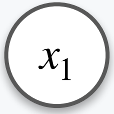
DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Gradient Descent and Backpropagation

Back Propagation Introduction

Back Propagation Introduction



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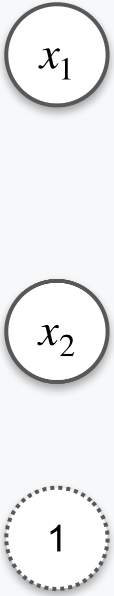

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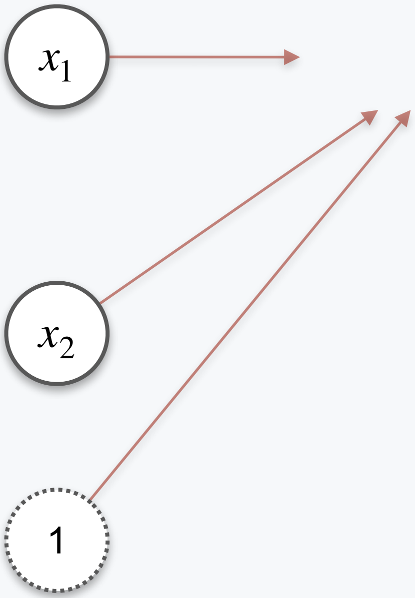

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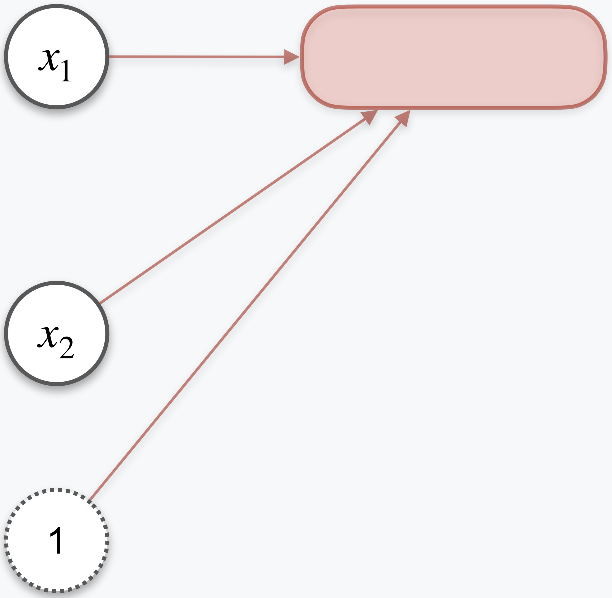

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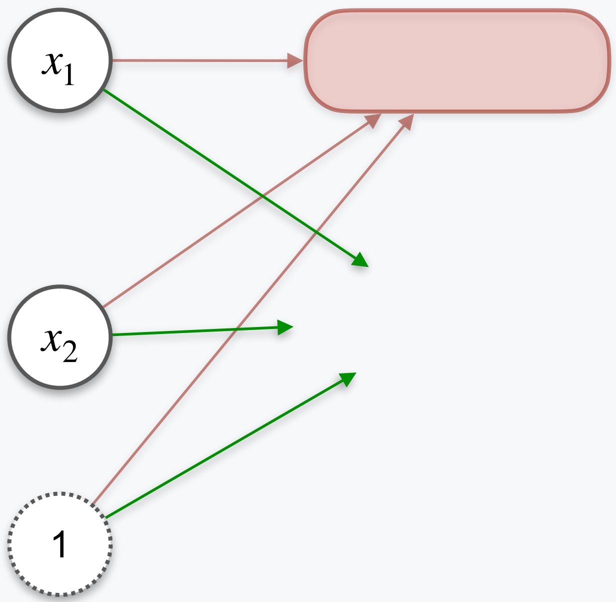

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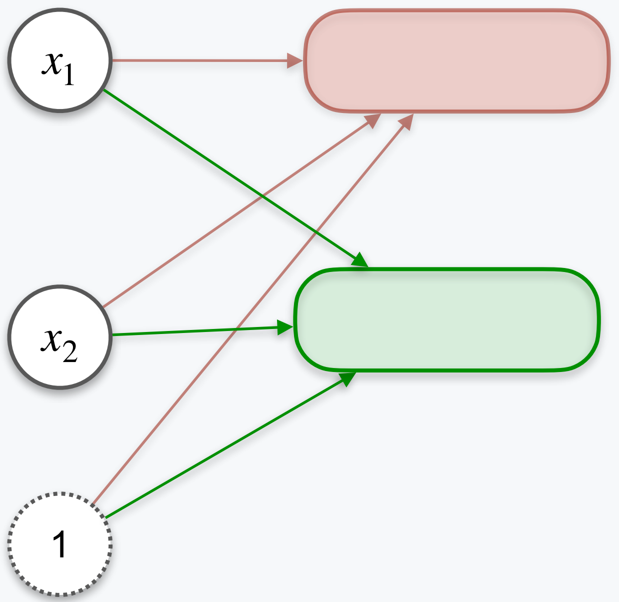

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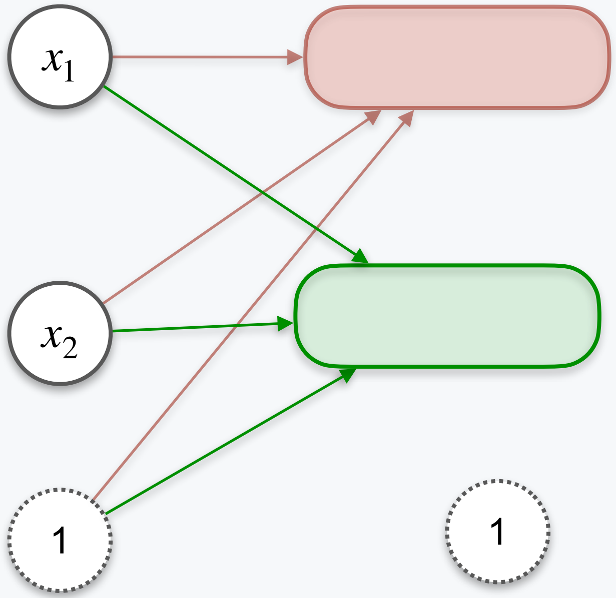

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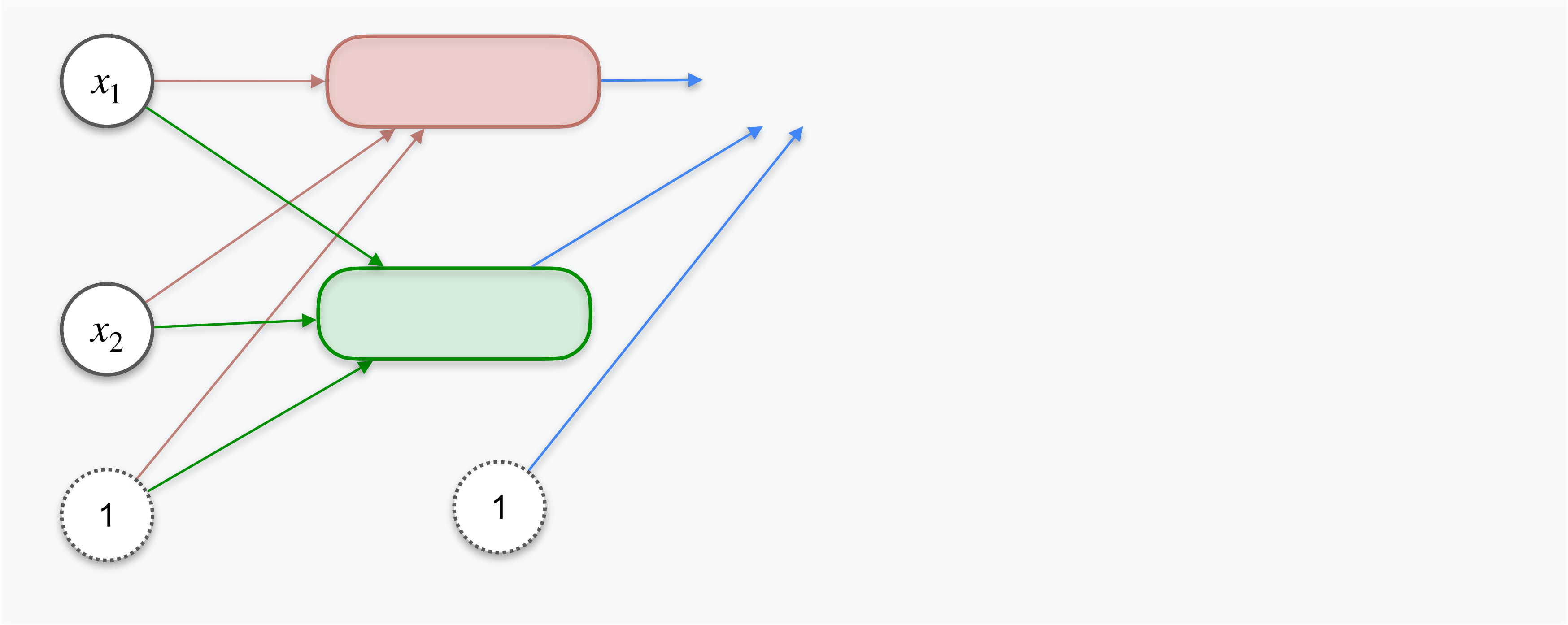

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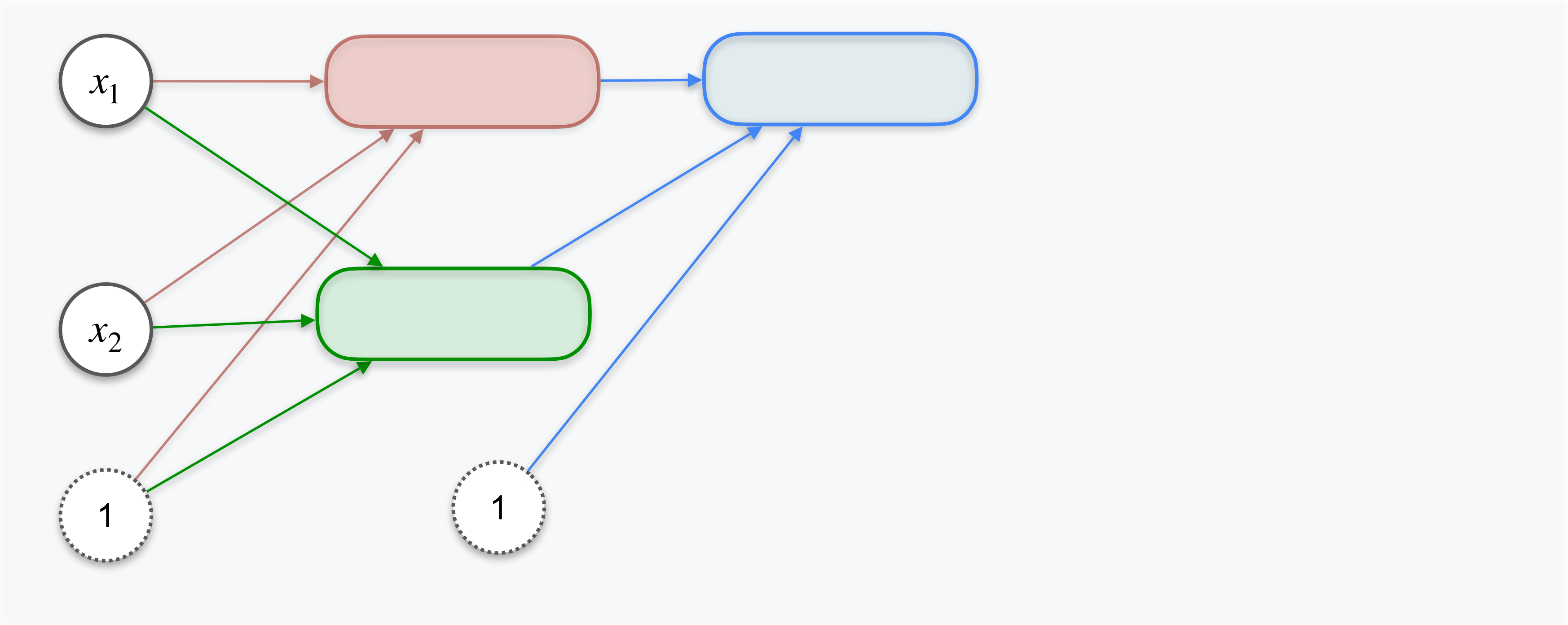

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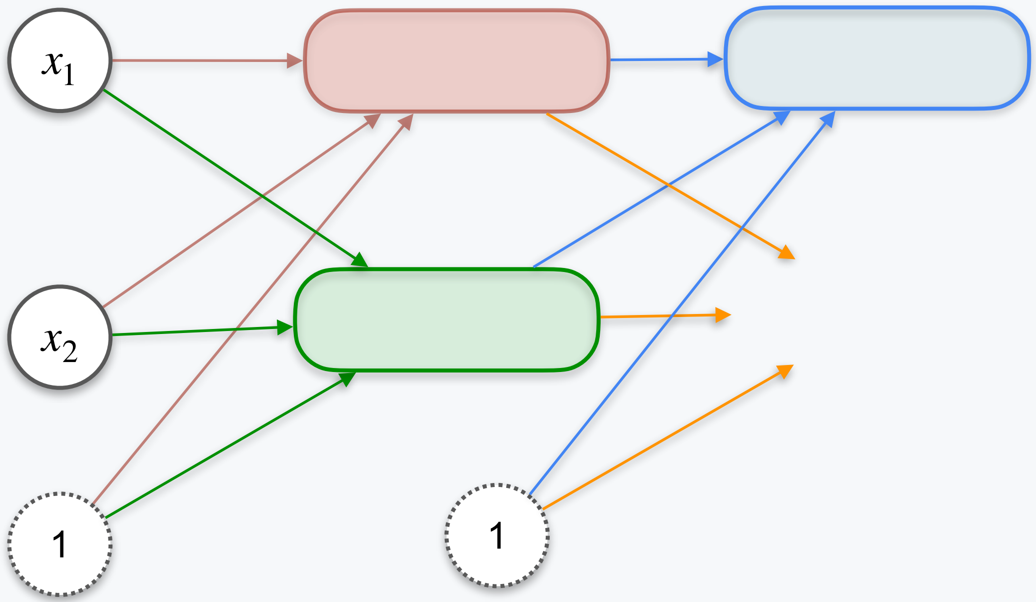

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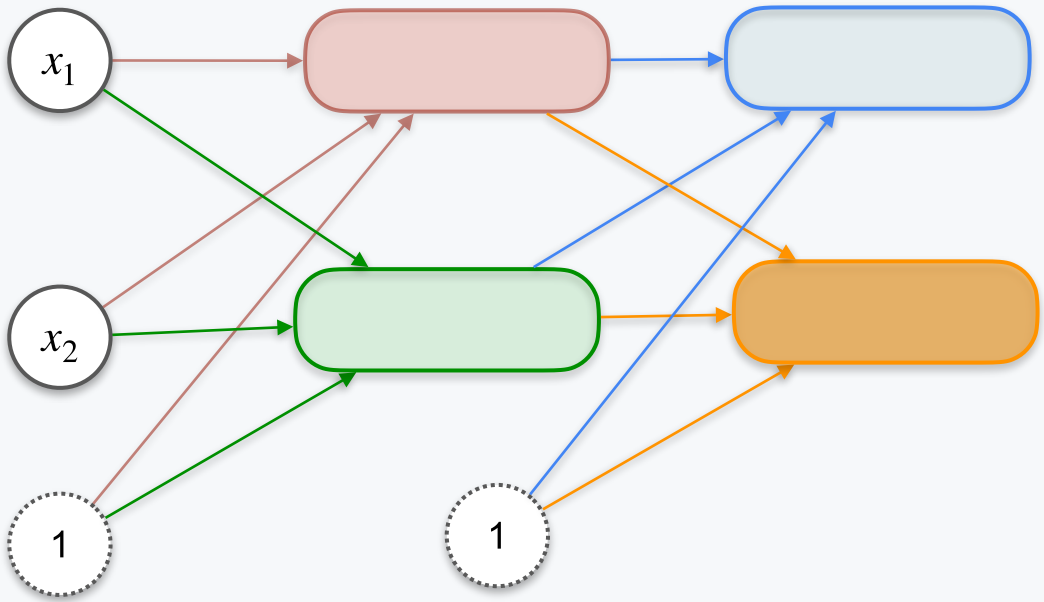

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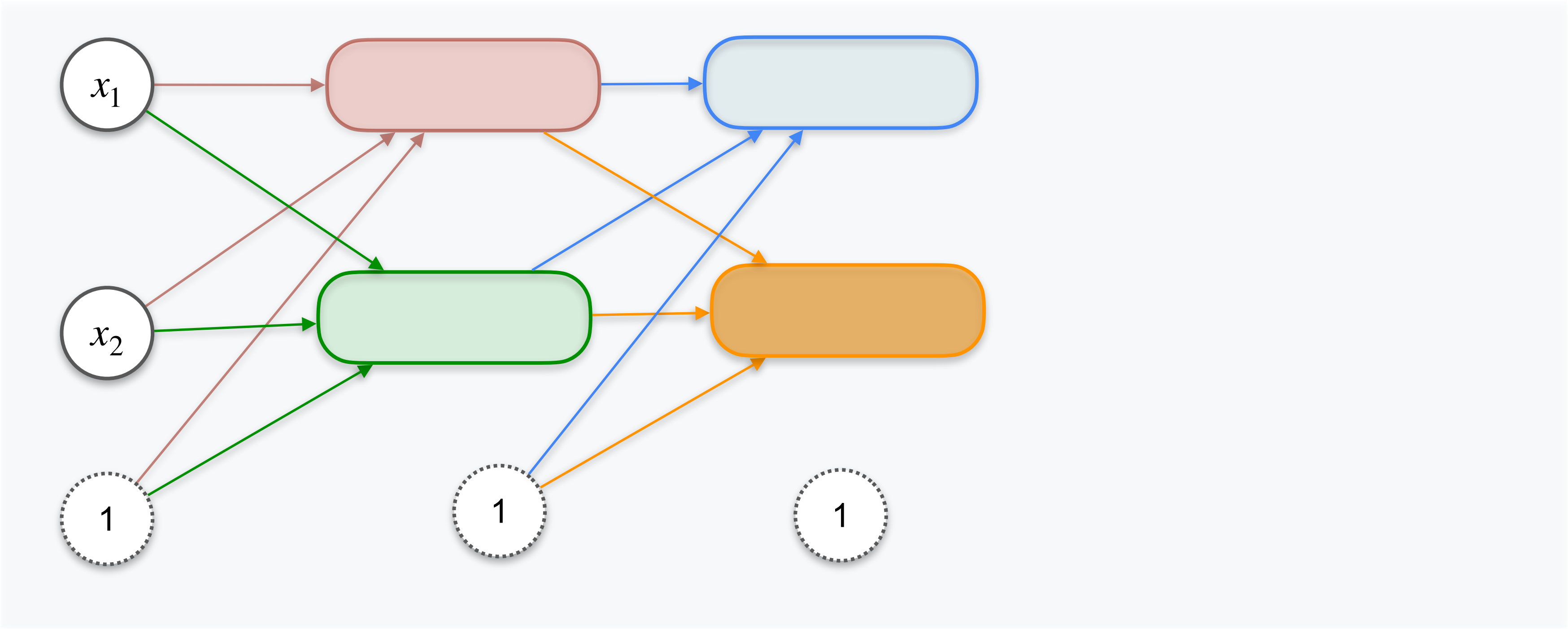

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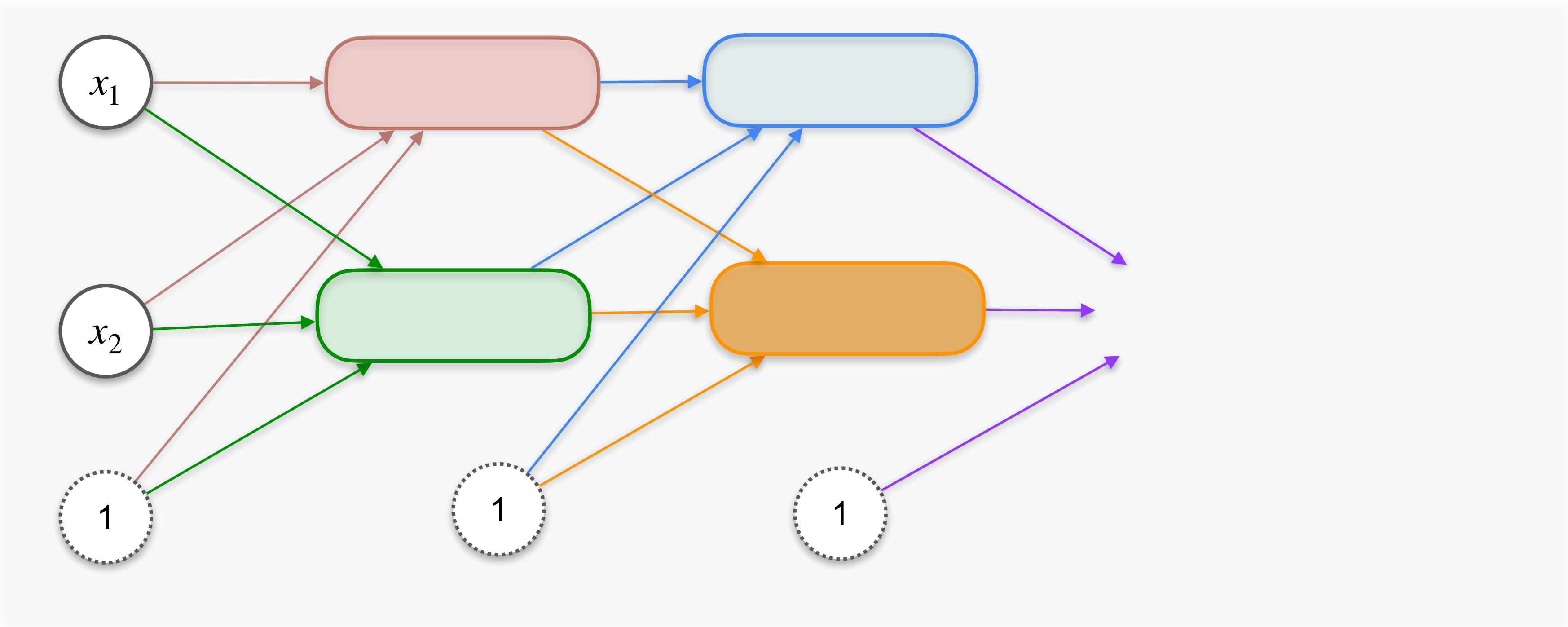

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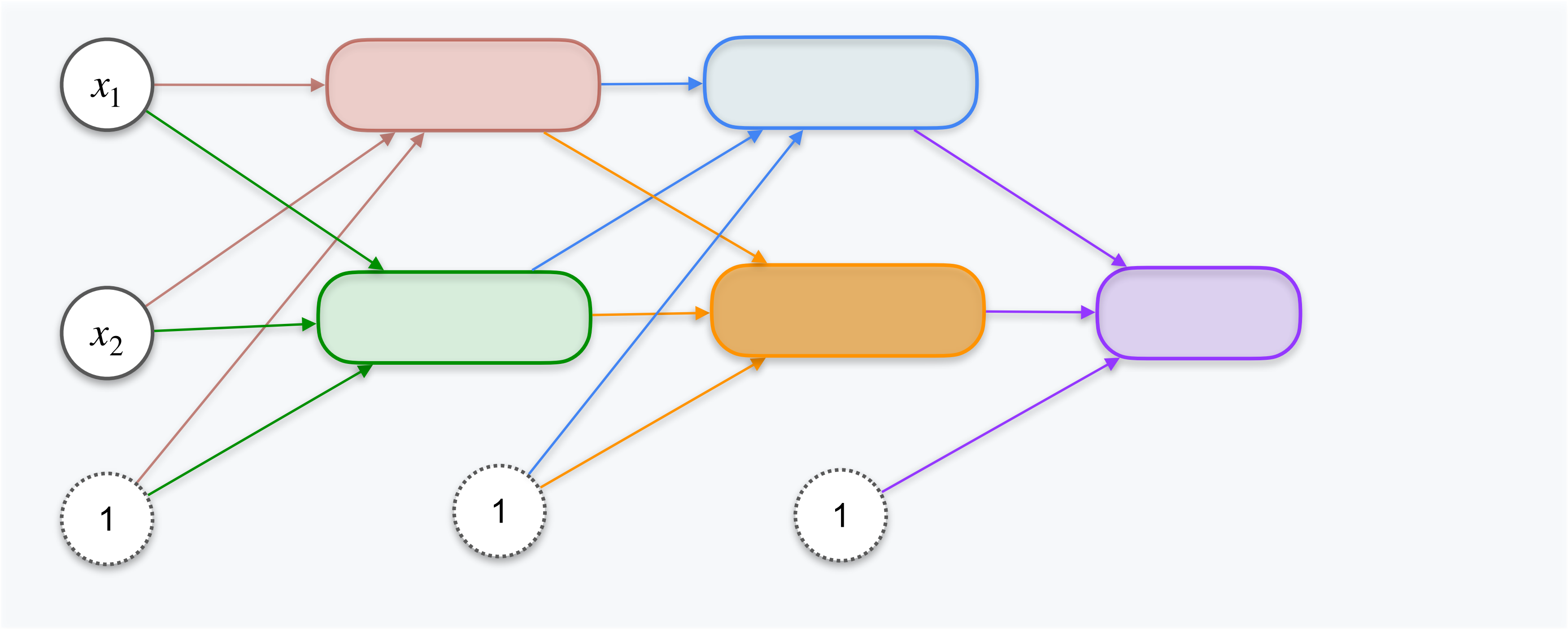

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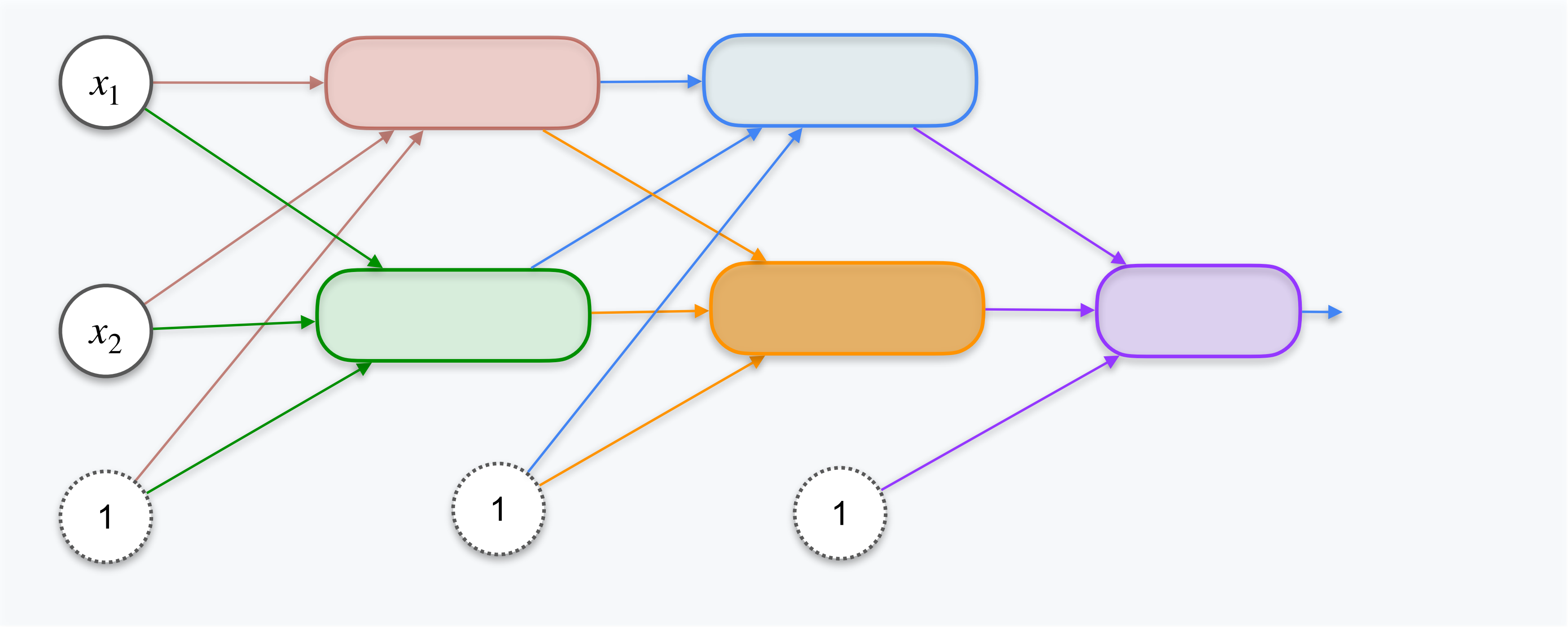

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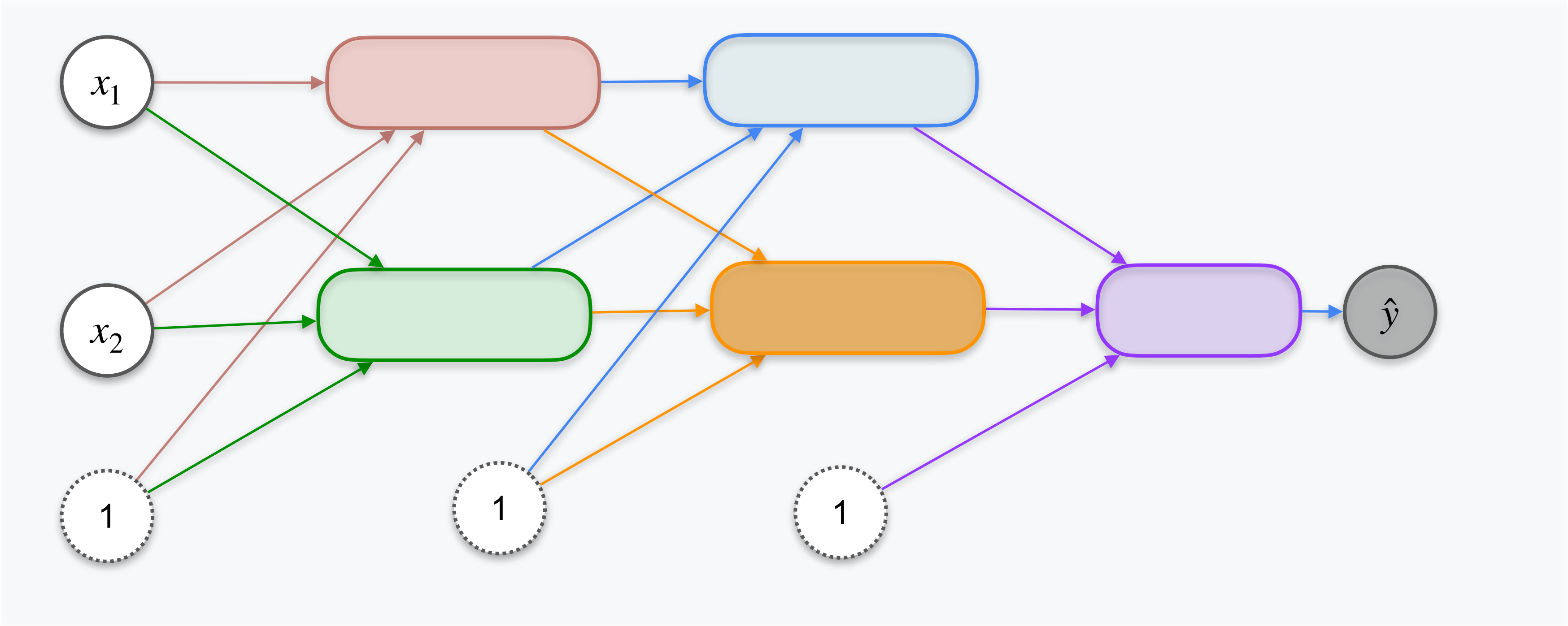

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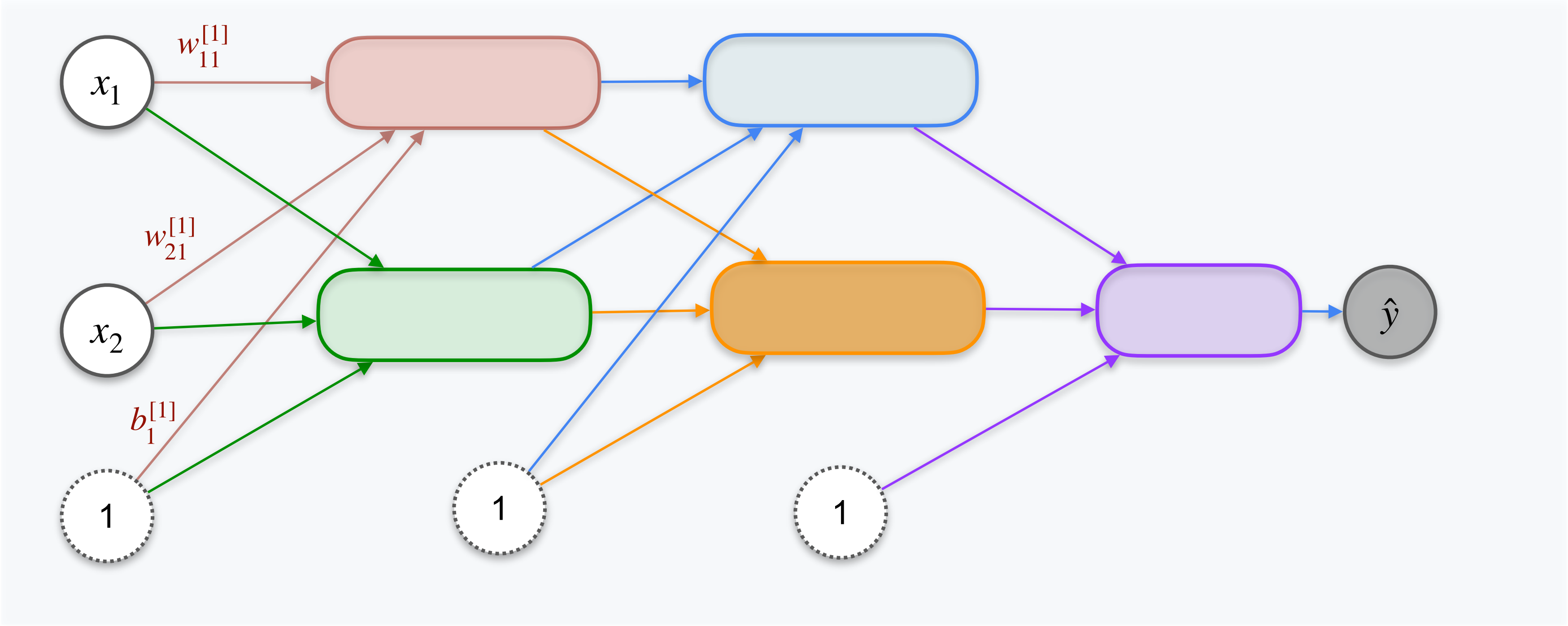

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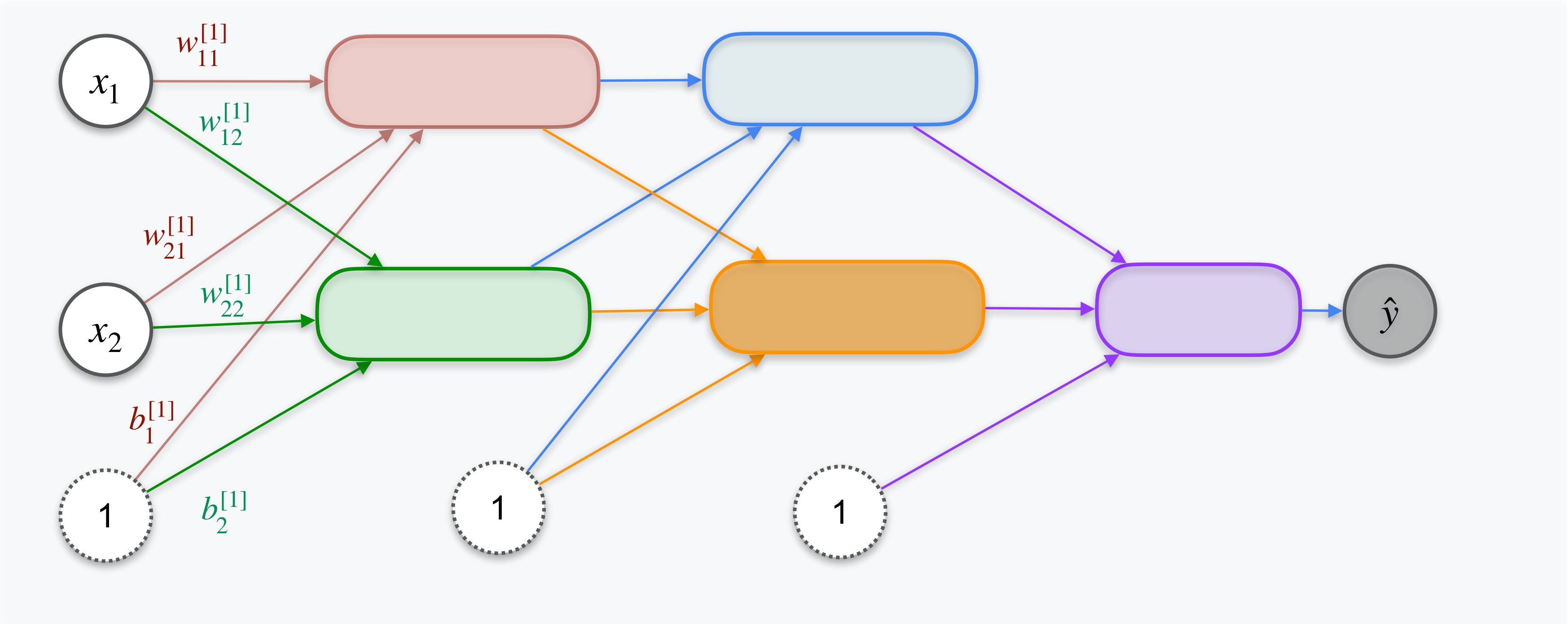

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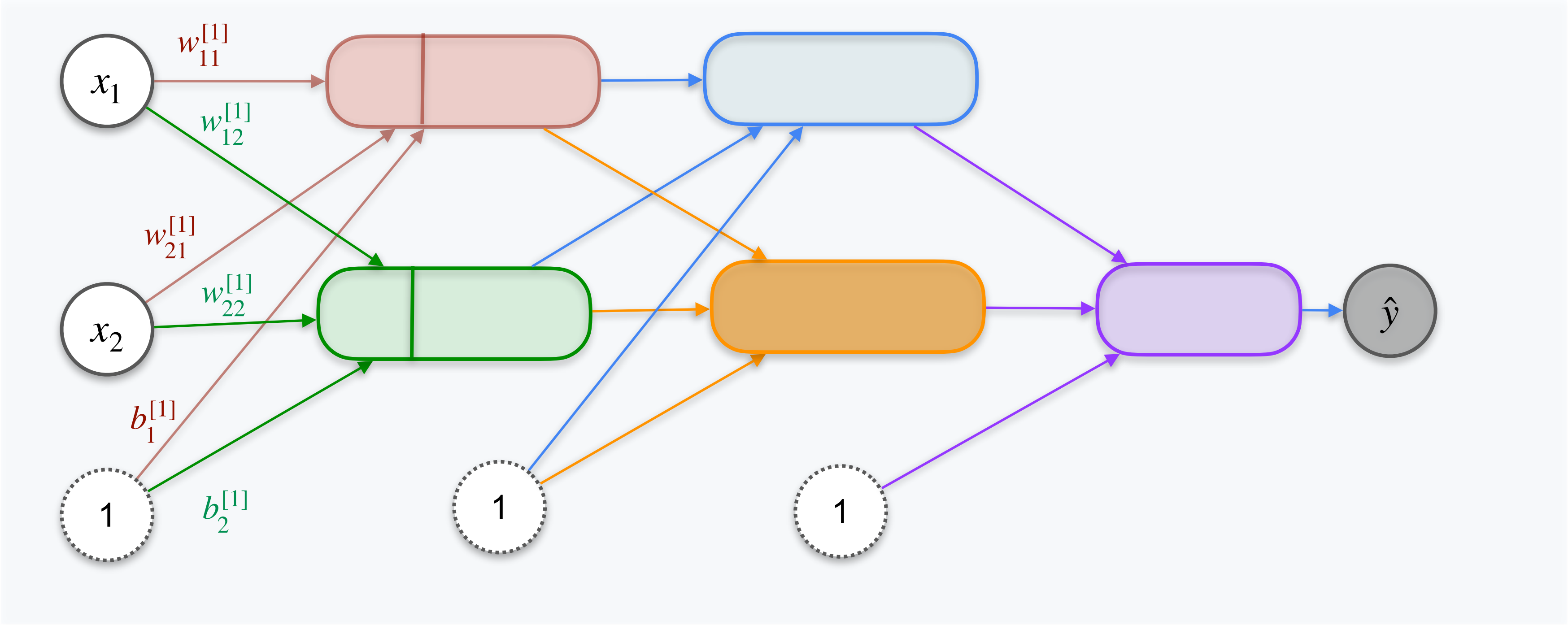

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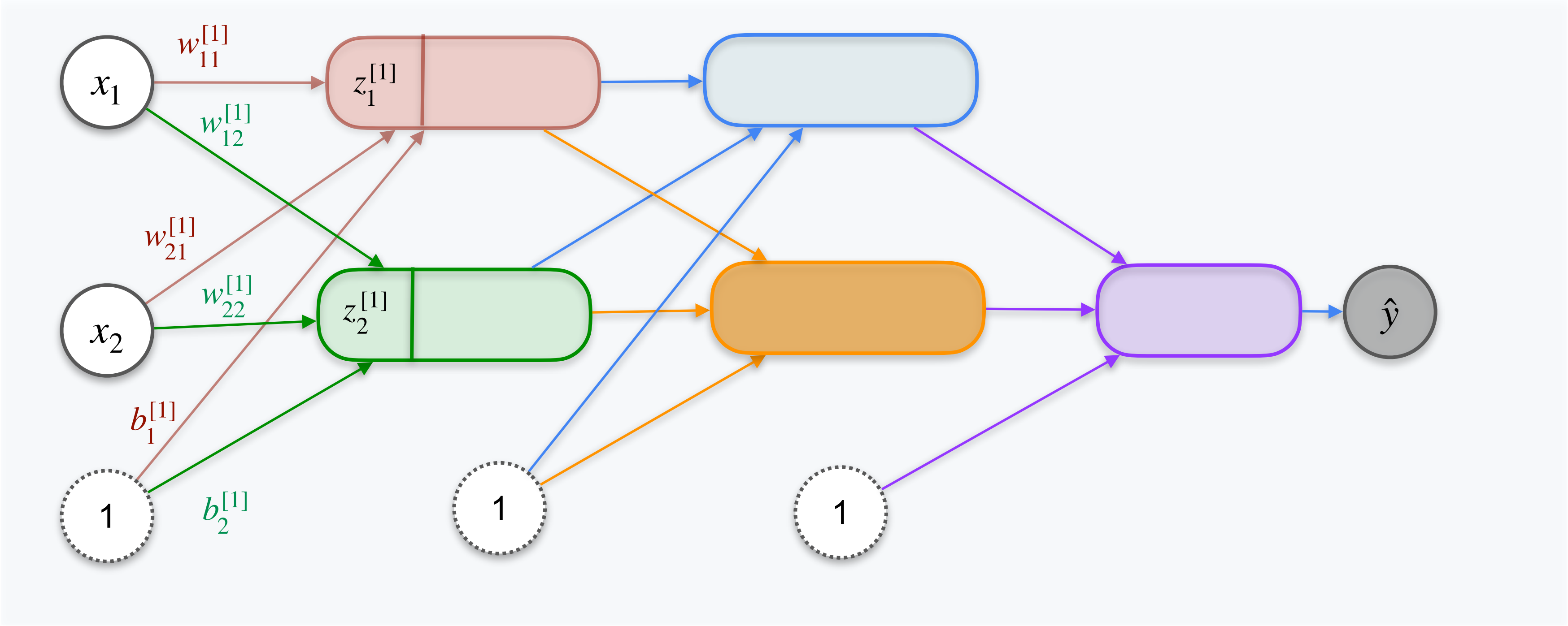

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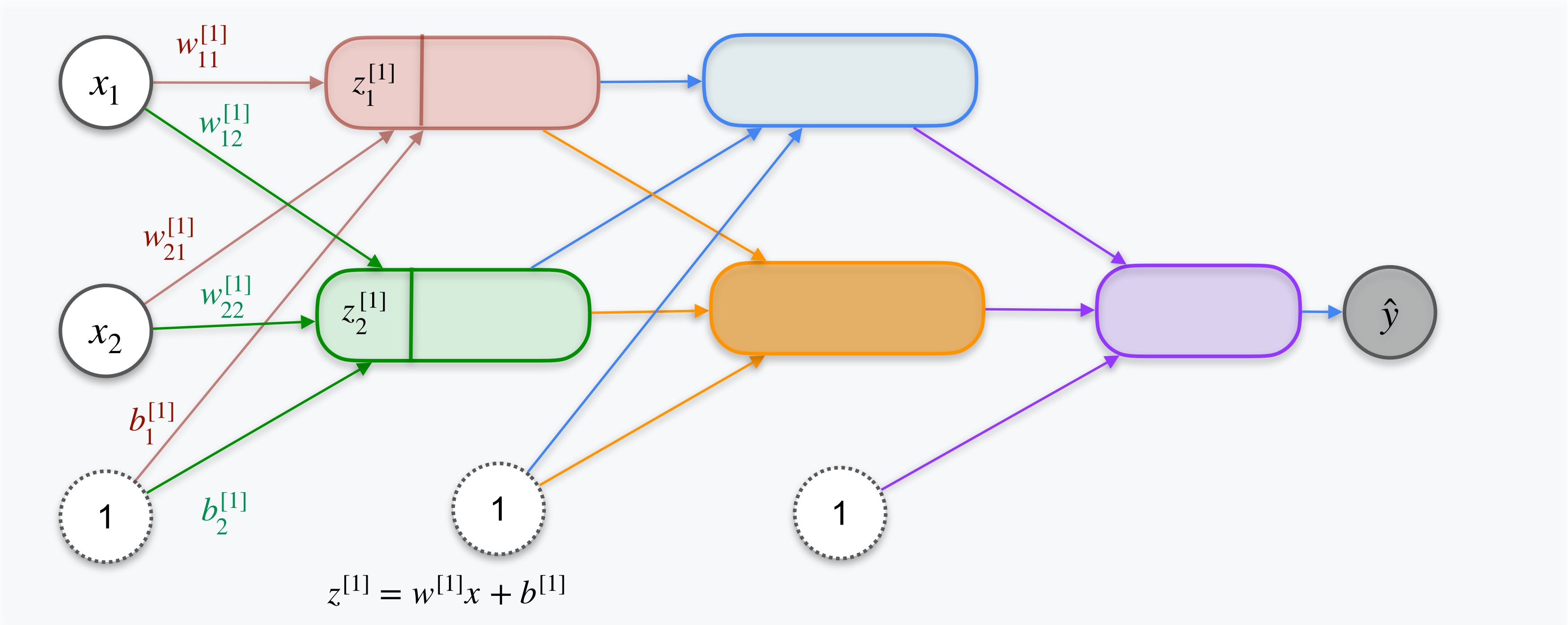

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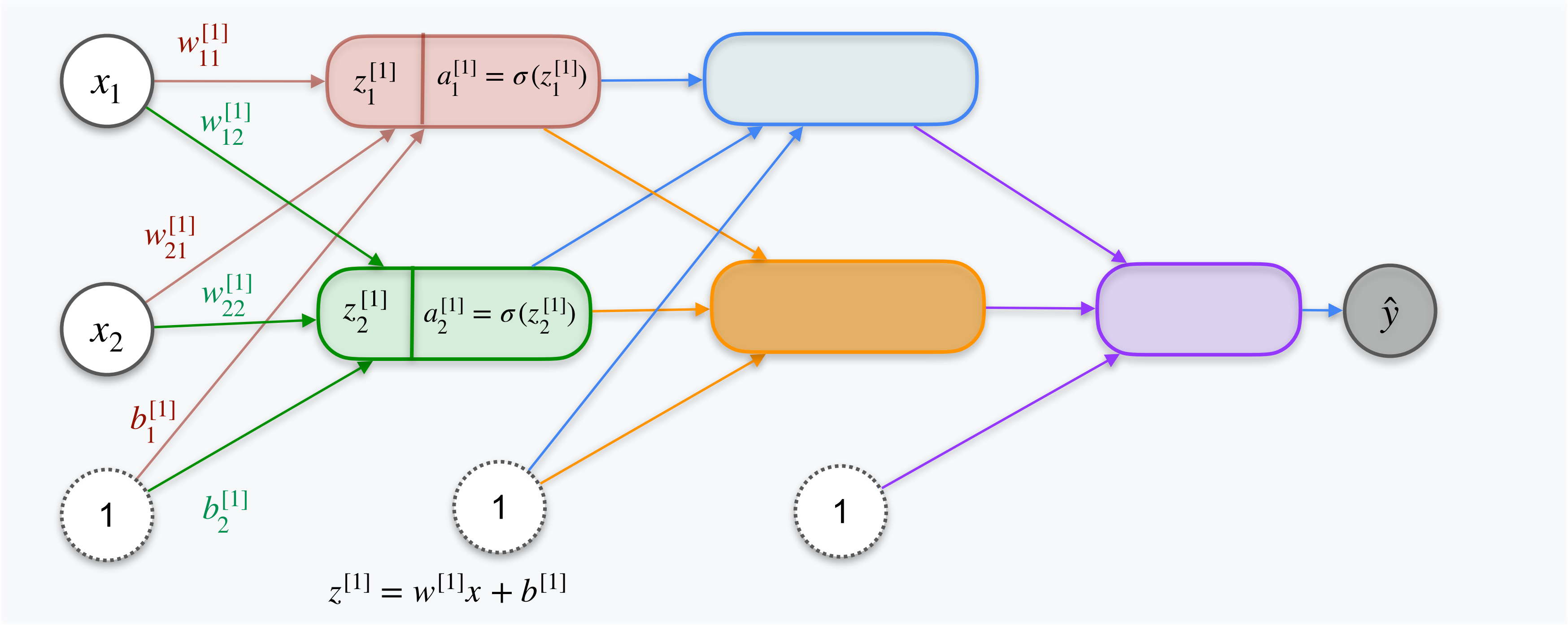

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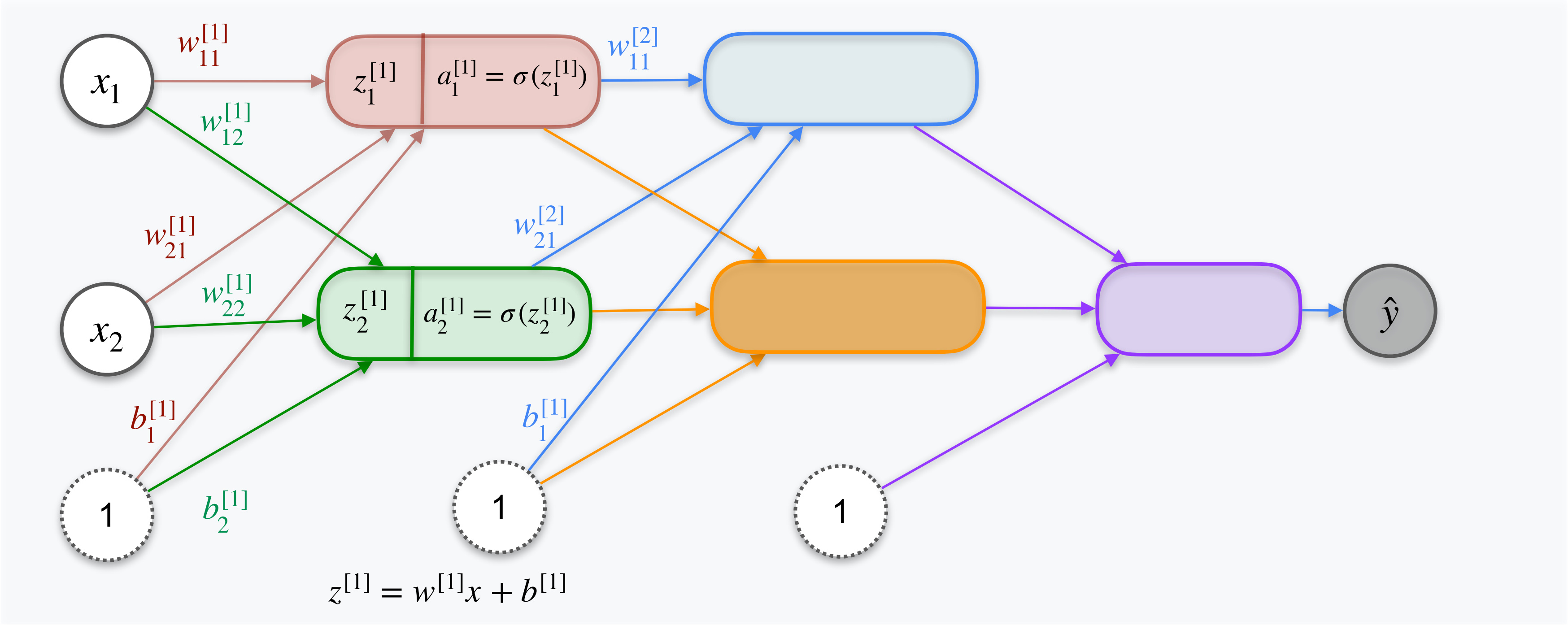

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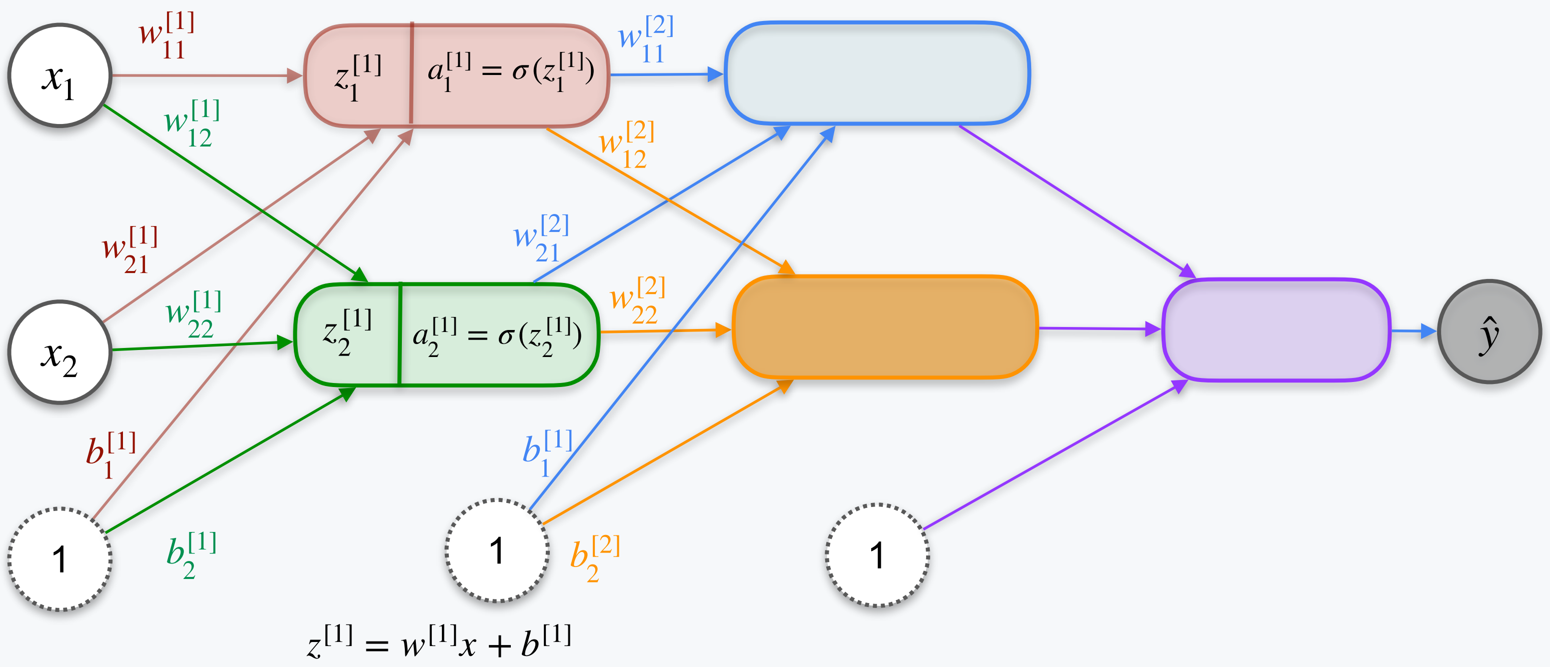

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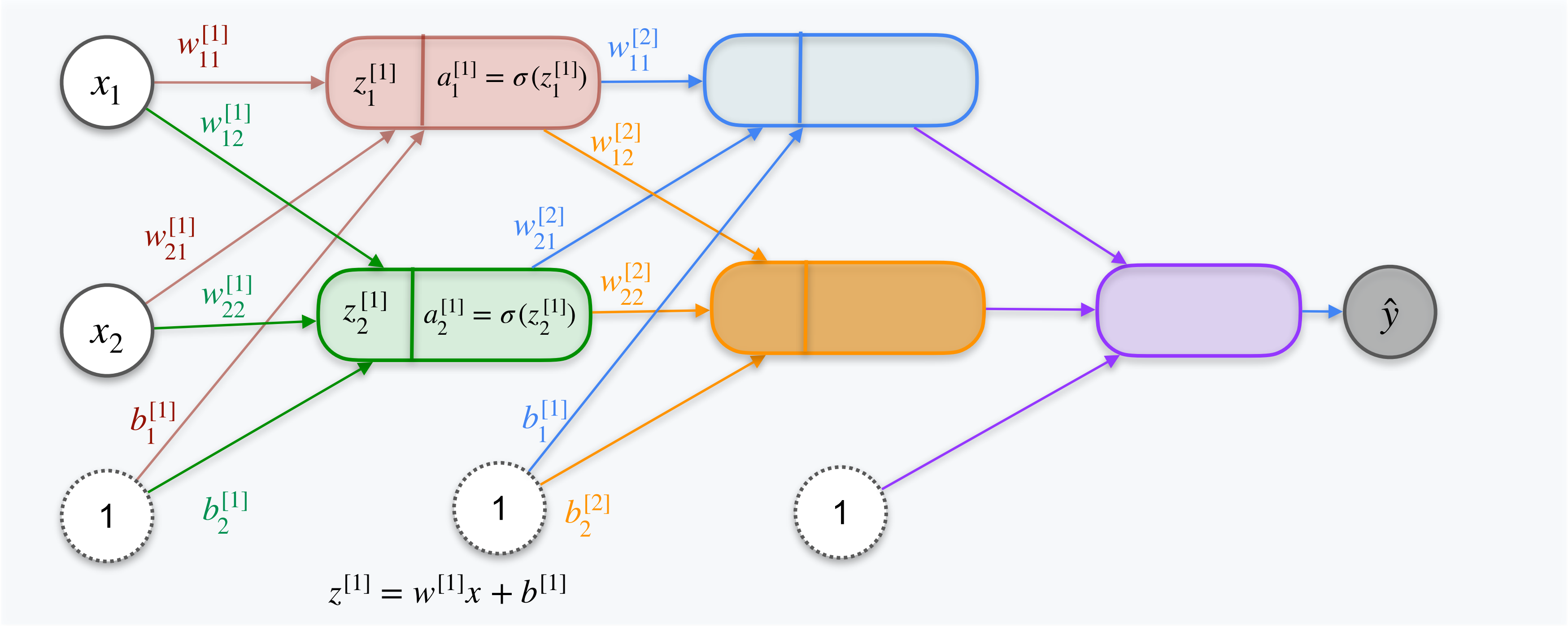

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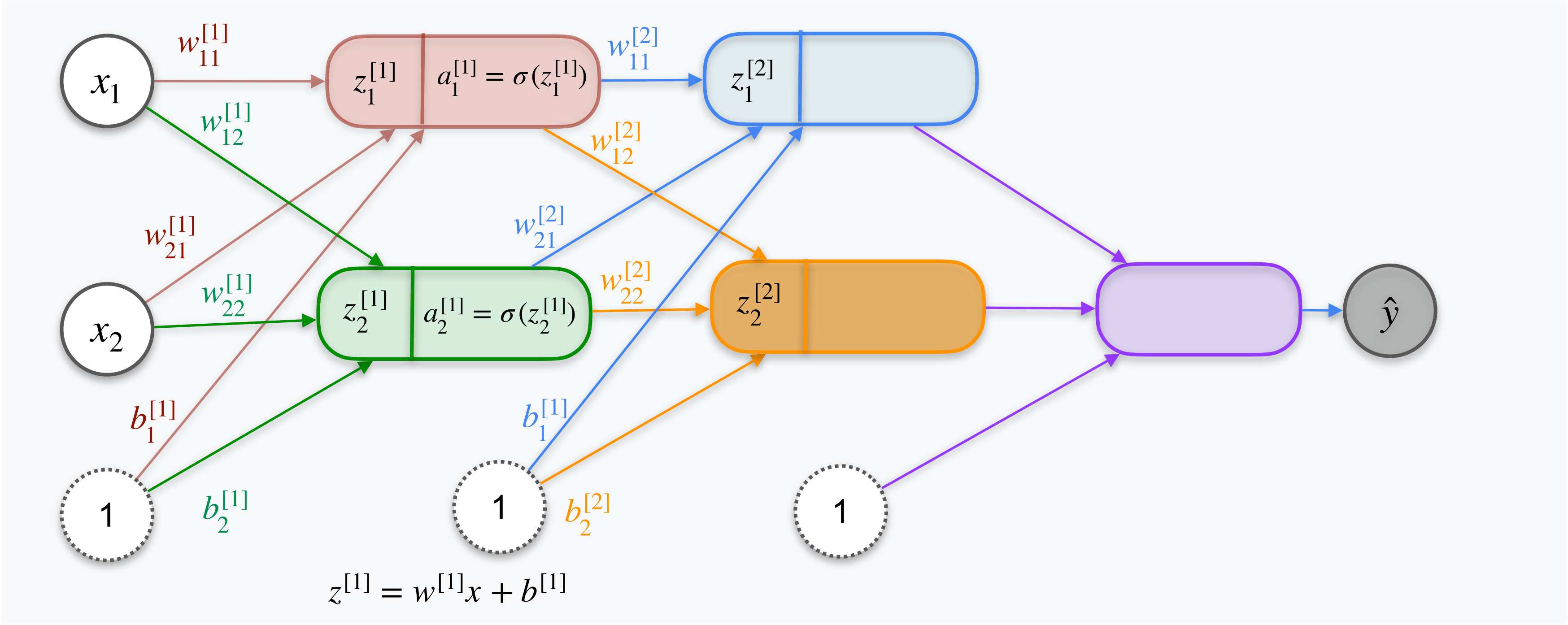

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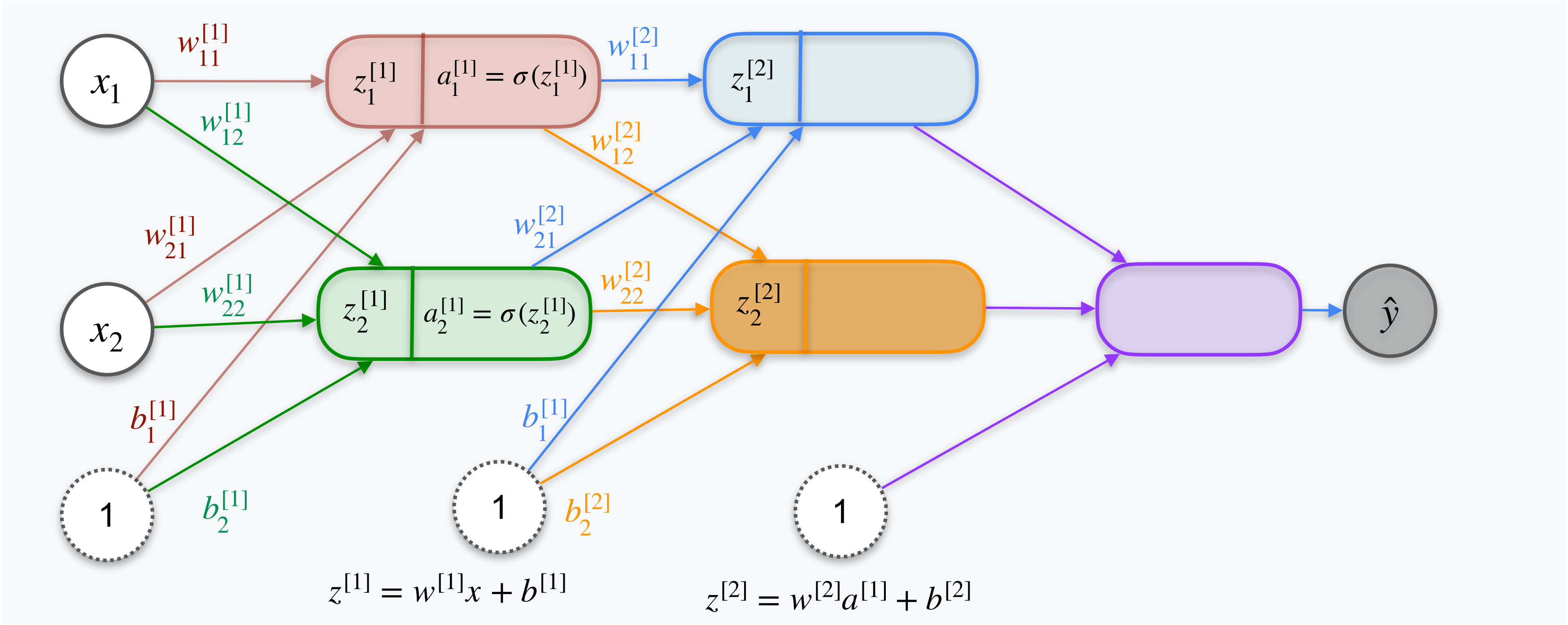

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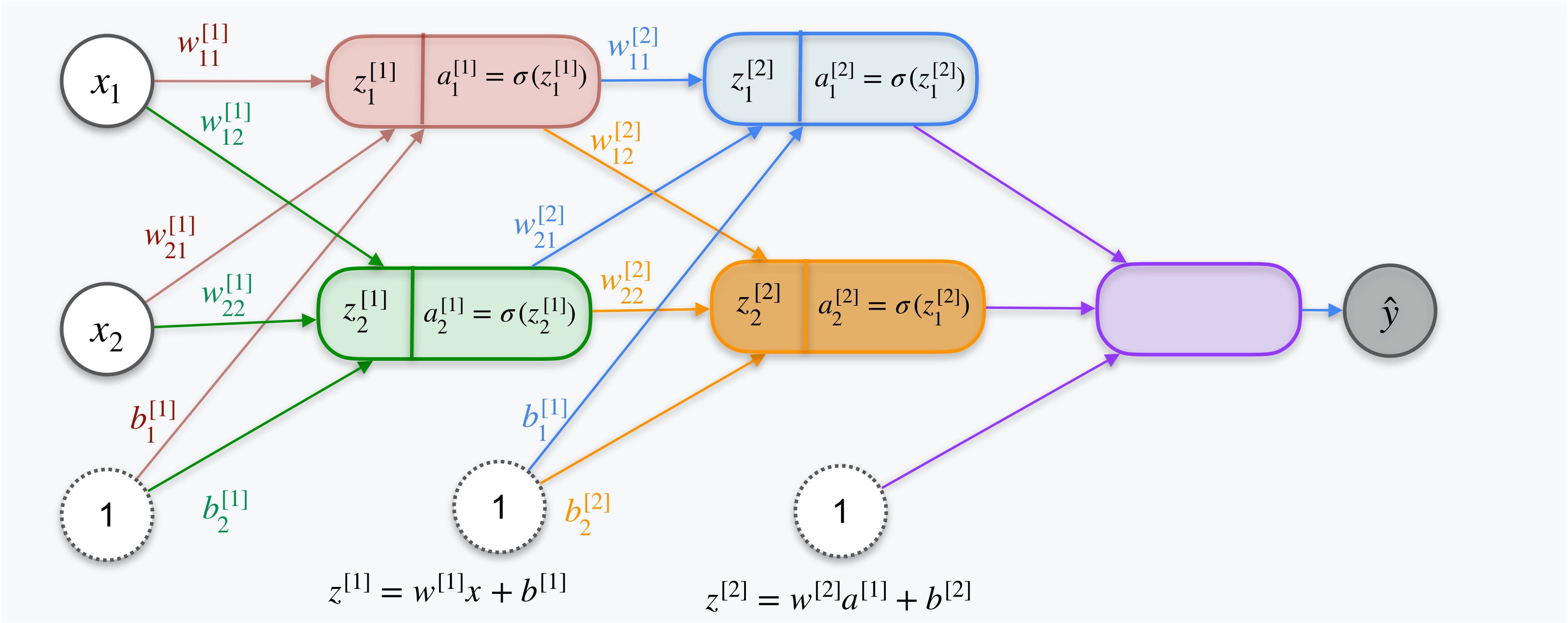

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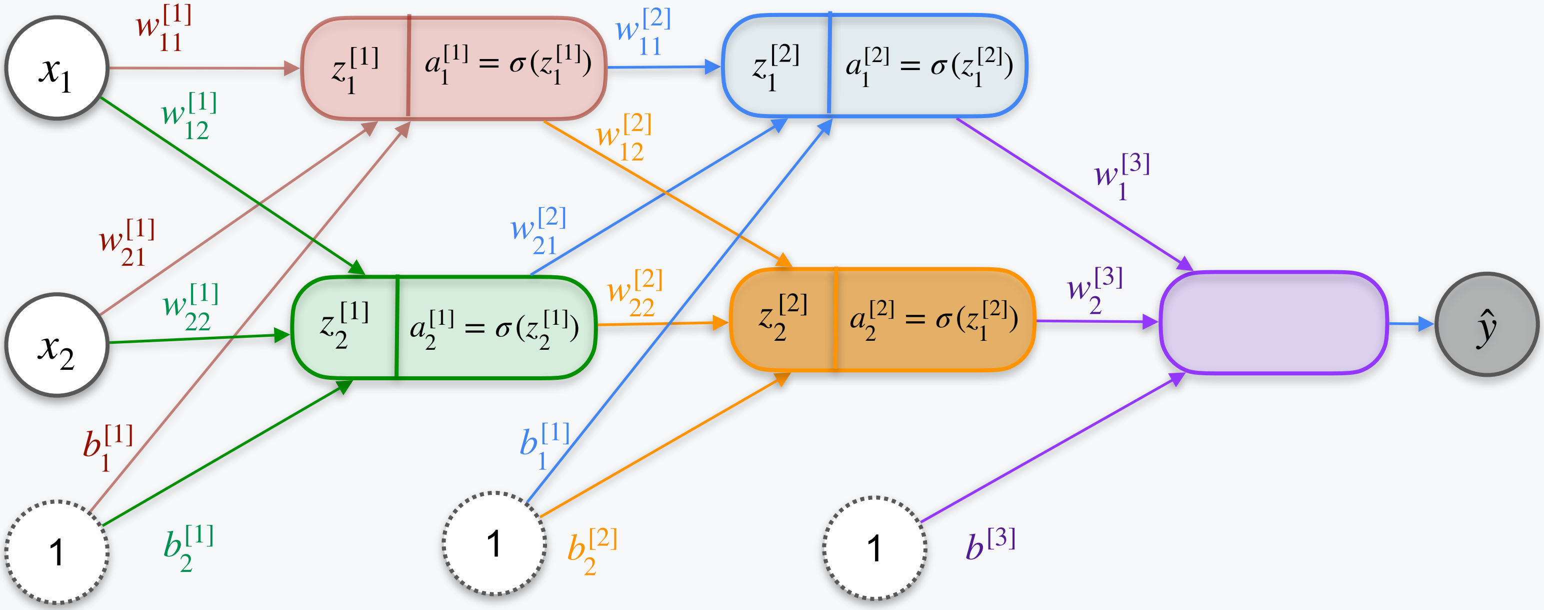

$z^{[1]} = w^{[1]}x + b^{[1]}$

$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$

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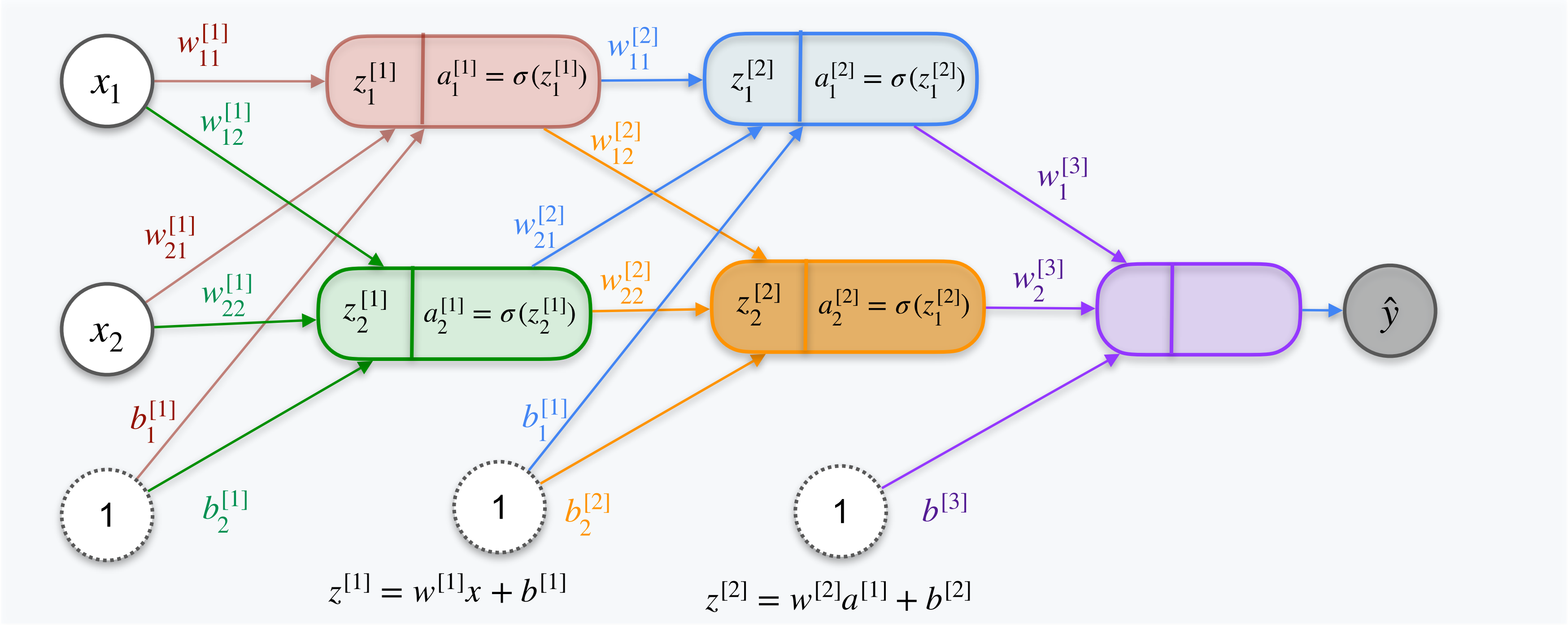

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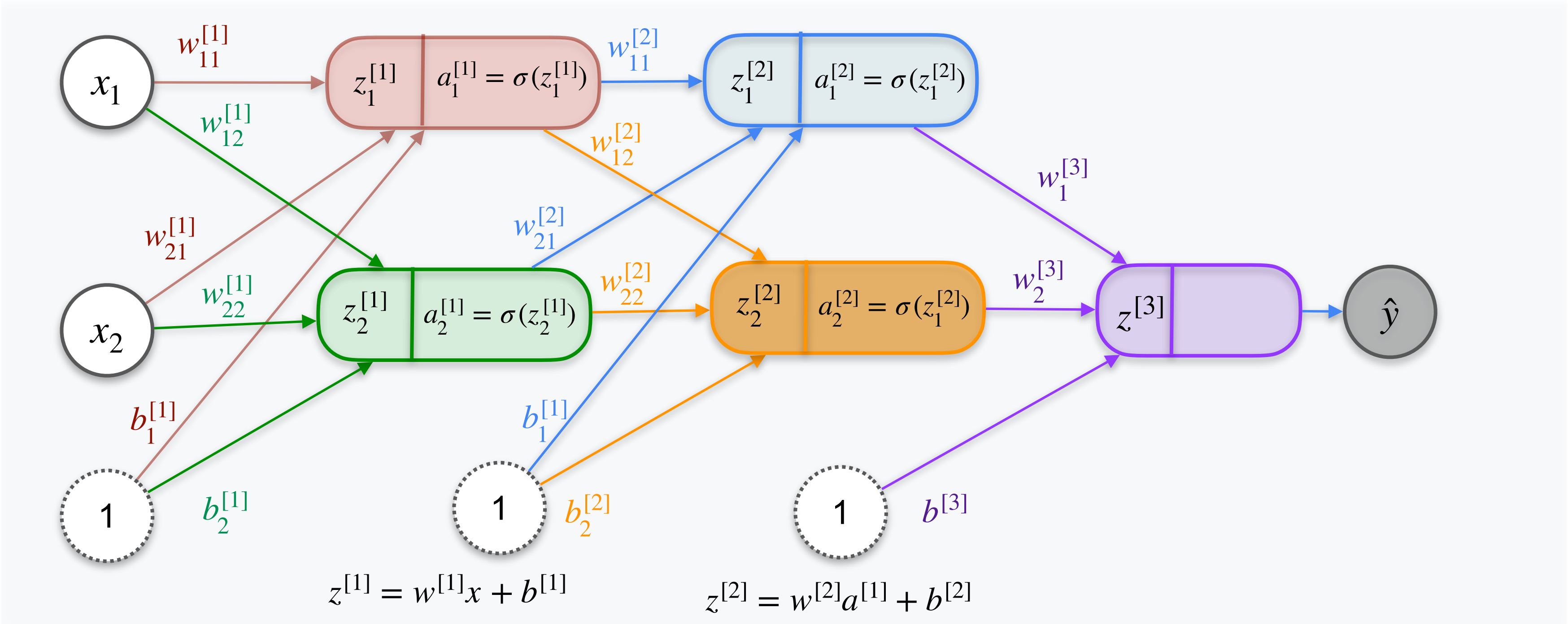

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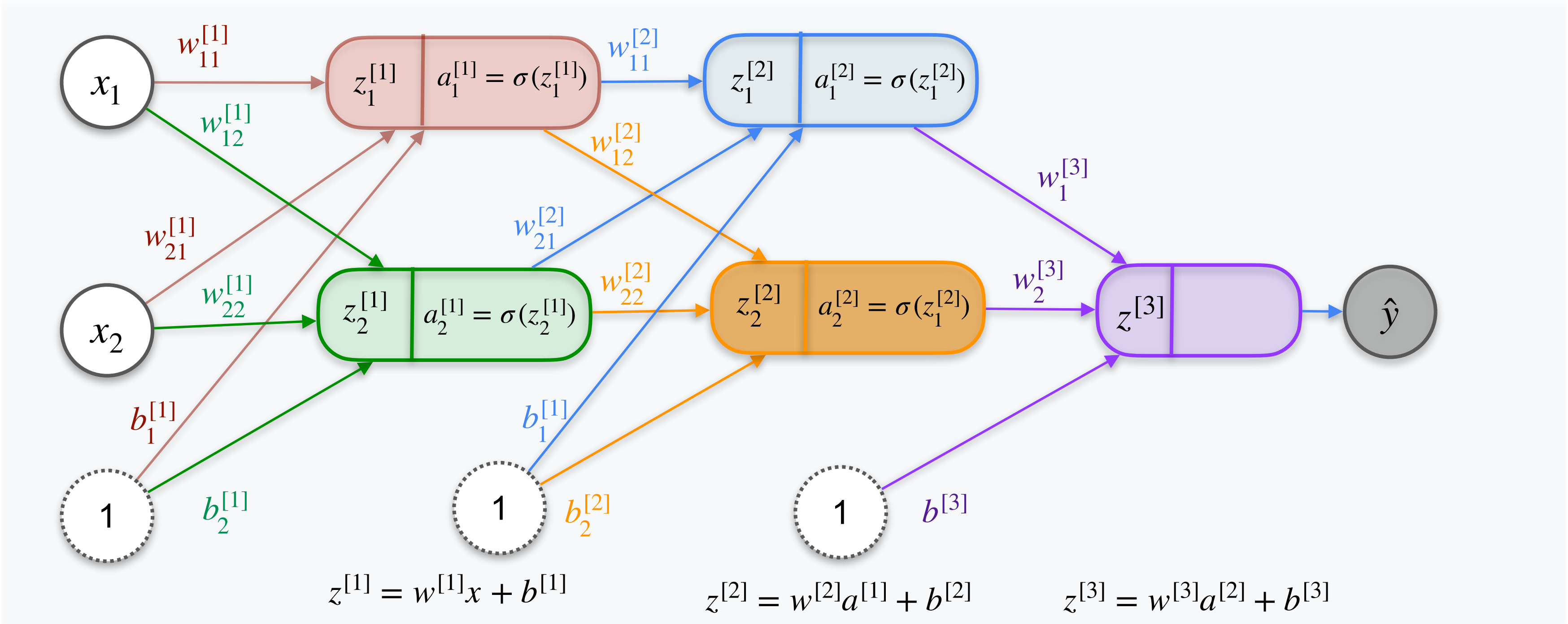

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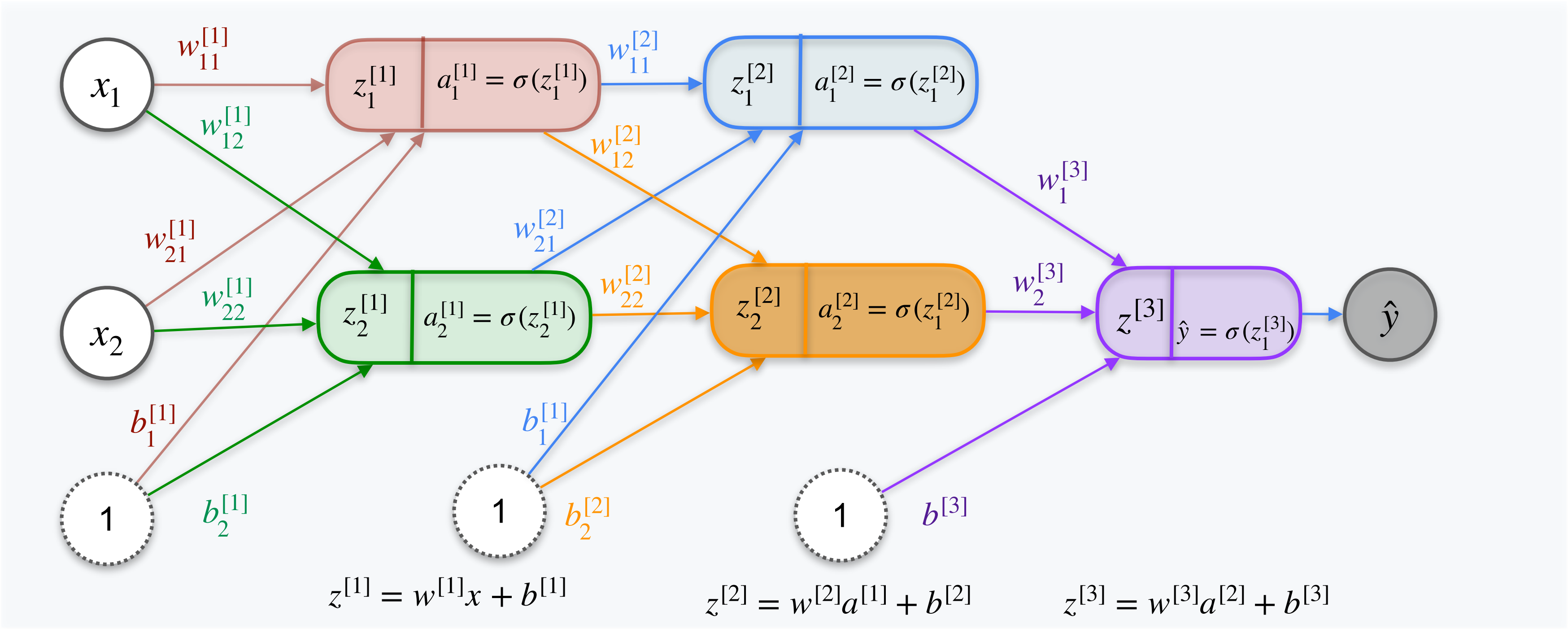

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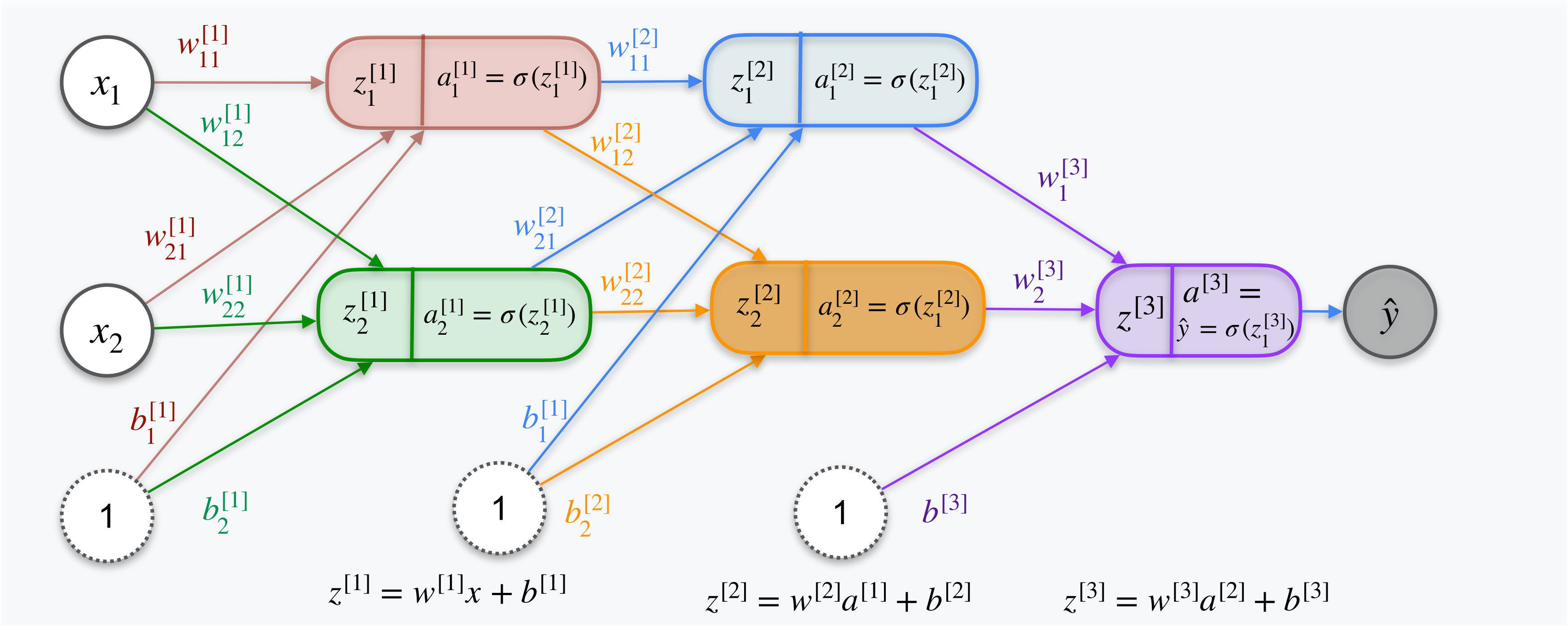

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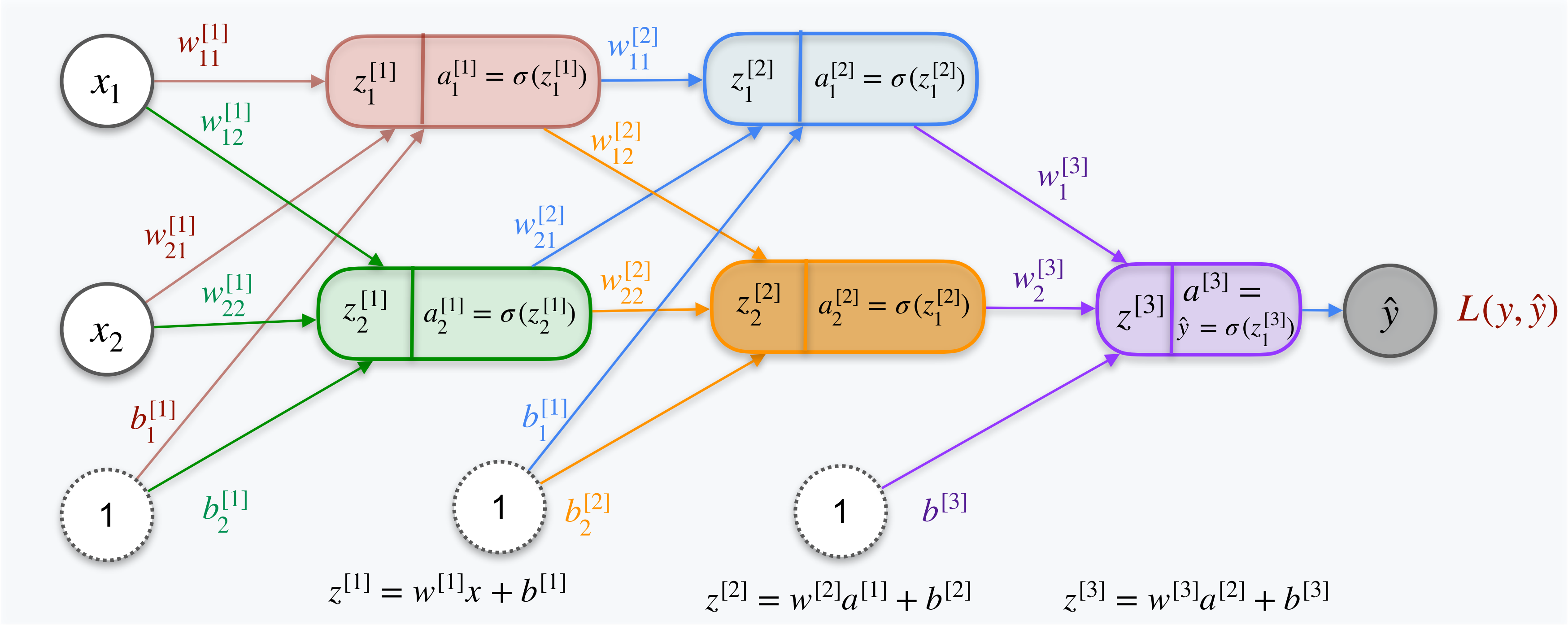

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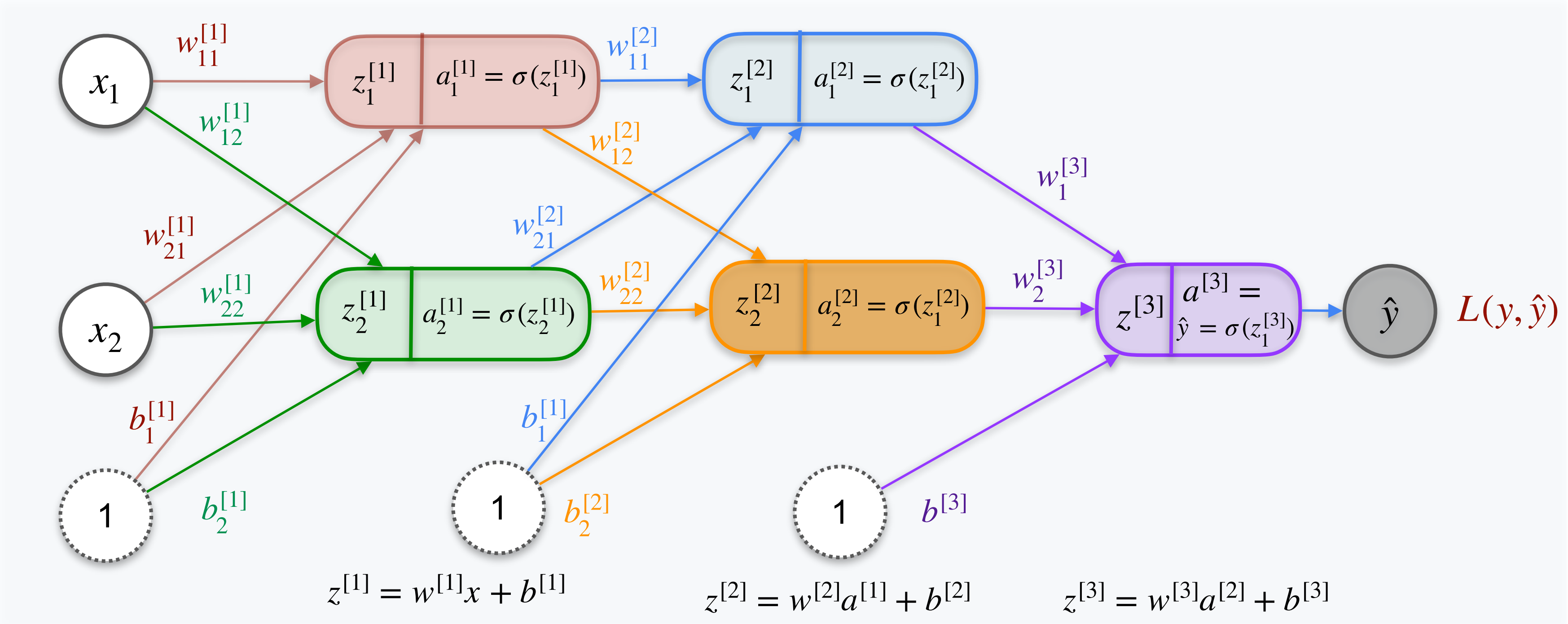

$L(y, \hat{y})$

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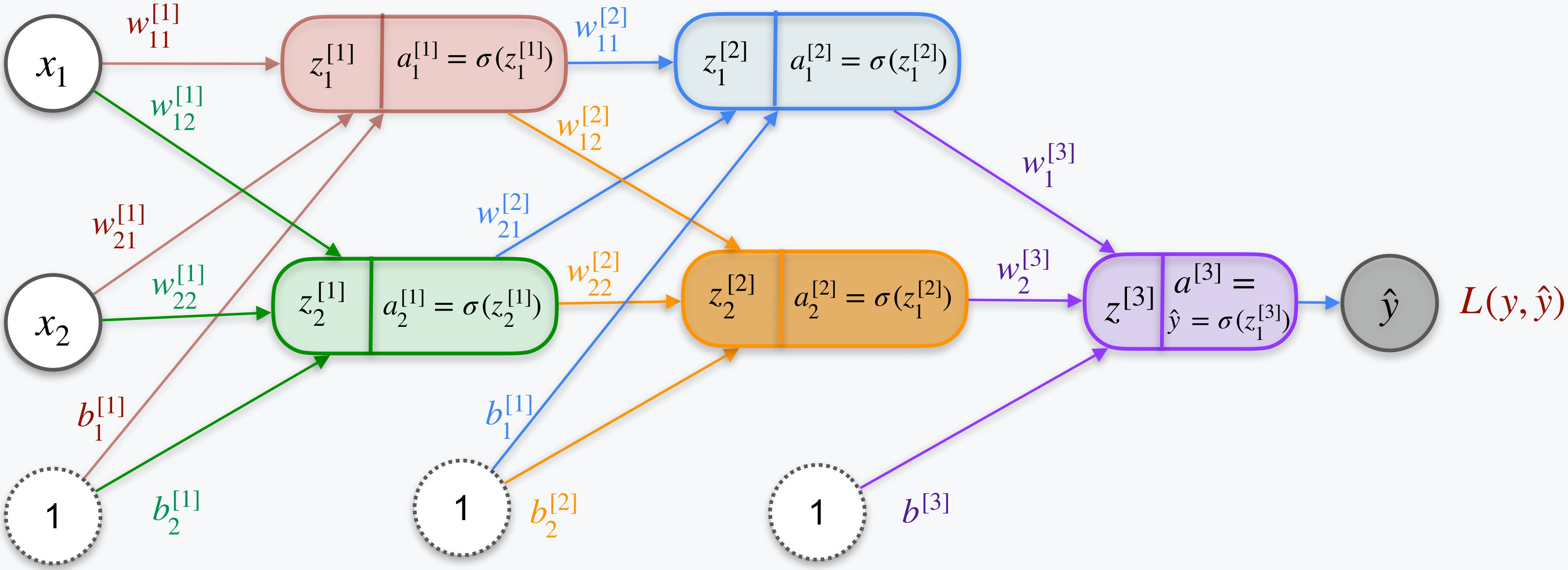

$z^{[1]} = w^{[1]}x + b^{[1]}$

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$$L(y, \hat{y}) = -ylog(\hat{y}) - (1 - y)log(1 - \hat{y})$$

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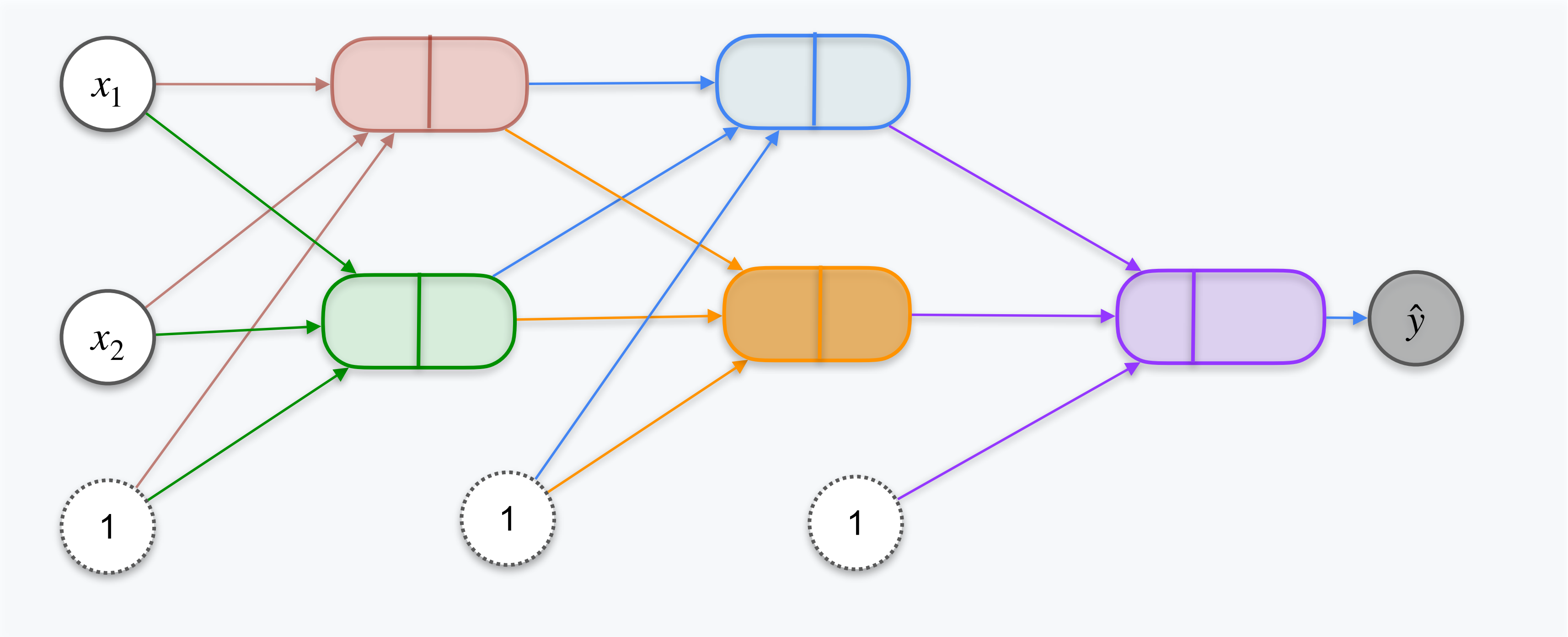

