28 marks

Instructions:
1. Provide justification/steps for each solution. Answers without reasoning will be awarded 0 marks.
2. Answer questions in order. Answer all parts of a question at one place.
3. Label x,y axis and other significant parameters in any plot you draw.

Question 1 (a) Consider an element E with the following V-I characteristics:

$$i_E = \begin{cases} a v_E & \text{if } v_E > 2\,V \\ 0 & \text{if } v_E \leq 2\,V \end{cases}$$

3 marks

with i_E and v_E denoting the current and voltage through/across the device E. Is the element linear? You can treat the current as output and voltage as input to the device.

for linearity
if i_{E1} is output of V_{E1}
& I_{E2} is output of V_{E2}
then $i_{E1}+i_{E2}$ should be output of $V_{E1}+V_{E2}$
Hence for device E

$i_{E1} = a v_{E1}\,\mathbb{1}(v_{E1}>2)$

$i_{E2} = a v_{E2}\,\mathbb{1}(v_{E2}>2)$

$i_{E3} = a v_{E3}\,\mathbb{1}(v_{E3}>2)$

whereas $I_{E1}+I_{E2} = a v_{E1}\,\mathbb{1}(v_{E1}>2) + a v_{E2}\,\mathbb{1}(v_{E2}>2)$

Now if $V_{E1}>2$, $V_{E2}<2$ $\Rightarrow v_{E3}>2$

then $i_{E1} = a v_{E1}$ $\quad i_{E2} = 0$

$i_{E1} + i_{E2} = a v_{E1}$

while $i_{E3} = a v_{E3} = a v_{E1} + a v_{E2} \neq a v_{E1}$
$= i_{E1}+i_{E2}$

$\boxed{\text{Hence it is not linear.}}$

A simple ~~ex~~ counter example can be given.
take $V_{E1} = 3\,V$, $V_{E2} = 1\,V \Rightarrow v_{E3} = 4\,V$

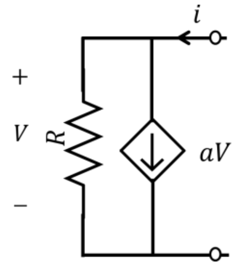
$\downarrow$

$i_{E1} = 3a \quad i_{E2} = 0 \quad i_{E3} = 4a$

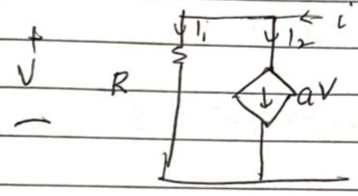
Hence $i_{E3} \neq i_{E1}+i_{E2}$.

(b) Consider the below circuit. Is this sub-circuit equivalent to a resistor? If yes, write down the value of its resistance.

3 marks



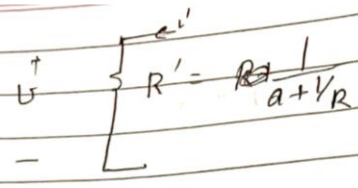
two circuits are equivalent if there IV characteristics is the same. Let us compute its IV charac..



$i = I_1 + I_2$

$= \frac{V}{R} + aV = V\left(a + \frac{1}{R}\right)$

Consider the following circuit



For this circuit,

$i = v\left(a + \frac{1}{R}\right)$

Hence two circuit are equivalent

Yes, this circuit is equivalent to a resistor.

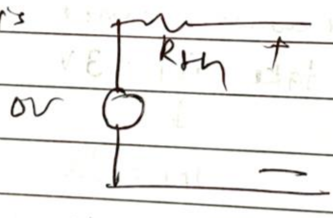
Value $= \frac{1}{a + 1/R} = \frac{R}{Ra + 1}$

Approach 2

this can also be done by computing thevenin equivalent. which will come out to be a simple resistor. marks can be given if student has computed this correctly

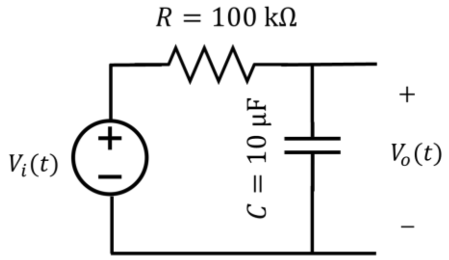
$V_{th} = 0 \quad R_{th} = \frac{R}{Ra + 1}$

