months vary from contract to contract and are chosen by the exchange to meet the needs of the market participants.

- At any given time, contracts trade for the closest delivery month and a number of subsequent delivery months.
- The exchange specifies when trading in a particular month's contract will begin.
- The exchange also specifies the last day on which trading can take place for a given contract.
- Trading generally ceases a few days before the last day on which delivery can be made.

Prices, Positions and Limits

The exchange further defines how prices will be quoted: the currency and precision. The exhange also specifies daily price movement limits for most contracts. The contract is said to be:

- limit down if it has moved down by the limit from yesterday's closing price.
- limig up if it has moved up by the limit from yesterday's closing price.

A *limit move* is a move in the either direction that crosses the limit. Normally, trading ceases for the day when a contract is limit up or limit down, but, in some instances, the exchange has the authority to step in and change the limits. The daily price limits prevent large movements in price from occuring because of speculative excesses, but they can become a barrier to trading when the price of an underlying commodity is in fact shifting rapidly; price limits remain controversial in terms of their effect on the futures market. **Position limits** are the maximum number of contracts that an speculator (any investor?) may hold. These limits prevent the speculators from exercising undue influence on the market.

2.2 Convergence of Future Price and Spot Price

The futures price converges to the spot price of the underlying asset as the delivery period comes closer; at the delivery period the future price is practically equal to the spot price. The reason for this is the opportunity for arbitrage while the prices are far apart:

- If cost(futures)>cost(asset), traders will short futures contracts, delivering with the assets bought at a lower cost.
- If cost(futures)<cost(asset), companies interested in acquiring the asset will enter long positions in futures contracts instead of buying the asset, thus making the futures more valuable.

2.3 The Operation of Margins

One of the key roles of the exchange is to organize trading so that contract defaults are avoided.

Daily Settlements

This refers to practice of adjusting the investor's *margin account* at the end of a day to reflect the investor's loss or gain. The account also contains deposits known as *initial margins*, which need to be deposited on entering futures contracts. The investor's profit at the end of the day is given by:

$$\sum_{i\in[n]} (\text{spot price of asset i} - \text{strike price of futures contract of asset i in long position}) \tag{1}$$

and negative terms for futures in short positions. The margin account is altered by this amount at the end of each day. Hence, initial margins are determined by the asset's price and the expected variation. Note that the

previous days alteration is first undone, to avoid the incorrect accumulation or gain/loss brought about by a constant difference between the future's strike price and the spot price. A trade is first settled at the close of the day on which it takes place. It is then settled at the close of trading on each subsequent day. Note that money actually moves from the investor's margin account to their broker, and then to traders in the opposite position who are profiting in the other direction because of the gap between the future's strike price and the spot price.
We also have the notion of a **maintenance margin**, whose role is to ensure the margin account's balance doesn't go into negative values. The investor is rung up when hte margin account's balance falls below this value and the investor is expected to top up the balance to the initial margin level by the end of the next day. The extra funds provided in the margin account are known as **variation margin**; in the absence of a variation margin, the broker closes out the position.

3 Misc. Summaries

3.1 Cash and Carry Arbitrage

It's a market neutral strategy combining:

- purchase of a long position in an asset such as a stock or commodity
- sale (short) of a position in a futures contract on that same underlying asset.

The strategy seeks to exploit pricing inefficencies between spot and futures markets for an asset. We "carry" the asset for physical delivery until the expiry date of the futures contract. The strategy involves the risk of expenses associated with physically "carrying" an asset until expiry, but the risk of market movement, the major component in any regular long or short trade is mitigated. For profits, the futures contract must be theoretically priced above the stock.

Strategy is viable only if cash inflow from the short futures exceeds the acquisition costs and carrying costs on the long asset position.

While arbitrage in non-physical assets, such as stock indices, requires only financing costs, (as opposed to arbitrage in physical markets, which also requires storage and insurance of the associated commodities), the barrier to entry is also lower than that of arbitrage in physical markets. Thus, more players enter the market and the spread between futures and spot prices are very low, leading to fewer opportunities to profit.

3.2 The Futures Pricing

3.2.1 The Pricing Formula

The difference between the futures and spot price is called **the basis or the spread**. The spot-future parity refers to the difference that arises between spot and futures prices due to variables such as interest rates, dividends, time to expiry, etc. It's the mathematical equation involving the spot price and the futures price.

$$\text{FP} = \text{SP}^*(1+r_f) \text{ - d}$$

where r_f is the risk free rate (for the entire year) and d is the dividend expected to be paid out between now and the expiry. As r_f is for the entire year, it has to be scaled as (x is the number of days to expiry):

$$\text{FP} = \text{SP}^*(1+r_f \tfrac{x}{365}) \text{ - d}$$

Note that the FP as evaluated above is the **Fair Value**. The actual value (on the exchange) is calld **Market Value**. The difference between the two arises due to market costs such as transaction charges, taxes, margins etc.