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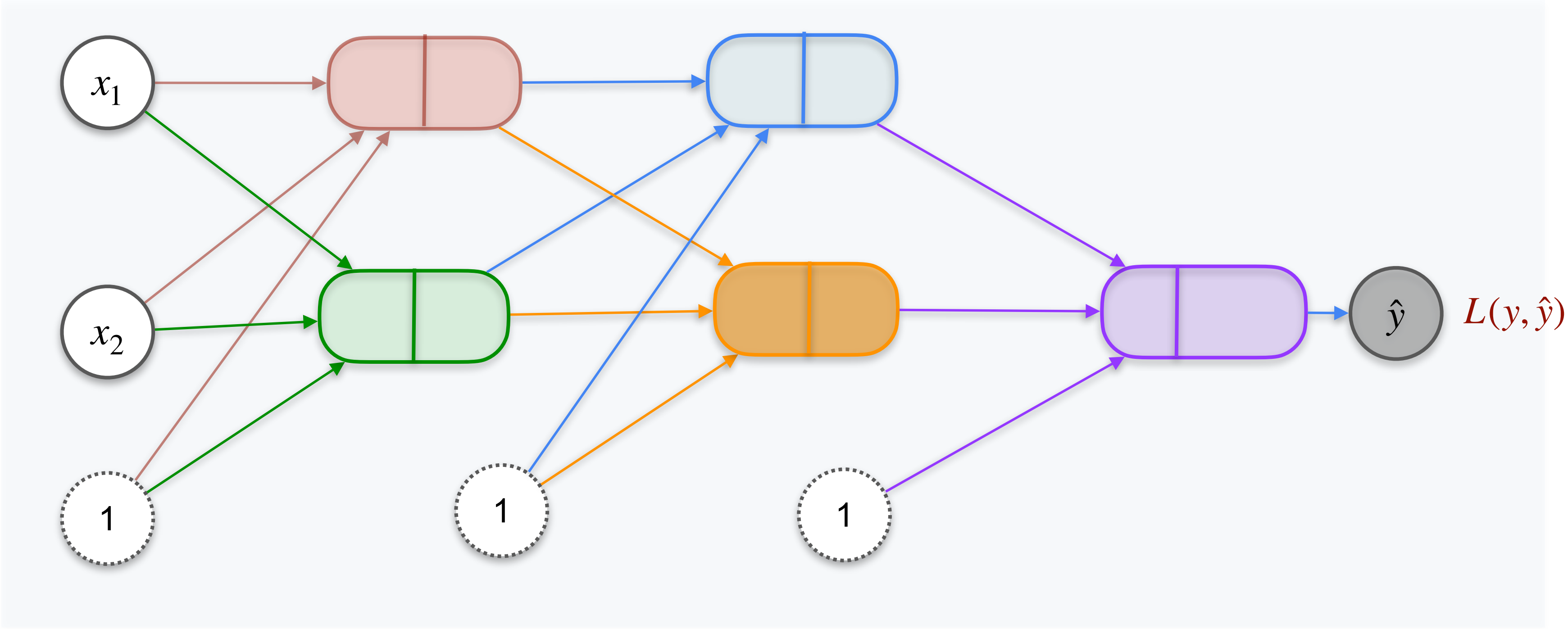

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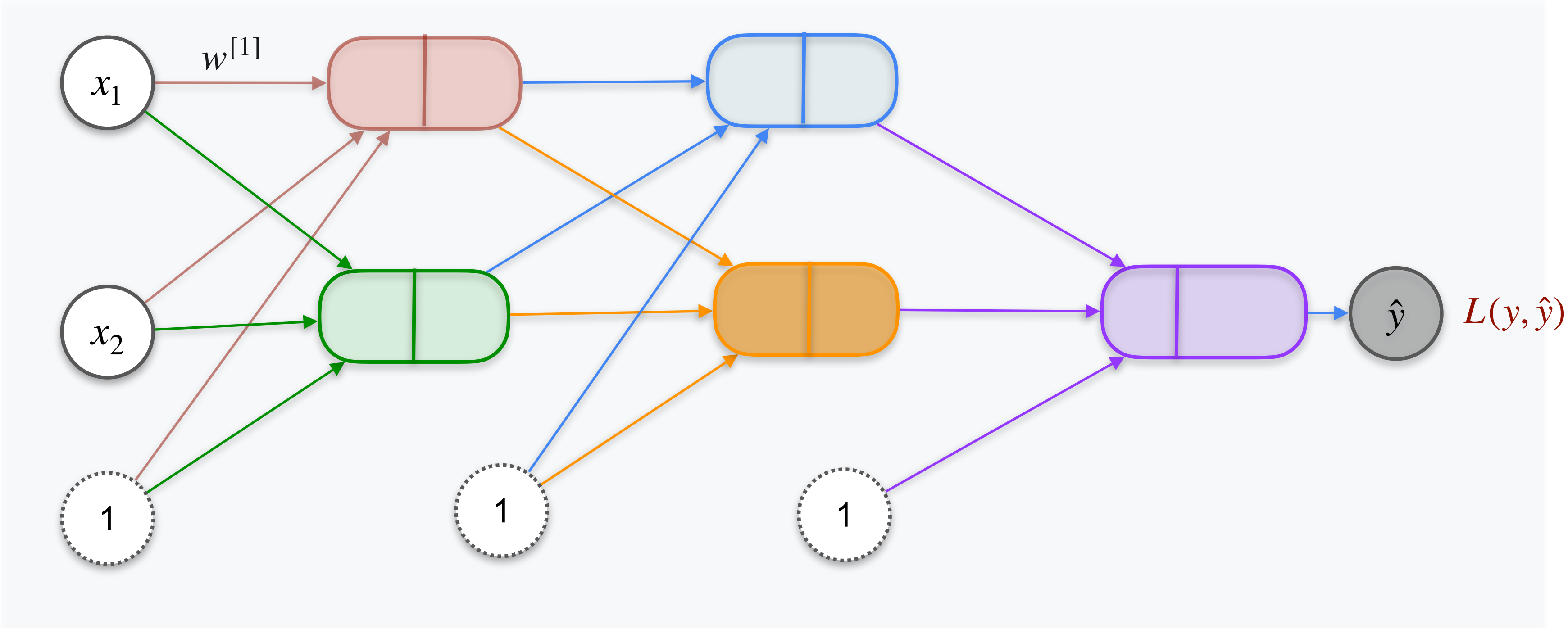

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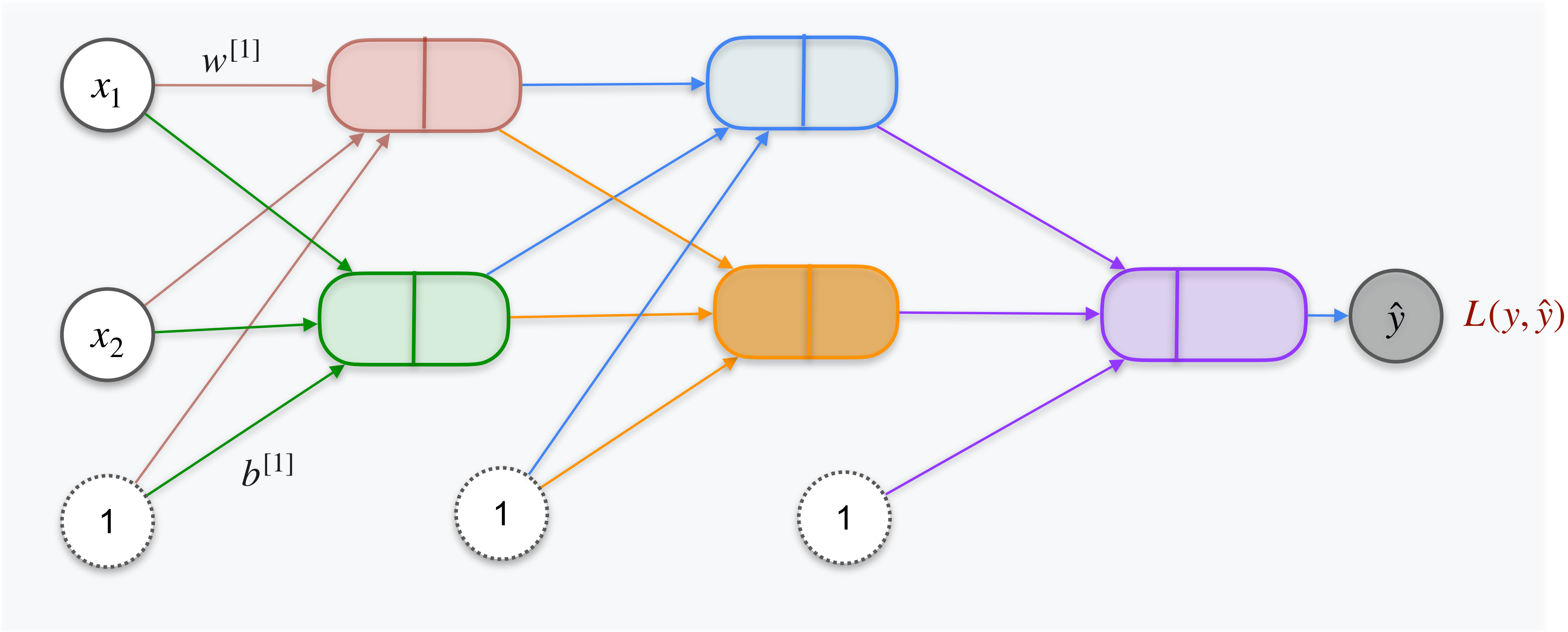

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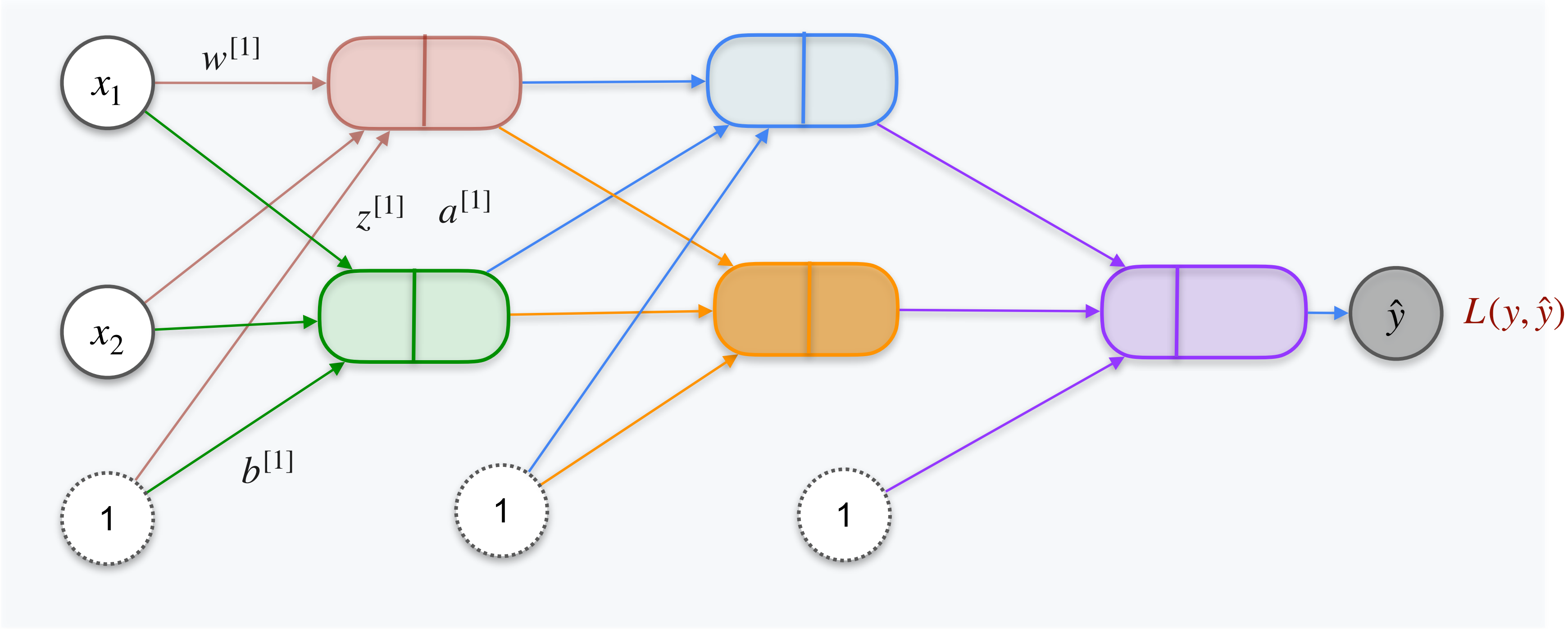

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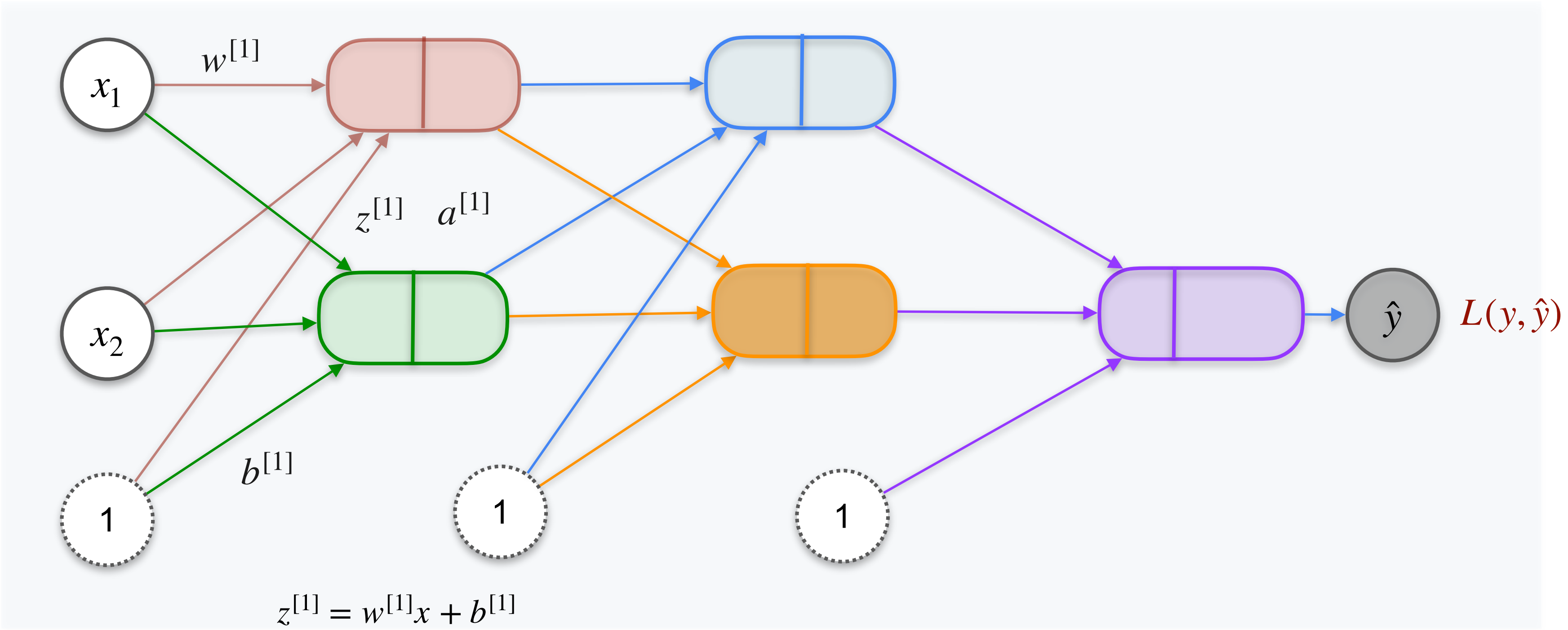

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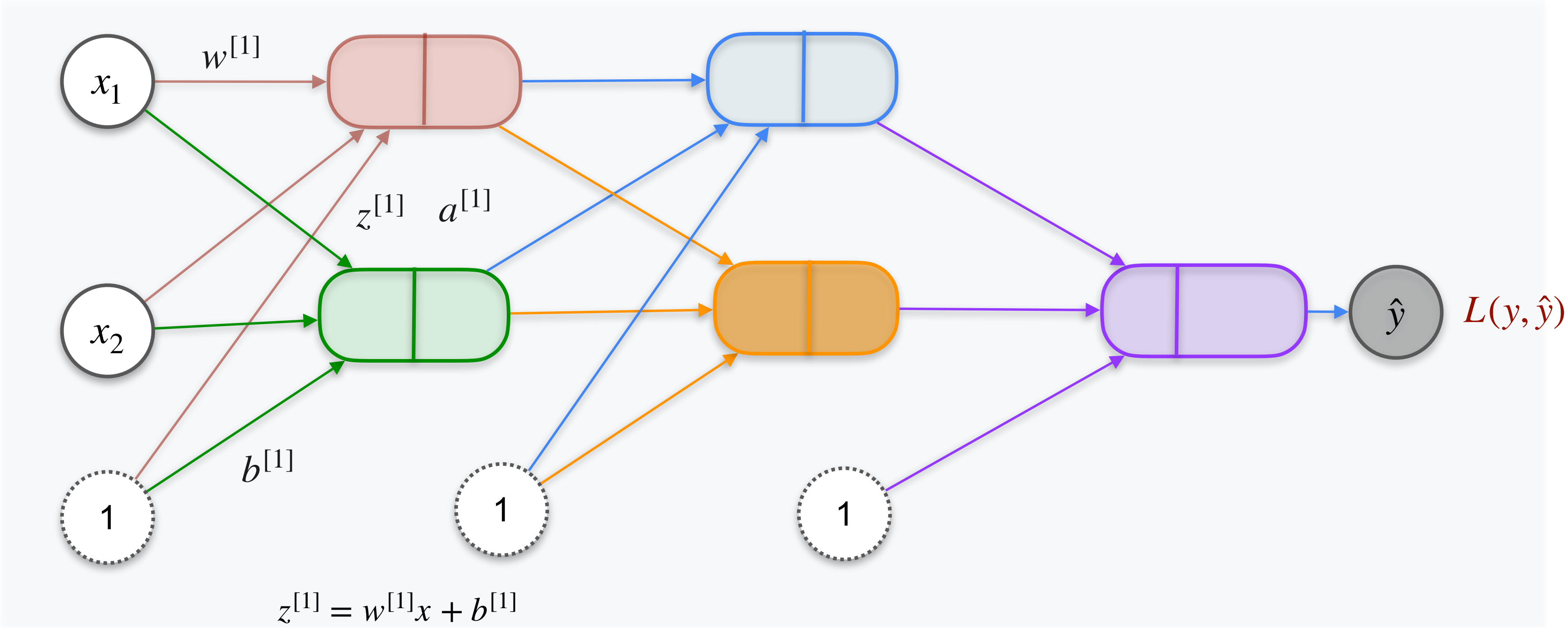

$$z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$

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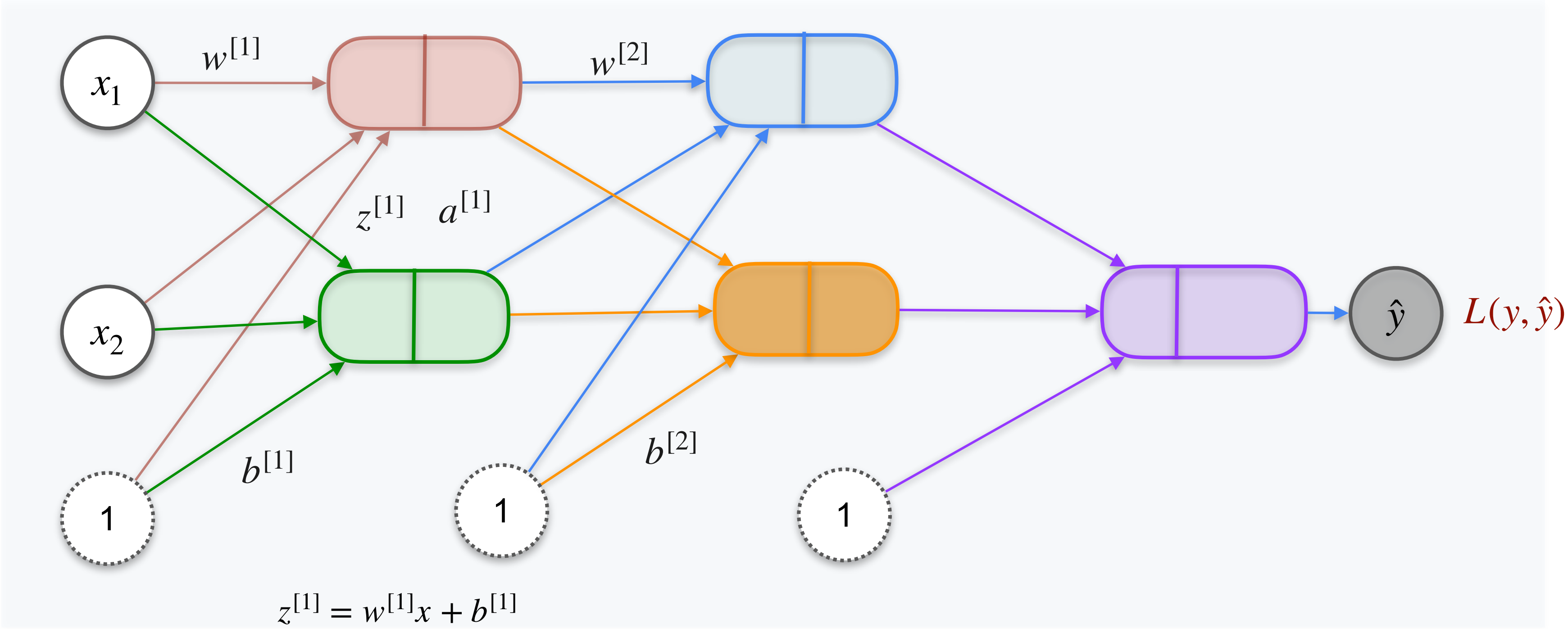

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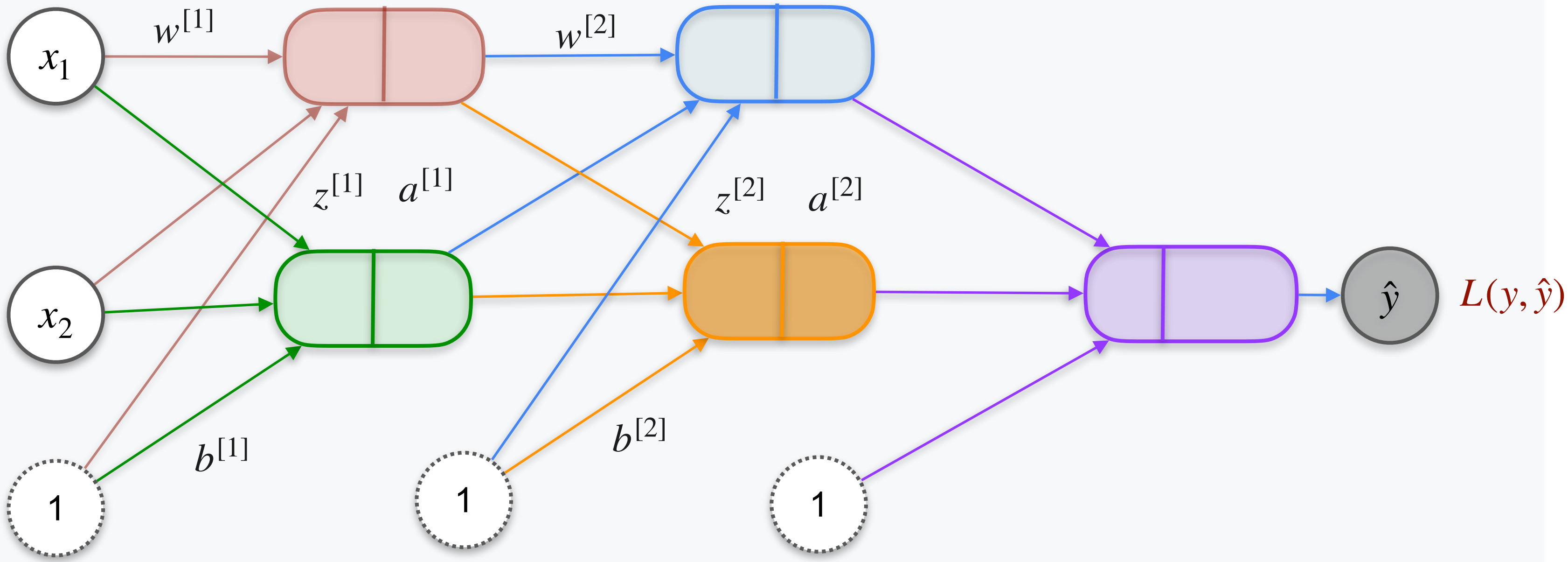

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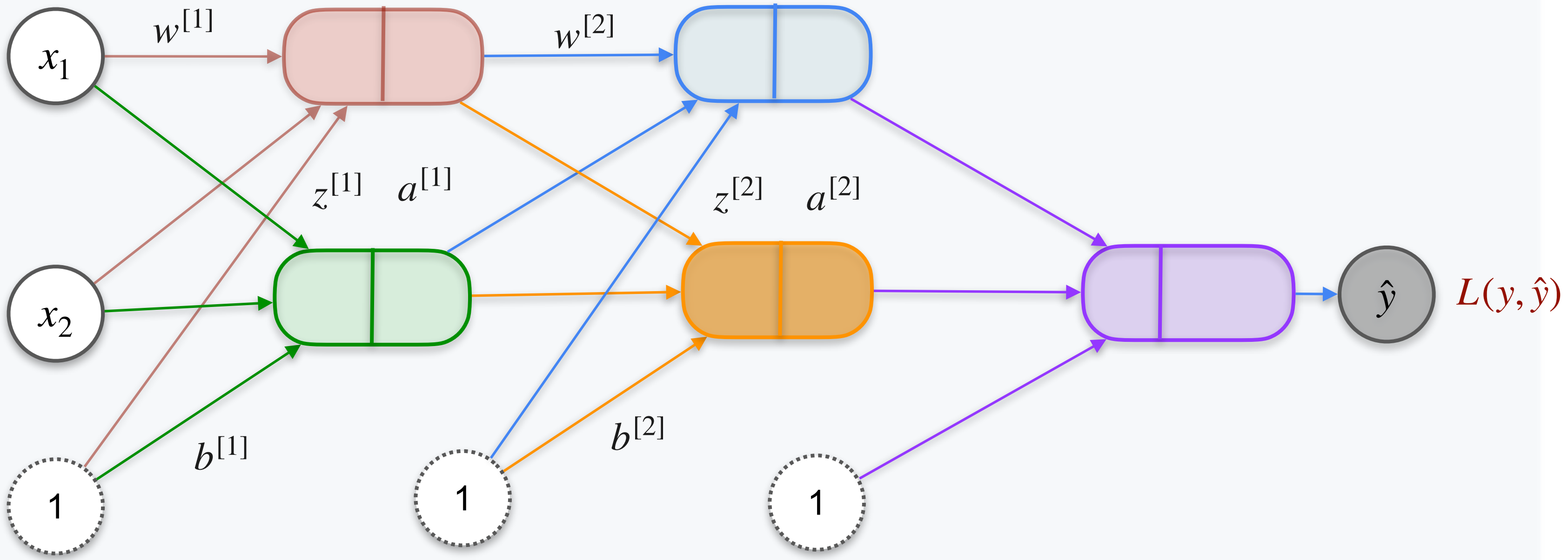

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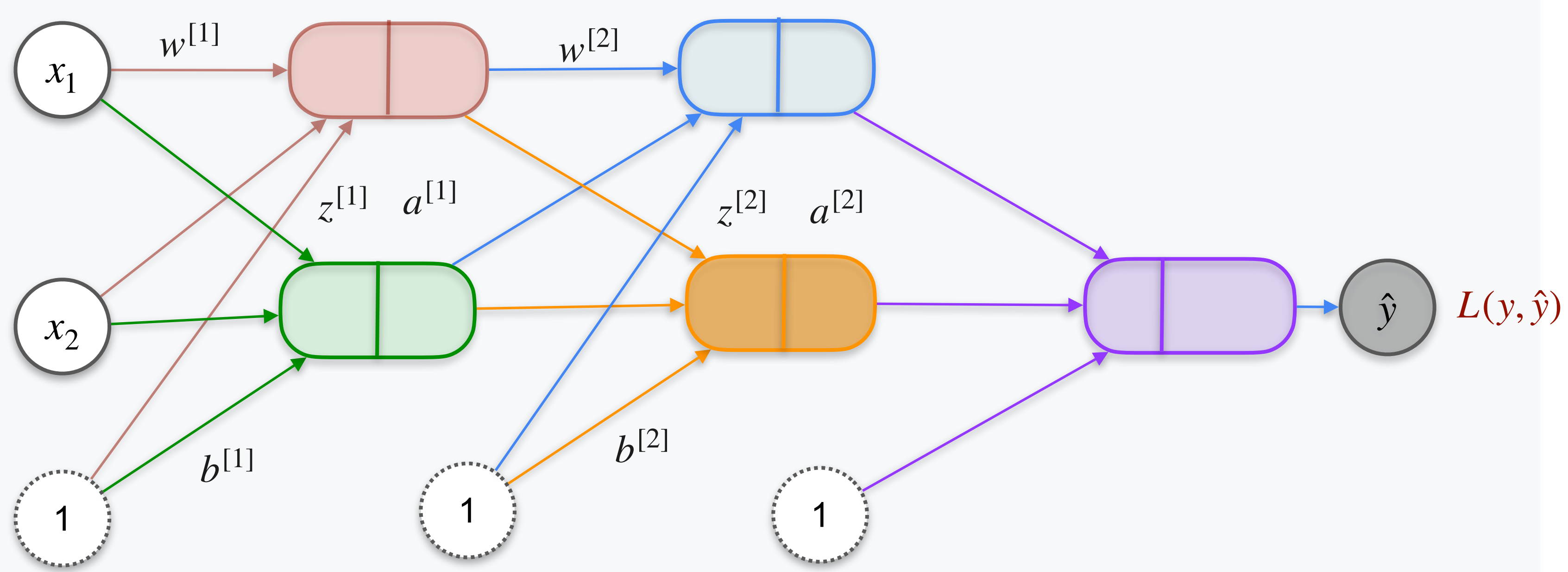

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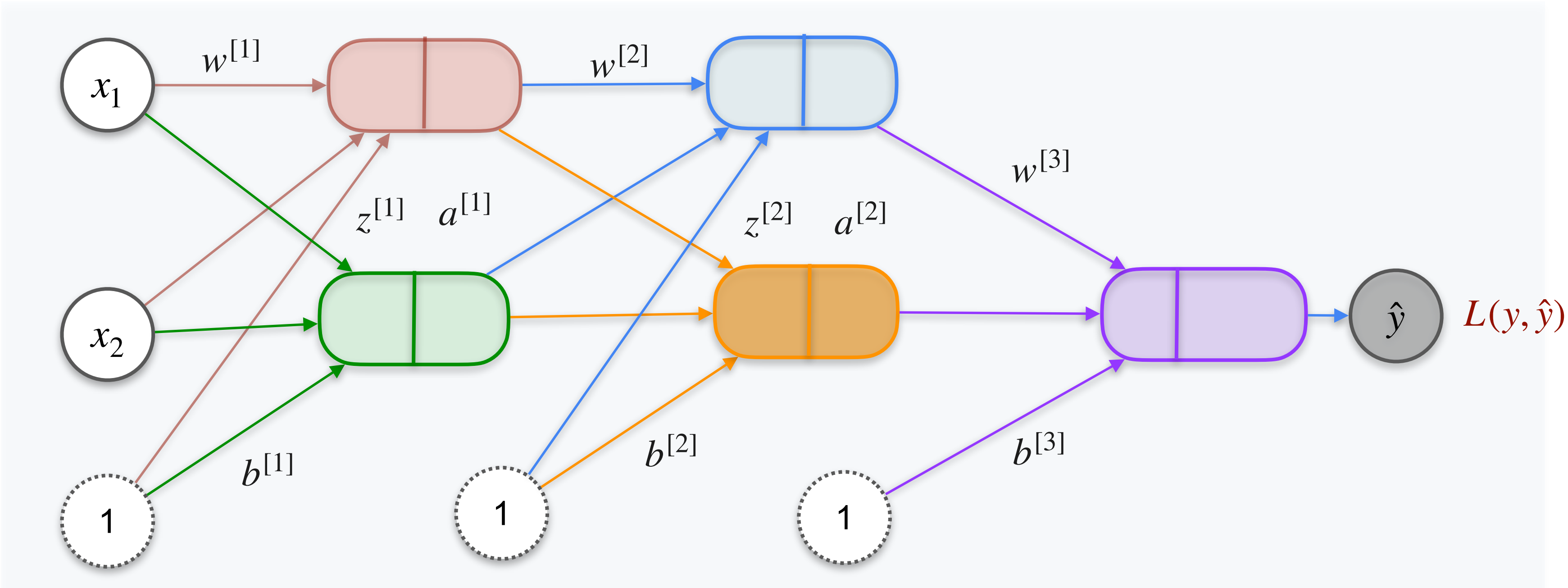

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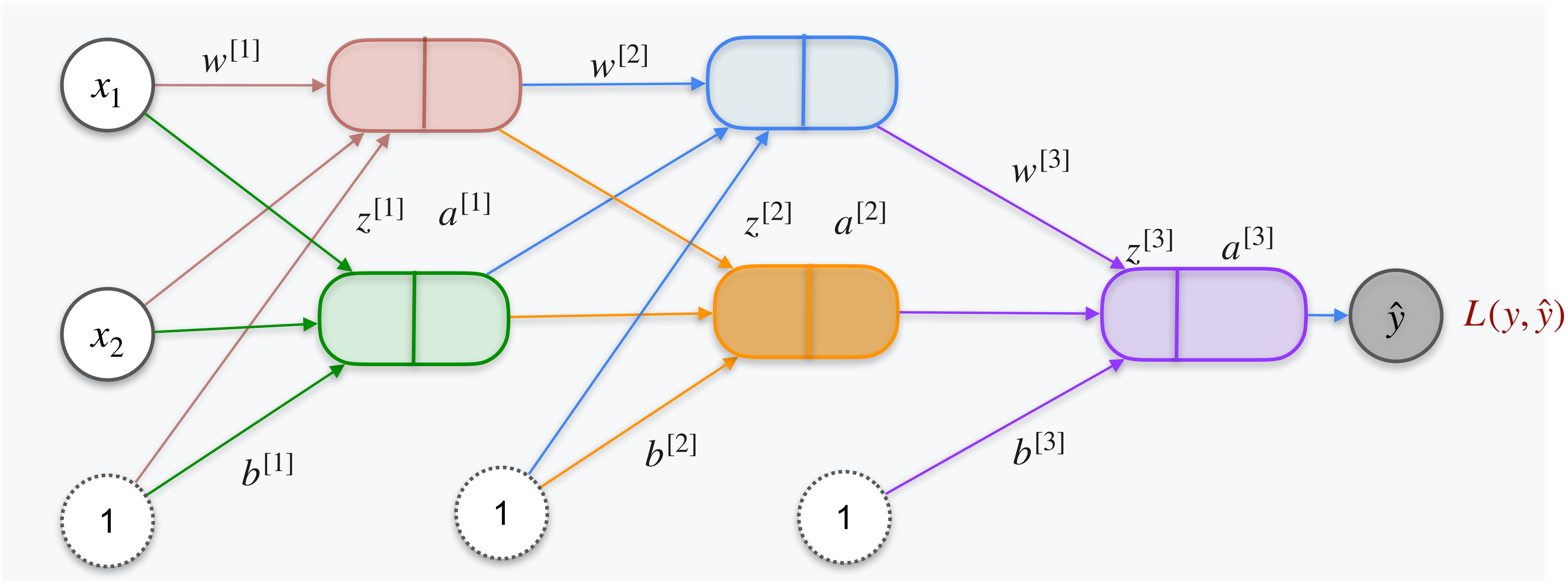

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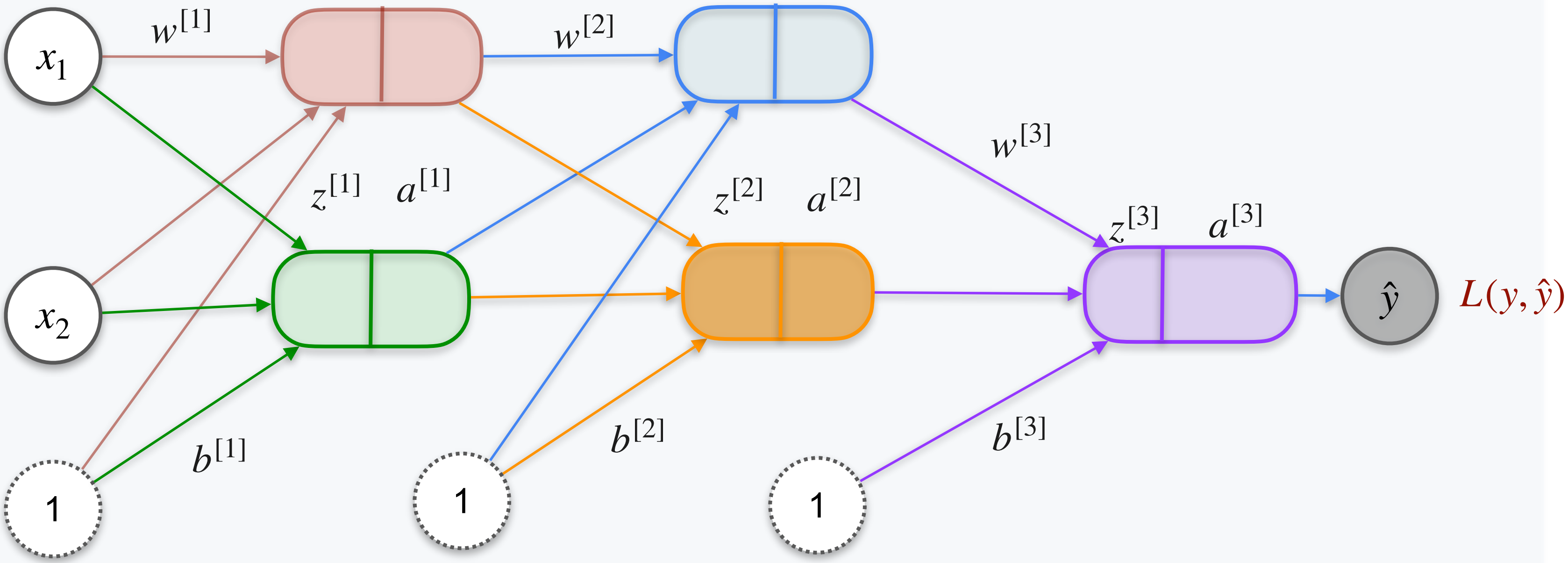

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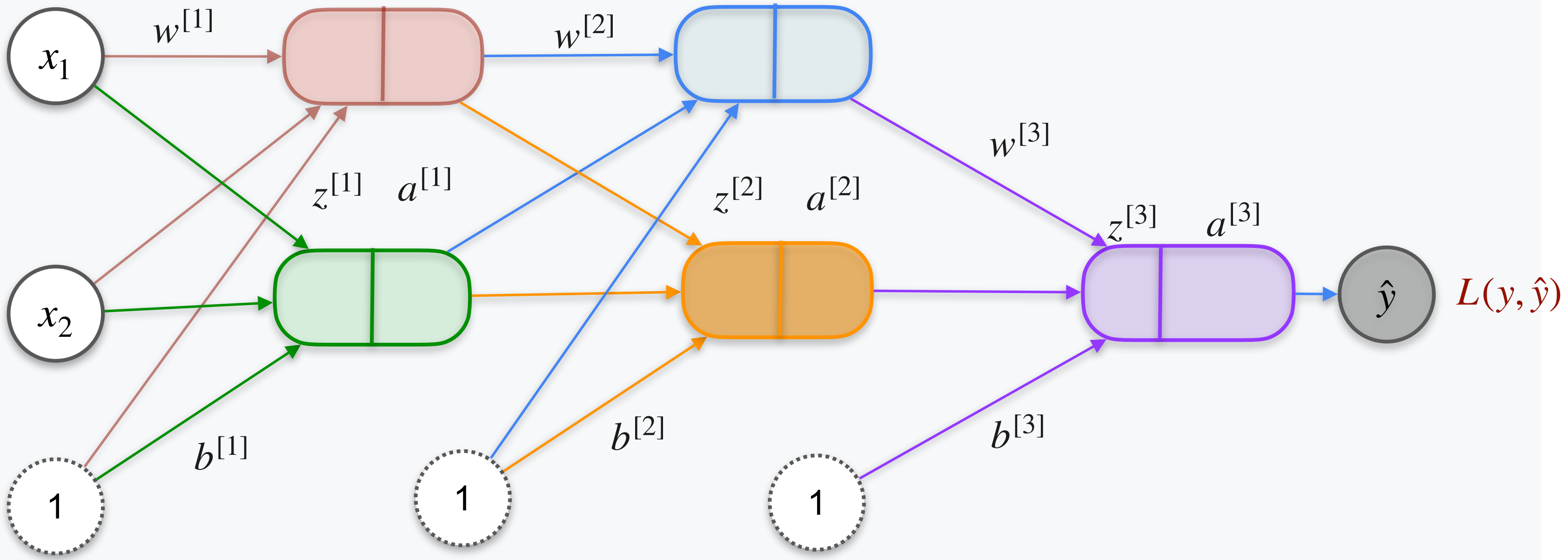

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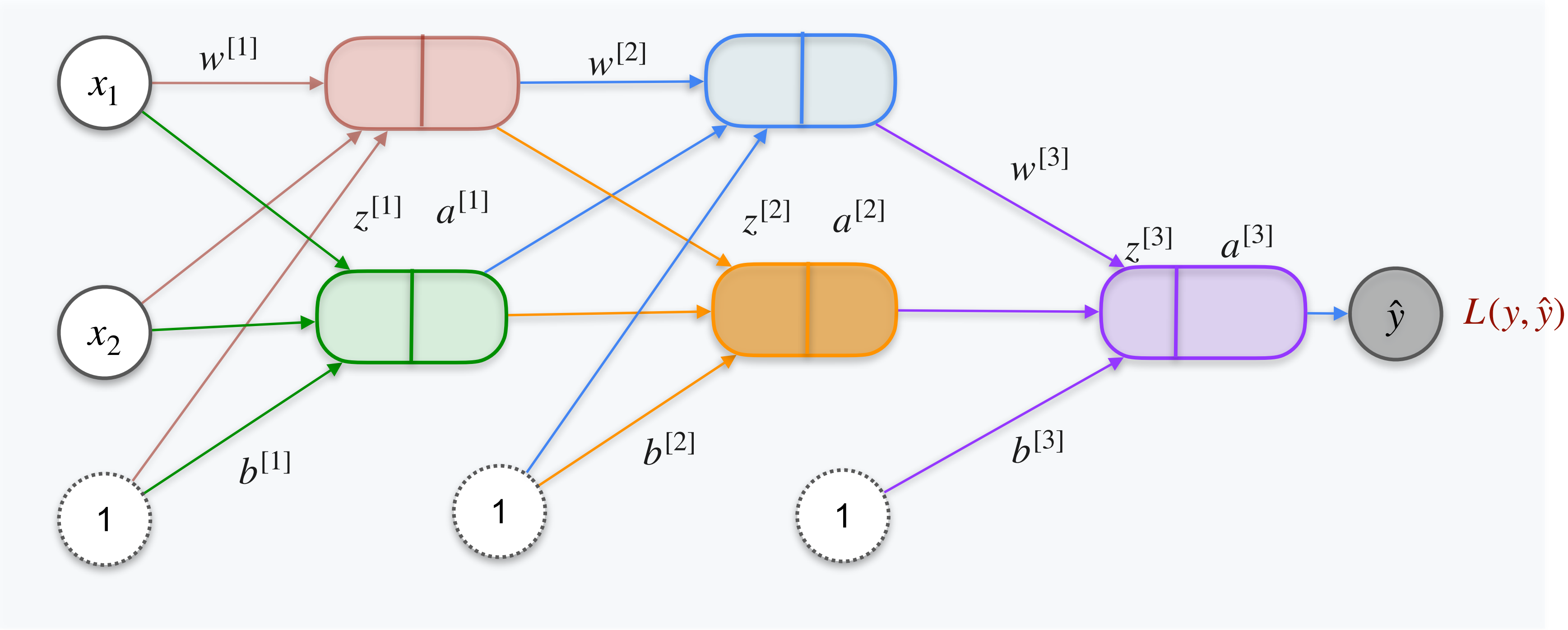

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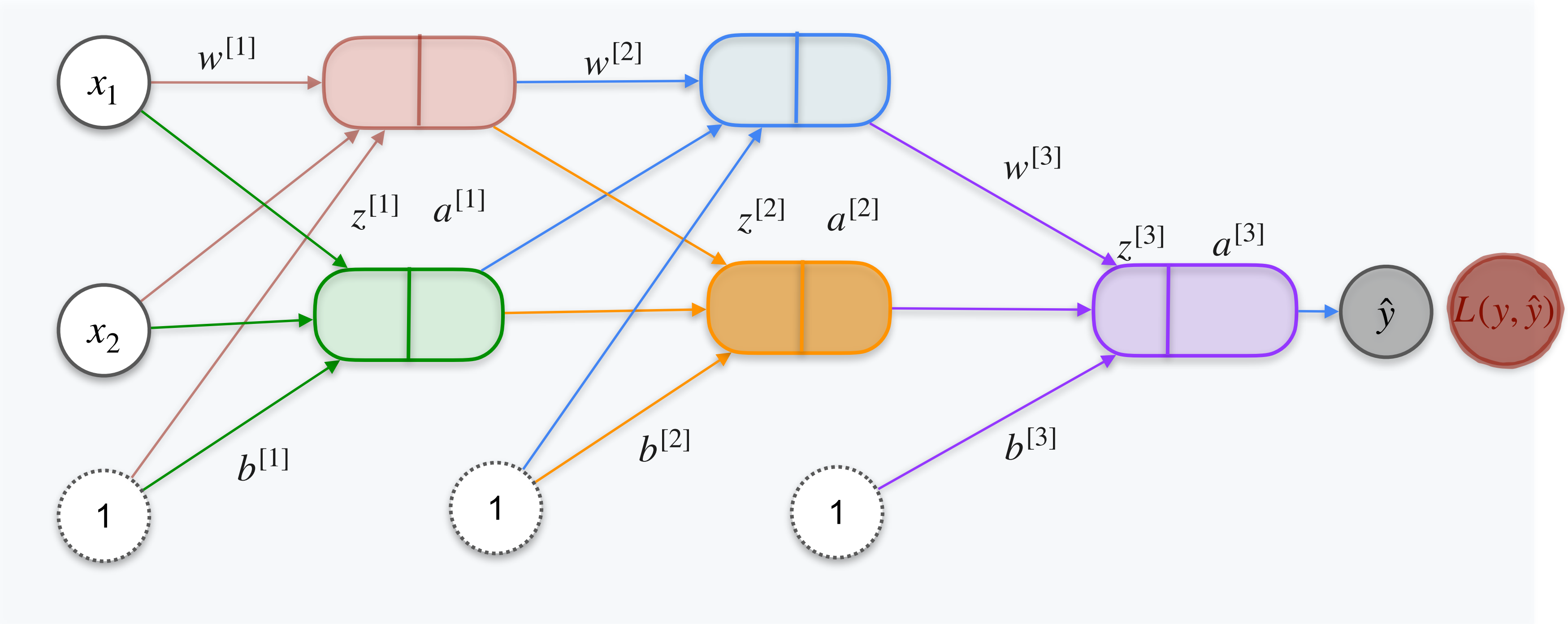

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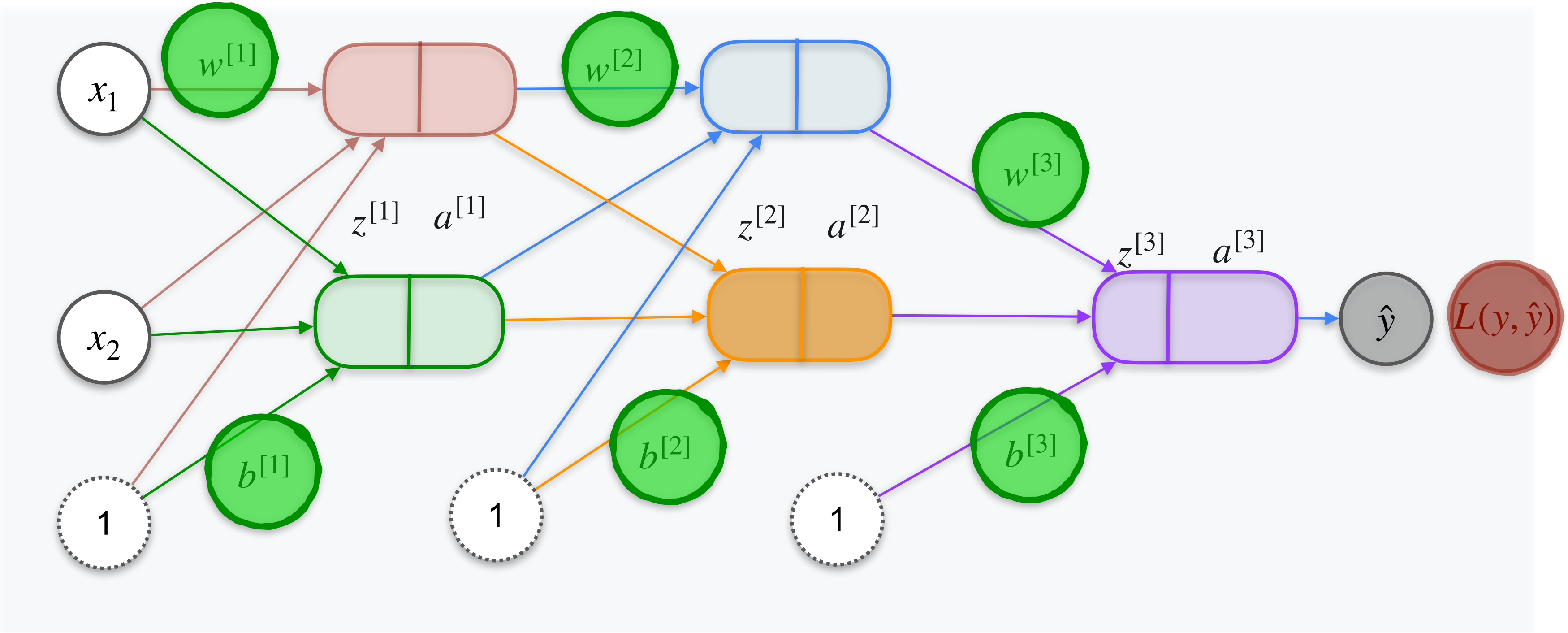

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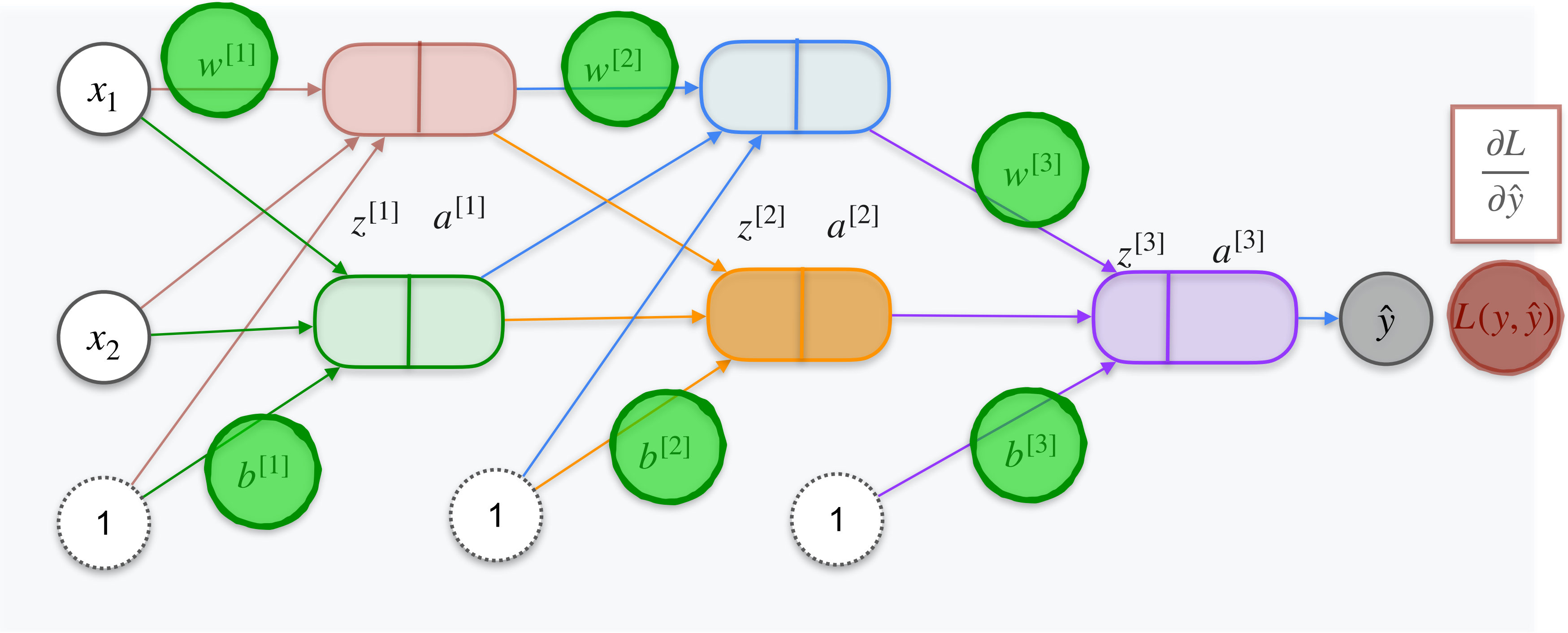

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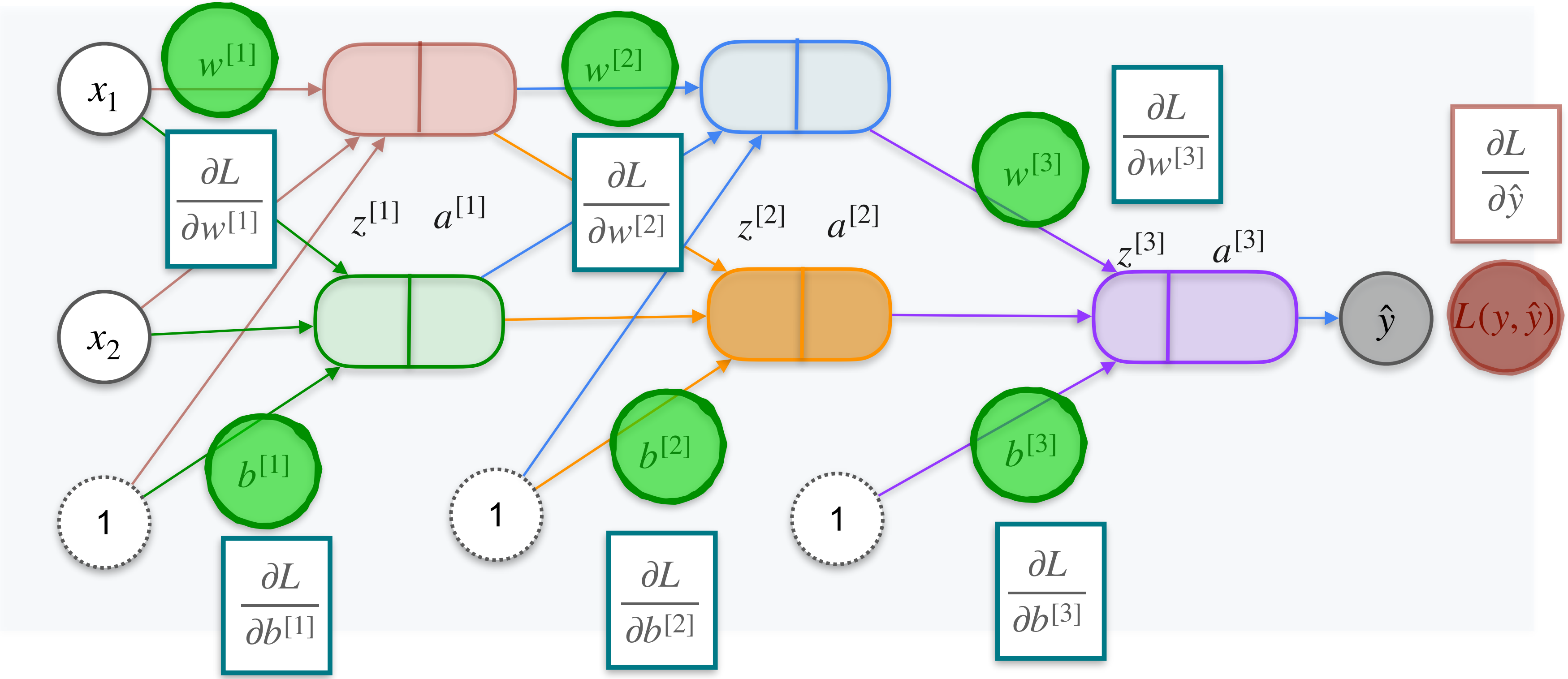

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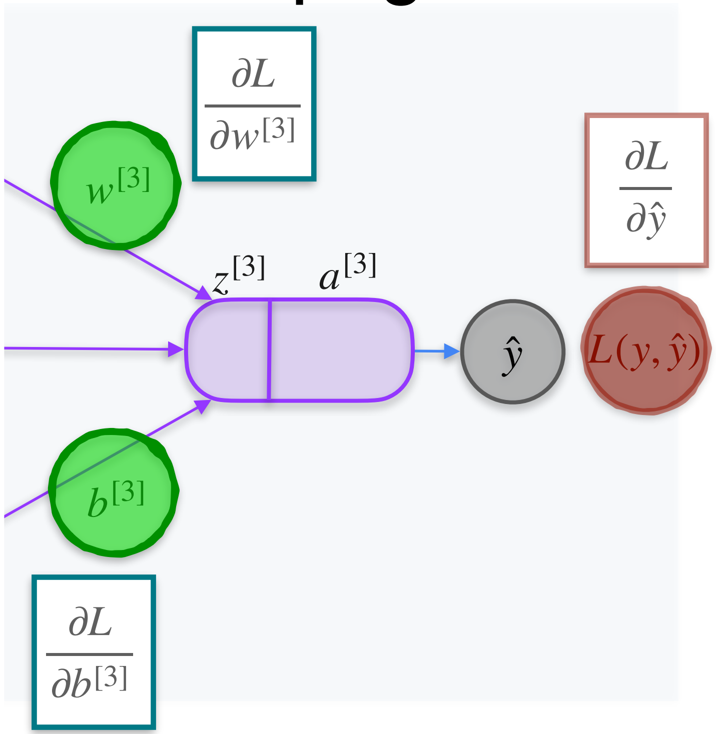

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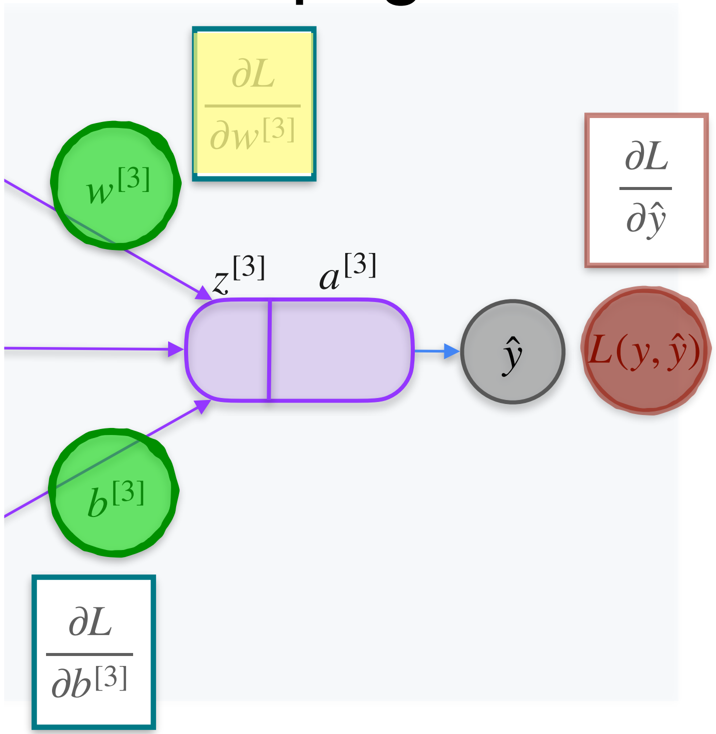

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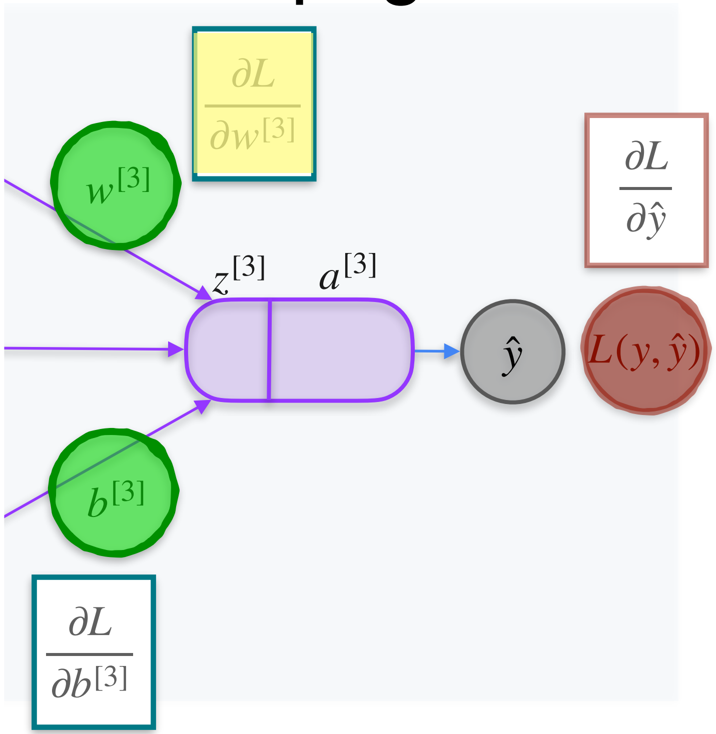

$$\frac{\partial L}{\partial w^{[3]}}$$

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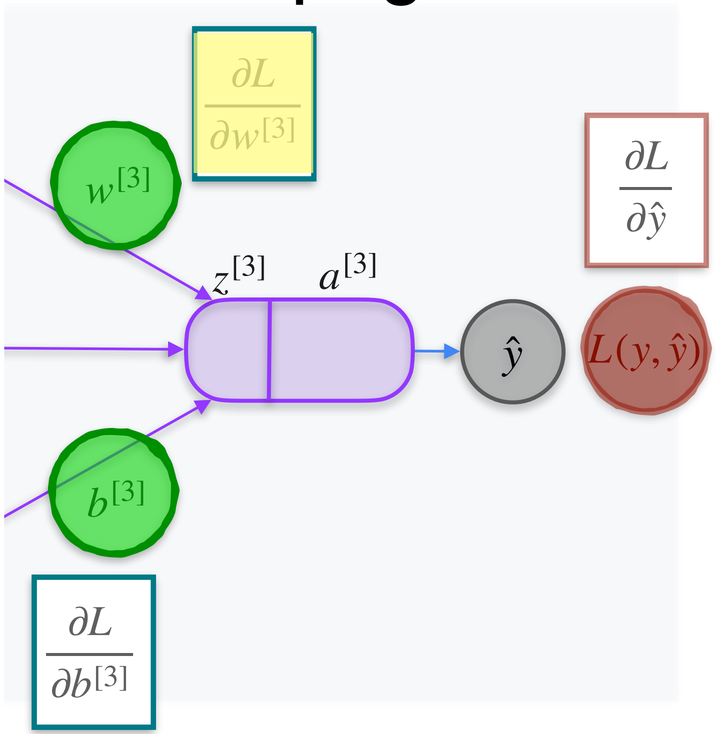

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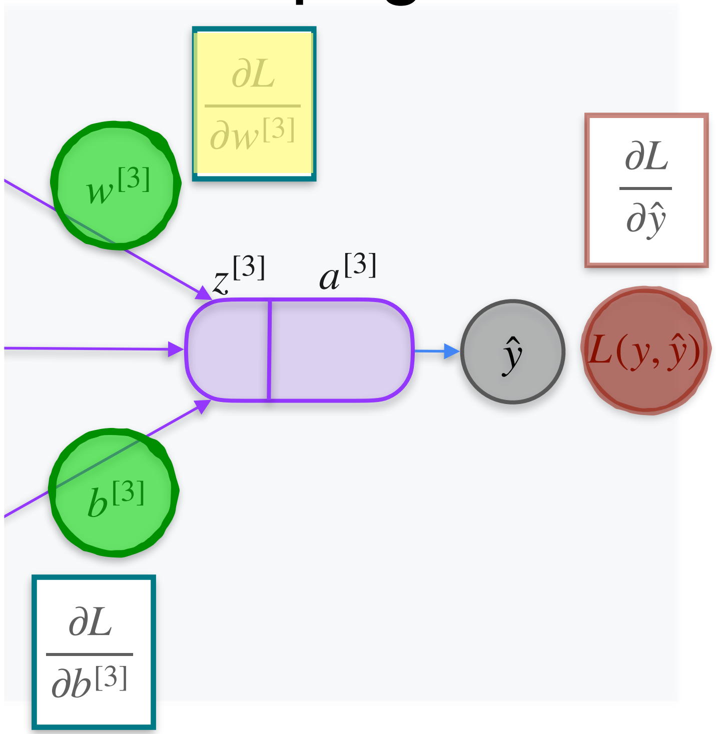

$$\frac{\partial L}{\partial w^{[3]}} \qquad \frac{\partial a^{[3]}}{\partial z^{[3]}} \qquad \frac{\partial L}{\partial a^{[3]}}$$

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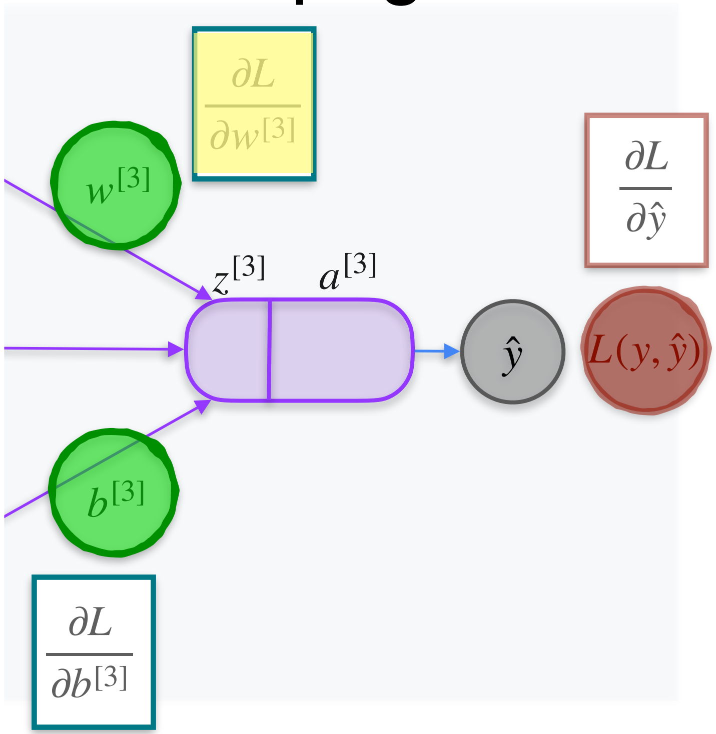

$$\frac{\partial L}{\partial w^{[3]}} \qquad \frac{\partial z^{[3]}}{\partial w^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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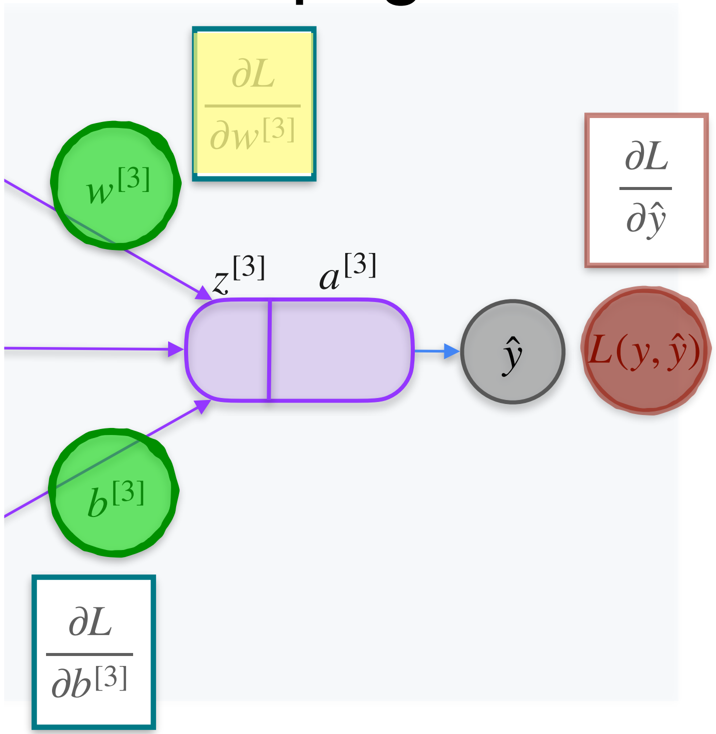

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

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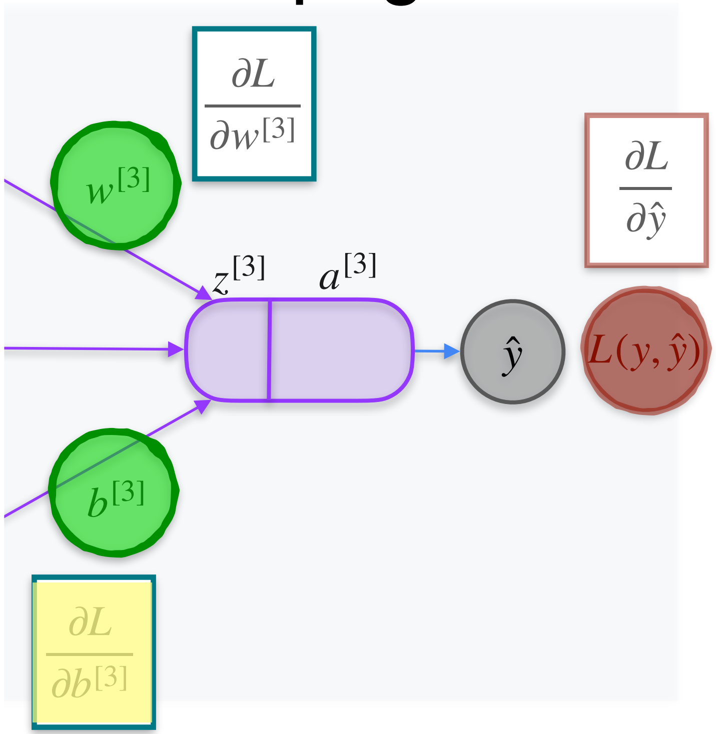

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

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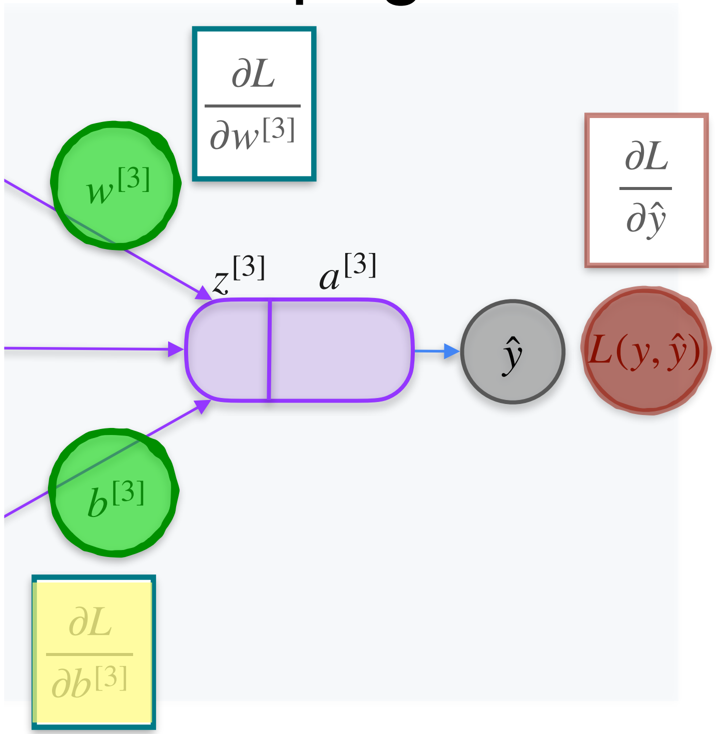

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}}$$

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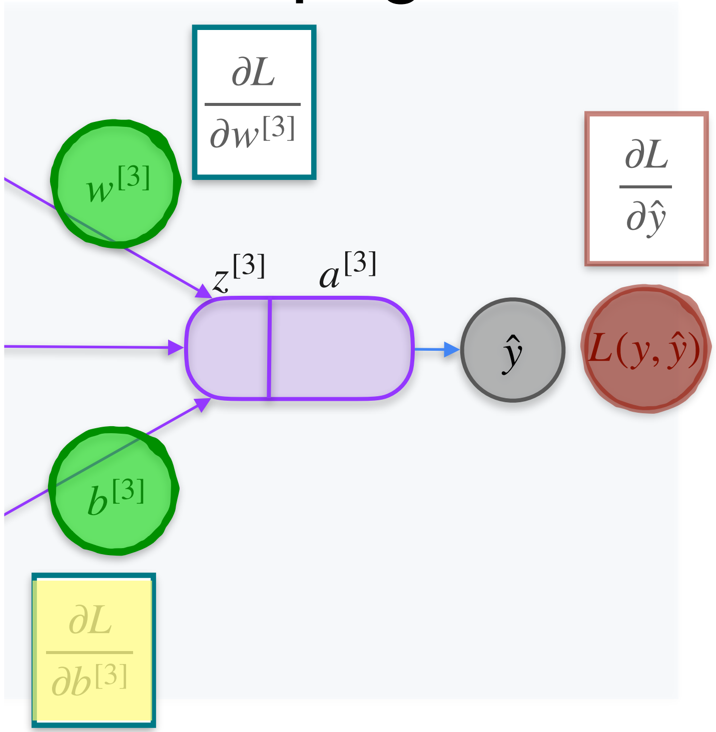

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} \quad \frac{\partial z^{[3]}}{\partial b^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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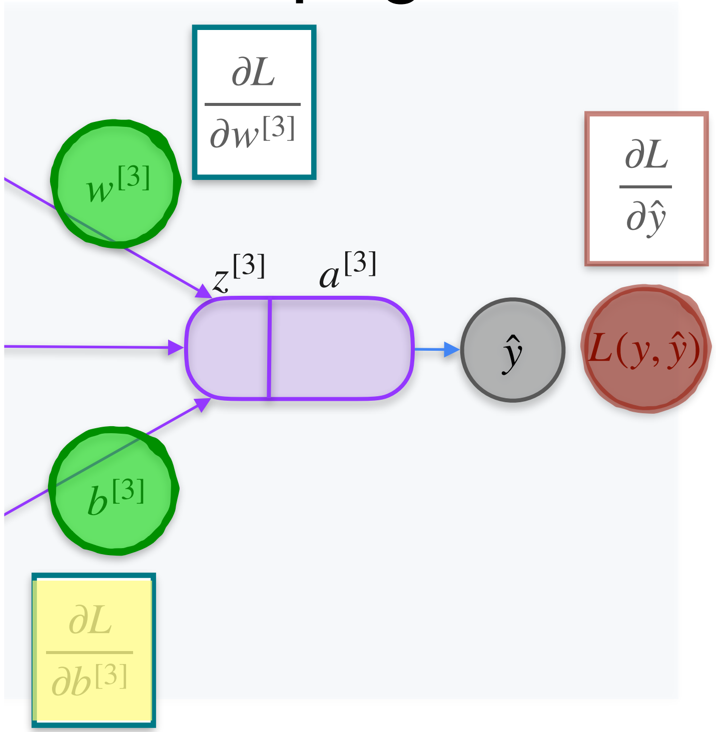

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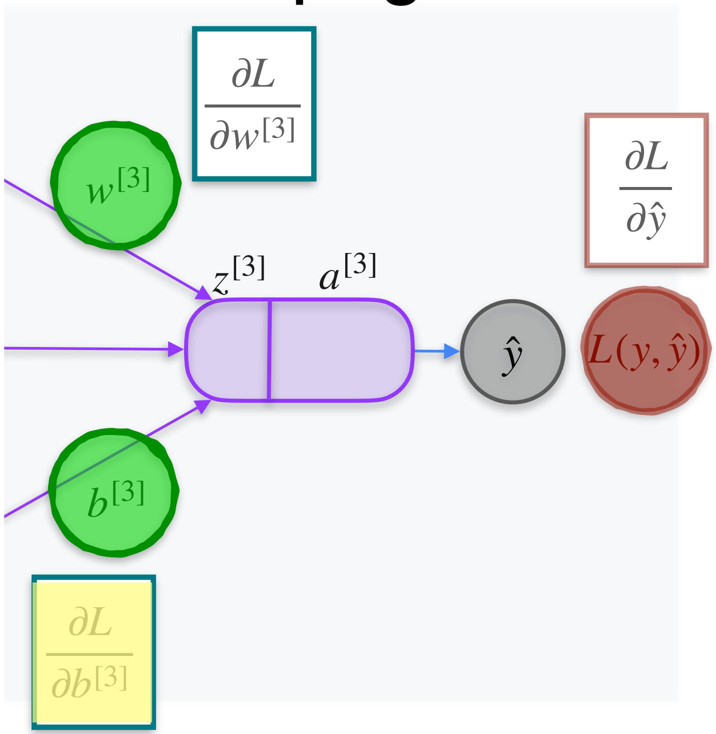

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} = \frac{\partial z^{[3]}}{\partial b^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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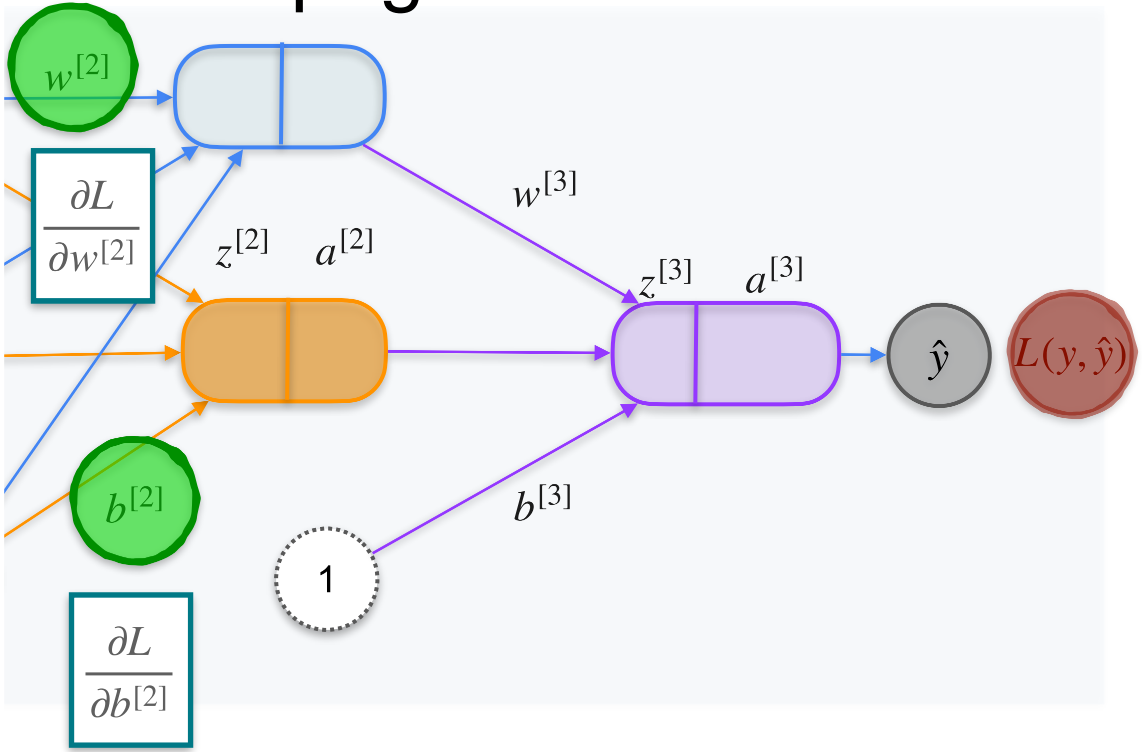

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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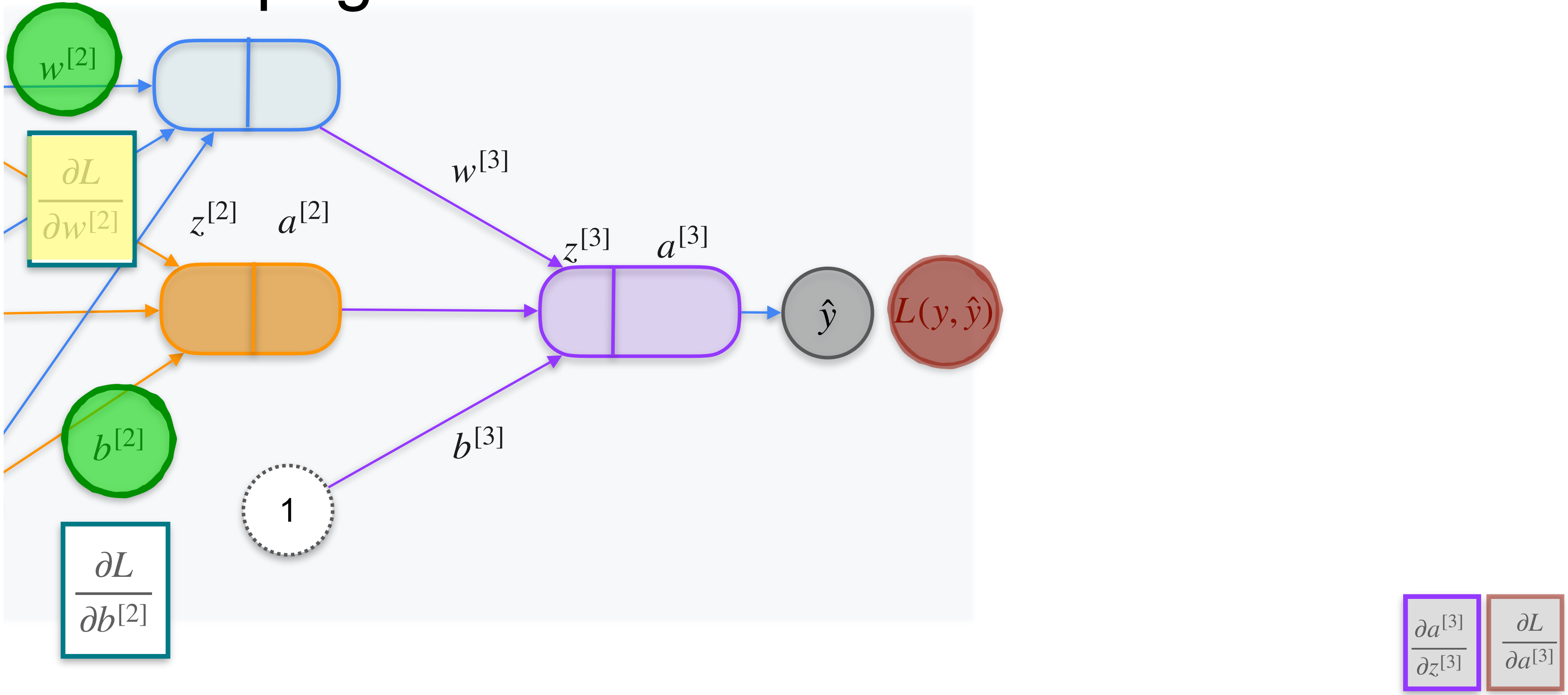

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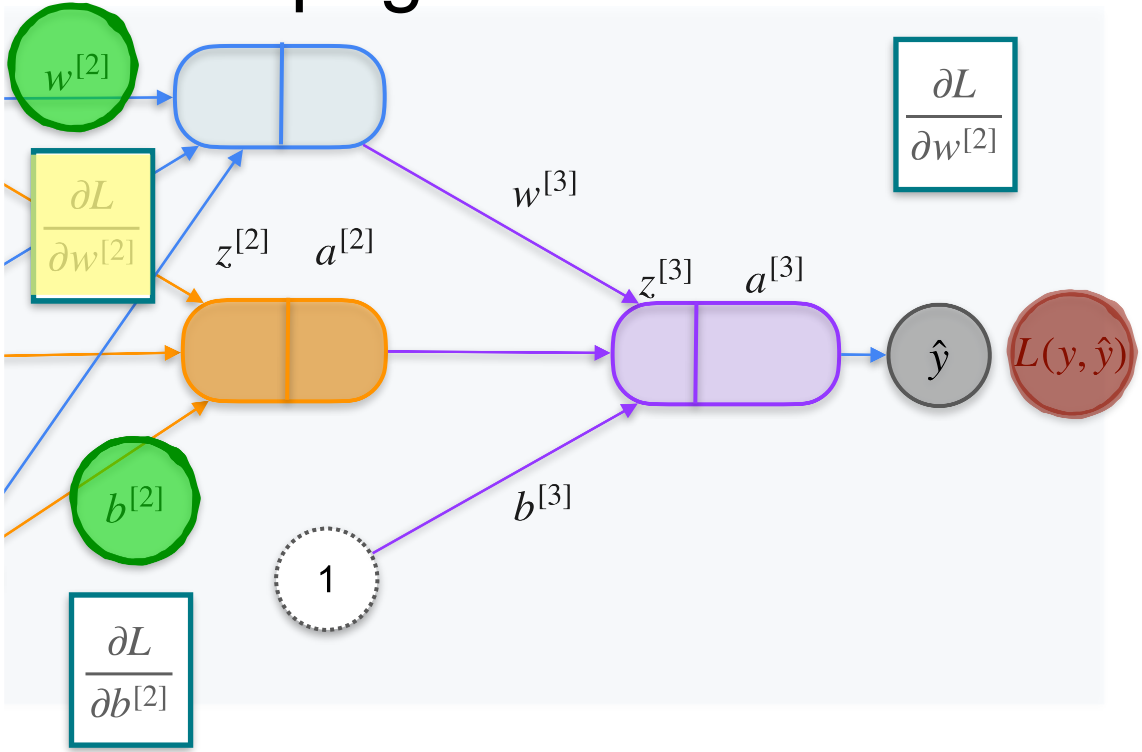

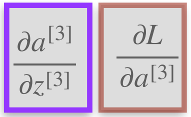

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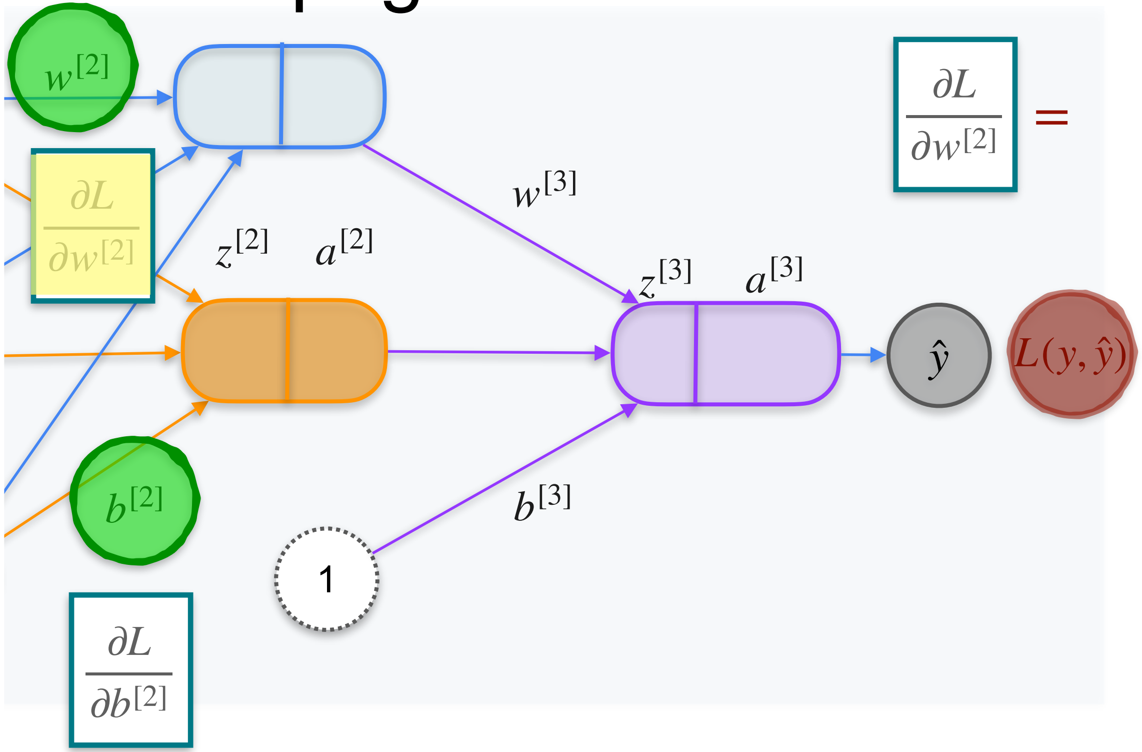

$$\frac{\partial L}{\partial w^{[2]}} =$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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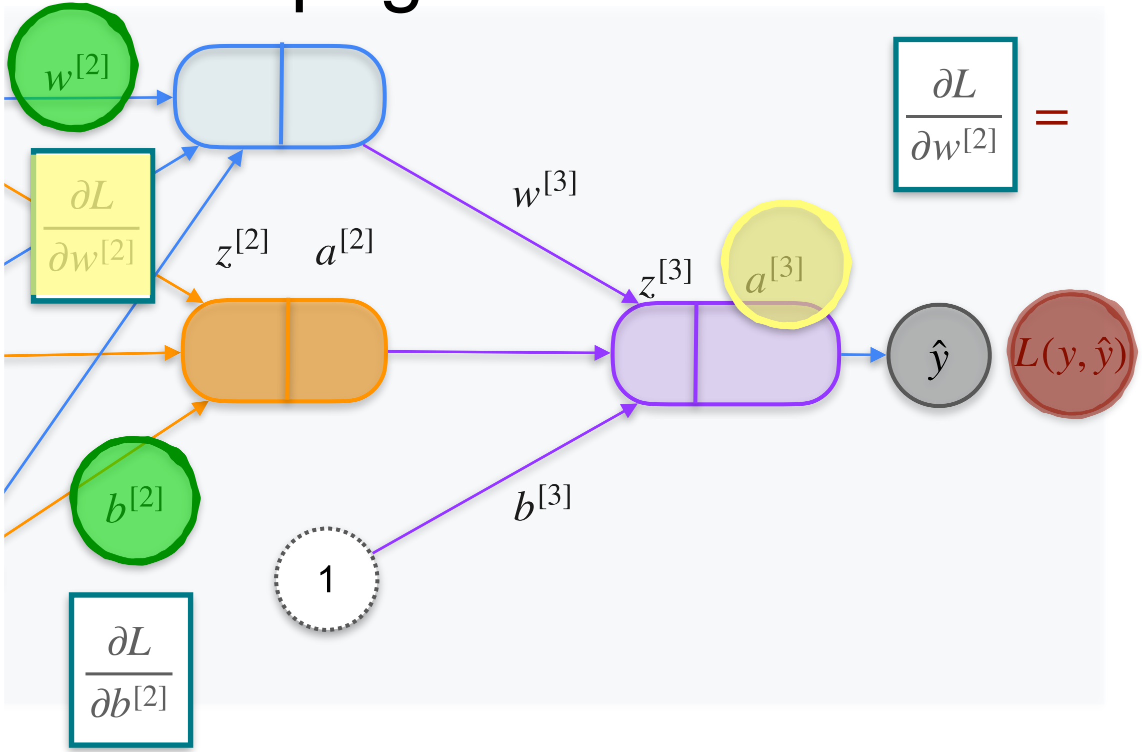

$$\frac{\partial L}{\partial w^{[2]}} =$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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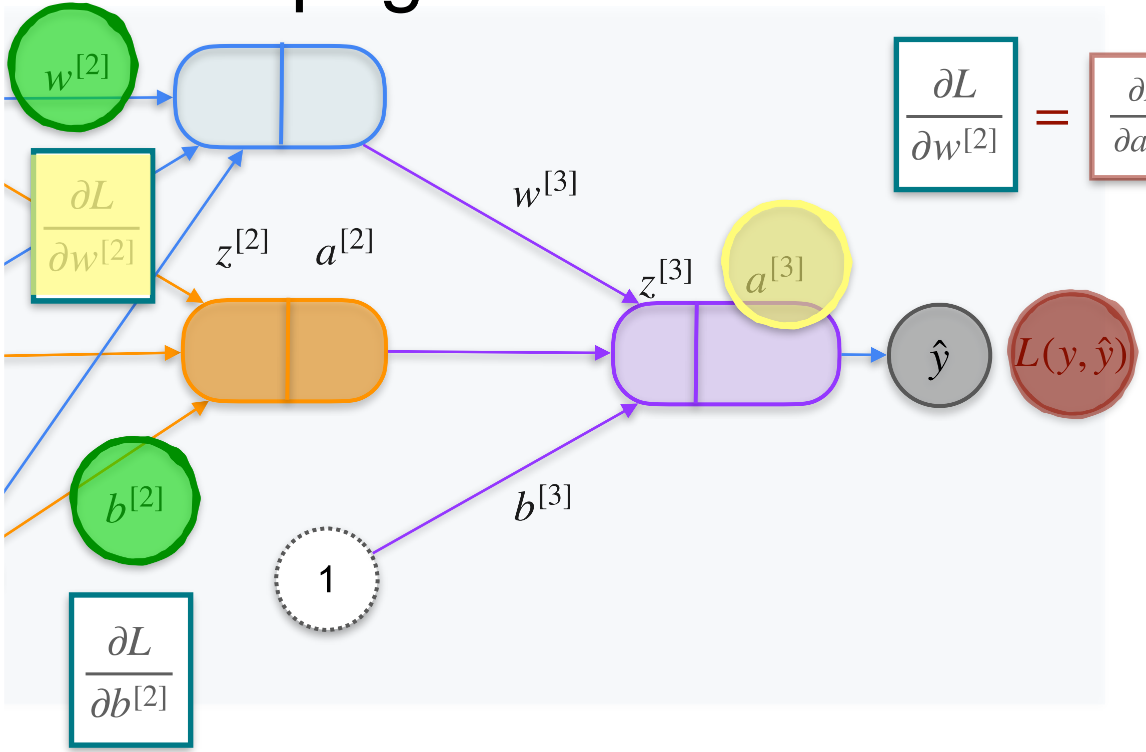

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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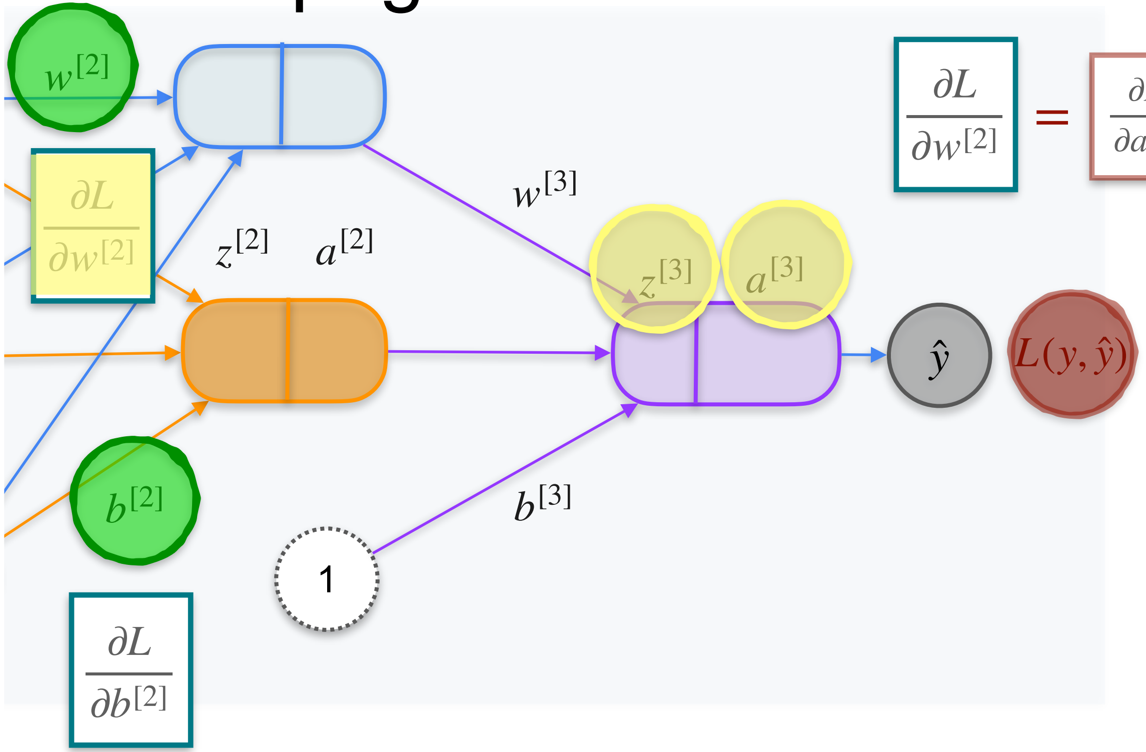

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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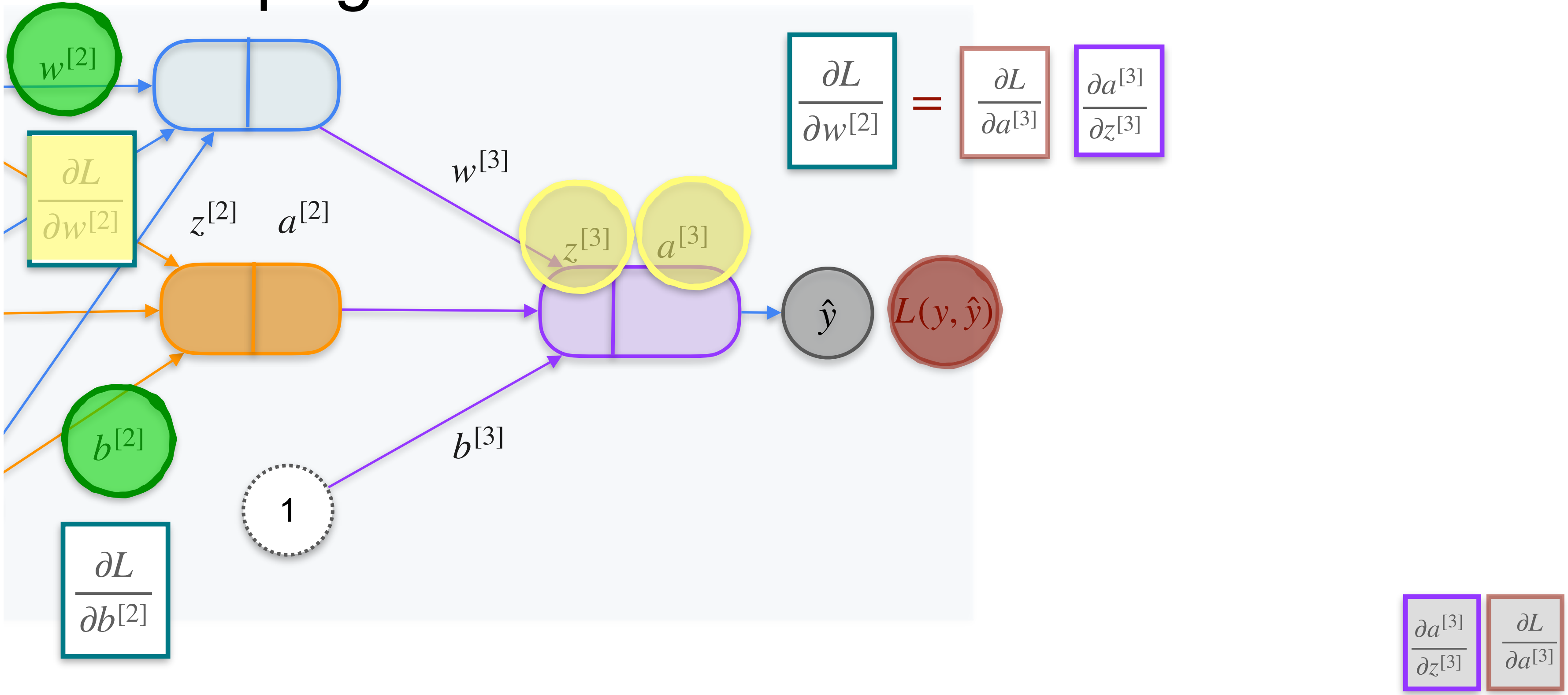

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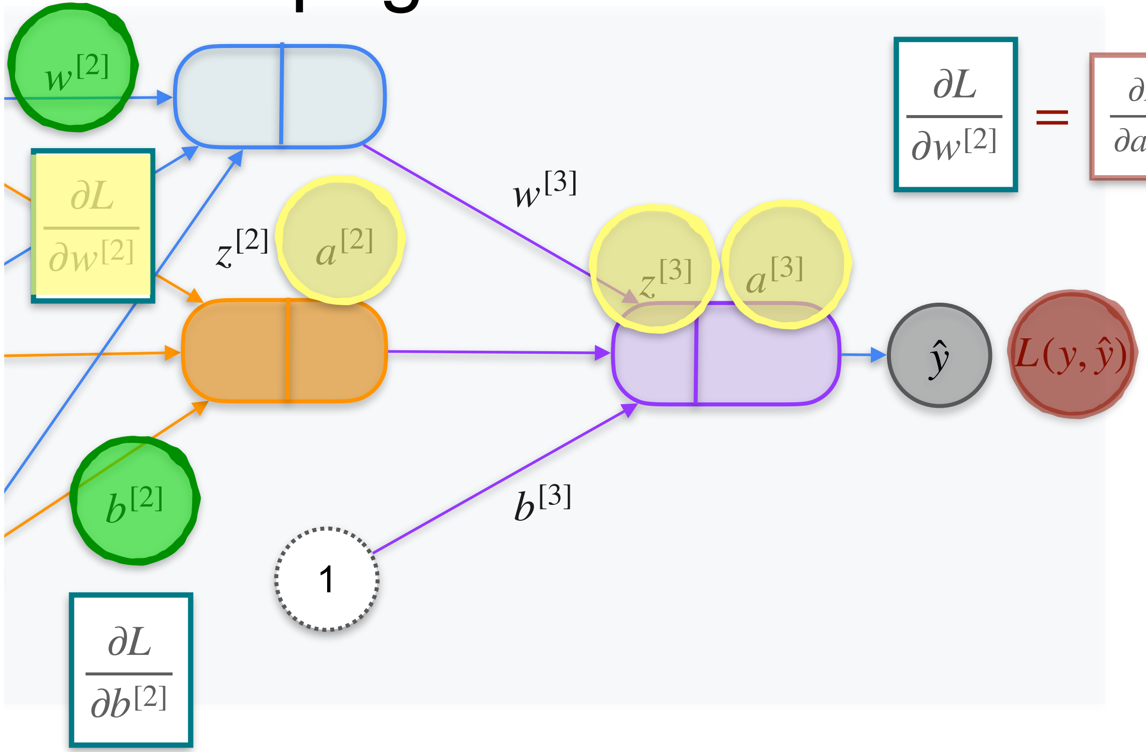

