



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 4

Bases

Span

Orthogonal and orthonormal bases

Orthogonal and orthonormal matrices

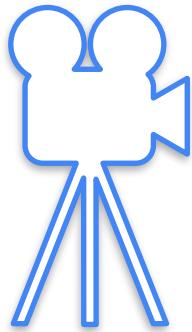


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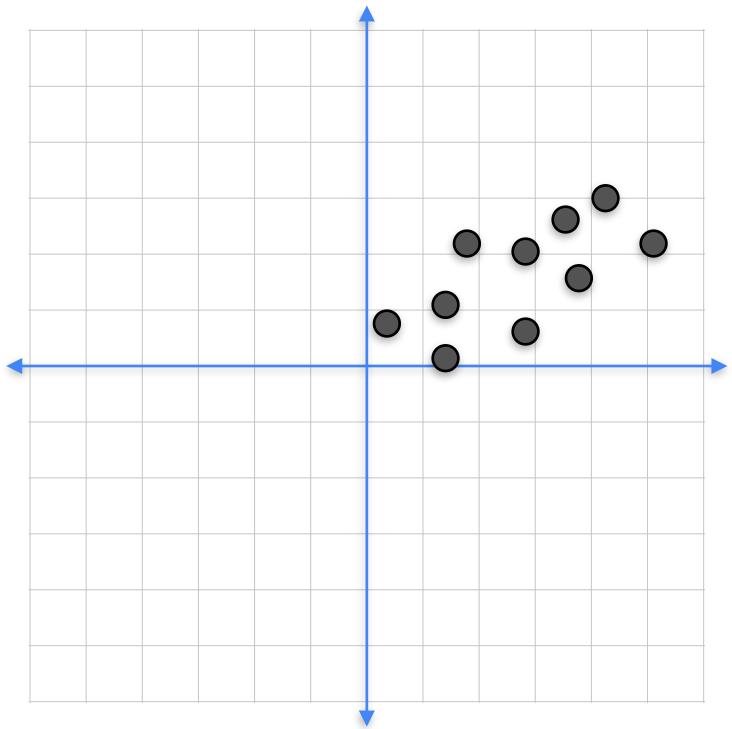
Determinants and Eigenvectors

