EE 200 Signals, Systems & Networks 2023

End Sem Exam (B)

21/11/2023
30 Marks
120 min

1 (a) Find the Laplace Transform of the unit step $u(t) = u_0(t) \leftrightarrow U_0(s)$. Use the convolution property to find $U_k(s)$ for $u_k(t) = t^k u(t); k > 0$. Sketch the pole-zero plots and ROC for $U_0(s), U_1(s)$. 3

(b) Let $h(t) = \begin{cases} 0; & t < 0 \\ 1-t; & 0 \le t < 2 \\ -1; & t \ge 2 \end{cases}$. Sketch $h(t)$. Write $h(t)$ as a linear combination of suitably scaled and time shifted $h_k(t) = u_k(t); k \ge 0$. Finally, find $H(s)$ using the results of $1(a)$ above. Sketch the poles and zeroes and the ROC for $H(s)$. 3

(c) Build the complete block diagram of $h(t)$ by interconnecting the components (i) unit delay blocks ($y(t) = x(t-1)$); (ii) α-gain blocks ($y(t) = \alpha x(t)$) and (iii) unit step response blocks ($y(t) = x(t) * u(t)$). 4

2 (a) Find the FT of the signum function, given by $\operatorname{sgn}(t) = \begin{cases} t/|t|; & t \neq 0 \\ 0; & t = 0 \end{cases}$. 3

(b) $x(t)$ is real valued and causal. Find $X_o(\omega)$ in terms of $X_e(\omega)$. 3

(c) Use the extended CTFT and the convolution property to find $X(\omega)$ for $x(t)$ shown in Fig.2. Calculate $|X(\omega)|, \underline{/X(\omega)}$. 4

3 (a) Look at Fig.3. If $v_C(0) = -3\,\text{V}$, find and sketch $v_{RL}(t), i_{RL}(t), i_{RS}(t), e_C(t), p_{RS}(t), p_{RL}(t)$ over $-1 < t < 3$s. (follow polarities/directions shown) if the switch is closed at $t = 0$. $e_C(t), p_{RS}(t), p_{RL}(t)$ are capacitor energy, power in R_S, power in R_L respectively. Find $v_L(\infty), i_L(\infty)$. Identify the time instant at which the capacitor is uncharged. Make the origins of the time axis in all the 6 plots lie in the same vertical line. 6

(b) If we repeated the above problem for the case when $v_C(0) = 6\,\text{V}$, which of the above plots would be affected? Identify the plots that would remain the same and replot the ones which would change. 4

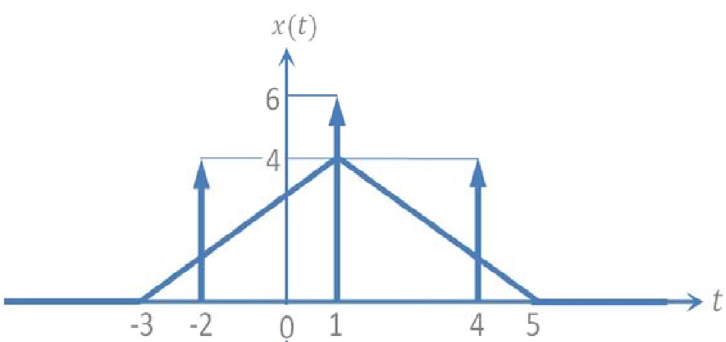


Fig.2

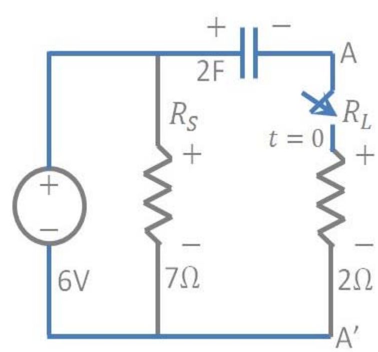


Fig.3

SOLUTIONS

$1(a)$ $U_0(s) = \int_0^\infty e^{-st}dt = [e^{-st}/(-s)]|_0^\infty = 1/s$. Since $u(t)$ is causal, $ROC_{U(s)} : \sigma > 0$.
Next, $u_1(t) = tu(t) = u_0(t) * u(t) \rightarrow U^2(s) = s^{-2} : \sigma > 0$ because $u_1(t)$ is causal.
Generalizing, $u_k(t) = u_{k-1}(t) * u(t) = (t/k)u_{k-1}(t)$. So, $t^k u(t) = k!u_k(t) \rightarrow U_k(s) = k!/s^{k+1}$. All $u_k(t); k \geq 0$ are causal, so, $ROC_{U_k(s)}; \sigma > 0$.

$1(b)$ $h(t) = u(t) - tu(t) + (t-2)u(t-2) = u_0(t) - u_1(t) + (t-2)u_1(t-2)$.
$H(s) = 1/s - 1/s^2 + e^{-2s}/s^2$, because $(t-2)u(t-2) = u_1(t-2) \rightarrow e^{-2s}/s^2$.
Single pole at $s = 0$ plus double pole at $s = 0$. Zeroes along the $j\omega$ axis at $\omega = n\pi; -\infty < n < \infty$ due to the numerator term $(1 - e^{-2s})$. Pole - zero cancellation at $s = 0$ leaves only a double zero at $s = 0$.