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \; \frac{\partial a^{[3]}}{\partial z^{[3]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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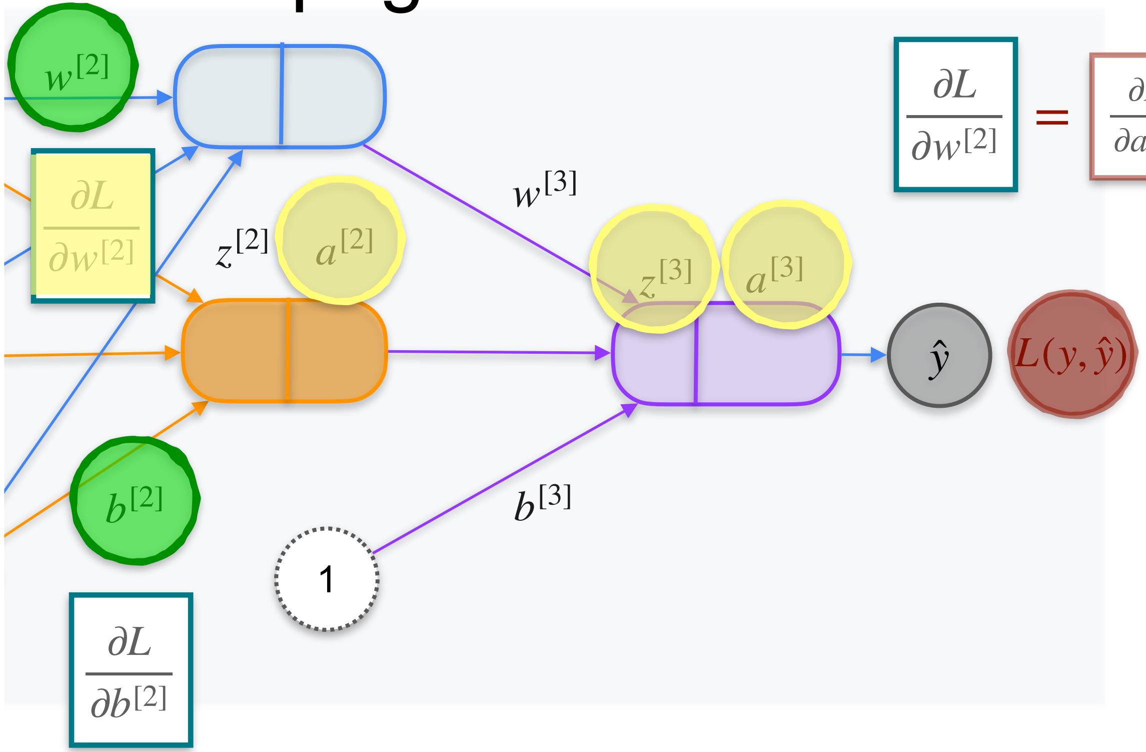

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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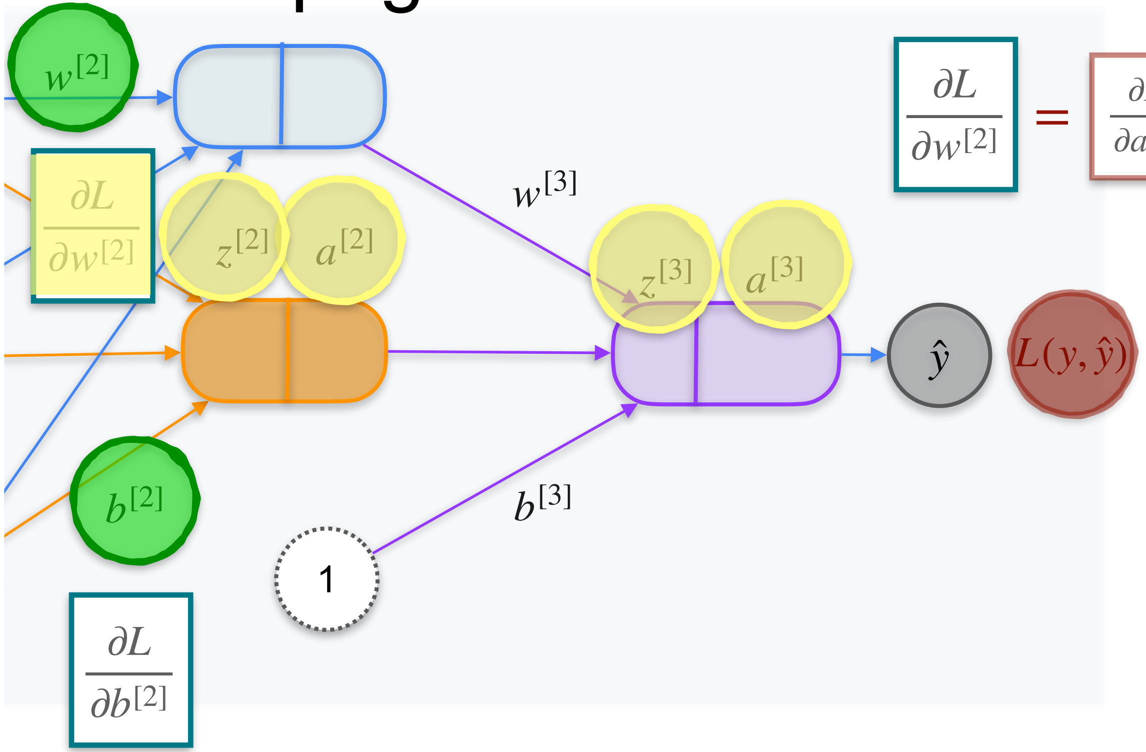

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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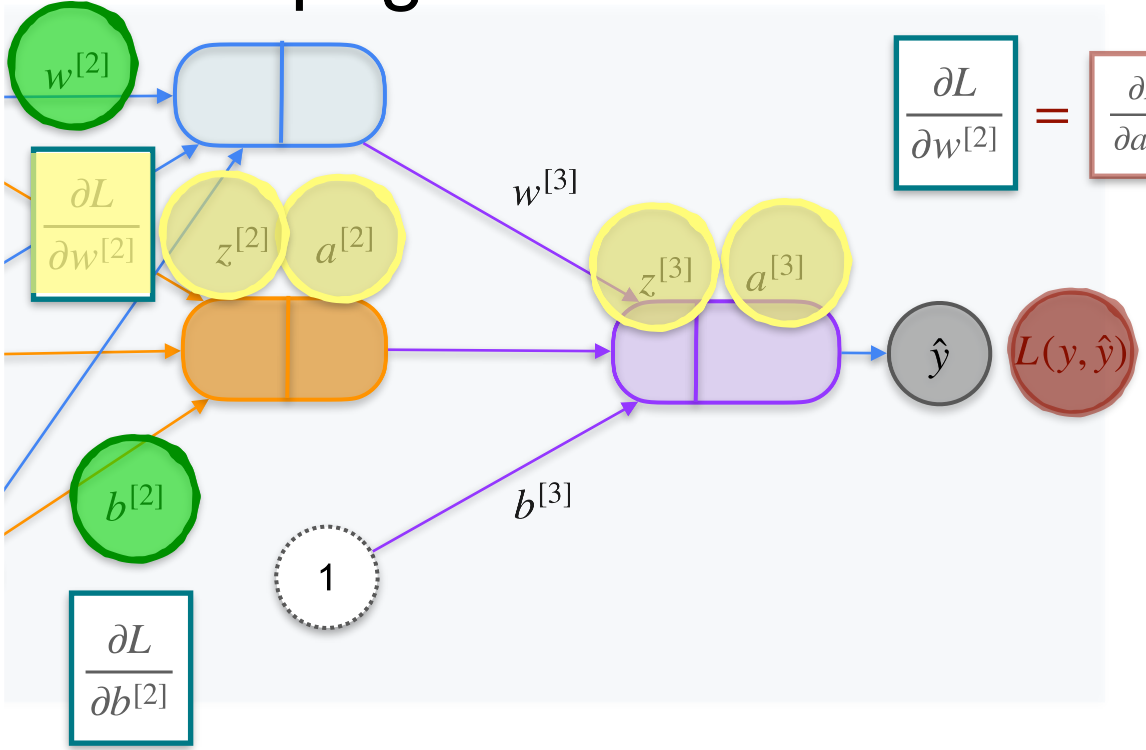

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}} \frac{\partial a^{[2]}}{\partial z^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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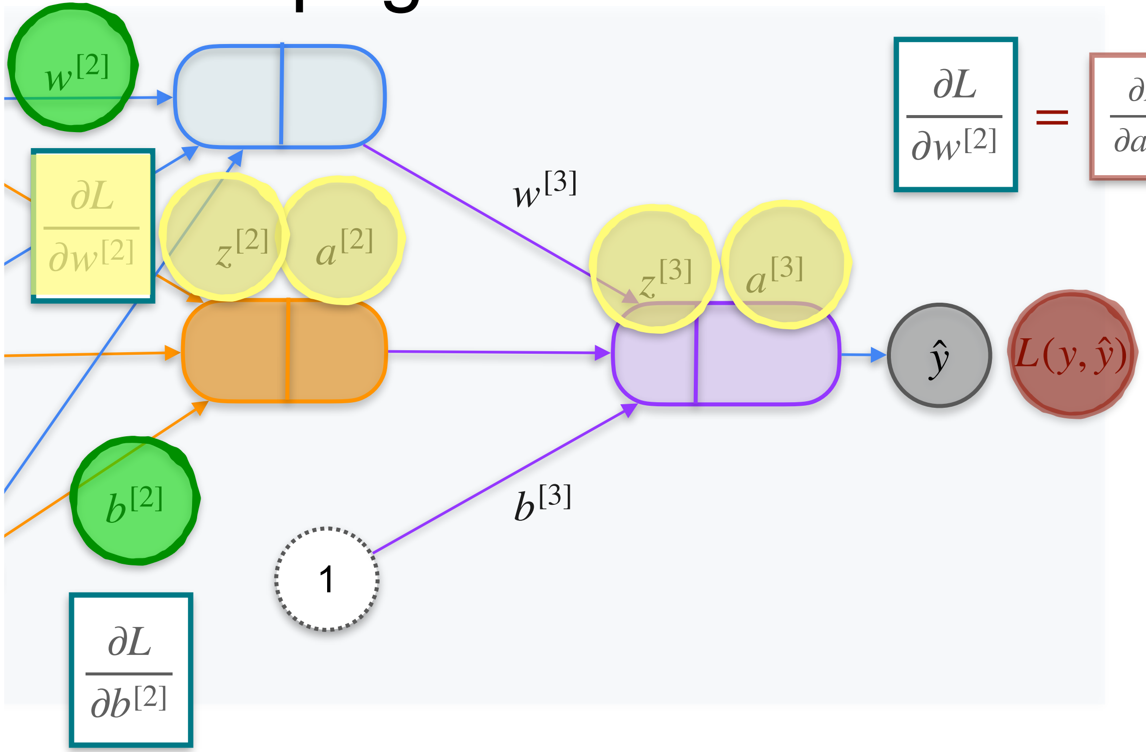

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}} \frac{\partial a^{[2]}}{\partial z^{[2]}} \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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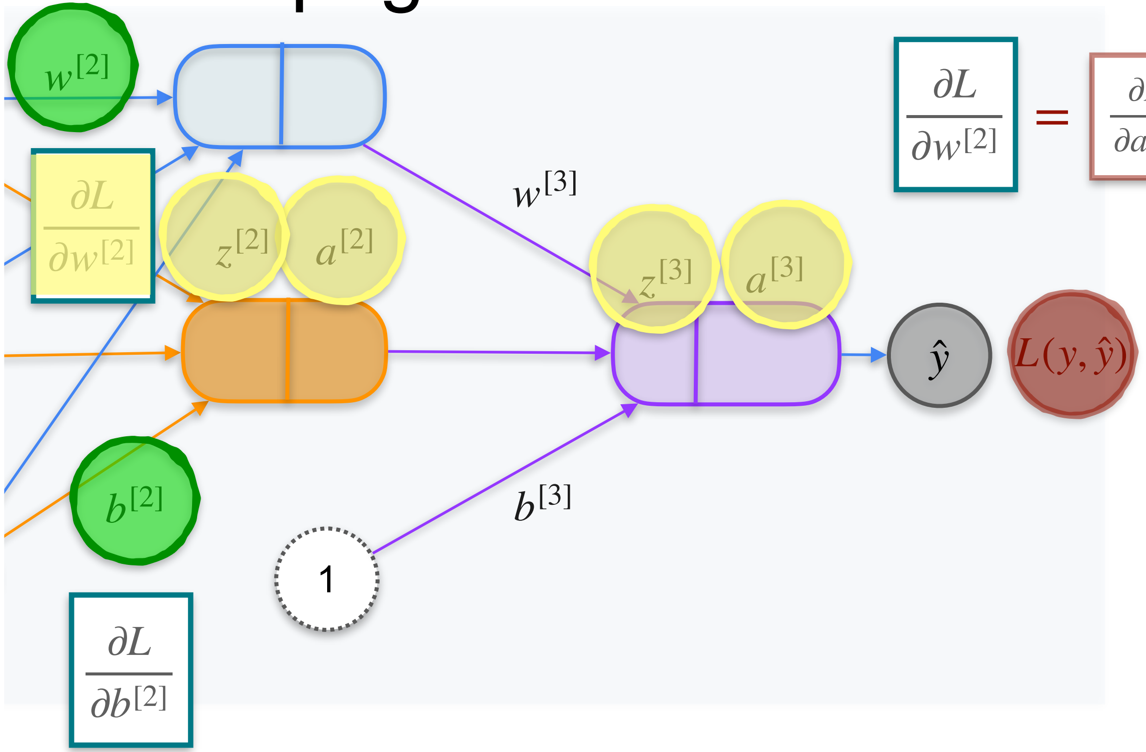

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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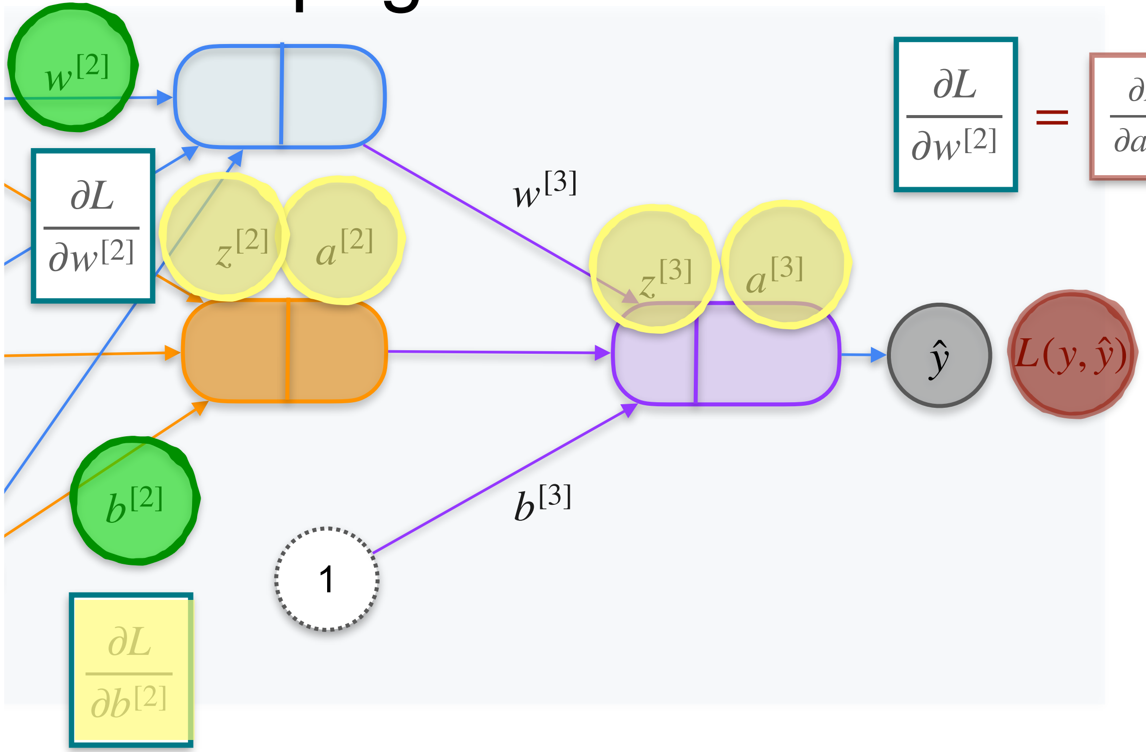

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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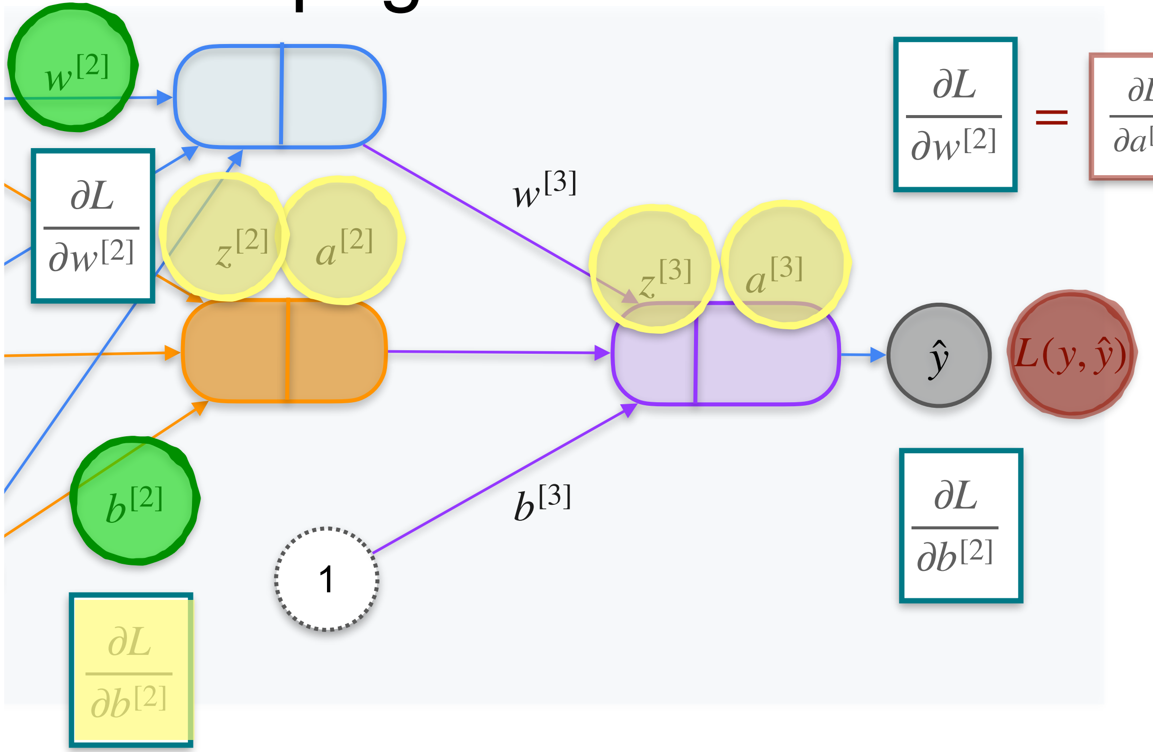

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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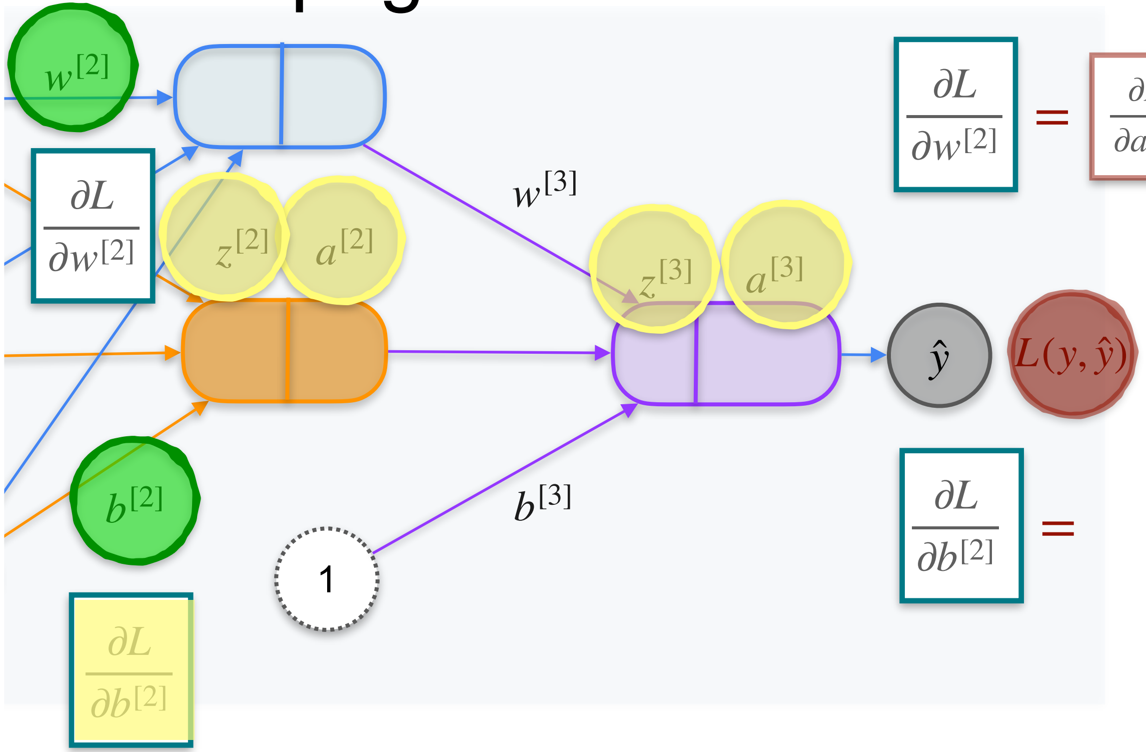

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} =$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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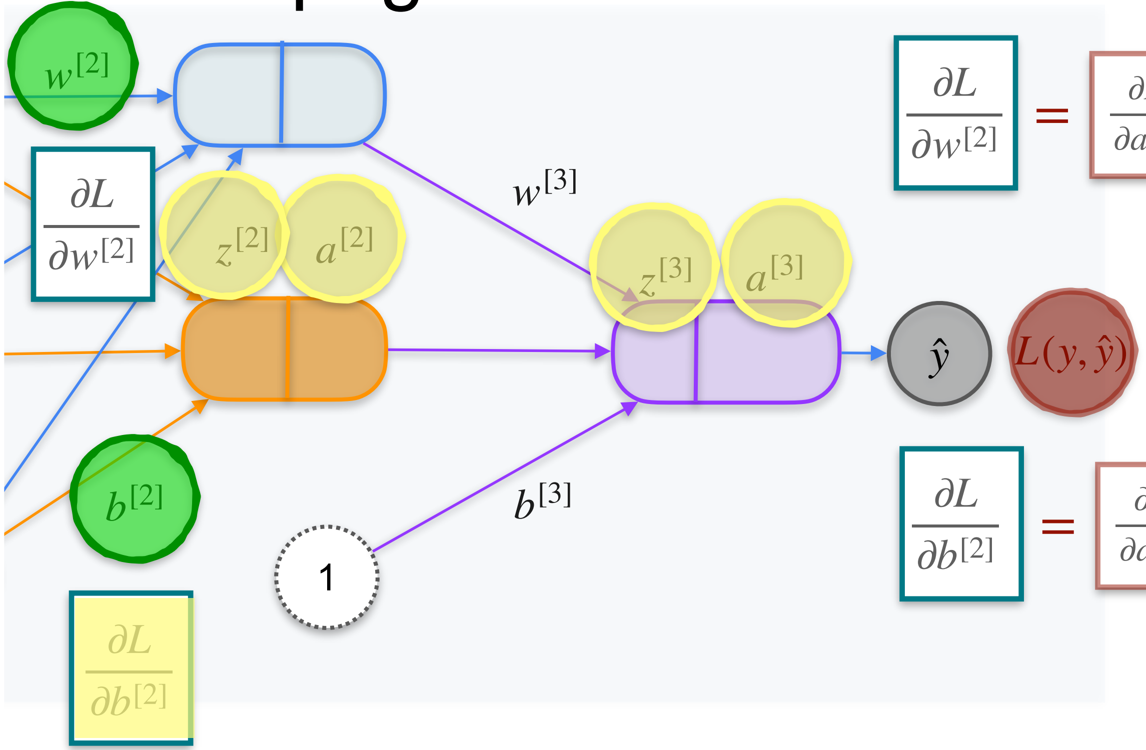

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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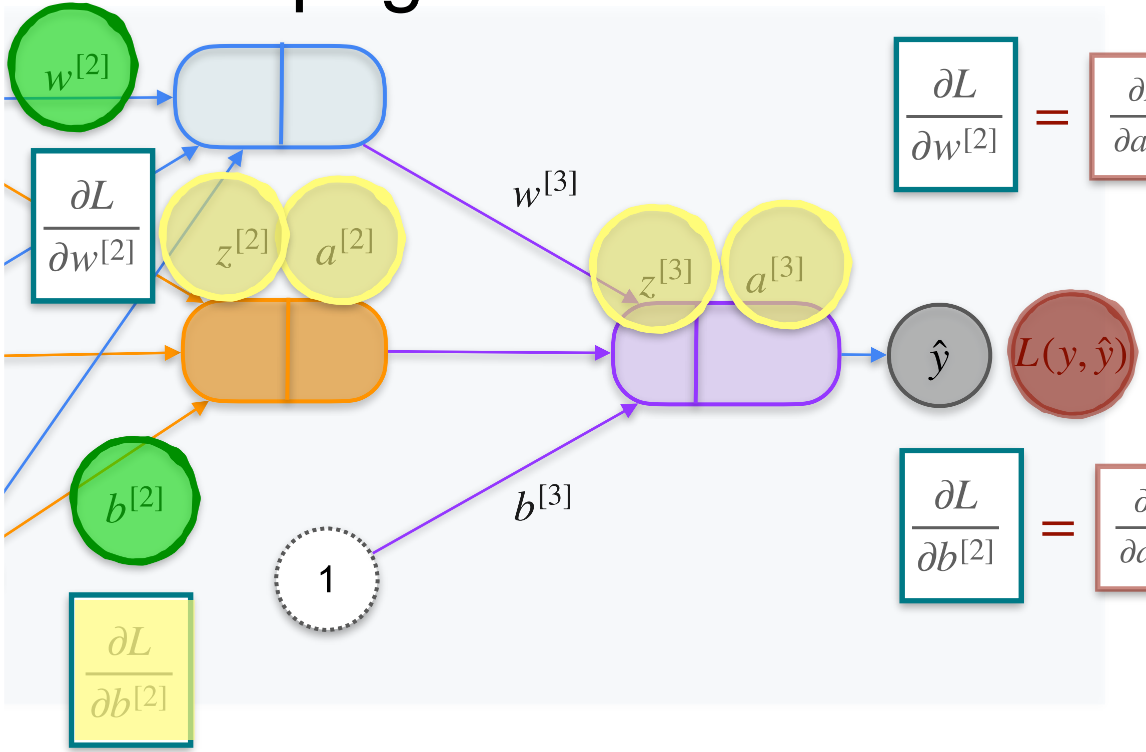

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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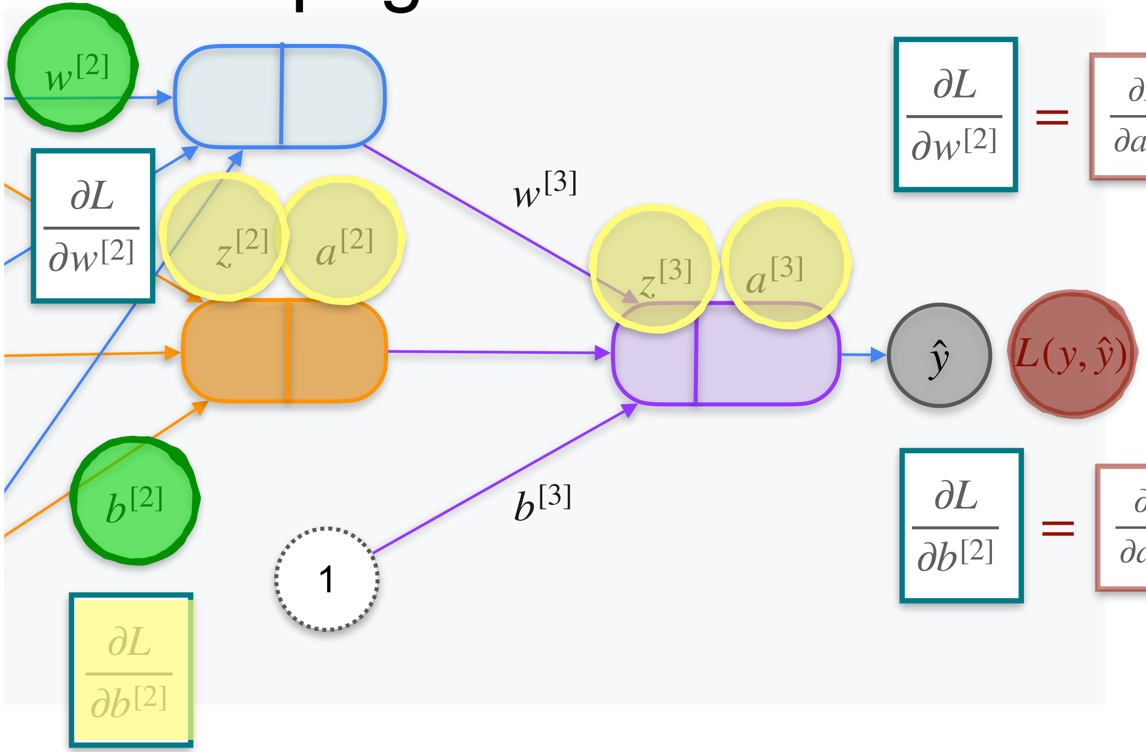

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial z^{[3]}}{\partial a^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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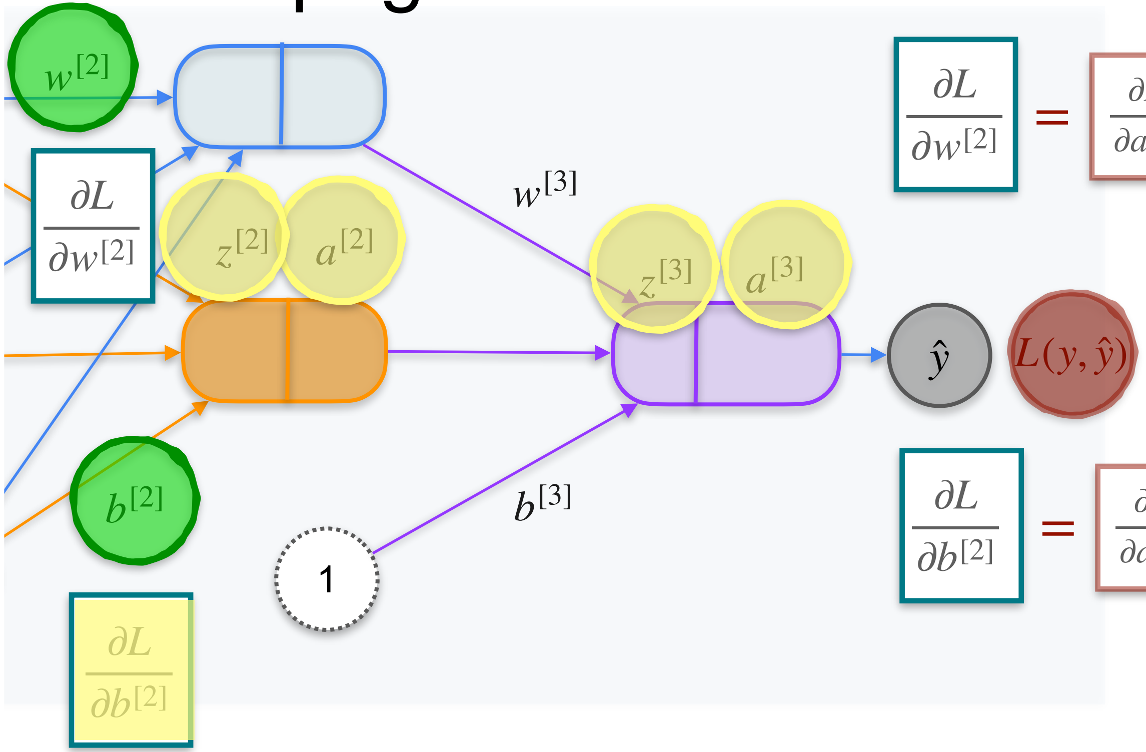

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}} \frac{\partial a^{[2]}}{\partial z^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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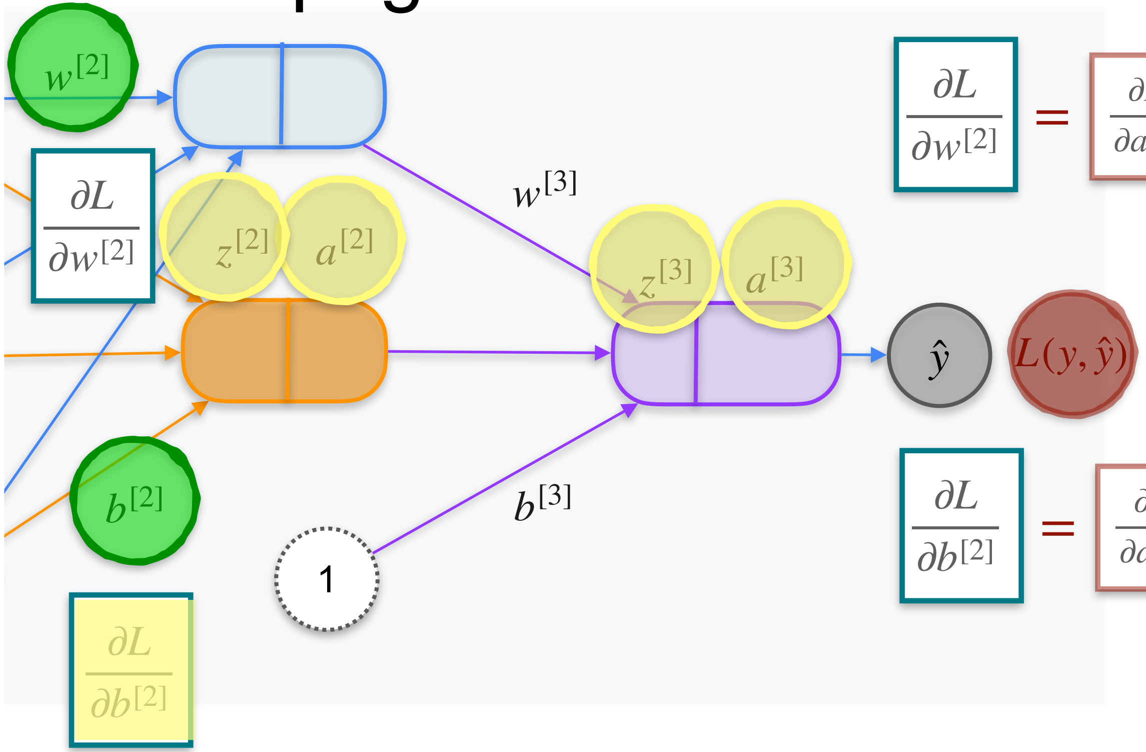

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial z^{[3]}}{\partial a^{[2]}} \quad \frac{\partial a^{[2]}}{\partial z^{[2]}} \quad \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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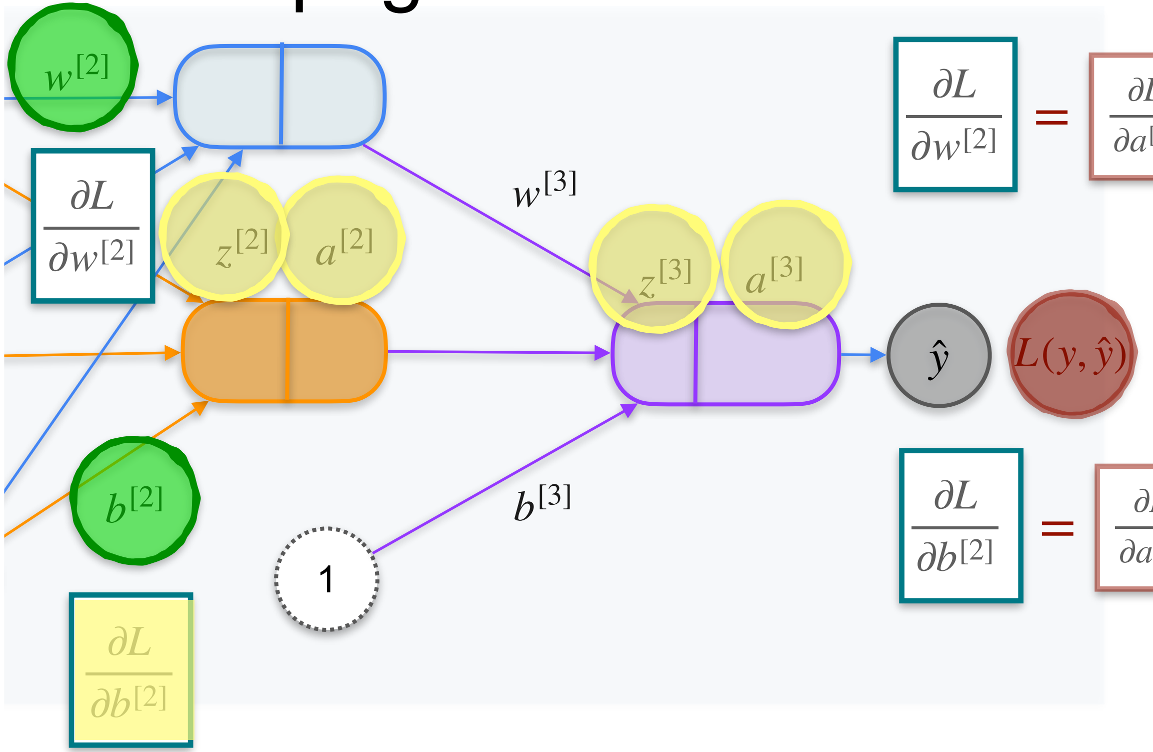

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$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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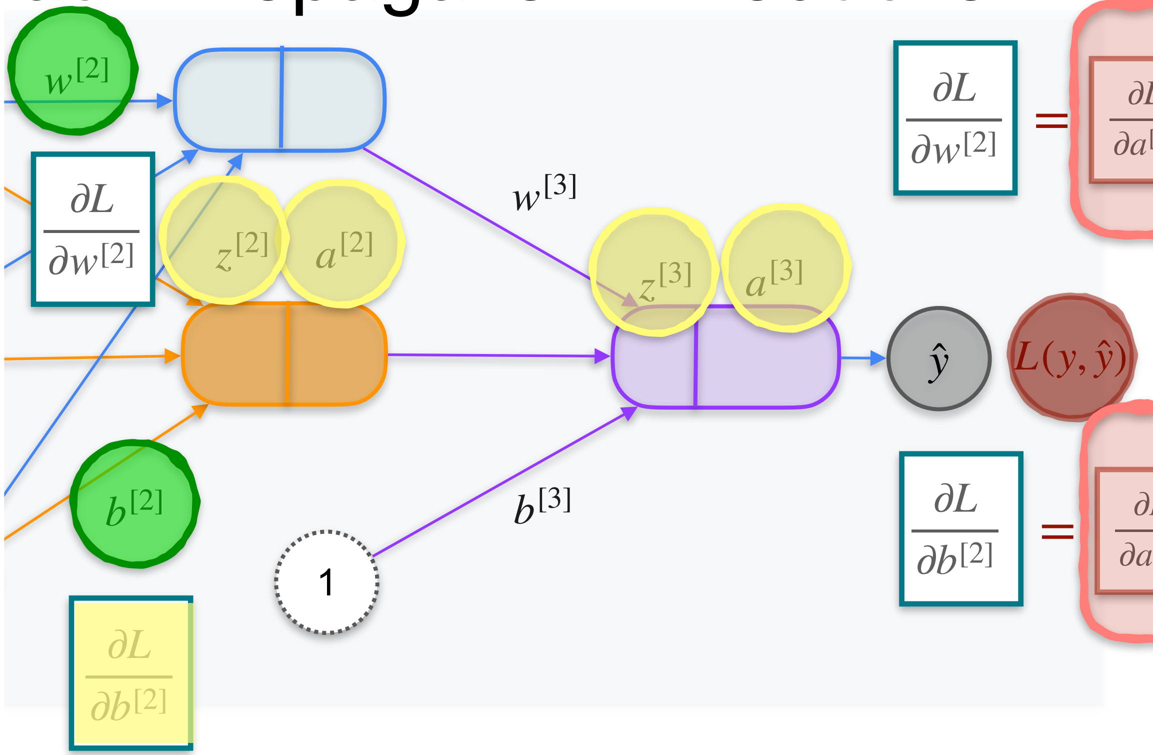

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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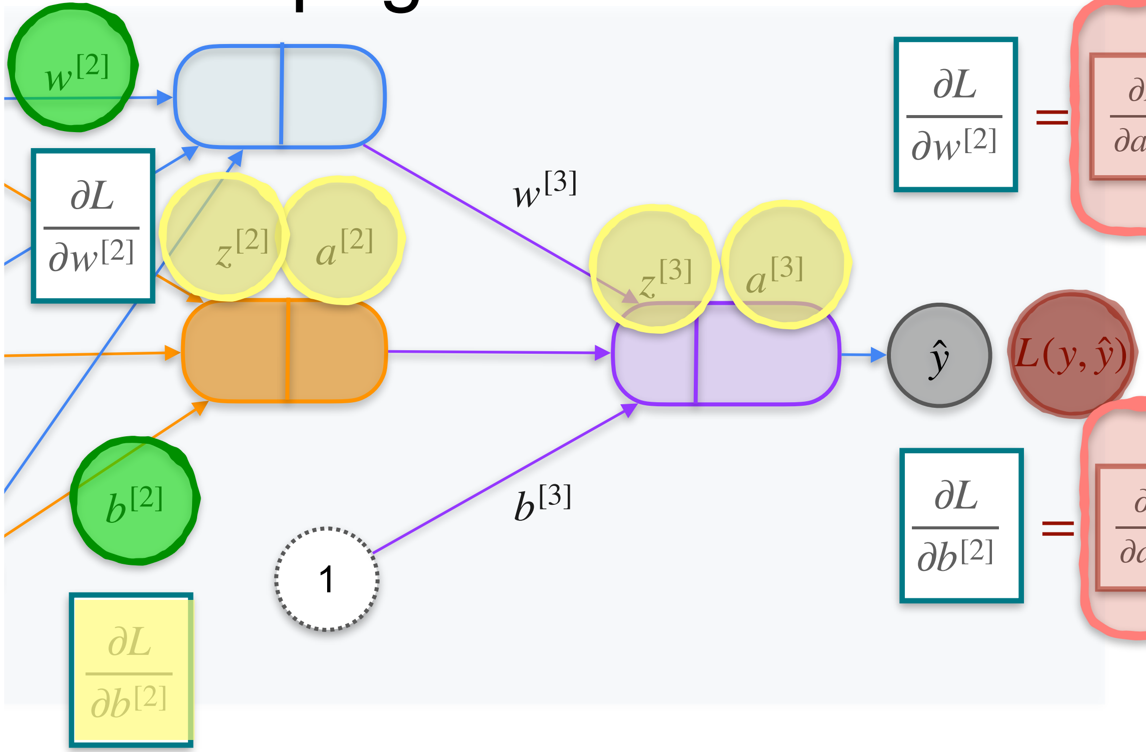

$$\frac{\partial L}{\partial w^{[2]}} = \boxed{\frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \boxed{\frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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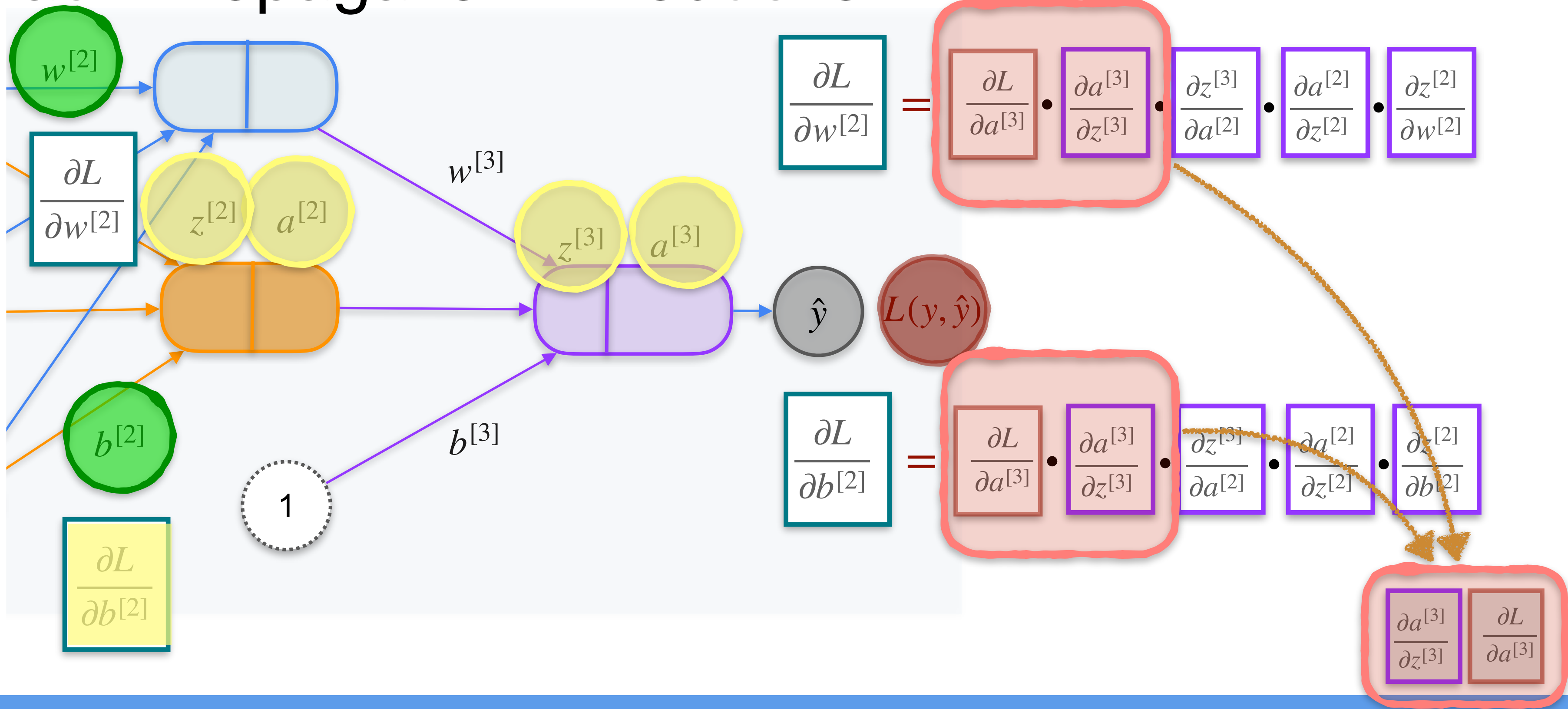

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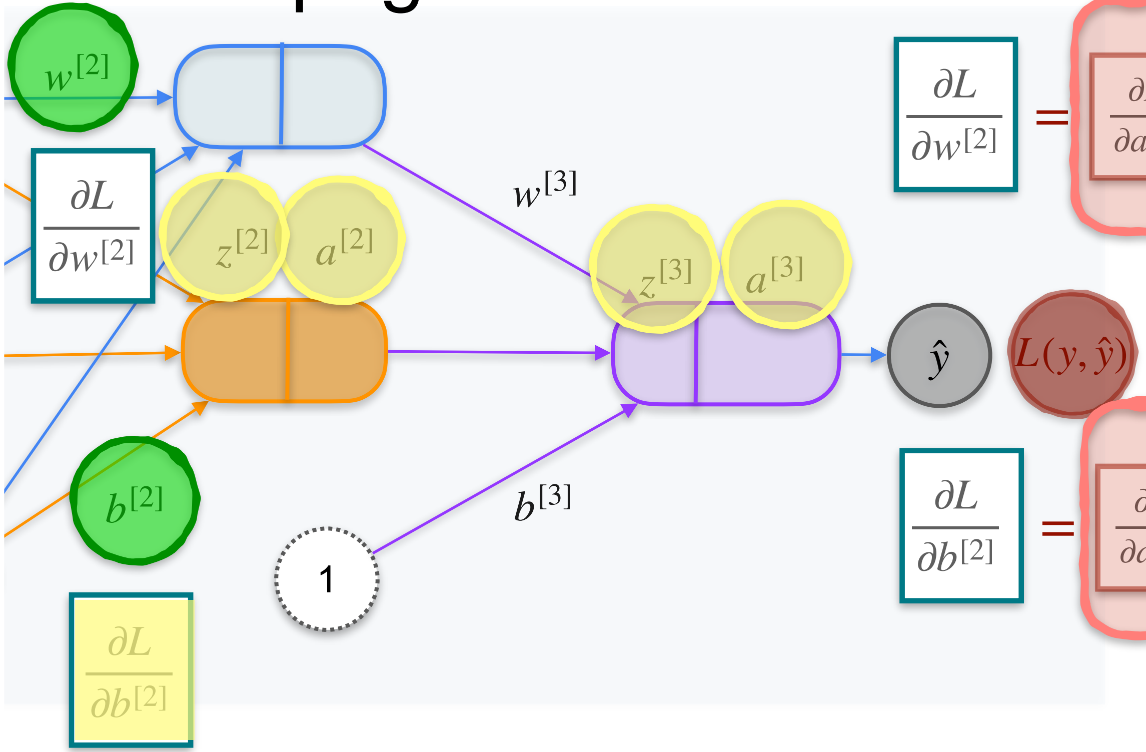

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$\frac{\partial a^{[2]}}{\partial z^{[2]}} \qquad \frac{\partial a^{[3]}}{\partial z^{[3]}} \; \frac{\partial L}{\partial a^{[3]}}$$

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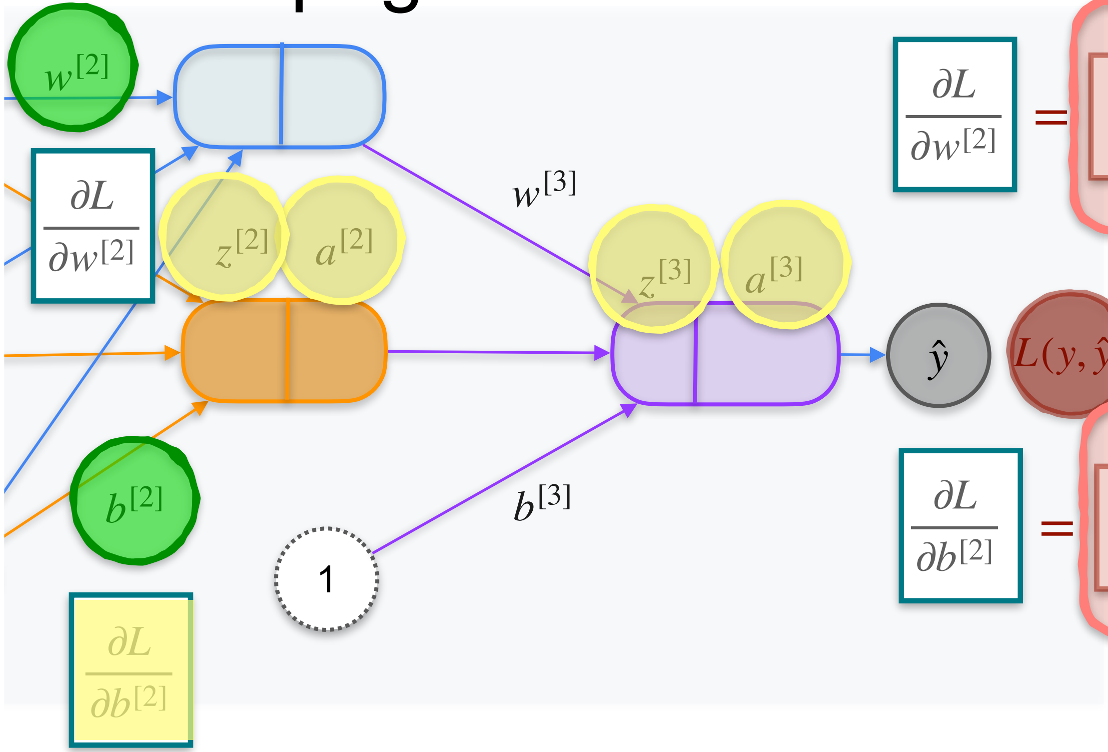

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$\frac{\partial a^{[2]}}{\partial z^{[2]}} \quad \frac{\partial z^{[3]}}{\partial a^{[2]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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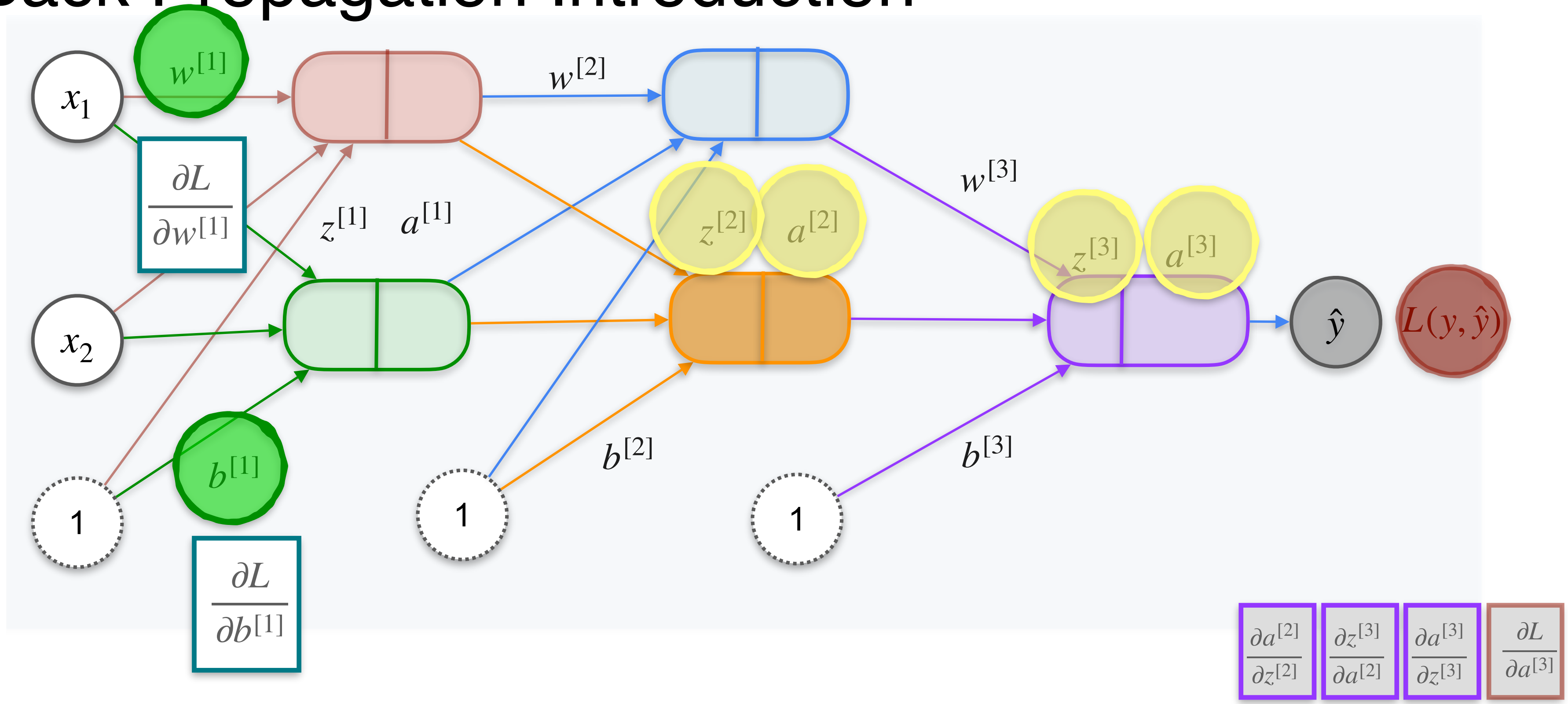

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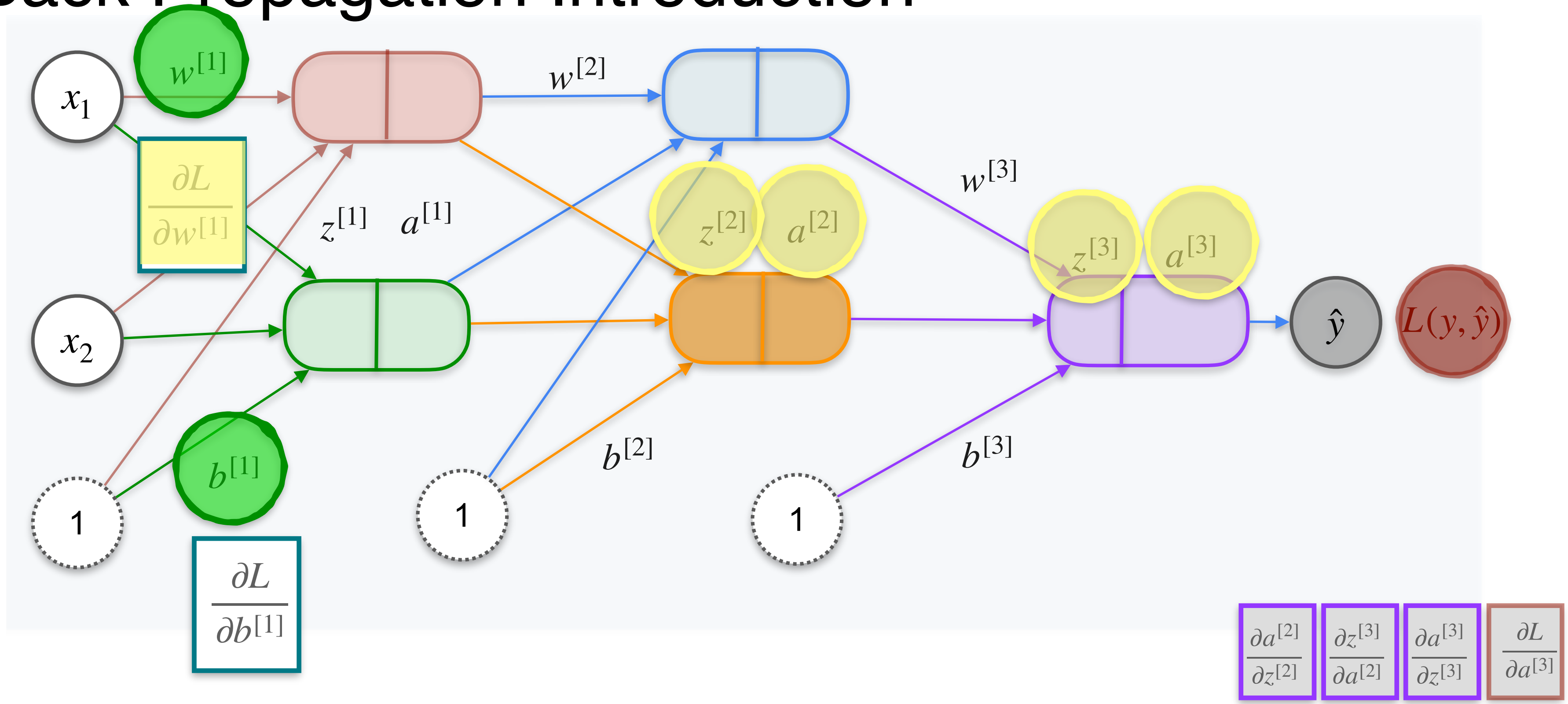

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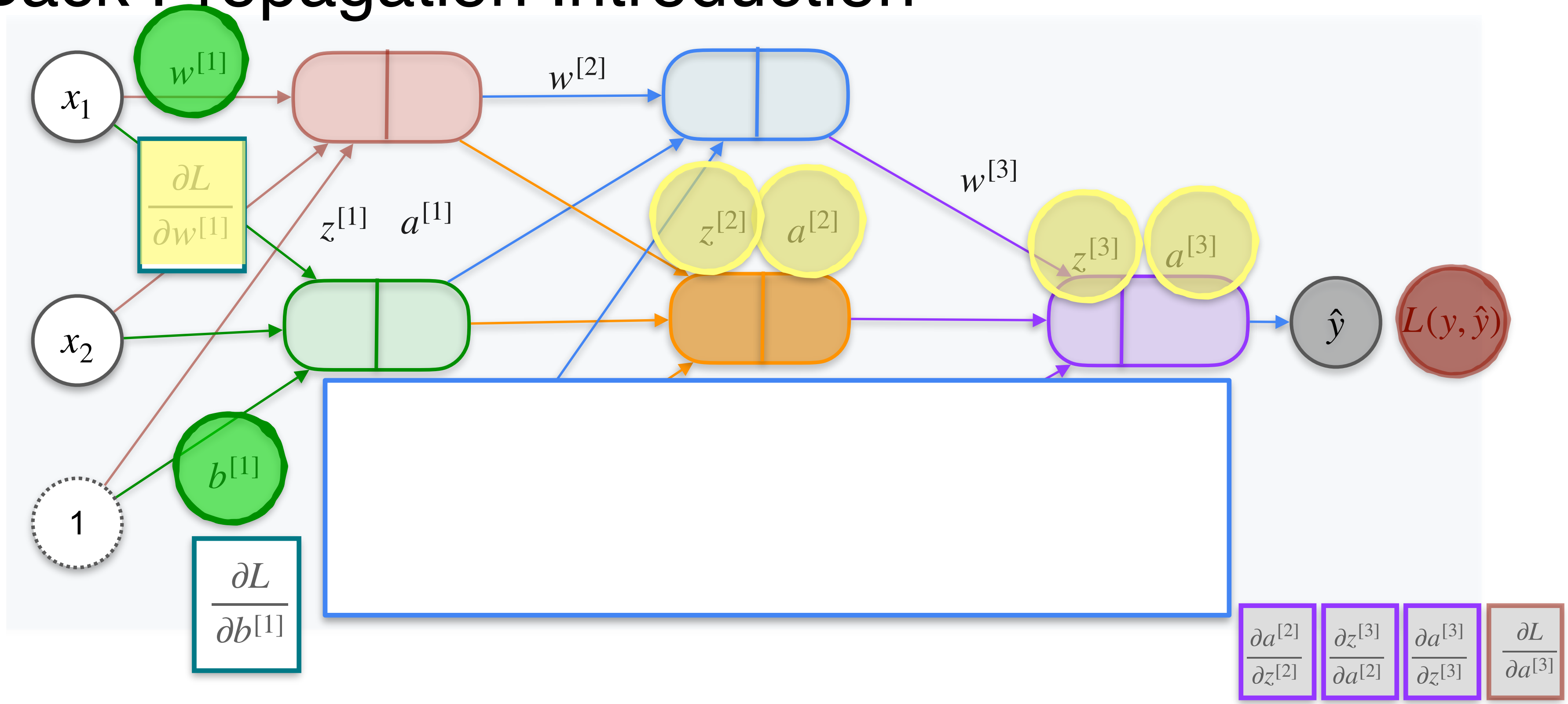

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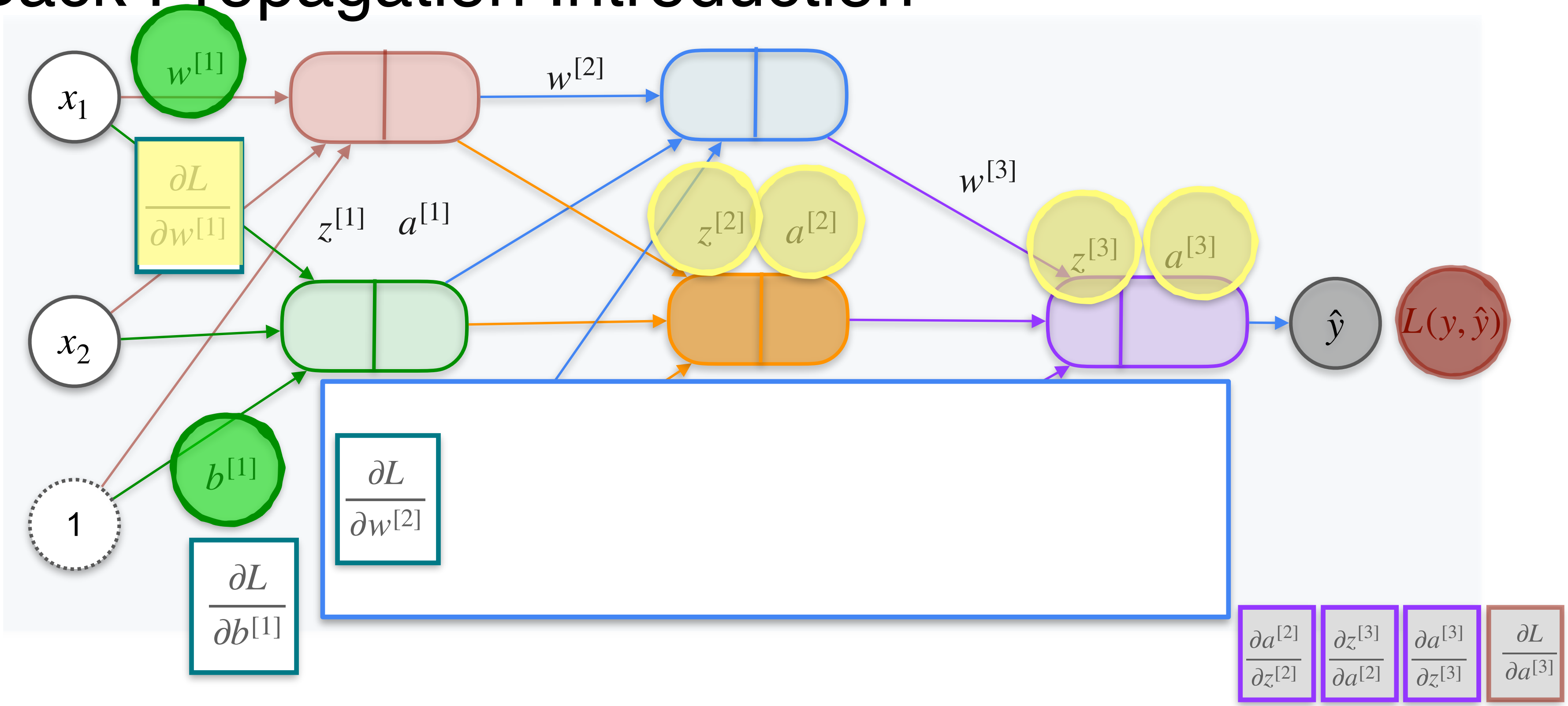

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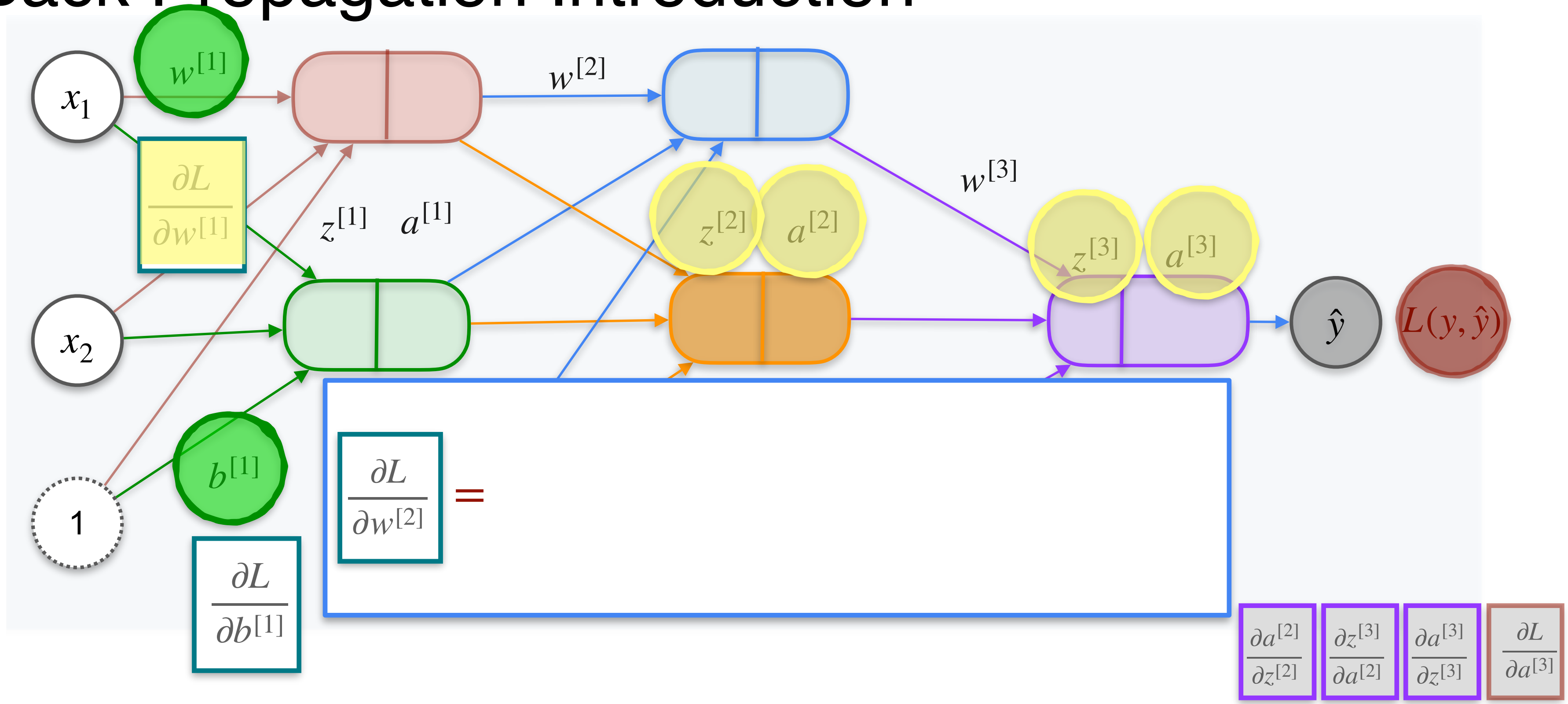

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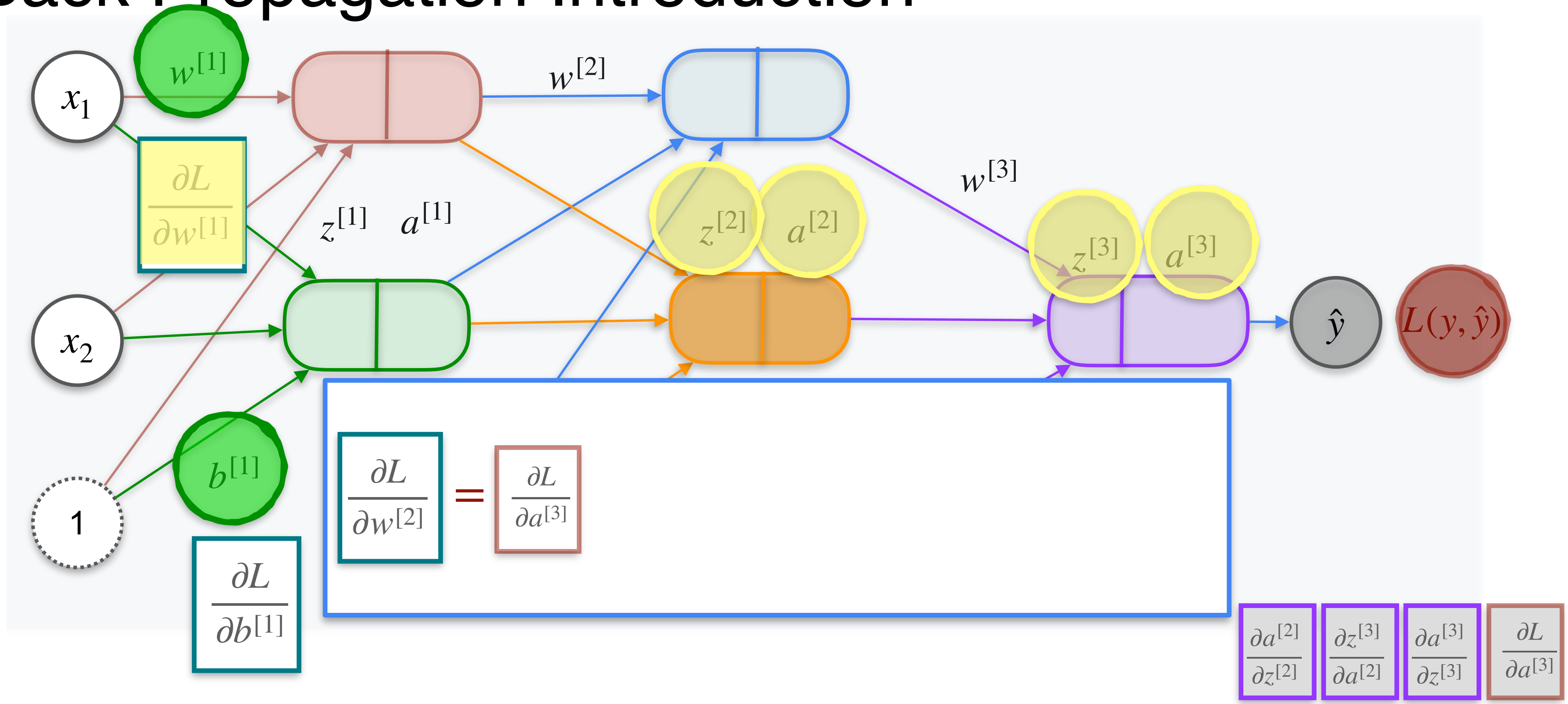

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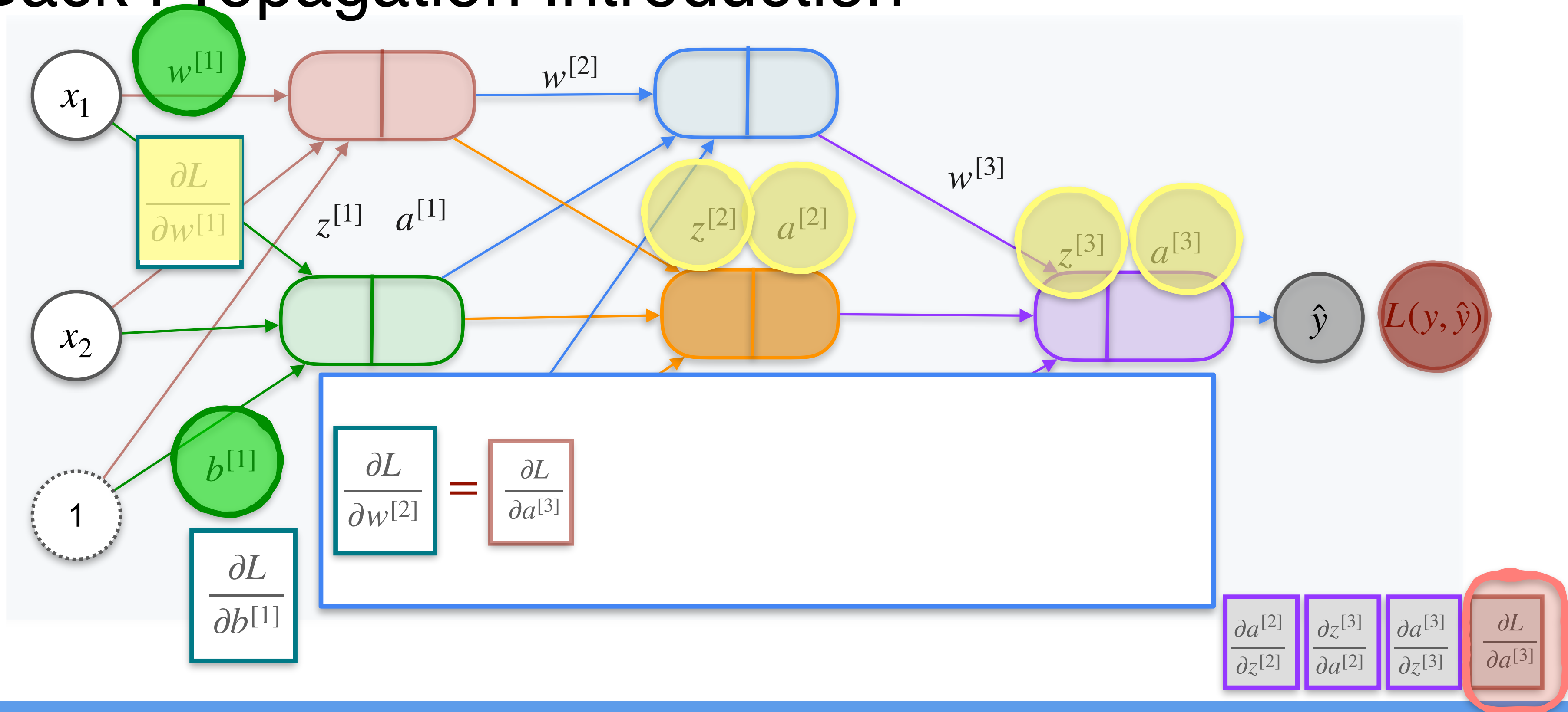

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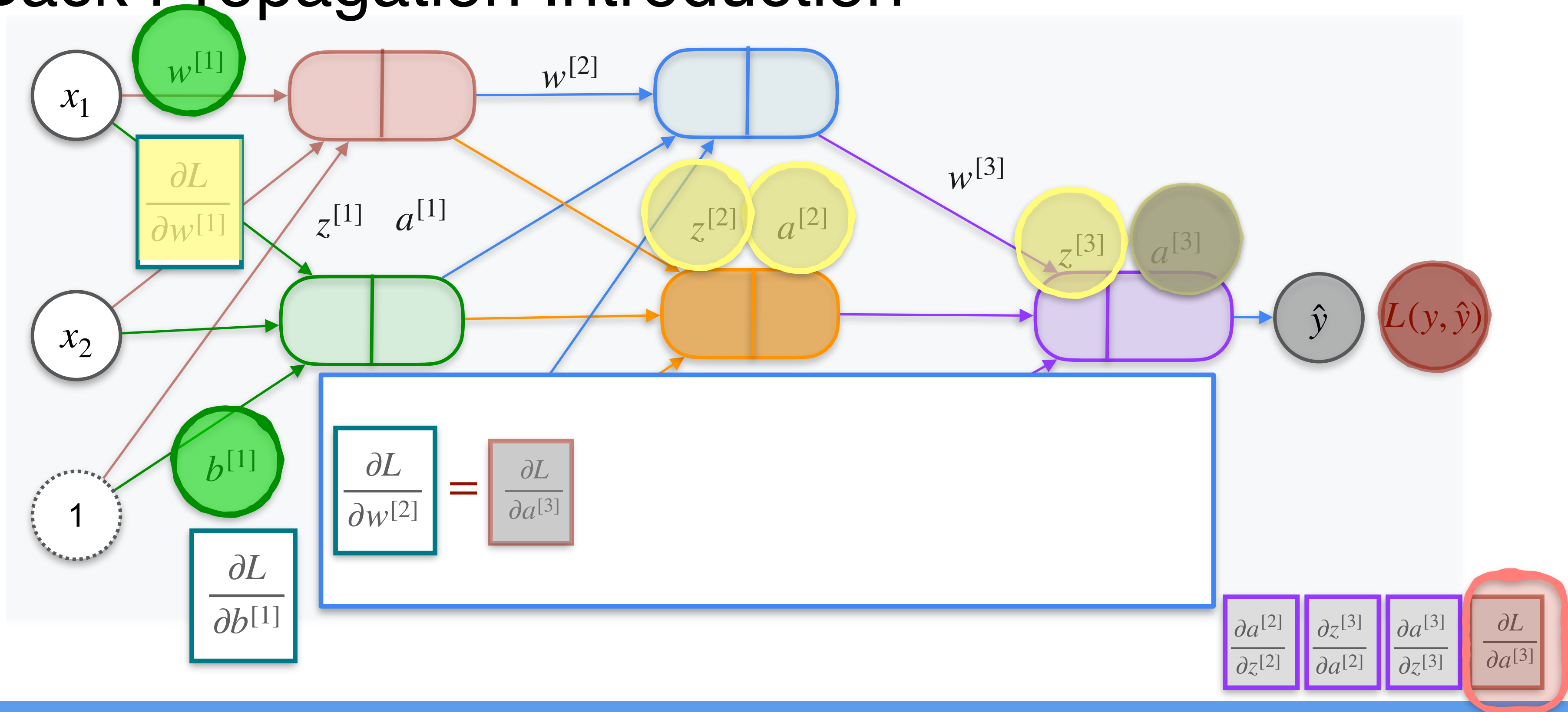

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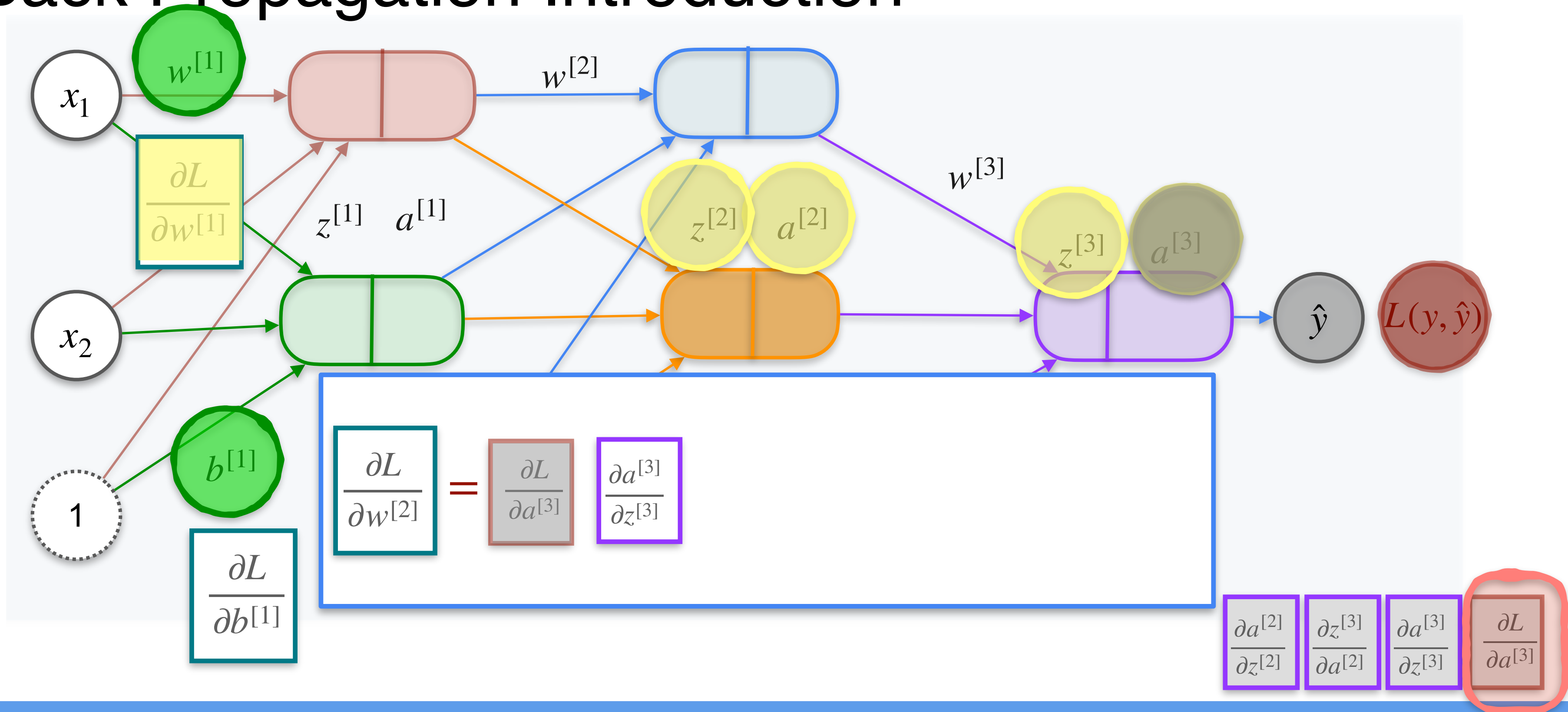

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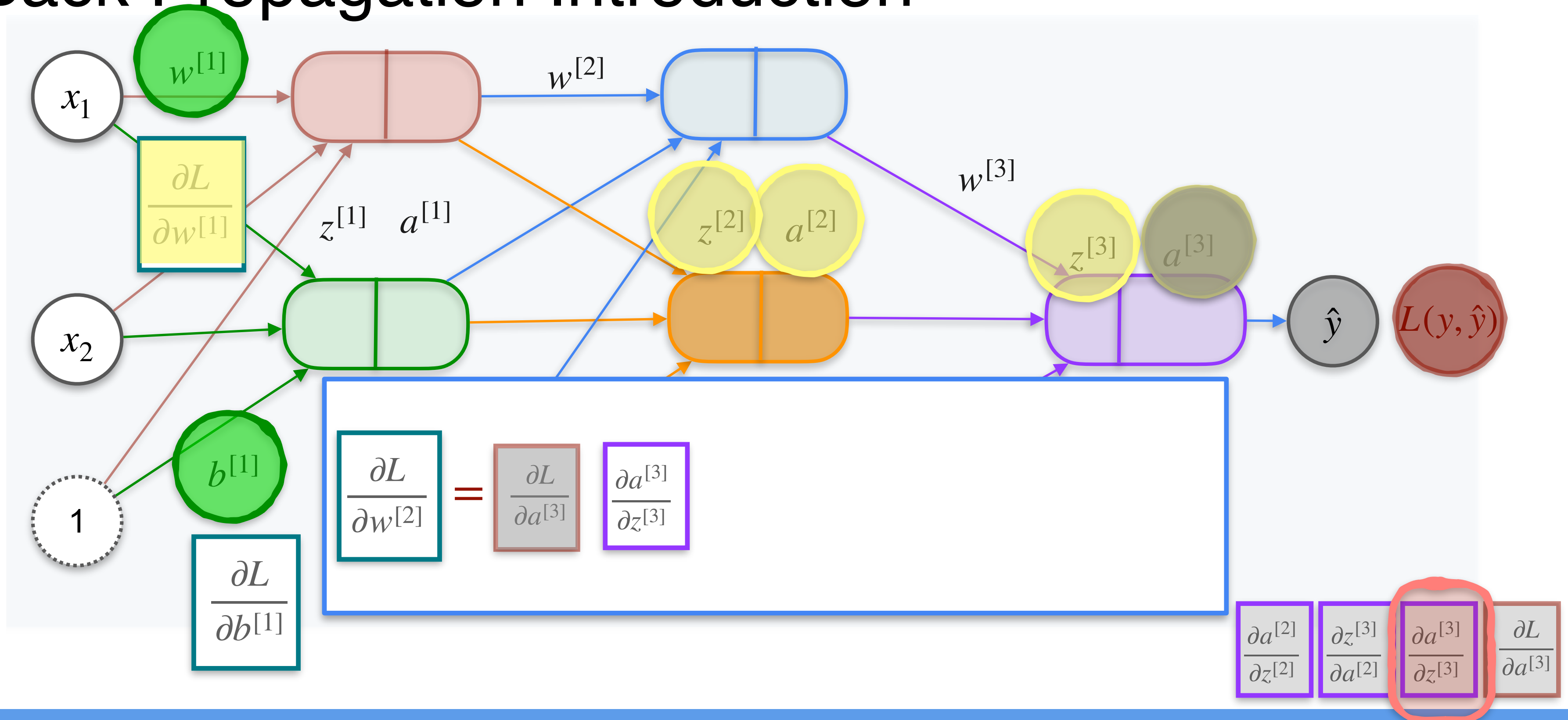

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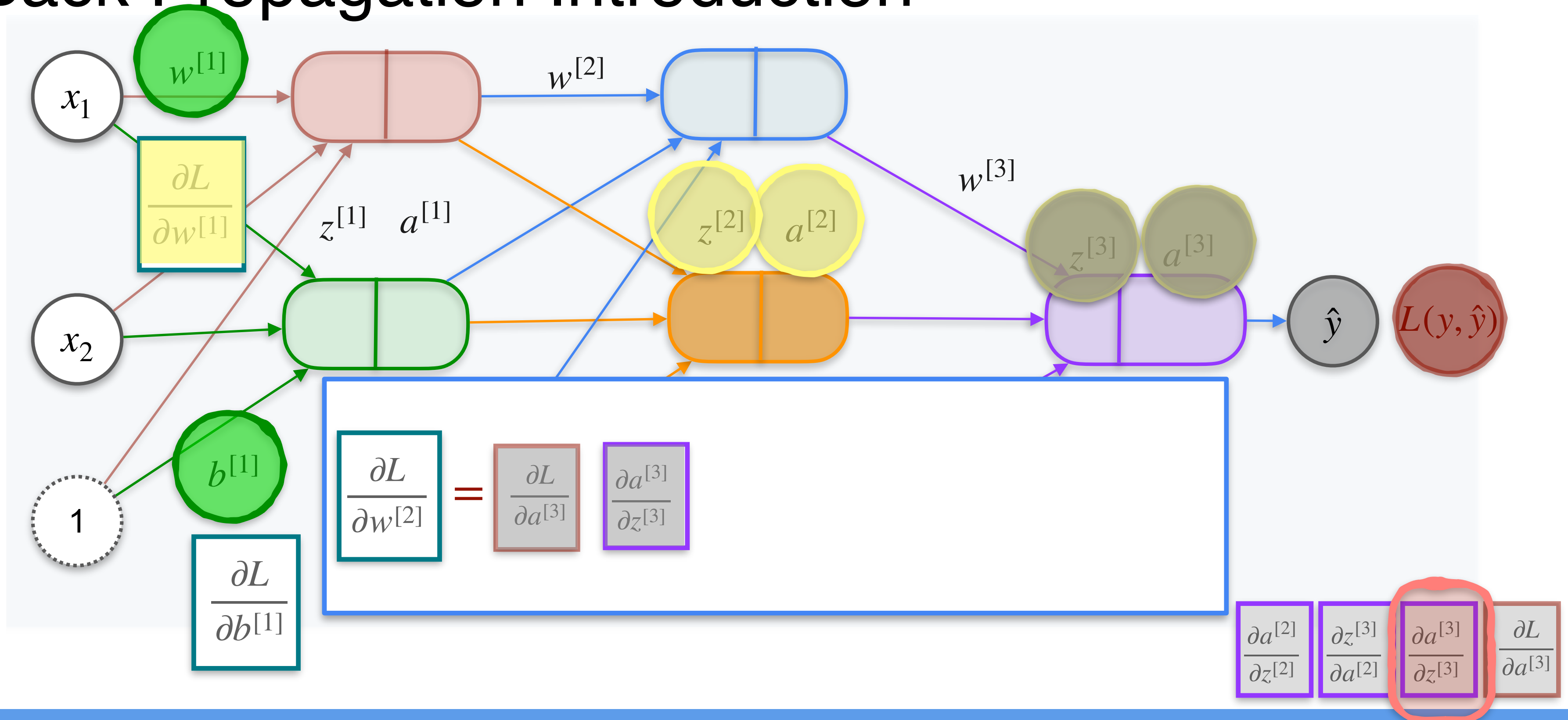

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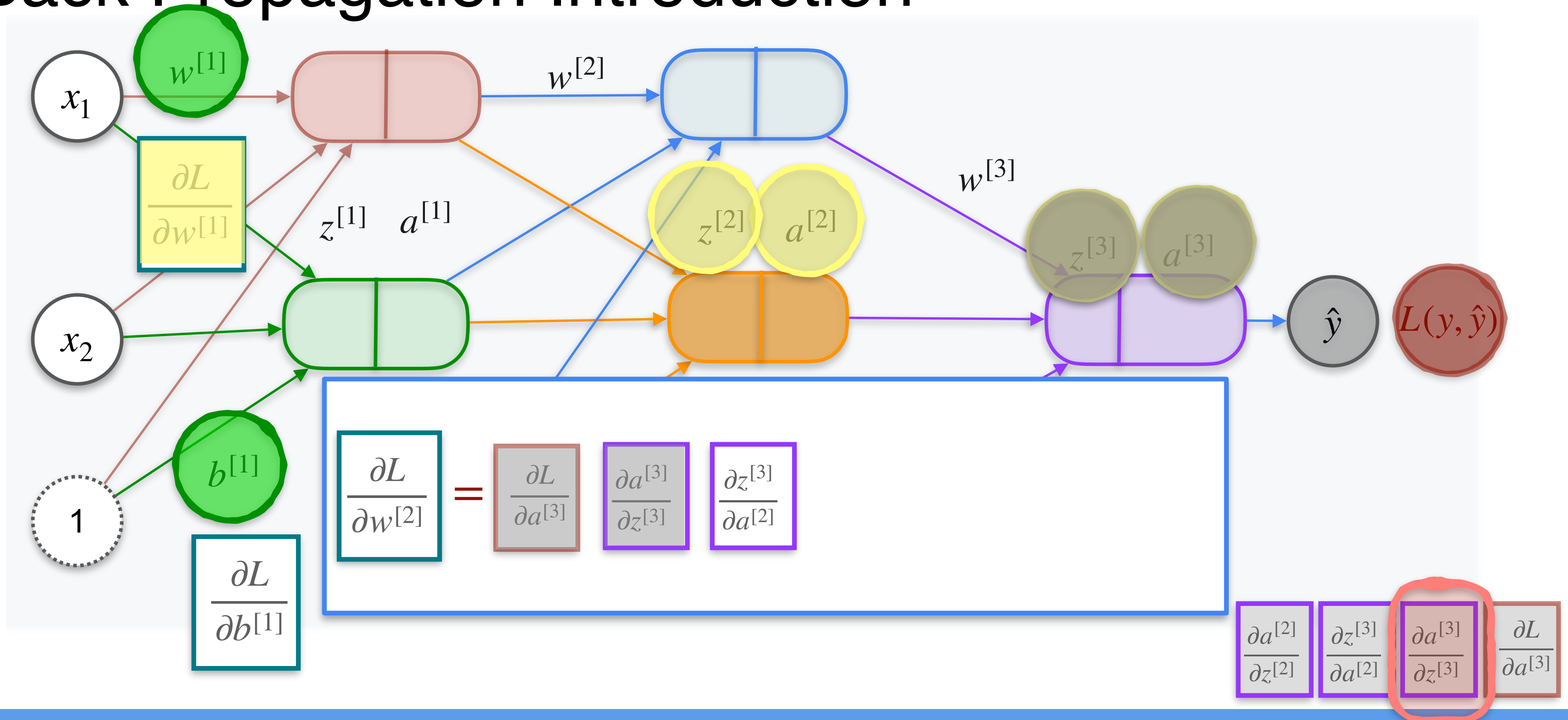

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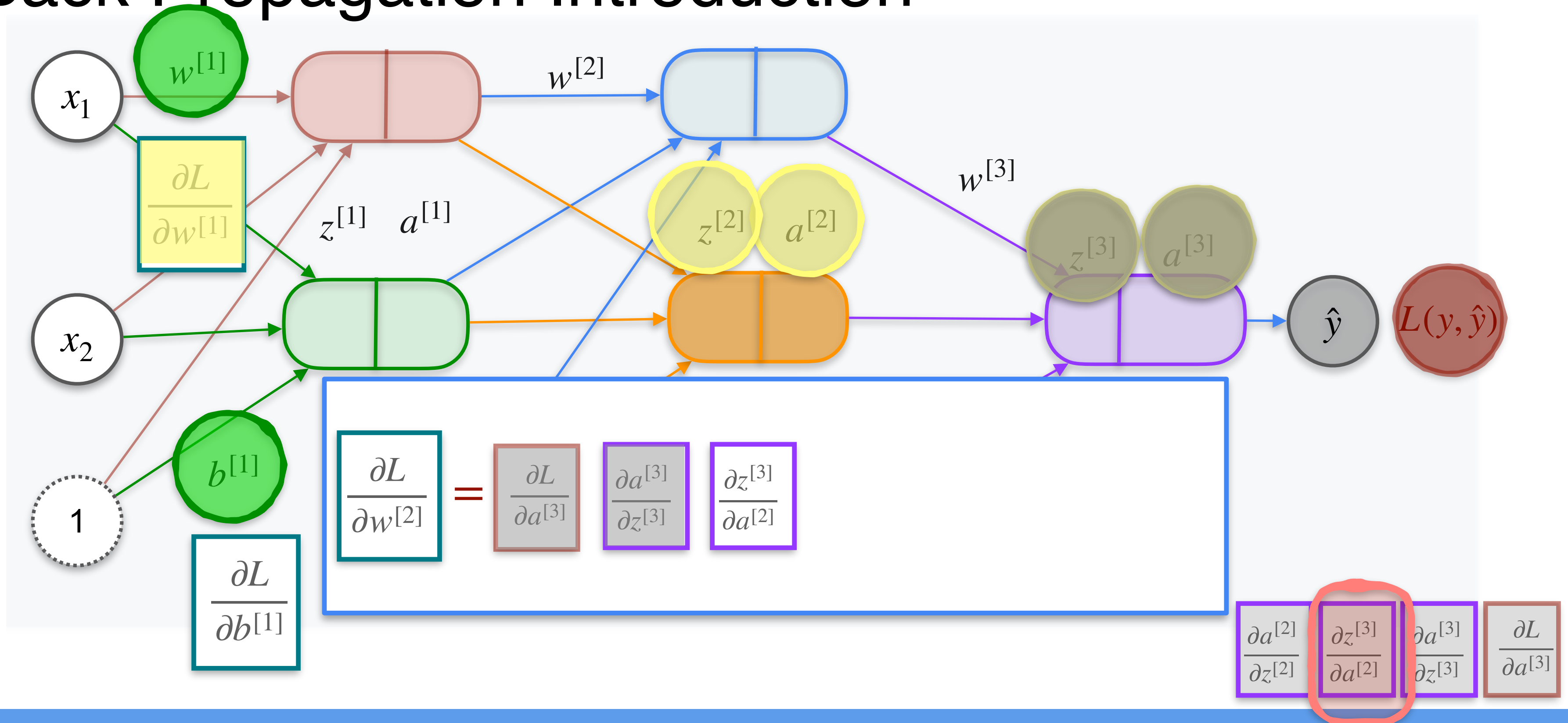

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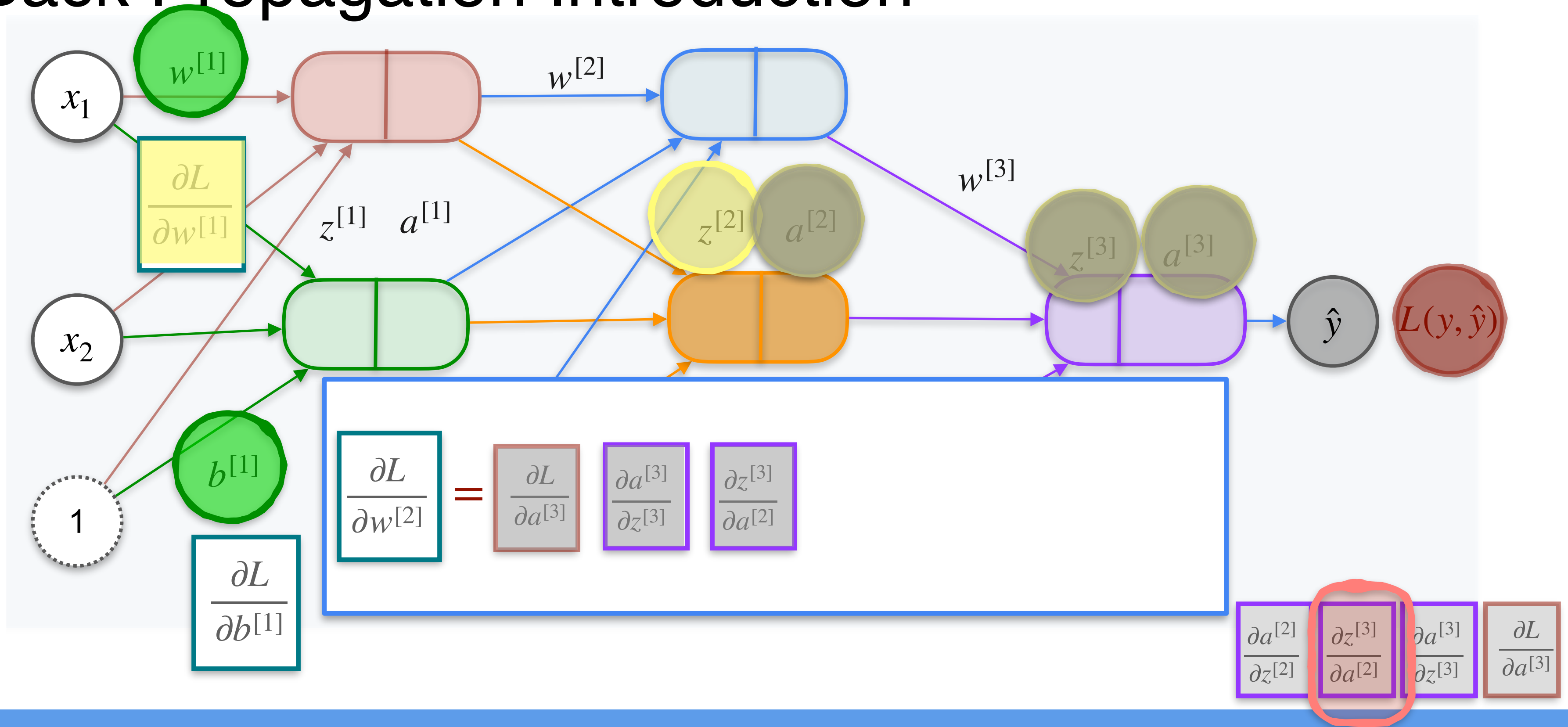

