| Name: | Roll. No.: | Section: | Duration: 55 mins<br>Max. marks: 30<br>No. of questions: 2 |
|---|---|---|---|
| | | | |

# Mid-sem Exam – Part A

ESc201A

**Write your answers here only.**

Q 1. (a) $N$ bulbs of resistance $r$ each are connected in series in a straight line to form a setup of length $l$, driven by a DC supply. Our multimeter reads the currents and voltages at a rate 1000 samples per second. Which of the following assumptions need to be satisfied if we were to replace this setup of all the bulbs with a single resistance $Nr$ ? (Mark √ against them) (5 marks)

- ✓ There should be no charge leakage from the setup
- The charge leakage from the setup should be constant
- There should be no flux leakage from the setup
- ✓ The flux leaking from the setup should be constant
- ✓ The length $l$ must be less than 300 km
- The length $l$ must be less than 3 km

$d = c \times T$

$= c/f$

$= \frac{3\times10^8 \text{ m/s}}{1000 \text{ samples/s}}$

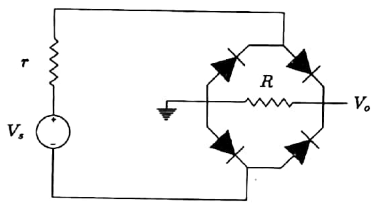
$= 3\times10^5$ m

$= 300$ km

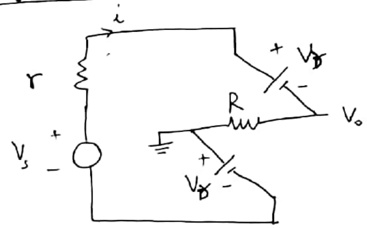
Q 1. (b) Consider the following circuit. Assume constant voltage diode model ($V_\gamma = 0.7\,V, r_D = 0$).

i. Draw $V_o$ (y-axis) vs $V_s$ (x-axis, +ve and -ve). Mark all the points clearly. (5 marks)

ii. Write the expression for $V_o$ in terms of given quantities and $V_s$. (5 marks)




When $V_s > 0$ & $i > 0$




using KVL,

$-V_s + ir + V_\gamma + V_o + V_\gamma = 0$

Also, $i = V_o/R$

$\therefore\ V_o + 2V_\gamma + \frac{V_o r}{R} = V_s$

$$\boxed{V_o = \frac{V_s - 2V_\gamma}{\left(1+\frac{r}{R}\right)}}$$

$i = \frac{V_o}{R}$ is $> 0$ when

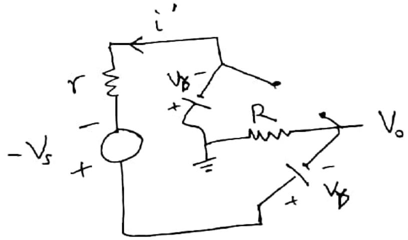
$V_s - 2V_\gamma > 0$

$\Rightarrow V_s > 2V_\gamma$

when $V_s \in (0, 2V_\gamma)$, then $i = 0$

$\therefore\ \boxed{V_o = iR = 0}$

when $V_s < 0$ & $i' > 0$




using KVL,

$-V_s - i'r - V_\gamma - V_o - V_\gamma = 0$

and $V_o = i'R$

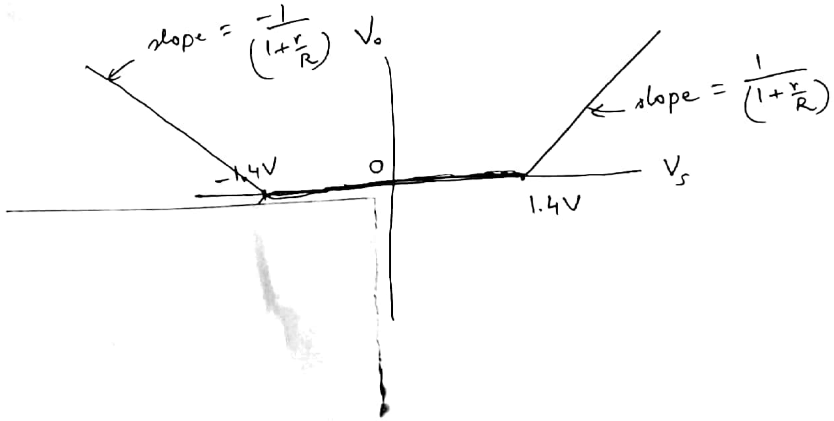
$\therefore\ \boxed{V_o = \frac{-V_s - 2V_\gamma}{\left(1+\frac{r}{R}\right)}}$

