MSO203B MSO203 Part B

SIDDHARTH GARG

TOTAL POINTS

8 / 9

QUESTION 1

1 Part B Method of Characteristics 8 / 9

✓ **+ 1 pts** $$\dot{x}(t)=1,\, x(0)=s$$

✓ **+ 1 pts** $$\dot{y}(t)=-3,\, y(0)=0$$

✓ **+ 1 pts** $$\dot{z}(t)=z^4,\, z(0)=1$$

If the ode with initial condition is written then only give full '1' mark for rubric 1,2,3.

✓ **+ 1 pts** $$x(t) = t+s$$

✓ **+ 1 pts** $$y(t) = -3t$$

✓ **+ 1 pts** $$z(t) = (\frac{1}{1-3t})^{1/3}$$

+ 1 pts $$u(x,y) = (\frac{1}{1+y})^{1/3}$$ for $$y> -1$$

If $$y> -1$$ is not written, give '0'.

✓ **+ 1 pts** $$x= -\frac{y}{3} + s$$

projected characteristics does not intersect as they are family of parallel straight line.

✓ **+ 1 pts** For the same reason projected characteristic passes through all points.

+ 0 pts Completely wrong/ Not Attempted.

+ 9 pts Completely correct

QUIZ: MSO-203 (PART- B)

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Q4 [9-marks] Consider the following problem:

$u_x - 3u_y = u^4, \; u(x,0) = 1$

a) Find a solution of the above problem using method of characteristics.

b) Does two different projected characteristics intersects? Does there exist any point in $\mathbb{R}^2$ through which no projected characteristics passes?

Sol. (a) ① Initializing parameters: $x = f(s) = s$, $z = g(s) = 1$, $y = h(s) = 0$; where Γ is the curve parameterized by s and Γ here is the x-axis.

$z = u(x,y)$

② Characteristic equations:

$a(x,y,u) = \dot{x}(t) = 1 \qquad x(0) = s$ —— ①

$b(x,y,u) = \dot{y}(t) = -3 \qquad y(0) = 0$ —— ②

$c(x,y,u) = \dot{z}(t) = z^4 \qquad z(0) = 1$ —— ③

③ Solving characteristics

From ① $\quad x(t) = t + s$ —— ④

From ② $\quad y(t) = -3t$ —— ⑤

From ③ $\quad \displaystyle\int \frac{\dot{z}(t)}{z^4}\,dt = \int dt$

$\Rightarrow \dfrac{z^{-3}}{-3} = t + c$

From initial condition

$\dfrac{1}{-3} = c + 0$

$\Rightarrow c = -\dfrac{1}{3}$

$\therefore \dfrac{-1}{3z^3} = t - \dfrac{1}{3}$

$\Rightarrow z^3 = \dfrac{-1}{3t-1} = \dfrac{1}{1-3t}$

Substituting value of $y(t) = -3t$

$\Rightarrow z^3 = \dfrac{1}{1+y} \Rightarrow z = \left(\dfrac{1}{1+y}\right)^{1/3}$

$\therefore \boxed{u(x,y) = \left(\dfrac{1}{1+y}\right)^{1/3}}$

(b) From ④ and ⑤

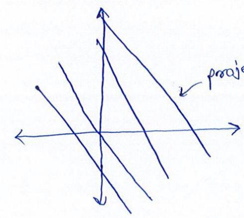
$x(t) = -\dfrac{y(t)}{3} + s$. —— ⑥

Let $x(t) = -\dfrac{y(t)}{3} + s_1$ and $x(t) = -\dfrac{y(t)}{3} + s_2$ be two projected characteristic.

On subtracting we get

$s_1 = s_2$

$\therefore$ Two different characteristics donot intersect.



projected characteristics are parallel with slope $= -3$

There exist no point in $\mathbb{R}^2$ through which no projected characteristic passes. This is because we can always a parameter s in ⑥ for any given value of x and y.