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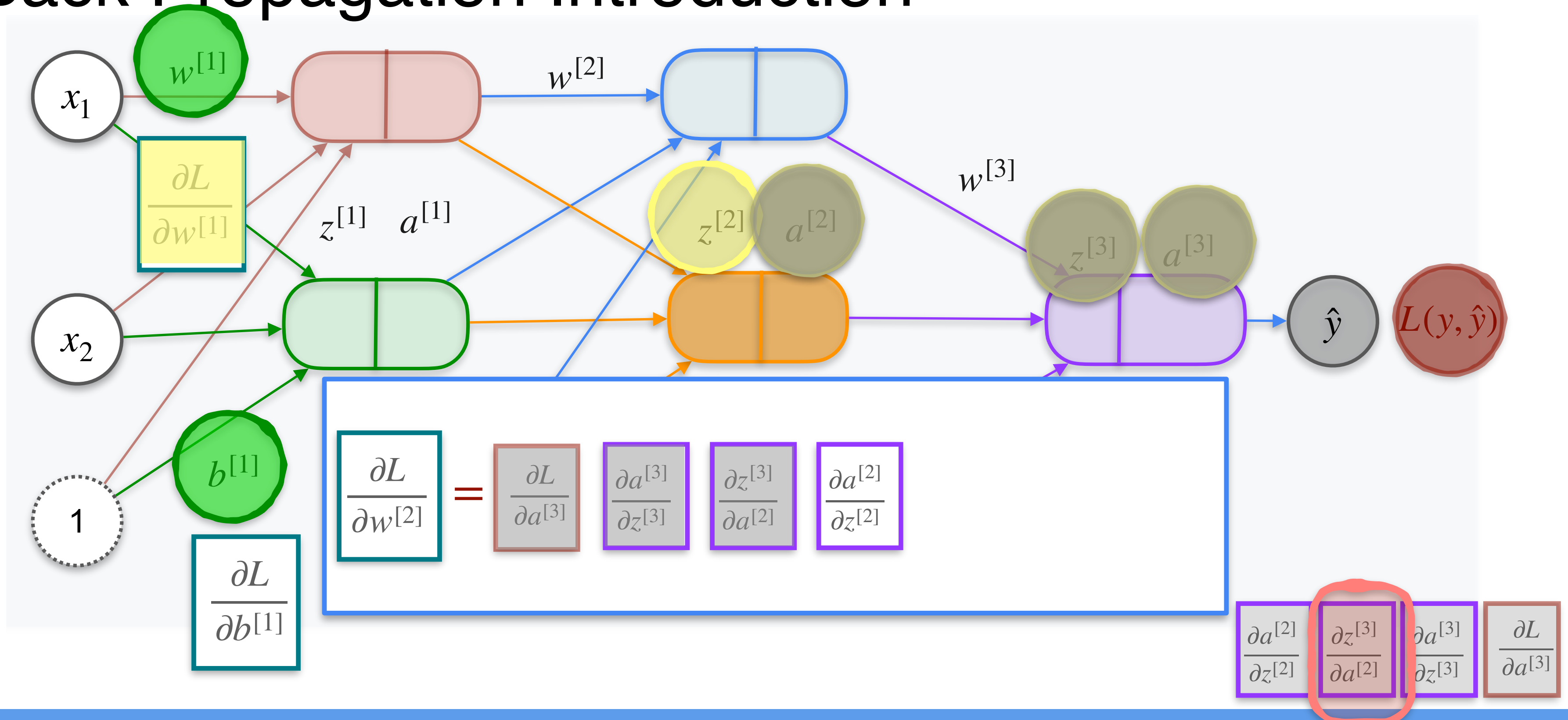

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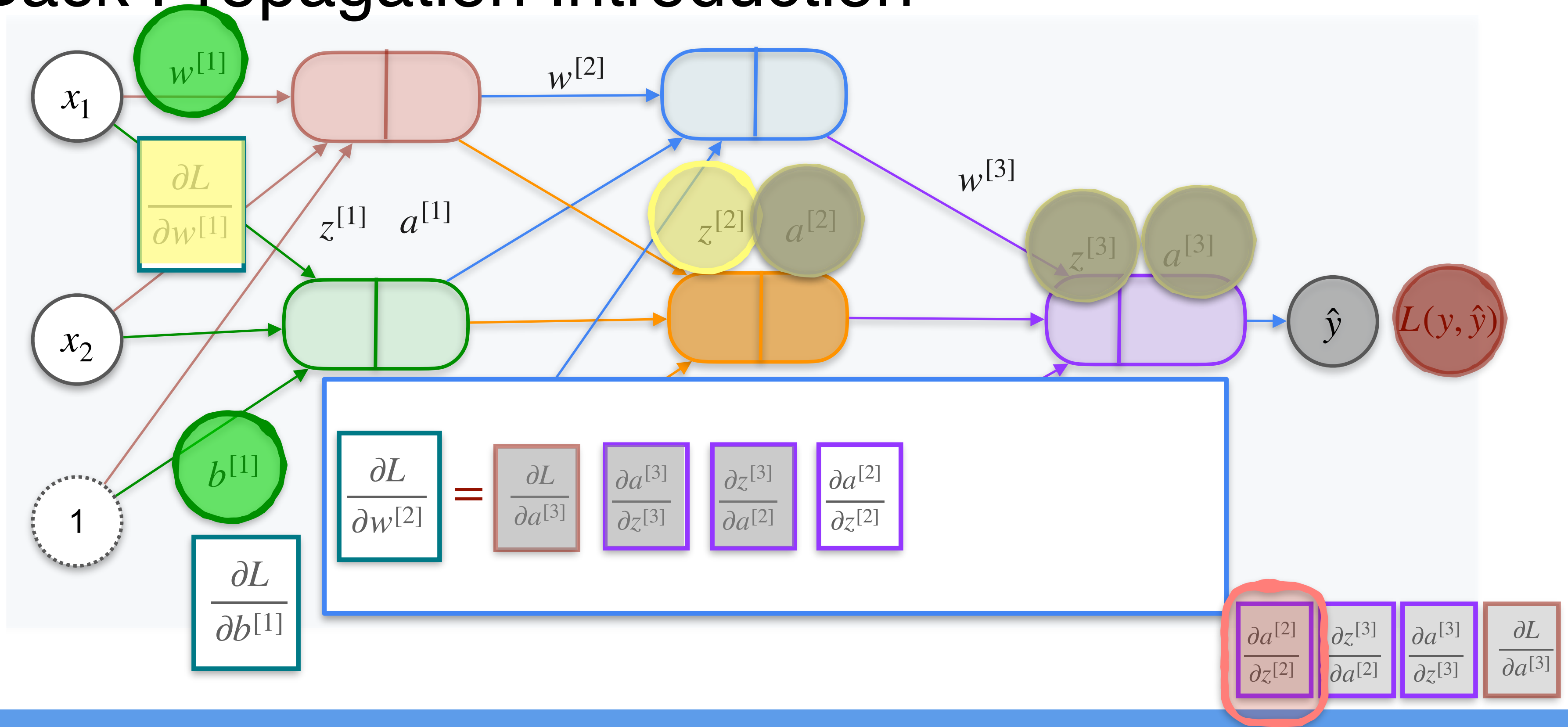

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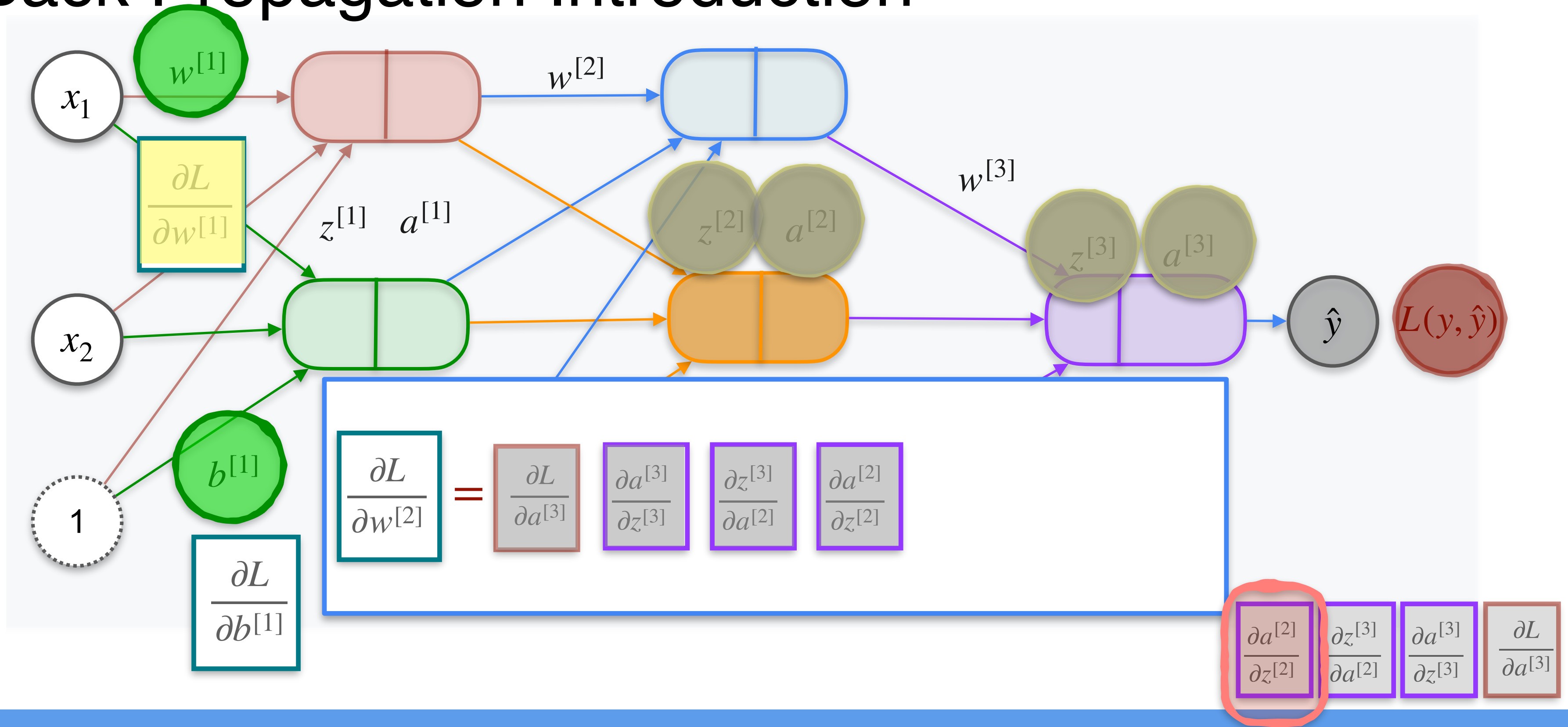

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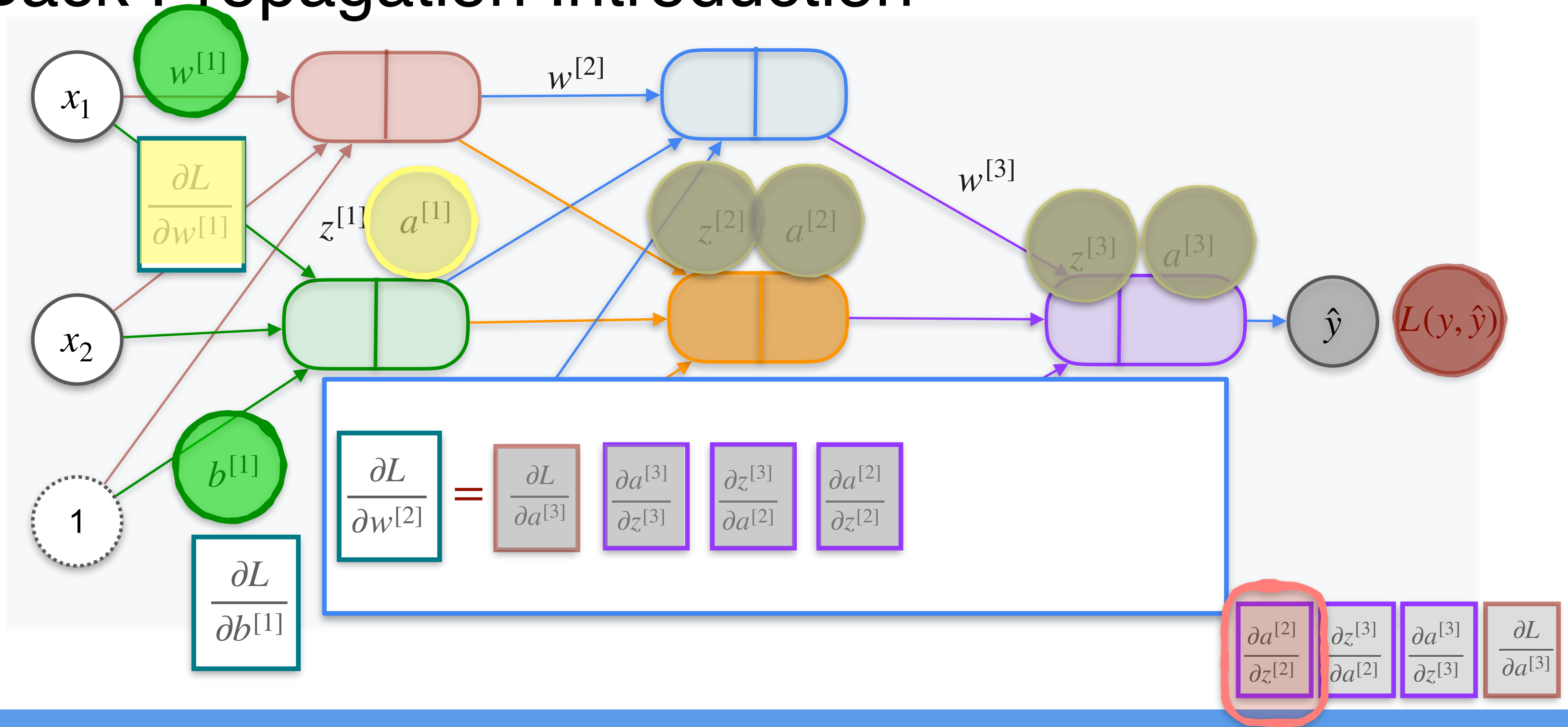

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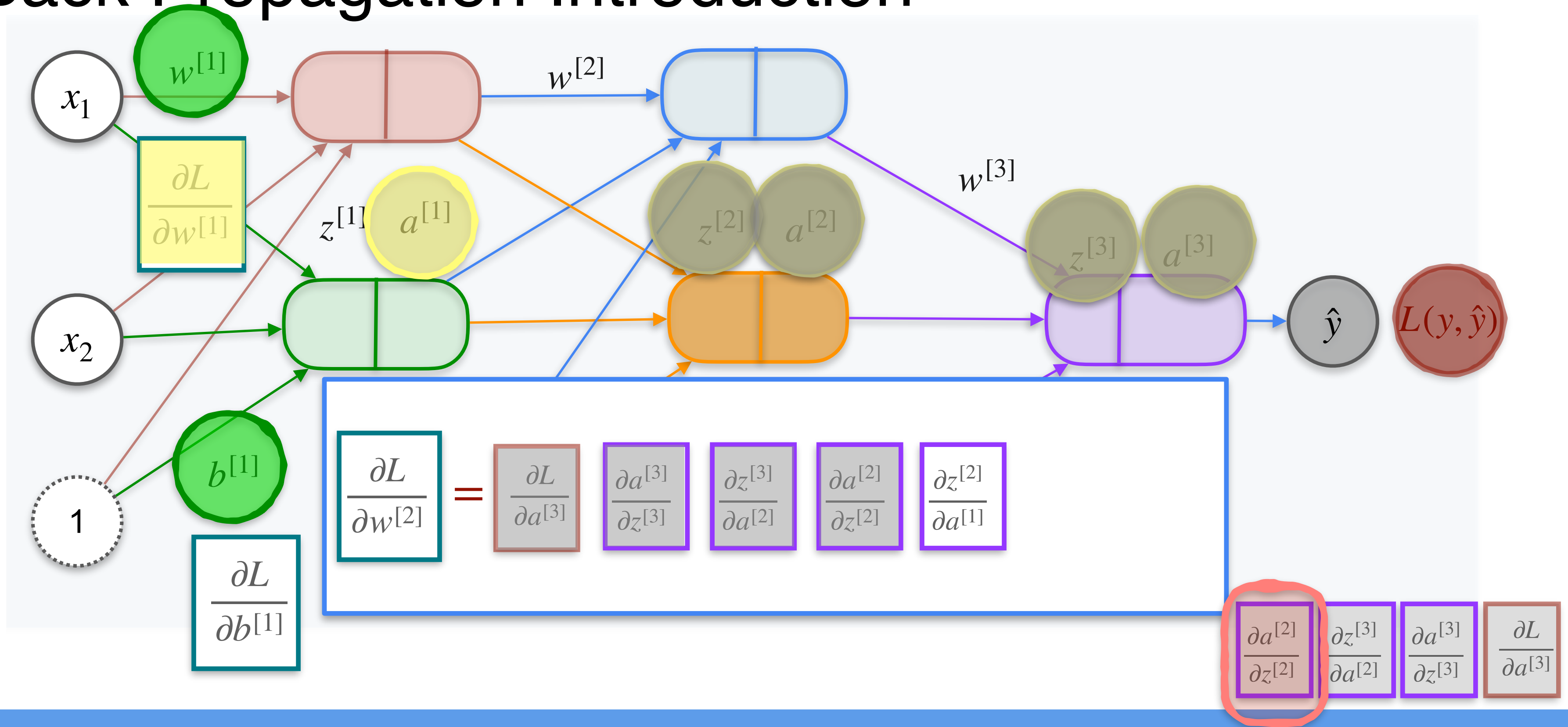

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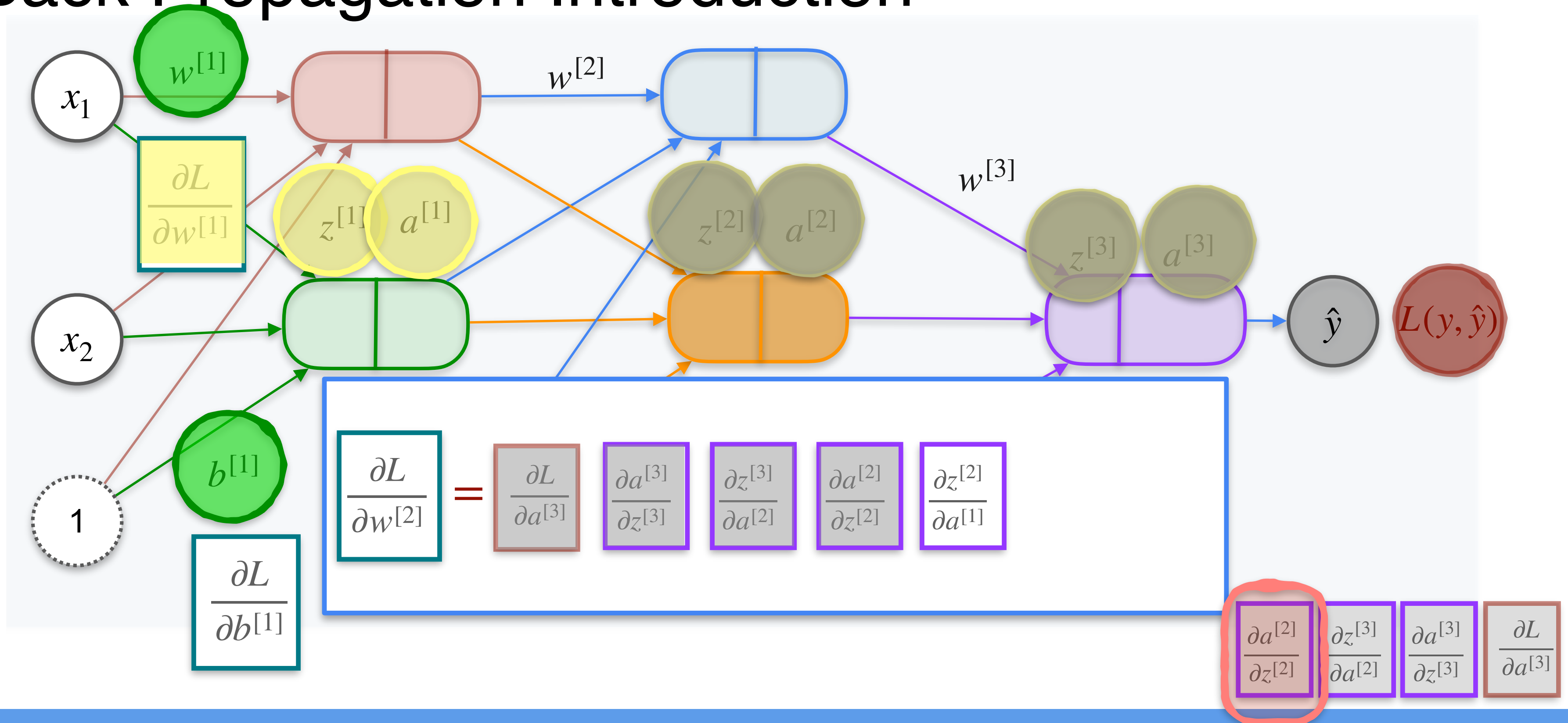

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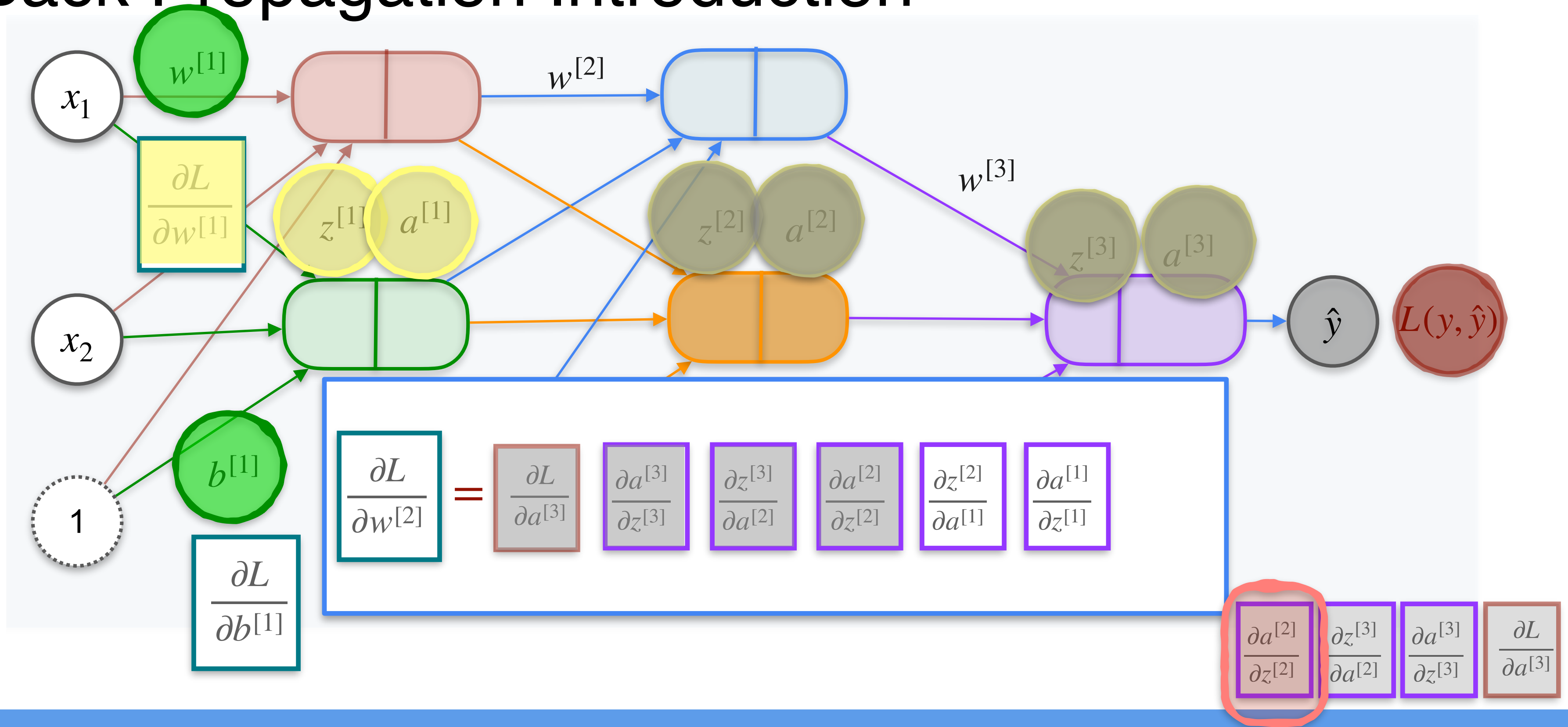

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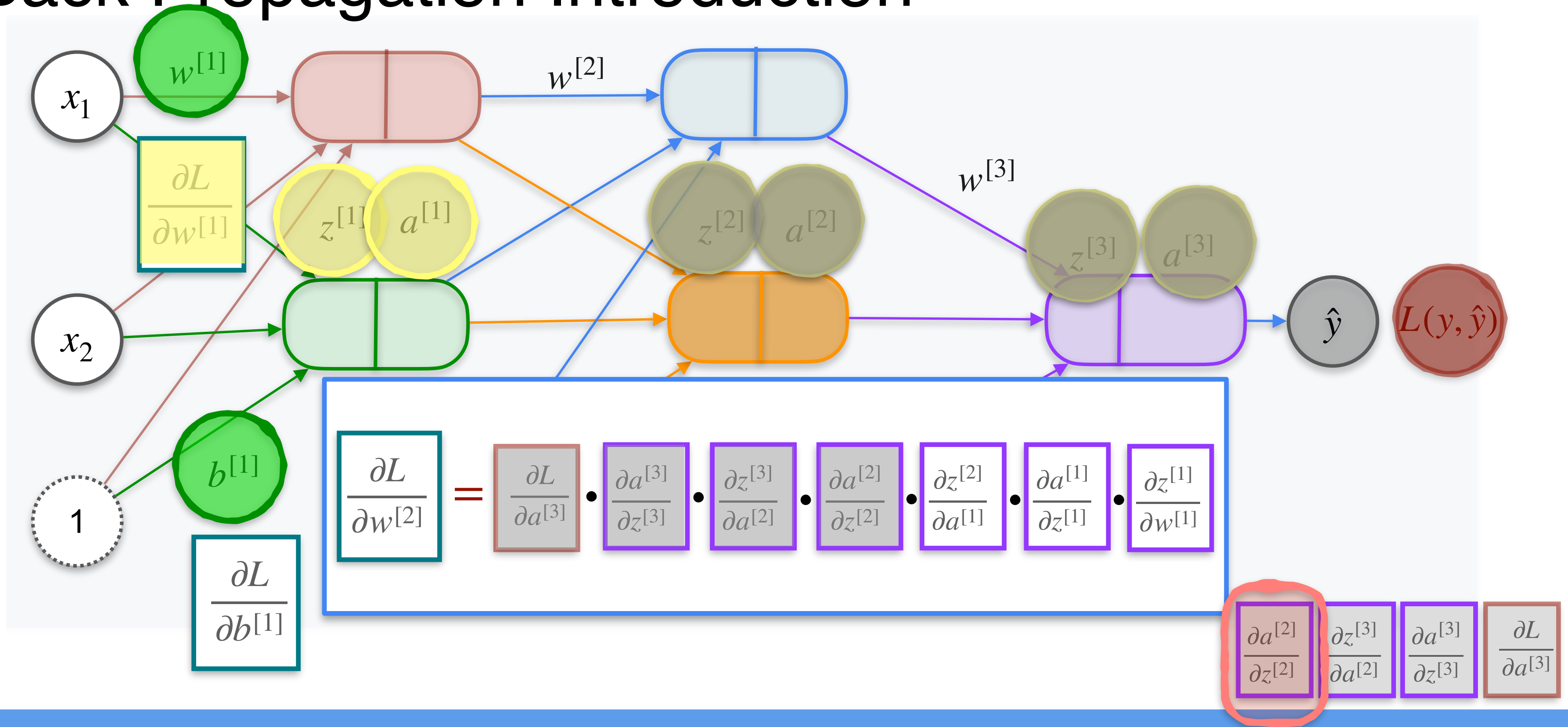
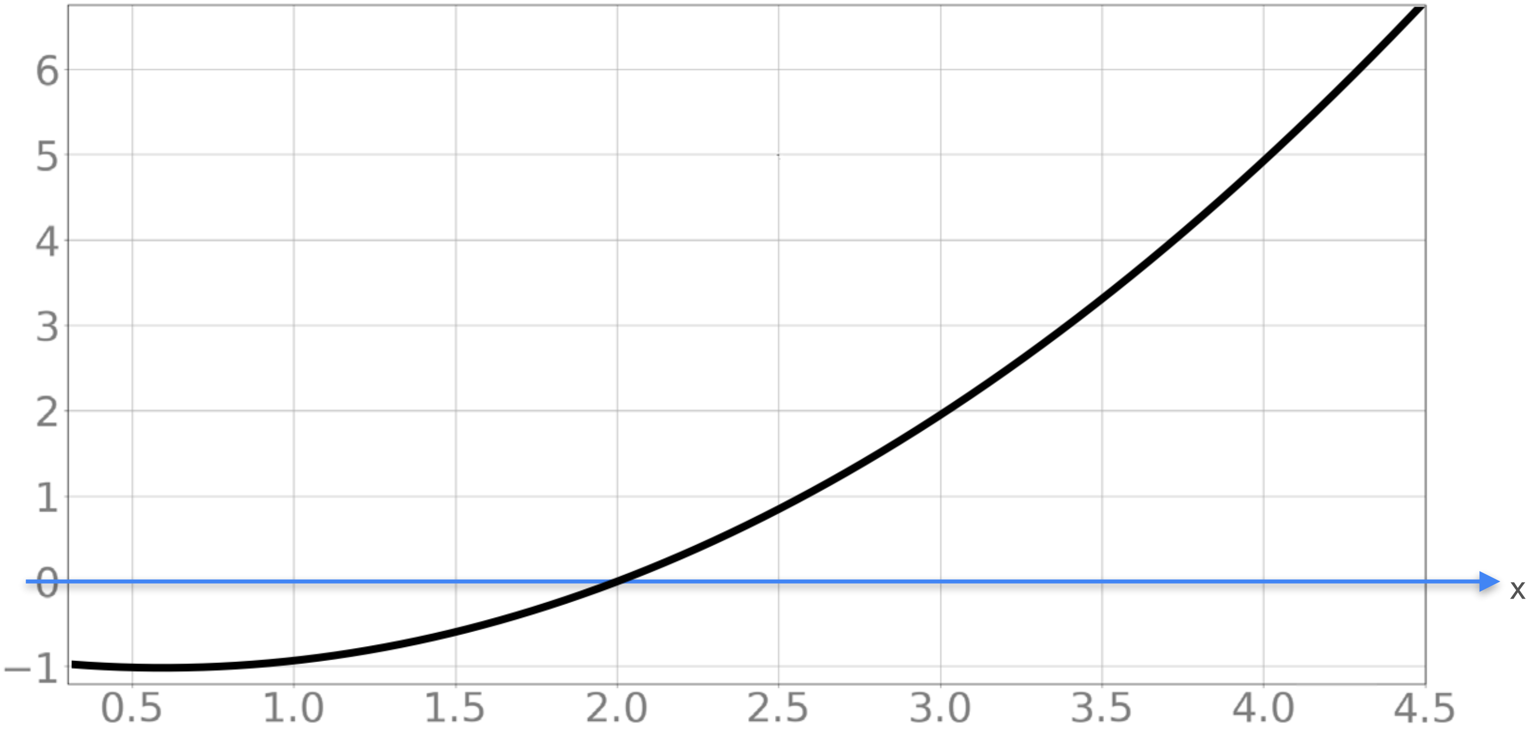


Optimization in Neural Networks and Newton's Method

Newton's method

Newton's Method

Newton's Method



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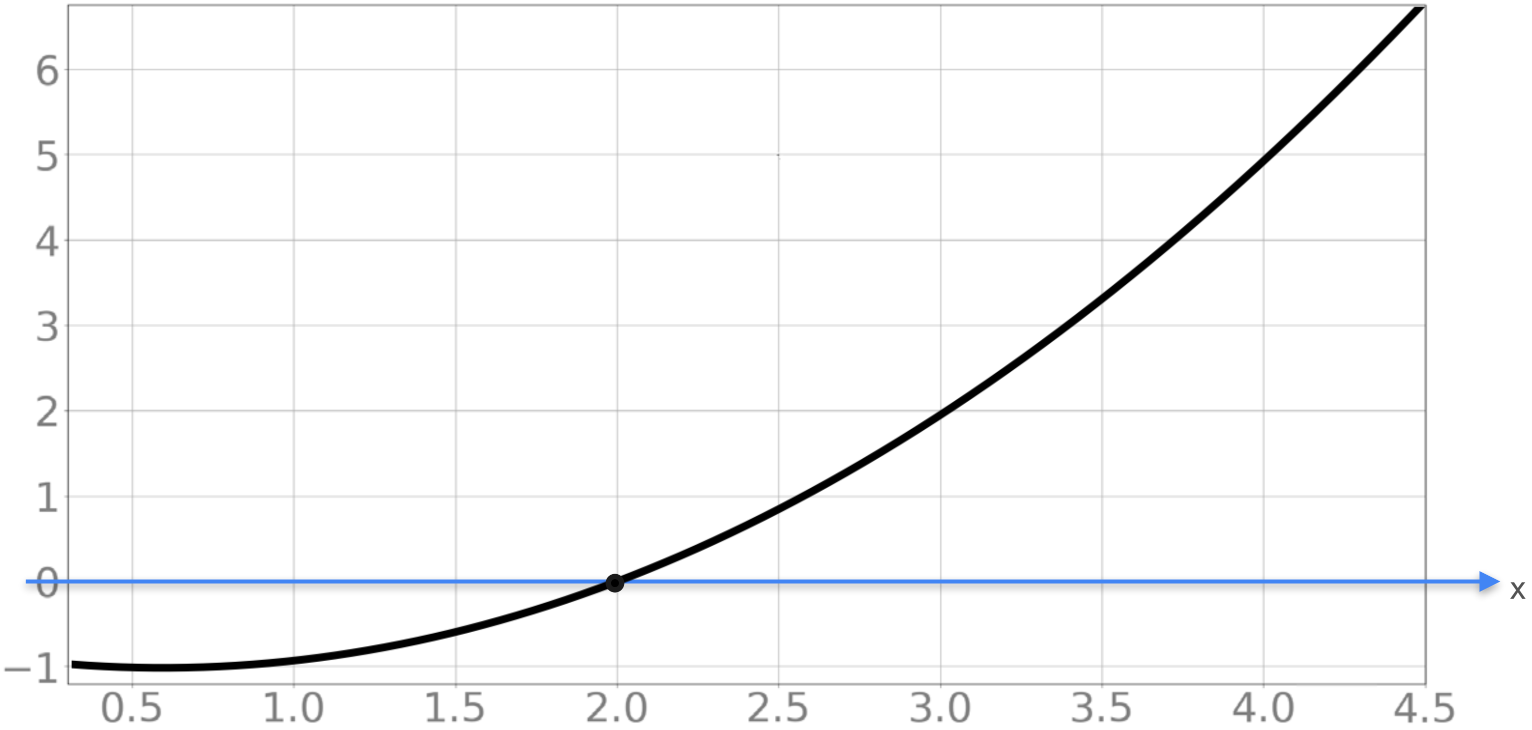

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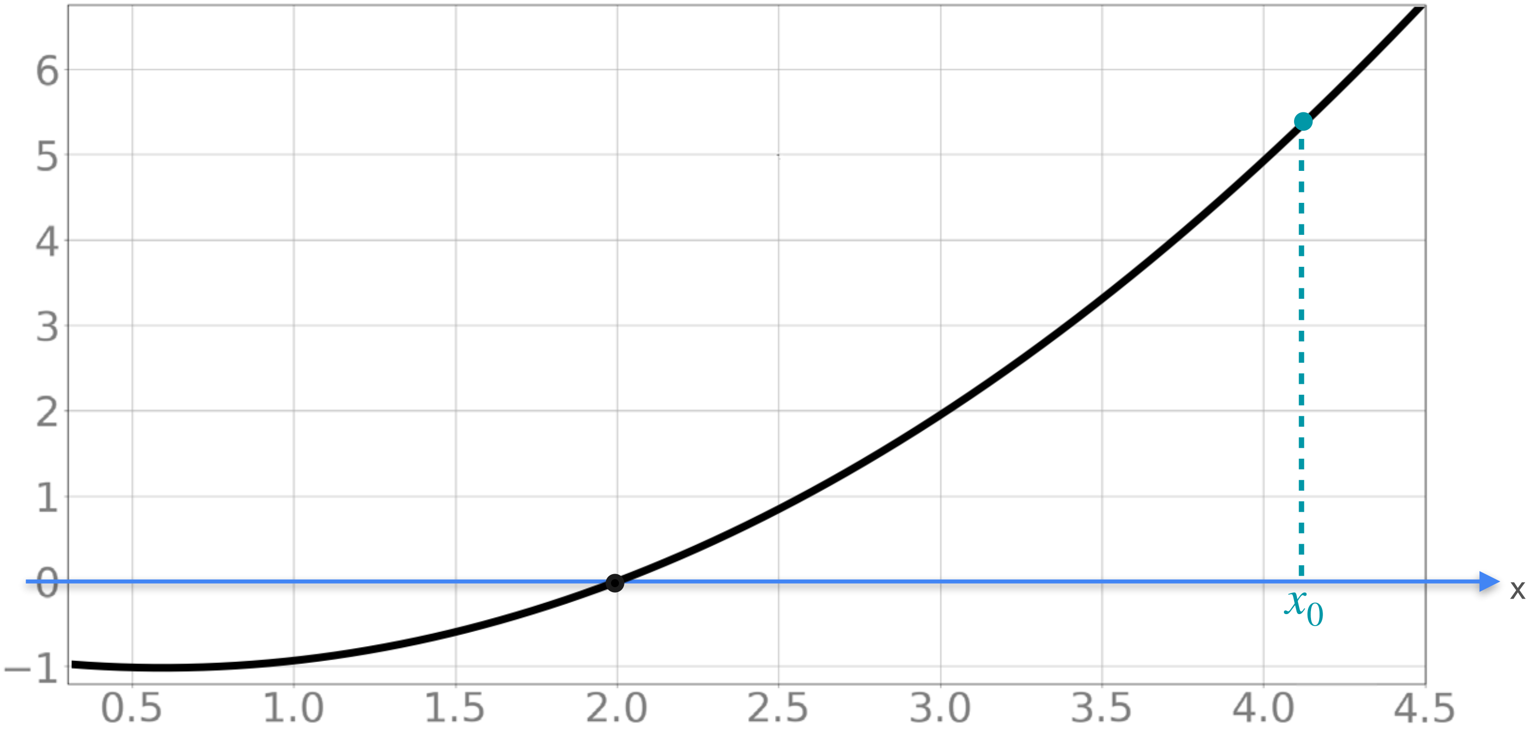

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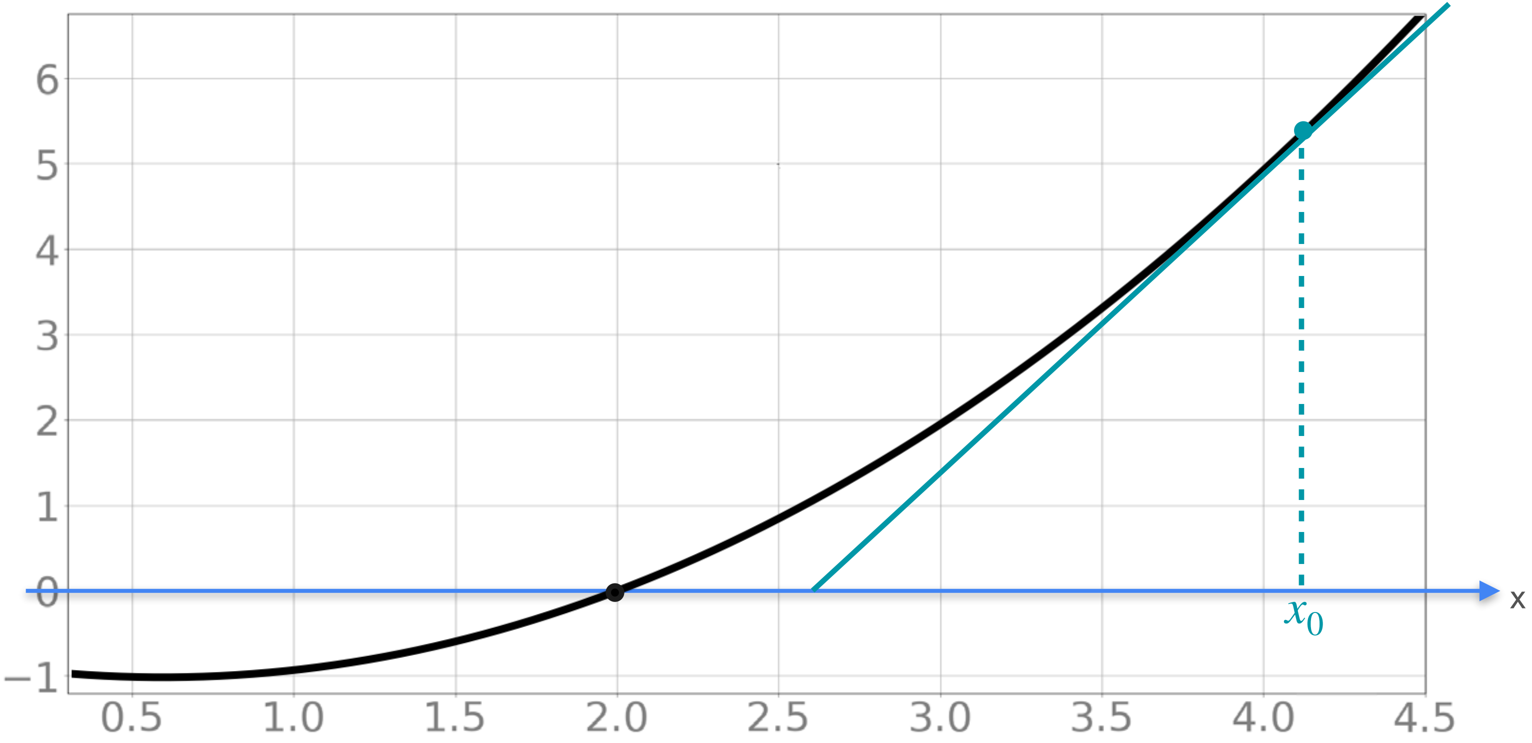

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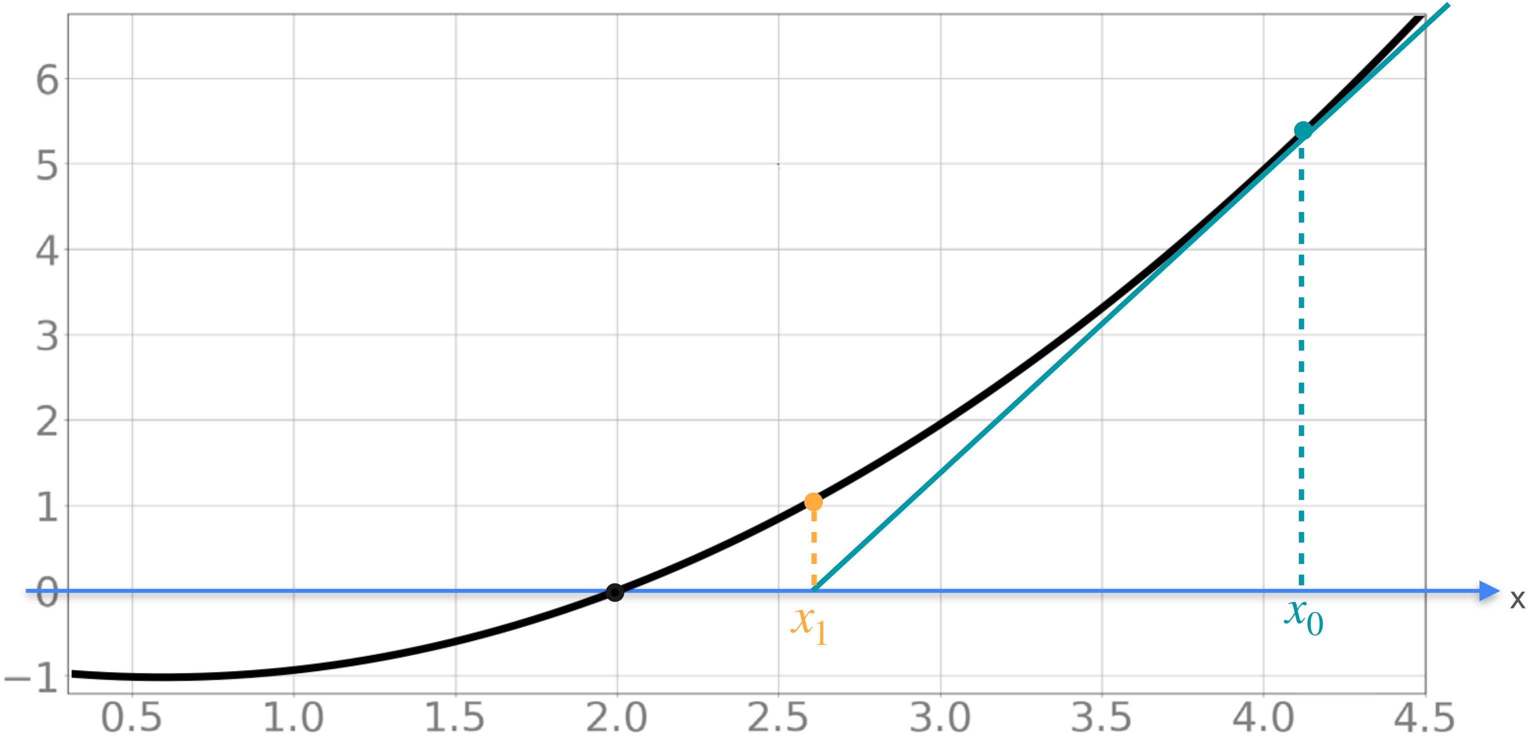

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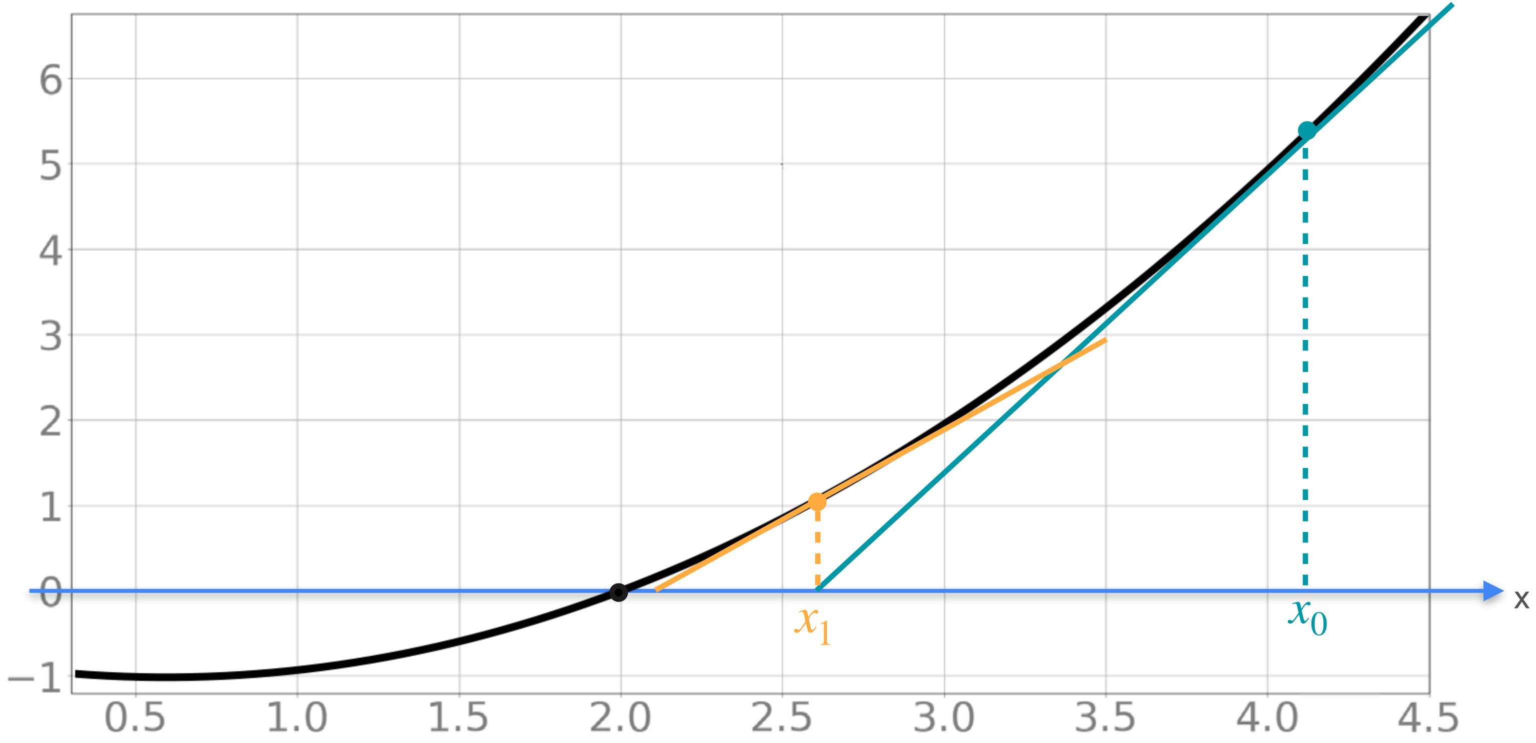

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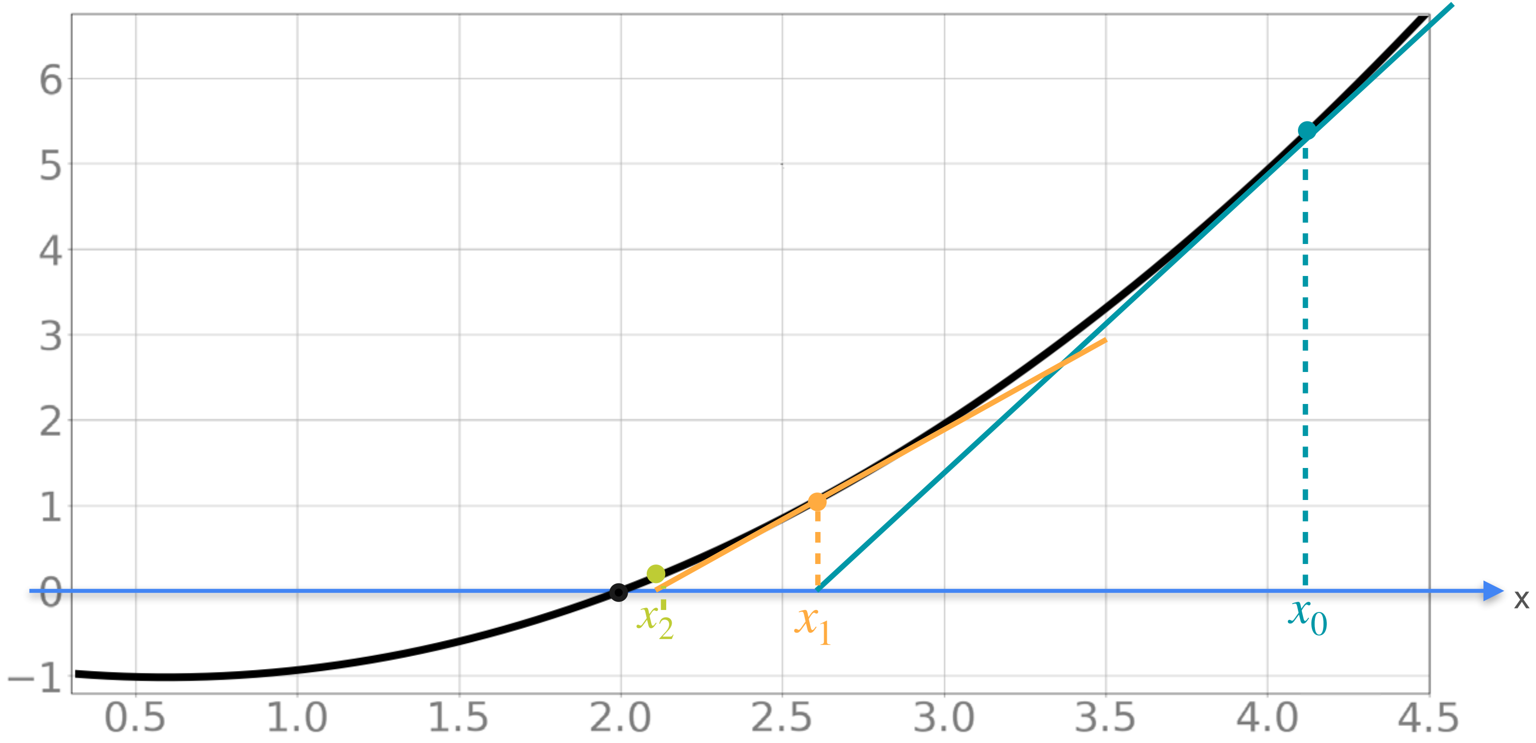

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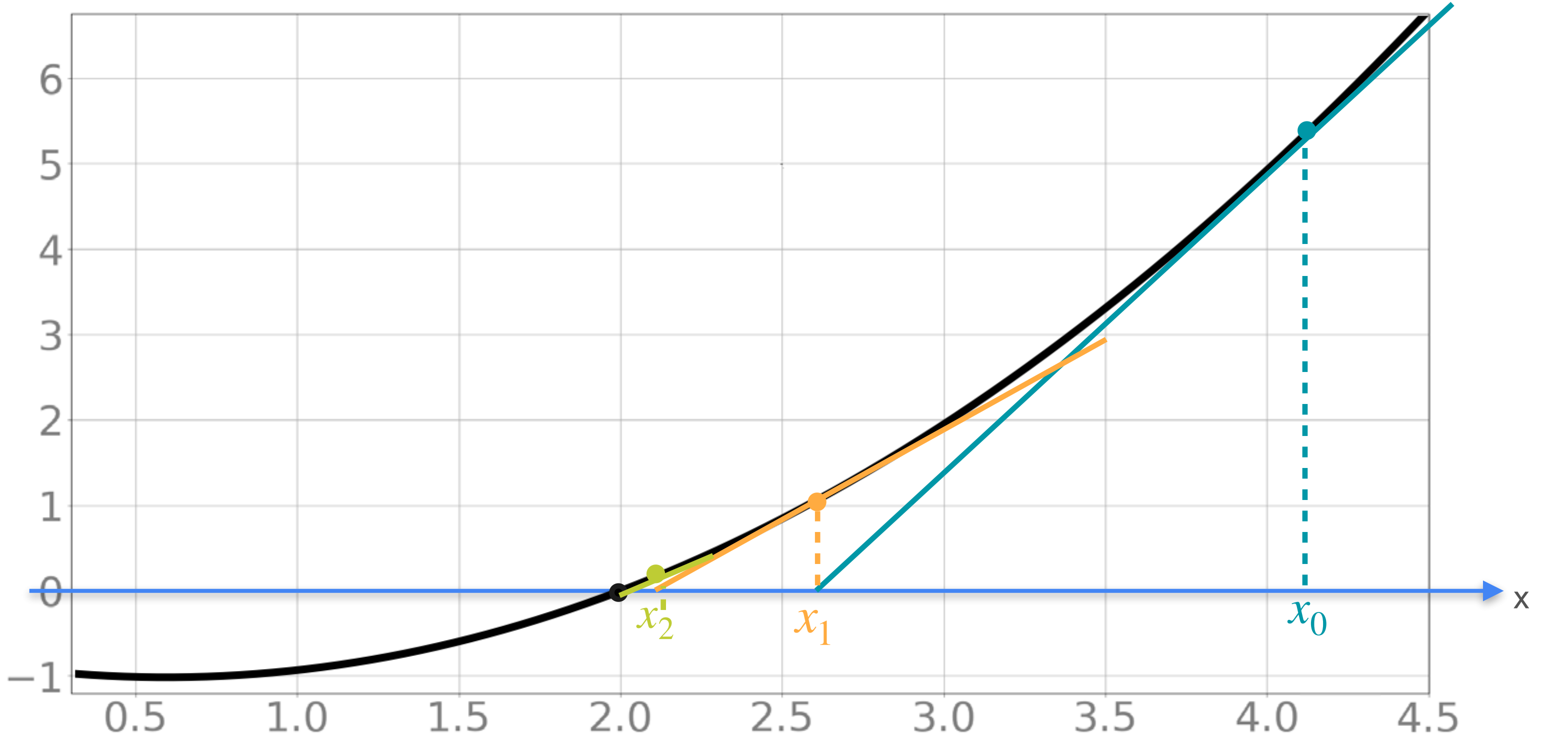

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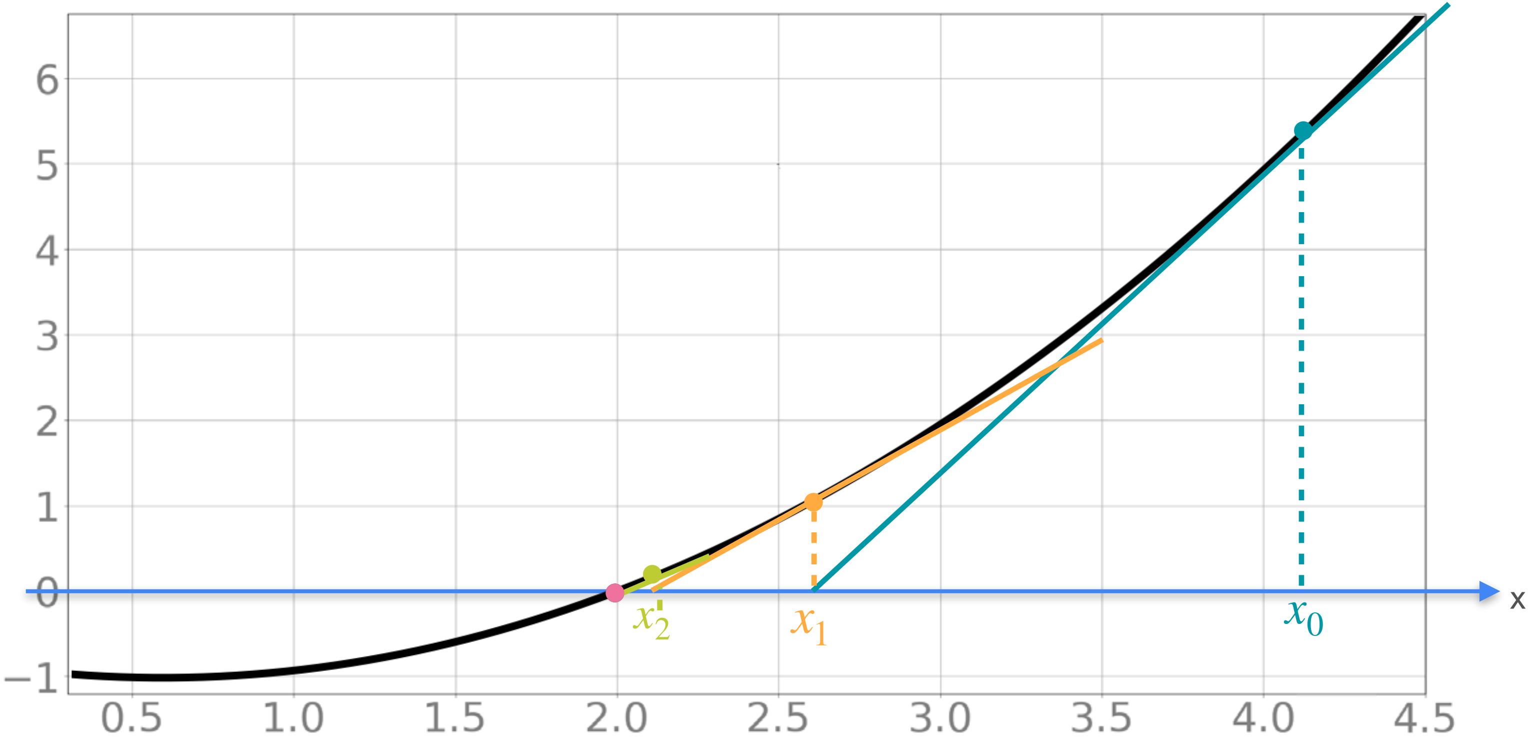
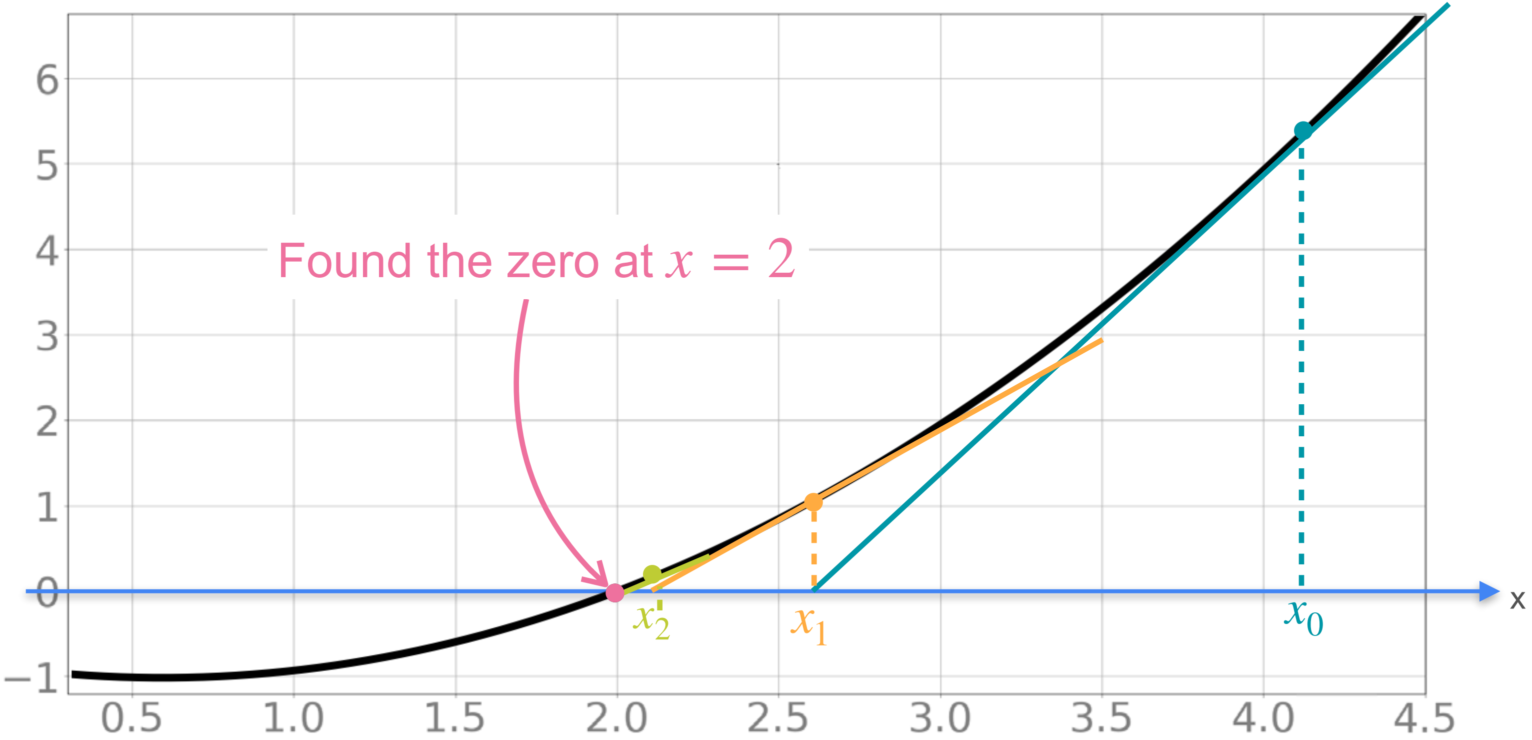


Newton's Method



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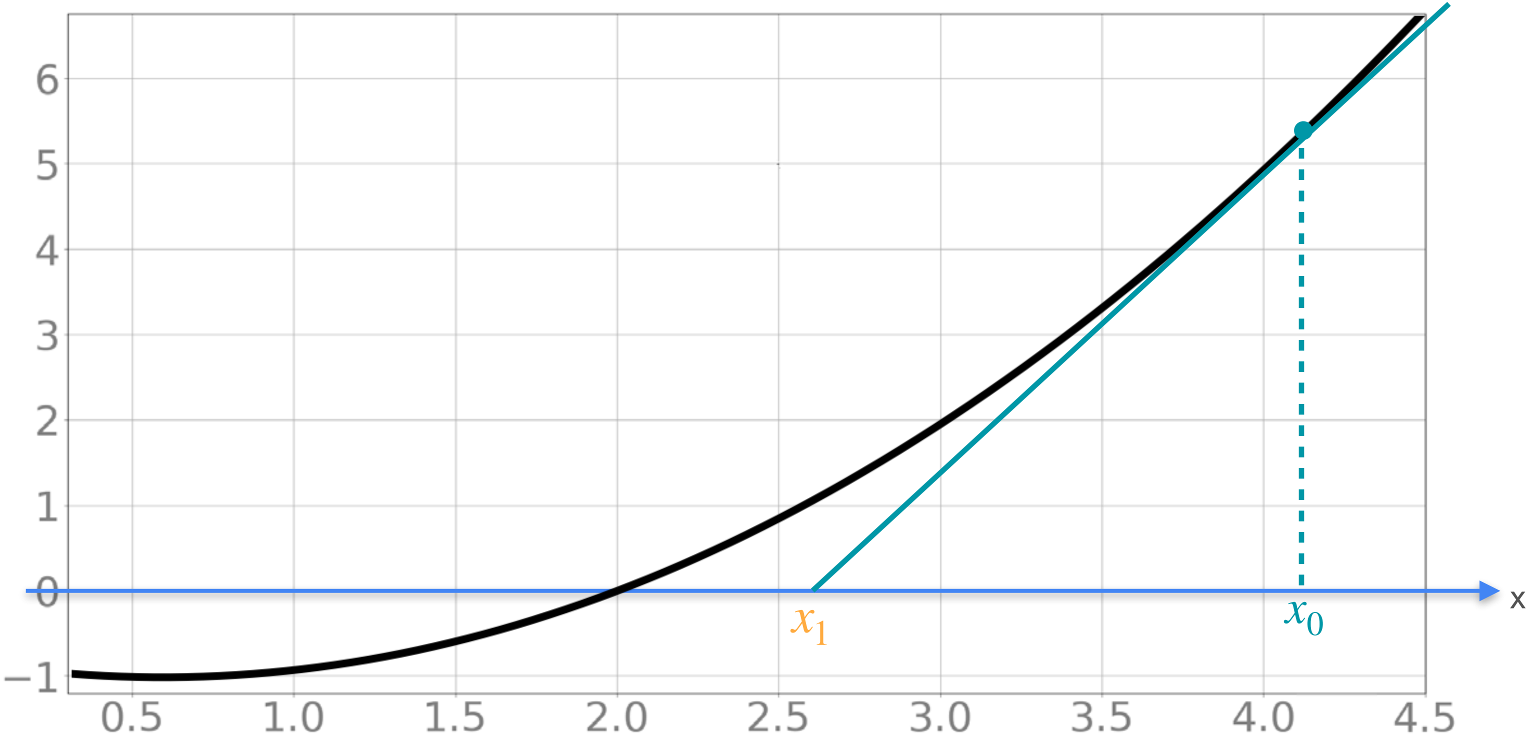

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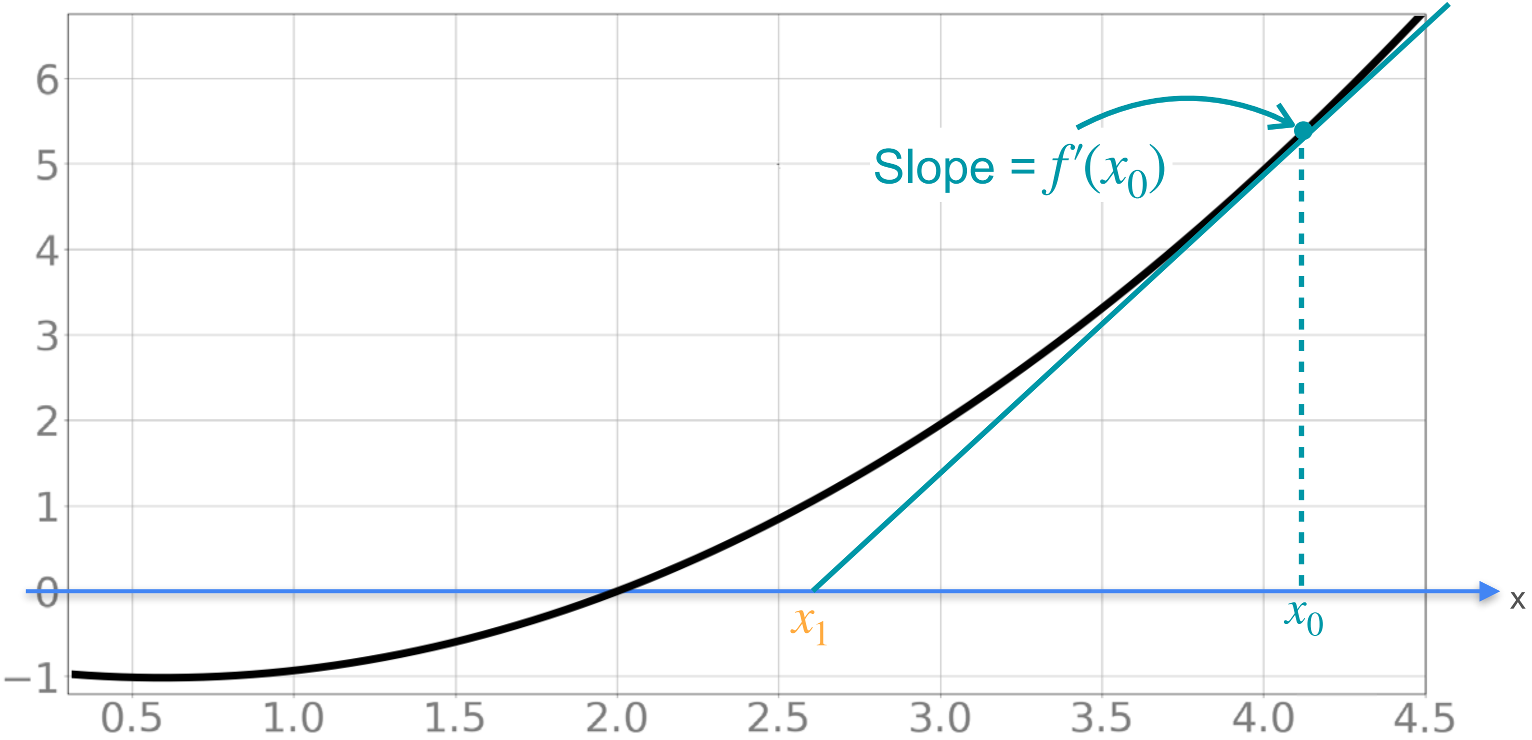

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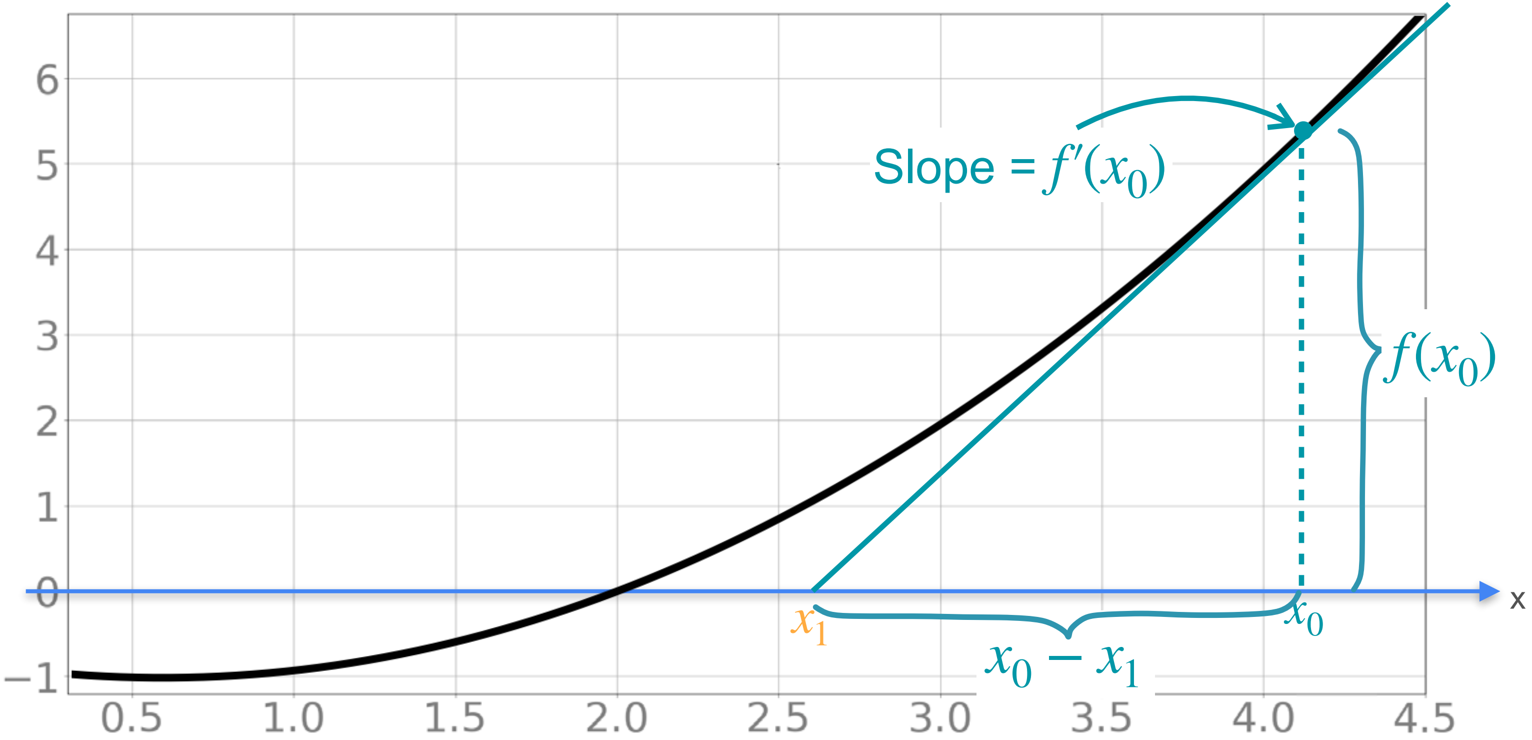

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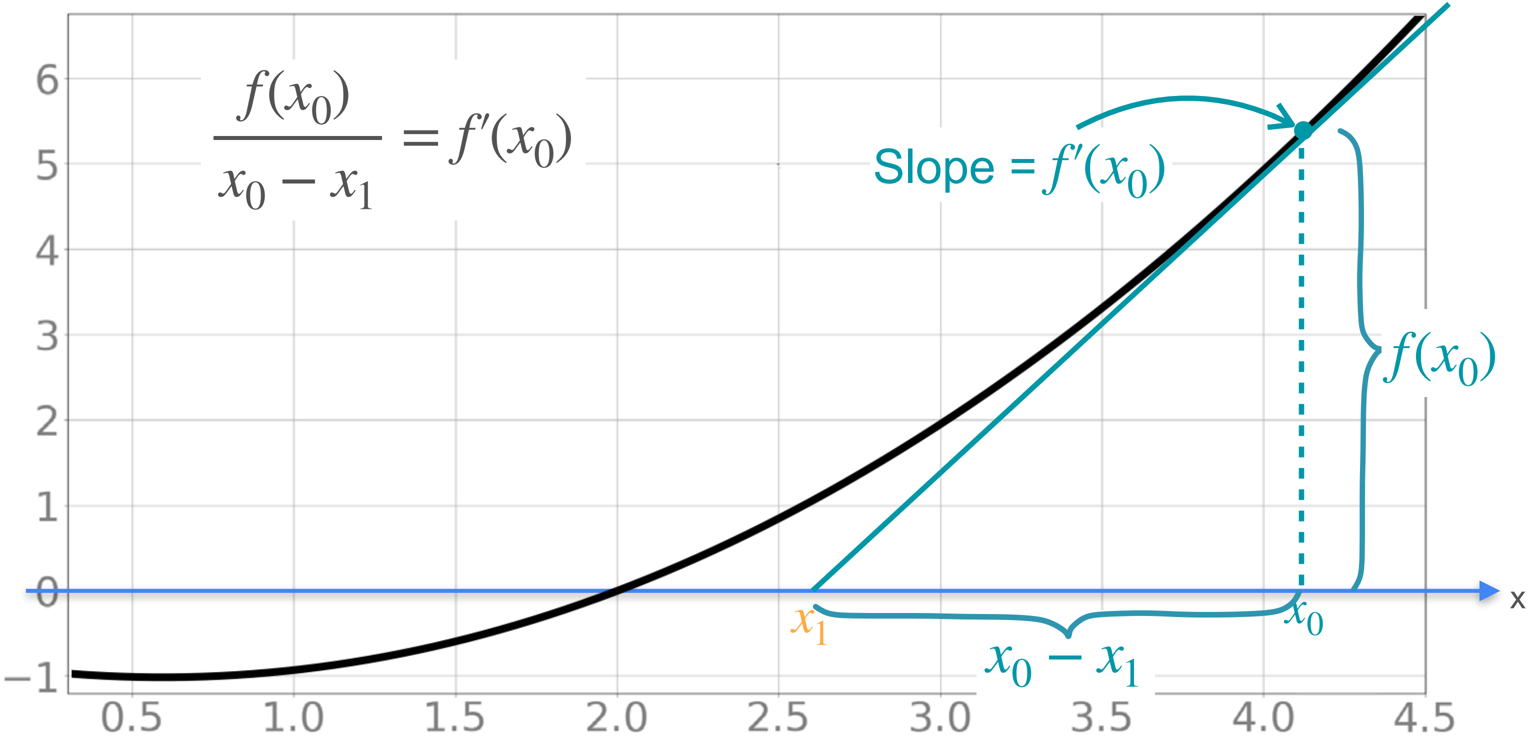

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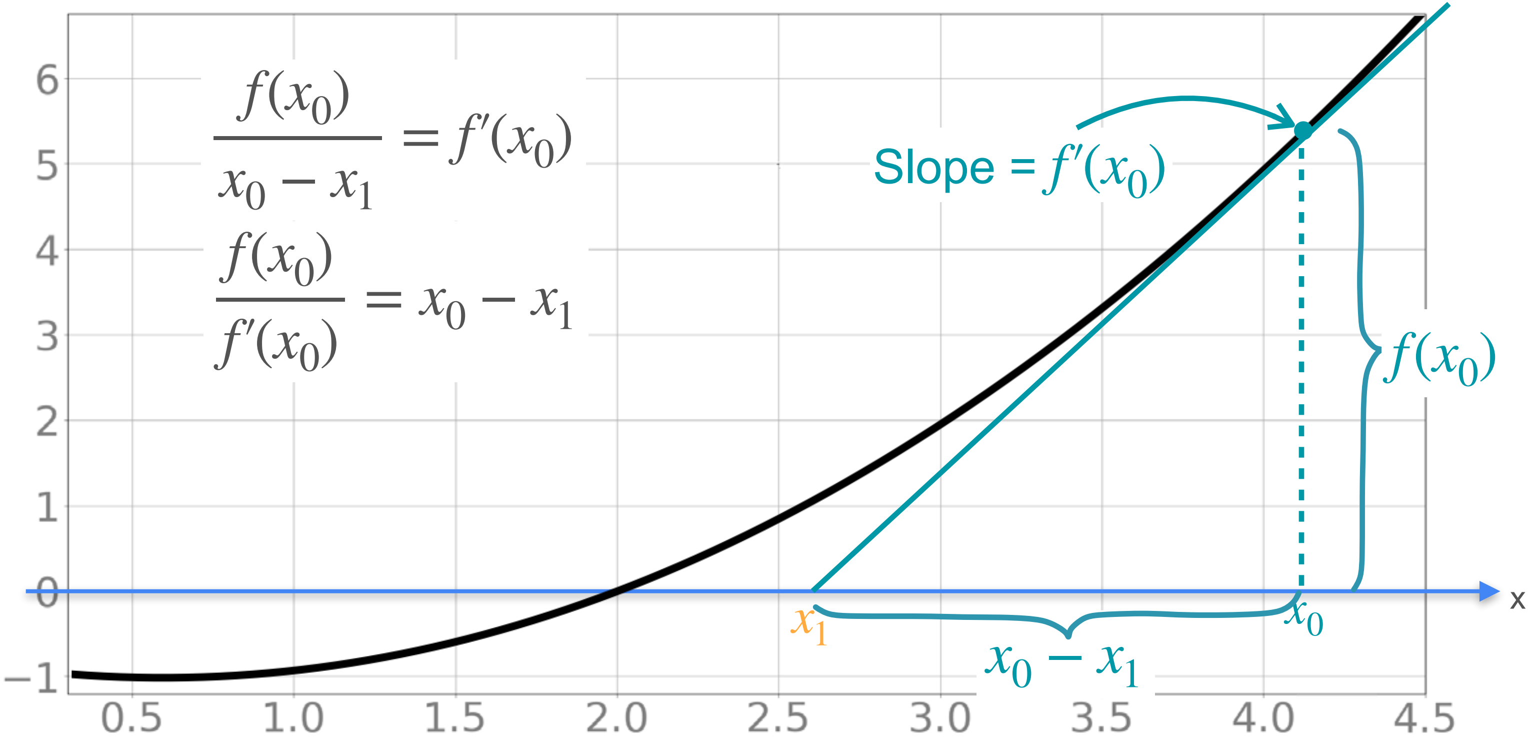

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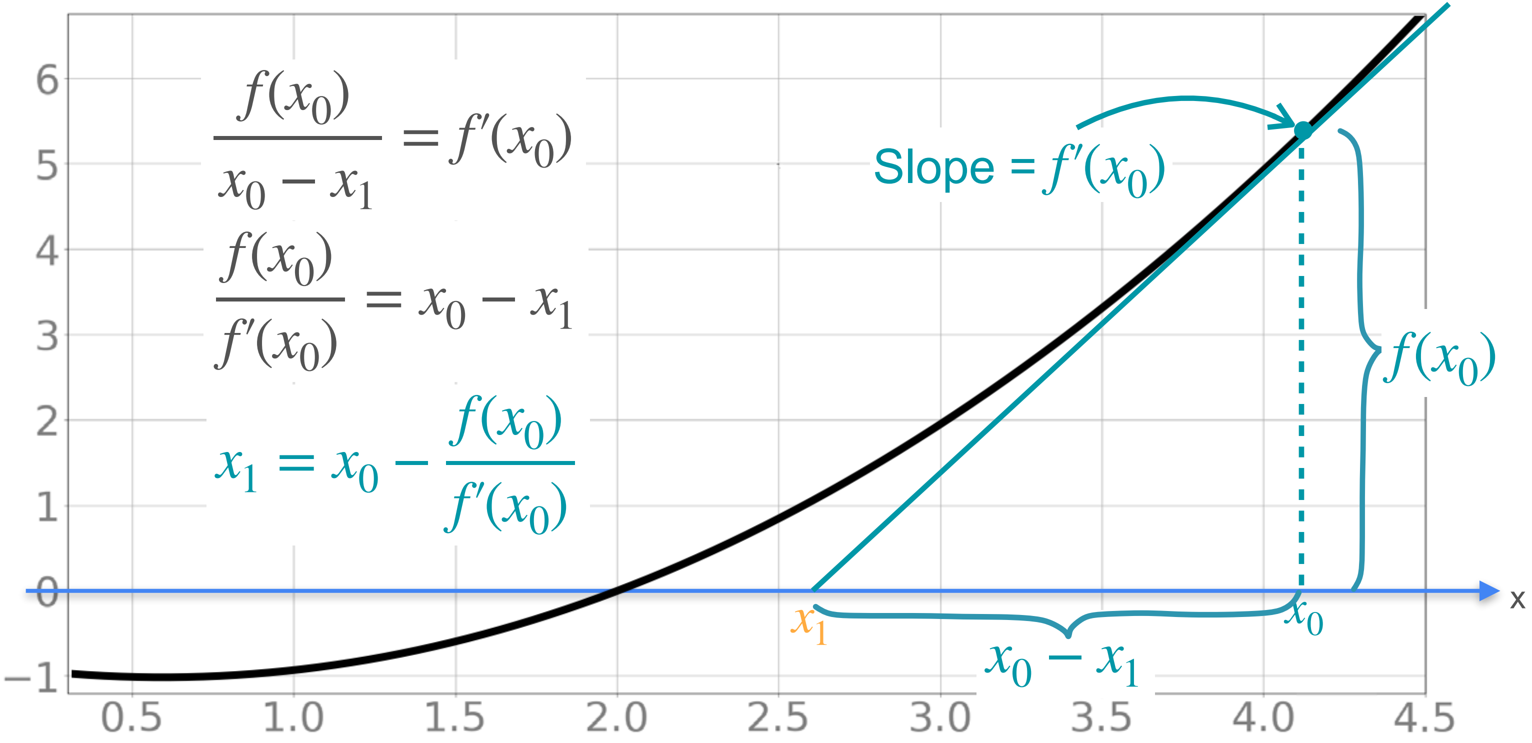

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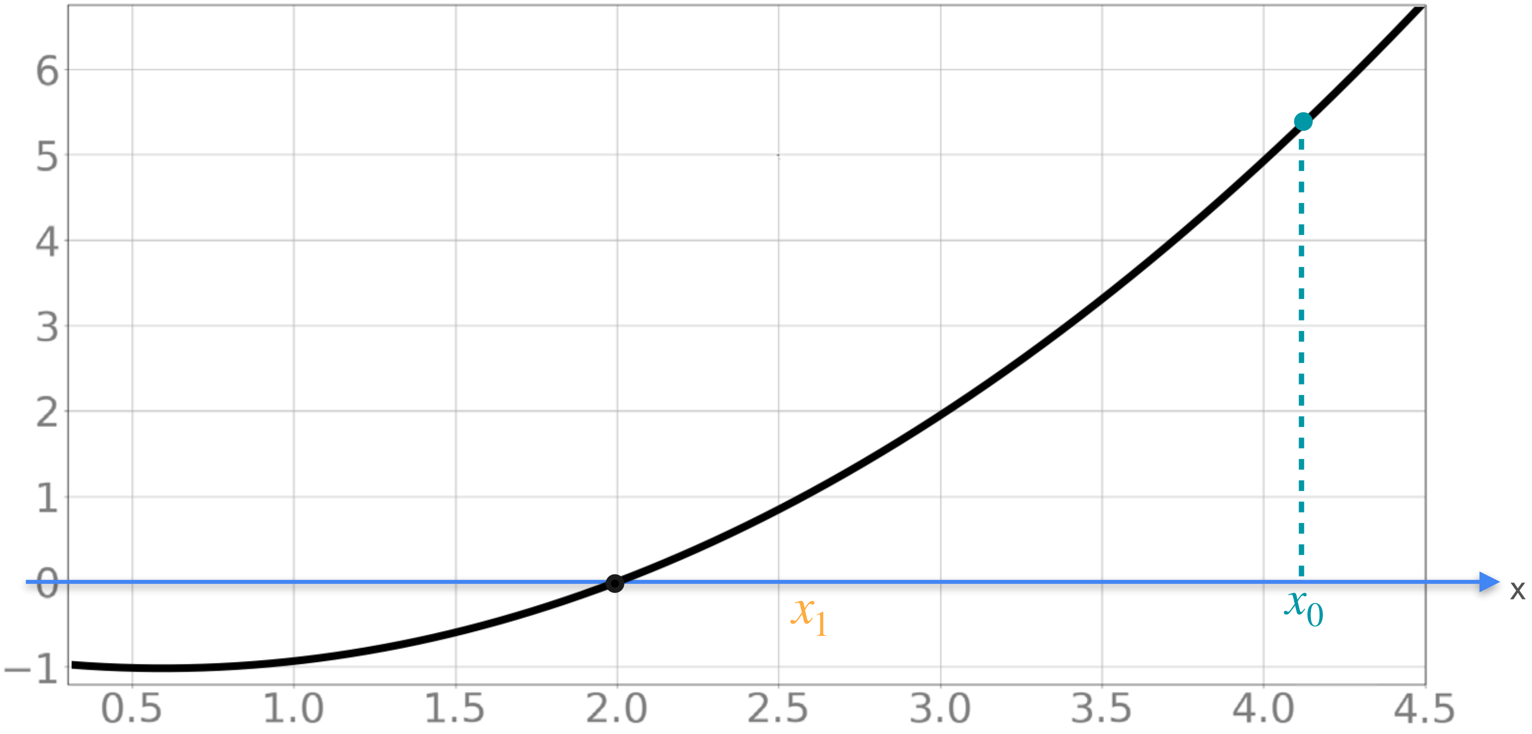

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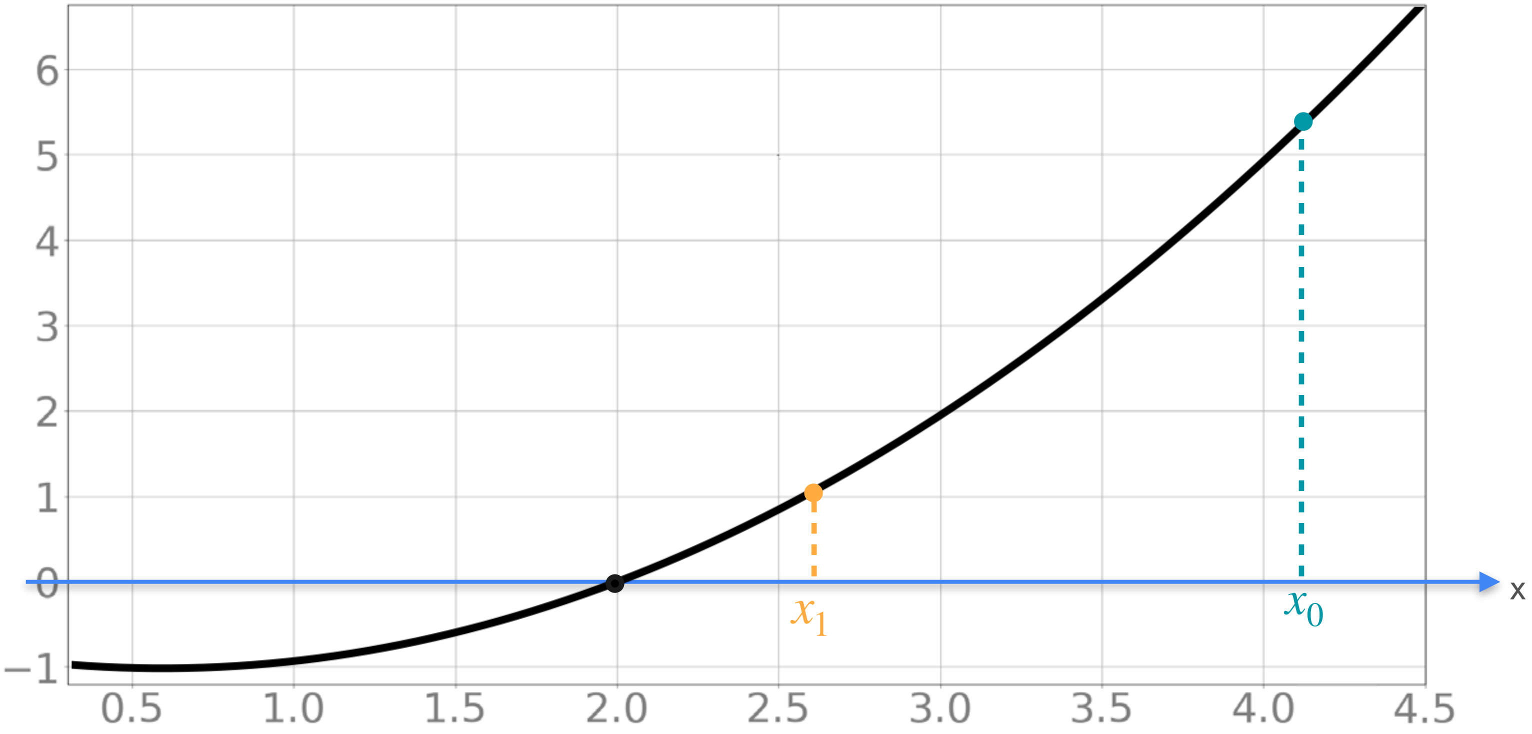

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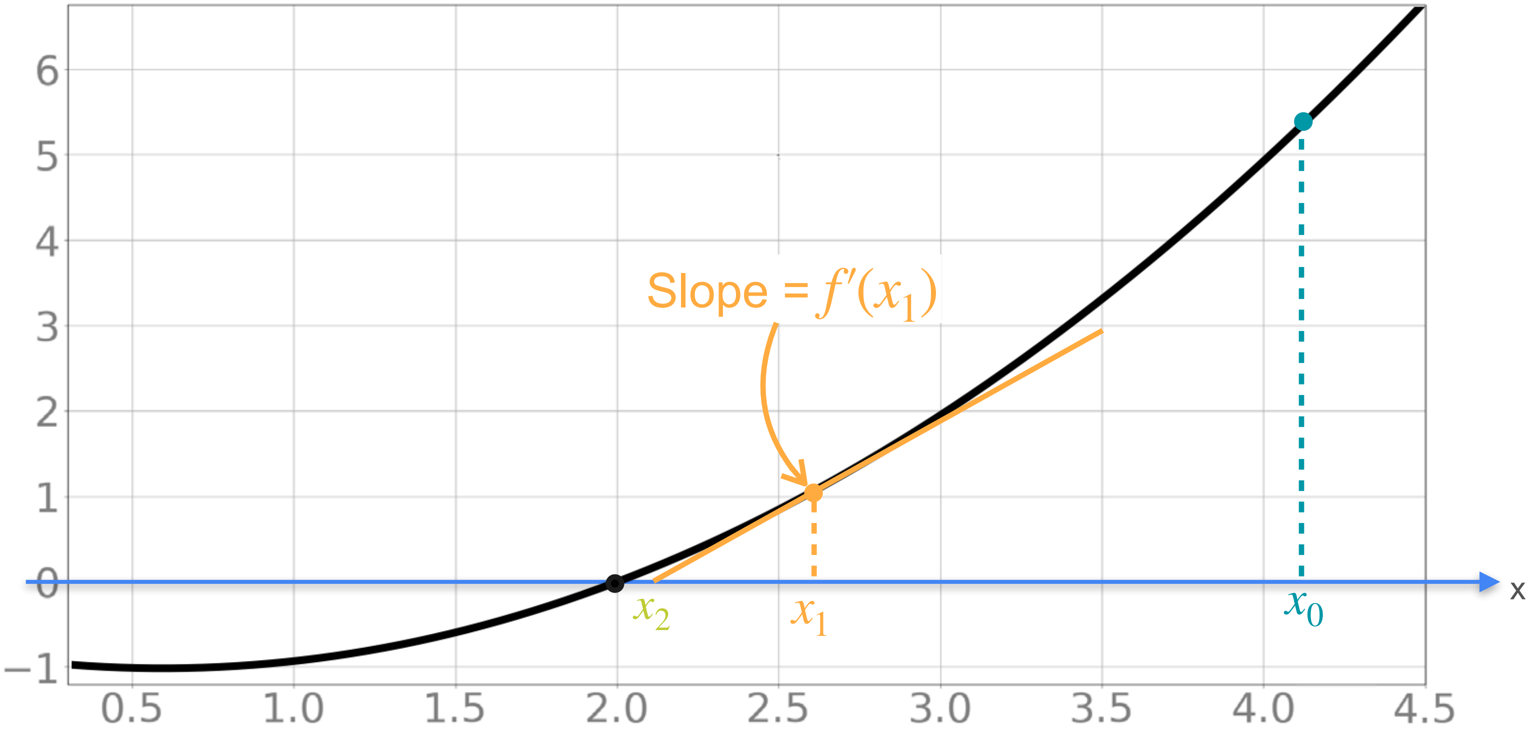

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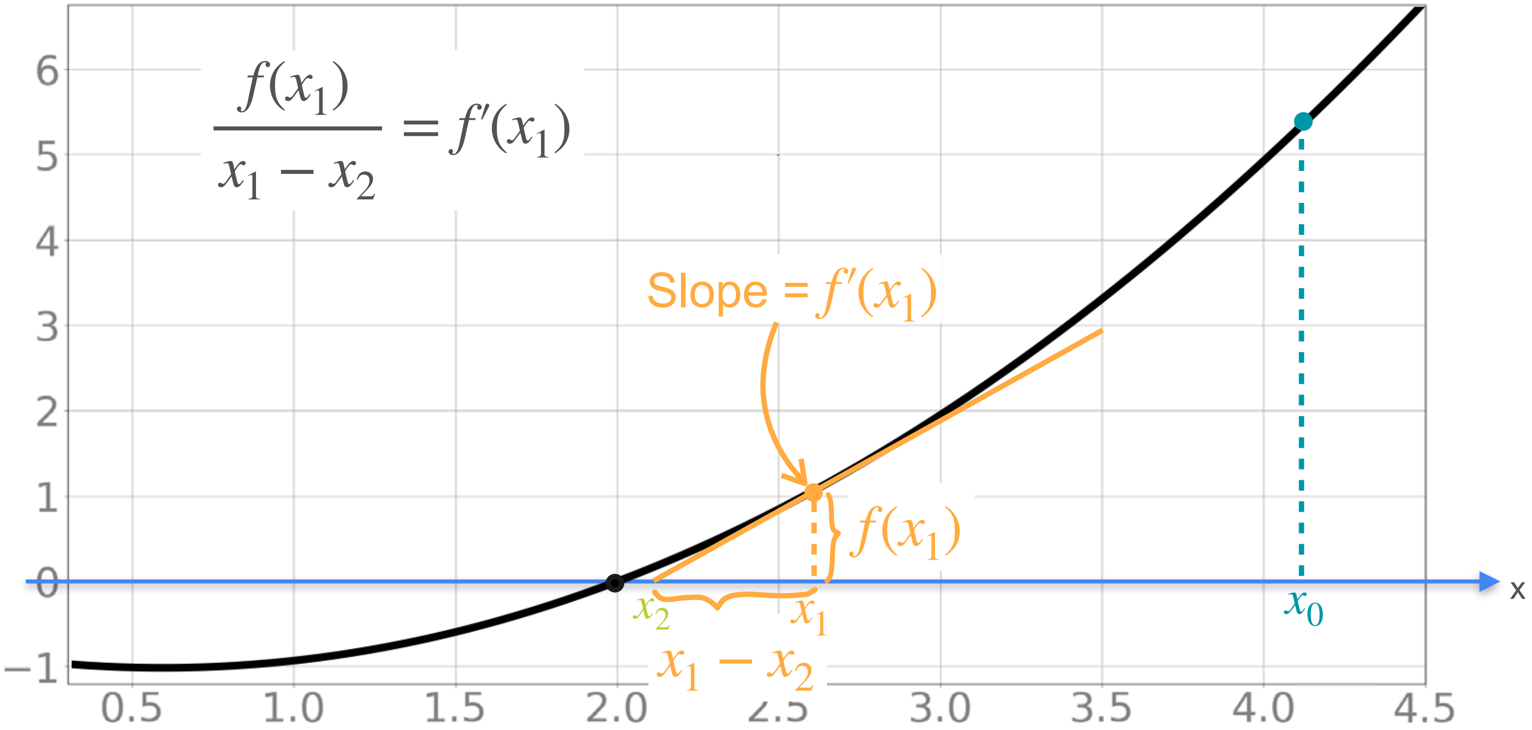