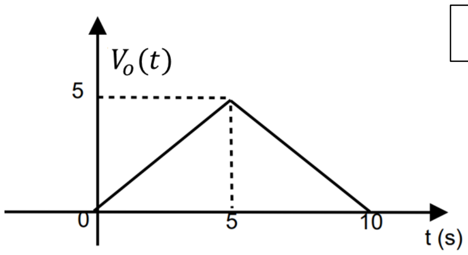
equivalent is



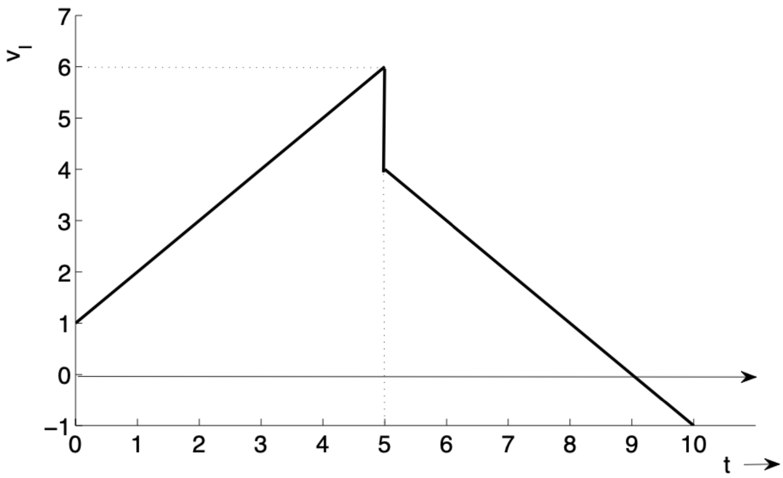
Question 2 (a) Consider the RC circuit shown below (left). The output $V_o(t)$ is shown in right. Compute and draw the input signal $V_i(t)$ as a function of t, that must be applied to produce this output signal.

3 marks





Solution: We have $V_I = RC\frac{dV_0}{dt} + V_0$. $RC = 1\ s$. For $0 < t < 5$, we have $V_0 = t \implies V_I = t + 1$. For $5 < t < 10$, we have $V_0 = 10 - t \implies V_I = 9 - t$. $V_I(t)$ is plotted as below.



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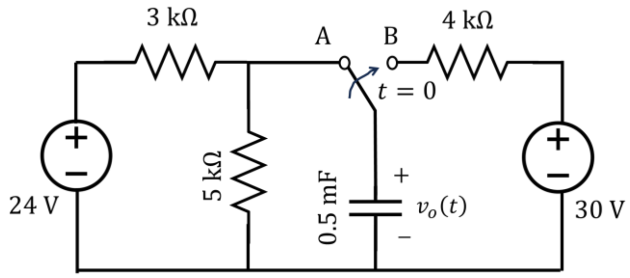
if x,y axis are not properly labeled or other significant parameters are not given, marks will be deducted.

(b) The switch in the circuit below has been in the position A for a long time. At $t = 0$, the switch moves to B. Detemine $v_o(t)$ for $t > 0$. Calculate its value at $t = 1s$.

3 marks



Solution:

For $t < 0$, the switch is at position A. The capacitor acts like an open circuit to dc, but v is the same as the voltage across the 5-kΩ resistor. Hence, the voltage across the capacitor just before $t = 0$ is obtained by voltage division as

$$v(0^-) = \frac{5}{5+3}(24) = 15 \text{ V}$$

Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 15 \text{ V}$$

For $t > 0$, the switch is in position B. The Thevenin resistance connected to the capacitor is $R_{Th} = 4$ kΩ, and the time constant is

$$\tau = R_{Th}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s}$$

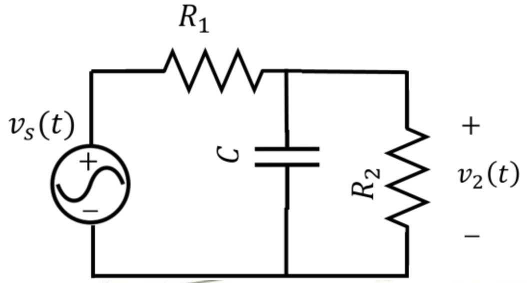
Since the capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30$ V. Thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$
$$= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t}) \text{ V}$$

