End-Sem Exam : MSO 203B (PDE)

20th November 2018 : 9AM-11AM

Guidelines

- After you finish with your paper please wait for the invigilator to arrive at your desk at which time you provide your answer sheet. If you fail to do so and your paper is lost we will not be held responsible.
- Please be seated till the end even if you have finished with your exam early. Leave only after the invigilators confirm that all papers have been accounted for.
- Problem 1 had to be done from scratch without assuming any formula.
- Any extra sheet should have your roll no and name. If you fail to do so we can't accept that as a valid document.
- Please state any important result which you are using to find solution.

Problems

1. Reduce the problem

$$u_{xx} + u_{xy} + u_{yy} + u_x = 0$$

to its canonical form. [10 Marks]

2. Find the solution of the problem

$$y^3 u_{xx} + u_y - y u_{yy} = 0$$
$$u(x,y) = x \text{ on } \{x + \tfrac{y^2}{2} = 4\} \text{ for } 2 \leq x \leq 4$$
$$u(x,y) = \tfrac{x^2}{2} \text{ on } \{x - \tfrac{y^2}{2} = 0\} \text{ for } 0 \leq x \leq 2$$

provided $y > 0$. [10 Marks]

3. Prove or Disprove: The problem

$$-\Delta u(x,y) = 0 \text{ in } \Omega$$
$$u(x,y) = \exp(x^2+y^2) \text{ on } \partial\Omega$$

admits an unique solution with $u(x_0,y_0) = \frac{1}{2}$ provided Ω is an open bounded domain in $\mathbf{R}^2$ and $(x_0,y_0) \in \Omega$. [5 Marks]

4. Define $A := \{(x,t) : 0 < x < \pi, 0 < t < T\}$ and let u solves

$$u_t - u_{xx} = 0 \text{ in } A$$
$$u(0,t) = 0,\ u(\pi,t) = 0,\ 0 \le t \le T$$
$$u(x,0) = \sin^2 x,\ 0 < x < \pi$$

Using maximum principle show that $0 \le u(x,t) \le e^{-t}\sin x$ in A.

[10 Marks]

5. Use Separation of Variable to solve the following:

$$u_t - 4u_{xx} = 0;\ x \in (0,l),\ t > 0$$
$$u(0,t) = 0,\ u(l,t) = 0,\ t \ge 0$$
$$u(x,0) = f(x),\ 0 \le x \le l$$

provided f is a smooth bounded function. [10 Marks]

— END —