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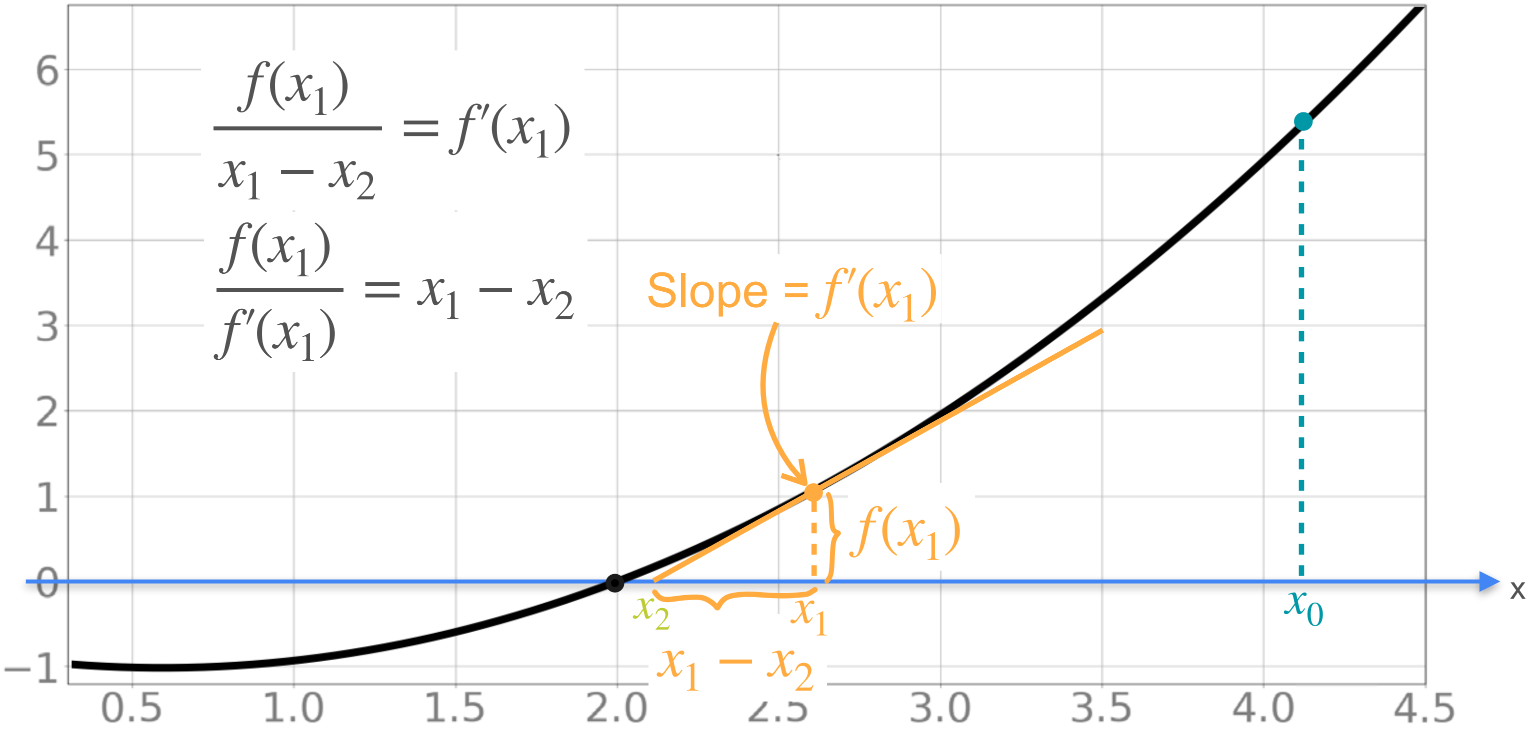

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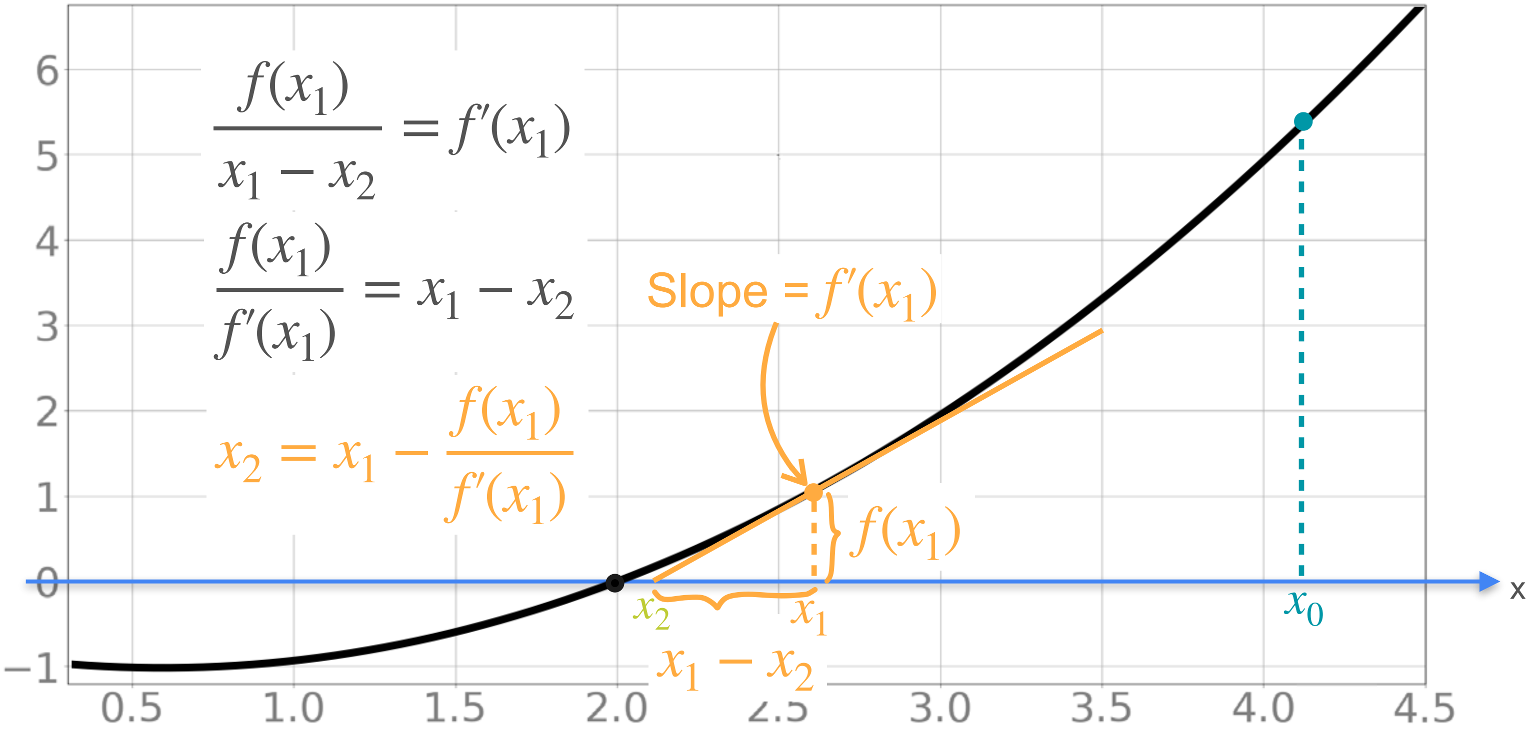

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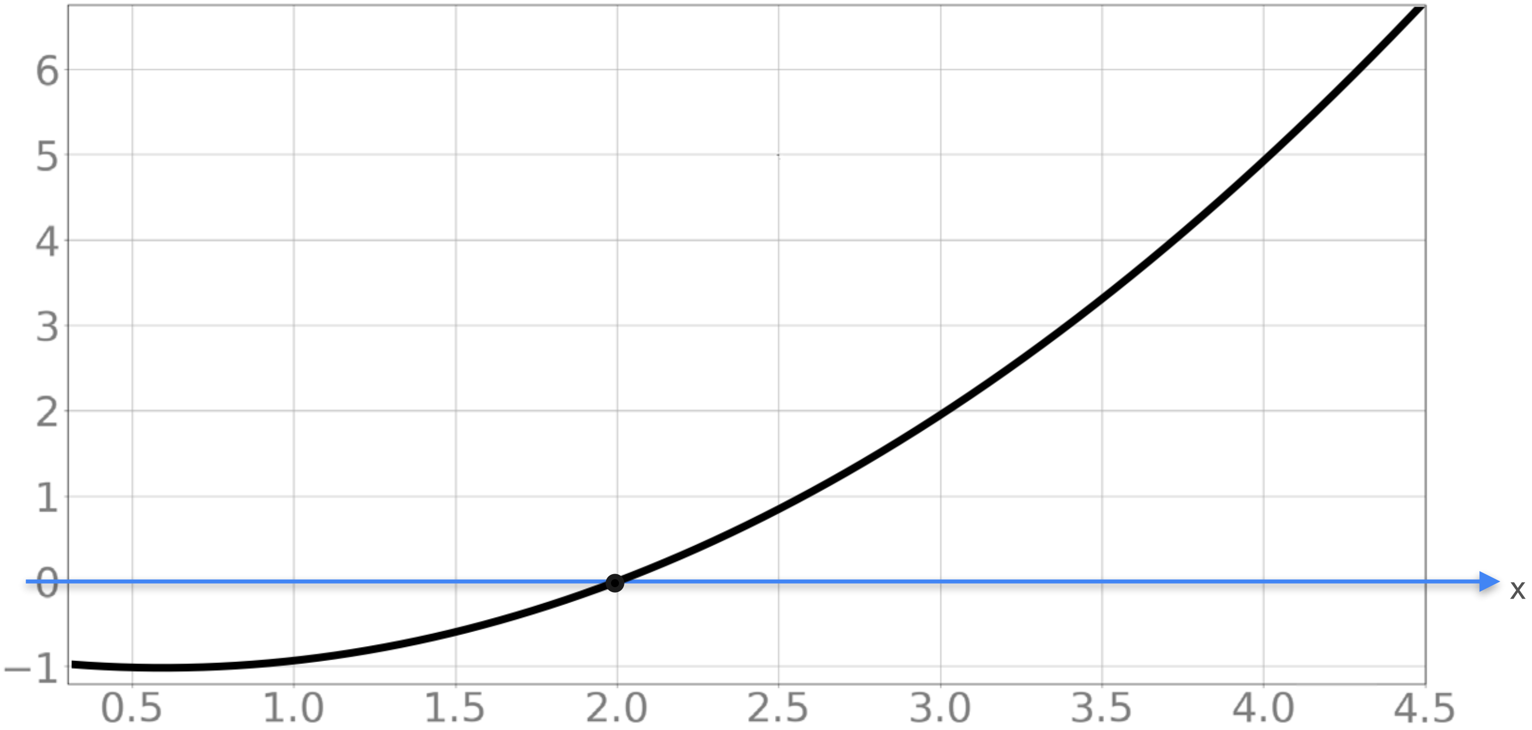

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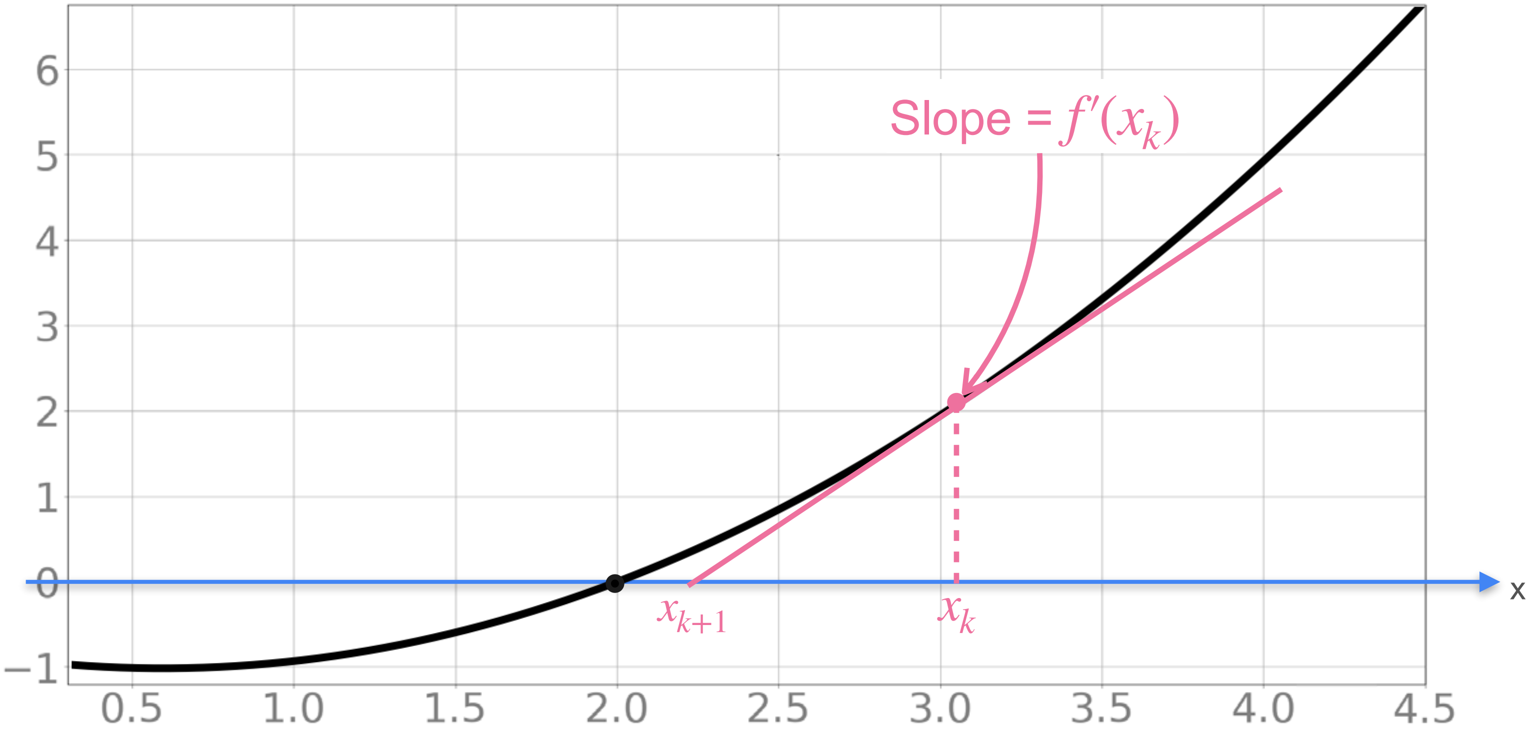

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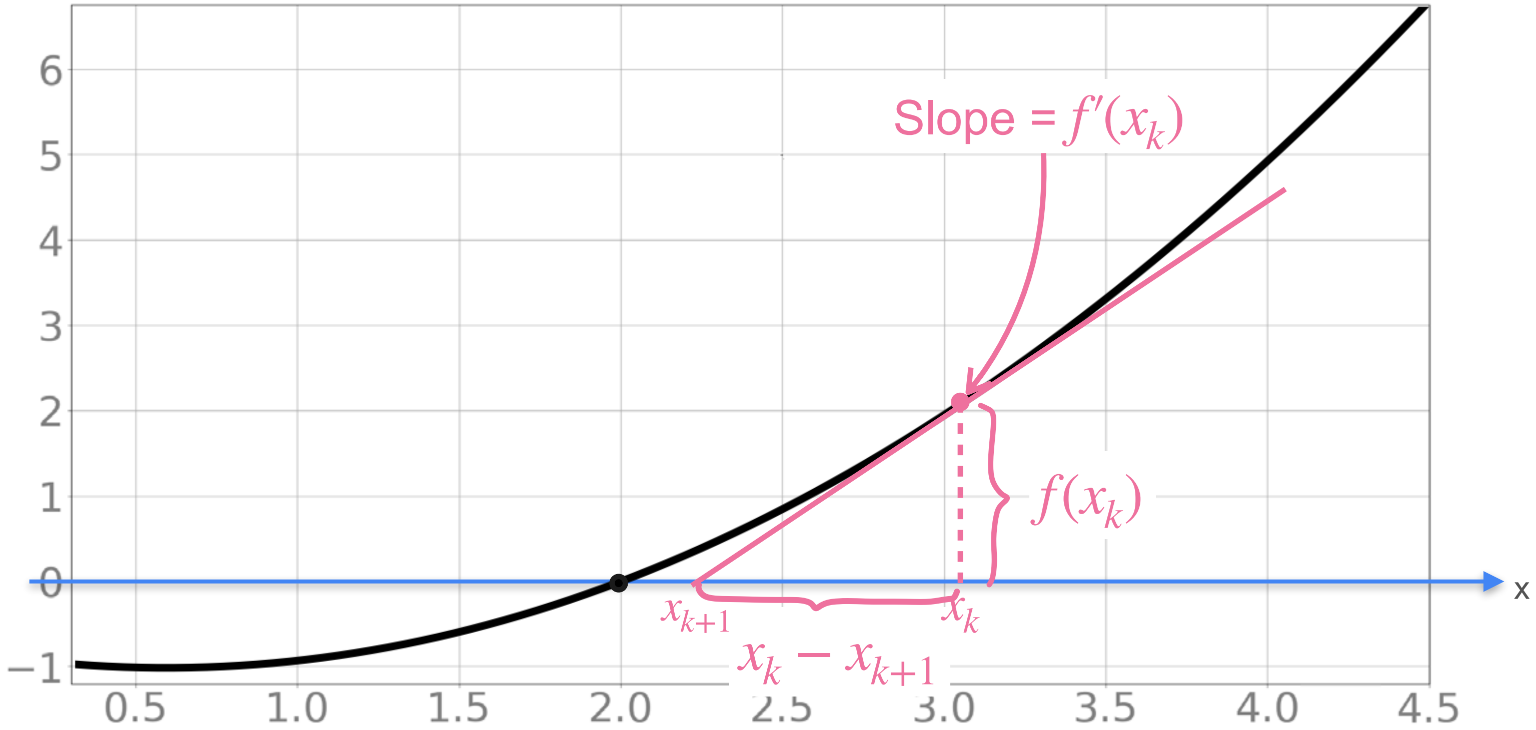

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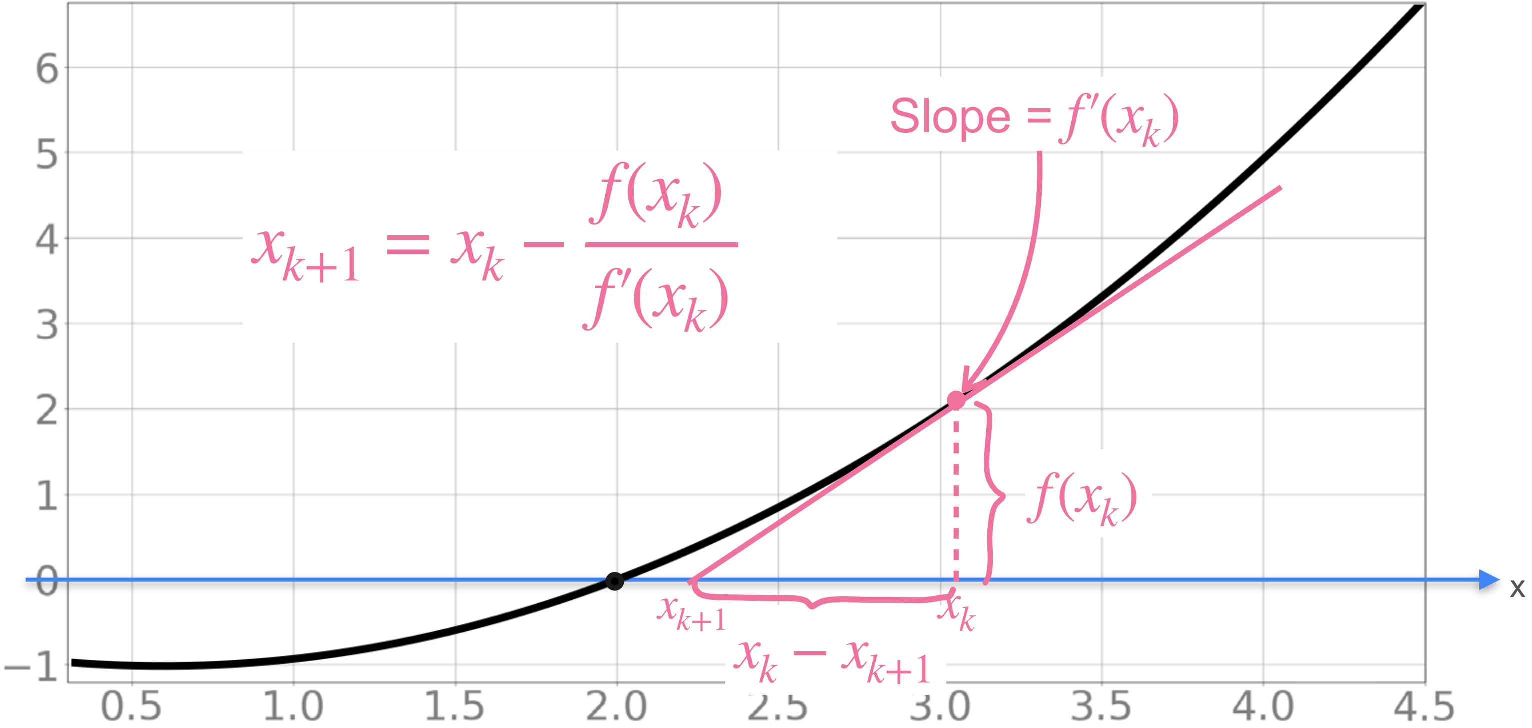

Newton's Method for Optimization

Newton's Method for Optimization

Newton’s Method for Optimization

Newton’s method

Goal: find a zero of $f(x)$

Newton's Method for Optimization

Newton's method

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NM for Optimization

Goal: minimize $g(x) \Rightarrow$ find zeros of $g'(x)$

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NM for Optimization

Goal: minimize $g(x)$ $\Rightarrow$ find zeros of $g'(x)$

$$f(x) \mapsto g'(x) \qquad f'(x) \mapsto (g'(x))'$$

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0

NM for Optimization

Goal: minimize $g(x)$ $\Rightarrow$ find zeros of $g'(x)$

$$f(x) \mapsto g'(x) \qquad f'(x) \mapsto (g'(x))'$$

1) Start with some x_0

Newton’s Method for Optimization

Newton’s method

Goal: find a zero of $f(x)$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

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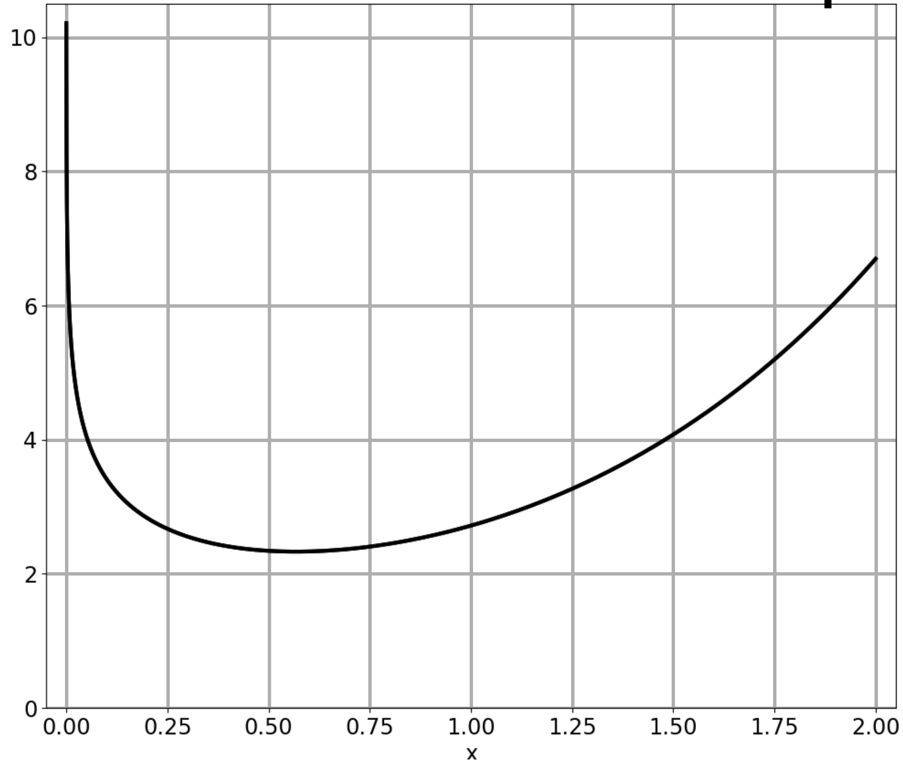
DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Newton's method: An example

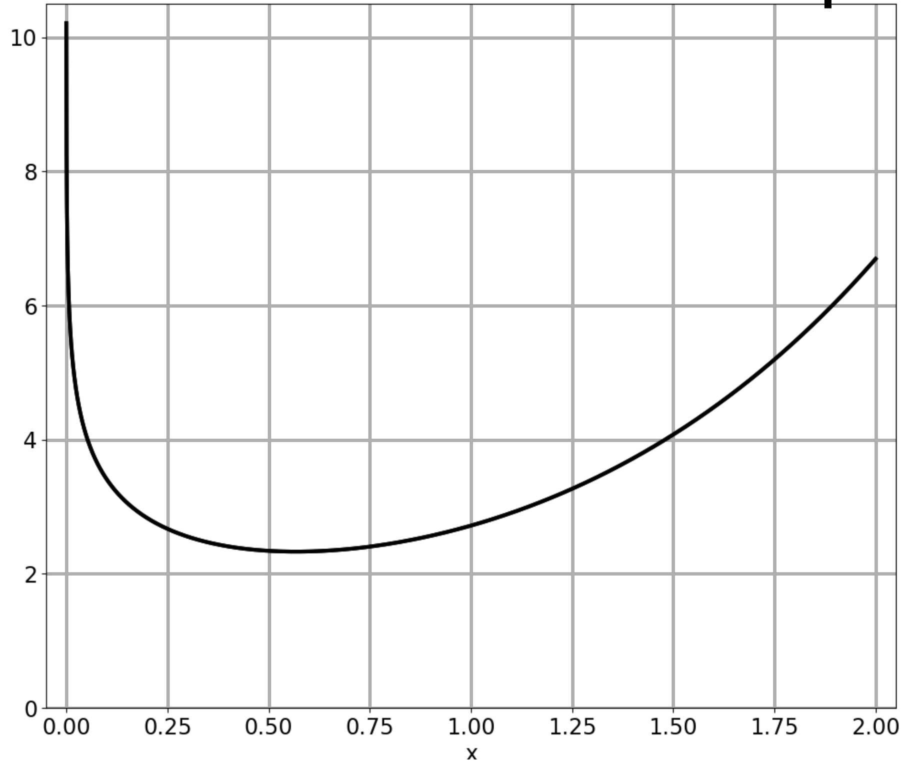
Newton’s Method for Optimization

Newton's Method for Optimization



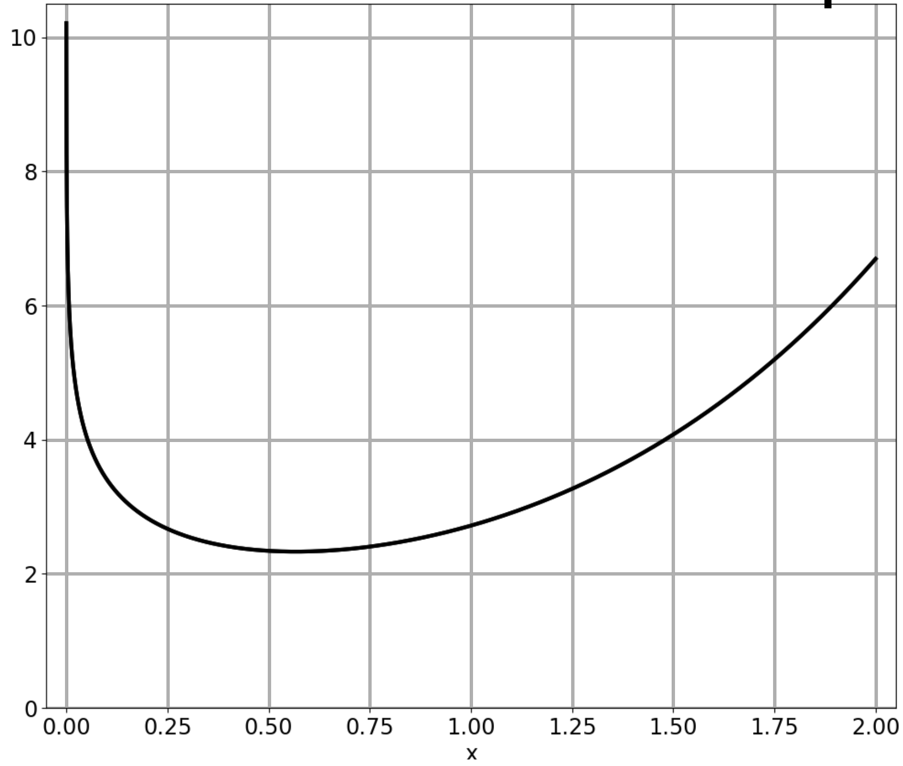
$$g(x) = e^x - \log(x)$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

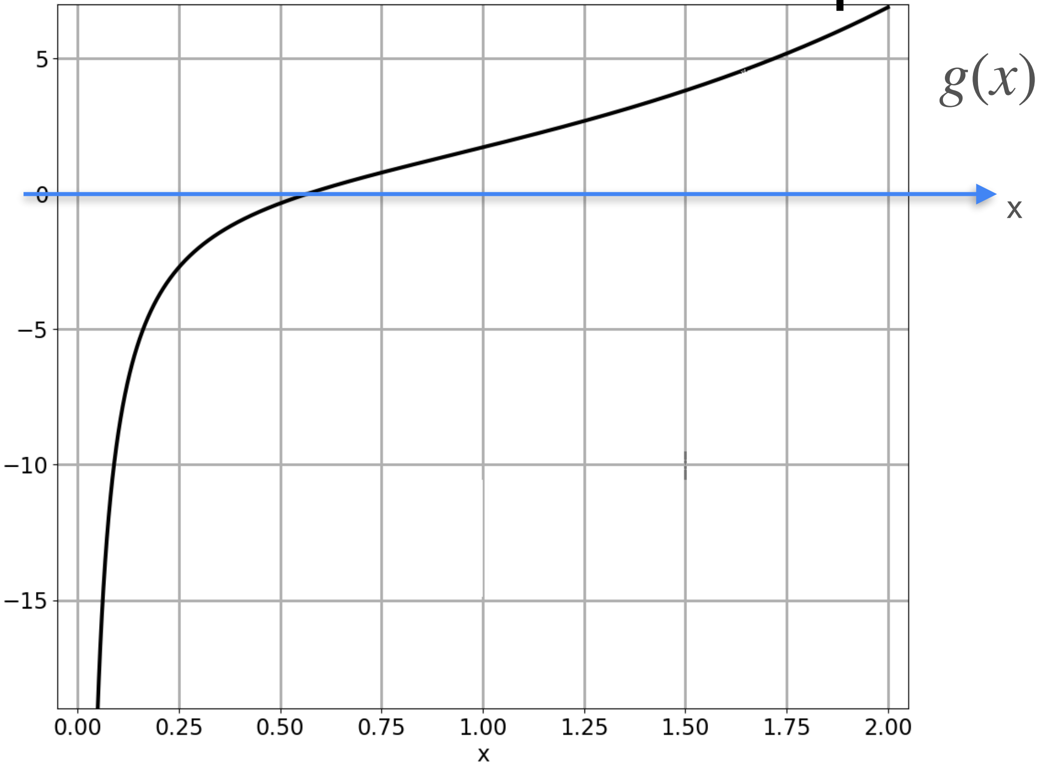
Newton's Method for Optimization



$g(x) = e^x - \log(x)$ $\quad g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

Newton’s Method for Optimization



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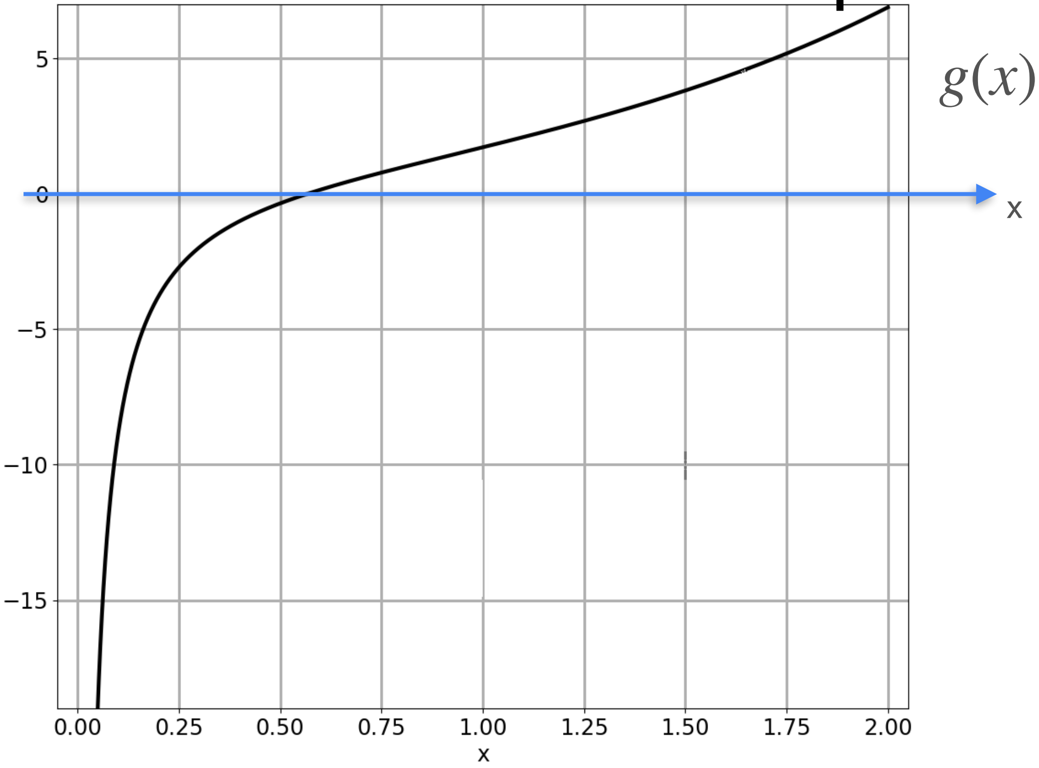
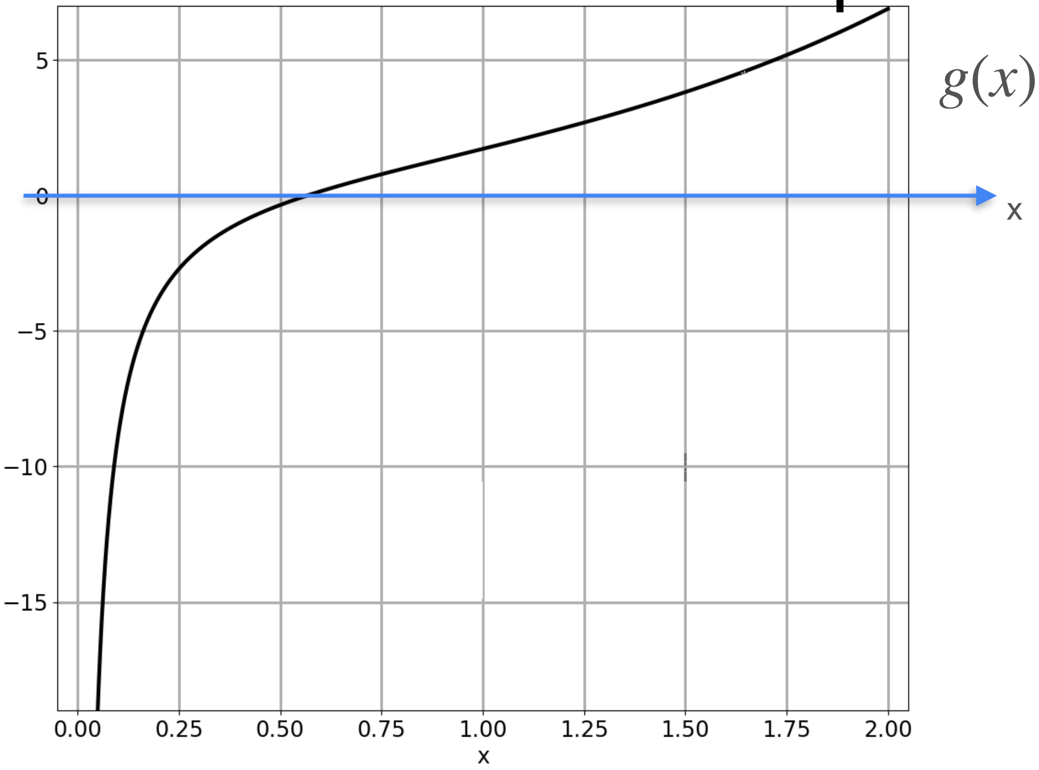


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

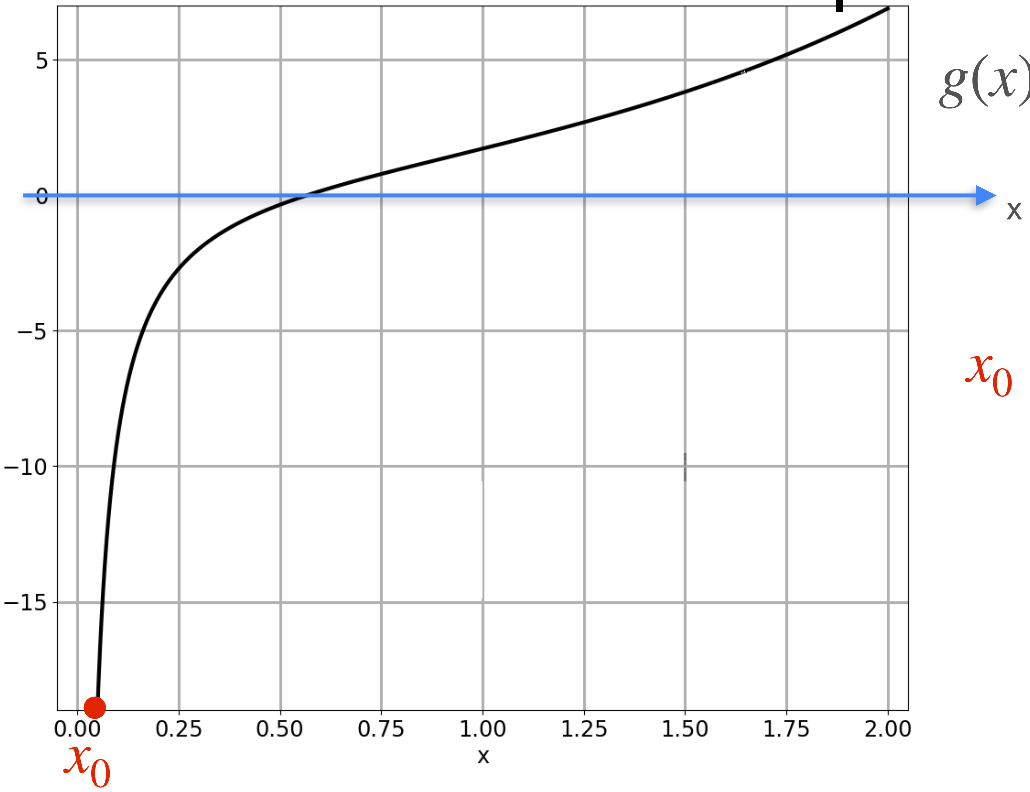


$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

Newton's Method for Optimization



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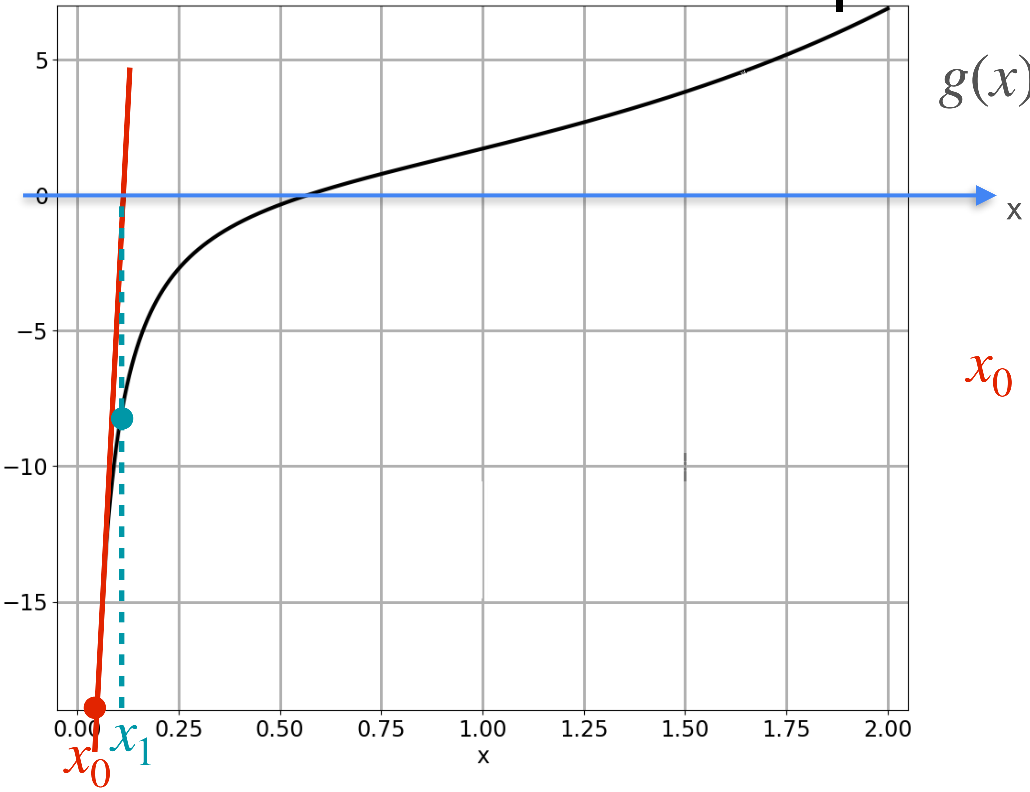

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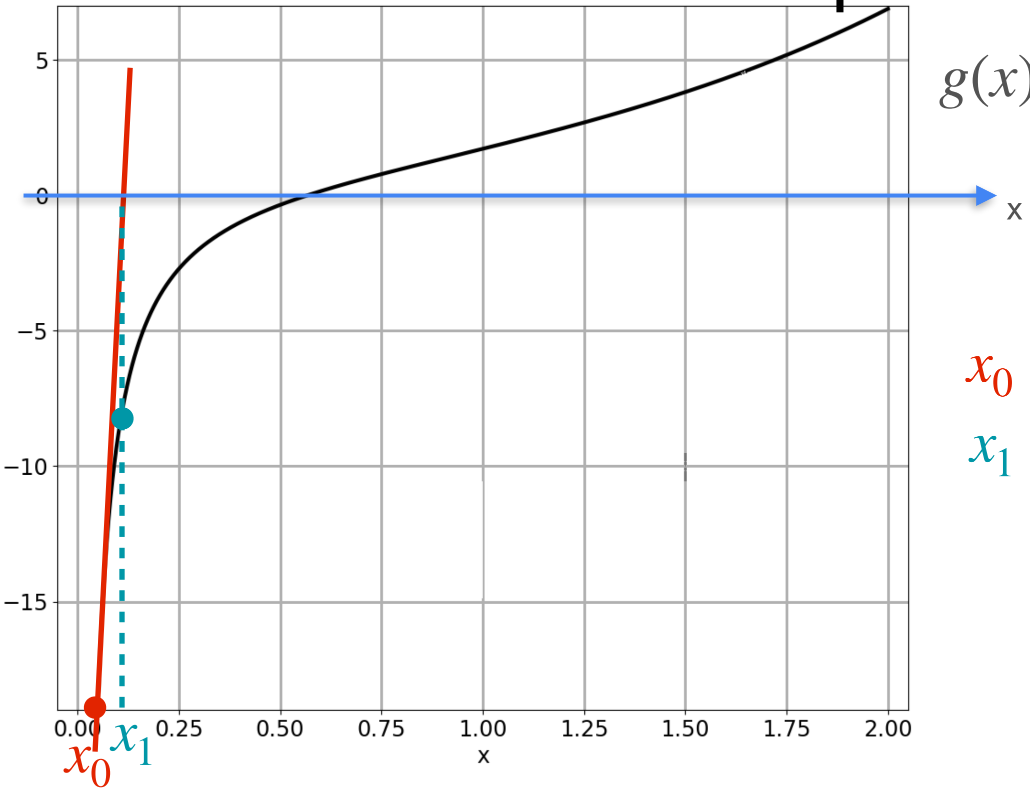

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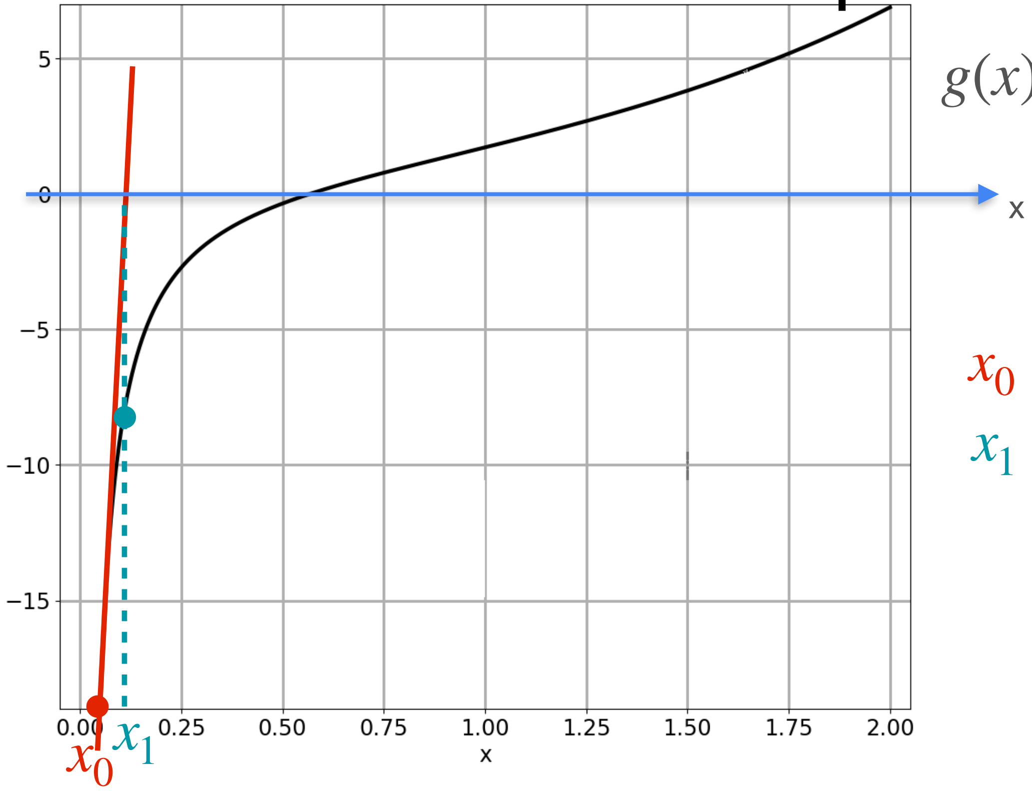
$$x_0 = 0.05$$

$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$

$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)}$$

Newton's Method for Optimization



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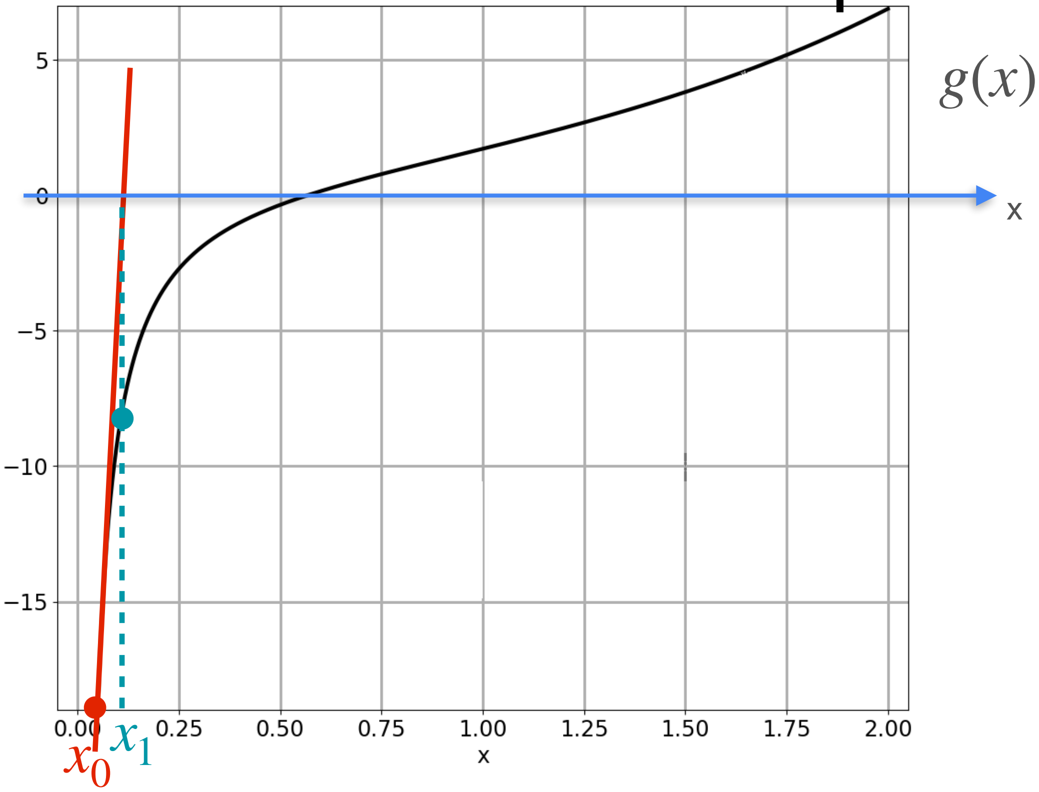
$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$

$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)} = 0.097$$

Newton's Method for Optimization



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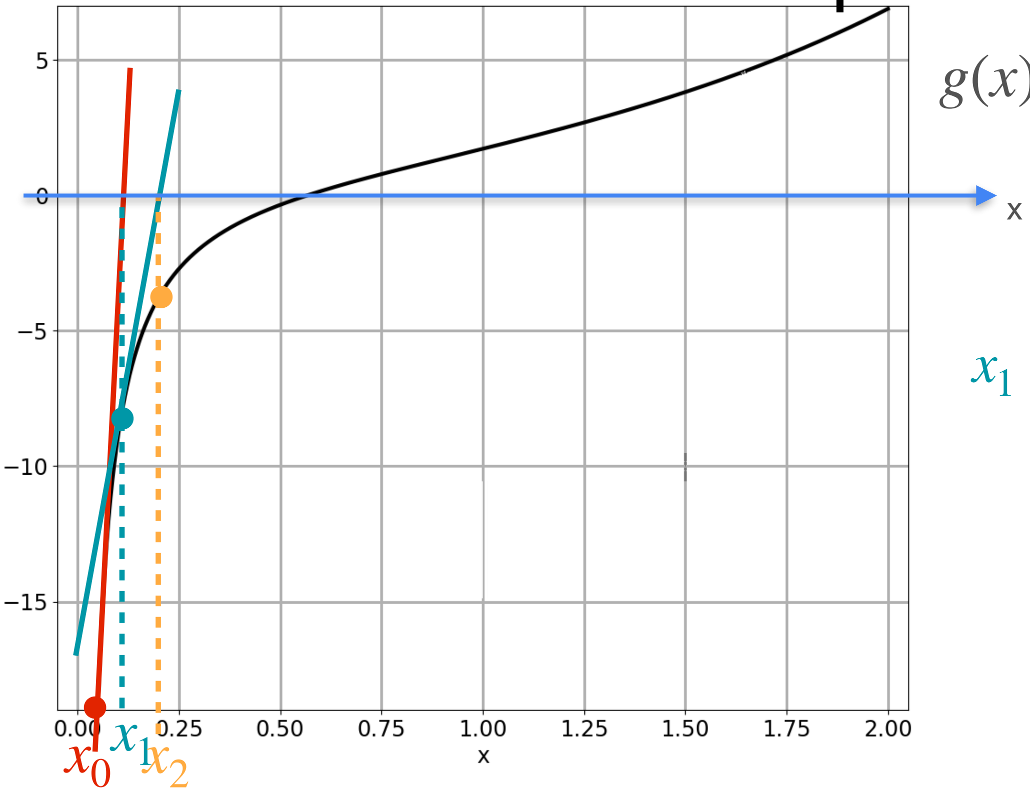
Newton's Method for Optimization



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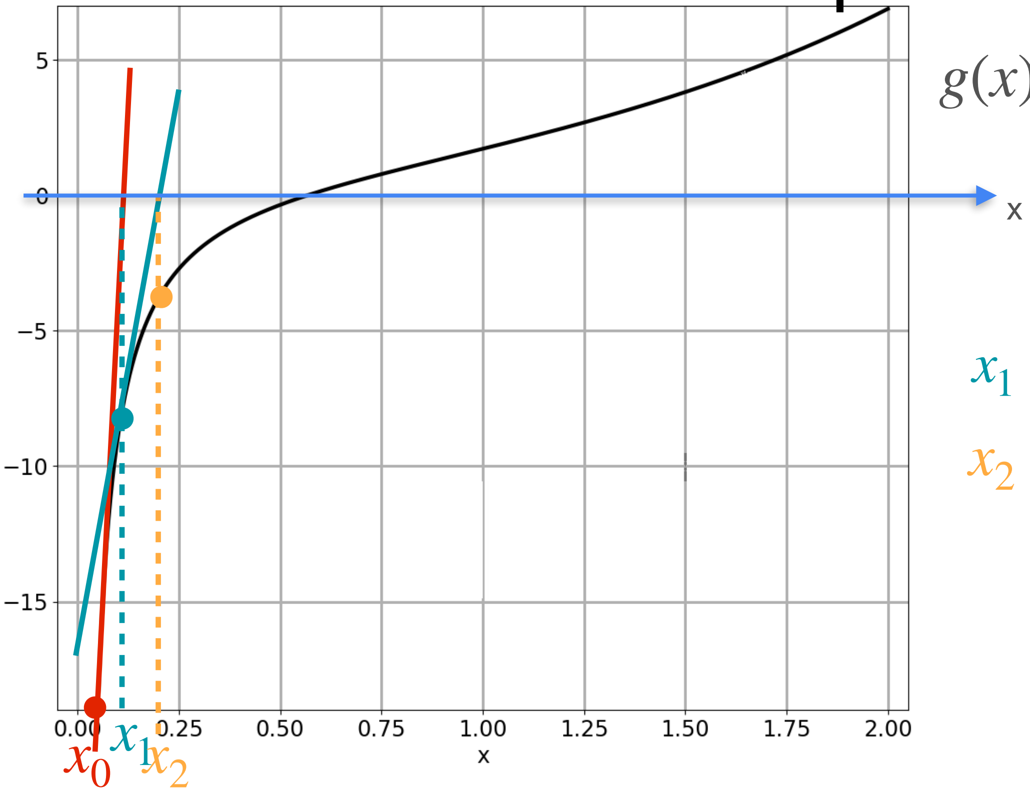
Newton's Method for Optimization



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$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_1 = 0.097$

$$x_2 = x_1 - \frac{g'(x_1)}{(g'(x_1))'}$$

$$= 0.097 - \frac{\left(e^{0.097} - \frac{1}{0.097}\right)}{\left(e^{0.097} + \frac{1}{0.097^2}\right)} = 0.183$$

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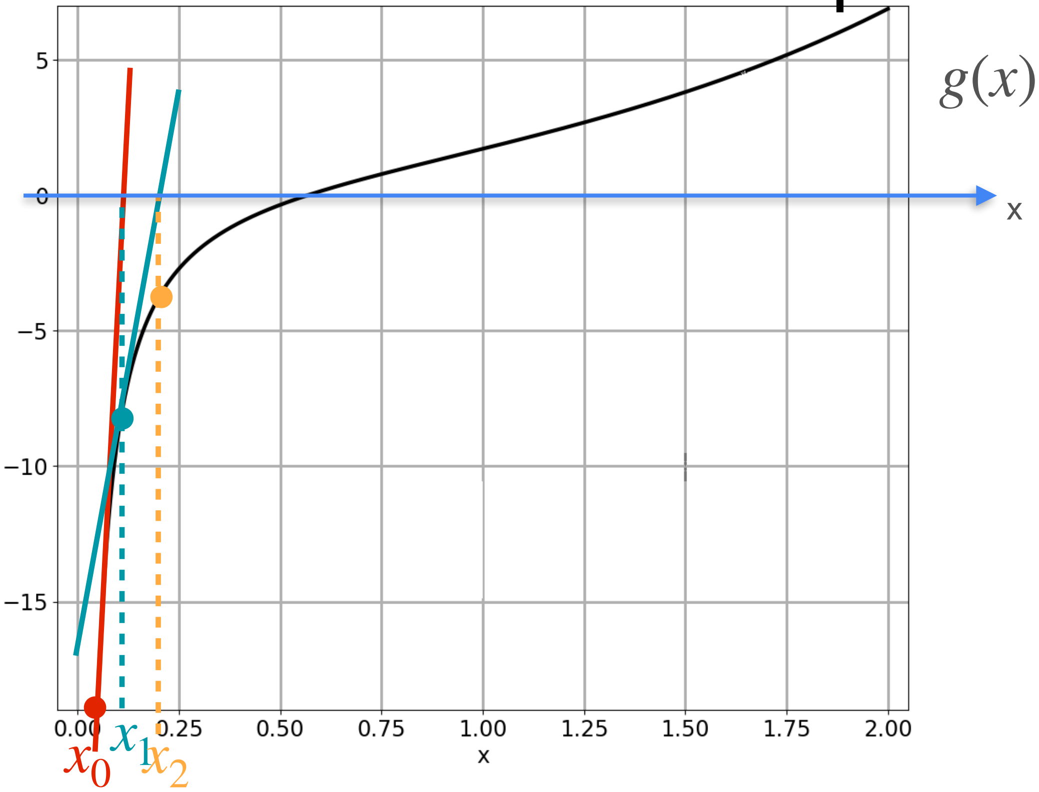
Newton's Method for Optimization

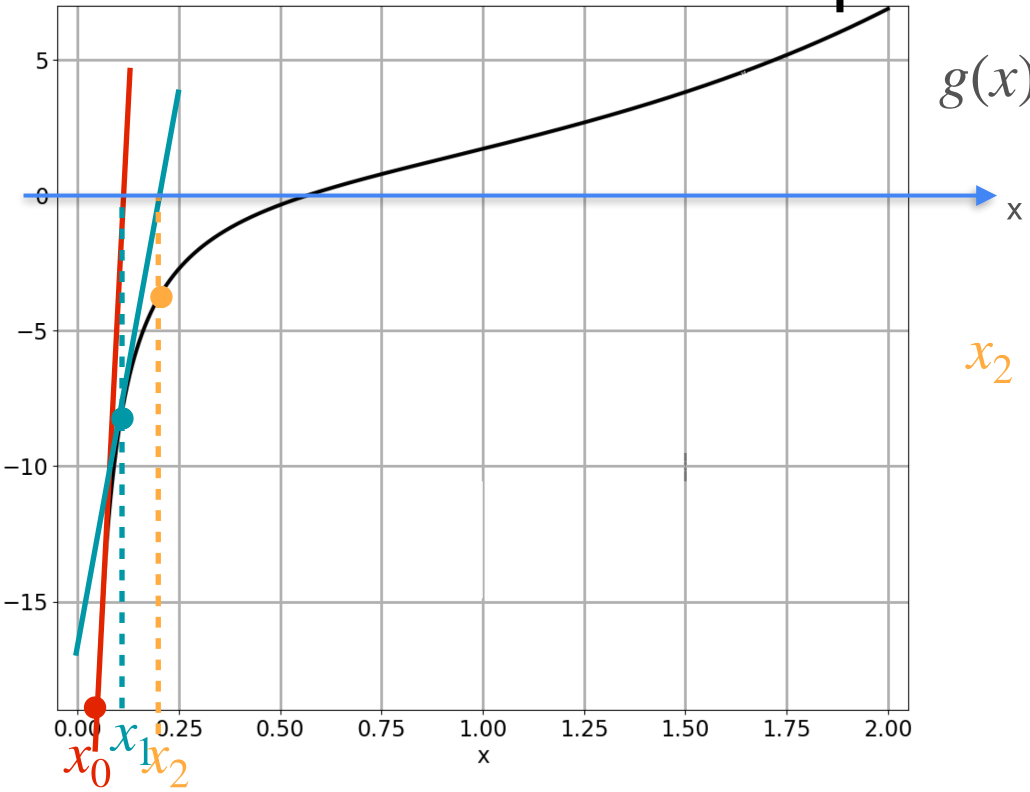


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



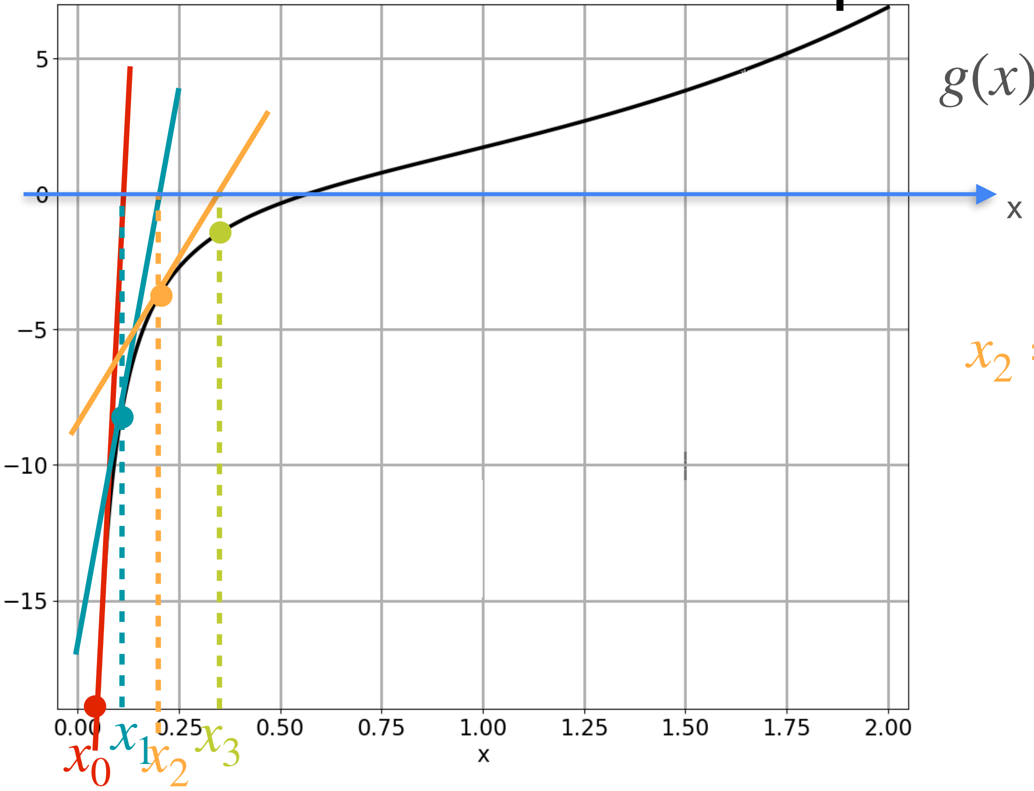
$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

Newton's Method for Optimization



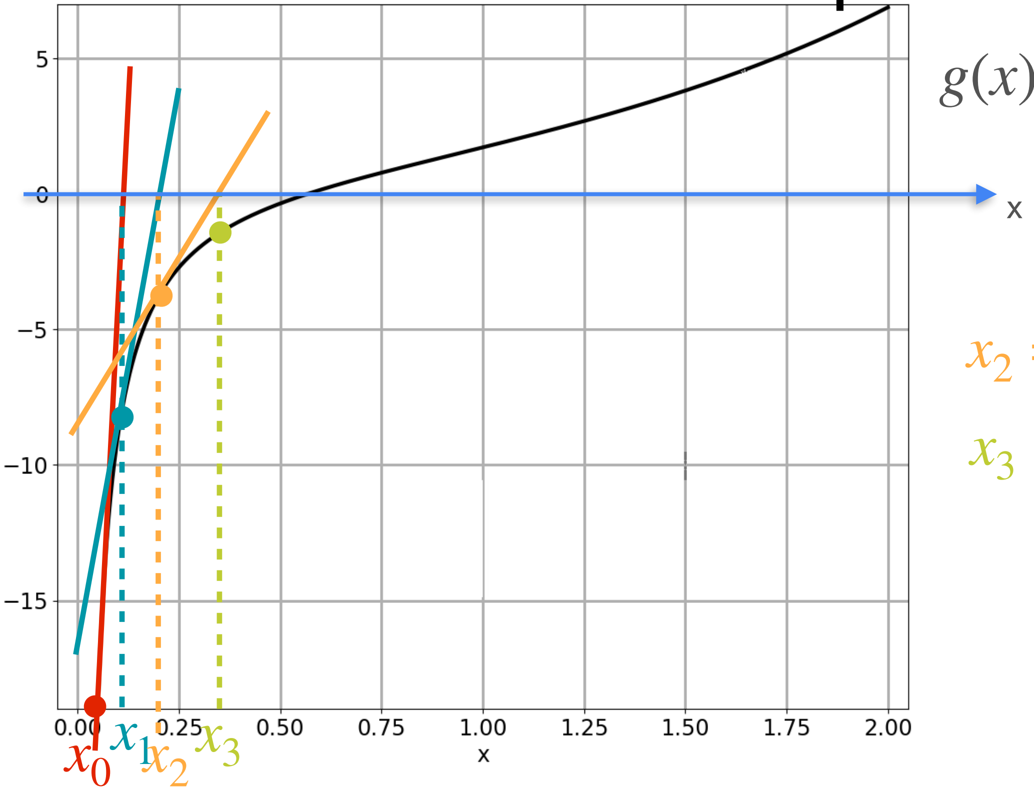
$g(x) = e^x - \log(x)$ $\quad g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)}$$

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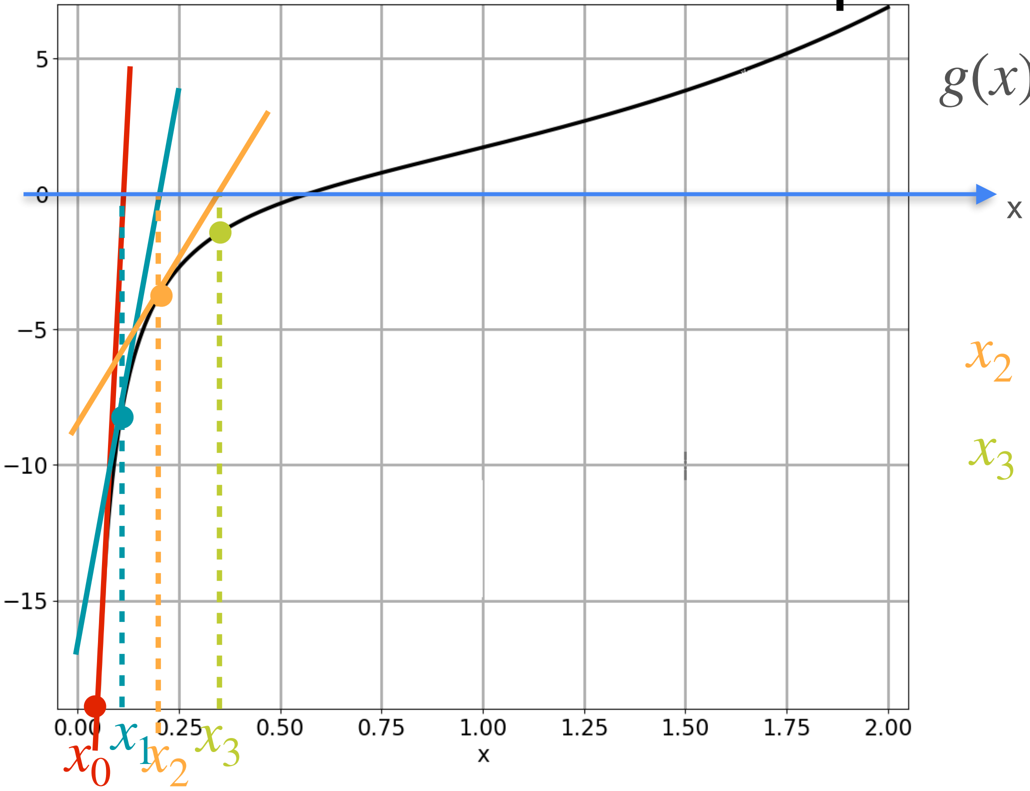
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

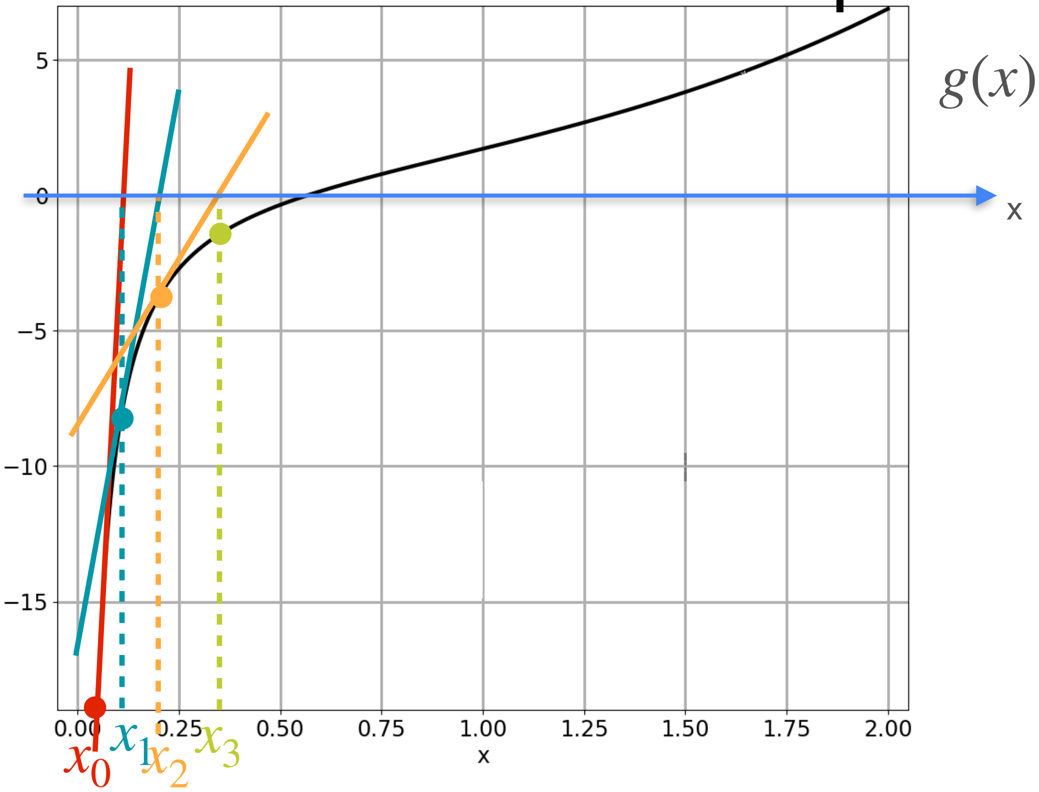
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)} = 0.320$$

Newton's Method for Optimization

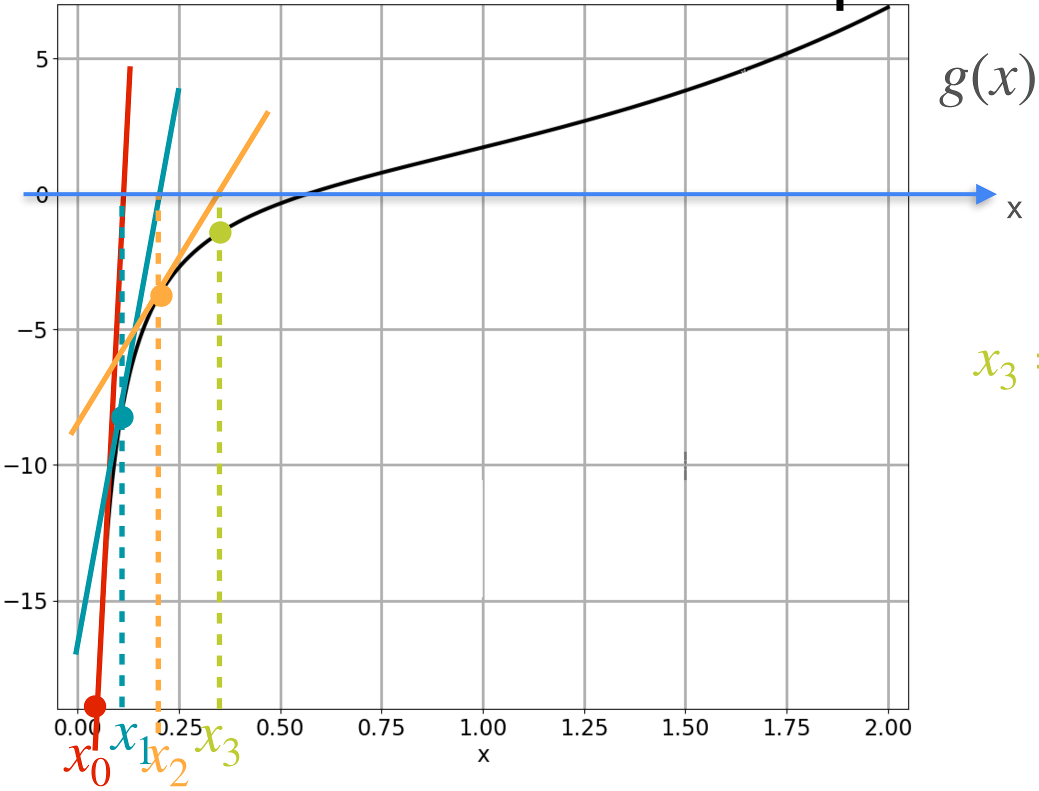


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton’s Method for Optimization



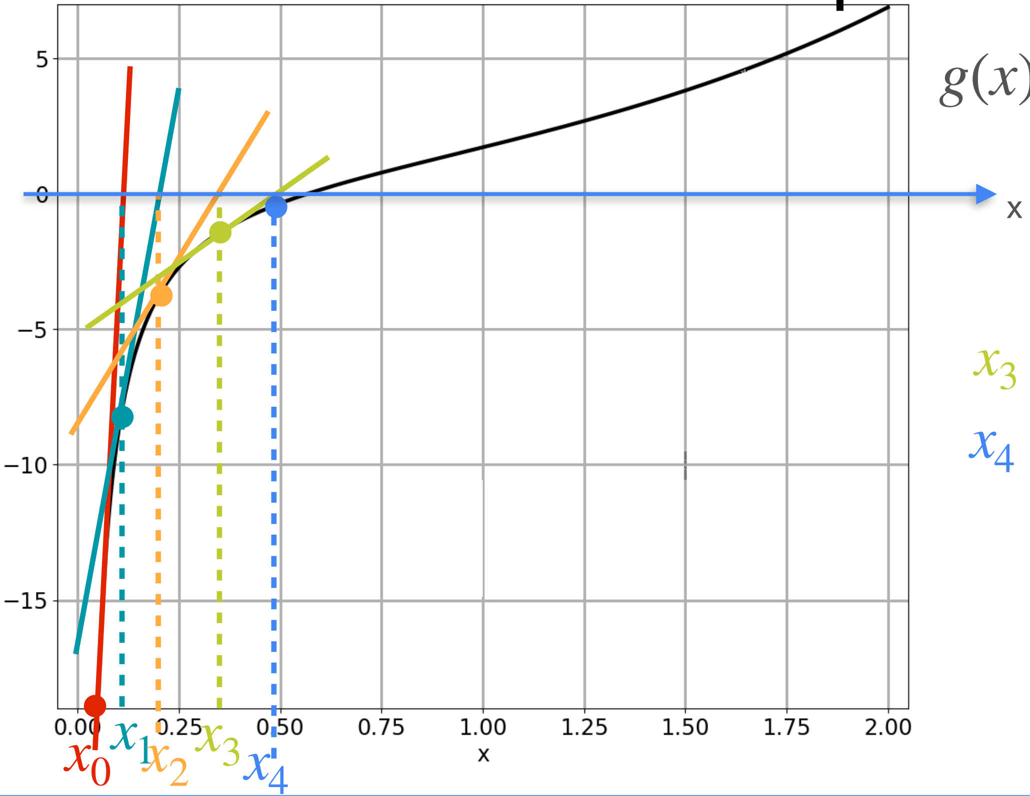
$g(x) = e^x - \log(x)$ $\quad$ $g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_3 = 0.320$

Newton's Method for Optimization



$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

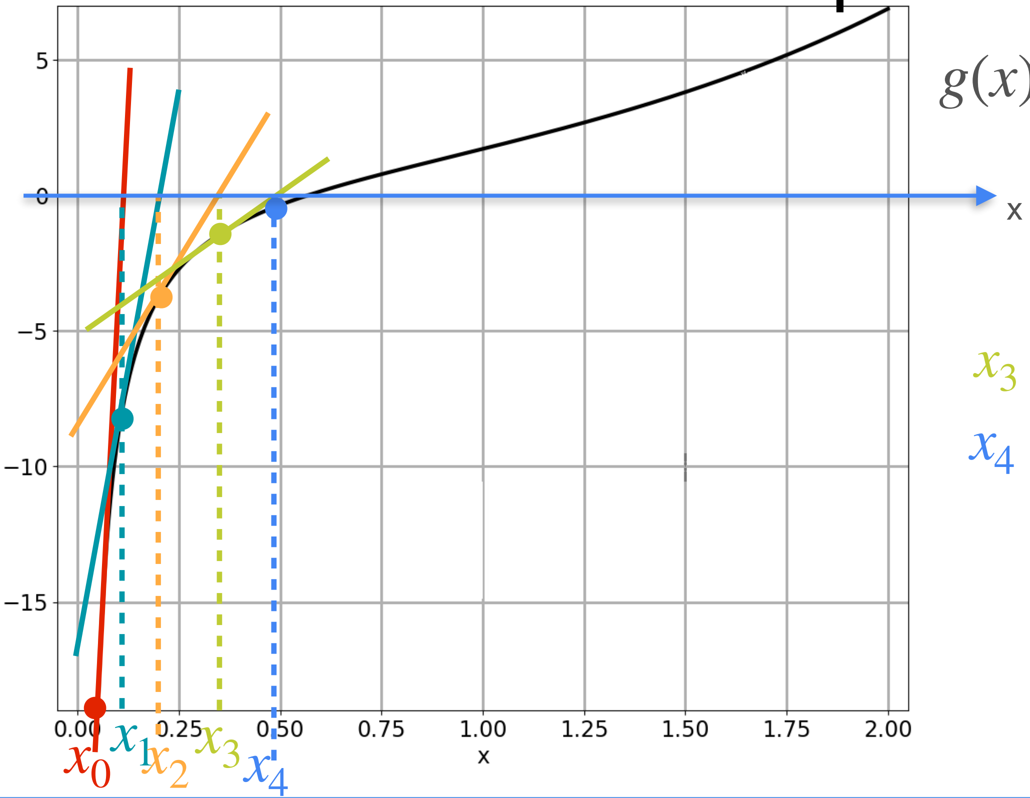
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_3 = 0.320$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)}$$

Newton's Method for Optimization



$g(x) = e^x - \log(x)$ $\quad$ $g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

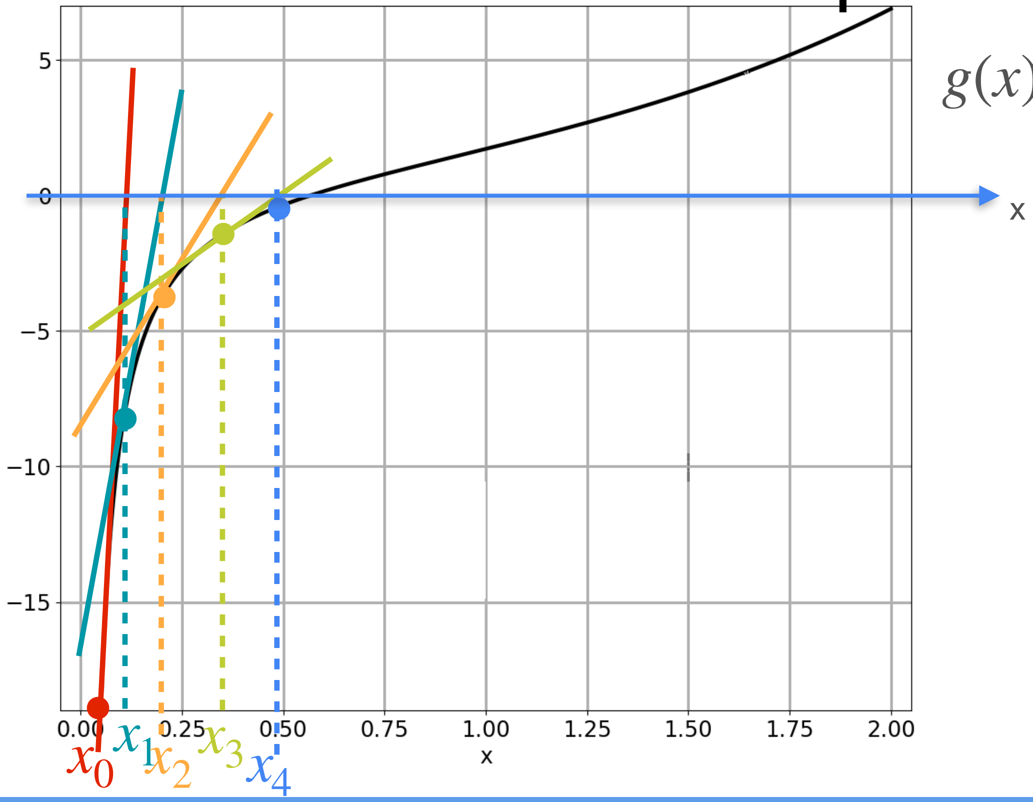
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_3 = 0.320$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)} = 0.477$$

Newton's Method for Optimization

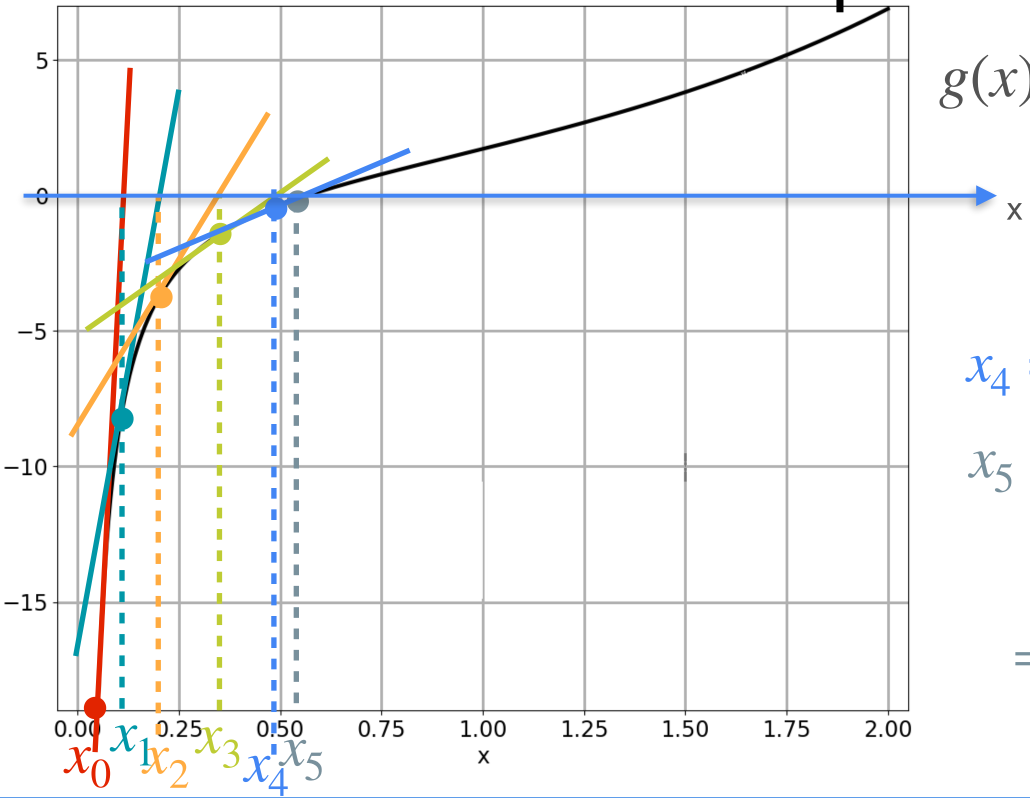


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



$g(x) = e^x - \log(x)$ $\quad g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

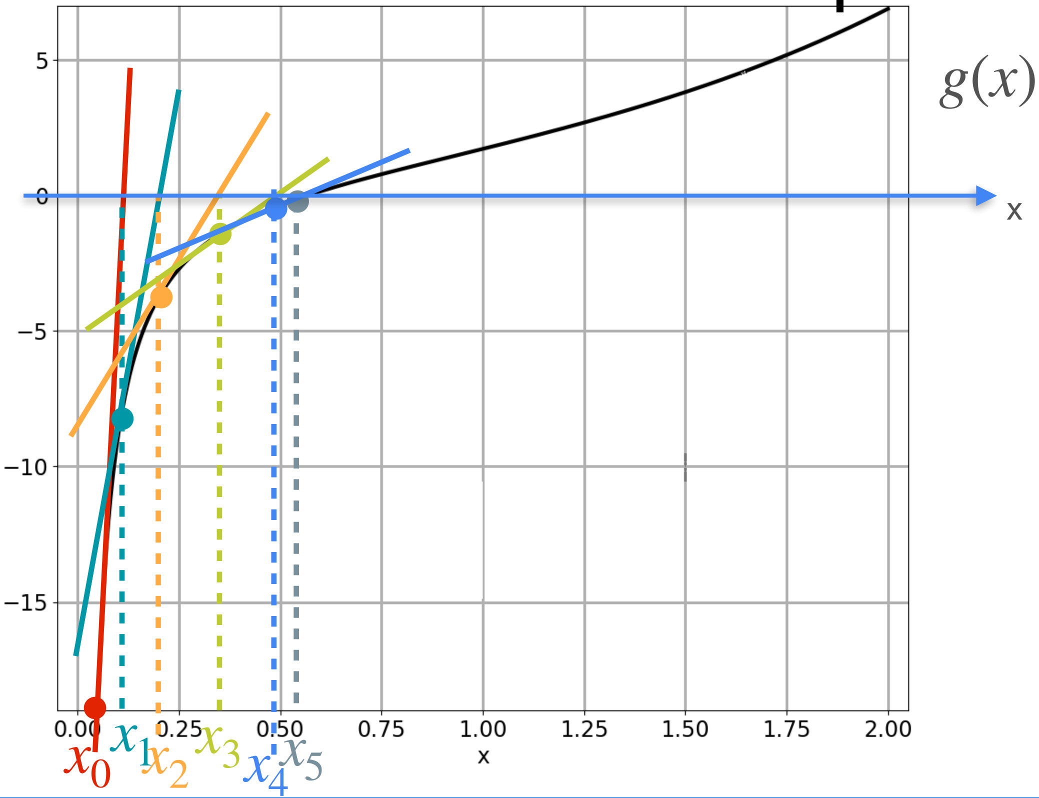
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_4 = 0.477$

$$x_5 = x_4 - \frac{g'(x_4)}{(g'(x_4))'}$$

$$= 0.447 - \frac{\left(e^{0.447} - \frac{1}{0.447}\right)}{\left(e^{0.447} + \frac{1}{0.447^2}\right)} = 0.558$$

Newton's Method for Optimization

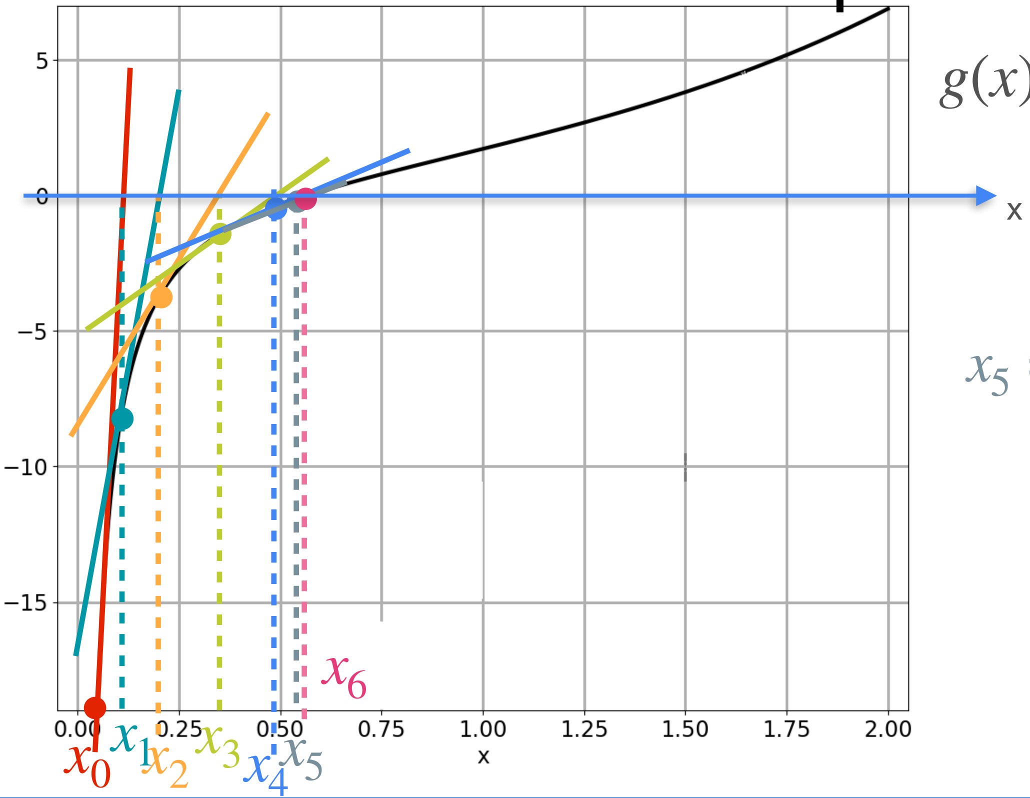


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton’s Method for Optimization



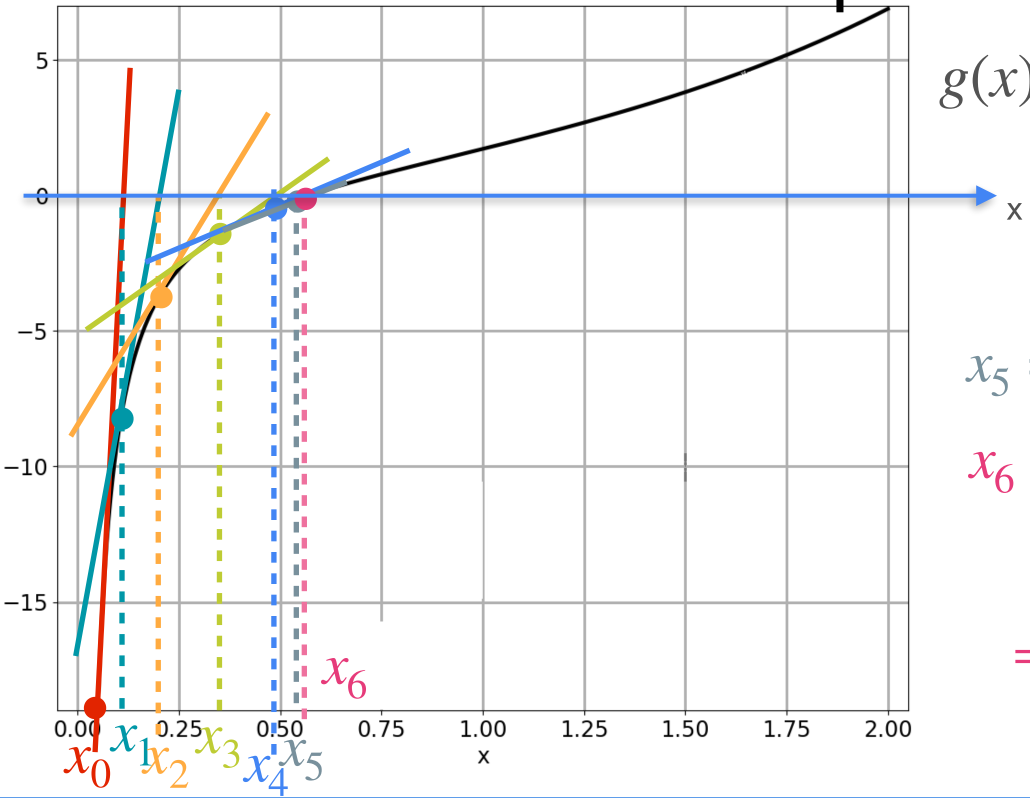
$g(x) = e^x - \log(x)$ $\quad g'(x) = e^x - 1/x$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_5 = 0.558$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

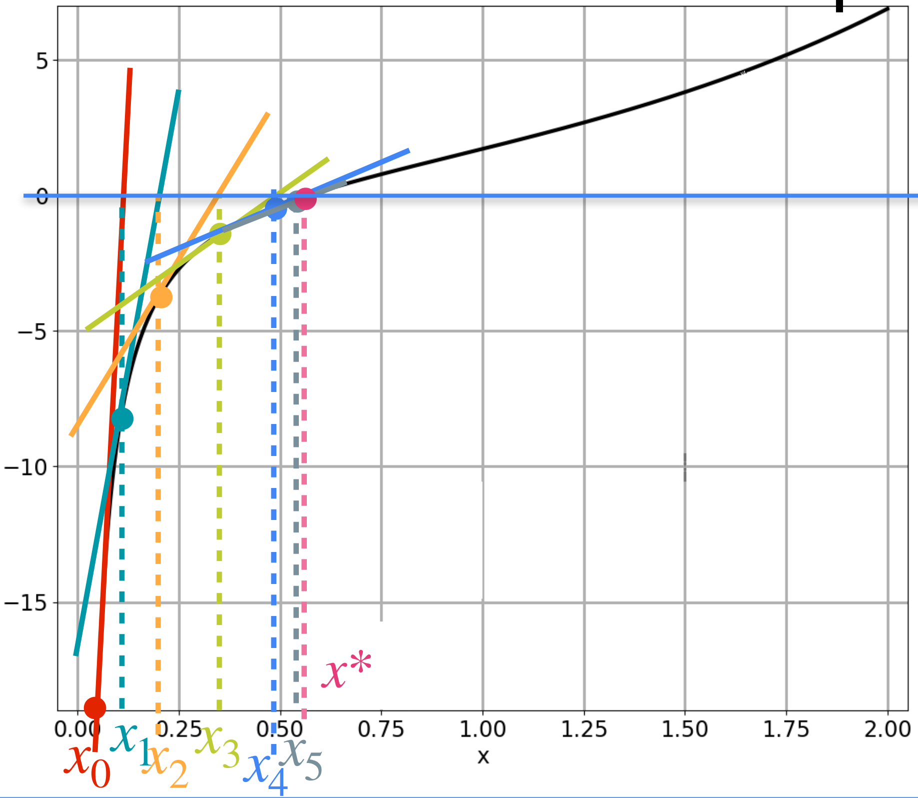
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x_6 = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x^* = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)} = 0.567$$

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Optimization in Neural Networks and Newton's Method

The second derivative

Second Derivative

Second Derivative

Newton's method:

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$$

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$$??

Second Derivative

Newton's method: $x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$??

Second derivative

Second Derivative

Newton's method: $x_{k+1} = x_k - \dfrac{g'(x_k)}{\boxed{(g'(x_k))'}}$??

Second derivative

Leibniz notation: $$\frac{d^2 f(x)}{dx^2} = \frac{d}{dx}\left(\frac{df(x)}{dx}\right)$$

Second Derivative

Newton's method: $x_{k+1} = x_k - \dfrac{g'(x_k)}{\boxed{(g'(x_k))'}}$??