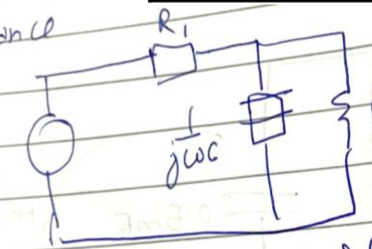
At $t = 1$,

$$v(1) = 30 - 15e^{-0.5} = 20.9 \text{ V}$$

Question 3 (a) Consider the following circuit. The circuit is fed with input $v_s(t) = a_1\cos(\omega_1 t) + a_2\cos(\omega_2 t + \phi)$. What will the voltage $v_2(t)$ across the resistor R_2? **4 marks**



3 the impedance model is



Let us compute Response separately for

$a_1 \cos \omega_1 t \longrightarrow V_s = a_1\angle 0$

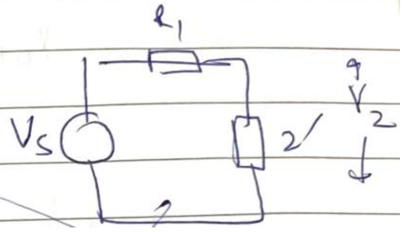
and

$a_2 \cos \omega_2 t + \phi \qquad V_s = a_2\angle\phi$

equivalant impedance for $\frac{1}{j\omega C} \parallel R_2$ is

$$Z' = \frac{1}{\frac{1}{R_2} + j\omega C}$$

$$= \frac{R_2}{1 + R_2\omega C j}$$



output across R_2

$$V_2 = V_s \frac{Z'}{Z' + R_1}$$

$$= \frac{V_s \quad R_2/(1+jR_2\omega C)}{\frac{R_2}{1+jR_2\omega C} + R_1}$$

$$= \frac{V_s R_2}{R_2 + R_1 + j(R_1 R_2 \omega C)}$$

$$= \frac{V_s R_2}{\sqrt{(R_2+R_1)^2 + R_1^2 R_2^2 \omega^2 C^2}} \angle \tan^{-1}\left(-\frac{R_1 R_2 \omega C}{R_1 + R_2}\right)$$

Let us call $A(\omega) = \frac{R_2}{\sqrt{(R_1+R_2)^2 + R_1^2 R_2^2 \omega^2 C^2}}$

$$\theta(\omega) = \tan^{-1}\left(-\frac{R_1 R_2 \omega C}{R_1 + R_2}\right)$$

Hence the output is

$a_1 \cos \omega_1 t \rightarrow V_s = a_1 \angle 0$

output $V_{o1} = a_1 A(\omega_1) \angle -\theta(\omega_1)$

$a_2 \cos(\omega_2 t + \phi) \rightarrow V_s = a_2 \angle \phi$

$V_{o2} = a_2 A(\omega_2) \angle -\theta(\omega_2) + \phi$

output

$a_1 A(\omega_1) \cos(\omega_1 t - \theta(\omega_1))$

$+ a_2 A(\omega_2) \cos(\omega_2 t + \cancel{\phi} - \theta_2(\omega_2))$

Numerical value.

$\cancel{A(\omega_1)} = \cancel{\frac{R_2\; 4\times10^3}{}}$

$$A(\omega) = \frac{R_2/R_1}{\sqrt{\left(1+\frac{R_2}{R_1}\right)^2 + (R_2 \omega C)^2}}$$

$\theta(\omega)$

$= \tan^{-1} \frac{-R_2 \omega C}{1 + R_2/R_1}$

(b) Now consider $R_1 = 1\mathrm{k}\Omega$, $R_2 = 2\mathrm{k}\Omega$ and $C = 0.5\mu\mathrm{F}$. Further, $a_1 = a_2 = 1, \omega_1 = 1kHz, \omega_2 = 10kHz, \phi = 45^{o}$. Write down the value of $v_2(t)$ for these parameter values.

