MSO203B PART C

OM SHRIVASTAVA

TOTAL POINTS

28 / 30

QUESTION 1

1 Question 1 6 / 8

- **+ 0 pts** Incorrect/not attempted
- **+ 2 pts** As it is a Regular Sturm Liouville problem, so all the eigen values are real.
- ✓ **+ 2 pts** *For $$\lambda<0$$ the general solution is given by $$Ae^{kt}+Be^{-kt}$$ and it is shown that $$\lambda<0$$can't be an eigen value.*
- ✓ **+ 2 pts** *For $$\lambda=0$$ general solution is given by u(t)=At+B. The eigen pair is given by (0,constant).*
- ✓ **+ 1 pts** *For $$\lambda>0 $$, ($$n^2\pi^2, Acos(n \pi x))$$ is an eigen pair. 1 mark for writing the first component correctly.*
- ✓ **+ 1 pts** *1 mark for writing the second component of the eigen pair correctly.*
- **+ 8 pts** Completely Correct.

QUESTION 2

2 Question 2 10 / 10

- ✓ **+ 10 pts** *Everything correct*
- **+ 1 pts** u(x,t)=X(x)T(t)
- **+ 0 pts** Completely wrong
- **+ 1 pts** $$\frac{X(x)''}{X(x)}+\frac{T''(t)}{T(t)}+\lambda=0$$
- **+ 2 pts** $$\frac{X''(x)}{X(x)}=-\lambda-\frac{T''(t)}{T(t)}=\mu$$ or $$\frac{T''(t)}{T(t)}=-\lambda-\frac{X''(x)}{X(x)}=\mu$$
- **+ 1 pts** $$X'(0)=X'(1)=0$$ or $$T'(0)=T'(1)=0$$
- **+ 2 pts** $$\mu_{n}=-n^{2}\pi^{2}$$: $$n=0,1,2,...$$
- **+ 1 pts** $$T'_{n}(0)=T'_{n}(1)=0$$ or $$X'(0)=X'(1)=0$$
- **+ 2 pts** $$\lambda_{{n},{m}}=\pi^{2}(n^{2}+m^{2})$$: $$n,m=0,1,2,...$$

QUESTION 3

3 Question 3 and 4 together 12 / 12

- ✓ **+ 6 pts** *Q3- fully Correct*
- **+ 3 pts** Q3:partially correct with no wrong answer
- **+ 1 pts** Q3: Not attempted
- **+ 0 pts** Q3: Wrong option selected
- ✓ **+ 6 pts** *Q4: FULL Y CORRECT*
- **+ 3 pts** Q4: partially correct answer with no wrong answer
- **+ 1 pts** q4: not attempted
- **+ 0 pts** q4: wrong answer selected

END SEMESTER: MSO-203 (PART- C) Submitting time- 8:30 PM

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Q1 [8] Find the eigenpairs of the $-u'' = \lambda u$ *on* $(0,1), u'(0) = u'(1) = 0$.

⇒ By the standard method

Case I: $\lambda > 0$ $\quad \lambda = m^2 \Rightarrow u'' + m^2 u = 0$

Soul Solution of it e is $(A\sin mx + B\cos mx)$

$u = A\sin mx + B\cos mx$

$u' = mA\cos mx - mB\sin mx$

$u' = m(A\cos mx - B\sin mx)$

$u'(0) = mA = 0$

so $A=0$ since $\underline{m \neq 0}$

$u'(1) = m(-B\sin m) = 0$

here $m \neq 0$, $B \neq 0$ ... for non-trivial solution.

~~$A = \frac{2\pi}{\pi}$~~ $m = n\pi$ ... here n = integer

$\Rightarrow u = -n\pi B\sin mx$

$u = -n\pi B\sin n\pi x$ here $m = n\pi$, $\lambda = (n\pi)^2$

Also we can see that n is natural integer hence λ is real eigenvalue.

Case II: $\lambda = 0$, $u'' = 0$

$\boxed{u = Ax + B}$

$u' = A$

so we get $A=0$ since $u'(0) = A = 0 = u'(1)$

$\Rightarrow u = B$ and $\lambda = 0$, here again we can see $\lambda = 0$ is real eigen value

Case III: $\lambda < 0$ $\quad \lambda = -m^2$

$u'' = m^2 u$

$u = Ae^{mx} + Be^{-mx}$

$u' = mAe^{mx} - Bme^{-mx}$

$u'(0) = (A-B)m = 0 \rightarrow \underline{A = B}$

$u'(1) = m(Ae^{m} - Be^{-m}) = 0 \Rightarrow A(e^{2m} - 1) = 0$

So if $e^{2m} = 1 = e^{2n\pi i}$

$m = n\pi i$

$\boxed{m^2 = -n^2\pi^2}$

this will contradict fact that $\lambda < 0$ $\lambda = -m^2 < 0$ since $m^2 > 0$. So $-m^2 > 0$ hence $\lambda > 0$.