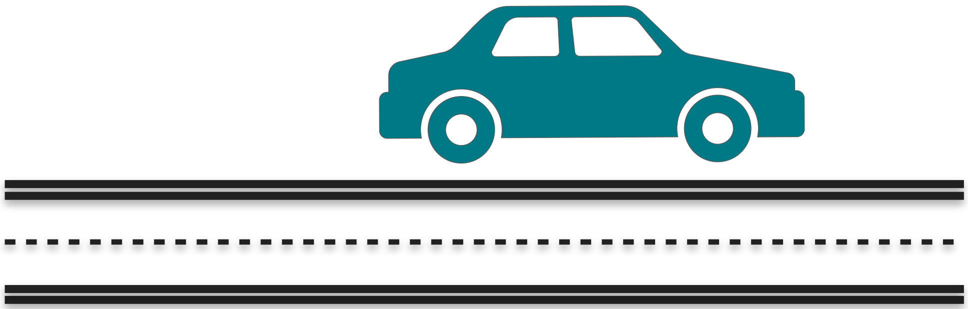
Second derivative

Leibniz notation: $\dfrac{d^2 f(x)}{dx^2} = \dfrac{d}{dx}\left(\dfrac{df(x)}{dx}\right)$

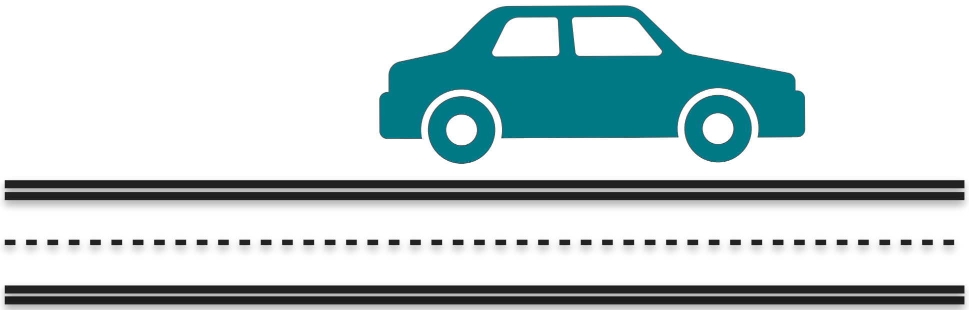
Lagrange notation: $f''(x)$

Understanding Second Derivative

Understanding Second Derivative

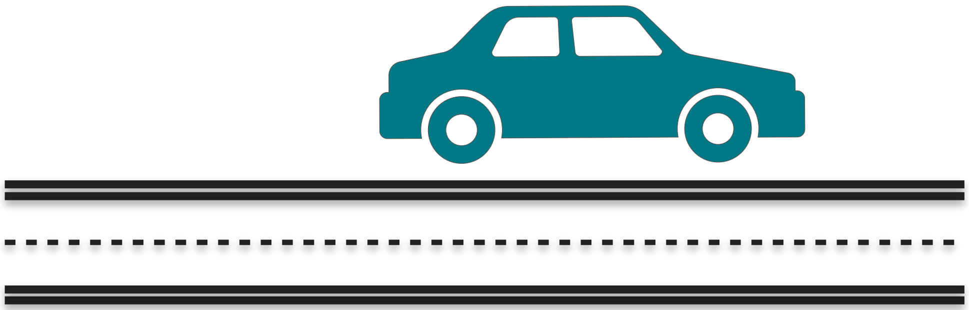


Understanding Second Derivative



x Distance

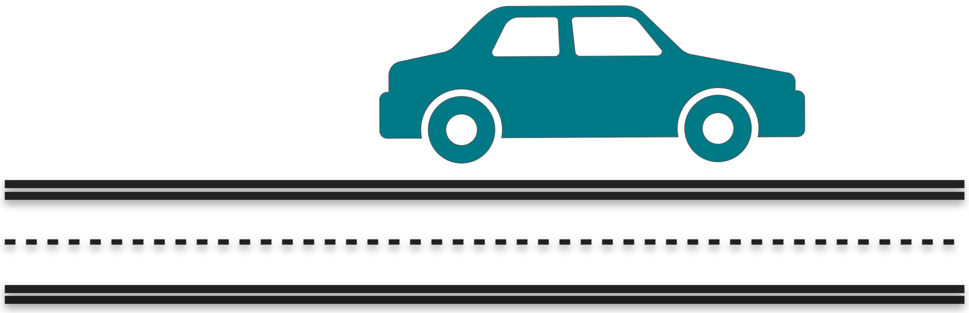
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative



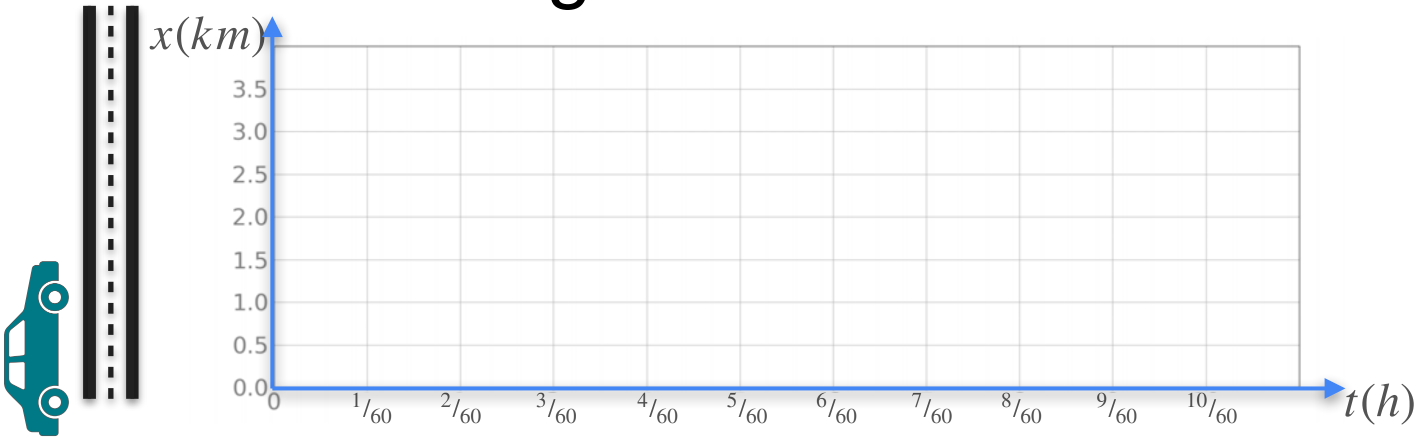
x Distance

v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$

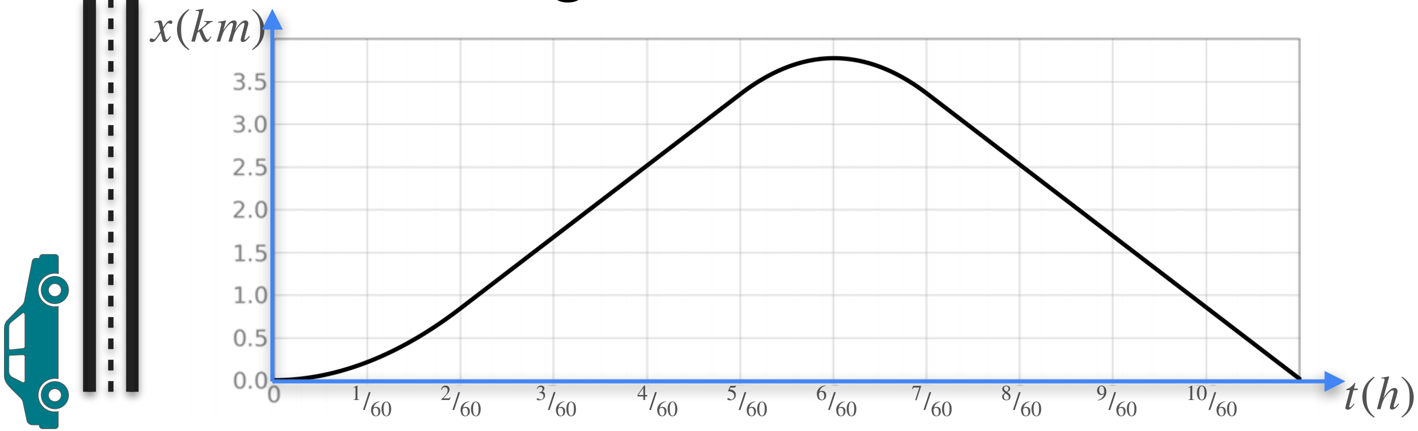
Understanding Second Derivative

Understanding Second Derivative



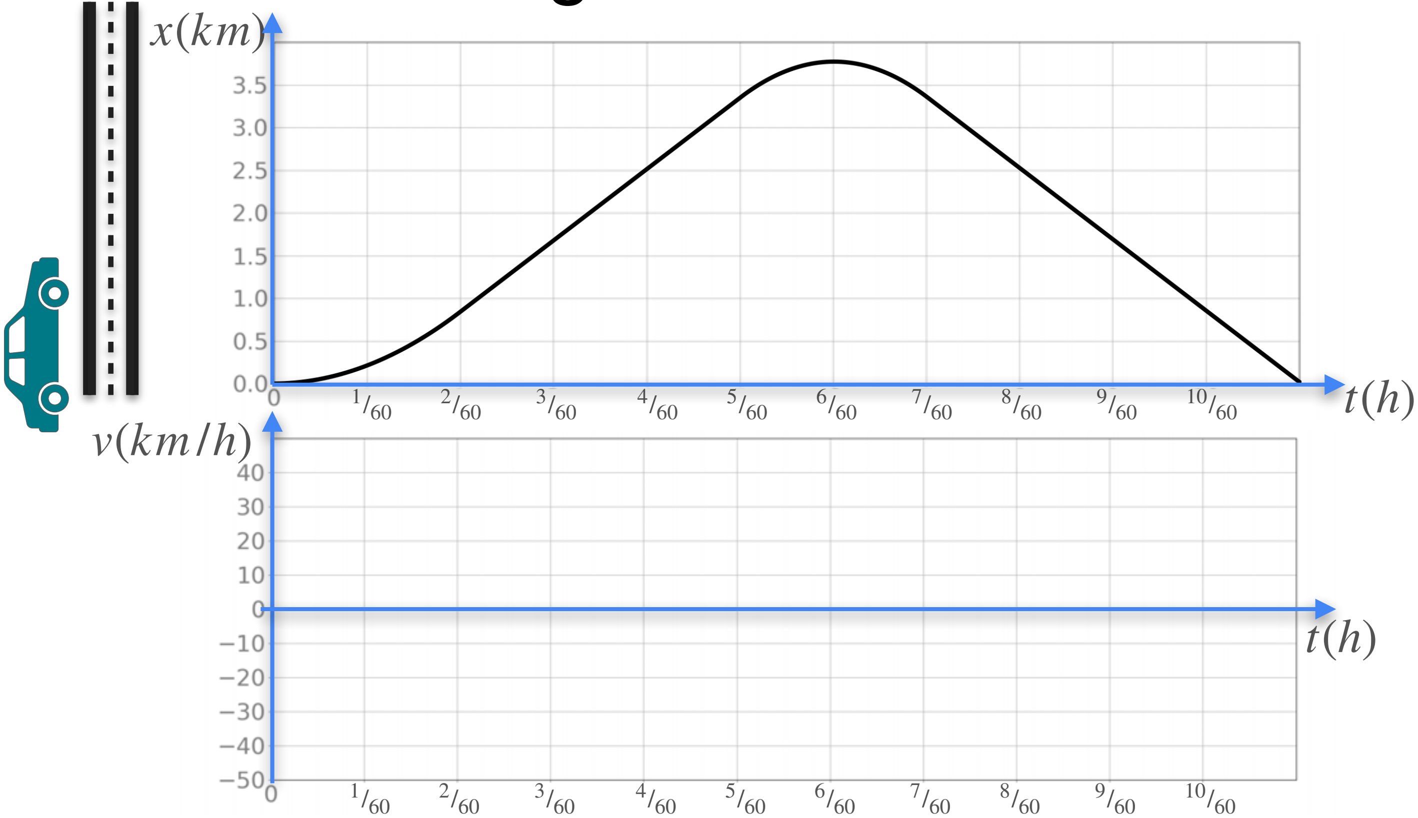
x Distance

Understanding Second Derivative



x Distance

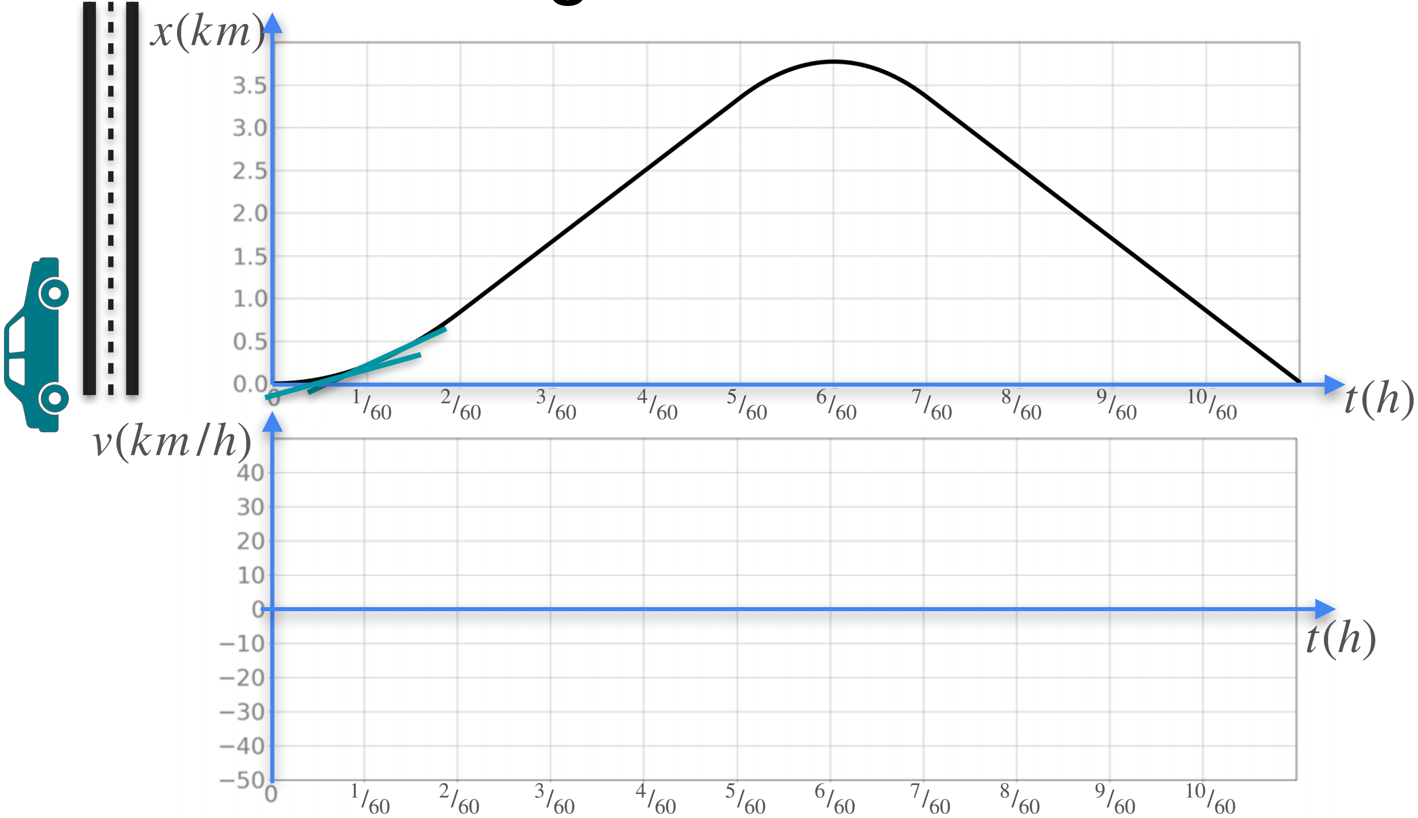
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

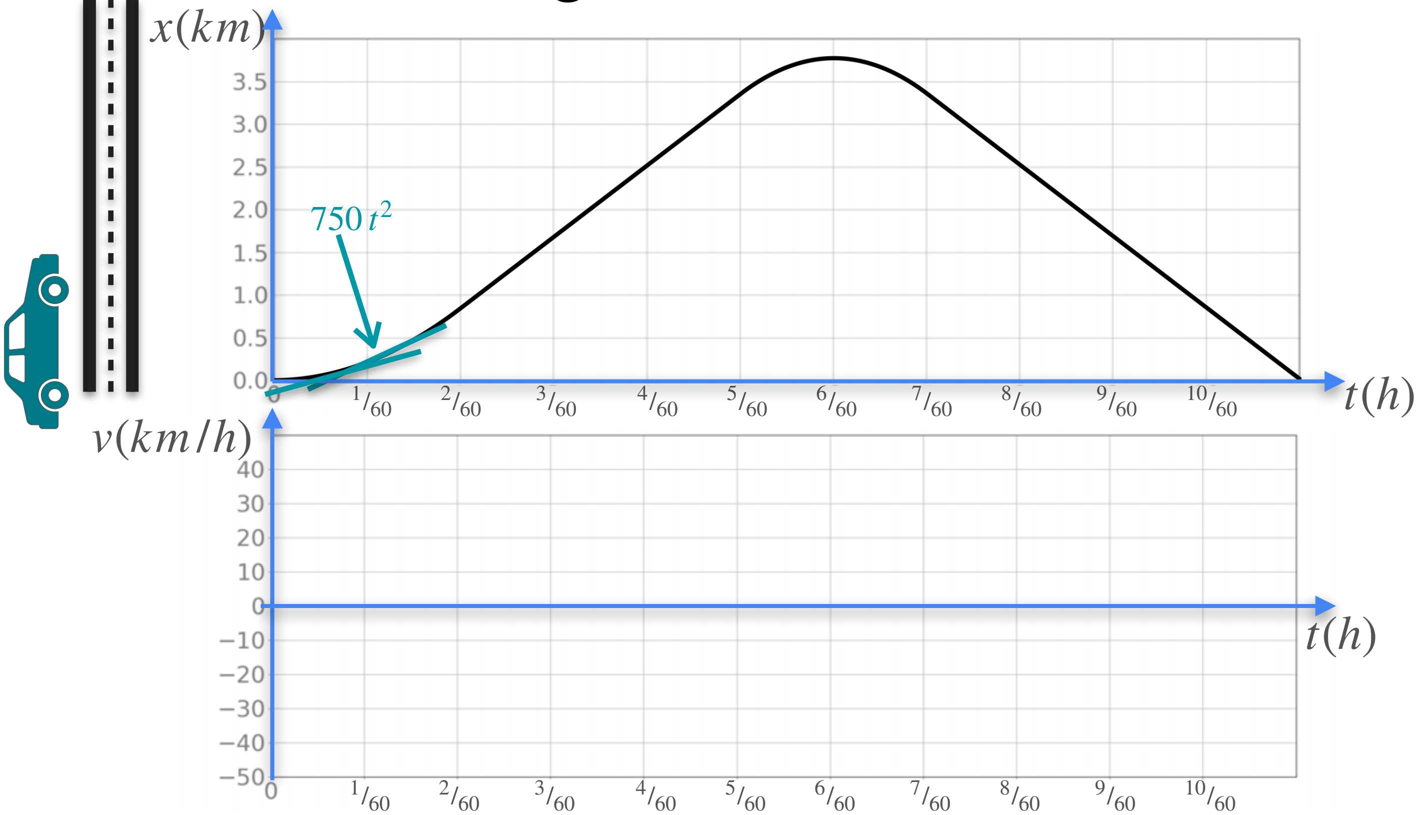
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

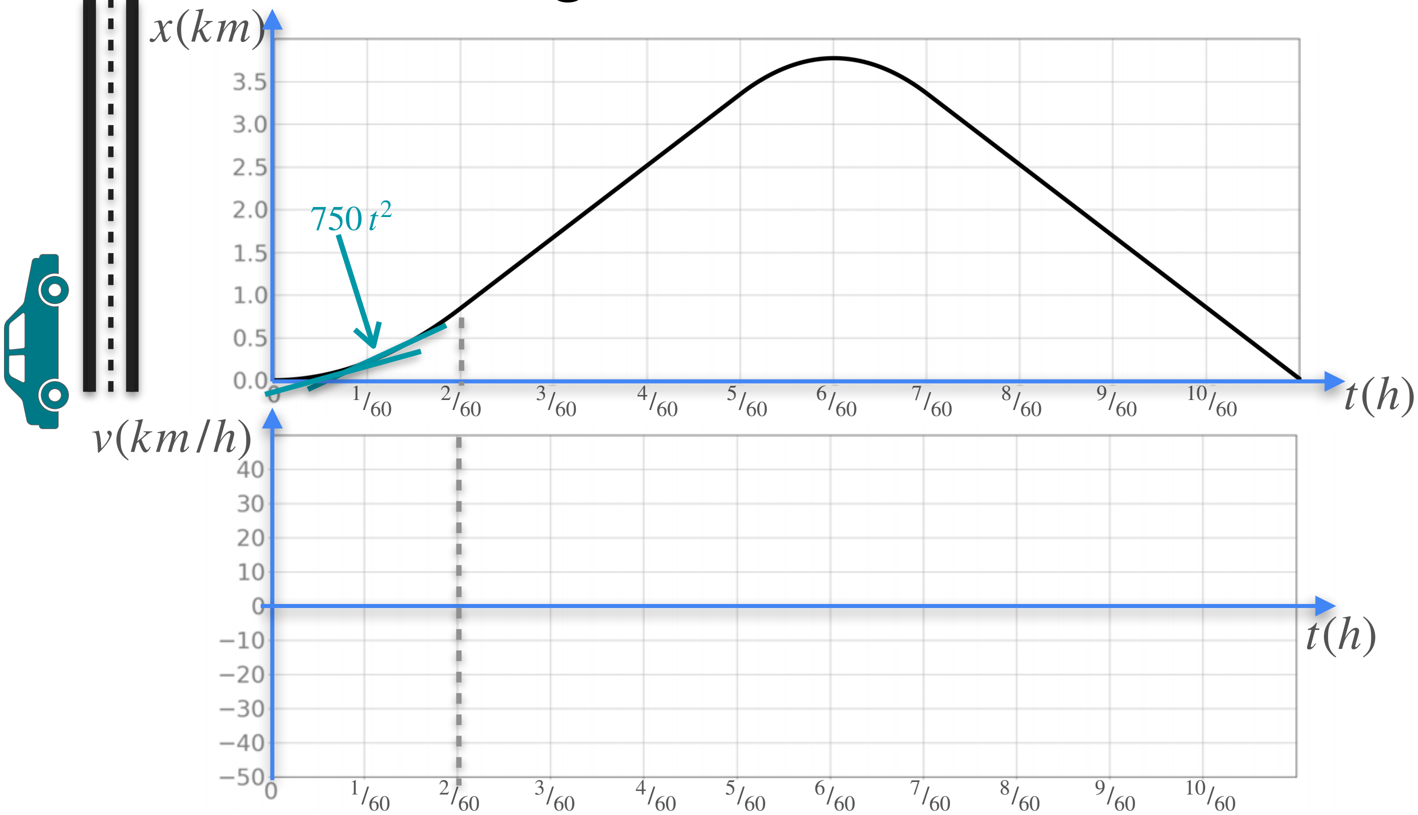
Understanding Second Derivative



x Distance

v Velocity $\dfrac{dx}{dt}$

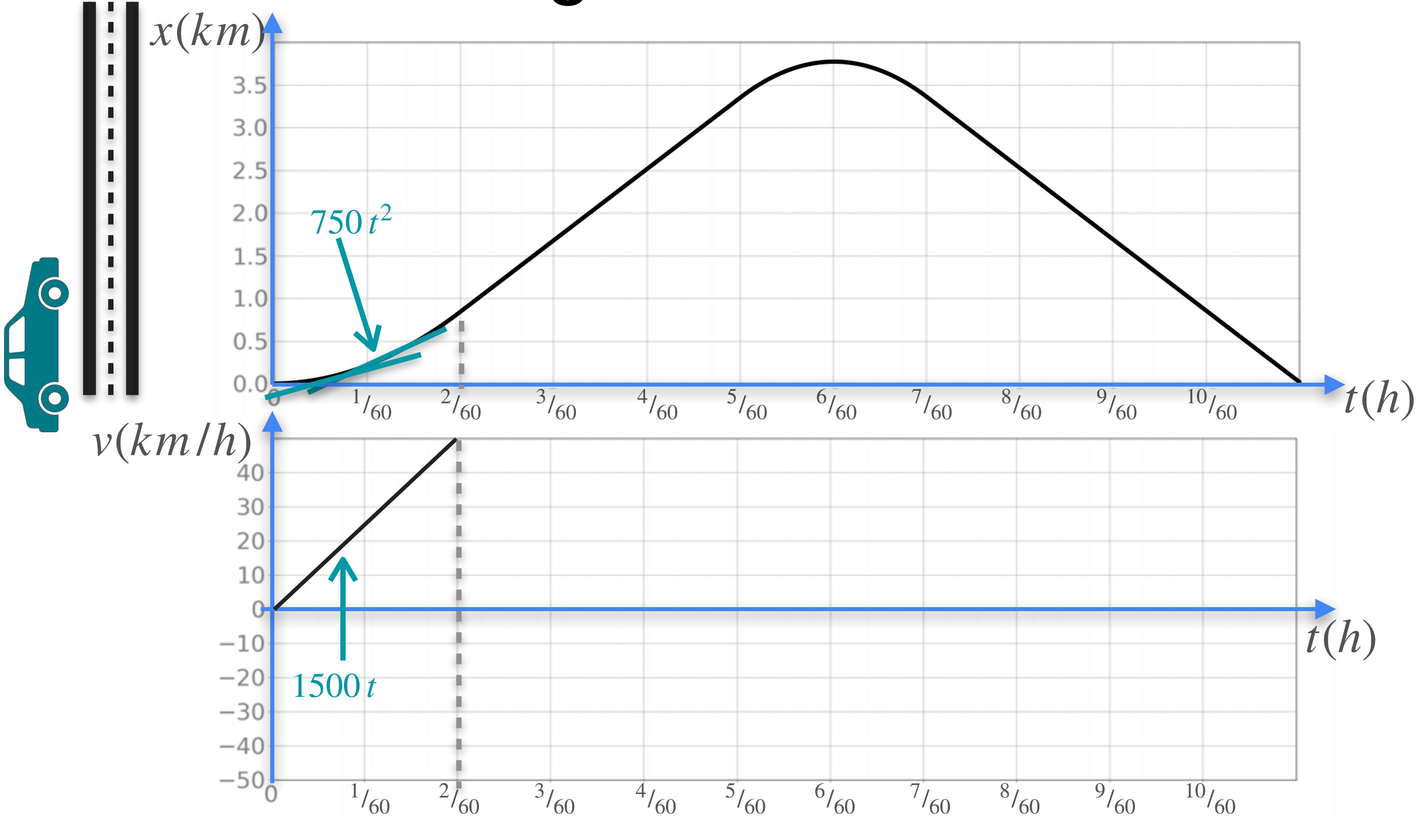
Understanding Second Derivative



x Distance

v Velocity $\dfrac{dx}{dt}$

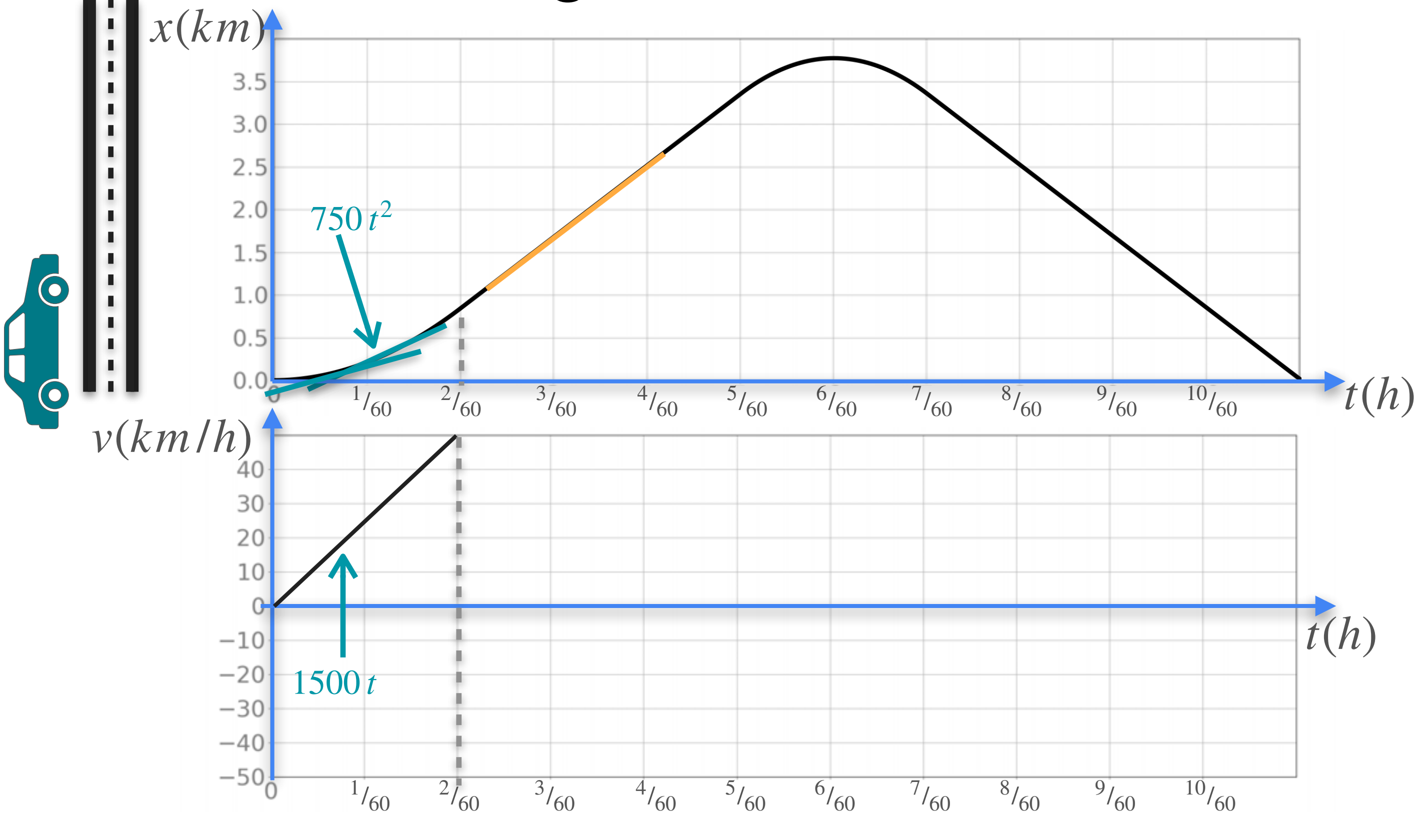
Understanding Second Derivative



x Distance

v Velocity $\dfrac{dx}{dt}$

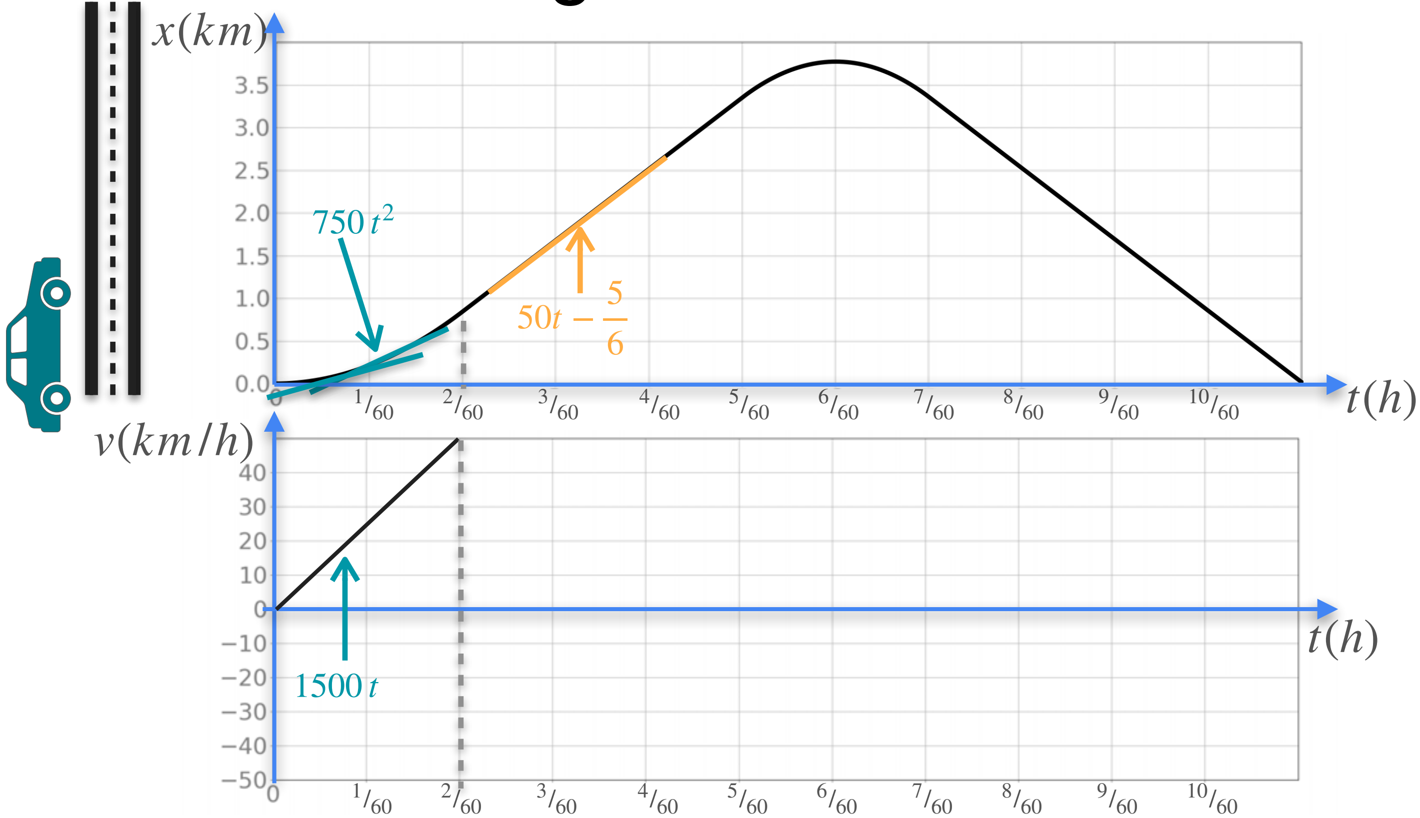
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

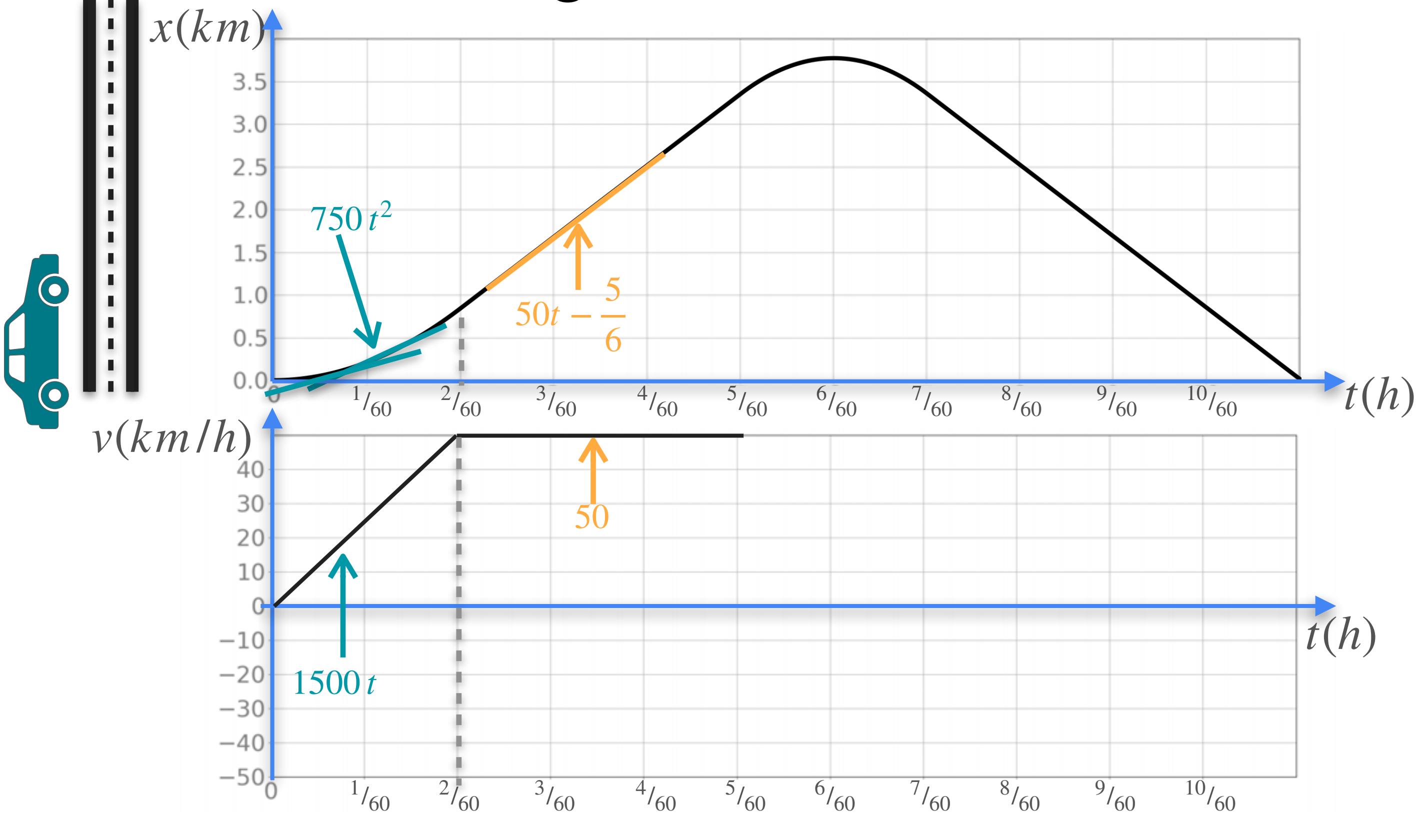
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

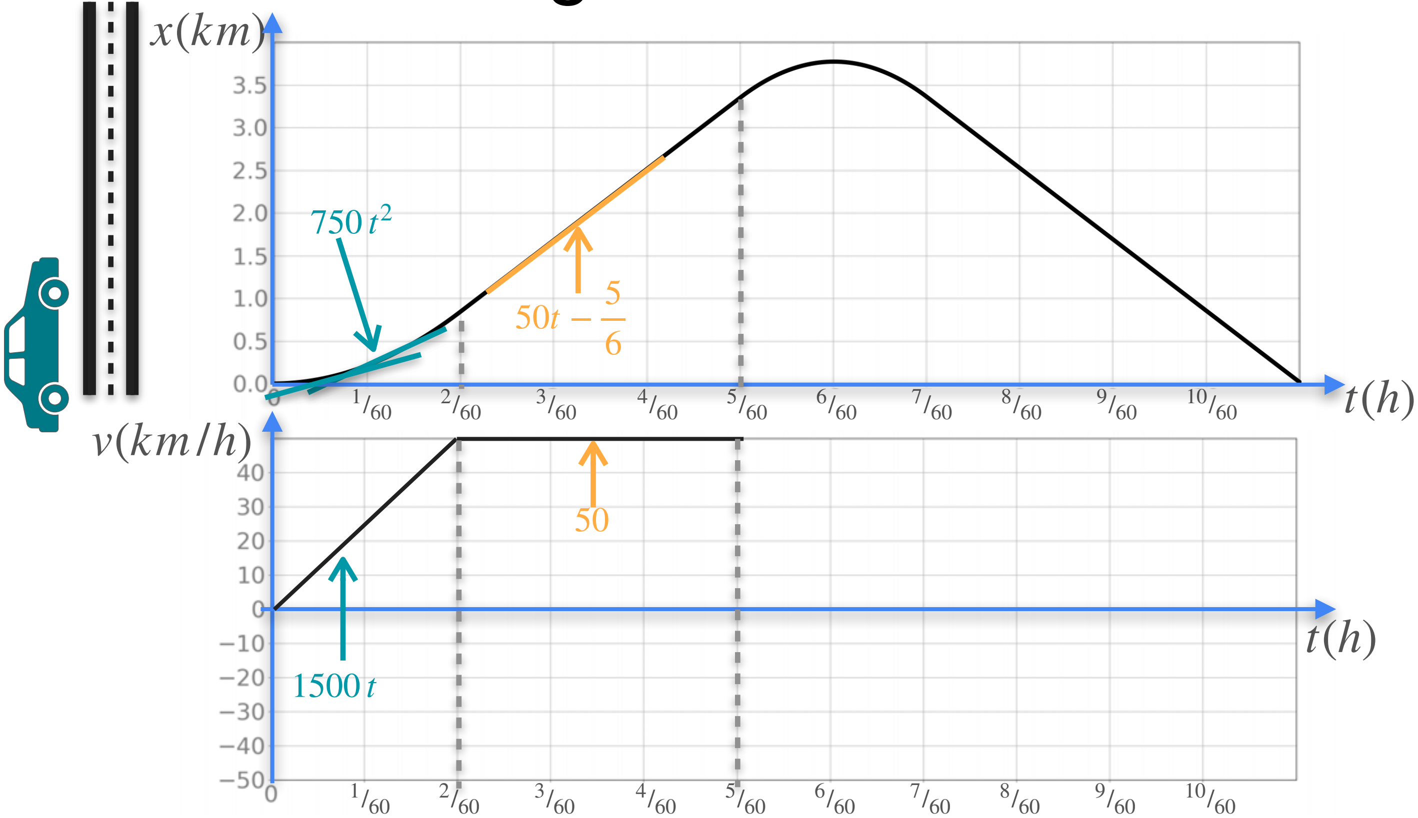
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

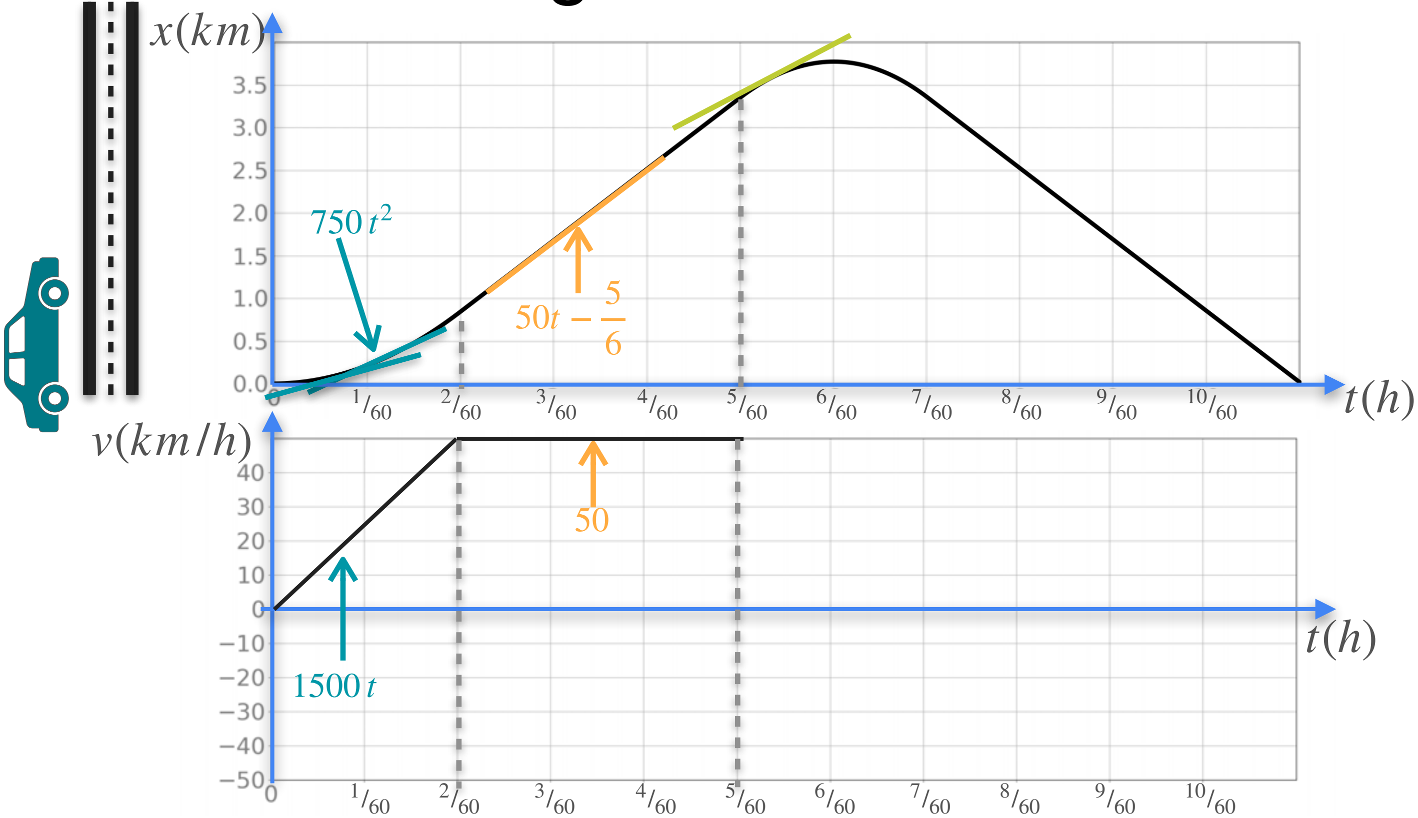
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

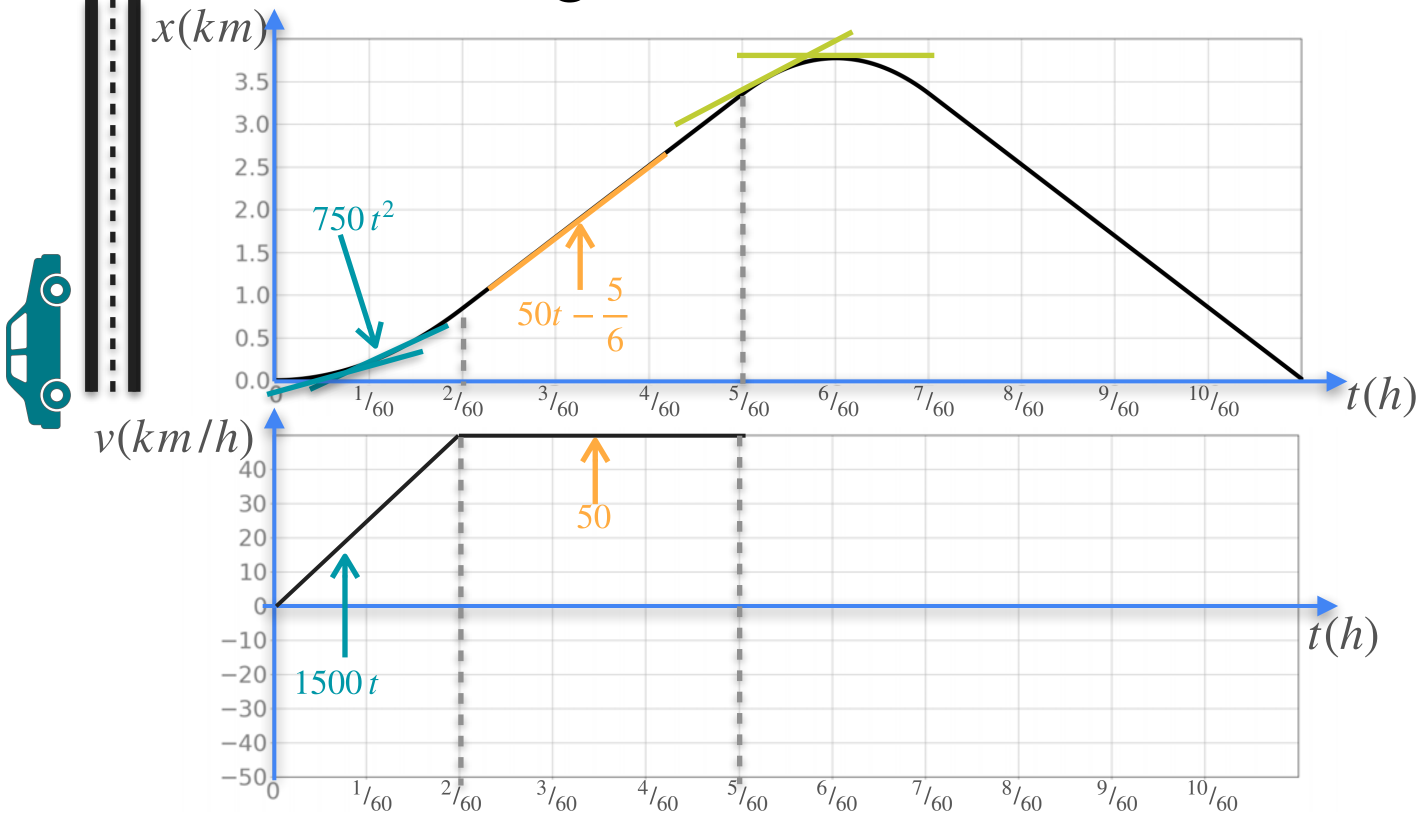
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

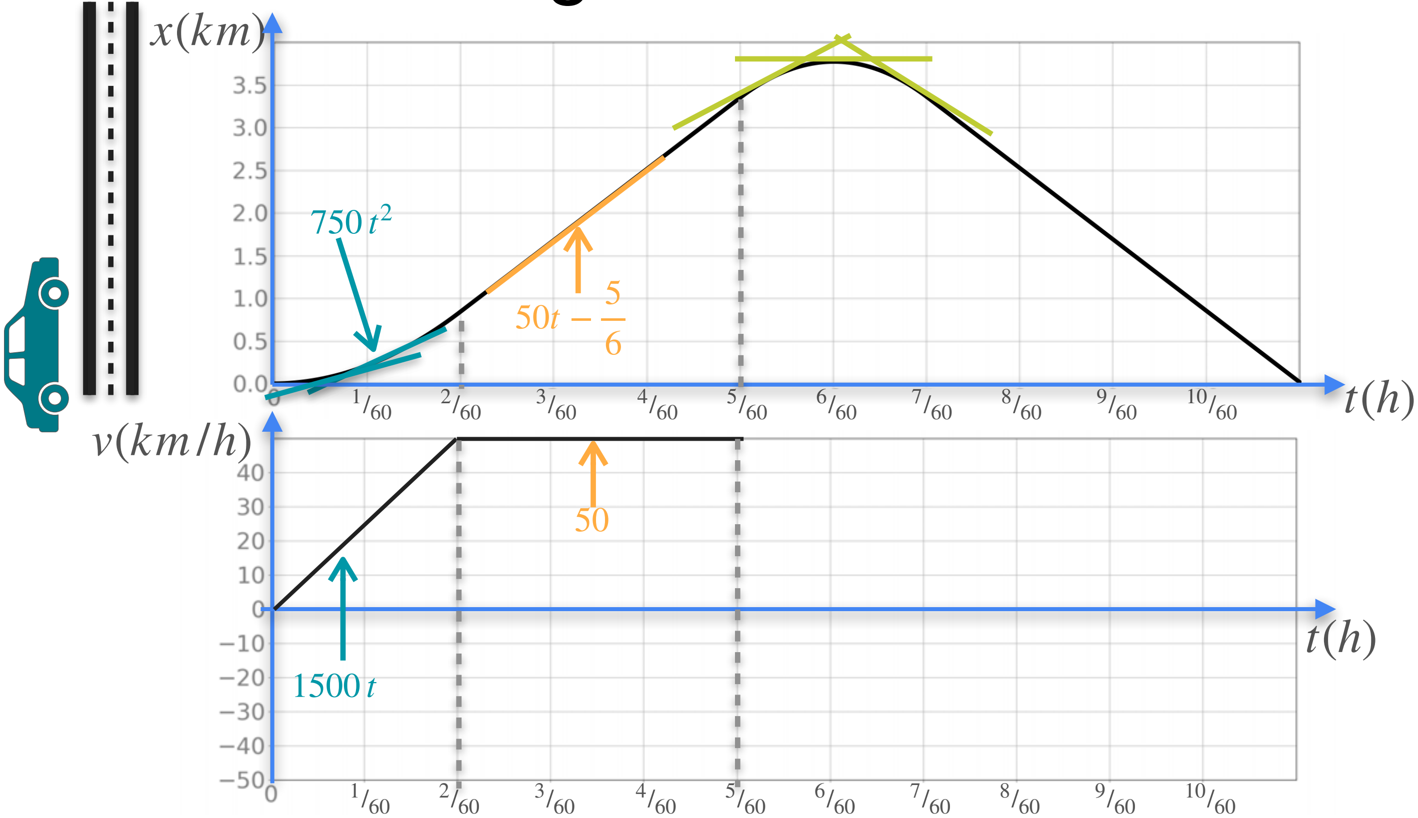
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

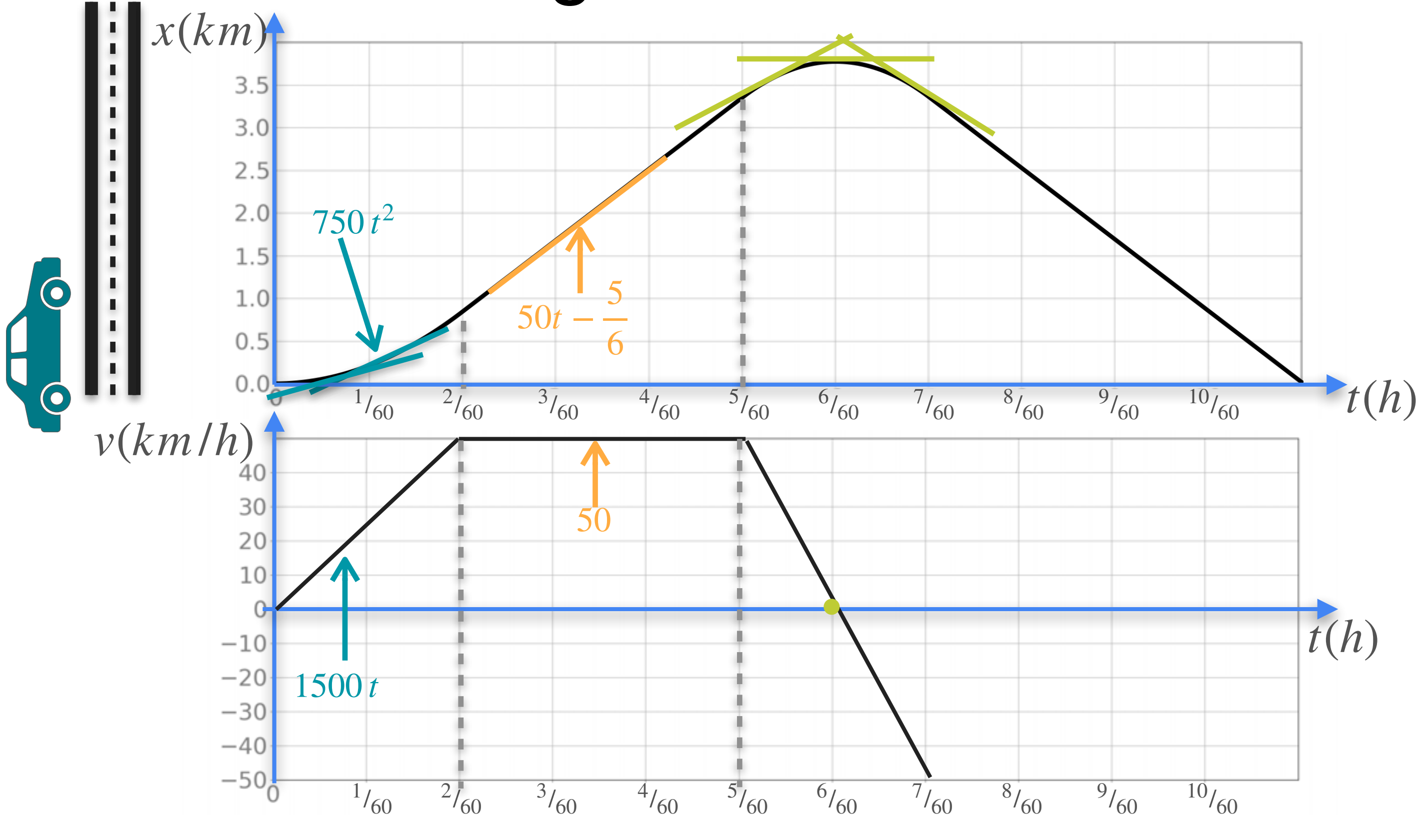
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

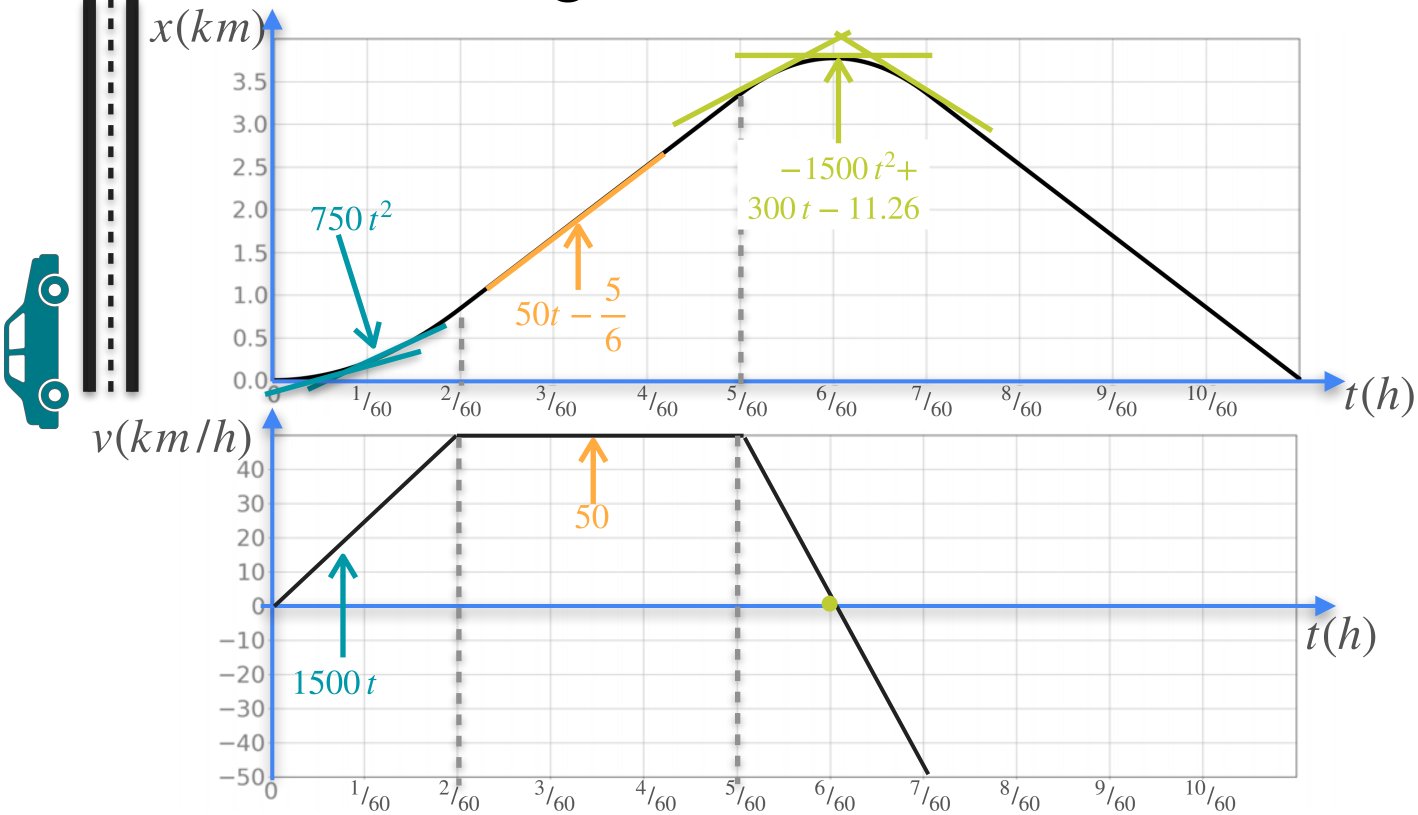
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

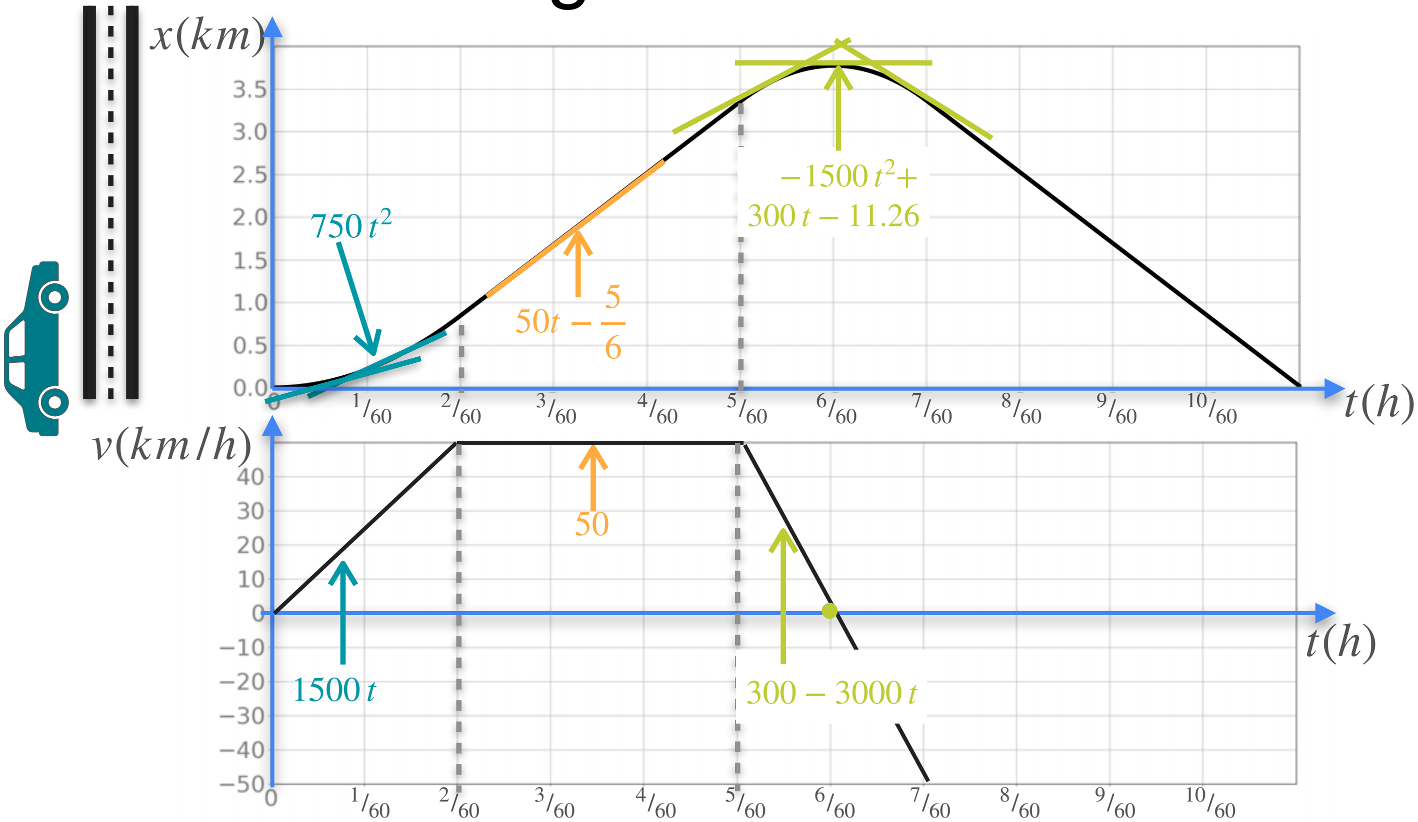
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

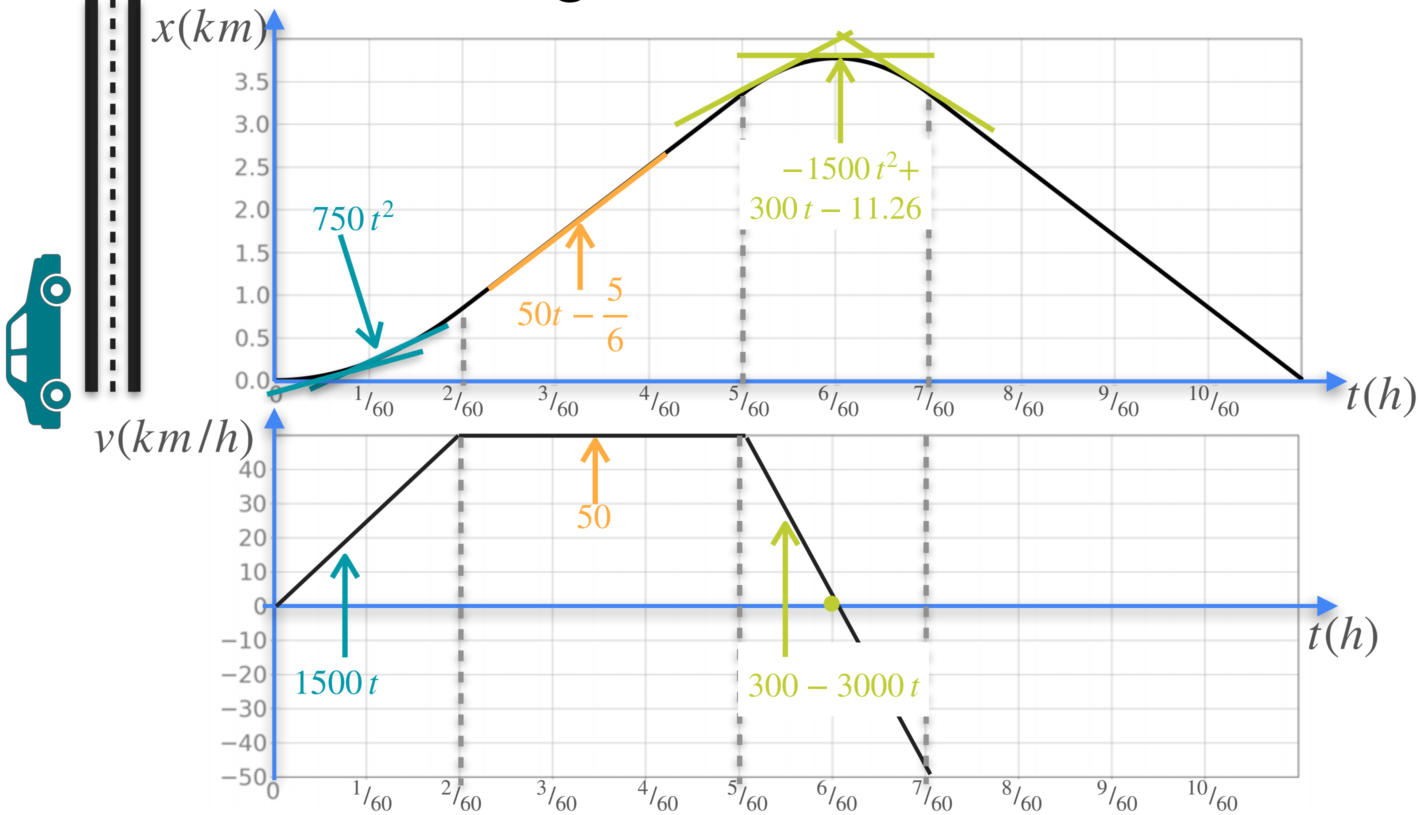
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

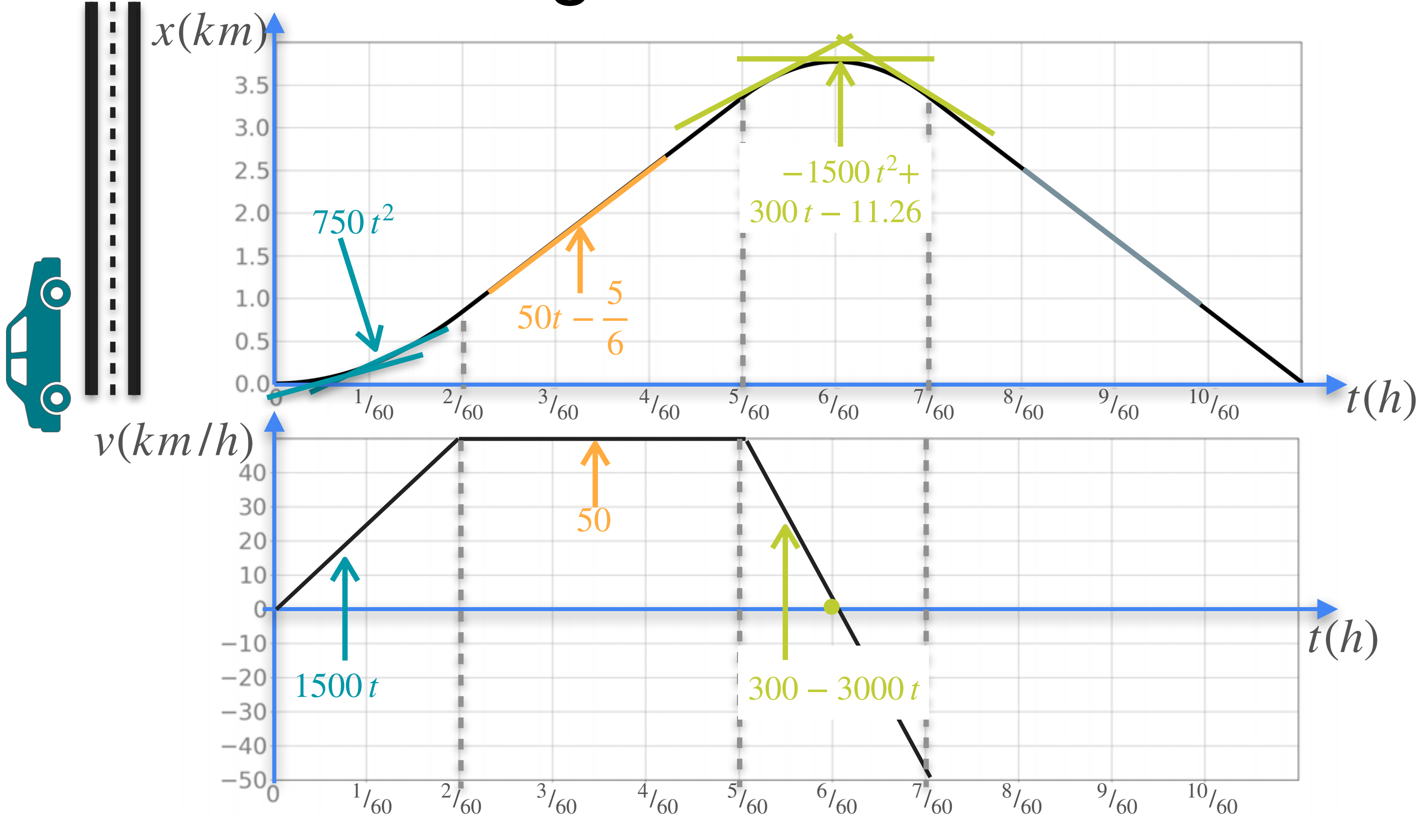
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

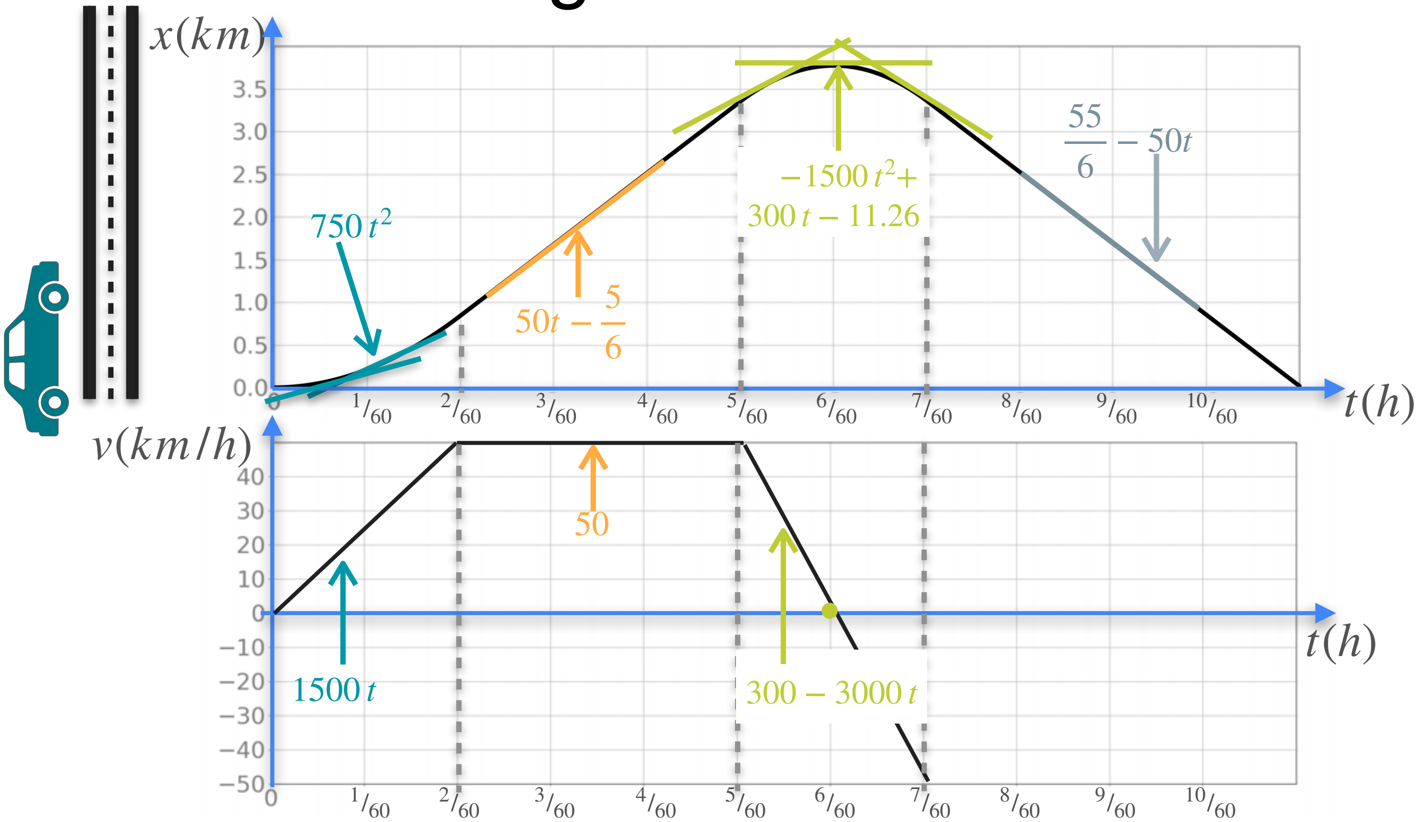
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

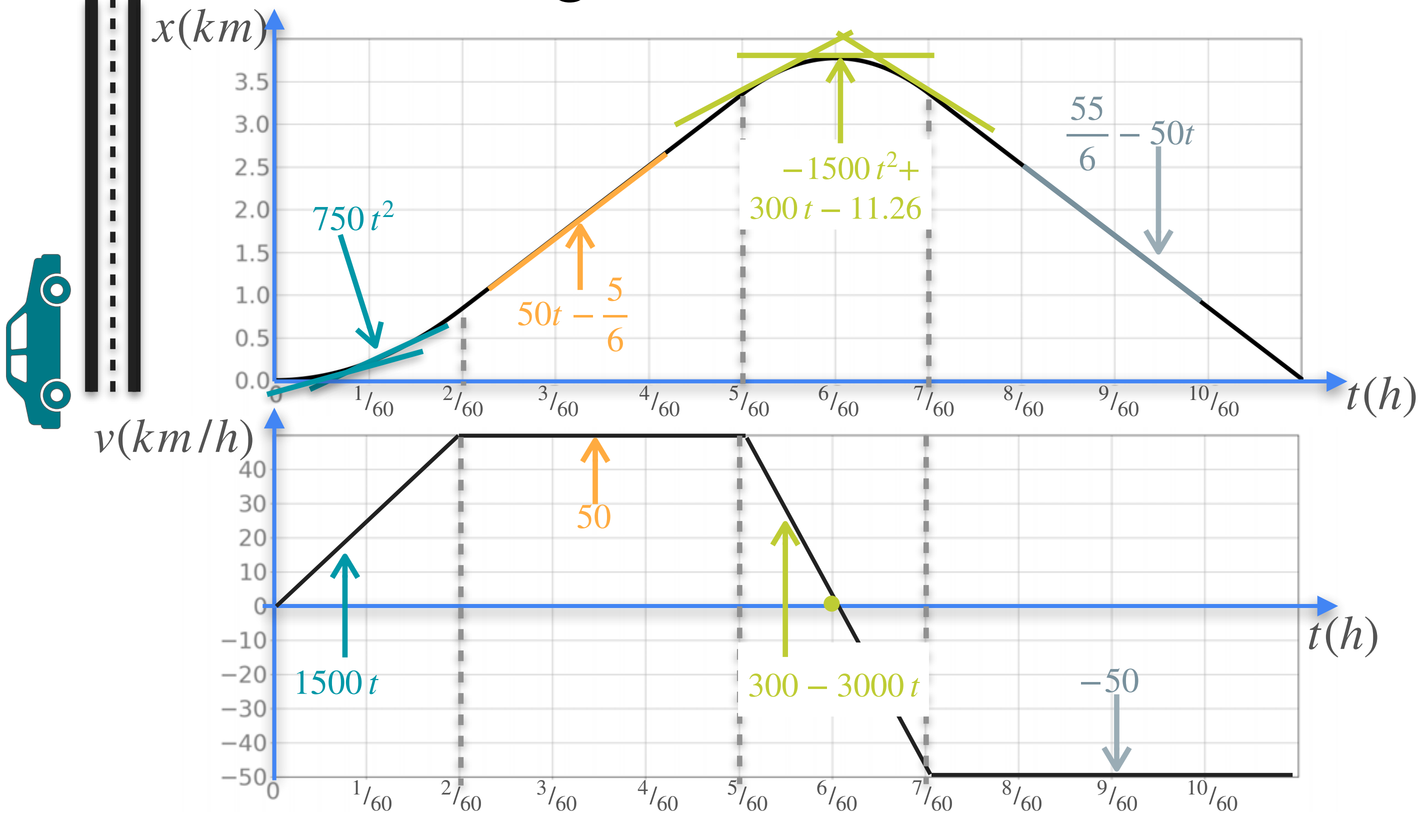
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

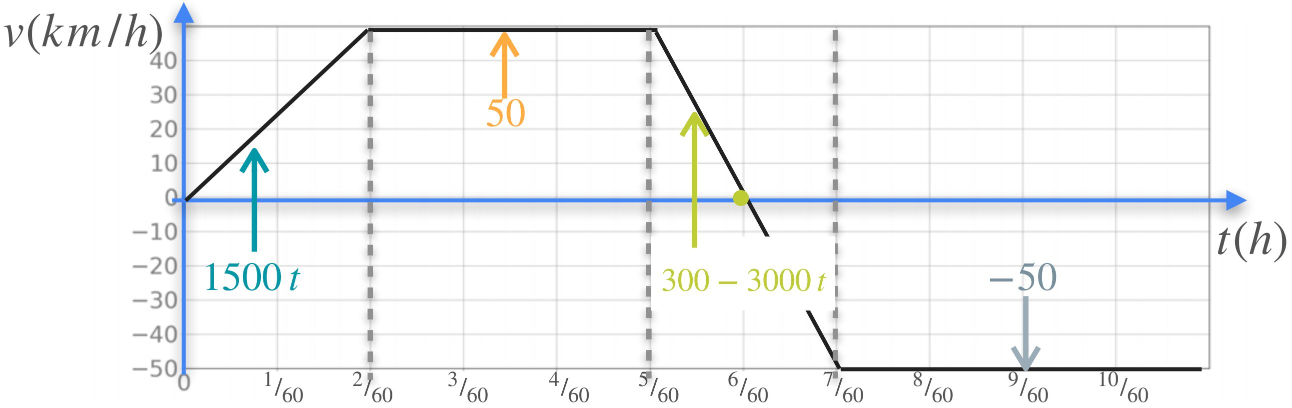
Understanding Second Derivative



x Distance

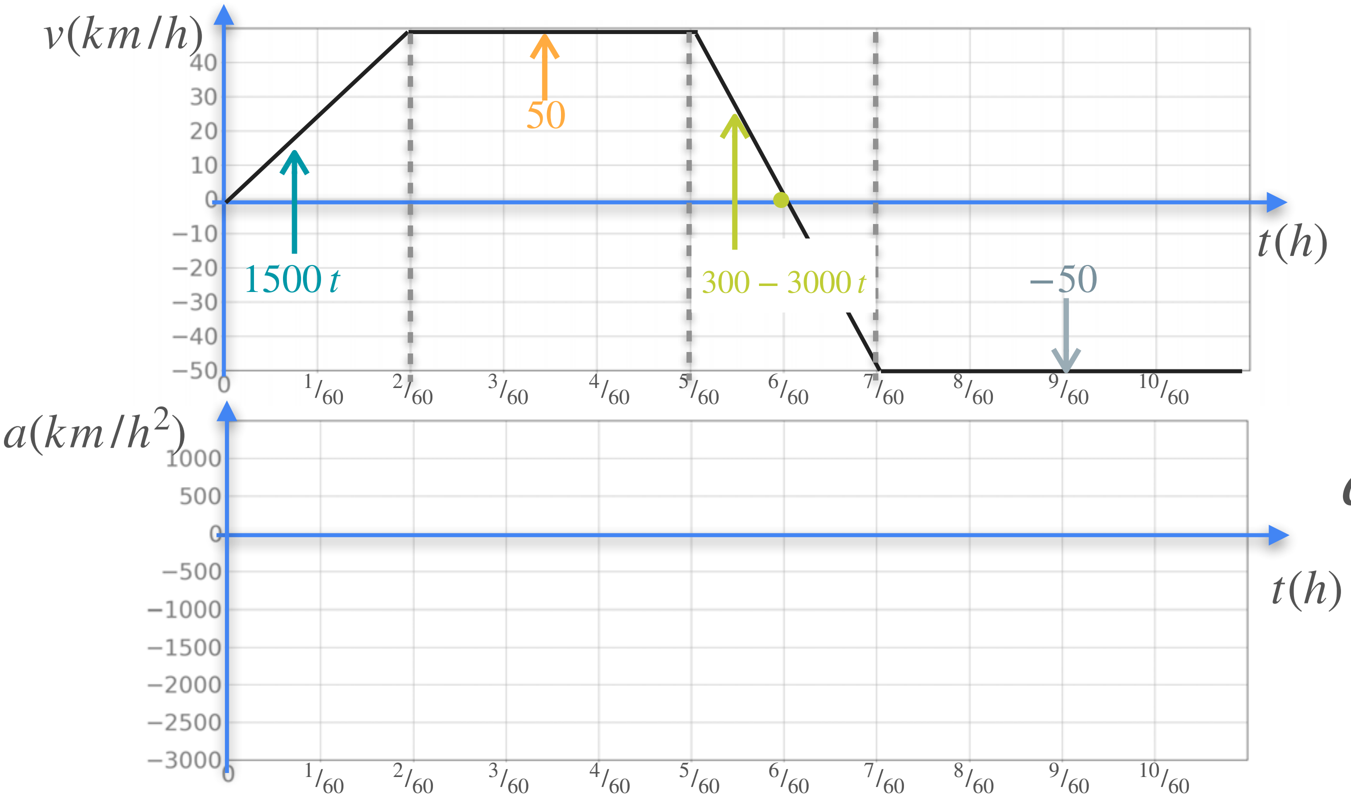
v Velocity $\frac{dx}{dt}$

Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

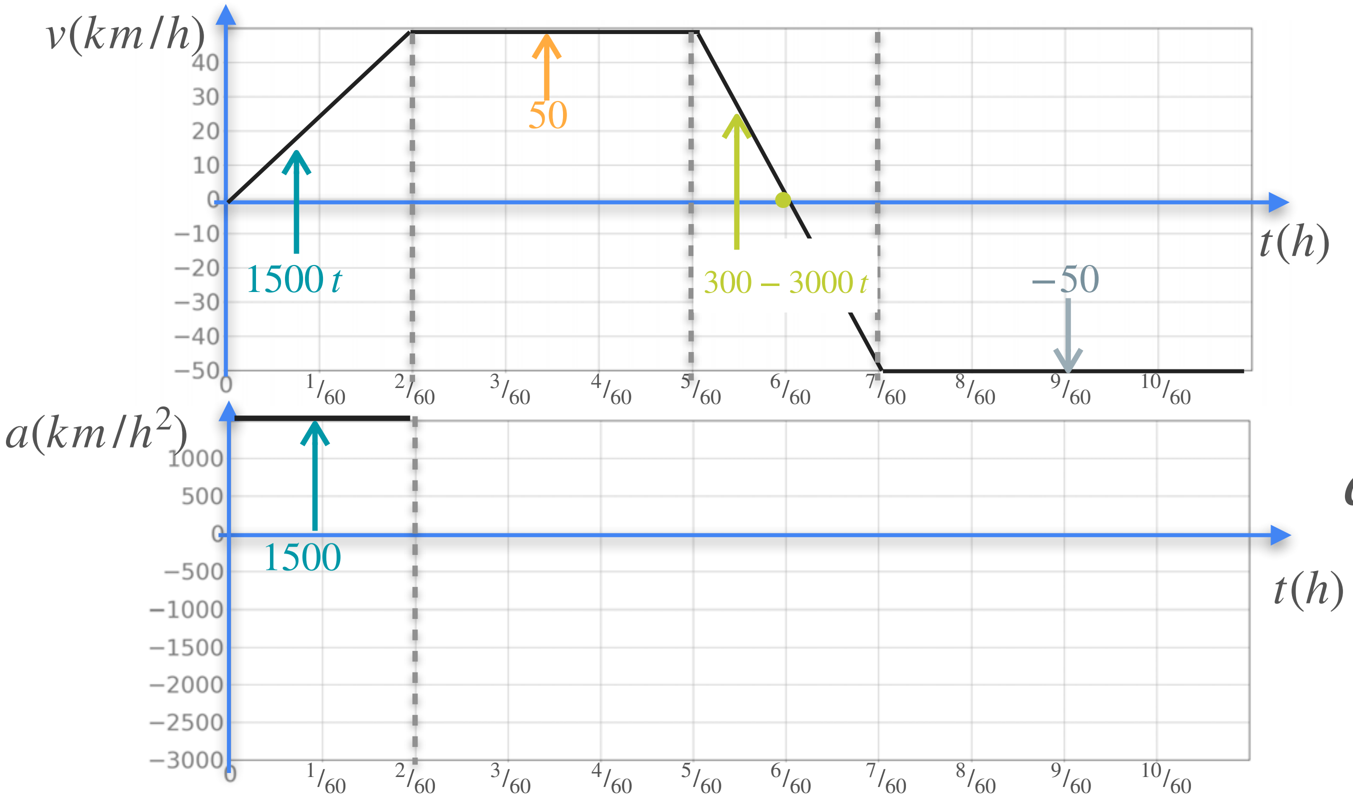
Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

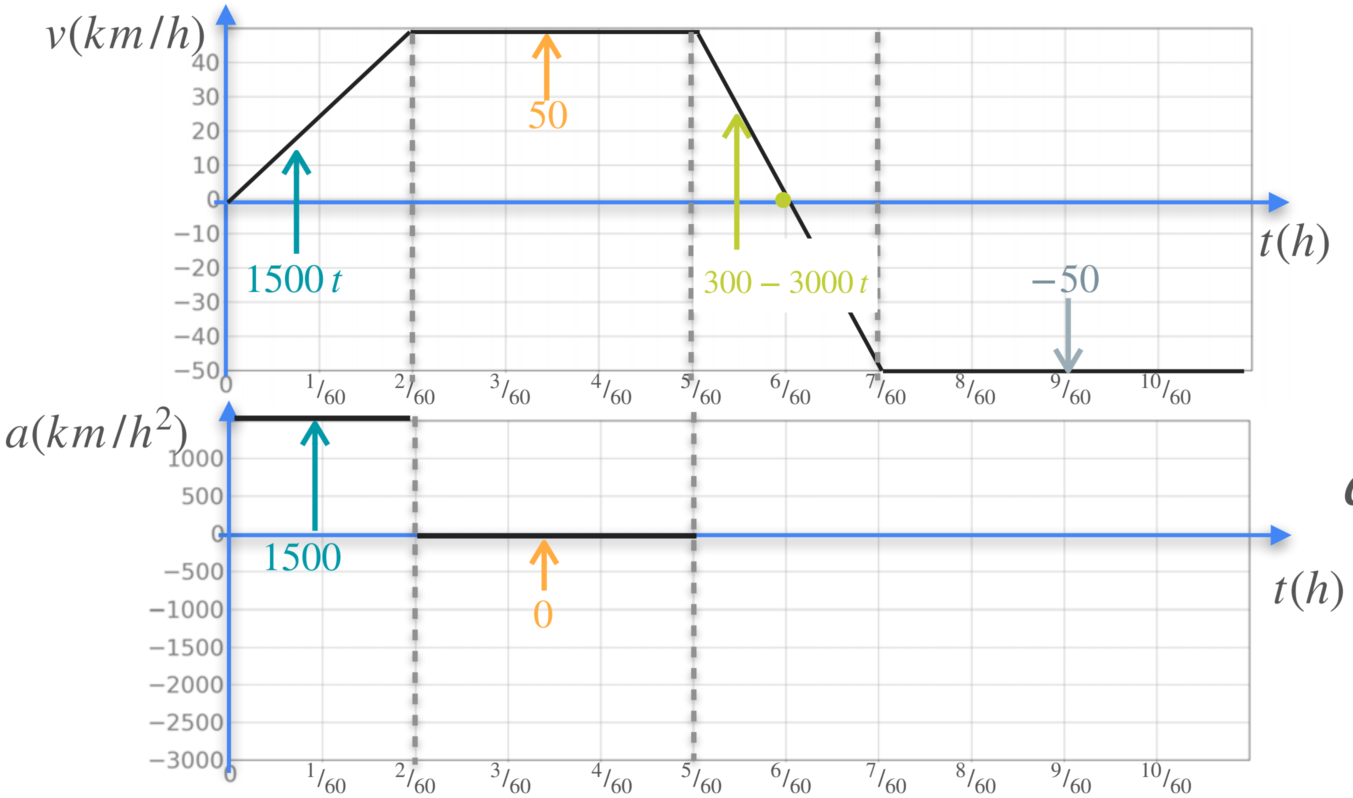
Understanding Second Derivative



v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt}$

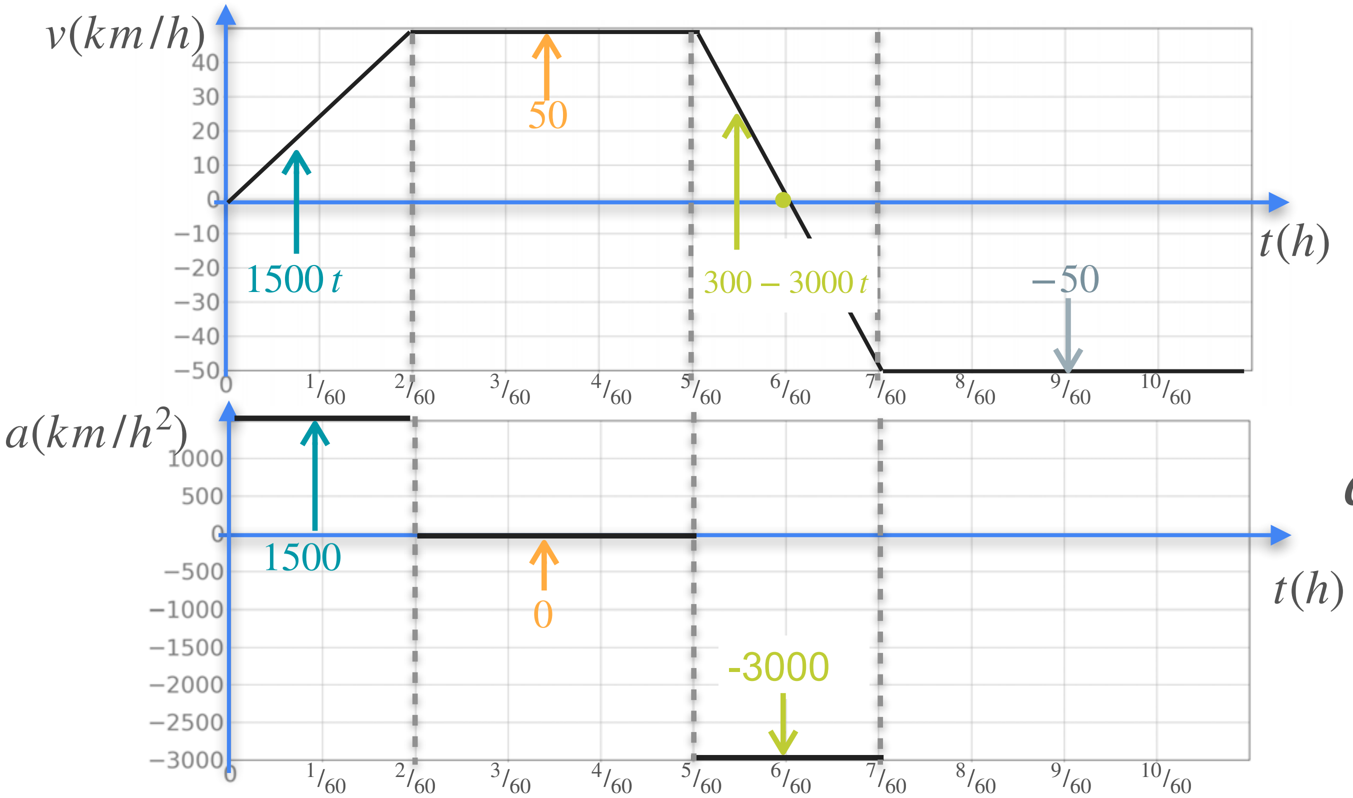
Understanding Second Derivative



v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt}$

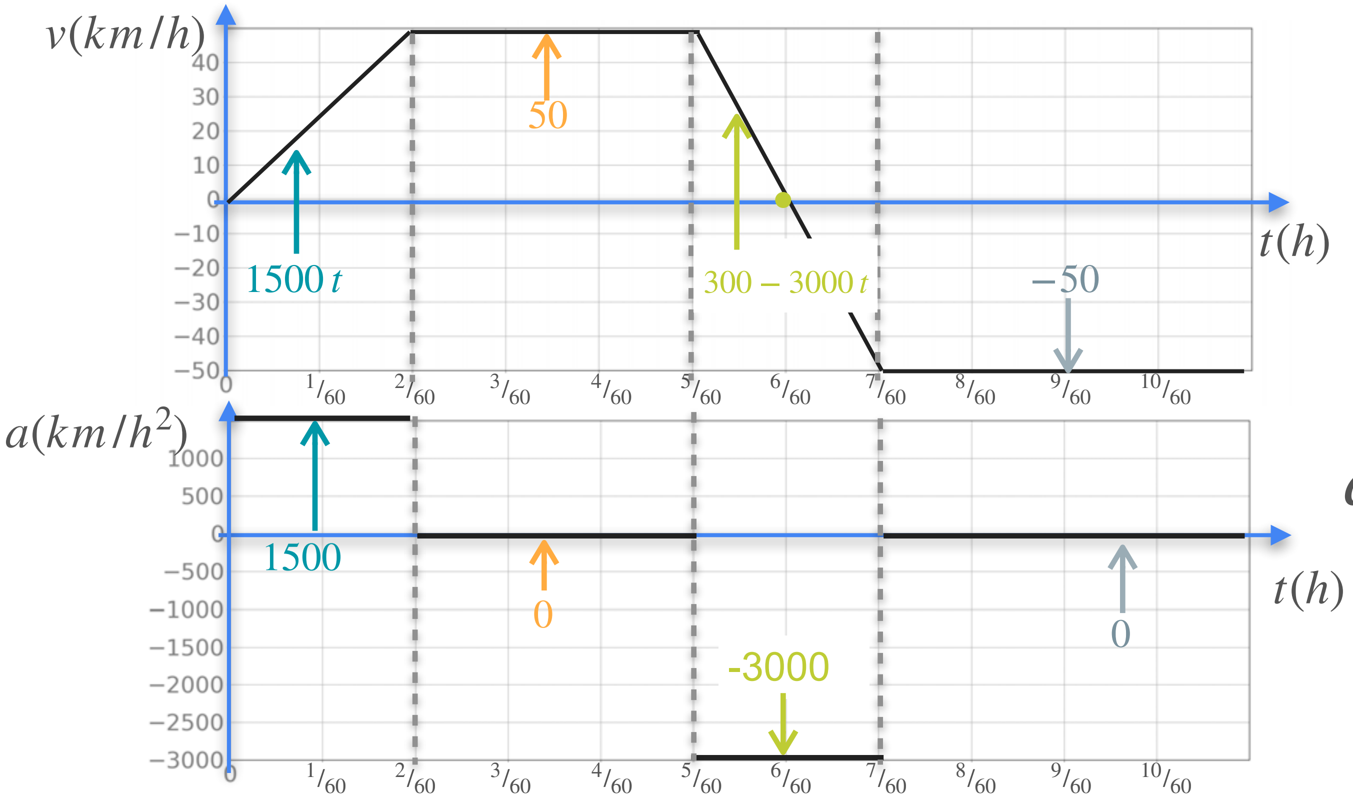
Understanding Second Derivative



v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt}$

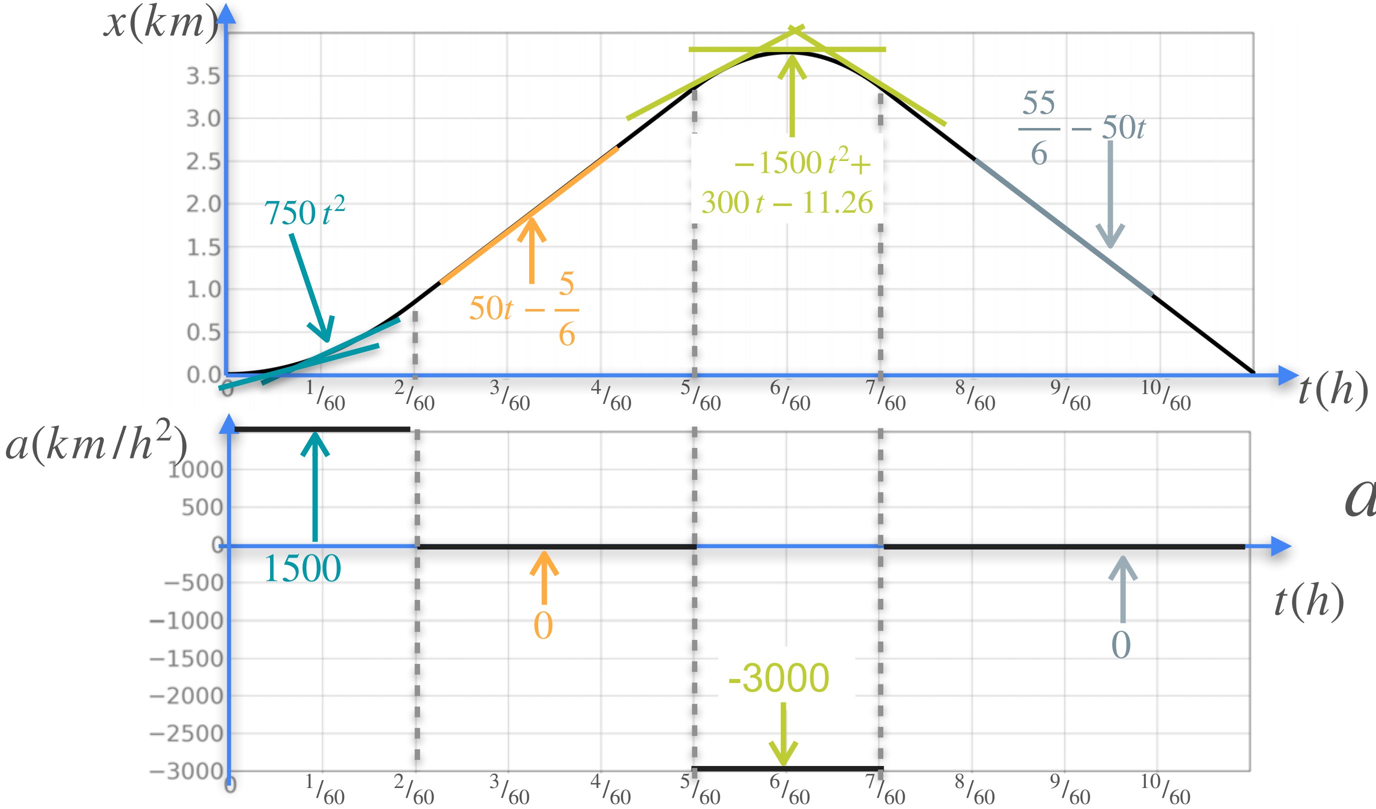
Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

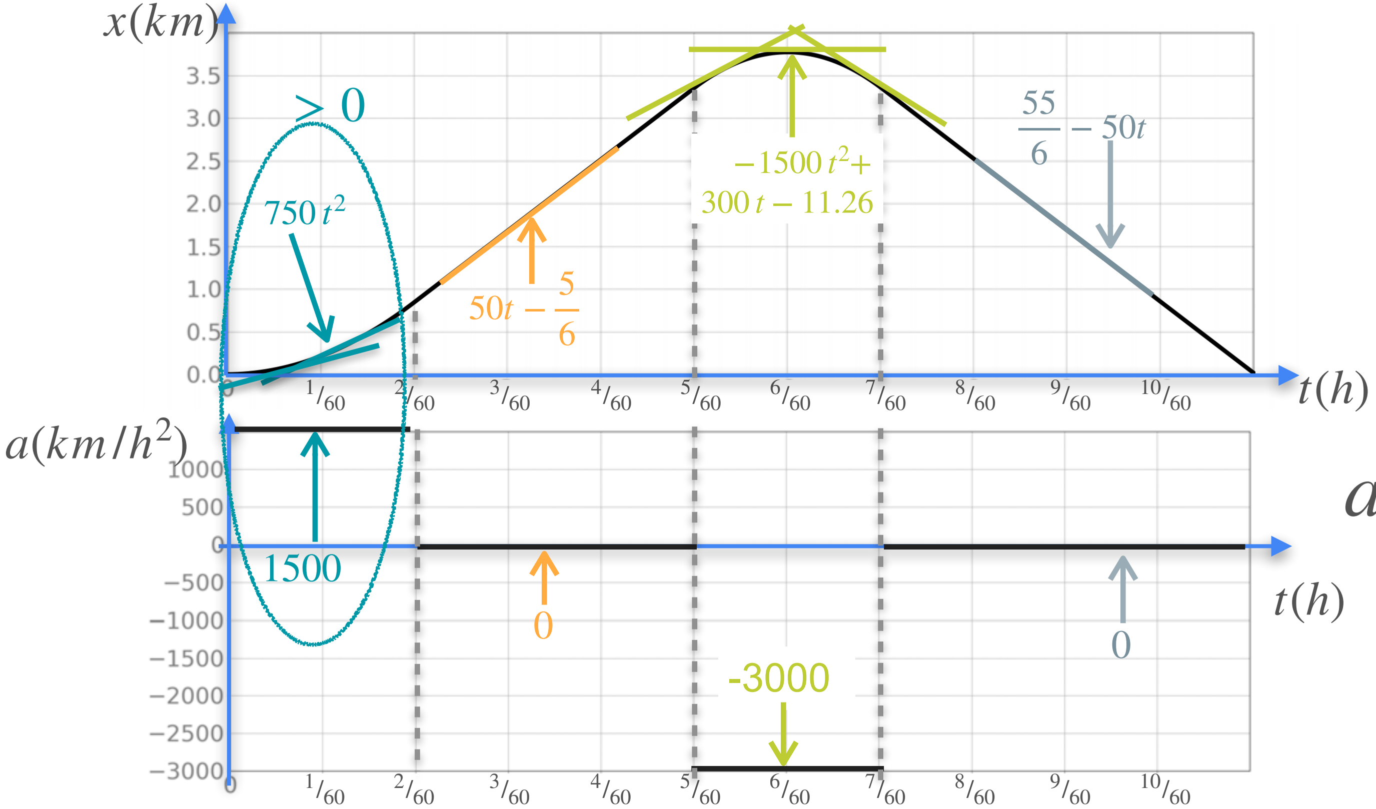
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