$i' = \frac{V_o}{R}$ is $> 0$ when $-V_s - 2V_\gamma > 0$

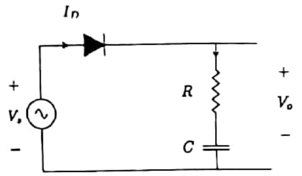
$\Rightarrow V_s < -2V_D$

when $V_s \in (-2V_\gamma, 0)$, then $i' = 0$

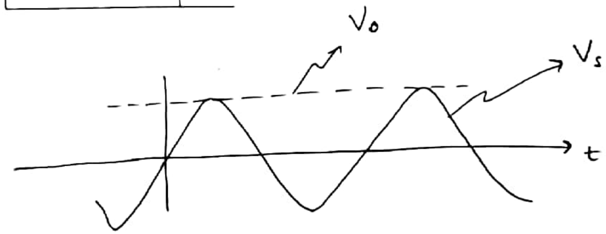
$\boxed{V_o = i'R = 0}$



**Q 2. (a)** In the single wave rectifier circuit, a student connected R in series with C, as shown below. Draw how will $V_o$ look like with respect to time. (3 marks)




if someone draws Vo
about R, we can give marks.
It will be 0 V in steady state.
No need to give or deduct marks for transient response.



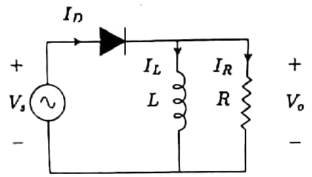

C is charged and $I_D = 0$ always.
C does not get discharged.

**Q 2. (b)** In the single wave rectifier circuit, a student connected an inductor $L$ instead of a capacitor. See figure below. Assume an ideal diode ($V_\gamma = 0, r_D = 0$) to answer the following:

i. Assuming the diode to be ON, plot the currents in the inductor, resistor and diode, as a function of time. (4 marks)

ii. Mark the time instance when the diode will turn OFF. (Hint: direction of diode current). Calculate this time instance. (4 marks)

iii. During the OFF state of the diode, plot the currents in the inductor, resistor and diode, as a function of time. (4 marks)




$V_s = V_m \cos\omega t$

Assuming ~~Ass~~ the diode is ON:-

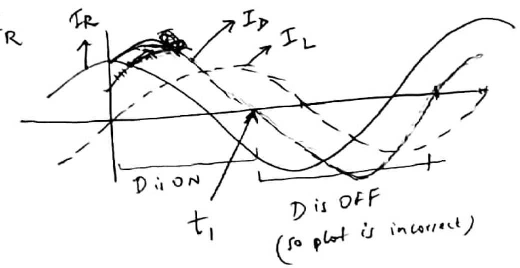
i) a) $L\frac{dI_L}{dt} = V_o = V_s = V_m\cos(\omega t) \Rightarrow I_L = \int \frac{V_m}{L}\cos\omega t\, dt$

$\Rightarrow \boxed{I_L = \frac{V_m}{\omega L}\sin\omega t}$

b) $I_R R = V_o = V_s = V_m\cos(\omega t)$

$\Rightarrow I_R = \frac{V_m}{R}\cos\omega t$

c) $I_D = I_L + I_R$




ii) $t_1$ is the time instance when the diode will turn OFF.

$$\frac{V_M}{R}\cos\omega t_1 + \frac{V_m}{\omega L}\sin\omega t_1 = 0 \quad ; \quad \omega t_1 \in \left(\frac{\pi}{2}, \pi\right)$$

$$\frac{\omega L}{\sqrt{(\omega L)^2 + R^2}}\cos\omega t_1 + \frac{R}{\sqrt{(\omega L)^2+R^2}}\sin\omega t_1 = 0$$

$$\cos(\omega t_1 - \phi) = 0$$

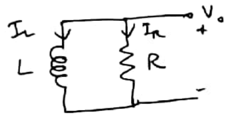
$$\tan\phi = \frac{R}{\omega L}$$

$$\omega t_1 - \phi = \frac{\pi}{2}$$

$$\omega t_1 = \frac{\pi}{2} + \tan^{-1}\frac{R}{\omega L}$$

$$\boxed{t_1 = \frac{1}{\omega}\left(\frac{\pi}{2} + \tan^{-1}\frac{R}{\omega L}\right)}$$

