Assignment 6 Solutions

1 $x(t) = x(t-T)$ holds for all t, including $t = T$ for example. Thus, $x(t) = x(t-T)$ and $x(t-T) = x(t-2T)$ so that $x(t) = x(t-2T)$. Assume $x(t) = x(t-kT)$, then $x(t-kT) = x(t-(k+1)T) = x(t)$. The proof is completed by induction. A similar proof can be made for $x[n] = x[n-kN]$.

(i) $e^{j\theta} = e^{j(\theta+2\pi)}$. Hence $e^{j2t} = e^{j(2\pi+2t)} = e^{j2(t+\pi)}$

Hence it is periodic with period π

(ii) $e^{j(6t-30^\circ)} = e^{j(6t-30^\circ-2\pi)} = e^{j(6(t-\pi/3)-30^\circ)}$

Periodic with period $\pi/3$.

(iii) $e^{2-j5t} = e^{2}e^{-j5t}$. periodic due to e^2 being a constant

(iv) $e^{-j2t} + e^{j5t}$. Components have periods of π and $2\pi/5$. ie, periods π and 0.4π. LCM is 2π. So the sum has a period of 2π

(v) $e^{j2t} + e^{j\pi t}$. Component periods are π and 2. One is irrational, the other is rational. Hence LCM $= \infty$. Hence non periodic.

2 Straightforward matter of substituting the signals in the expressions for $x_e(t)$, $x_o(t)$.

3 (i) $u[n] - u[n-5]$. Finite energy E_∞. zero (finite) average power P_∞

(ii) $\cos(4\pi t + 60^\circ)$. Periodic, hence $P_\infty < \infty$, E_∞ not finite.

(iii) $2^{-|t|}e^{(-3-4\pi)t} = [2^{-|t|}e^{-3t}]e^{-j4\pi t}$. First part of the product is unsymmetric about $t=0$: decays for $t>0$ but for $t<0$, it evaluates to $2^{t}e^{-3t}$ which grows larger as $t \to -\infty$ because e^{-3t} is stronger than 2^{t}. Hence the product $2^{-|t|}e^{-3t}e^{-j4\pi t}$ grows as $t \to -\infty$ (because the third factor is periodic) Hence both E_∞ and P_∞ are not finite.

when a signal has nonzero and finite P_∞, $E_\infty \to \infty$.
When E_∞ is finite and nonzero, $P_\infty = 0$.

4. Not true in general. Possibilities include

(1) $p(t) = p_1(t) \cdot p_2(t)$ when $x_1(t) x_2(t) = 0$

(2) $p(t) < p_1(t) \cdot p_2(t)$ when $x_1(t) x_2(t) < 0$

(3) $p(t) > p_1(t) \cdot p_2(t)$ when $x_1(t) x_2(t) > 0$

Example for case (1), $x_1[n] = 1 + (-1)^n$; $x_2[n] = 1 + (-1)^{n-1}$

case (2), $x_1[n]$ as above; $x_2[n] = -x_1[n]$

case (3) $x_1[n]$ as above; $x_2[n] = x_1[n]$

Condition for equality: $x_1[n] x_2[n] = 0$ or $x_1(t) x_2(t) = 0$

$E_{(t_1,t_2)} = \int_{t_1}^{t_2} p(t)\,dt = \int_{t_1}^{t_2} |x(t)|^2 dt$. Condition for

$E_{(t_1,t_2)} = E_{1:(t_1,t_2)} + E_{2:(t_1,t_2)}$ comes out to be

$\int_{t_1}^{t_2} x_1(t) x_2^*(t)\,dt = 0$. This condition is called <u>orthogonality</u>

For $E_{(t_1,t_2)} < E_{1:(t_1,t_2)} + E_{2:(t_1,t_2)}$. Condition: $\int_{t_1}^{t_2} x_1(t) x_2^*(t)\,dt < 0$

For x_1, x_2 real, this comes to $\int_{t_1}^{t_2} x_1(t) x_2(t)\,dt < 0$ (negatively correlated)

For $E_{(t_1,t_2)} > E_{1:(t_1,t_2)} + E_{2:(t_1,t_2)}$, x_1, x_2 must be positively correlated: $\int_{t_1}^{t_2} x_1(t) x_2(t)\,dt > 0$.

5. By the nature of the decomposition, $x_e(t), x_o(t)$ are orthogonal: $(x_e(t) + x_o(t))^2 = x_e^2(t) + x_o^2(t)$. Hence $E_\infty = E_{e:\infty} + E_{o:\infty}$ and $P_\infty = P_{e:\infty} + P_{o:\infty}$.

6.

	(Less) Memory	Causality	Stability	Linearity	Time Invariance
(a)	✓	✓	✗	✓	✗
(b)	✓	✓	✓	✓	✗
(c)	✗	✗	✓	✓	✗
(d)	✗	✗	✓	✗	✗
(e)	✓	✓	✓	✗	✓
(f)	✗	✗	✓	✓	✗
(g)	✗	✗	✓ *	✓	✗
(h)	✗	✗	✓ **	✓	✓

* Assuming that $x(t)$ contains no impulses.

** Assuming the d_k ; $k = -3, \ldots, +3$ to be finite constants.