$1(c)$ Block Diagram shown below

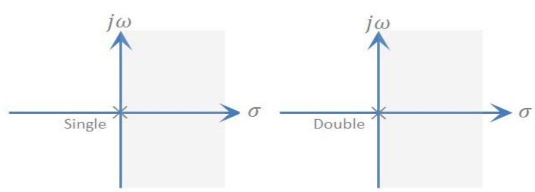


Fig.1(a)

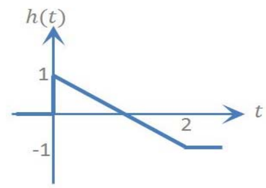


Fig.1(b)

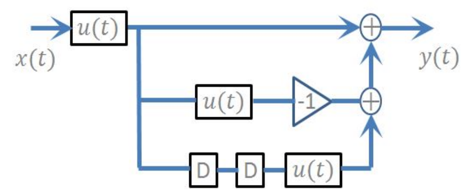


Fig.1(c)

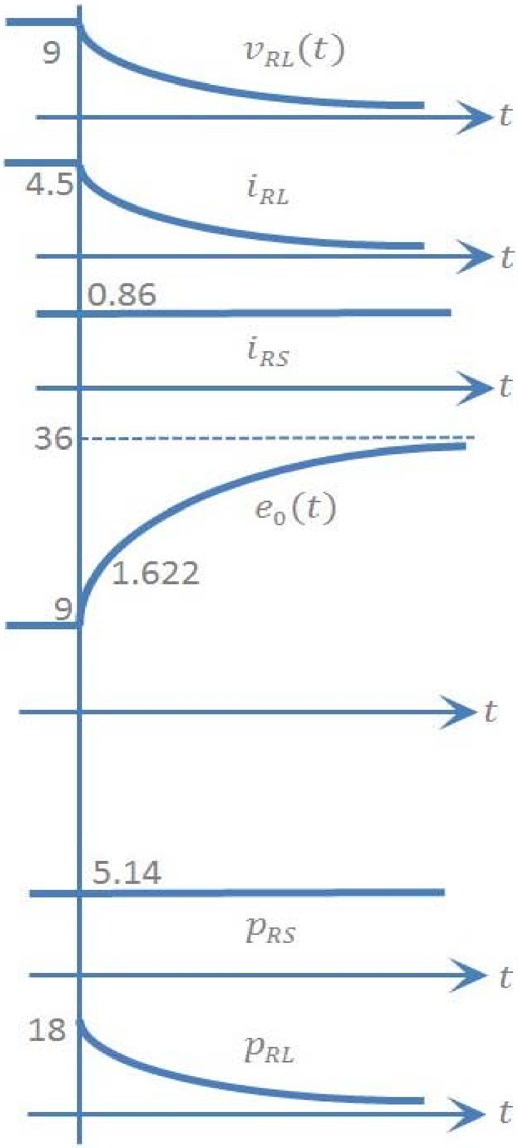
$2(a)$ $\text{sgn}(t) = u(t) - u(-t) \rightarrow [1/jw + \pi\delta(\omega)] - [1/j(-\omega) + \pi\delta(-\omega)] = 2/jw$.

$2(b)$ Let $x(t) \rightarrow X(\omega)$. Then, for $t < 0, x_e(t) = -x_o(t)$, because $x(t) = 0; t < 0$ and for $t > 0, x_e(t) = x_o(t) = \frac{1}{2}x(t)$. So $x_o(t) = x_e(t)\text{sgn}(t)$ and $x_e(t) = x_o(t)\text{sgn}(t)$. Thus, $X_e(\omega) = \frac{2}{j\omega} * X_o(\omega)$.

$2(c)$ First shift $x(t)$towards the left by 1, to get $x'(t) = x(t+1)$. Rest of the analysis will be for $x'(t)$. Express $x'(t)$ as $x'_c(t) + x'_d(t)$, where $x'_c(t)$ is just the triangular function without the impulses and $x'_d(t)$ consists of only the 3 impulses. Then $x'_c(t) = x'_r(t) * x'_r(t)$, where $x'_r(t) = \begin{cases} 1; & |t| \leq 2 \\ 0; & |t| > 2 \end{cases}$ has been designed to satisfy both the support and area constraints for $x'_r(t) * x'_r(t) = x'_c(t)$. So, $X'_r(\omega) = (2\sin 2\omega)/\omega$ and $X'_c(\omega) = X'^2_r(\omega)$. Next, the impulse pair $4\delta(t \pm 3) \rightarrow 8\cos 3\omega$ and the single impulse $6\delta(t) \rightarrow 6$. Overall, $X'(\omega) = \frac{4}{\omega^2}\sin^2 2\omega + 8\cos 3\omega + 6$, and $X(\omega) = X'(\omega)e^{-j\omega}$. So $X(\omega) = |X'(\omega)|\angle[-\omega] = \left[\frac{4}{\omega^2}\sin^2 2\omega + 8|\cos 3\omega| + 6\right] \angle[-\omega]$.

$3(a)$ $v_C(t) = (v_C(0) - v_C(\infty))e^{-t/R_L C} + v_C(\infty)$;
$v_{RL} = V_S - v_C$; $i_{RL} = v_{RL}/R_L$; $e_C = Cv_C^2/2$; $p_{RL} = v_{RL}^2/R_L$; $p_{RS} = V_S^2/R_S$;
$v_{RL}(\infty) = i_{RL}(\infty) = 0$;
Time at which $v_C = 0$ is $-R_L C \ln[-v_C(\infty)/(v_C(0) - v_C(\infty))] = 1.622\,\text{s}$.

$3(b)$ $v_C(t) = 6\,\text{V}$; $v_{RL}(t) = i_{RL}(t) = p_{RL}(t) = 0\,\text{V}$, $e_C(t) = 36\,\text{J}$ are changed
$i_{RS}(t) = V_S/R_L$; $p_{RS}(t) = V_S^2/R_L$ are unchanged.



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$v_{RL}(t)$
t
4.5
i_{RL}
t
0.86
i_{RS}
t
36
$e_0(t)$
1.622
9
t
5.14
p_{RS}
t
18
p_{RL}
t