2 marks

Numerical value

At ω_1: $R_2\omega_1 C = 2\times10^3 \times 10^3 \times \frac{1}{2}\times10^{-6} = 1$

$A(\omega_1) = \frac{2}{\sqrt{9+1}} = \frac{2}{\sqrt{10}}$

$\theta(\omega_1) = \tan^{-1}\left(\frac{-1}{3}\right) =$

At ω_2: $R_2\omega_2 C = 10$

$A = \frac{2}{\sqrt{9+100}} = \frac{2}{\sqrt{109}}$

$\theta_2(\omega) = \tan^{-1}\left(-\frac{10}{3}\right)$

18.4

output

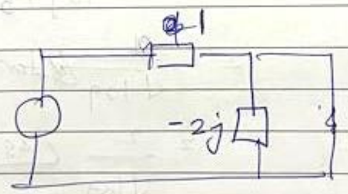
$$v_2(t) = \frac{2}{\sqrt{10}}\cos\left(\omega_1 t - \tan^{-1}\left(\tfrac{1}{3}\right)\right) + \frac{2}{\sqrt{109}}\cos\left(\omega_2 t + 45^\circ - \tan^{-1}\left(\tfrac{10}{3}\right)\right)$$

this is 73 °

If the time domain signal is not given, significant marks will be deducted.

Approach-2

At ω_1



$z' = 2 \| -2j = \frac{2\times(-2j)}{2-2j}$

$\boxed{z' = \frac{-2j}{1-j}}$

$$V_2 = \frac{1\times \frac{-2j}{1-j}}{1 + \frac{-2j}{1-j}}$$

$j\cdot j = -1$

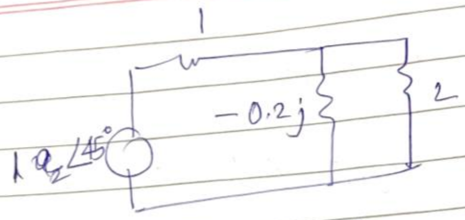
$-j\cdot -j = -1$

$$= \frac{-2j}{1-j-2j} = \frac{-2j}{1-3j}\times\frac{j}{j}$$

$$= \frac{2}{j+3}$$

$$= \frac{2}{\sqrt{1+9}}\angle -\tan^{-1} 1/3 = \frac{2}{\sqrt{10}}\angle -15^\circ$$

At $\omega_2 =$



$2' = \frac{-2 \times 0.2j}{2 - 0.2j} = \frac{-0.2j}{1-0.1j} = \frac{0.2}{j+0.1}$

$V'_{o_2} = 1\angle 45 \dfrac{0.2/(j+0.1)}{1 + \frac{0.2}{j+0.1}}$

$= 1\angle 45 \dfrac{0.2}{j+0.3}$

$= 1\angle 45 \dfrac{2}{10j+3}$

$= \dfrac{2}{\sqrt{109}} \angle 45^\circ - \tan^{-1}\left(\dfrac{10}{3}\right)$

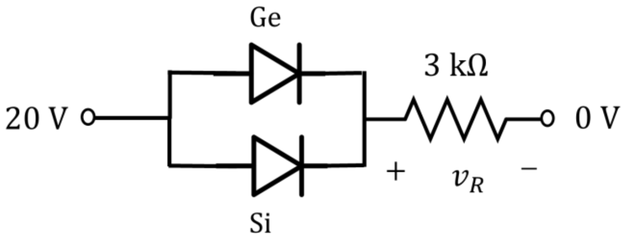
$= \dfrac{2}{\sqrt{109}} \angle 45 - 28^\circ = \dfrac{2}{\sqrt{109}} \angle 17^\circ$

$V' = \dfrac{2}{\sqrt{10}} \cos(10^3 t - 15^\circ)$

$+ \dfrac{2}{\sqrt{109}} \cos(10^4 t + 17^\circ)$

Question 4 Consider the following circuit consisting of two diodes, made of different semiconductor material (Si and Ge respectively). The Si diode has 0.7V cut-in voltage (V_γ) and 0 resistance while the Ge diode has 0.3V cut-in voltage and 0 resistance. Find the voltage v_R across the resistor. Compute the current through each diode.