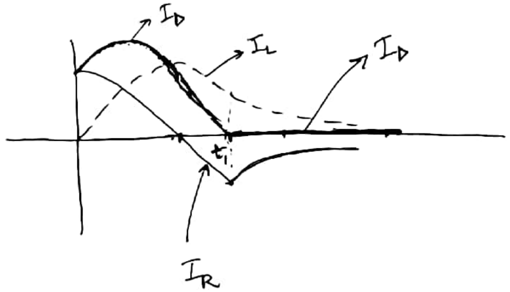
iii) When Diode ~~is~~ is off,




$I_L + I_R = 0 \Rightarrow I_R = -I_L$

and $I_L = I_{L,0}\, e^{-\frac{tR}{L}}$

here, $I_{L,0} = \frac{V_m}{\omega L}\sin(\omega t_1)$



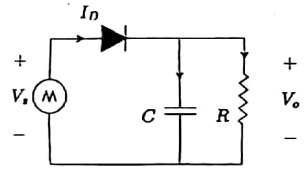
| Name: | Roll. No.: | Section: | Duration: 55 mins<br>Max. marks: 30<br>No. of questions: 2 |
|---|---|---|---|

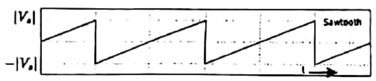
# Mid-sem Exam – Part B

ESc201A

**Write your answers here only.**

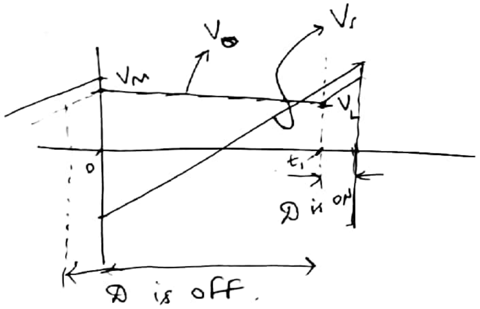
Q 3. (a) In the single wave rectifier circuit, a student connected a sawtooth voltage source, instead of a sinusoidal one, as shown below. The frequency of source is $f$ and the source voltage is plotted below. Assume a constant voltage diode model ($V_\gamma = 0.7\ V, r_D = 0$). Find the peak diode current. (5 marks)




$V_o = V_s - 0.7$

when $D$ is ON




$\therefore \boxed{V_M = V_s - 0.7}$

$V_c = V_s - 0.7$ when $D$ is ON

$$I_D = C\frac{dV_o}{dt} + \frac{V_o}{R}$$

$$= C\frac{2|V_s|}{T} + \frac{V_o}{R}$$

max $V_o = V_m$

$$\max I_D = \frac{2C|V_s|}{T} + \max\frac{V_o}{R} = \boxed{\frac{2C|V_s|}{T} + \frac{|V_s| - 0.7}{R}}$$

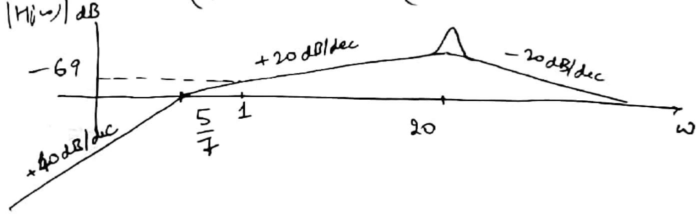
Q 3. (b) For the transfer function written below:

$$H(j\omega) = \frac{j\omega}{\left(2 + j\omega + \frac{400}{j\omega}\right)(5 + 7j\omega)}$$

i. Draw Bode magnitude plot (5 marks)

ii. Estimate the Q-factor for the resonance. (Hint: use asymptotic assumptions wherever needed) (5 marks)

i) $H(j\omega) = \dfrac{(j\omega)^2}{((j\omega)^2 + 2j\omega + 400)(5 + 7j\omega)}$




for $(5 + 7j\omega)$

$\omega_c = \frac{5}{7}$

for $2 + j\omega + \frac{400}{j\omega}$

$\omega_c^2 = 400$

$\boxed{\omega_c = 20}$

At $\omega = 1$,

$|H(j\omega)| = \dfrac{1}{400 \times 7}$

$20\log_{10}|H(j\omega)| = -20\log_{10}(2800)$

$= -69$

ii) $Q$ factor $= \dfrac{\omega_c}{\text{bandwidth}}$

To compute bandwidth:

$$\frac{1}{\sqrt{2^2 + \left(\omega_1 - \frac{400}{\omega_1}\right)^2}} = \frac{1}{\sqrt{2}} \times \underbrace{\frac{1}{2}}_{\text{max. value (at } \omega_c)}$$


$$\left(\omega_1 - \frac{400}{\omega_1}\right)^2 = 2^2$$

$$\omega_1^2 \pm 2\omega_1 - 400 = 0$$

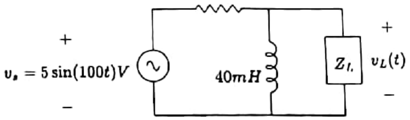
$$\Rightarrow \omega_1 = \frac{\pm 2 + \sqrt{4 + 4 \times 400}}{2}$$