Machine learning motivation

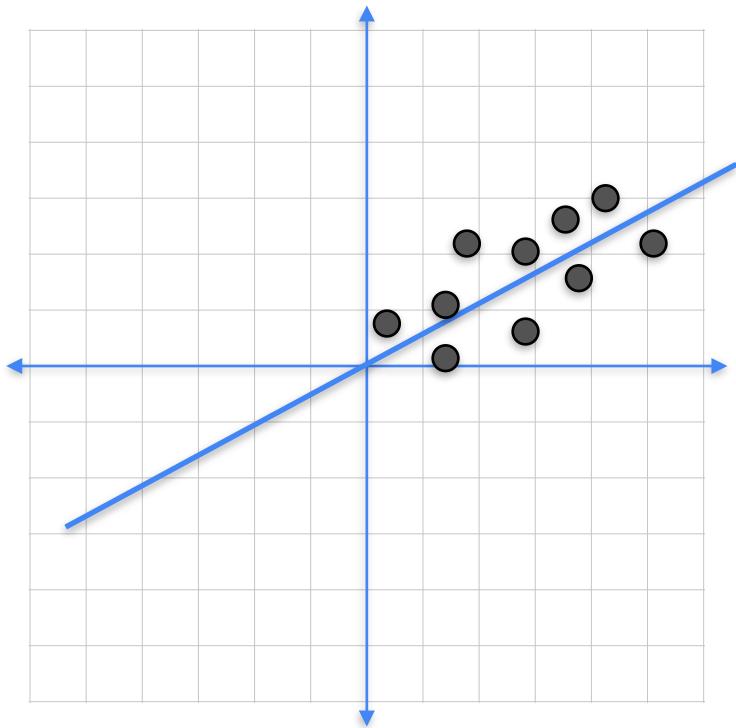
PCA



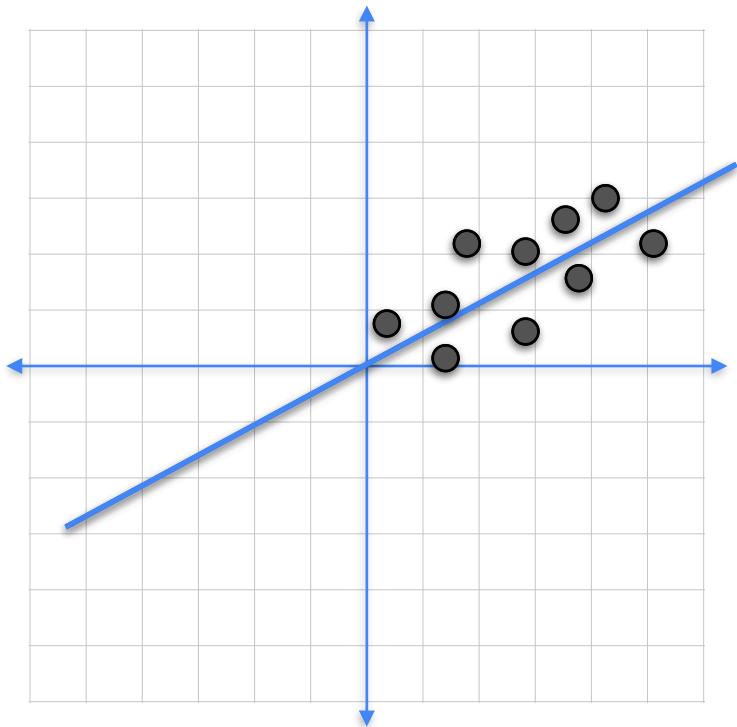
Principal Component Analysis



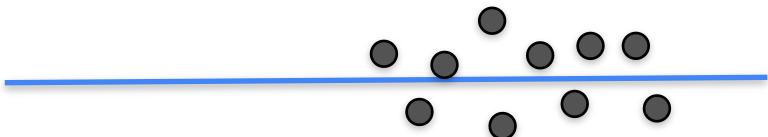
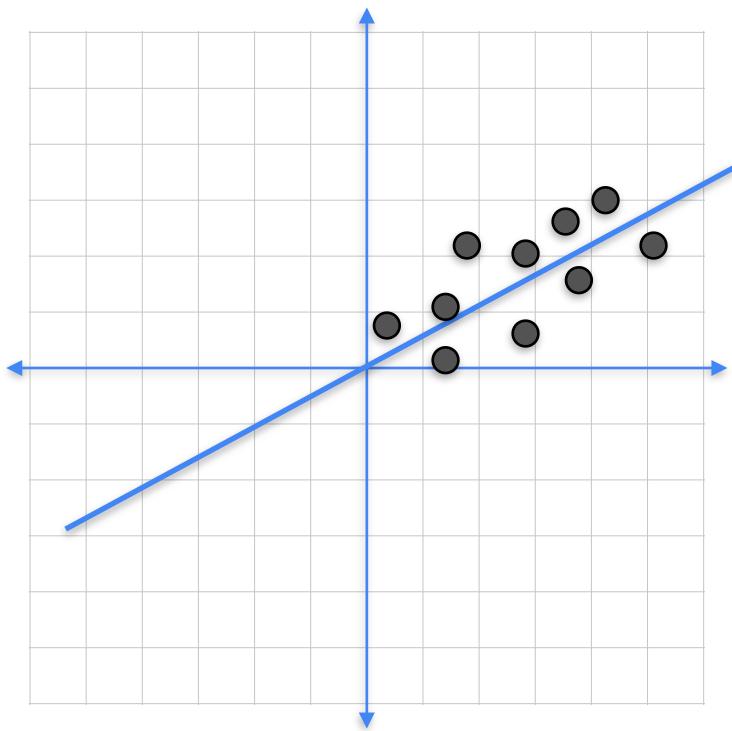
Principal Component Analysis



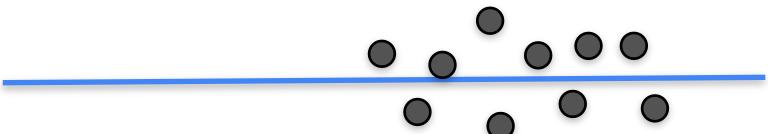
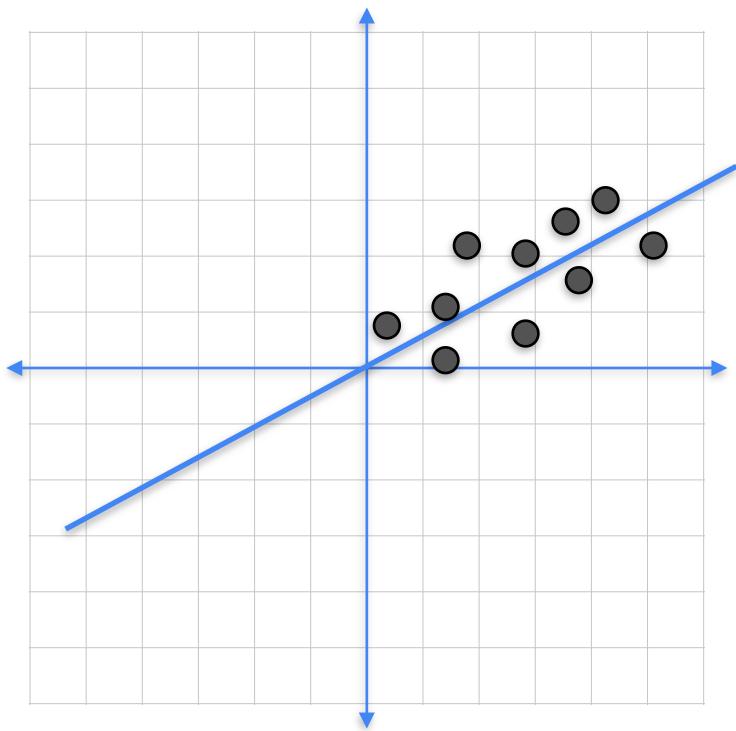
Principal Component Analysis