3 marks



It appears that when the applied voltage is switched on, both the diodes will turn "on". But that is not so. When voltage is applied, germanium diode (V0 = 0.3 V) will turn on first and a level of 0.3V is maintained across the parallel circuit.

The silicon diode never gets the opportunity to have 0.7 V across it and, therefore, remains in open circuit

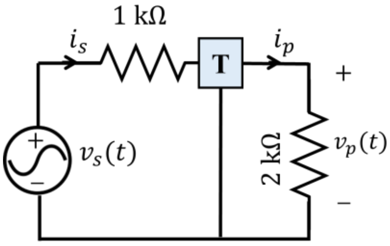
$$v_R = 20 - 0.3 = 19.7V$$

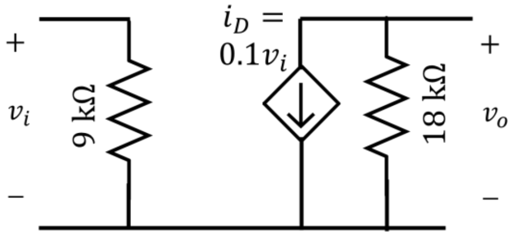
$$i_{Ge} = \frac{v_R}{R} = \frac{19.7}{3000} = 6.566\,mA$$

$$i_{Si} = 0\,mA$$

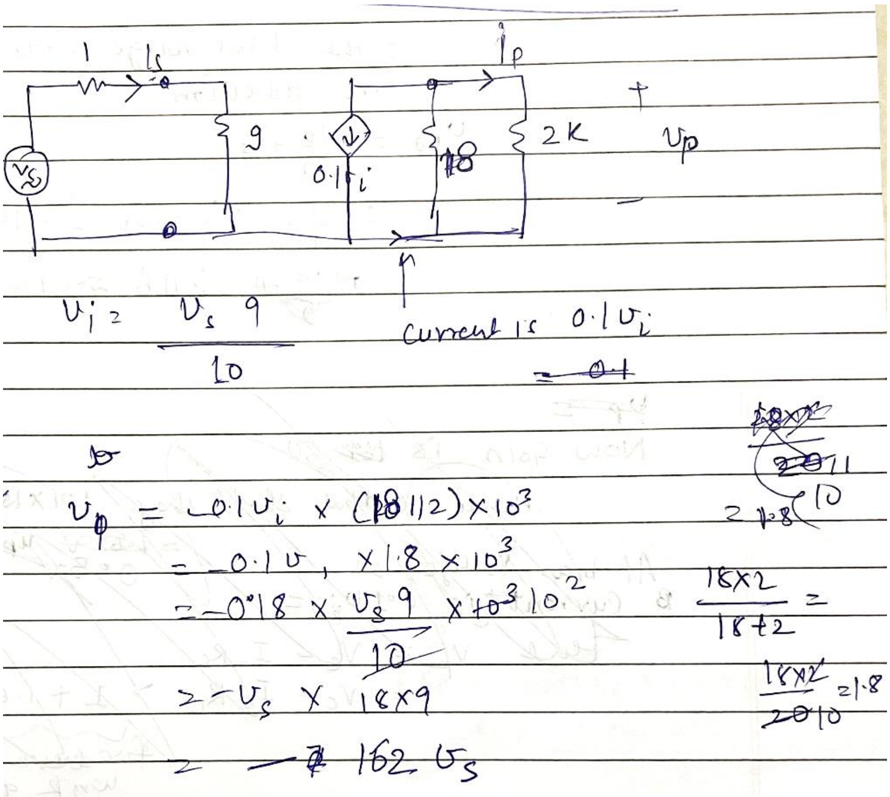
Question 5 (a) Determine the voltage gain v_p/v_s and current gain i_p/i_s of the circuit shown below on the left. The model for device T is shown below on the right.

