Notes from John C. Hull's famed textbook.

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1 Chapter 1: Introduction

A **derivative** can be defined as a financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables. The variables underlying a derivative are often the prices of traded assets. A stock option is a derivative whose price is dependent on the price of the stock. Note that derivatives can be dependant on almost any variable: from the price of hogs to the amount of snow in the Alpes.

1.1 Exchange-Traded Markets

A derivatives exchange is a market where individuals trade standardized contracts that have been defined by the exchange. Traditionally, derivative exchanges used the *open outcry system*; physically meeting up on the floor of the exchange, shouting and using a set of complicated hand signals. This is fortunately being replaced by electronic trading. The replacement has led to a growth in algorithmic trading, also known as blackbox trading, automated trading, high-frequency trading, or robo trading, or beep-boop-machine-make-money-out-of-thin-air trading. The last one is rarely used in the industry.

1.2 Over-the-counter Markets

This is an alternative to the exchange market and has become **much larger** than it in terms of the total volume of trading. Trades are done over the phone and usually between two fininsts or between a fininst and one of its clients (typically a corporate treasurer or a fund manager). Fininsts often act as **market makers** for the more commonly traded instruments; they are prepared to quote a bid price (to buy) and offer price (to sell). There is a credit risk associated with over-the-counter trade, while exchanges have organized themselves to eliminate virtually all credit risk.

1.3 Forward Contracts

A type of derivative that is an agreement to buy or sell an asset at a certain future time for a certain price (is a **forward contract**). In contrast, a **spot contract** is an agreement to buy or sell an asset today. These appear in over-the-counter trades. Each party involved in a forward contract can assume one of two positions:

1. the **long** position: agreeing to **buy** the asset at the specified date and price.

2. the **short** position: agreeing to **sell** the asset at the specified date and price.

The payoff from a **long** position in a forward contract on one unit of an asset is: $S_T - K$, where K is the delivery price (the one agreed to by both parties) and S_T is the spot price of the asset at maturity of the contract. The payoff in the **short** position is negative of that in the long position: $K - S_T$. As forward contracts cost nothing to enter, these payoffs are also the net gain or loss associated with the contract.

Forward contracts can be used in the landscape of stocks to benefit from expected changes in stock prices:

- If a stock is expected to drop ($\implies$ have a lower S_T in the near future), enter a forward contract to buy the stock in the future at $K <$ current price, having sold it now at the current price.

- If a stock is expected to rise (*implies* have a higher S_T in the near future), enter a forward contract to sell the stock in the future at $K > $ current price, having sold it now at the current price.

Both these points assume enough profit is made to justify any loans involved.

1.4   Futures Contracts

They are also agreements to buy/sell an asset at a certain time at a certain price, but are traded on an exchange. The commodities include pork bellies, live cattle, sugar, wool, lumber, copper, aluminum, gold, and tin. The financial assets include stock indices, currencies, and Treasury bonds. Future prices are regularly reported in the financial press.

1.5   Options

Options are traded both on exchanges and in the over-the-counter market. There are two types:

1. Call option: Gives the holder the right to buy the underlying asset by a certain date for a certain price.

2. Put option: Gives the holder the right to sell the underlying asset by a certain date for a certain price.

The specified price is called the **exercise price or strike price**, while the specified date is called the **expiration date or maturity**. Note that **American** options can be exercised at any date on or before the expiration, while **European** options can only be exercised on the expiration date.
The option gives the holder the right to exercise a trade; they don't have to actually make the trade. This freedom comes at a cost; precisely, the cost to acquiring an option.

1.5.1   Properties of Options

- The price of a call option decreases as the strike price increases, while the price of a put option increases as the strike price increases. (Is this because the prices is in general expected to drop, and hence buying at a higher price (call option with high strike price) is less preferred while selling at a higher price (put option with higher strike price) is more preferred?).

- However, both types of options become more valuable as their time to maturity increases.

Consider a case where an investor buys a call option with the asset as 100 shares of Google, the strike price as $520 per share and the expiration date as 18/12/10, at a price of $32 per share.

- The investor has spent $3200 in acquiring the option.

- If Google's share price doesn't rise above $520 by the expiration date, the investor has no reason to exercise the option (paying $520 for a share while the bid price is much below that) and has thus lost $3200.

- If before the expiration date, Google's share price rises above $520, the investor can exercise the option; buy the shares at $520, and sell them at the current price S_T, making a profit of $100(S_T - (520 + 32))$.

- Note that we're neglecting TVoM here.

The person who sells the put option is obliged to do the buying at the option's maturity (or before). Consider a case where an investor sells a put option with on 100 shares of Google with the strike price as $480 per share and the expiration date as 18/09/10, at a price of $22.2 per share.

- Immediately, the investor receives a cash inflow of $100 \times 22.2 = \$2220$

- Suppose Google's share price up to the expiration date remains above $480, the buyer of the option has no incentive to sell his shares at $480, and doesn't exercise the option.