So ~~only trivial~~ no negative eigenvalue is possible.

Eigen pair we get are $[(n\pi)^2, B\cos n\pi x]$ and $(0, [0, B])$
(eigen value, eigenfunction)

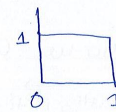
Where B is some arbitrary constant.

OR simply both can be represented as $[(n\pi)^2, B\cos n\pi x]$.

Q2 [10] Find eigenvalues of the problem $-\triangle u = \lambda u$ *in* Ω, *on* $(0,1)\times(0,1)$ and $u_y(x,0) = u_y(x,1) = 0, u_x(0,y) = u_x(1,y) = 0$.

⇒ $-\Delta u = \lambda u$ .... given

By separation of variable $u = F(x)G(y)$



$\Rightarrow (F''G + G''F) = -\lambda F(x)G(y)$

$\Rightarrow \frac{F''}{F} + \frac{G''}{G} = -\lambda$

$\Rightarrow -\frac{F''}{F} = \frac{G''}{G} + \lambda = K$ ↑ some constant since both are functions of different variables

$\Rightarrow F'' = -FK$ $\quad \boxed{K = m^2}$

let it has a solution $F = A\sin mx + B\cos mx$ ~~$Ae^{\sqrt{K}x} + Be^{-\sqrt{K}x}$~~

So ~~$F'(x) = A\sqrt{K}e^{\sqrt{K}x} + Be$~~ $F'(x) = Am\cos mx - Bm\sin mx$

$F'(0) = Am = 0$

$\boxed{A = 0}$

now $F'(1) = \cancel{Am\cos m} - Bm\sin m = 0 \Rightarrow \sin m = 0$

$m = n\pi$ .....where n = some integer

So now $k = m^2 = (n\pi)^2$

So we have $\frac{G''}{G} + \lambda = (n\pi)^2$

$\frac{G''}{G} = (n\pi)^2 - \lambda$

now solving for $\frac{G''}{G}$ we get let $(n\pi)^2 - \lambda = -\underbrace{z^2}_{\text{Eigenvalue}}$ — (i)

we get general solution as $A\cos zy + B\sin zy = G(y)$

$G'(y) = -Az\sin zy + Bz\cos zy$

$G'(0) = Bz = 0$

$\boxed{B=0}$ since $z \neq 0$.

$G'(1) = -Az\sin z = 0$

$z = q\pi$ ... q is some arbitrary integer.

So $G(y) = A\cos q\pi y$

Eigenvalue $= -(q\pi)^2$

So we get by (i) $(n\pi)^2 + (q\pi)^2 = \lambda$

$\lambda = (q^2+n^2)\pi^2$ ... Eigenvalue for the problem.

Or also we can write that

$U = F(x)G(y) = (A\sin mx + B\cos mx)(C\sin zy + D\cos zy)$

Putting initial condition in this manner

$U_x = (Am\cos mx - Bm\sin mx)(C\sin zy + D\cos zy)$

$U_x(0,y) = Am(C\sin zy + D\cos zy) = 0$

$\boxed{A=0}$

$U_x(1,y) = (-Bm\sin m)(C\sin zy + D\cos zy) = 0$

$m = (n\pi)$

so $k = (n\pi)^2$

Similarly we get By $U_y(x,0) = \boxed{C=0}$ .. similarly.

$U_y(x,1) = \sin zy = 0$

$z = q\pi$ .. q is integer

So Eigenvalue we took was $-z^2 = -(q\pi)^2$

By Equation (i) we have $(n\pi)^2 - \lambda = -z^2$

$\boxed{(n\pi)^2 + (q\pi)^2 = \lambda}$

So this would be the Eigenvalue of Neumann Laplace Eigenvalue Problem.

Each Multiple choice question has 6 marks. There can be more than one correct answers. Choosing only all correct answers will give you 6 marks. Partial correct answers with no wrong answers will give you 3 marks. Not answering the question will give you 1 marks. whenever wrong answer is selected you will get 0 marks. No justification is required for MCQ questions. Just put tick mark on the correct option (in your opinion) on the question itself.

Q3 Consider the function $f(x) = |\sin(x) + \cos(x)|$ on real line. Then,

a) f is a periodic function.

b) fundamental period of f is 2π.

c) f is a differentiable function.

d) fundamental period of f is π.

e) None of the above

Q4 Which of the following statements are correct

a) If $|f|$ is a periodic then f is not necessarily periodic.

b) If f is periodic then f is necessarily a continuous function.

c) The functions $f(t) = \sin(t)$ and $g(t) = \cos(2\pi t)$ are periodic, but $f + g$ is not a periodic function.

d) If f is a periodic functions then f^2 is also a periodic function.

e) None of the above