(−ve sign ignored $\because \omega_1 > 0$)

$$\therefore \omega_1' - \omega_1 = \frac{2 \times 2}{2} = 2$$

$$\therefore Q = \frac{20}{2} = \boxed{10}$$ Ans

**Q 4.** For the AC circuit shown below, voltage is given in terms of peak value.

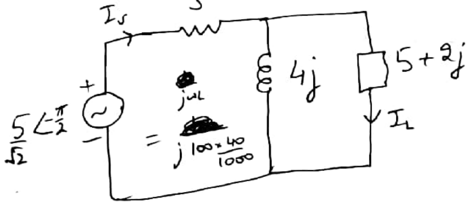



i. If $Z_L = 5 + 2j\ \Omega$, draw the phasor diagram showing $V_s$, $I_s$ (current leaving the +ve terminal of voltage source), $V_L$ and $I_L$ (current entering the +ve terminal of $Z_L$). Mark the amplitude and phase of each quantity. (6 marks)

ii. If $Z_L$ is variable, find the value of $Z_L$ such that the reactive power supplied to the load has maximum magnitude. Also, write the value of reactive power. (Hint: use Thevenin's equivalent for easier calculations) (9 marks)

i) $V_s = \frac{5}{\sqrt{2}} \angle -\frac{\pi}{2}$ V (rms)

$= -3.5j$




$I_s = \dfrac{5(-j)}{\sqrt{2}(4.3+2.4j)}$

$= -0.35 - 0.63j$

$= 0.7\angle -120°$ A (rms)

$Z_{net} = 3 + (4j \,||\, (5+2j)) = 3 + \dfrac{(4j)(5+2j)}{4j+5+2j}$

$= 3 + \dfrac{(-8+20j)}{(5+6j)}$

$= 4.3 + 2.4j$

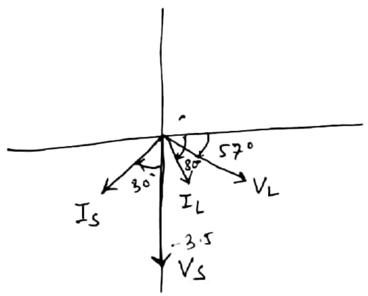
$V_L = V_s - 3I_s$

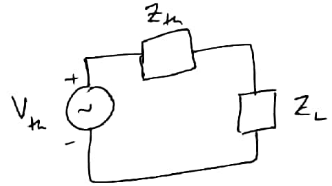
$= \frac{5}{\sqrt{2}}(-j) + 3(0.35 + 0.63j)$

$= 1.05 - 1.6j$ V (rms) $= 1.95\angle -57°$

$I_L = \dfrac{V_L}{5+2j} = 0.068 - 0.36j$ A (rms)

$= 0.37\angle -79°$




$V_{th} = \dfrac{4j}{4j+3} \cdot \dfrac{5(-j)}{\sqrt{2}}$

$= 1.7 - 2.3j = 2.8\angle -53°$

(Let $Z_L = R_L + jX_L$)

$Z_{th} = 3 \,||\, 4j$

$= 1.92 + 1.44j$

$= 2.4\angle 37°$

$Q_L = \text{Im}(V_L I_L^*)$

$= \text{Im}\left(V_L \dfrac{V_L^*}{Z_L^*}\right) = \text{Im}\left(\dfrac{|V_L|^2 Z_L}{|Z_L|^2}\right)$

$= \left|\dfrac{V_L}{Z_L}\right|^2 X_L = |I_L|^2 X_L$

$= \dfrac{|V_{th}|^2}{|Z_{th} + Z_L|^2} X_L$

$= \dfrac{|V_{th}|^2 \, X_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$

$\dfrac{\partial Q_L}{\partial R_L} = 0$ gives −ve $R_L$, which is not possible

$\therefore$ Q is min w.r.t. $R_L$ if $\boxed{R_L = 0}$

$\dfrac{\partial Q_L}{\partial X_L} = 0 \Rightarrow \dfrac{\partial}{\partial X_L}\left(\dfrac{X_L}{R_{th}^2 + (X_{th} + X_L)^2}\right) = 0$

$\Rightarrow R_{th}^2 + (X_{th} + X_L)^2 - X_L\, 2(X_{th} + X_L) = 0$

$R_{th}^2 + X_{th}^2 + \cancel{X_L^2} + \cancel{2X_{th}X_L} - \cancel{2X_{th}X_L} - \cancel{2}X_L^2 = 0$

$X_L^2 = X_{th}^2 + R_{th}^2$

$\therefore X_L^* = \sqrt{R_{th}^2 + X_{th}^2} = |Z_{th}|$

$\therefore \boxed{Z_L^* = |Z_{th}|j = 2.4j\ \Omega}$

$Q_L$ at $Z_L^* = \dfrac{(2.8)^2}{(4.3)^2} \times 2.4 = \boxed{1.02 \text{ VAR}}$