- Suppose Google's share price S_T is below $480 before the expiration date, the buyer of the option sells the shares back to the investor at $480, and buys them at S_T making a profit of: $100 \times (480 - (S_T + 22.2))$.

2

- In this case, the investor is forced to buy shares worth $S_T < 480$ at 480, and suffers a net loss of: $100 \times (480 - (S_T + 22.2))$.

- The investor's gain can be defined as 100 times $S_T + 22.2 - 480$ when $S_T < 480$ and as 22.2 when $S_T > 480$.

The diagrams below summarize the two analyses with the assumption of European options.
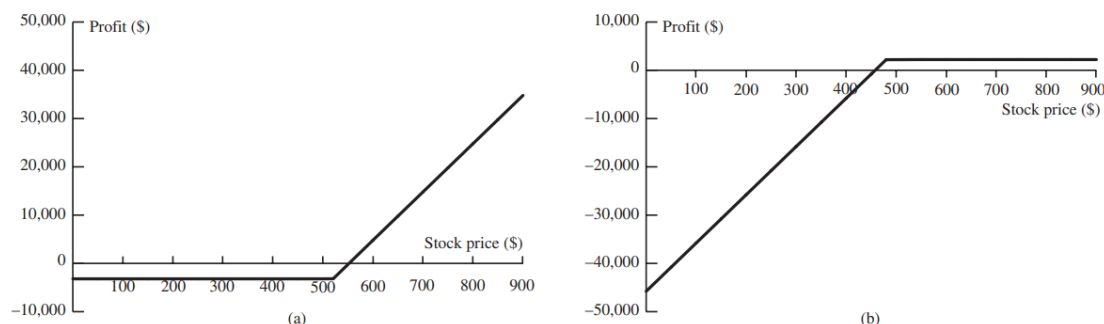
Note that there are four types of participants in the options market with the choice of put/call option and buying or selling the option. Buyers are said to take **long positions** and sellers are said to take **short positions**. Selling an option is also known as **writing an option**.

1.6 Types of Traders

Derivatives markets have been successful mainly because of the variety of traders they have attracted and their great deal of liquidity. When an investor wants to take one side of a contract, there is usually no problem in finding someone who is prepared to take the other side. Traders can be broadly divided into three categories:

1. Hedgers: use derivatives to reduce the risk they face from potential future movements in a market variable.

2. Speculators: use derivatives to bet on the future direction of a market variable.

3. Arbitrageurs: take offsetting positions in two or more instruments to lock in a profit.

1.7 Hedgers

Hedgers can use future contracts to, for example, hedge the risk of foreign exchange. Companies that agree to pay a fixed amount in a currency, say C2, different from their operating currency, say C1, can buy futures of the amount in C2 at a fixed price in C1; hence avoiding any loss due to fluctuations in the exchange rate between C1 and C2. Conversely, a company set to receive a payment in C2 in the future can sell futures of C1 at the current exchange rate to hedge the foreign exchange risk. It should be noted that the companies may have been better off without using the future contracts, but they could have been worse off too.
"The purpose of hedging is to reduce risk. There is no guarantee that the outcome with hedging will be better than the one without hedging."

1.7.1 Hedging with Options

We use options to hedge against the possible decline in the price of shares held by us. One method is to buy put options at a high enough strike price K to lower-bound our possible loss by $CP - K$, where CP is the current price of the shares.

- If the price drops below K, we exercise our option to sell the shares at $\$K$ and buy them back at the lower price.

- If the price doesn't drop below K, we don't exercise our option, we have lost the amount we paid for the option, but we've been protected from the risk.

We note that options provide a lower bound for the loss incurred by price dips, while retaining the possibilty to earn from price rises. Futures, however, fix the price to protect against price dips but also keep us from benefitting from price rises.

1.8 Speculators

Speculators are either betting on the price of an asset going down or going up.

1.8.1 With Futures

Suppose a speculator believes that C2 will be increasingly more valuable than C1 over the next few months. The speculator can:

- Purchase, say 250,000 of C2 right away and sell it when it becomes more valuable.

- Enter the long position on a future implying the purchase of 250,000 of C2 at a rate K lower than the expected exchange rate in the future. Then, on the expiration date, we purchase 250,000 of C2 at this lowered rate and immediately sell it for a profit given by: $250,000 \times (S_T - K)$

Note that if the futures's promised price is lower than the current exchange rate, the profit obtained is even larger than that obtained by the first option, while the loss is reduced in magnitude too. The futures market allows the speculator to take a large speculative position with a relatively small initial outlay.

1.8.2 With Options

Options allow for more profit (and equivalently, more loss, albeit capped at the capital) to be made with the same investment capital, than would be made by purchasing shares rightaway. This is because options themselves are much cheaper than shares. The only reason the profit/loss is magnified is because you can buy more options with the same capital right now than you can buy shares. Summary: Good outcomes are magnified, while with bad outcomes, the entire initial investment is lost.

1.8.3 Comparison

It's important to note that with futures, both the loss and gain are unbounded but with options, the loss is bounded below by the amount paid for the options.

1.9 Arbitrageurs