3 marks





Solution:



$v_i = \frac{v_s\,9}{10}$

current is $0.1v_i$

$v_p = -0.1v_i \times (18 \| 2) \times 10^3$

$= -0.1v_i \times 1.8 \times 10^3$

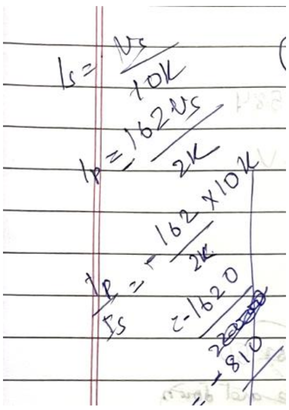
$= -0.18 \times \frac{v_s\,9}{10} \times 10^3$

$= -v_s \times 18 \times 9$

$= -162\,v_s$

$\frac{18 \times 2}{18+2} = \frac{18 \times 2}{20} = 1.8$

$$\frac{v_p}{v_s} = -162$$



$i_s = \frac{v_s}{10K}$

$i_p = \frac{162\,v_s}{2K}$

$\frac{i_p}{i_s} = \frac{-162 \times 10K}{2K}$

$= -1620$

-810

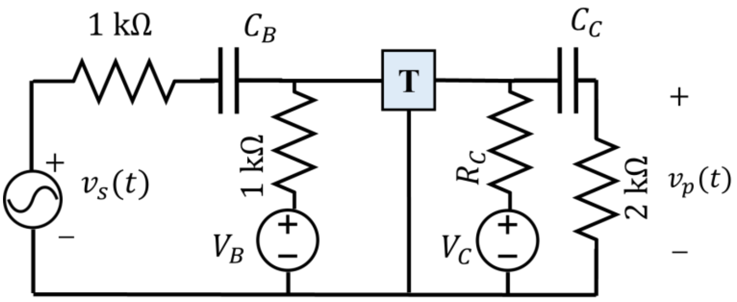
$$\frac{i_p}{i_s} = -810$$

(b) Suppose the model of T is changed. Now the current i_D is given as

$$i_D = \begin{cases} 0.1(v_i - 1) & \text{if } v_i > 1\,V, v_o > 1 \\ 0 & \text{otherwise} \end{cases}$$

Can the following circuit provide necessary bias for the device to operate and provide the $\frac{v_p}{v_s} = 50$ amplification gain to a small signal v_S of frequency $1kHz$ and peak-to-peak voltage $0.01V$?. If yes, compute the values of C_B, C_C, V_C, R_C, V_B.

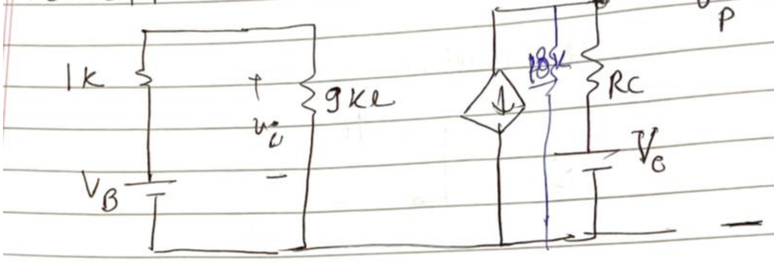
4 marks



Solution: CB/CC are selected such that are short circuit at $\omega = 1kHz$

$\frac{1}{\omega C} \gg R \sim 1k\Omega \Rightarrow C \gg \frac{1}{\omega \times 1k\Omega} = 1\mu F \Rightarrow$ C = 20μF should be sufficient