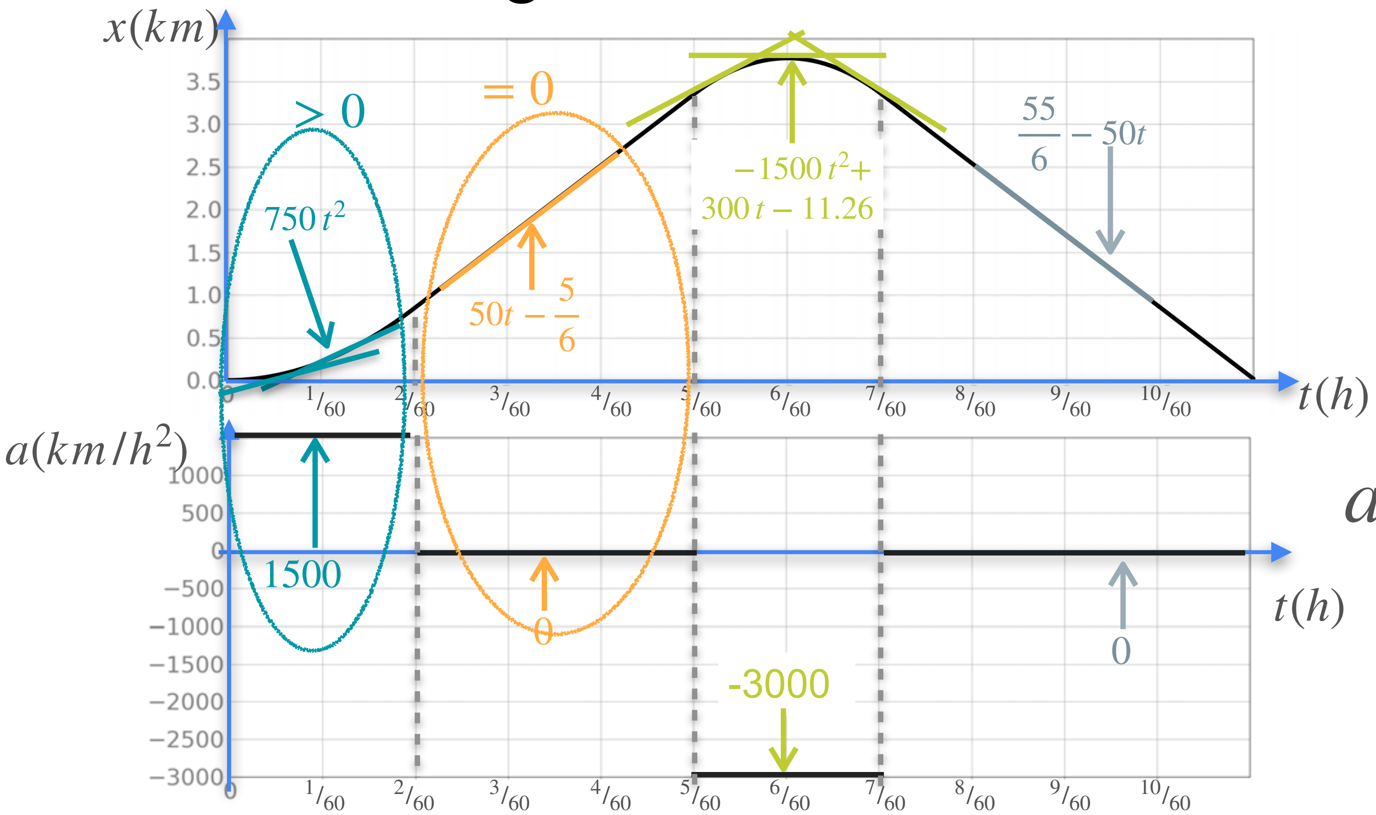
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

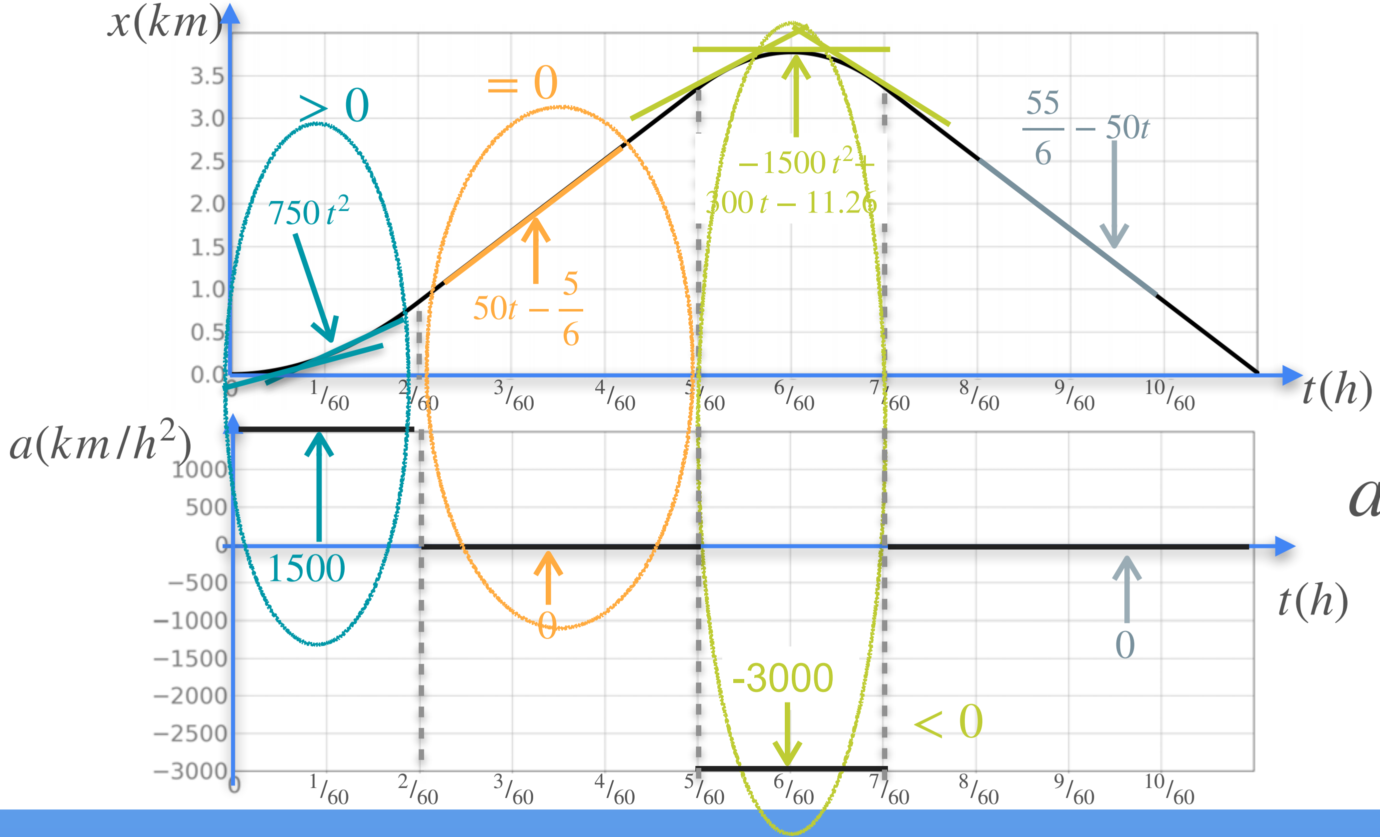
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

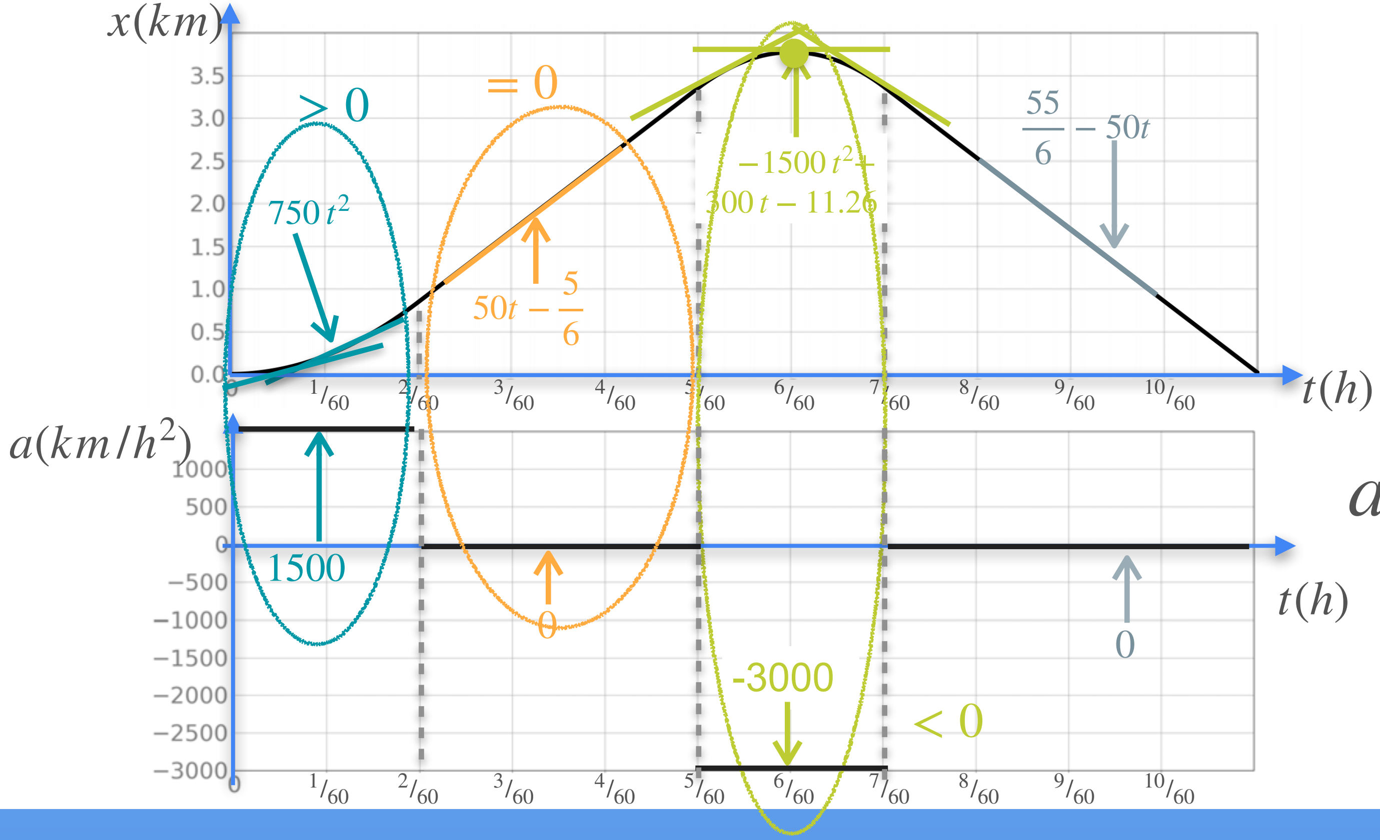
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

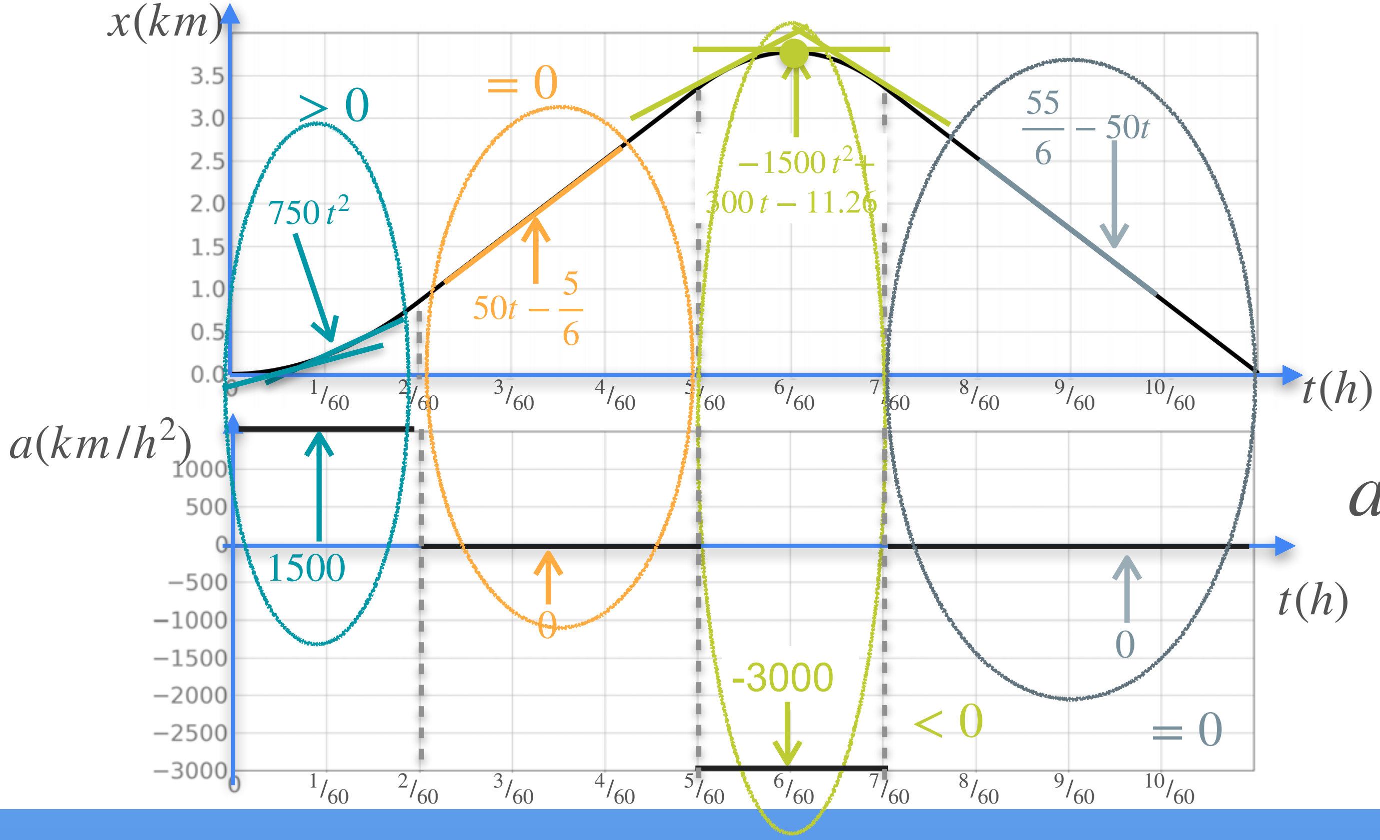
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

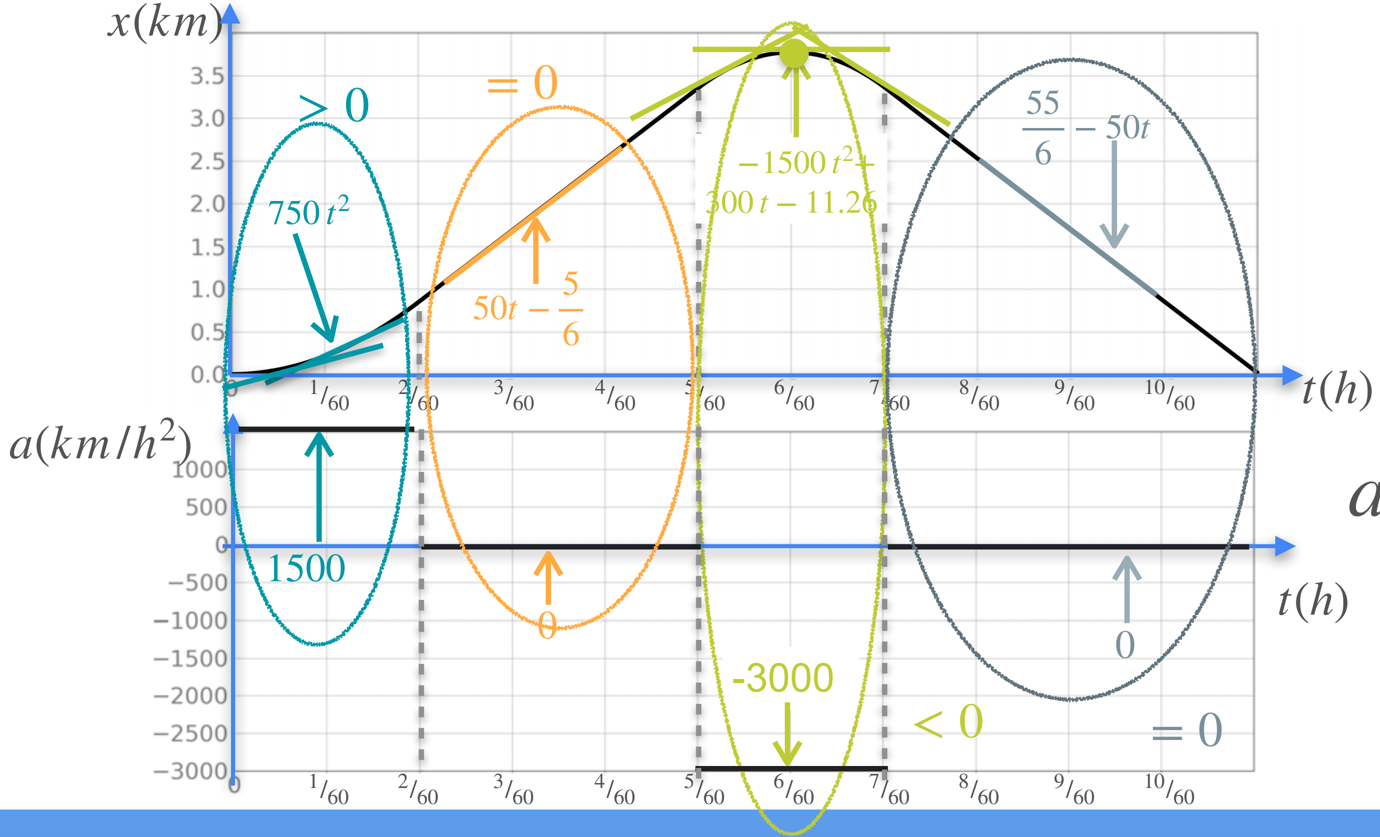
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative



x Distance

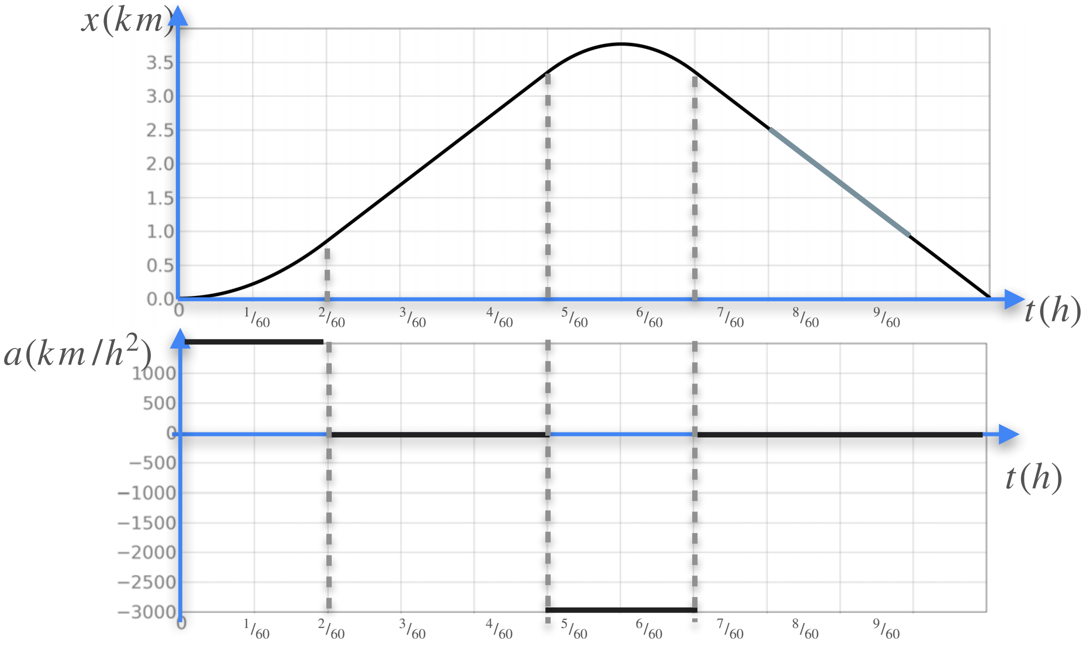
Second derivative tells us about the curvature

a Acceleration $\frac{d^2x}{dt^2}$

DeepLearning.AI

Curvature

Curvature

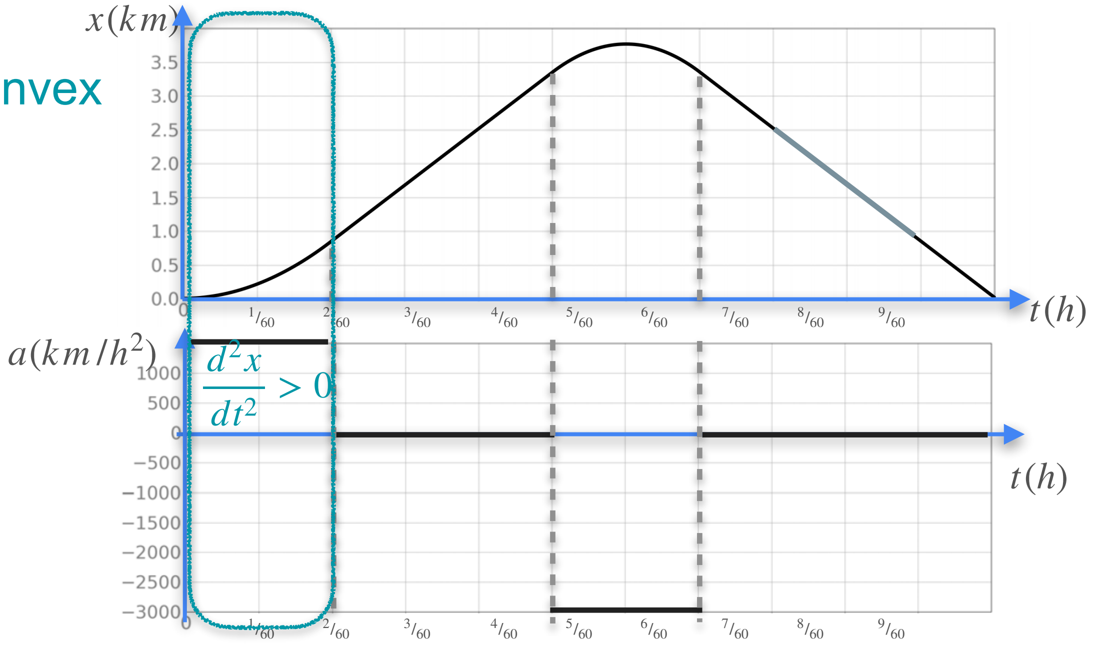


x(km)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1/60
2/60
3/60
4/60
5/60
6/60
7/60
8/60
9/60
t(h)
a(km/h^2)
1000
500
0
−500
−1000
−1500
−2000
−2500
−3000
0
1/60
2/60
3/60
4/60
5/60
6/60
7/60
8/60
9/60
t(h)

Curvature

$$\frac{d^2x}{dt^2} > 0$$

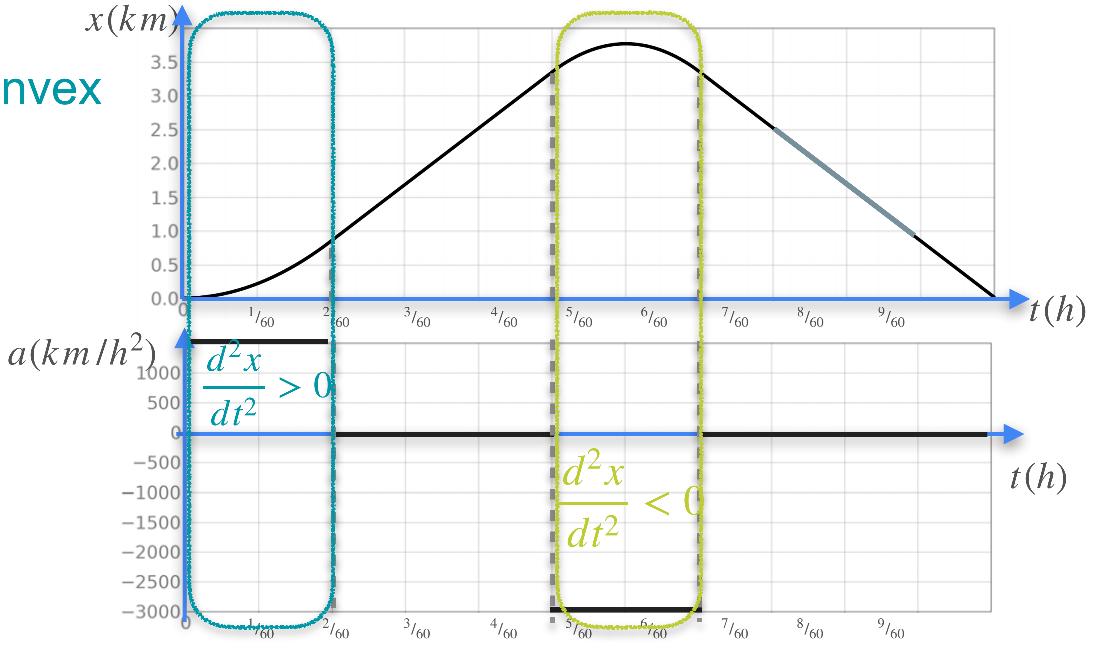
Concave up or convex



Curvature

$$\frac{d^2x}{dt^2} > 0$$ Concave up or convex

$$\frac{d^2x}{dt^2} < 0$$ Concave down

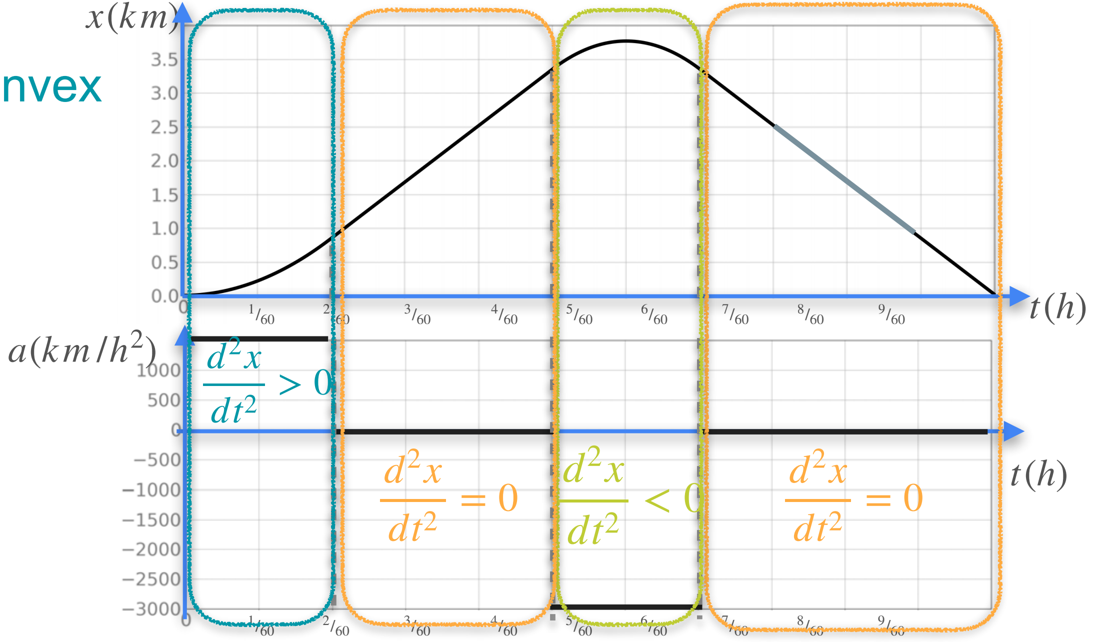


Curvature

$$\frac{d^2x}{dt^2} > 0$$ Concave up or convex

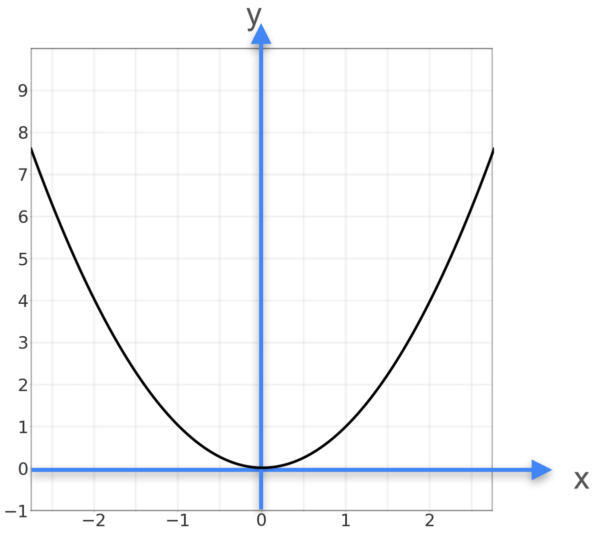
$$\frac{d^2x}{dt^2} < 0$$ Concave down

$$\frac{d^2x}{dt^2} = 0$$ Need more information

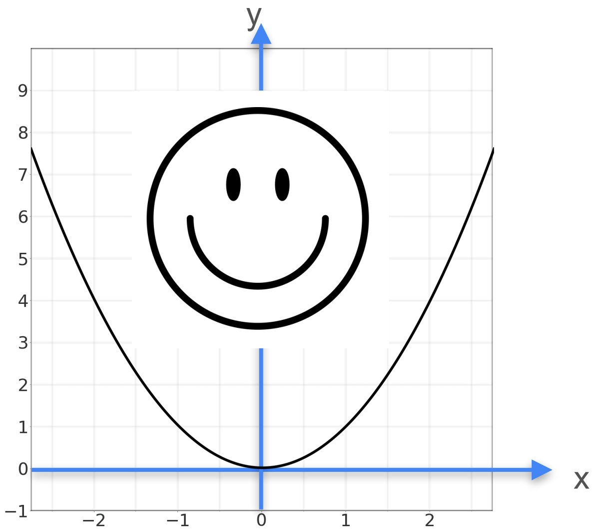


Curvature

Curvature



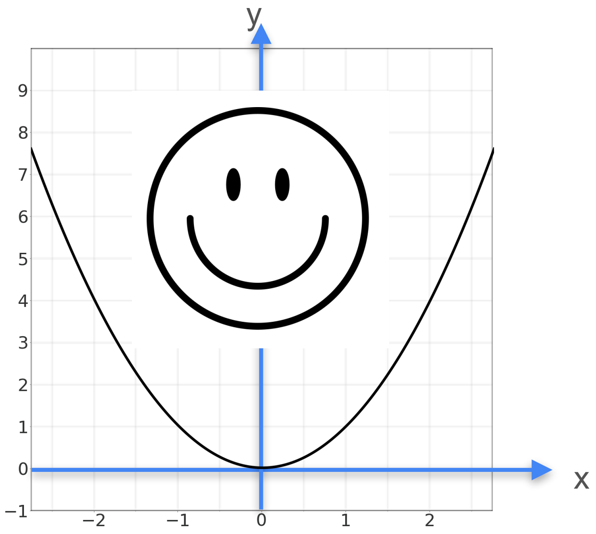
Curvature



Concave up or convex

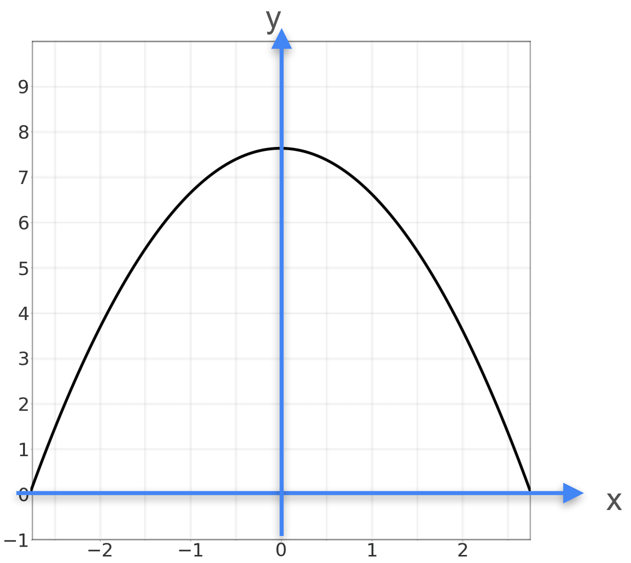
$$f''(0) > 0$$

Curvature

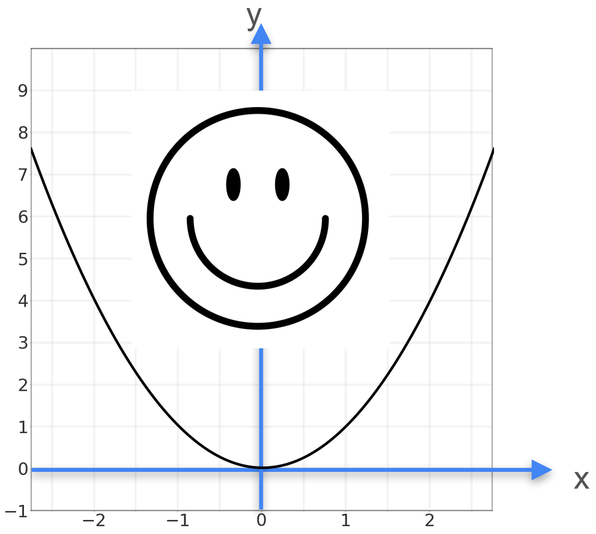


Concave up or convex

$$f''(0) > 0$$

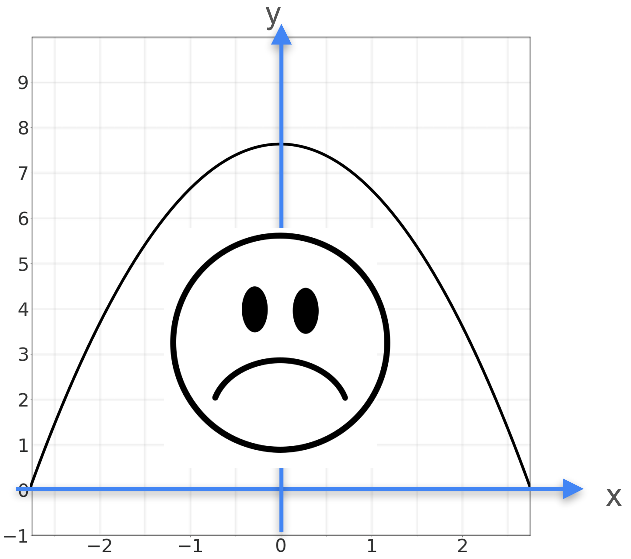


Curvature



Concave up or convex

$$f''(0) > 0$$



Concave down

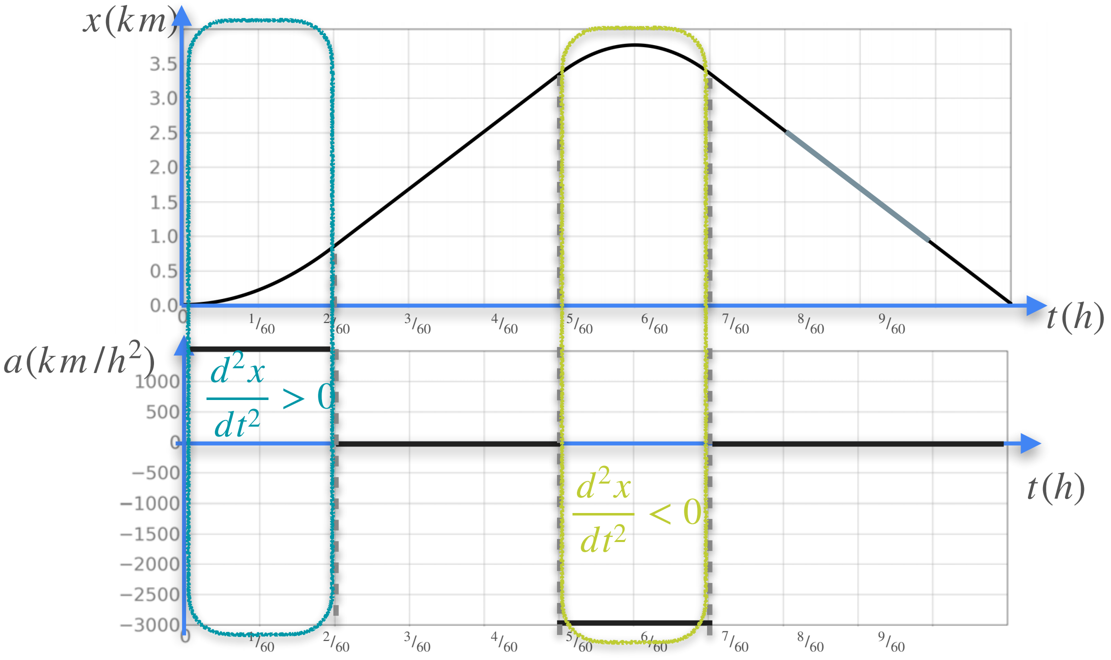
$$f''(0) < 0$$

Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$

$$\frac{d^2x}{dt^2} < 0$$

$$\frac{d^2x}{dt^2} = 0$$

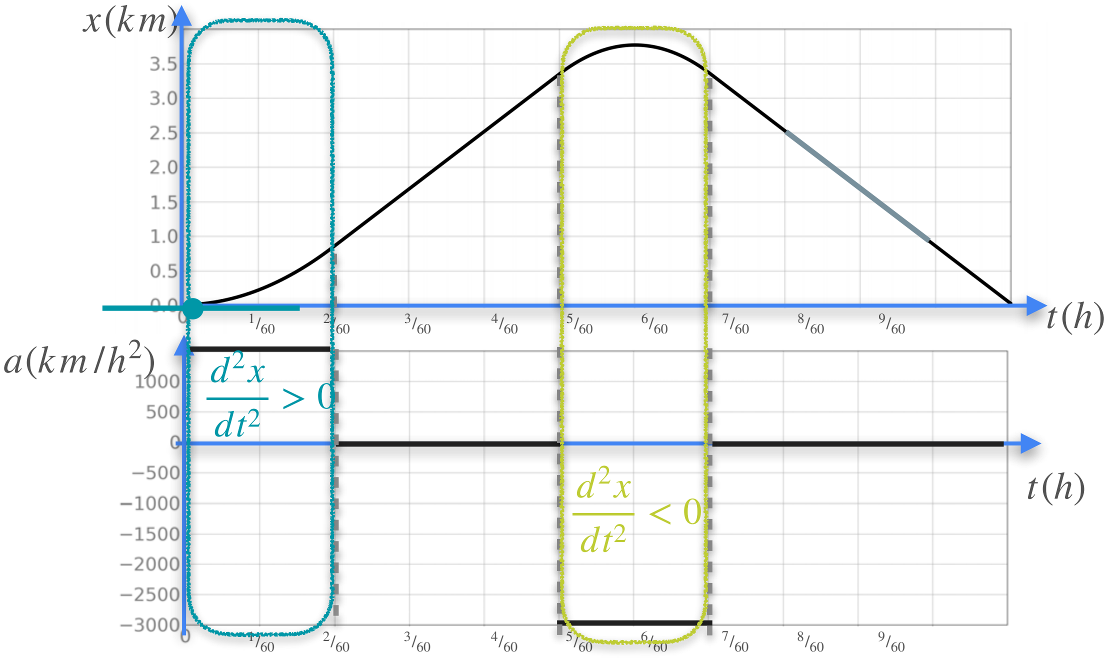


Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$

$$\frac{d^2x}{dt^2} = 0$$

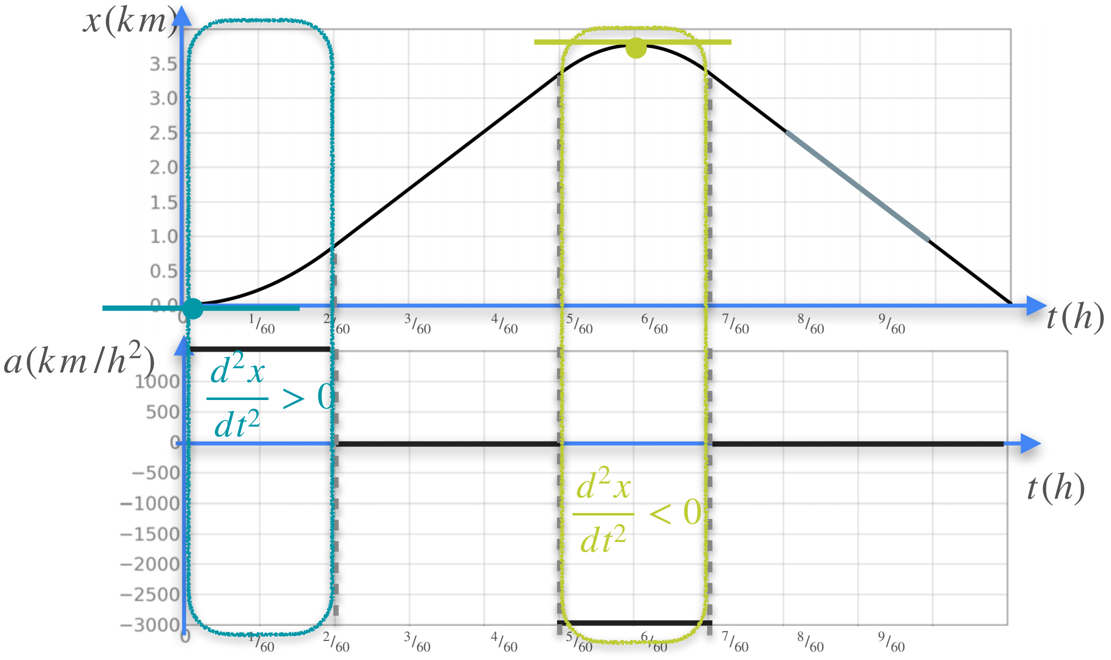


Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0 \qquad \text{(Local) Minimum}$$

$$\frac{d^2x}{dt^2} < 0 \qquad \text{(Local) maximum}$$

$$\frac{d^2x}{dt^2} = 0$$

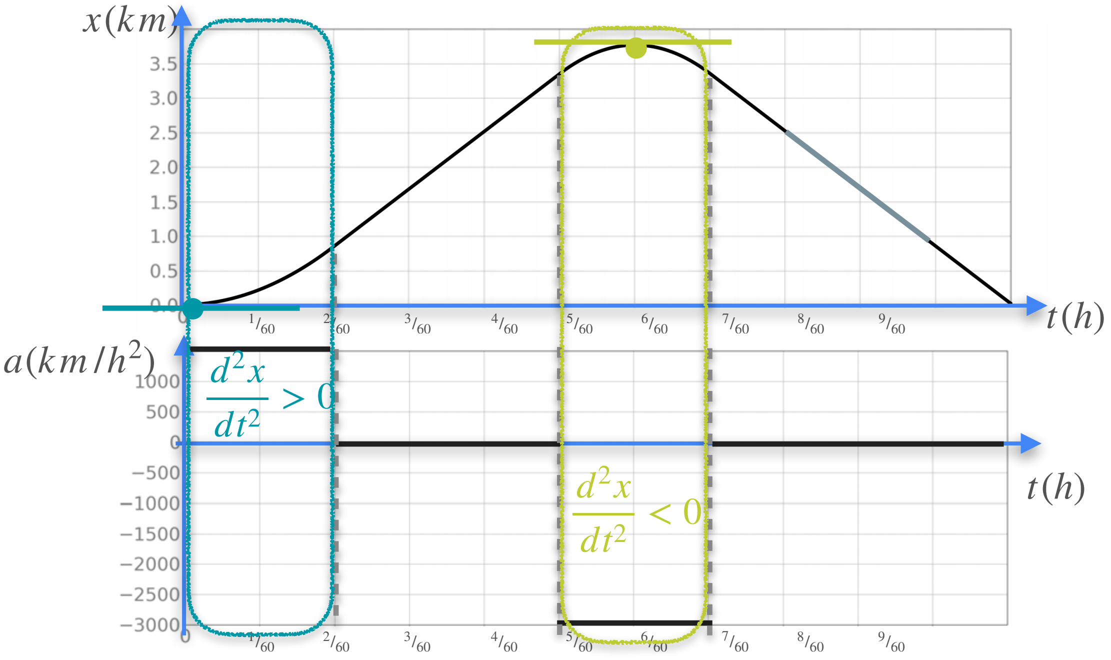


Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$ (Local) maximum

$$\frac{d^2x}{dt^2} = 0$$ Inconclusive



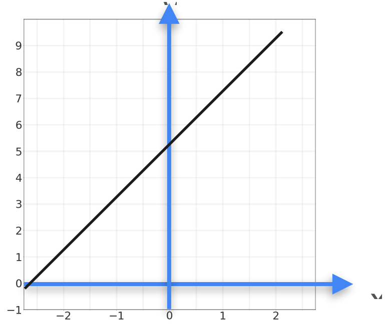
Curvature

First derivative

Second derivative

Curvature

First derivative



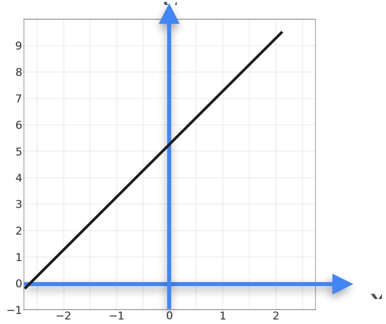
Increasing

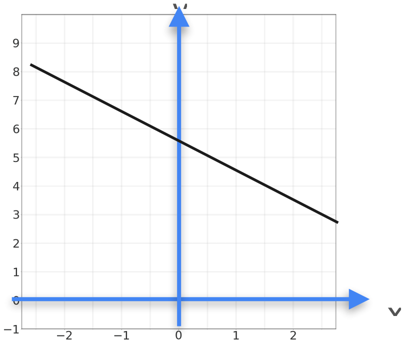
$$f'(0) > 0$$

Second derivative

Curvature

First derivative





Increasing

$f'(0) > 0$

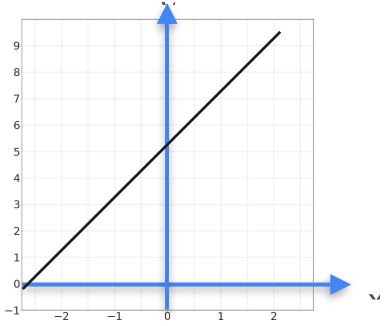
Decreasing

$f'(0) < 0$

Second derivative

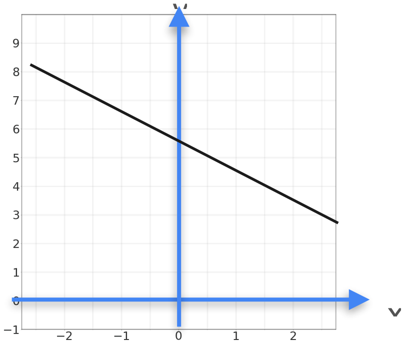
Curvature

First derivative



Increasing

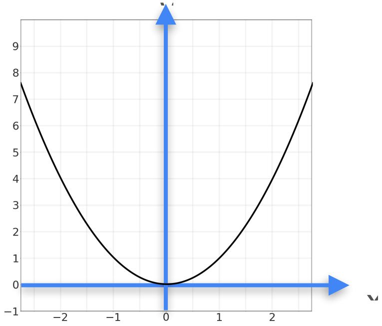
$$f'(0) > 0$$



Decreasing

$$f'(0) < 0$$

Second derivative

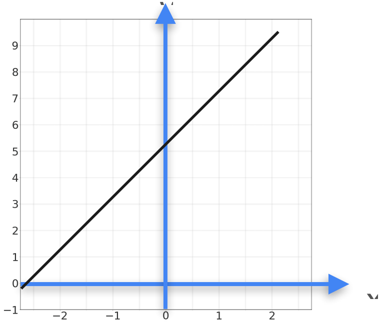


Concave up

$$f''(0) > 0$$

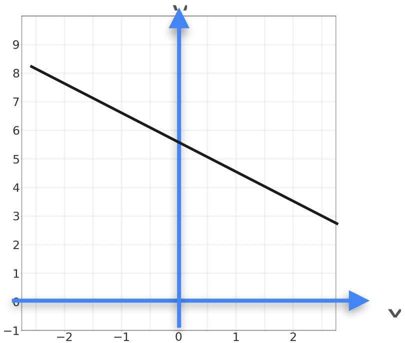
Curvature

First derivative



Increasing

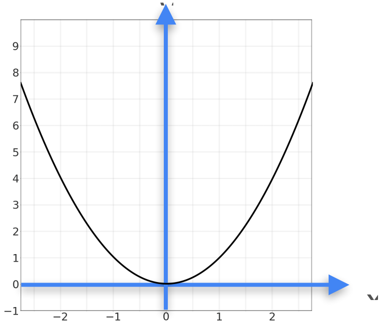
$$f'(0) > 0$$



Decreasing

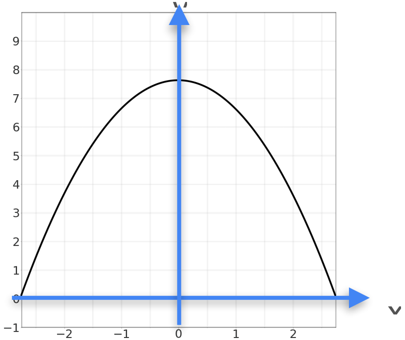
$$f'(0) < 0$$

Second derivative



Concave up

$$f''(0) > 0$$



Concave down

$$f''(0) < 0$$

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

The Hessian

Second Derivative

Second Derivative

	1 variable	2 variables

Second Derivative

	1 variable	2 variables
Function	$f(x)$	

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$

Second Derivative

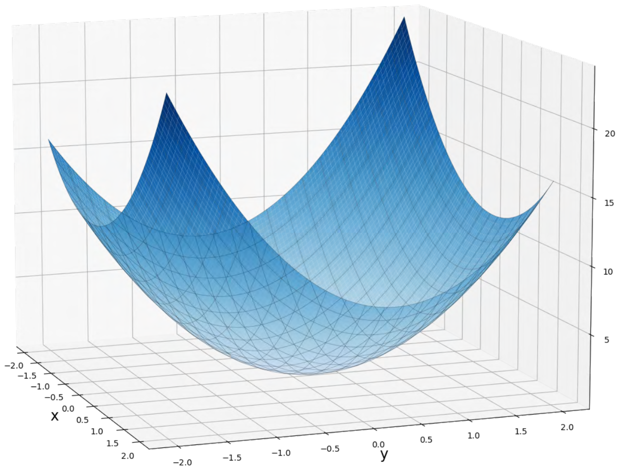
	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$		

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$	???	

Second Derivative

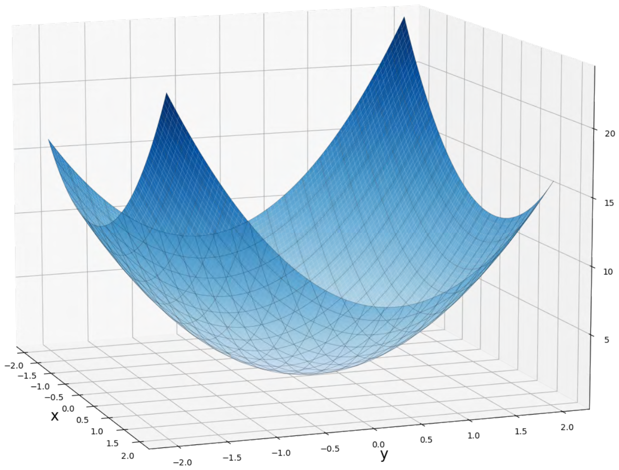
Second Derivative



$$f(x, y) =$$

$$2x^2 + 3y^2 - xy$$

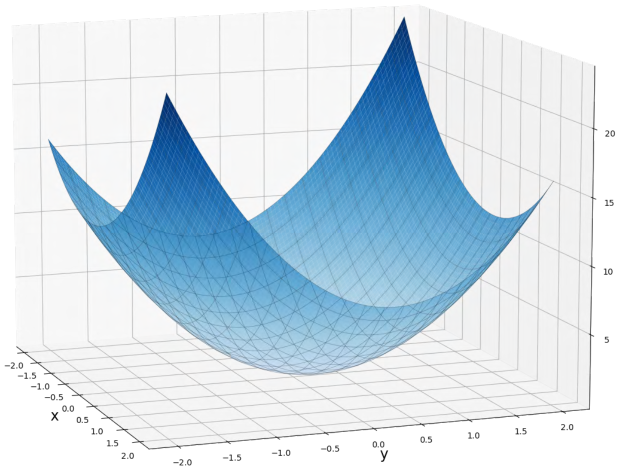
Second Derivative



$$f(x, y) =$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y$$

Second Derivative

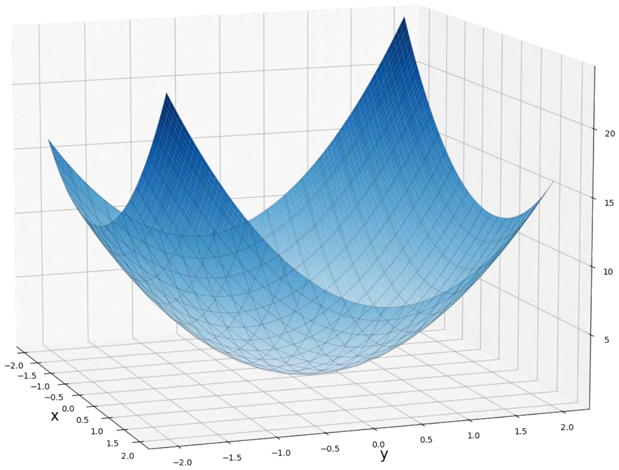


$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\xrightarrow{x} 4x - y$$

$$\xrightarrow{y} 6y - x$$

Second Derivative



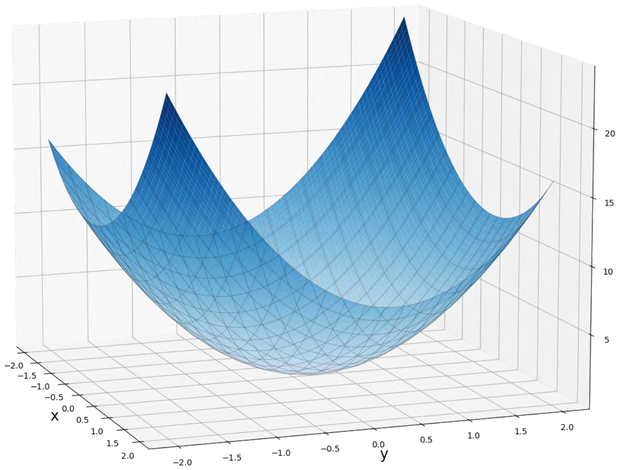
$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y \xrightarrow{x} 4$$

$$2x^2 + 3y^2 - xy \xrightarrow{y} 6y - x$$

Rate of change of $f'_x(x, y)$ w.r.t x

Second Derivative



$$f(x,y) = 2x^2 + 3y^2 - xy$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y$$

$$2x^2 + 3y^2 - xy \xrightarrow{y} 6y - x$$

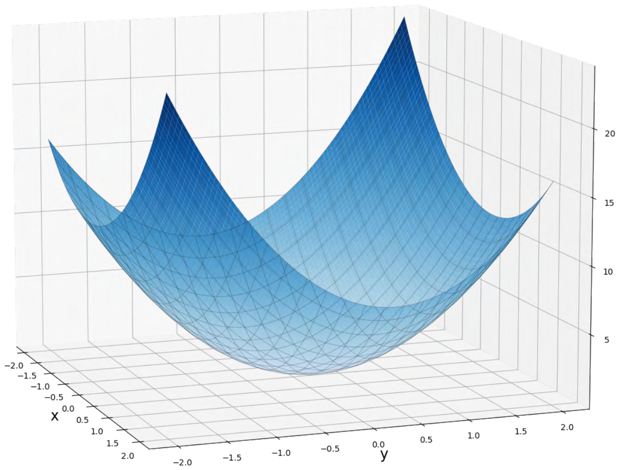
$$4x - y \xrightarrow{x} 4$$

Rate of change of $f'_x(x,y)$ w.r.t x

$$4x - y \xrightarrow{y} -1$$

Rate of change of $f'_x(x,y)$ w.r.t y

Second Derivative



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y \xrightarrow{x} 4$$

Rate of change of $f'_x(x, y)$ w.r.t x

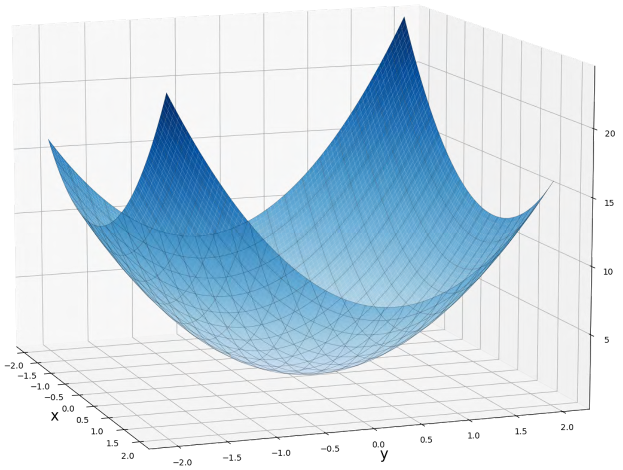
$$4x - y \xrightarrow{y} -1$$

Rate of change of $f'_x(x, y)$ w.r.t y

$$2x^2 + 3y^2 - xy \xrightarrow{y} 6y - x \xrightarrow{x} -1$$

Rate of change of $f'_y(x, y)$ w.r.t x

Second Derivative



$$f(x, y) = 2x^2 + 3y^2 - xy$$

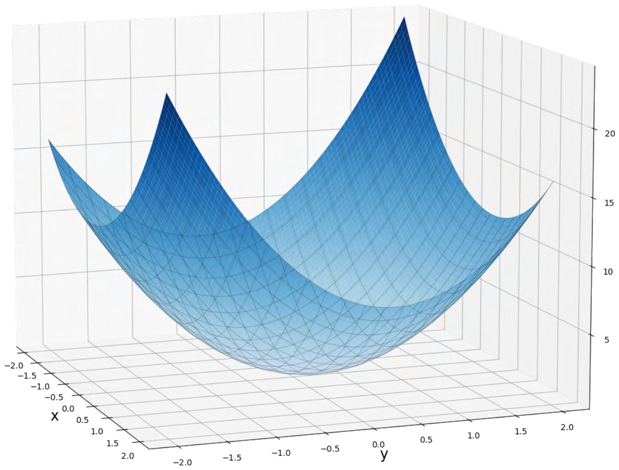
$\xrightarrow{x}$ $4x - y$

- $\xrightarrow{x}$ 4 Rate of change of $f'_x(x, y)$ w.r.t x
- $\xrightarrow{y}$ -1 Rate of change of $f'_x(x, y)$ w.r.t y

$\xrightarrow{y}$ $6y - x$

- $\xrightarrow{x}$ -1 Rate of change of $f'_y(x, y)$ w.r.t x
- $\xrightarrow{y}$ 6 Rate of change of $f'_y(x, y)$ w.r.t y

Second Derivative



$f(x, y) =$

$2x^2 + 3y^2 - xy$

$\xrightarrow{x}$ $4x - y$

- $\xrightarrow{x}$ 4 Rate of change of $f'_x(x, y)$ w.r.t x
- $\xrightarrow{y}$ $\boxed{-1}$ Rate of change of $f'_x(x, y)$ w.r.t y

$\xrightarrow{y}$ $6y - x$

- $\xrightarrow{x}$ $\boxed{-1}$ Rate of change of $f'_y(x, y)$ w.r.t x
- $\xrightarrow{y}$ 6 Rate of change of $f'_y(x, y)$ w.r.t y

What Do These Mean?

Rate of change of
$f_x(x, y)$ w.r.t x

Rate of change of
$f_y(x, y)$ w.r.t y

Rate of change of
$f_x(x, y)$ w.r.t y

Rate of change of
$f_y(x, y)$ w.r.t x

What Do These Mean?

Rate of change of
$f_x(x, y)$ w.r.t x

Rate of change of
$f_y(x, y)$ w.r.t y

Change in the change in the function
w.r.t tiny changes in x and y

Rate of change of
$f_x(x, y)$ w.r.t y

Rate of change of
$f_y(x, y)$ w.r.t x

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

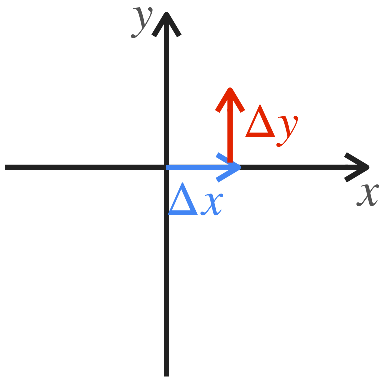
Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis



What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

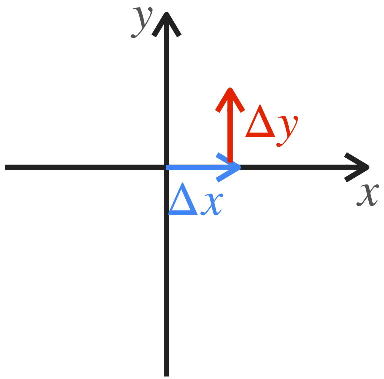
Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

2. They are the same!



What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

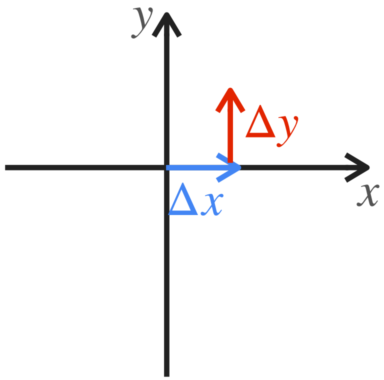
Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

2. They are the same!

(In most cases)



Notation

Rate of change of
$f'_x(x, y)$ w.r.t x

Rate of change of
$f'_y(x, y)$ w.r.t y

Rate of change of
$f'_x(x, y)$ w.r.t y

Rate of change of
$f'_y(x, y)$ w.r.t x

Notation

Leibniz's notation

Rate of change of $f'_x(x, y)$ w.r.t x

Rate of change of $f'_y(x, y)$ w.r.t y

Rate of change of $f'_x(x, y)$ w.r.t y

Rate of change of $f'_y(x, y)$ w.r.t x

Notation

	Leibniz's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$
Rate of change of $f'_x(x, y)$ w.r.t y	
Rate of change of $f'_y(x, y)$ w.r.t x	

Notation

	Leibniz's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial x^2}$
Rate of change of $f'_y(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial y^2}$
Rate of change of $f'_x(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial x \partial y}$
Rate of change of $f'_y(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial y \partial x}$

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial x^2}$	
Rate of change of $f'_y(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial y^2}$	
Rate of change of $f'_x(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial x \partial y}$	
Rate of change of $f'_y(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial y \partial x}$	

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial x^2}$	$f_{xx}(x, y)$
Rate of change of $f'_y(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial y^2}$	$f_{yy}(x, y)$
Rate of change of $f'_x(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial x \partial y}$	
Rate of change of $f'_y(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial y \partial x}$	

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial x^2}$	$f_{xx}(x, y)$
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Rate of change of $f'_x(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial x \partial y}$	$f_{xy}(x, y)$
Rate of change of $f'_y(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial y \partial x}$	$f_{yx}(x, y)$

Hessian Matrix

Hessian Matrix

$$f(x, y) =$$

$$\begin{tikzcd}
 & & 4 \\
 & 4x - y \arrow[ur, "x"] \arrow[dr, "y"'] & \\
2x^2 + 3y^2 - xy \arrow[ur, "x"] \arrow[ddr, "y"'] & & -1 \\
 & & -1 \\
 & 6y - x \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & 6
\end{tikzcd}$$

Hessian Matrix

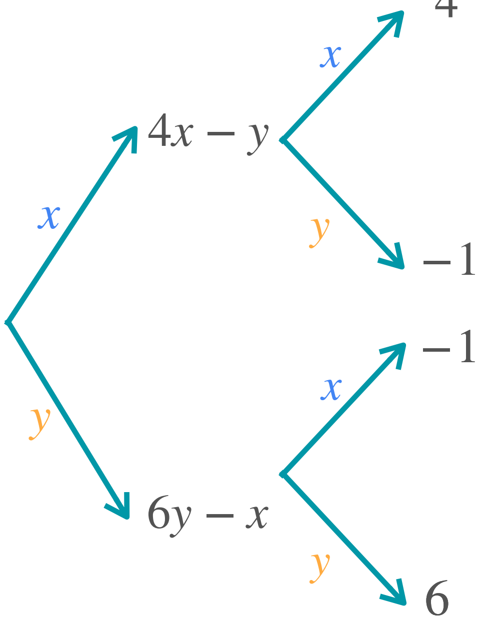
$$f(x, y) =$$

$$
\begin{tikzcd}
 & & 4 \\
 & 4x - y \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & -1 \\
2x^2 + 3y^2 - xy \arrow[uur, "x"] \arrow[ddr, "y"'] & & \\
 & & -1 \\
 & 6y - x \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & 6
\end{tikzcd}
$$

$$\begin{matrix} 4 \\ -1 \end{matrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$



$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Hessian Matrix

$$f(x,y) = 2x^2 + 3y^2 - xy$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y \xrightarrow{x} 4$$

$$4x - y \xrightarrow{y} -1$$

$$2x^2 + 3y^2 - xy \xrightarrow{y} 6y - x \xrightarrow{x} -1$$

$$6y - x \xrightarrow{y} 6$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \begin{matrix} -1 \\ 6 \end{matrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\begin{tikzcd}
 & & 4 \\
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Hessian Matrix

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& 6y - x \arrow[ur, "x"] \arrow[dr, "y"'] & \\
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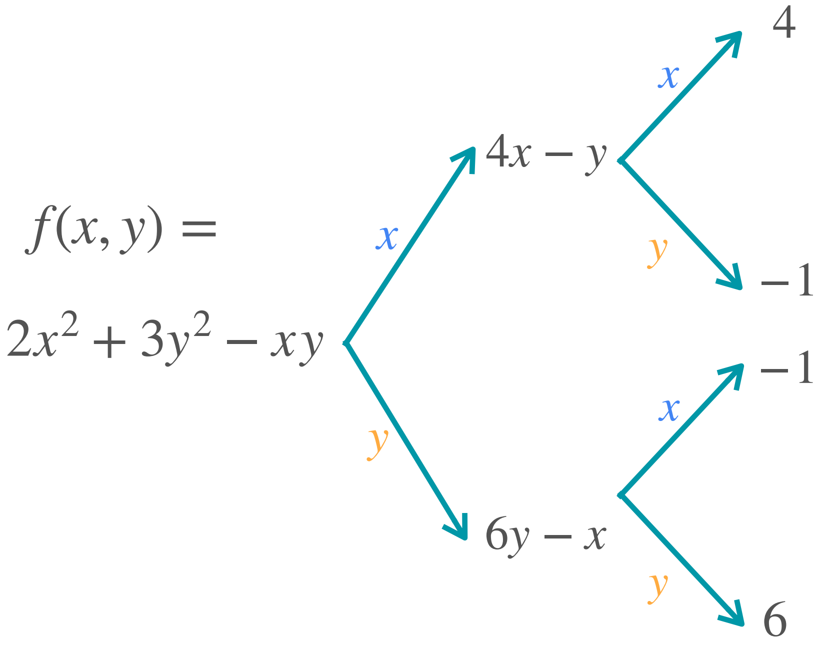
$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{matrix} 4 & -1 \\ -1 & 6 \end{matrix}$$

Hessian Matrix

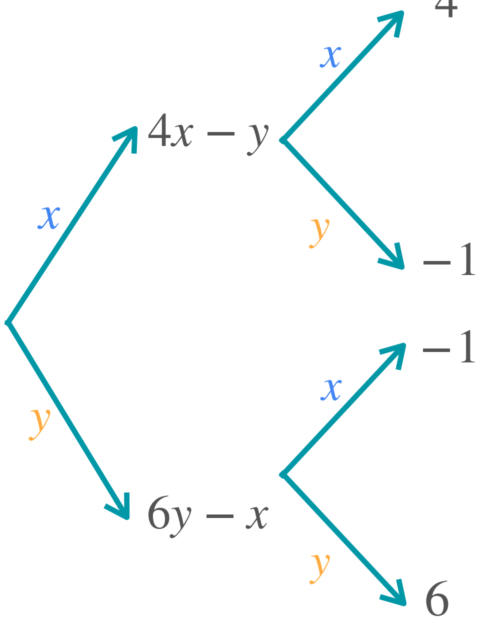


$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} \nabla f_x{}^T \\ \nabla f_y{}^T \end{bmatrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$



$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

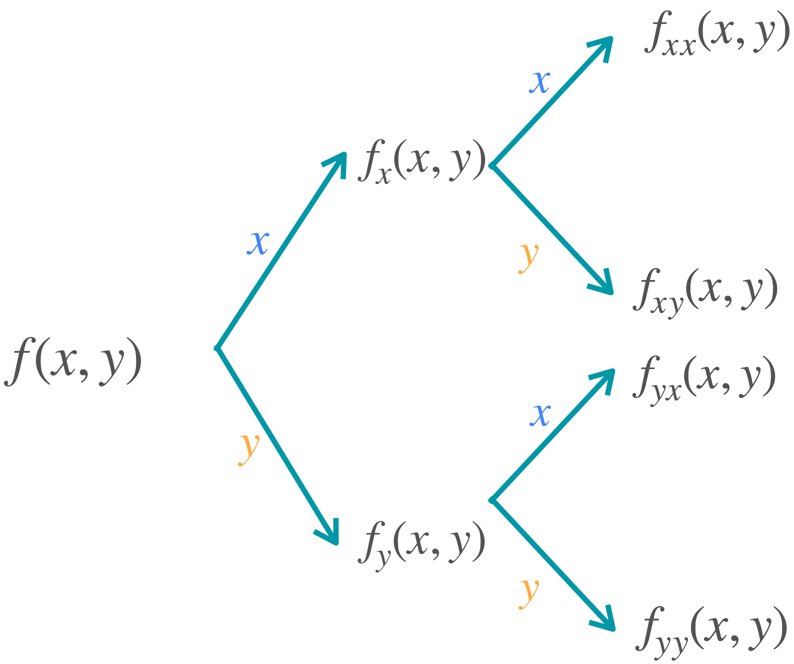
$$H = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} \nabla f_x^{\ T} \\ \nabla f_y^{\ T} \end{bmatrix}$$

Hessian matrix

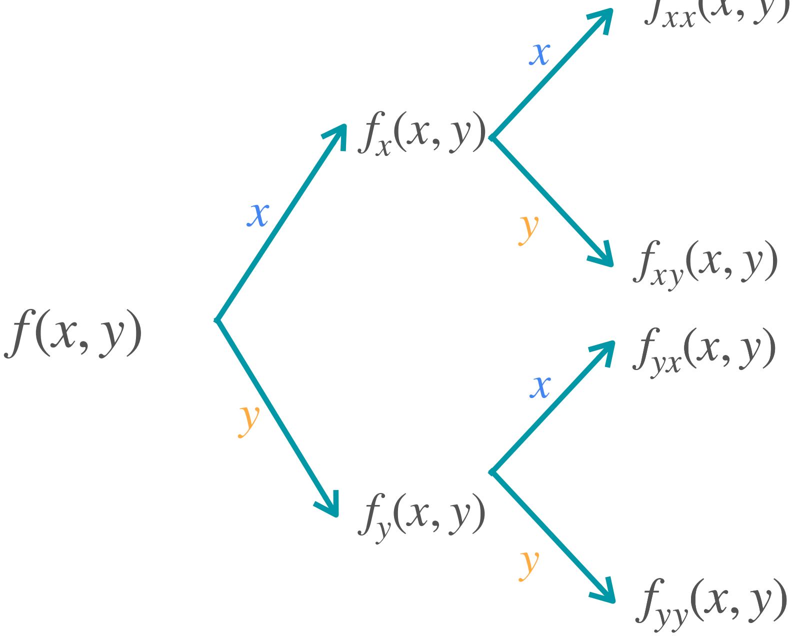
All information about second derivatives

Hessian Matrix - General Case

Hessian Matrix - General Case



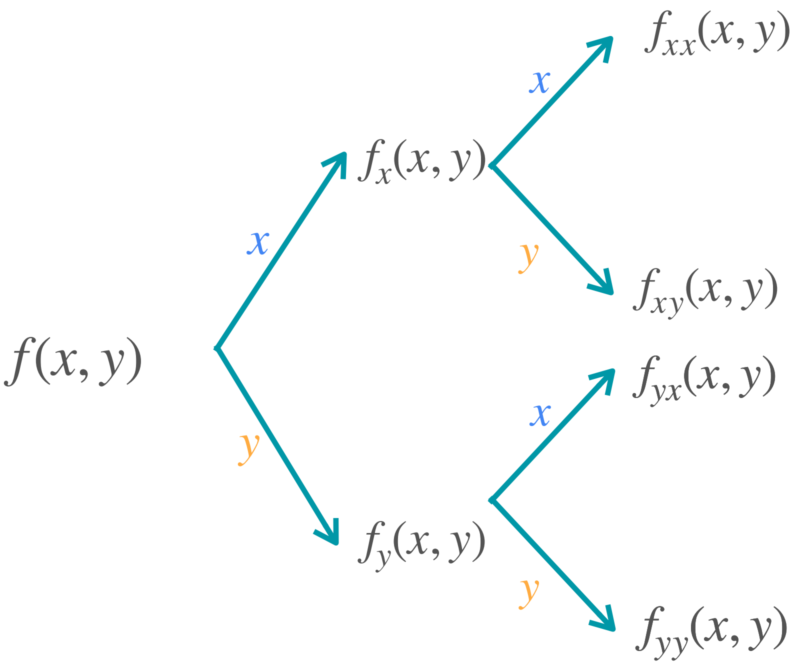
Hessian Matrix - General Case



$f_{xx}(x, y)$

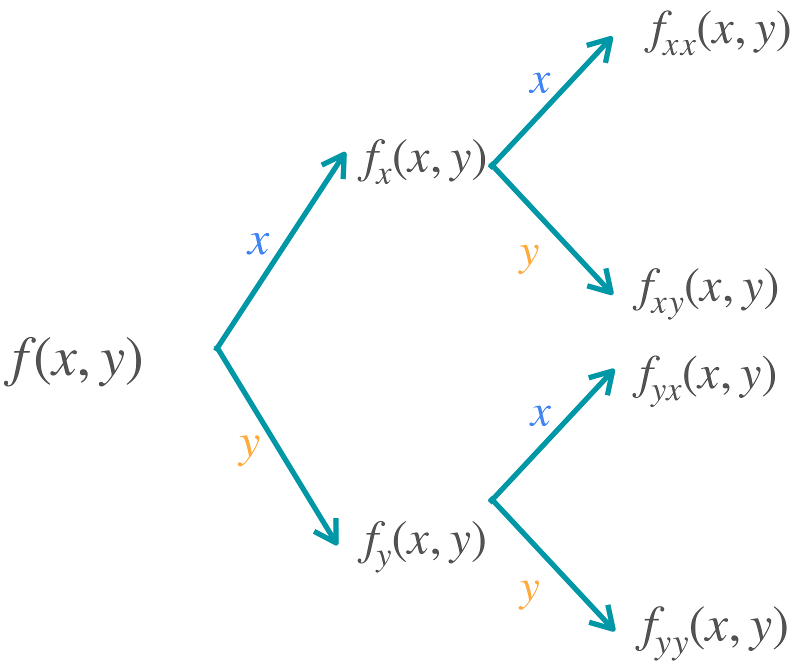
$f_{xy}(x, y)$

Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix}$$

Hessian Matrix - General Case

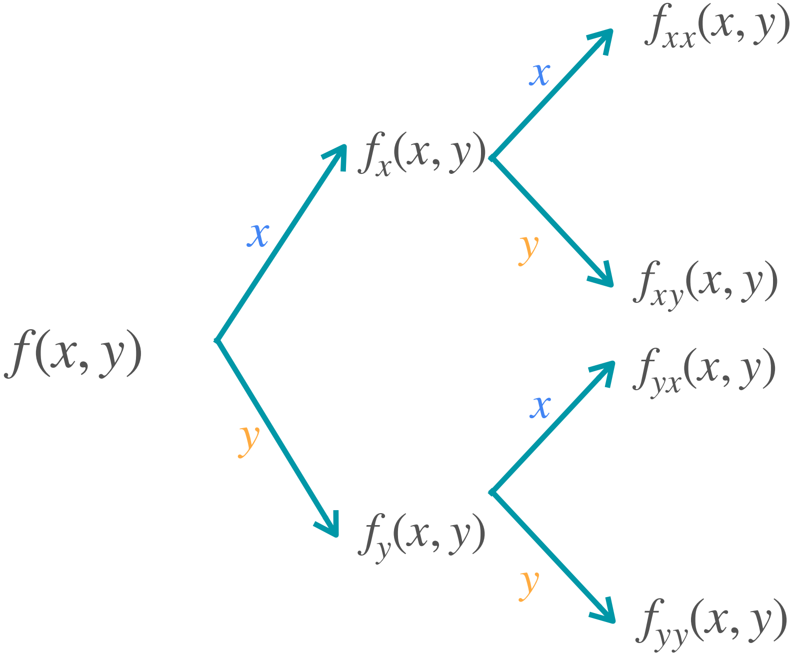


$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix}$$

$f_{yx}(x, y)$

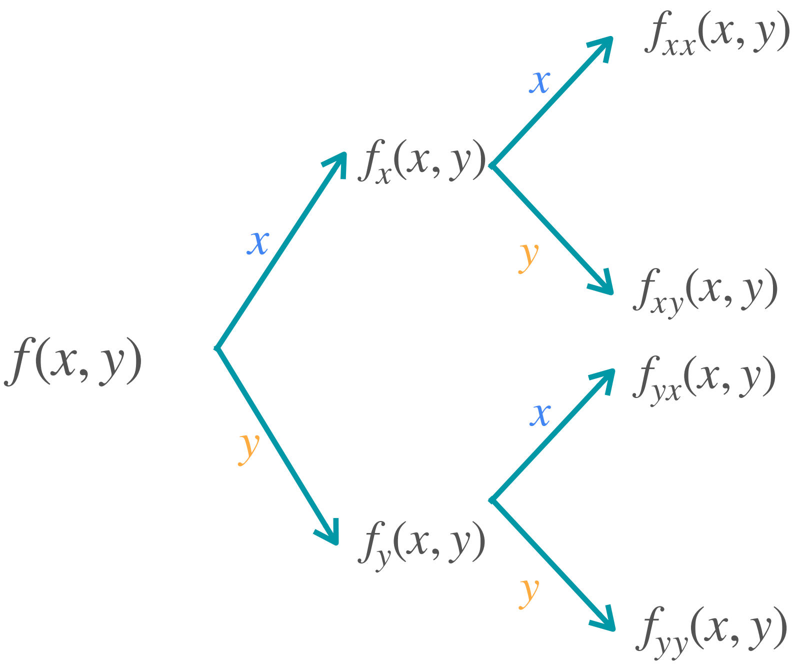
$f_{yy}(x, y)$

Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$\begin{matrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{matrix}$$

Hessian Matrix - General Case

$$
\begin{tikzcd}
 & & f_{xx}(x,y) \\
 & f_x(x,y) \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & f_{xy}(x,y) \\
f(x,y) \arrow[uur, "x"] \arrow[ddr, "y"'] & & \\
 & & f_{yx}(x,y) \\
 & f_y(x,y) \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & f_{yy}(x,y)
\end{tikzcd}
$$

$$\nabla f_x = \begin{bmatrix} f_{xx}(x,y) \\ f_{xy}(x,y) \end{bmatrix} \qquad \nabla f_y' = \begin{bmatrix} f_{yx}(x,y) \\ f_{yy}(x,y) \end{bmatrix}$$

$$\begin{bmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{bmatrix} = \begin{bmatrix} \nabla f_x^{\,T} \\ \nabla f_y^{\,T} \end{bmatrix}$$

Hessian Matrix - General Case

$$
\begin{tikzcd}
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 & f_y(x,y) \arrow[ur, "x"] \arrow[dr, "y"'] & \\
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\end{tikzcd}
$$

$$\nabla f_x = \begin{bmatrix} f_{xx}(x,y) \\ f_{xy}(x,y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x,y) \\ f_{yy}(x,y) \end{bmatrix}$$

$$H = \begin{bmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{bmatrix} = \begin{bmatrix} \nabla f_x{}^T \\ \nabla f_y{}^T \end{bmatrix}$$

Hessian matrix

All information about second derivatives

Second Derivative

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$		

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$	$H(x, y) = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix}$	

DeepLearning.AI

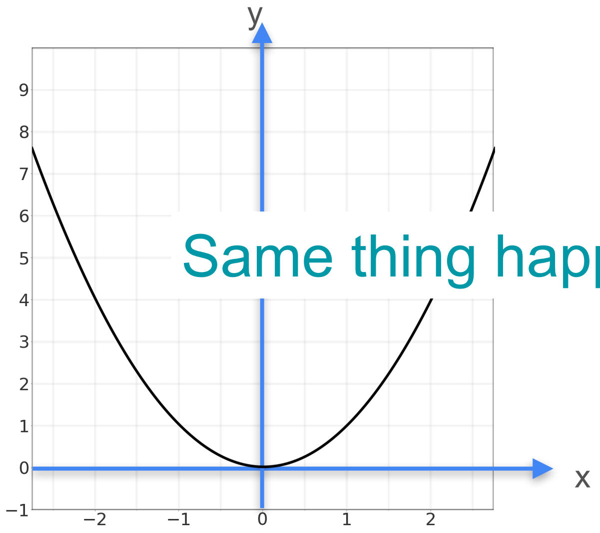
Optimization in Neural Networks and Newton's Method

Hessians and concavity

Remember…

Same thing happens for many variables!

Remember…

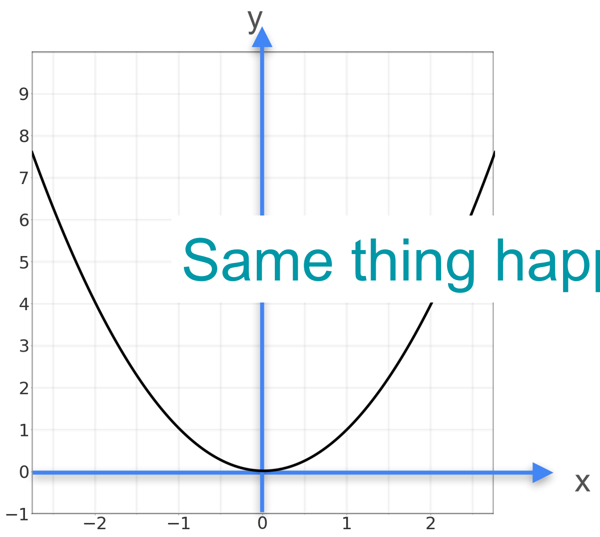


Same thing happens for many variables!

Concave up or convex

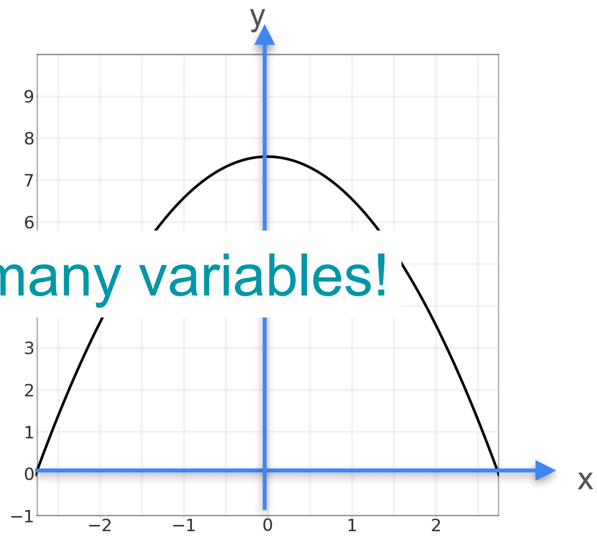
$$f''(0) > 0$$

Remember…



Concave up or convex

$$f''(0) > 0$$



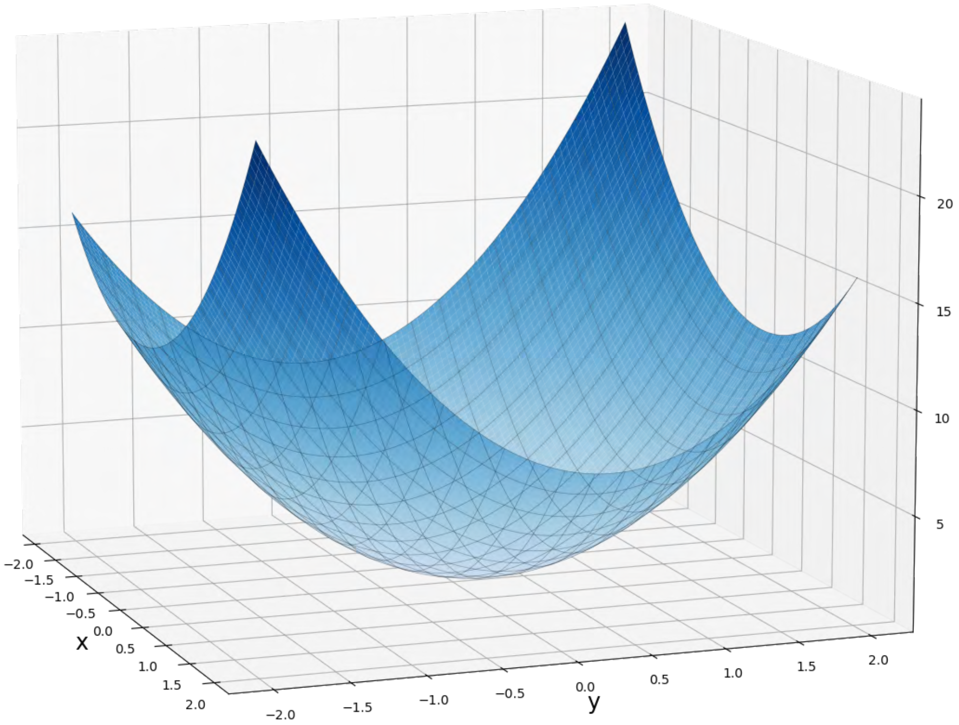
Concave down

$$f''(0) < 0$$

Same thing happens for many variables!

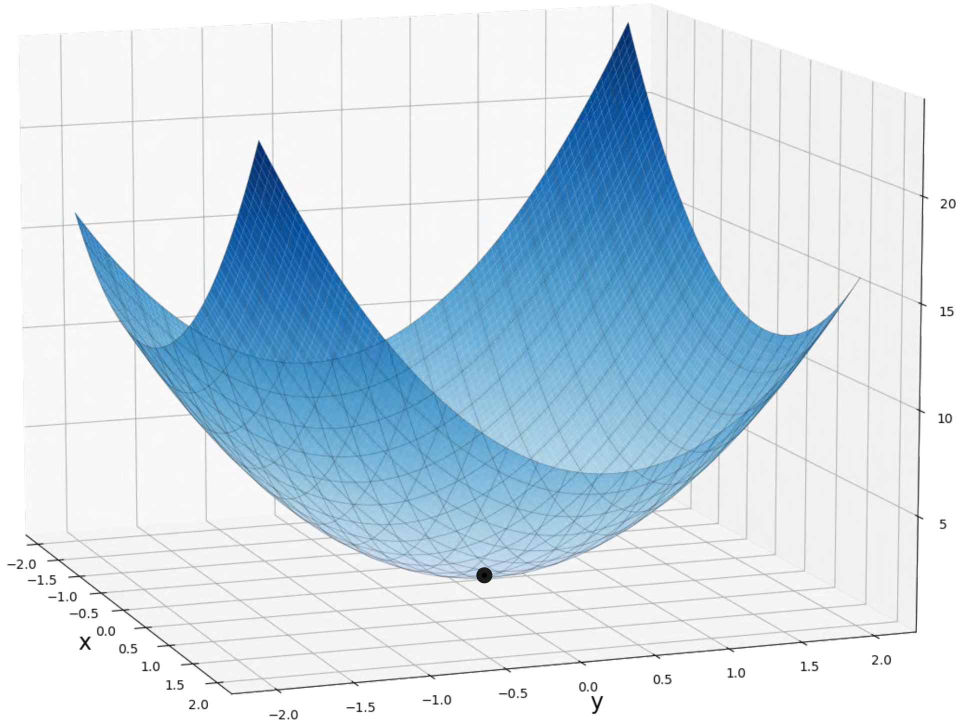
Concave Up

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

Concave Up



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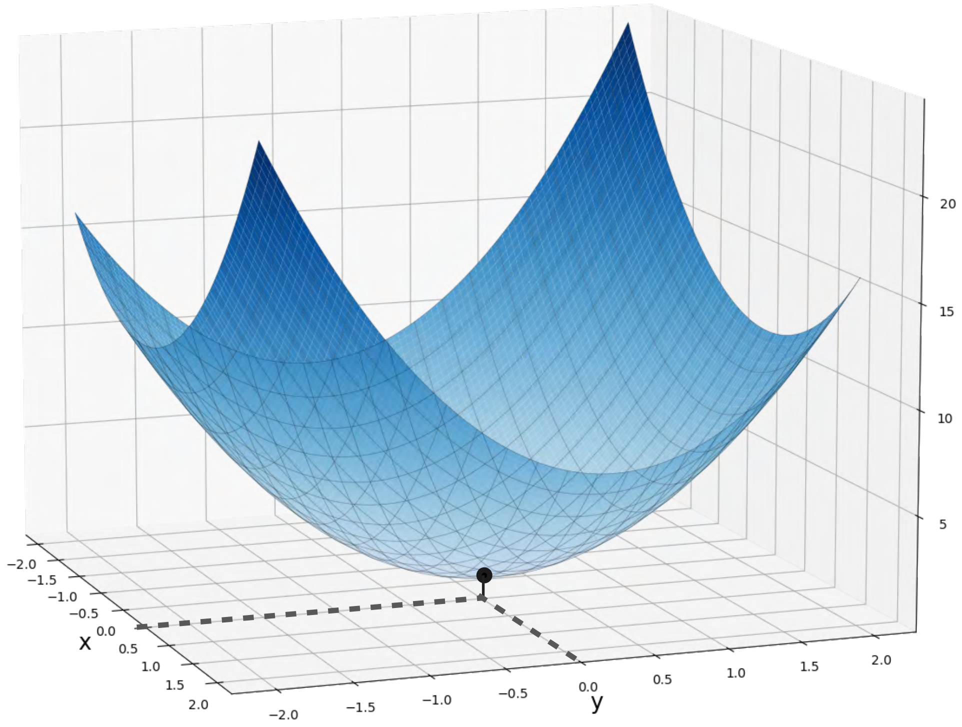
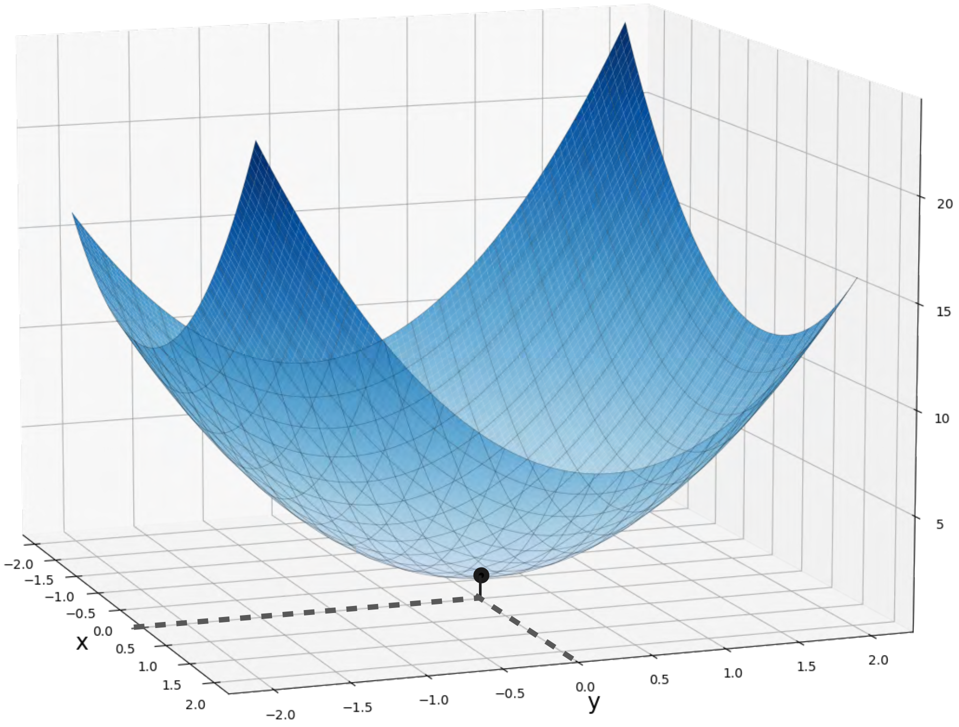
Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

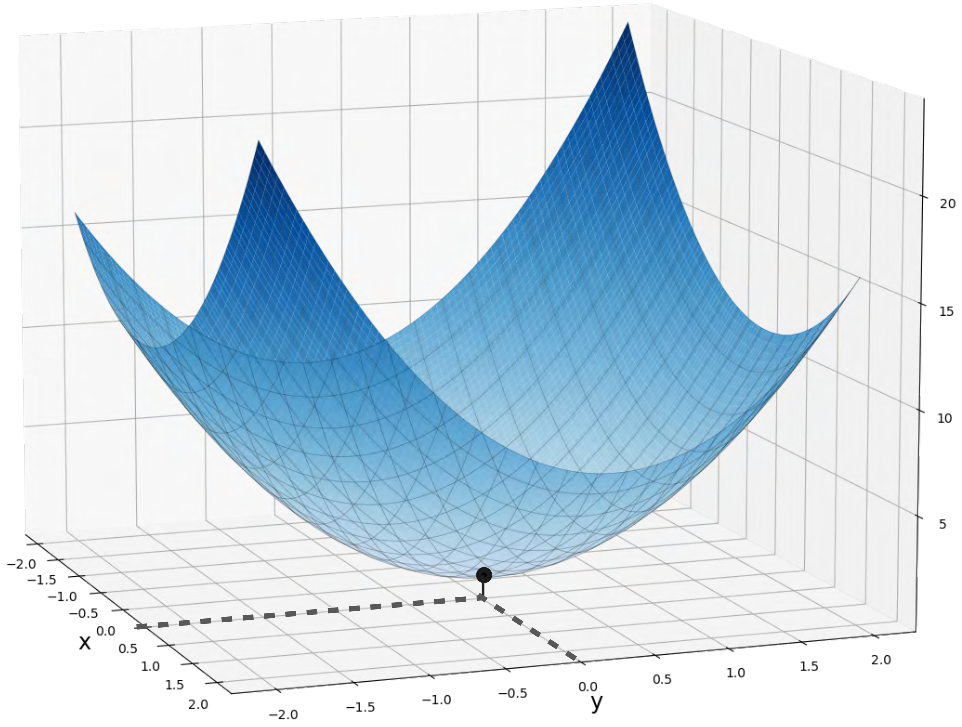
Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

Concave Up

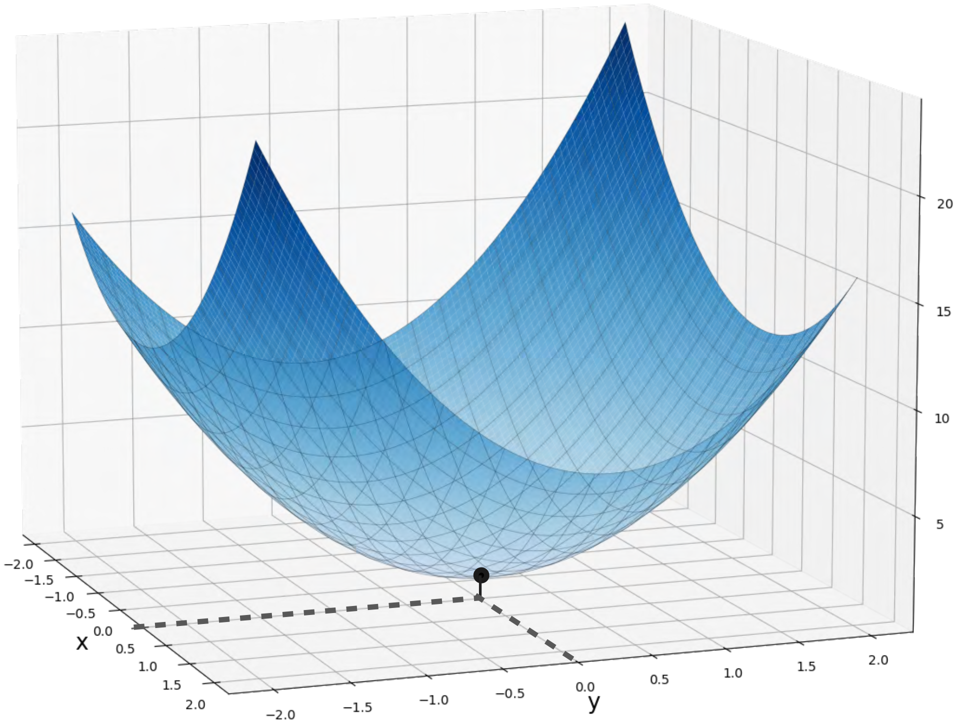


$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

Concave Up

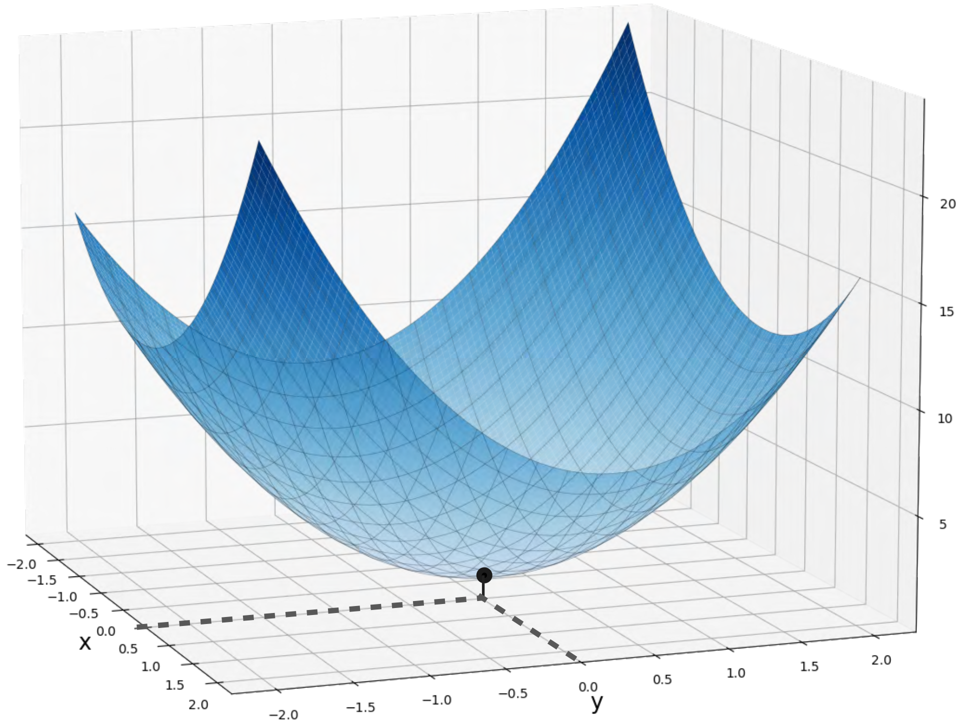


$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

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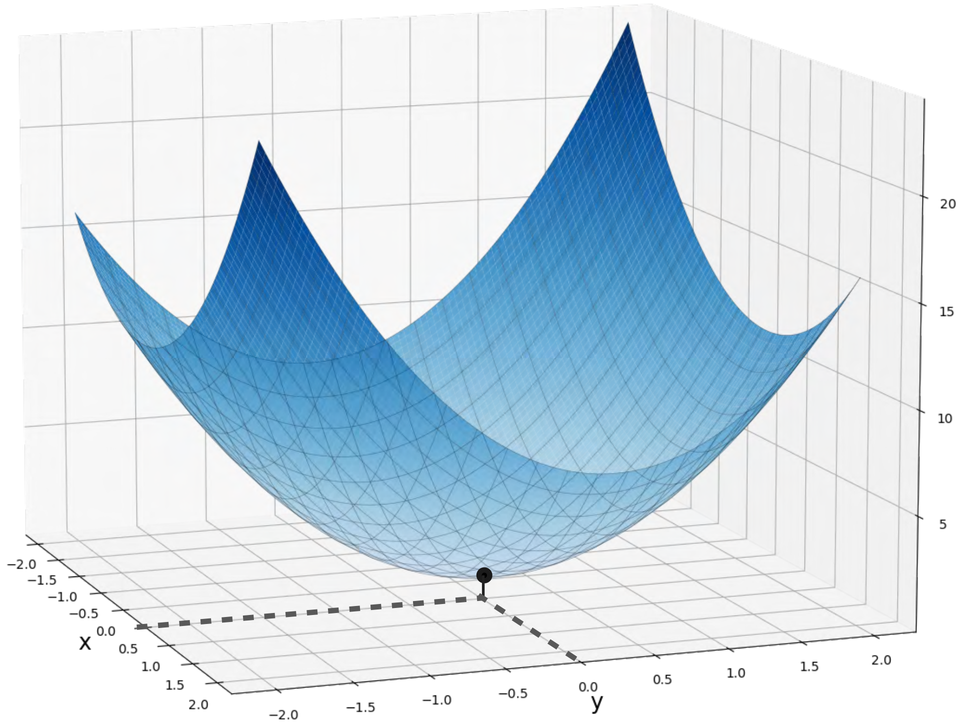
$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

Concave Up



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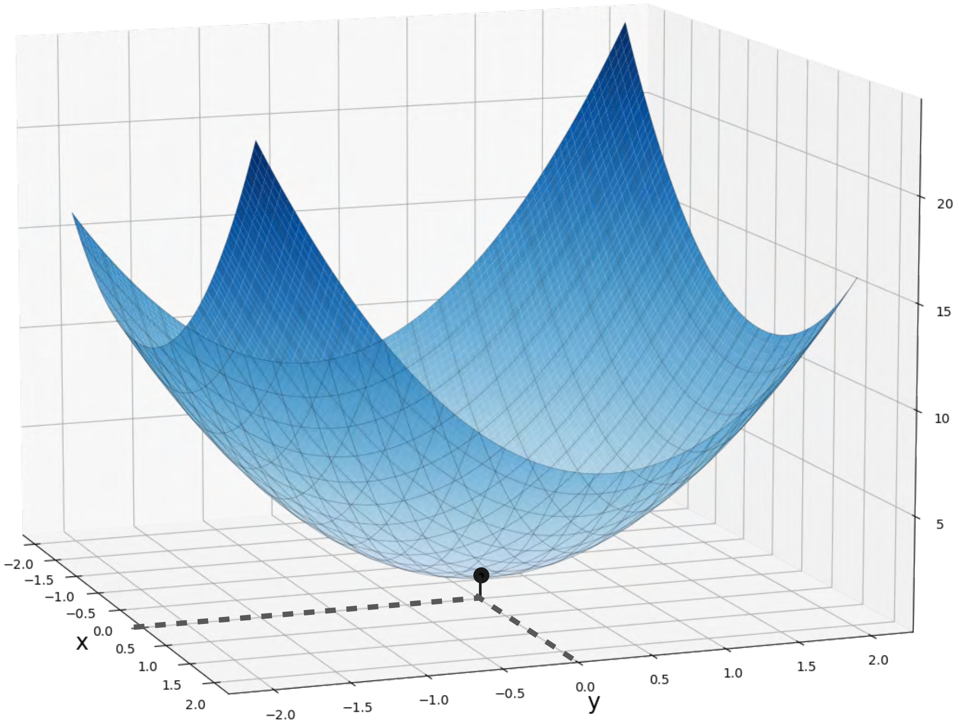
$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

Concave Up



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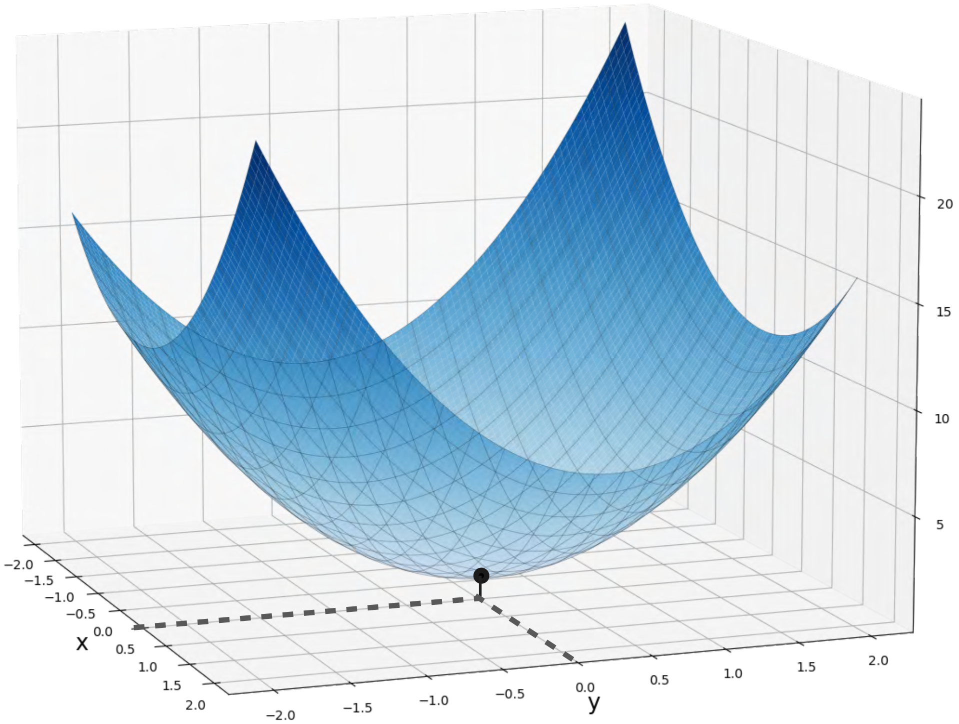
$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23 \rightarrow \lambda_1 = 6.41$$

Concave Up



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$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

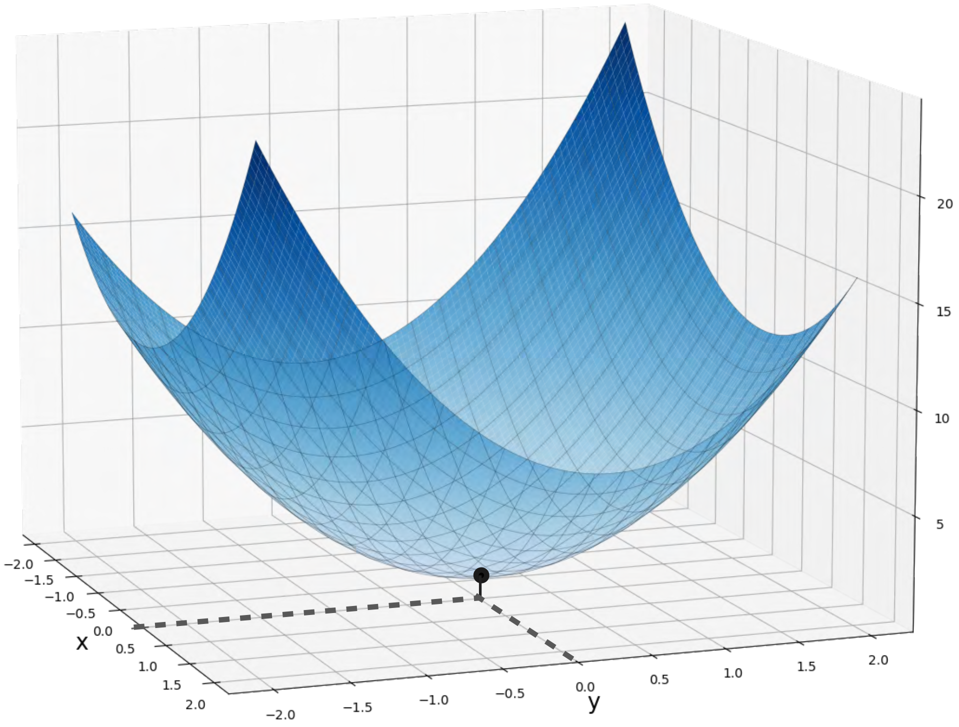
$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\lambda_1 = 6.41$$

$$\lambda_2 = 3.59$$

Concave Up



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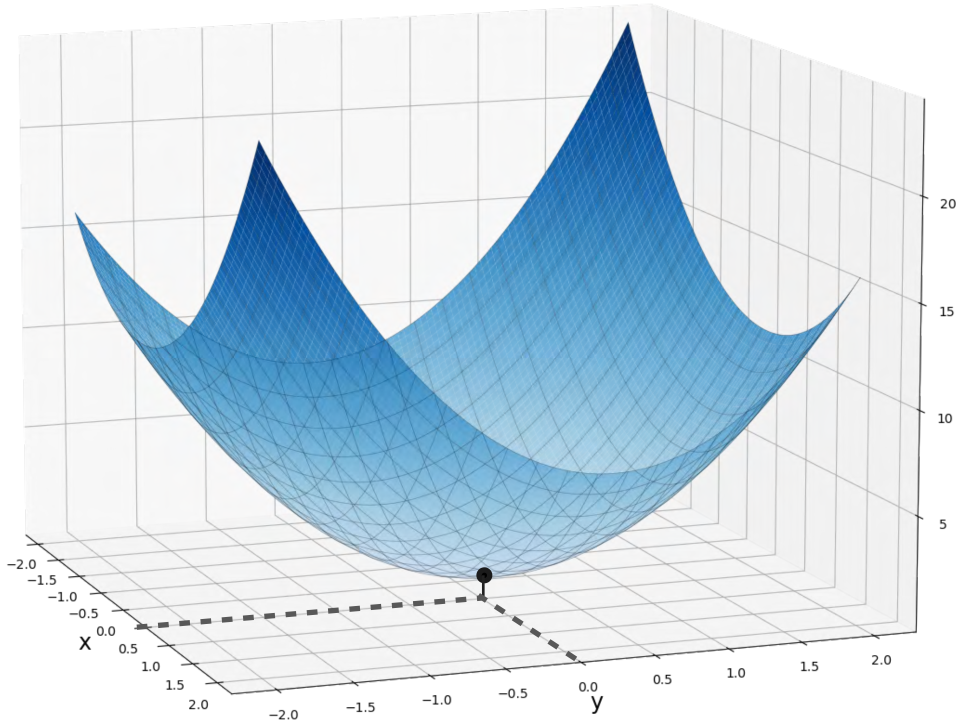
$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\boxed{\begin{aligned} \lambda_1 &= 6.41 \\ \lambda_2 &= 3.59 \end{aligned}}$$

> 0

Concave Up



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$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

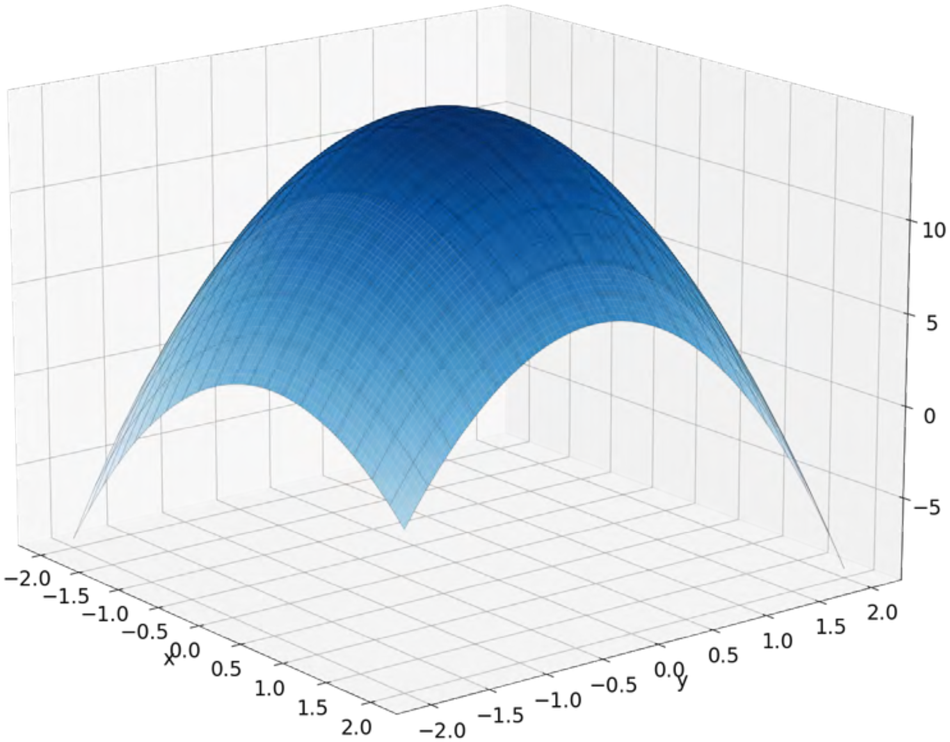
$$\boxed{\begin{matrix} \lambda_1 = 6.41 \\ \lambda_2 = 3.59 \end{matrix}}$$

> 0

$(0,0)$ is a minimum!

