Assignment 7

1 A continuous time signal $x(t)$ is said to be *periodic* with fundamental period T if $x(t) = x(t-T)$. Show that if this is true, $x(t) = x(t-kT)$ where k is any integer. For a discrete time signal $x[n]$, periodicity with period N implies that $x[n] = x[n-N]$. Show that in this case, $x[n] = x[n-kN]$ where again, k is any integer. Examine which of the following signals are periodic, and find their periods. $(i)\ e^{-j2t}$, $(ii)\ e^{j(6t-30^\circ)}$, $(iii)\ e^{2-j5t}$, $(iv)\ e^{-j2t}+e^{j5t}$, $(v)\ e^{j3t}+e^{j\pi t}$.

2 Any signal discrete or comntinuous time can be decomposed into so-called *even* and *odd* parts, given by $x_e[n] = (x[n]+x[-n])/2$ and $x_o[n] = (x[n]-x[-n])/2$, or as $x_e(t) = (x(t)+x(-t))/2$ and $x_o(t) = (x(t)-x(-t))/2$ respectively. The unit step signals $u(t), u[n]$ are defined respectively as $u(t) = 1; t \geq 0$ and $u[n] = 1; n \geq 0$. These functions are zero for $t, n < 0$. Find the even and odd parts of the following signals: $(i)\ u[n]-u[n-5]$, $(ii)\ \cos(4\pi t+60^\circ)$, $(iii)\ 2^{-|t|}e^{(3-j4\pi)t}$.

3 The *power* of a real signal $x(t)$ or $x[n]$ is given by the signal $p(t) = x^2(t)$ or $p[n] = x^2[n]$. More generally, for a complex signal, the power is defined as $|x(t)|^2$ or $|x[n]|^2$. The *energy* $E_{(t_1,t_2)}$ over interval (t_1,t_2) or $E_{[n_1,n_2]}$ over $[n_1,n_2]$, is given by $\int_{t_1}^{t_2} p(t)dt$ or $\sum_{n_1}^{n_2} p[n]$. Accordingly, $E_\infty = \int_{-\infty}^{\infty} p(t)dt$ or $E_\infty = \sum_{-\infty}^{\infty} p[n]$. Finally, we define *average* power of a signal as the following: $P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} p(t)dt$ or $P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{-N}^{N} p[n]$. Which of the following signals have a finite energy E_∞? $(i)\ u[n]-u[n-5]$, $(ii)\ \cos(4\pi t+60^\circ)$, $(iii)\ 2^{-|t|}e^{(-3-j4\pi)t}$. Which of them has finite average power P_∞? What happens to E_∞ when a signal has finite and nonzero P_∞? What happens to P_∞ when a signal has finite and nonzero E_∞?

4 Let $x_1(t), x_2(t)$ have powers $p_1(t), p_2(t)$. Let $x_1(t)+x_2(t) = x(t)$ and let its power be $p(t)$. In general, comment on whether $p(t) = p_1(t)+p_2(t)$ is true. If not, state what are the other possibilities, and give separate examples of signals for which each of the other possibilities happens. Find the condition required for equality to actually hold. If $E_{1:(t_1,t_2)}$ $E_{2:(t_1,t_2)}$ and $E_{(t_1,t_2)}$ represent the respective signal energies of $x_1(t), x_2(t), x(t)$, what are the conditions for $E_{1:(t_1,t_2)}+E_{2:(t_1,t_2)} = E_{(t_1,t_2)}$ to hold? Again, give separate examples of $x_1(t), x_2(t)$ for which $E_{1:(t_1,t_2)}+E_{2:(t_1,t_2)} > E_{(t_1,t_2)}$ and $E_{1:(t_1,t_2)}+E_{2:(t_1,t_2)} < E_{(t_1,t_2)}$.

5 For the even-odd decomposition of a signal, $x(t) = x_e(t)+x_o(t)$, examine the relation between the energy and average power E_∞, P_∞ of the signal to the sum of those respective quantities $E_{e:\infty}, E_{o:\infty}$ and $P_{e:\infty}, P_{o:\infty}$ of the even and odd components.

6 Which of the following systems are properly defined? For those that are properly defined, examine each system for the five principal properties of systems, namely, memory, causality, stability, linearity and time invariance. Use commutation diagrams where necessary.

$(a)\ y(t) = x(t)e^t$ $\quad (b)\ y[n] = \begin{cases} 2x[n]/n & n \neq 0 \\ x[n]; & n = 0 \end{cases}$ $\quad (c)\ y(t) = (x(t)-x(-t))/2$ $\quad (d)\ y[n] = \begin{cases} x[n]x[n+2]; & n > 0 \\ x^2[n]; & n = 0 \\ x[n]x[n-2]; & n < 0 \end{cases}$

$(e)\ y(t) = |x(t)|$ $\quad (f)\ y[n] = x[|n|]$ $\quad (g)\ y(t) = \int_{-T}^{T} x(t)dt$ $\quad (h)\ y[n] = \sum_{k=-3}^{+3} d_k x[n-k]$