DC Component



$$\rightarrow \frac{(v_B \pm v_s')9}{10} > 1$$

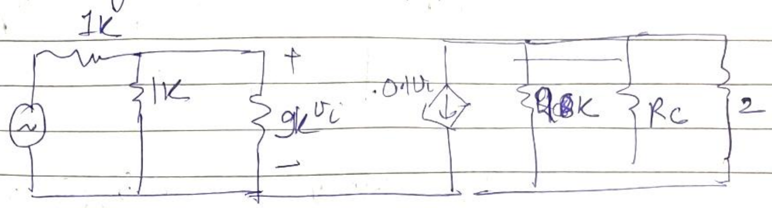
$$v_B \pm v_s' > \frac{10}{9}$$

$$v_B > \frac{10}{9} + v_s' \quad ✓$$

v_s' is the total voltage after ac addition

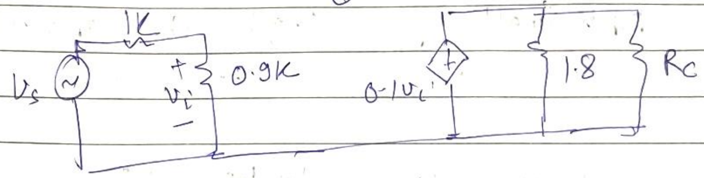
$$\boxed{\begin{aligned} v_{B0} &= \frac{10}{9} + v_s' \\ &\geq \frac{10}{9} + \frac{.9}{1.9} \times .01 = 1.11584 \\ &\approx 1.116 \approx 1.12V \end{aligned}}$$

Now do ac analysis



⇓

$\frac{9\times1}{9+1} = 0.9$



① Change in v_i due to v_s is

$$v_s' = \frac{0.9}{1.9} v_s$$ (will be used in DC analysis)

② gain $0.1\, v_i\, (1.8 \| R_c) / v_s$

$$= 0.1\, (1.8 \| R_c)\, \frac{.9}{1.9}\, v_s / v_s$$

$$= \frac{0.1 \times 9}{19} (1.8 \| R_c) = \cancel{162}\ 50$$

$$1.8 \| R_c = \frac{\cancel{162}\ 50 \times 19}{9 \times .1}$$

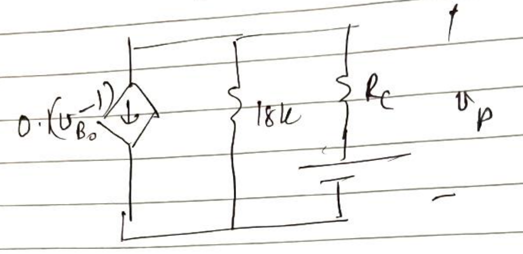
$$= \cancel{3420}\ 1.055K.$$

$$\frac{1.8 R_c}{R_c + 1.8} = 1.055$$

$$1.8 R_c = 1.055 R_c + 1.8 \times 1.055$$

$$\boxed{R_c = \frac{1.8 \times 1.055}{0.745} = 2.54k}$$

Now do DC analysis at output



$v_{B_0} = 1.12\ V$ $\qquad 0.1(v_{B_0}-1) =$

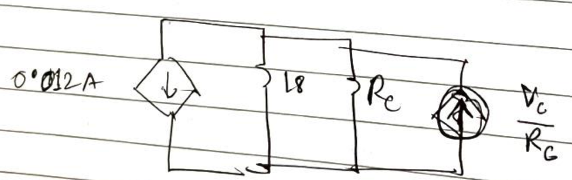
$0.1 \times .116 = 0.0116\ A$

Expected small signal change in output voltage

is $= .01 \times 50 = 0.5\ V$ up and down.

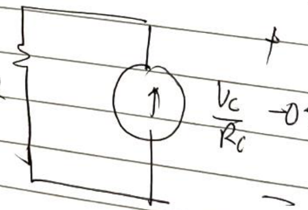
Hence $v_p > 1 + 0.5 = 1.5$ — (1)

Using norton equivalent



⇔

$2.23 = 18 \| R_c$



Voltage is

$$v_p = \left(\frac{V_c}{R_c} - 0.0116\right) \times 2.23K$$

$$\left(\frac{V_c}{R_c} - .0116\right) \times 2.23k = 1.5$$

$$\frac{V_c}{2.54K} - .0116 = \frac{1.5}{2.23k} = 0.67 \times 10^{-3}$$

$$\frac{V_c}{2.54k} = .0116 + .00067 = 0.01227$$

$.0116$
$.0014$
$.013$

$$\boxed{V_c = .013 \times 2.54K = 31.16\ V}$$