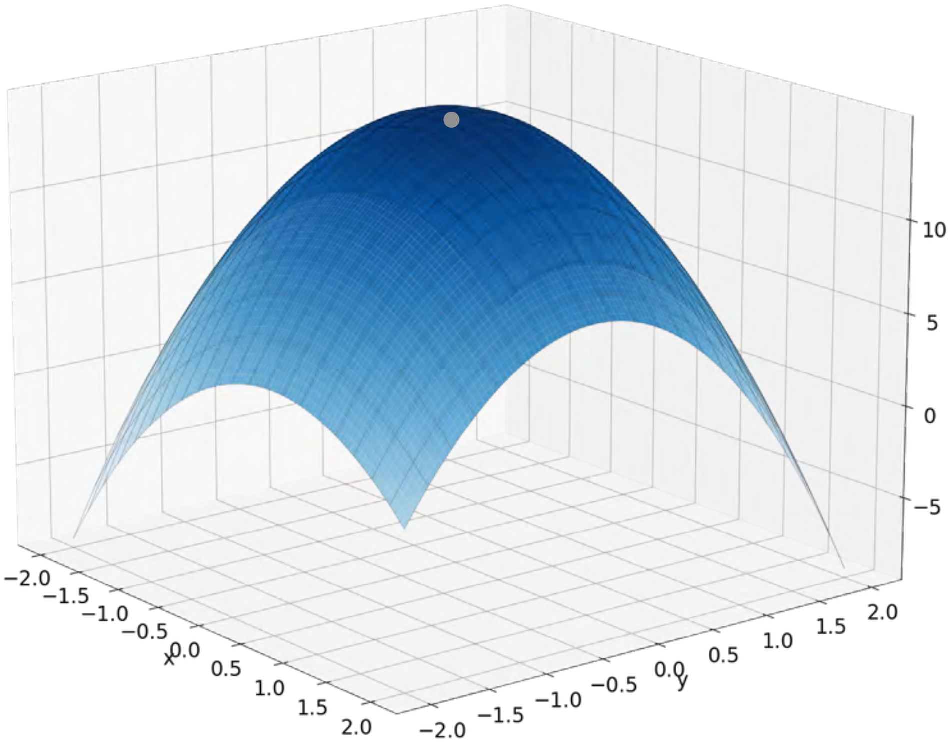
Concave Down

Concave Down



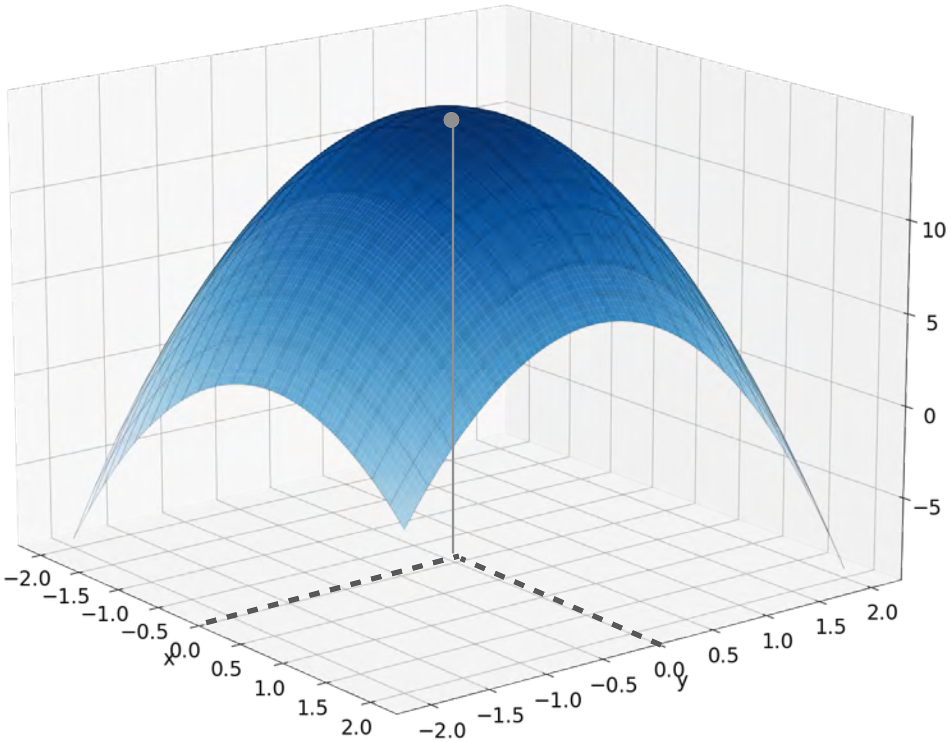
$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

Concave Down



$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

Concave Down



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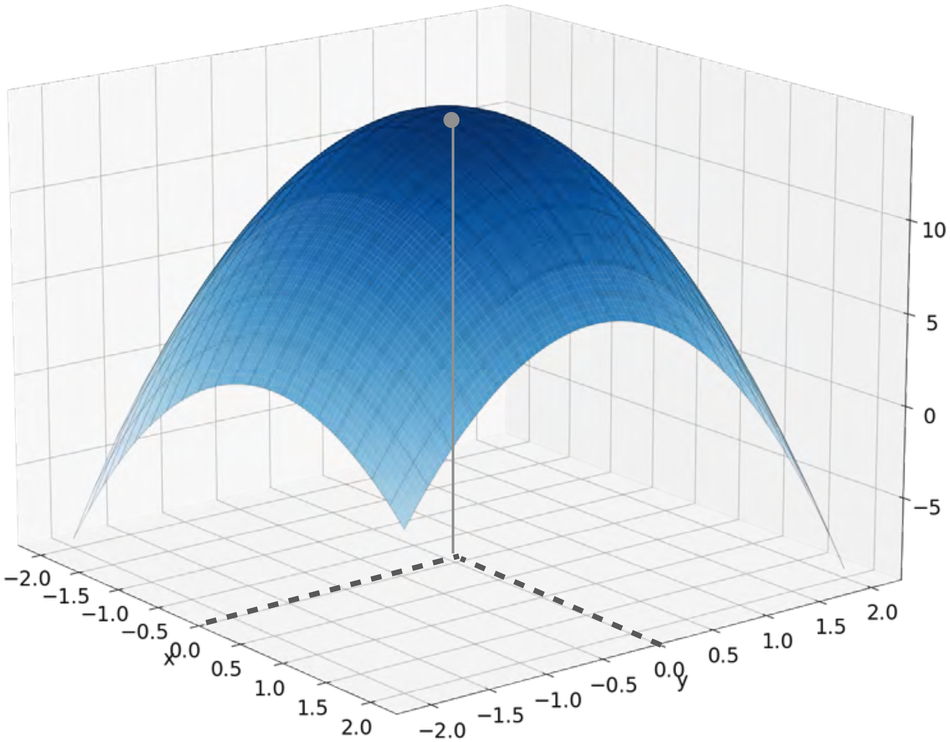
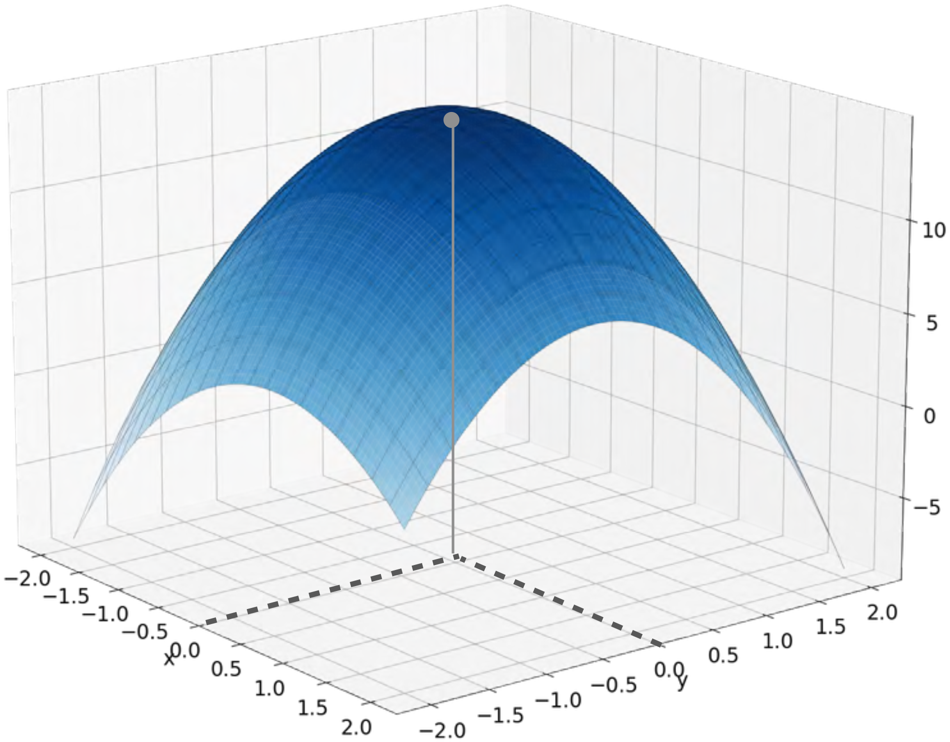
Concave Down



$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

Concave Down



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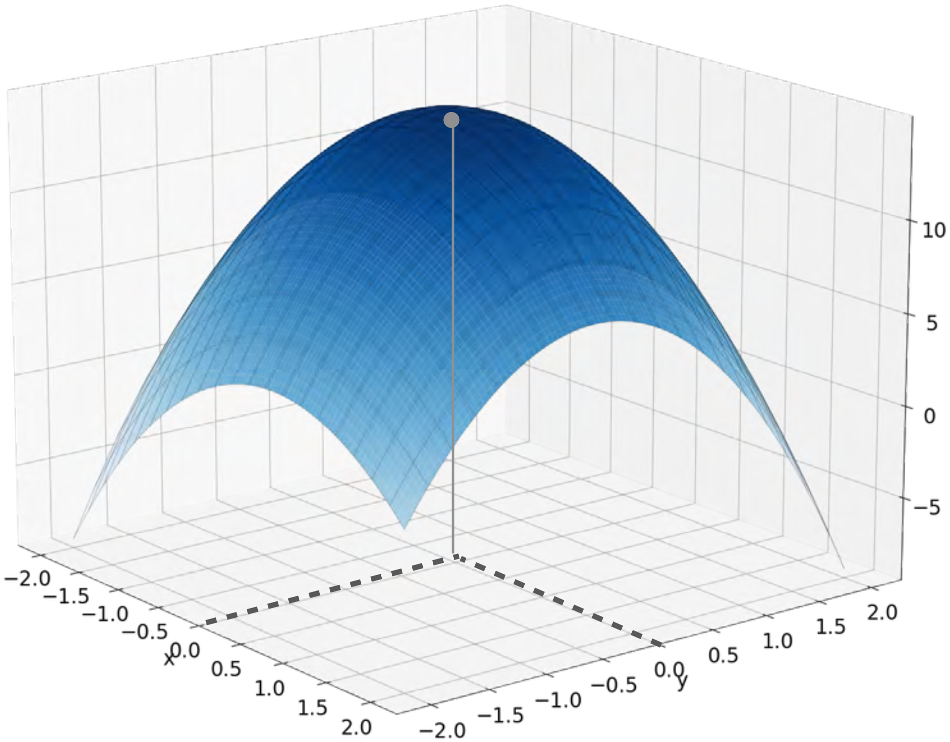
Concave Down



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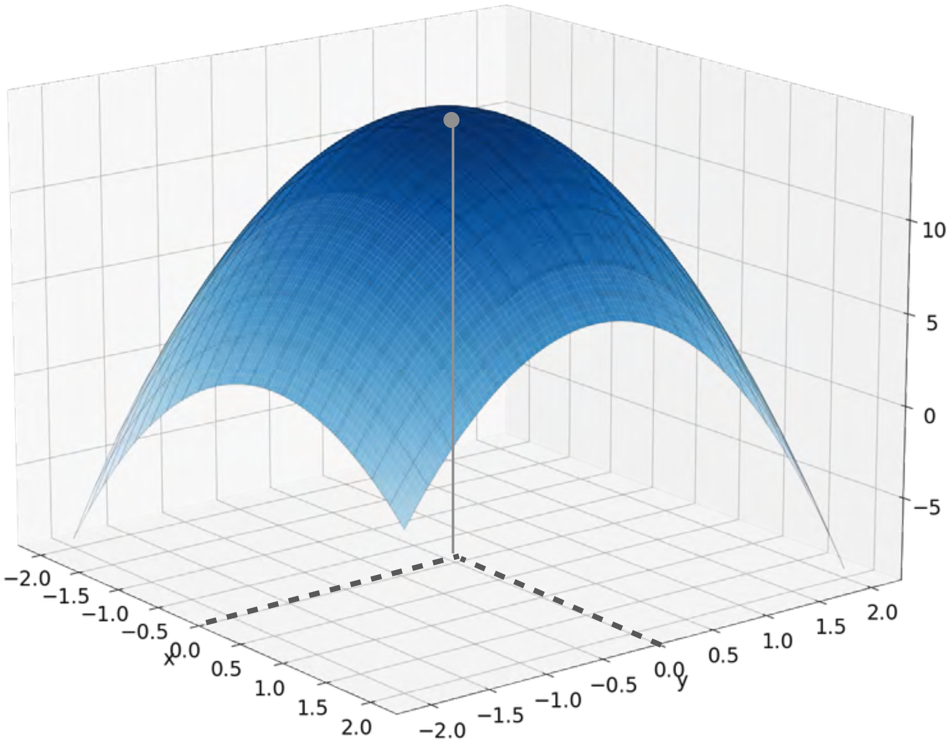
$$\det(H(0,0) - \lambda I) =$$

Concave Down



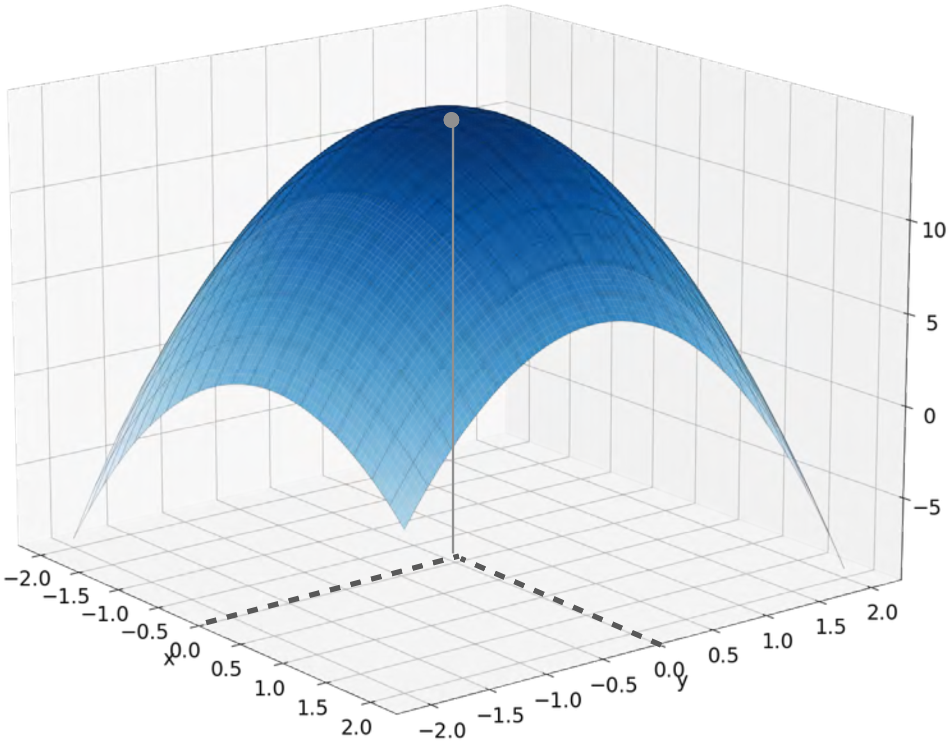
$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$
$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

Concave Down



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$$= \lambda^2 + 10\lambda + 23$$

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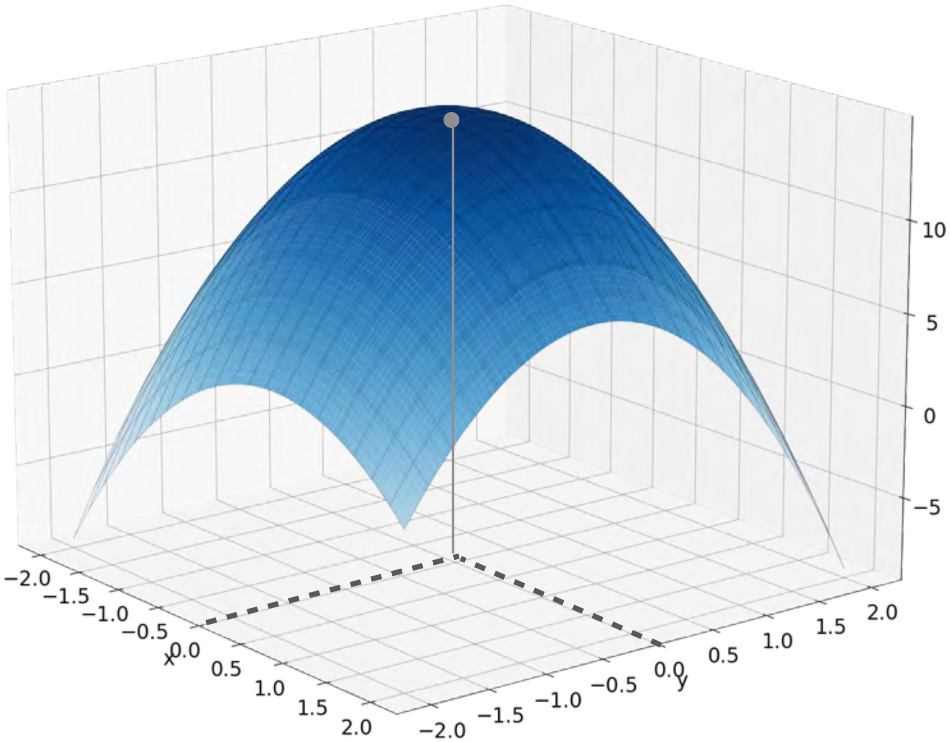
Concave Down



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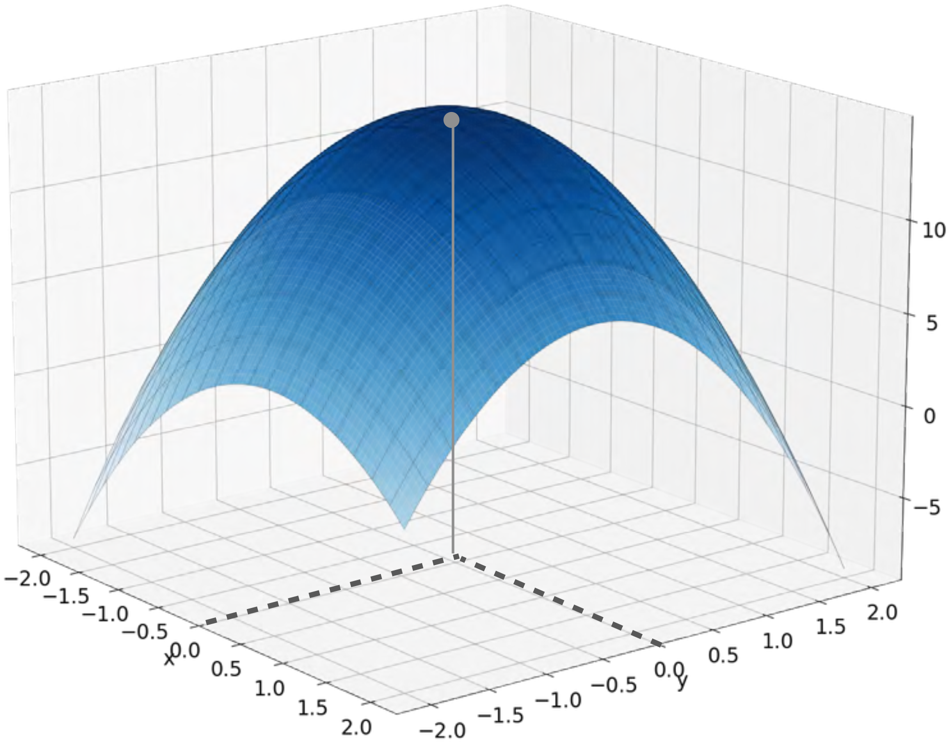
$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23 \rightarrow \lambda_1 = -3.49$$

Concave Down



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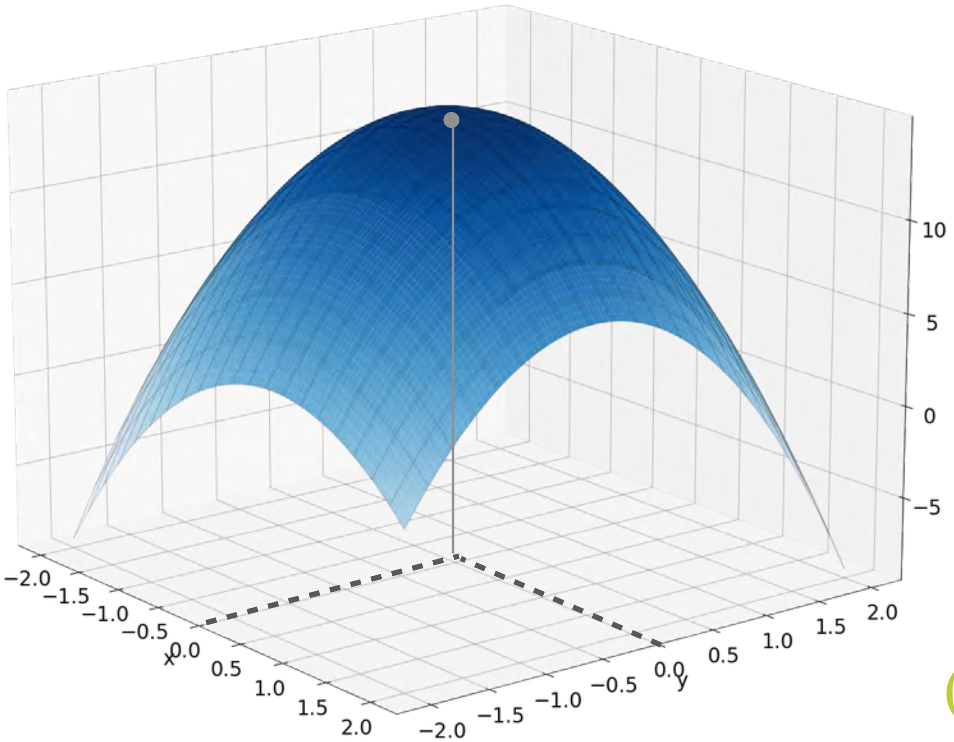
$$= \lambda^2 + 10\lambda + 23$$

$$\lambda_1 = -3.49$$

$$\lambda_2 = -6.41$$

Concave Down



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$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

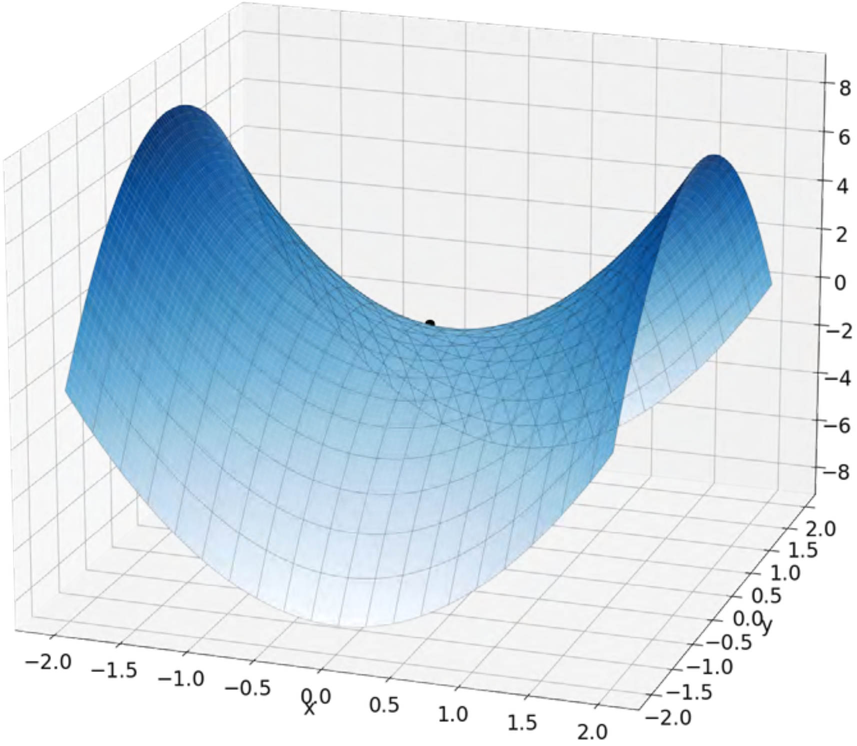
$$\boxed{\begin{aligned} \lambda_1 &= -3.49 \\ \lambda_2 &= -6.41 \end{aligned}}$$

< 0

(0,0) is a maximum!

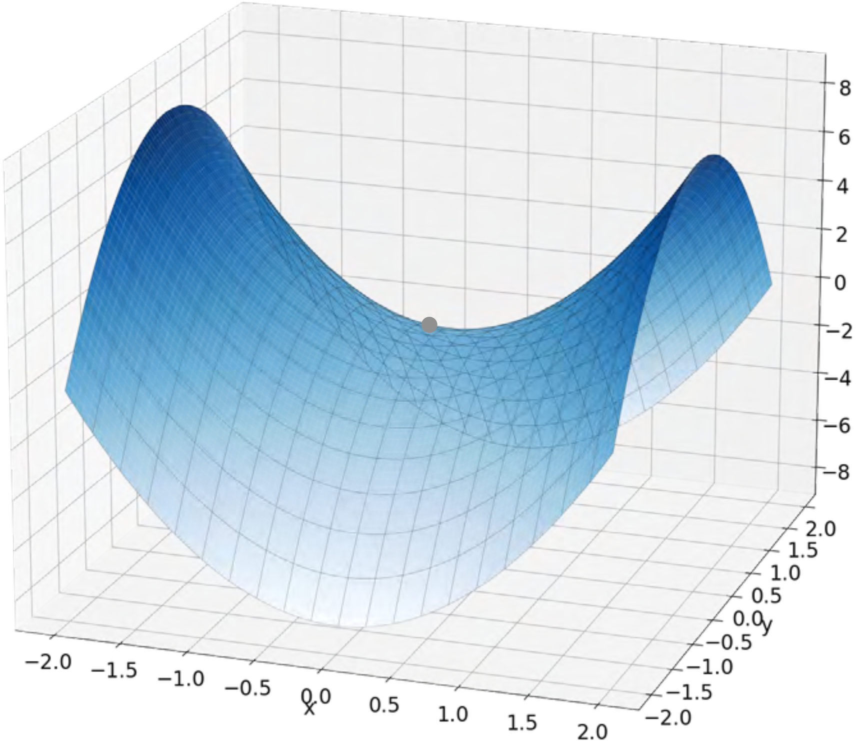
Saddle Point

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

Saddle Point



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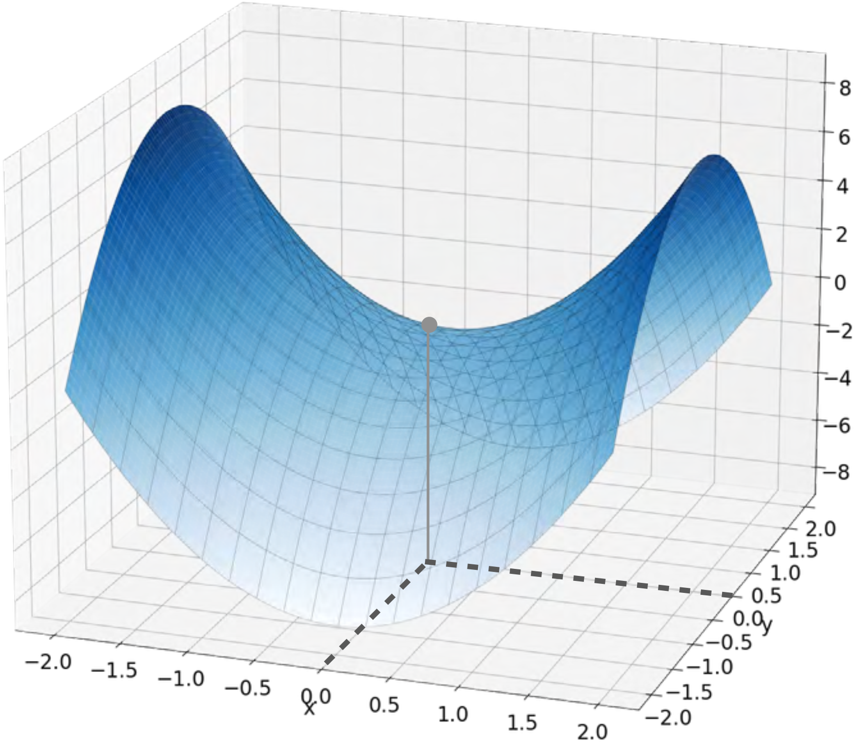

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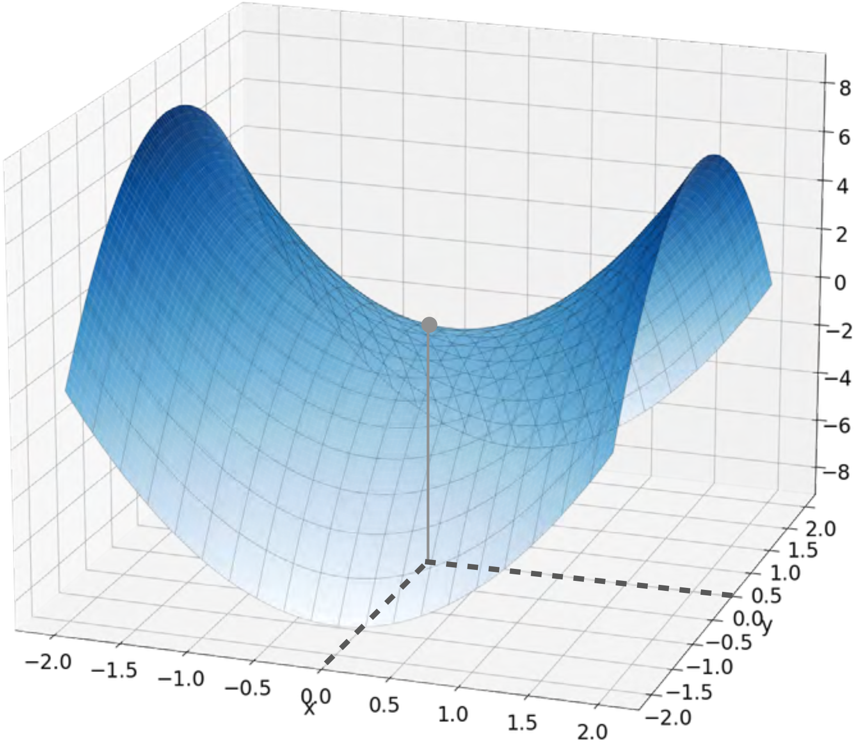
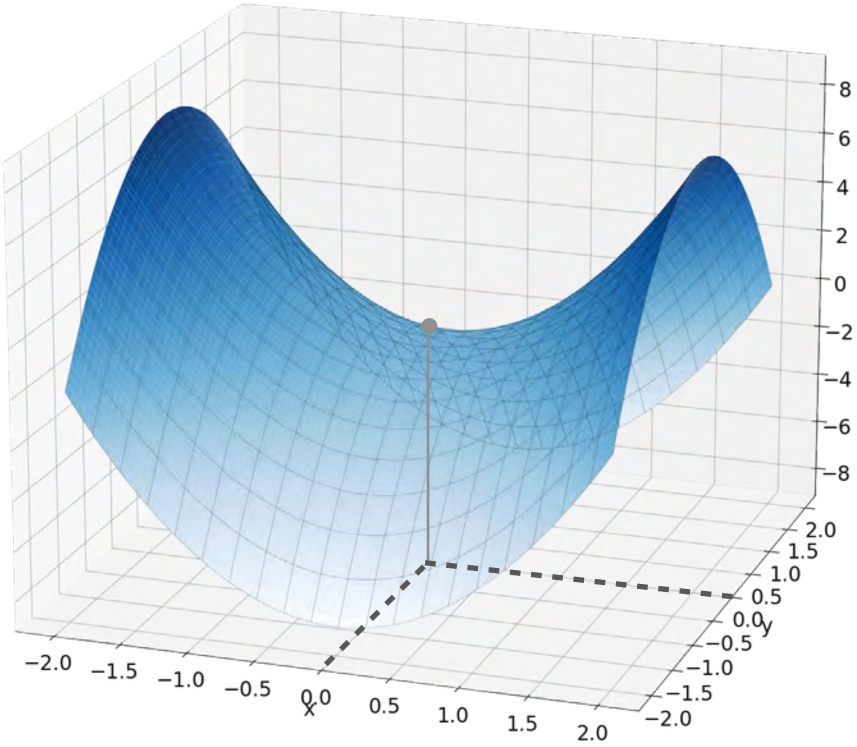
Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

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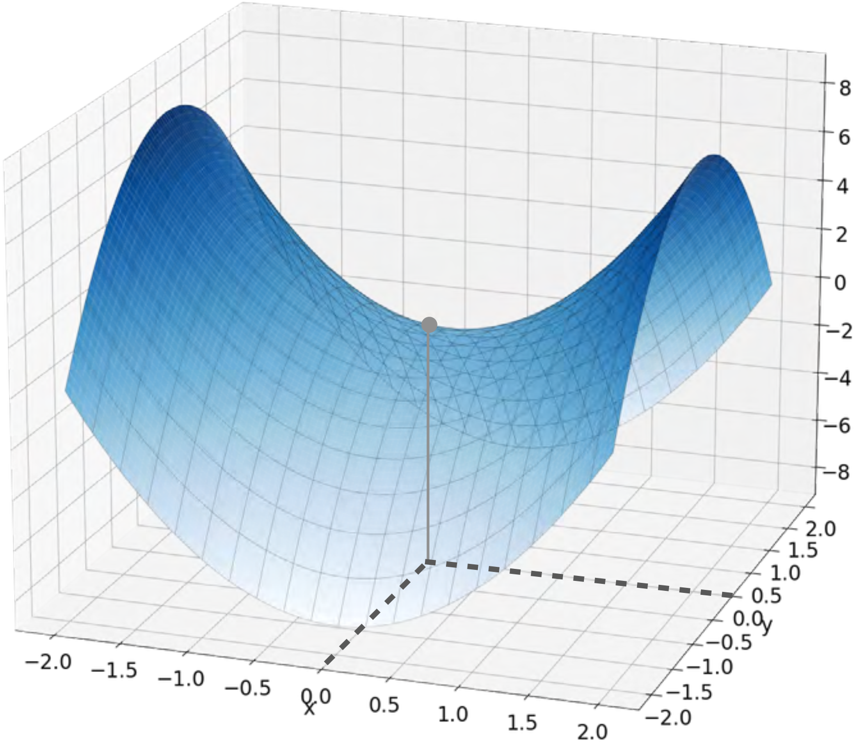
Saddle Point



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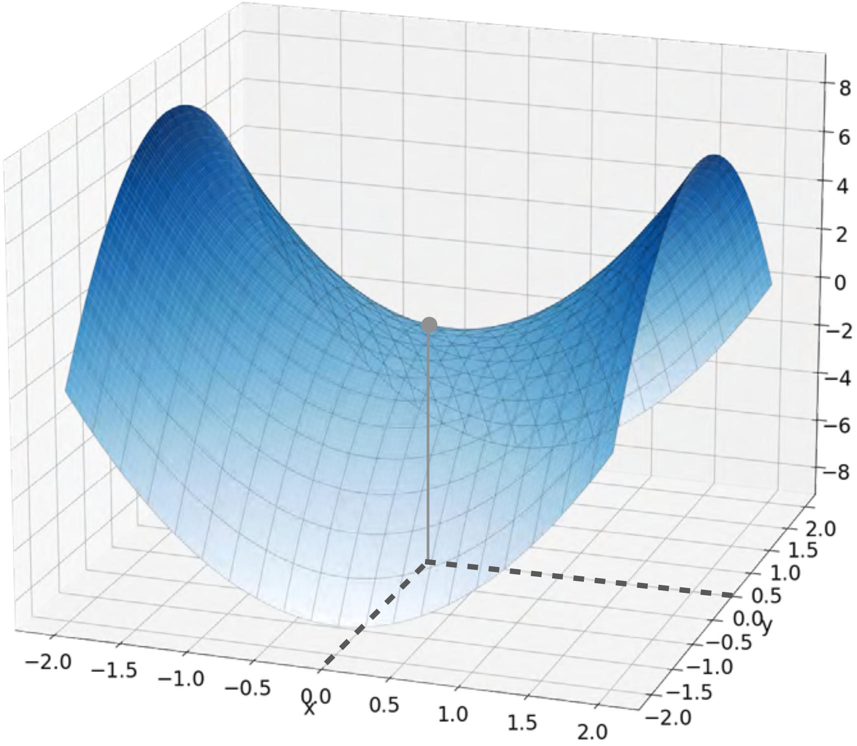
Saddle Point



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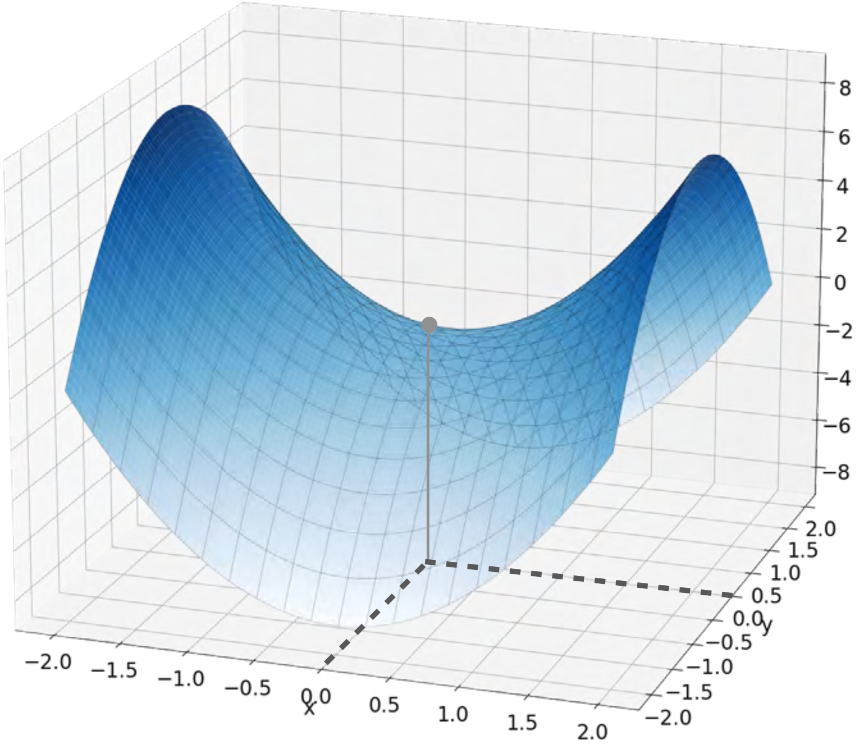
$$(4 - \lambda)(-4 - \lambda) - 0$$

$$\lambda_1 = -4$$

Saddle Point



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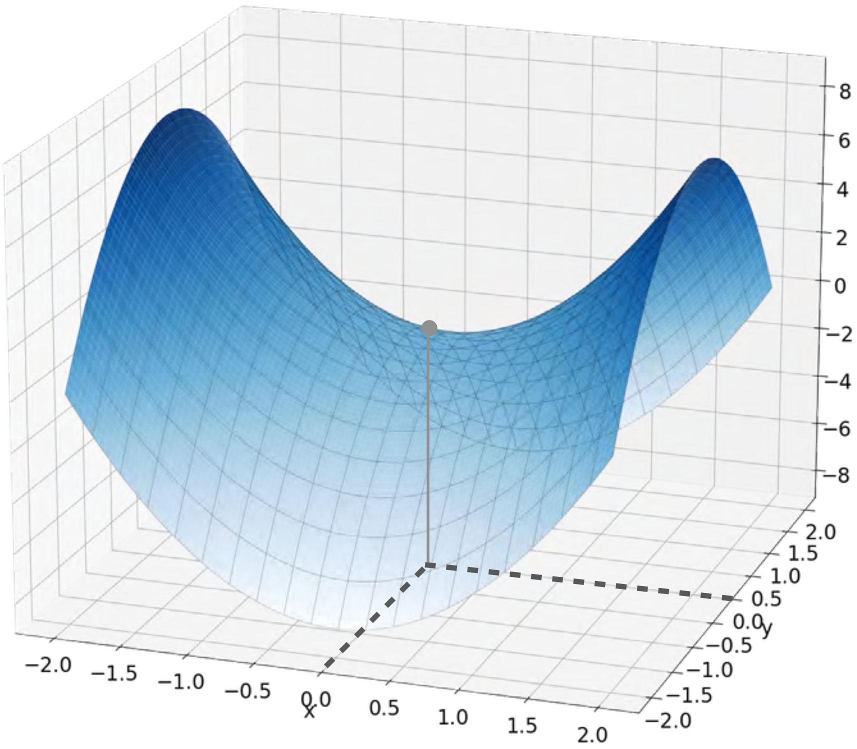
$$(4 - \lambda)(-4 - \lambda) - 0$$

$$\lambda_1 = -4$$

$$\lambda_2 = 4$$

Saddle Point



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$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

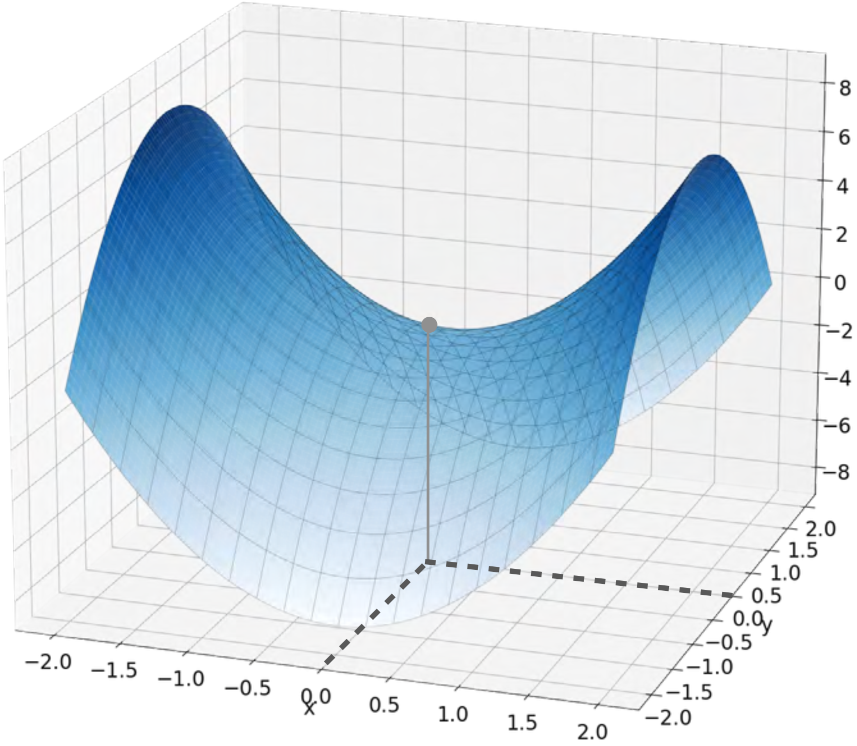
< 0

$$\boxed{\lambda_1 = -4}$$

$$\lambda_2 = 4$$

(0,0) is saddle point

Saddle Point



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$$(4 - \lambda)(-4 - \lambda) - 0$$

< 0

$$\boxed{\lambda_1 = -4}$$

$$\boxed{\lambda_2 = 4}$$

> 0

(0,0) is saddle point

Summary

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	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$

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(Local) minima	Happy face $f''(x) > 0$		

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	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	

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	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
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	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
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Need more information	$f''(x) = 0$		

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(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
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Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$ Or some $\lambda_i = 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
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Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$ Or some $\lambda_i = 0$	Some $\lambda_i > 0$ and some $\lambda_j < 0$ OR At least one $\lambda_i = 0$

Optimization in Neural Networks and Newton's Method

DeepLearning.AI

Newton's method for two variables

Newton's Method

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = \boxed{x_k} - \frac{f'(x_k)}{f''(x_k)}$$

Newton’s Method

1 variable

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Newton's Method

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Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} -$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - \boxed{f''(x_k)^{-1}} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k)$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - \boxed{f''(x_k)^{-1}}\,\boxed{f'(x_k)}$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k)\ \nabla f(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \; \nabla f(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \ \nabla f(x_k, y_k)$$

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2 variables

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$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} \cancel{- \nabla f(x_k, y_k) \ H^{-1}(x_k, y_k)}$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \underbrace{H^{-1}(x_k, y_k)}_{2\times 2} \underbrace{\nabla f(x_k, y_k)}_{2\times 1}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \cancel{\nabla f(x_k, y_k) H^{-1}(x_k, y_k)}$$

Newton's Method

2 variables

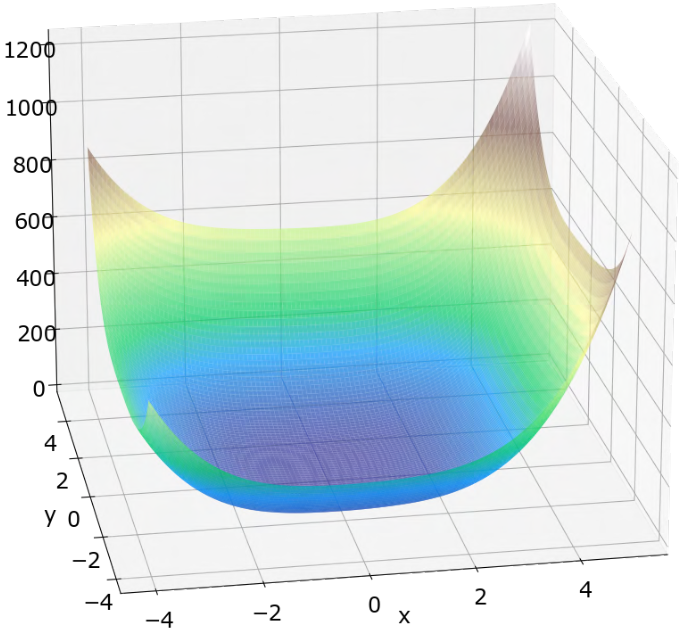
$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \underbrace{H^{-1}(x_k, y_k)}_{2\times 2} \underbrace{\nabla f(x_k, y_k)}_{2\times 1}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \cancel{\nabla f(x_k, y_k) H^{-1}(x_k, y_k)}$$

When working with 2 variables the order is crucial!

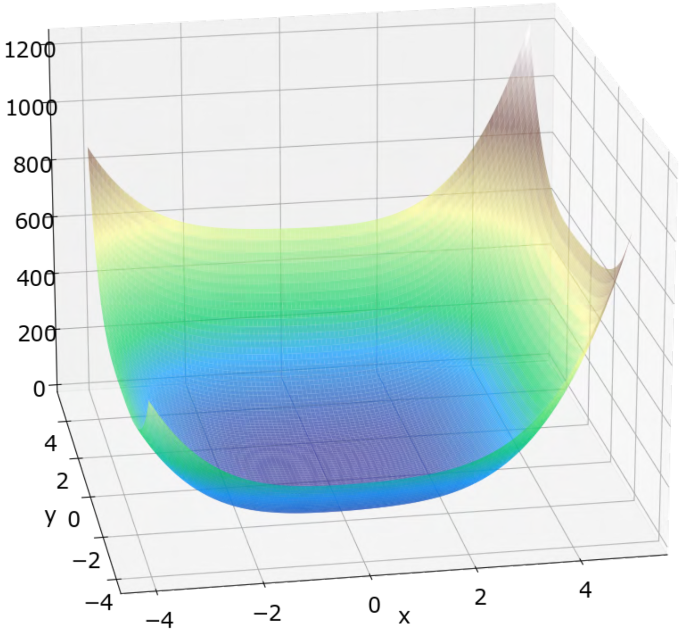
An Example

An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

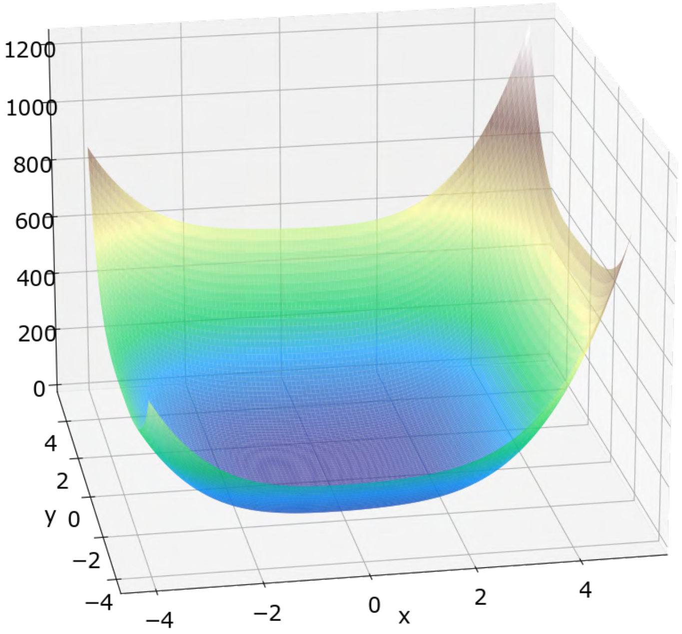
An Example



$$f(x,y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x,y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy$$

An Example



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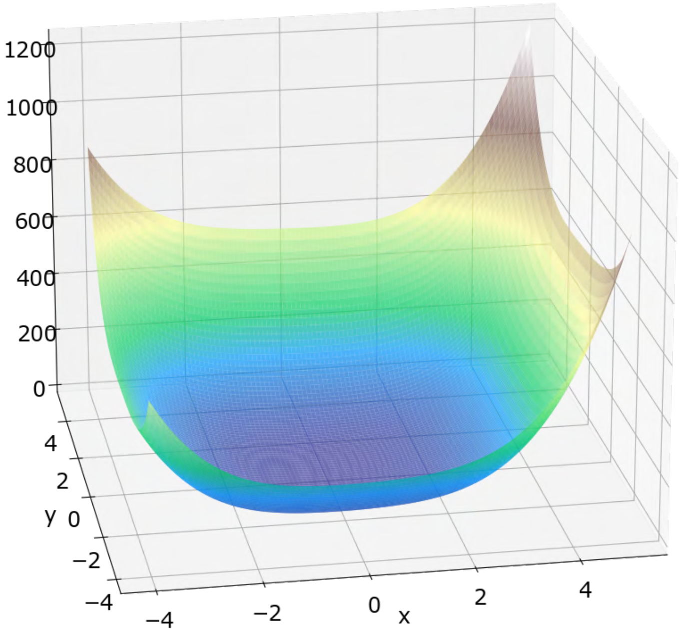
$$f(x, y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy$$

$$f(x, y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2$$

An Example

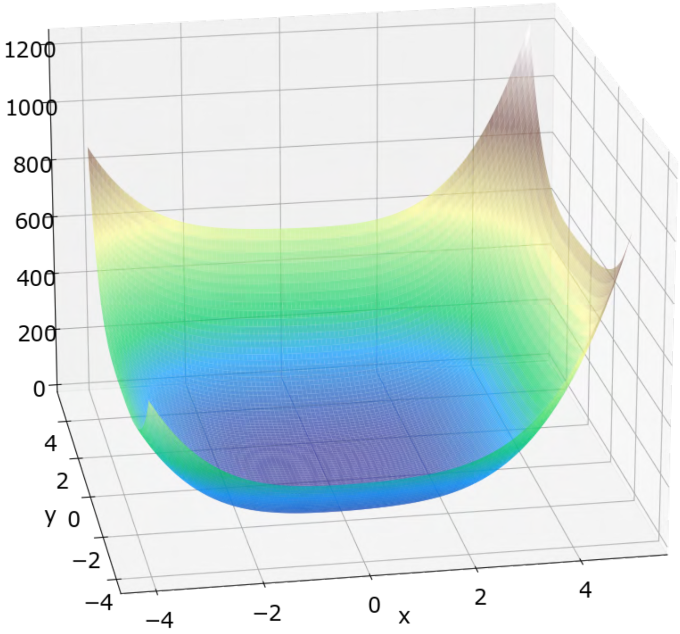


$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x, y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy \xrightarrow{x} 12x^2 + 8 - 0.4y$$

$$f(x, y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2$$

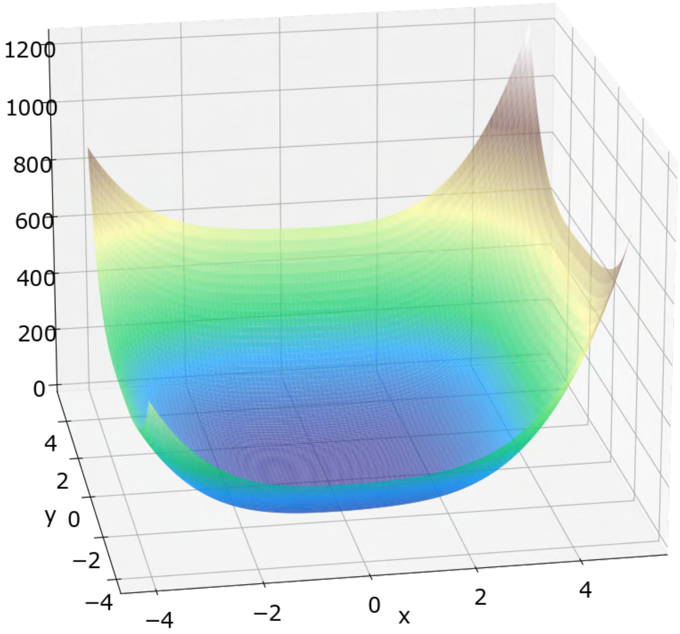
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$\begin{tikzcd}
 & & 12x^2 + 8 - 0.4y \\
 & 4x^3 + 8x - y - 0.4xy \arrow[ur, "x"] \arrow[dr, "y"'] & \\
f(x, y) \arrow[ur, "x"] \arrow[ddr, "y"'] & & -1 - 0.4x \\
 & & \\
 & 3.2y^3 + 4y - x - 0.2x^2 &
\end{tikzcd}$$

An Example



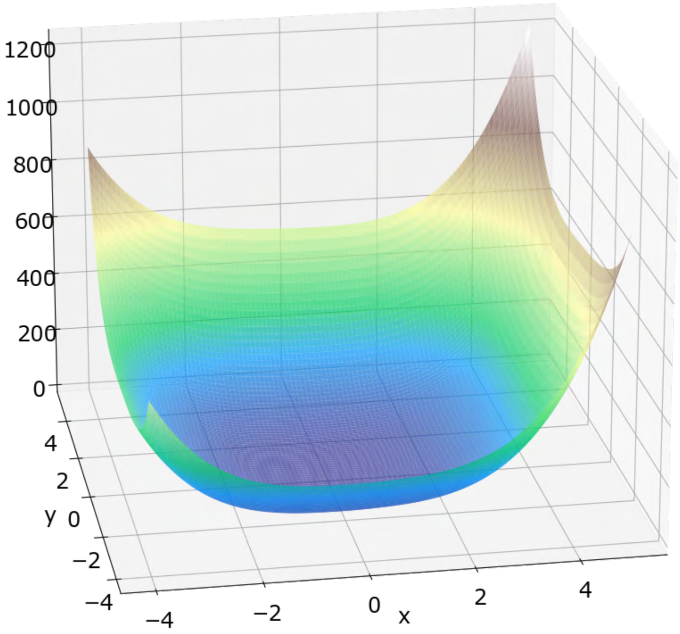
$$f(x,y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x,y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy \xrightarrow{x} 12x^2 + 8 - 0.4y$$

$$4x^3 + 8x - y - 0.4xy \xrightarrow{y} -1 - 0.4x$$

$$f(x,y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2 \xrightarrow{x} -1 - 0.4x$$

An Example



$$f(x,y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

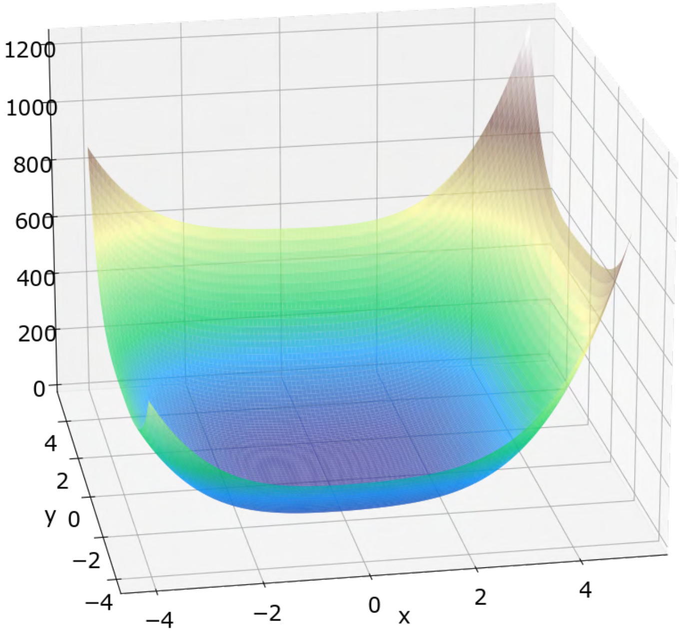
$$f(x,y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy \xrightarrow{x} 12x^2 + 8 - 0.4y$$

$$4x^3 + 8x - y - 0.4xy \xrightarrow{y} -1 - 0.4x$$

$$f(x,y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2 \xrightarrow{x} -1 - 0.4x$$

$$3.2y^3 + 4y - x - 0.2x^2 \xrightarrow{y} 9.6y^2 + 4$$

An Example



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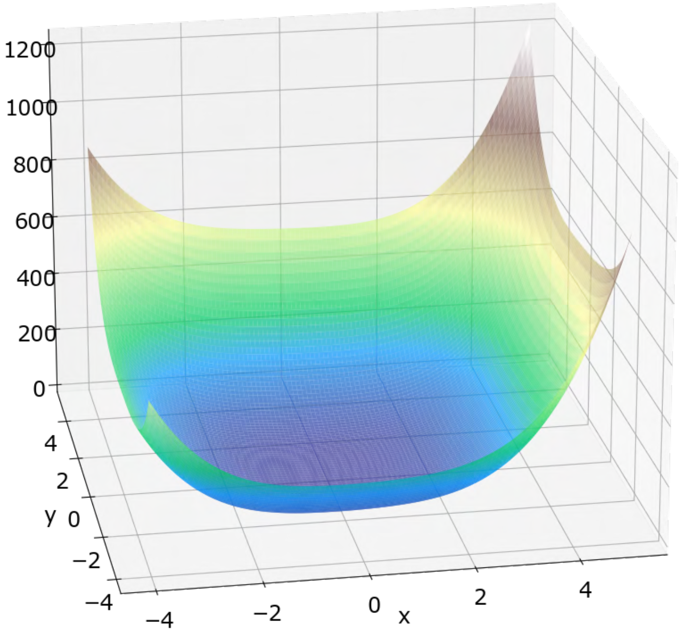
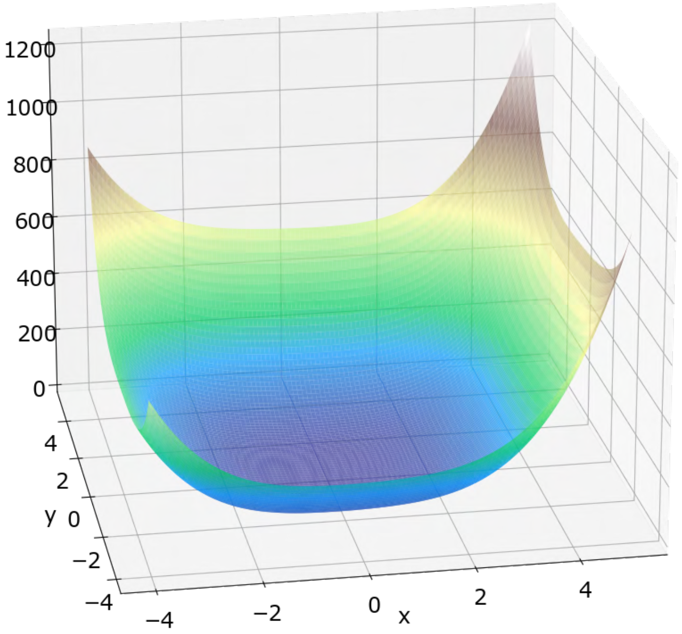


$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$\nabla f(x, y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

An Example

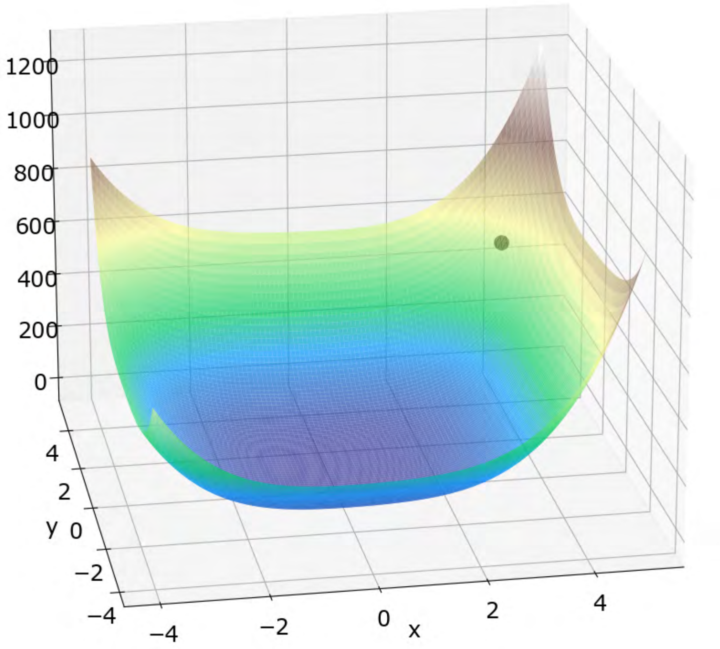


$$f(x,y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

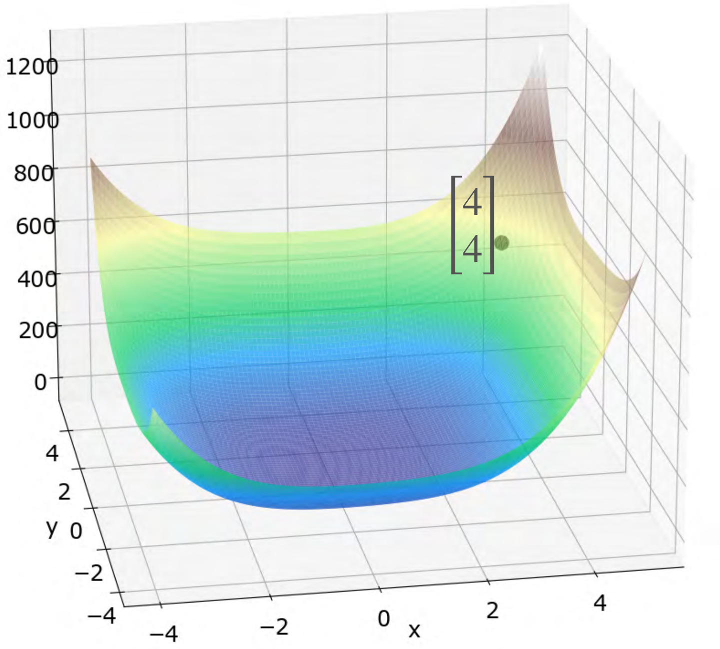
$$\nabla f(x,y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

$$H(x,y) = \begin{bmatrix} 12x^2 + 8 - 0.4y & -1 - 0.4x \\ -1 - 0.4x & 9.6y^2 + 4 \end{bmatrix}$$

An Example

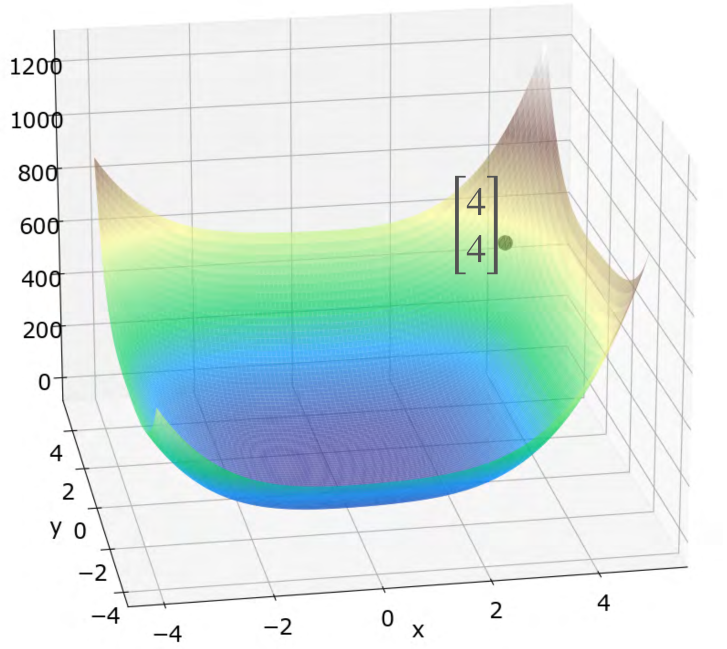


An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

An Example



Start at some point $\begin{bmatrix}x_0\\y_0\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix}277.6\\213.6\end{bmatrix}$$

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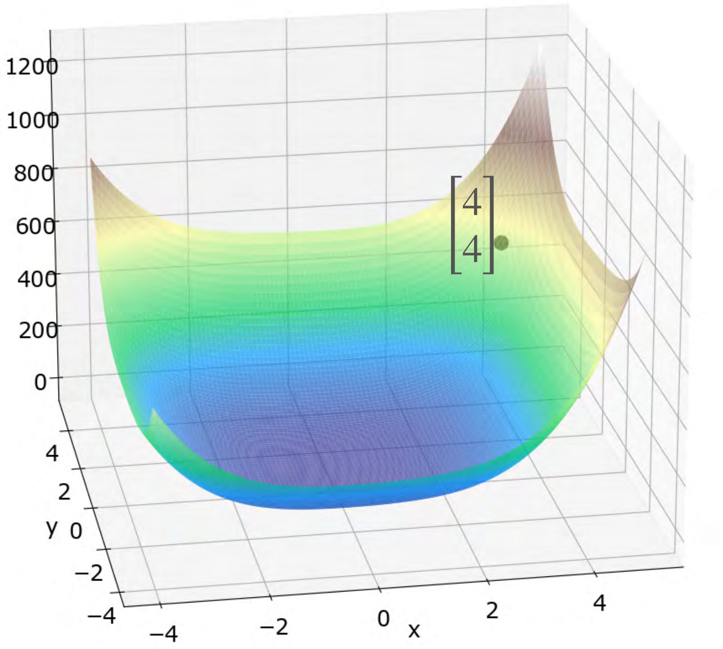
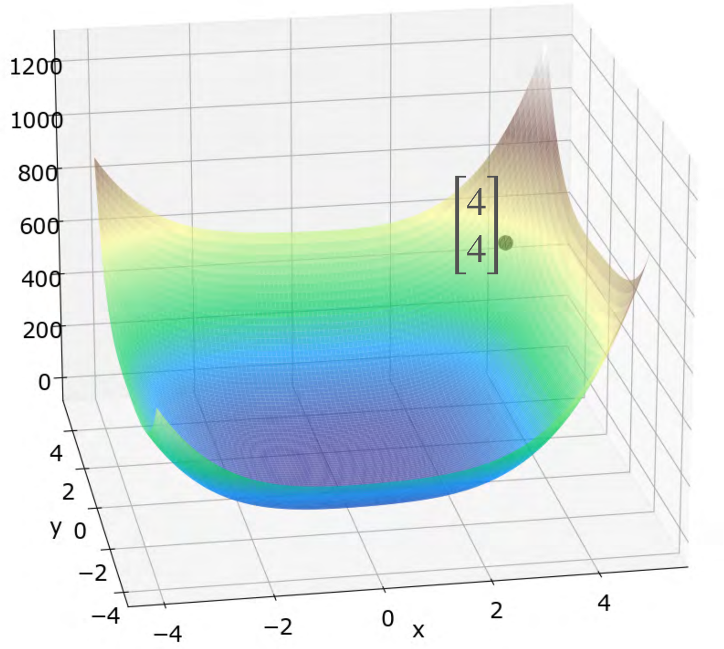
An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \qquad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

An Example

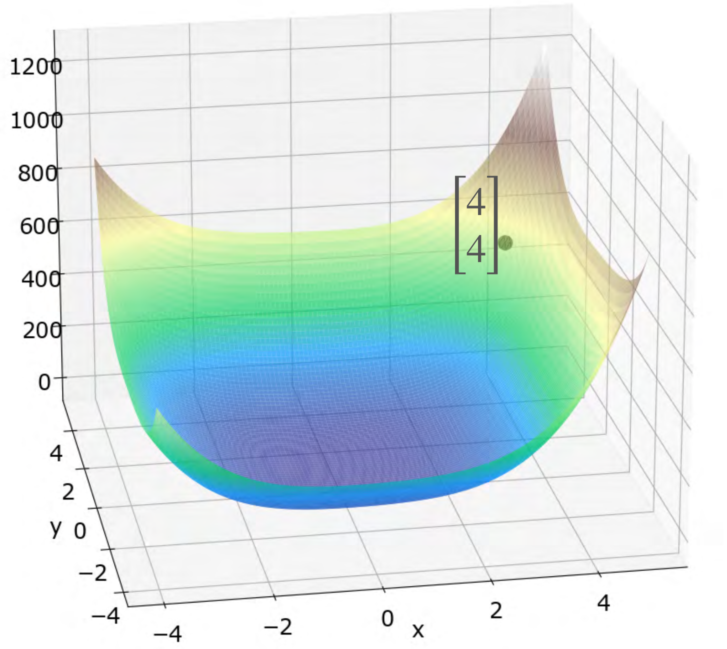


Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \qquad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} -$$

An Example



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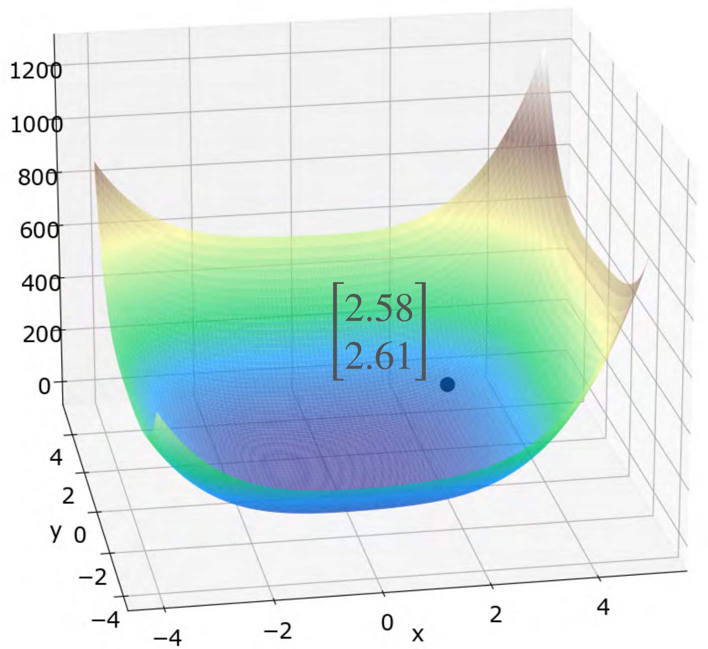
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}^{-1} \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix}$$

An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \qquad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

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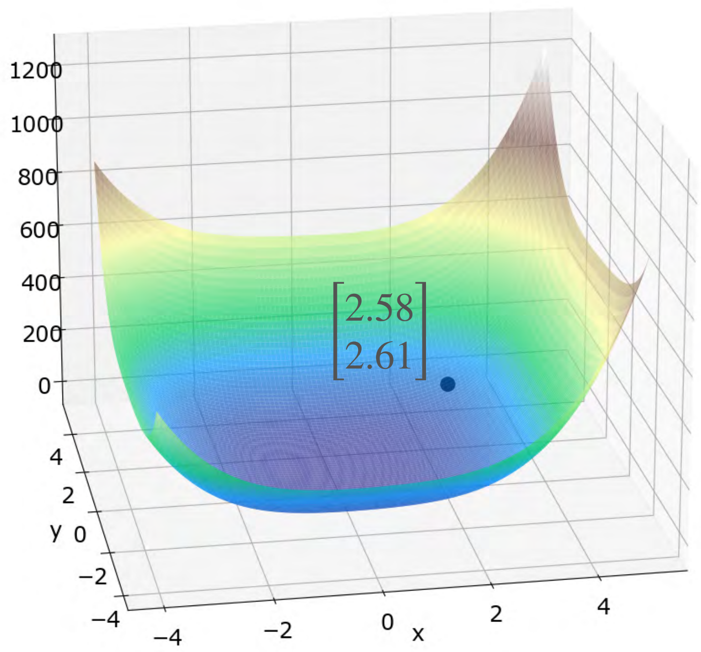
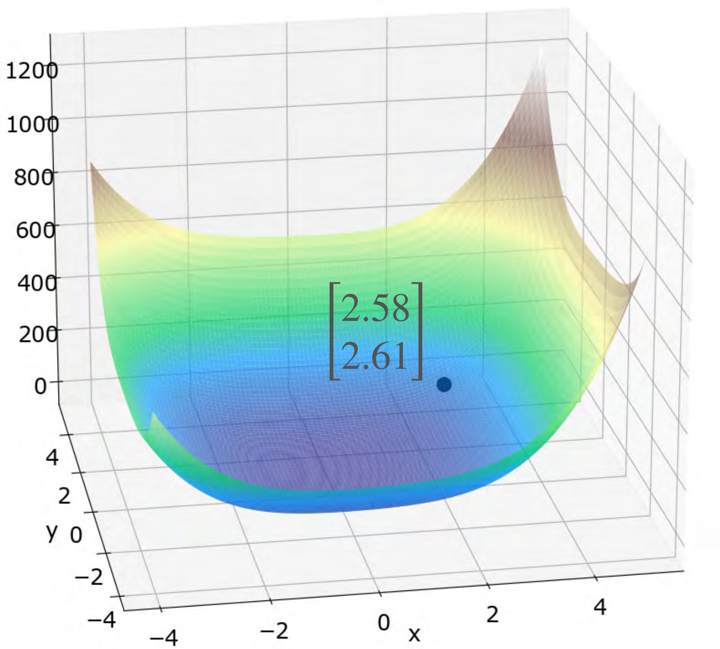
$$= \begin{bmatrix} 2.58 \\ 2.62 \end{bmatrix}$$

An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

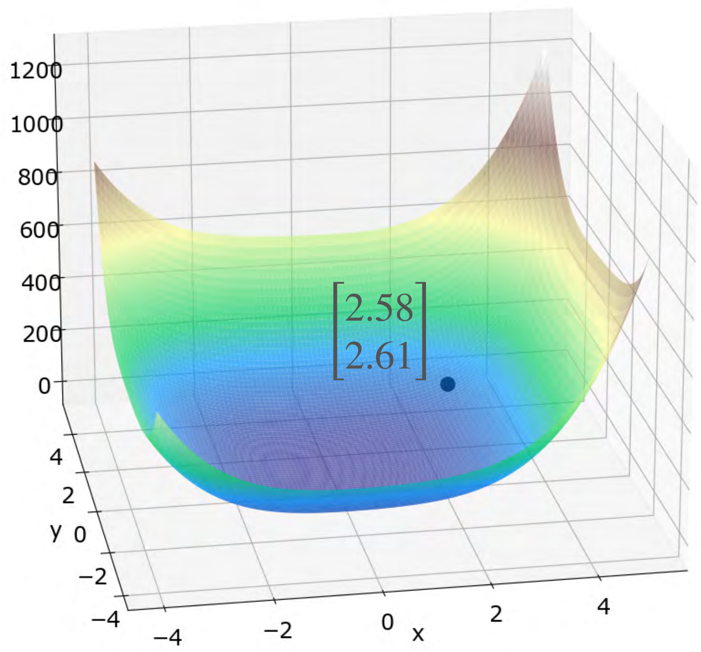
An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix}$$

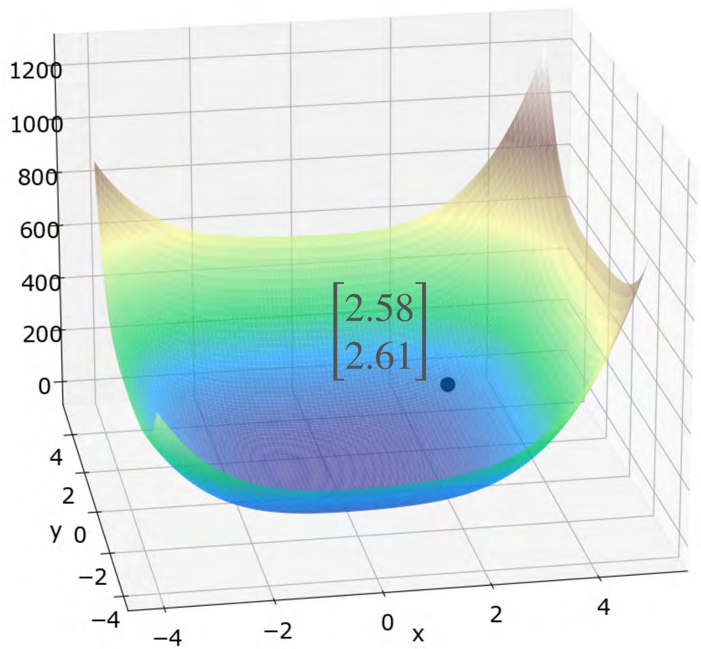
An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

An Example



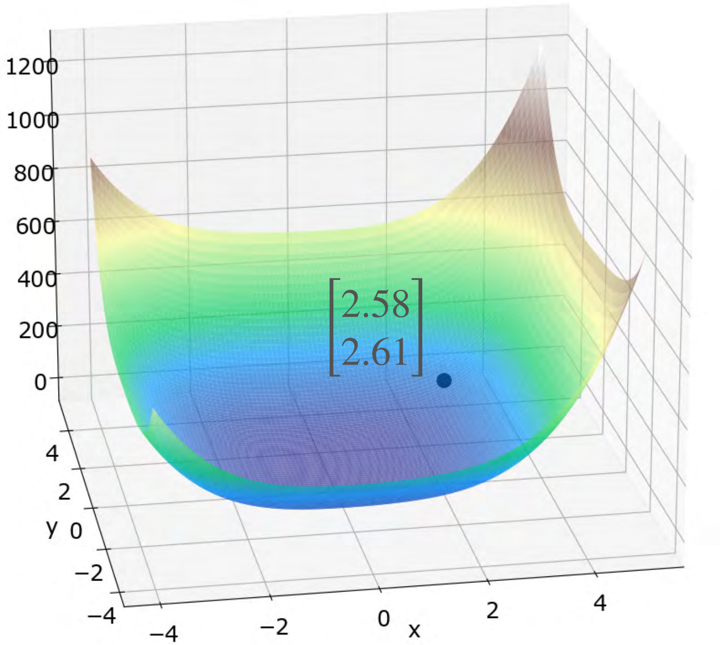
$$\begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} -$$

An Example

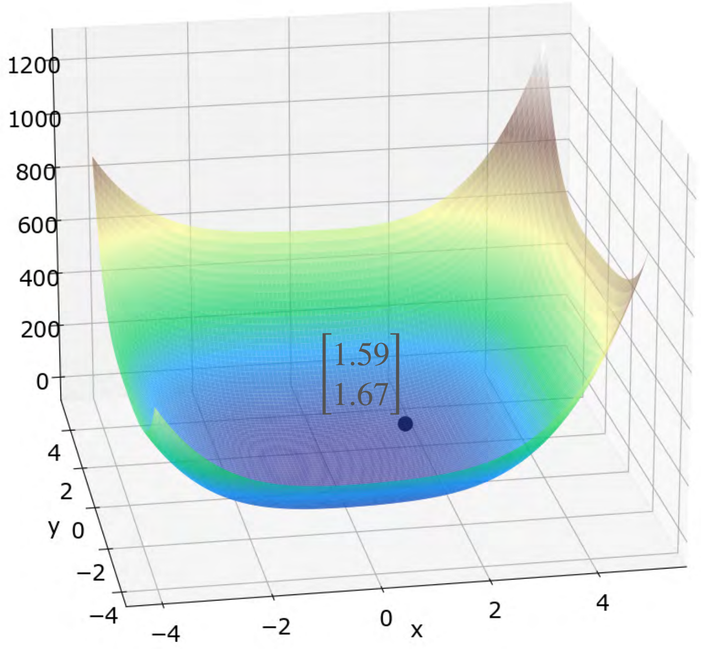


$$\begin{bmatrix}x_1\\y_1\end{bmatrix}=\begin{bmatrix}2.58\\2.61\end{bmatrix}$$

$$\nabla f(2.58, 2.61)=\begin{bmatrix}84.25\\63.4\end{bmatrix}\quad H(2.58, 2.61)=\begin{bmatrix}86.83 & -2.032\\-2.032 & 69.39\end{bmatrix}$$

$$\begin{bmatrix}x_2\\y_2\end{bmatrix}=\begin{bmatrix}2.58\\2.61\end{bmatrix}-\begin{bmatrix}86.83 & -2.032\\-2.032 & 69.39\end{bmatrix}^{-1}\begin{bmatrix}84.25\\63.4\end{bmatrix}$$

An Example



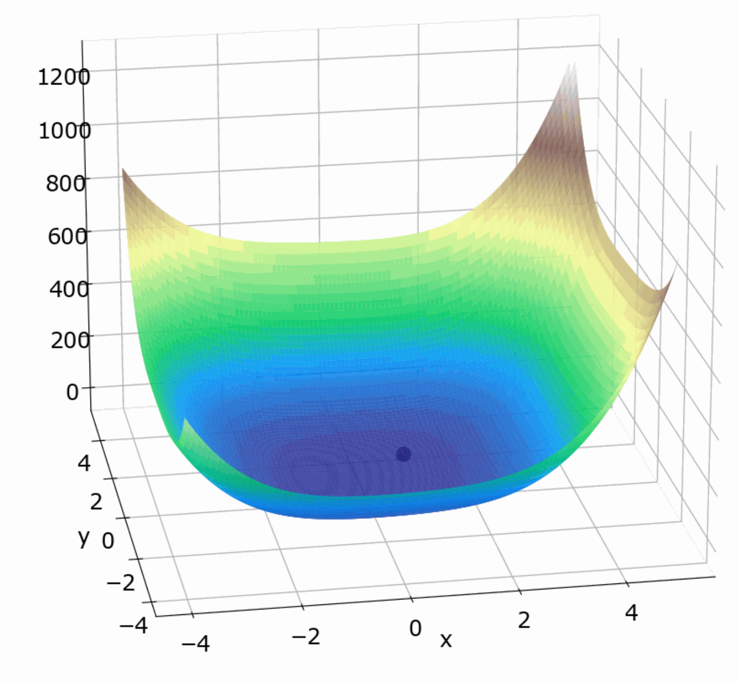
$$\begin{bmatrix}x_1\\y_1\end{bmatrix} = \begin{bmatrix}2.58\\2.61\end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix}84.25\\63.4\end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix}86.83 & -2.032\\-2.032 & 69.39\end{bmatrix}$$

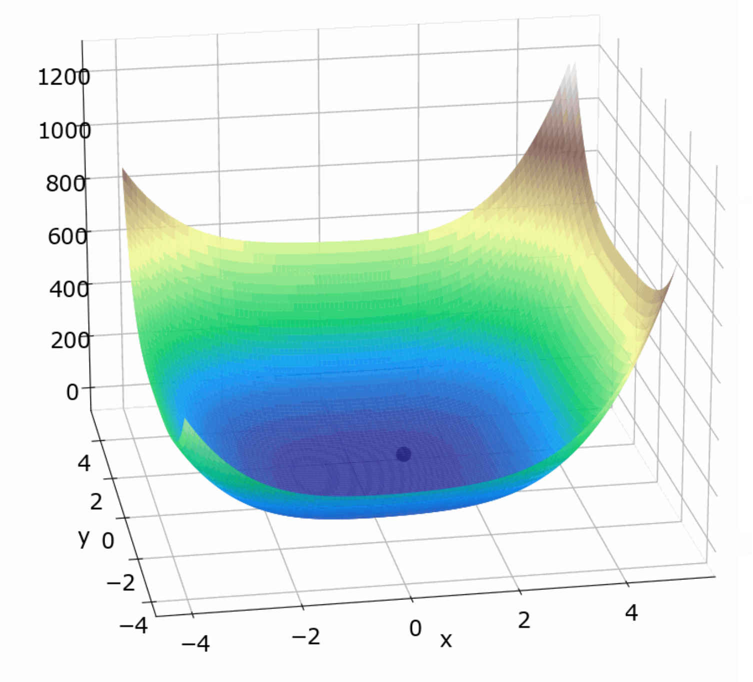
$$\begin{bmatrix}x_2\\y_2\end{bmatrix} = \begin{bmatrix}2.58\\2.61\end{bmatrix} - \begin{bmatrix}86.83 & -2.032\\-2.032 & 69.39\end{bmatrix}^{-1} \begin{bmatrix}84.25\\63.4\end{bmatrix}$$

$$= \begin{bmatrix}1.59\\1.67\end{bmatrix}$$

An Example

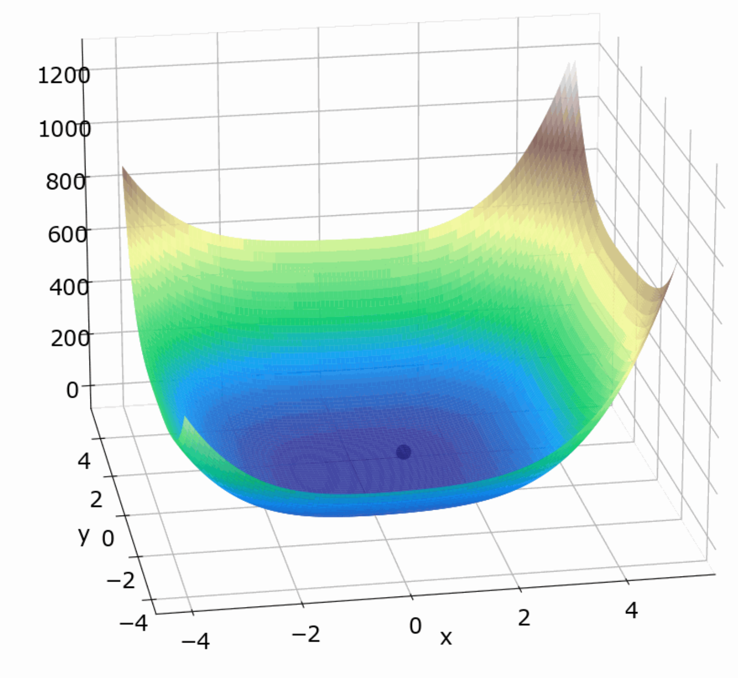


An Example



Repeat until you are close enough to the actual zero!

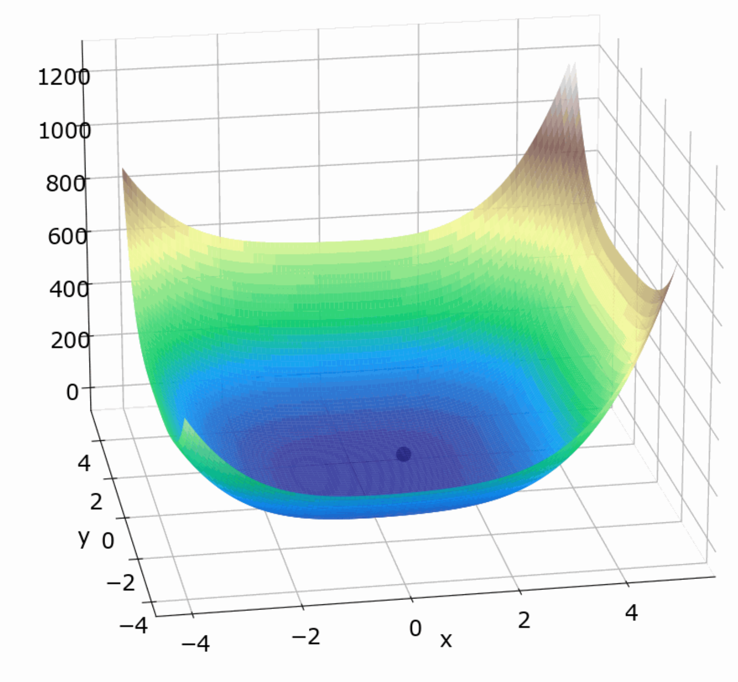
An Example



Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

An Example

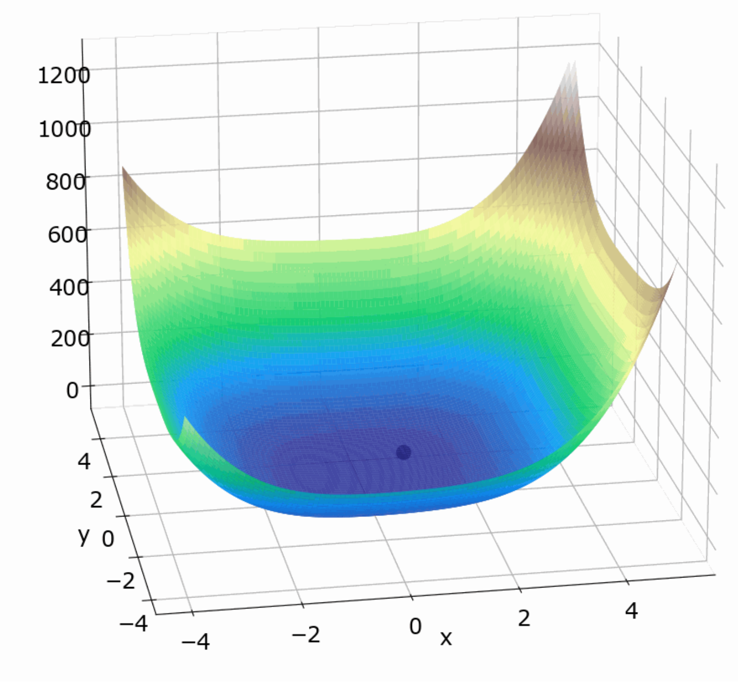


Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15\ 10^{-17} \\ -2.05\ 10^{-17} \end{bmatrix}$$

An Example



Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15\ 10^{-17} \\ -2.05\ 10^{-17} \end{bmatrix}$$

$$\begin{bmatrix} x* \\ y* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Optimization in Neural Networks and Newton's Method

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Conclusion