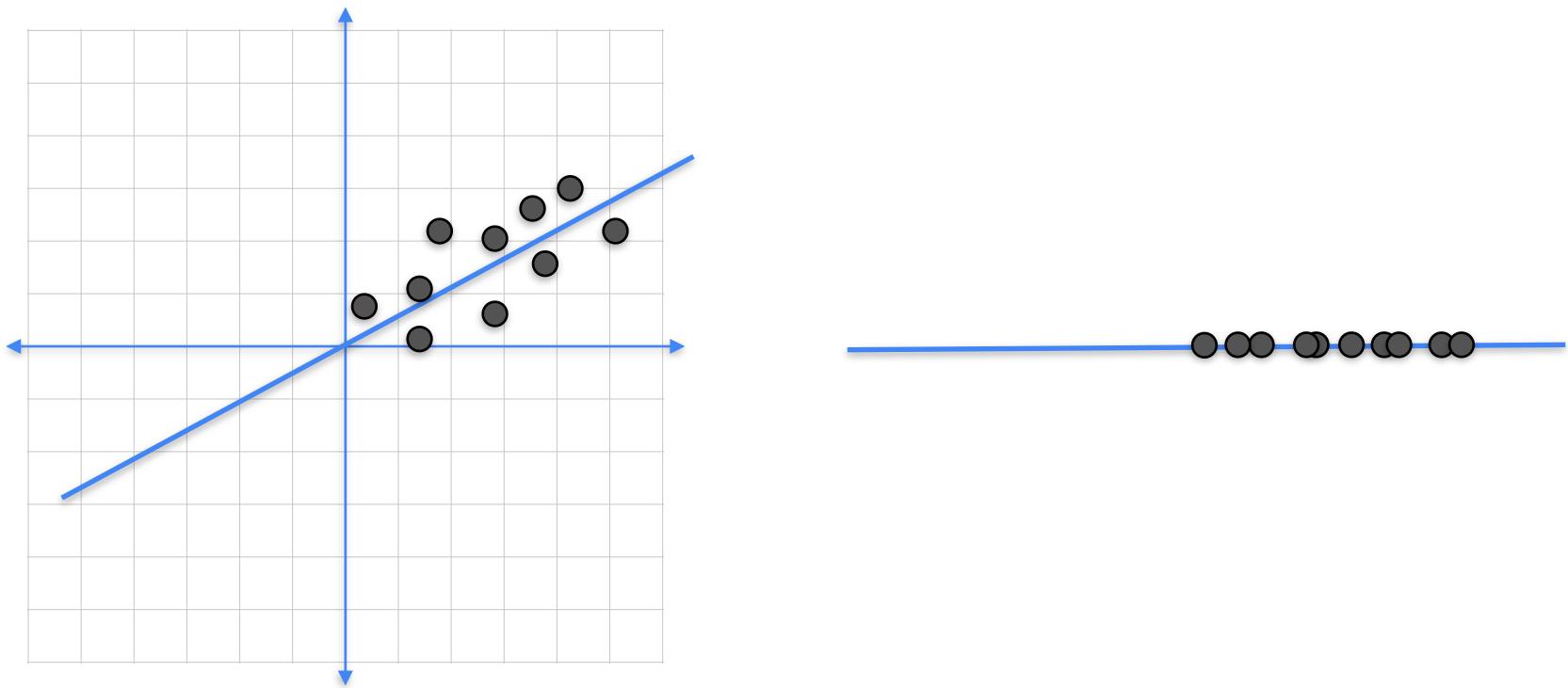
Principal Component Analysis



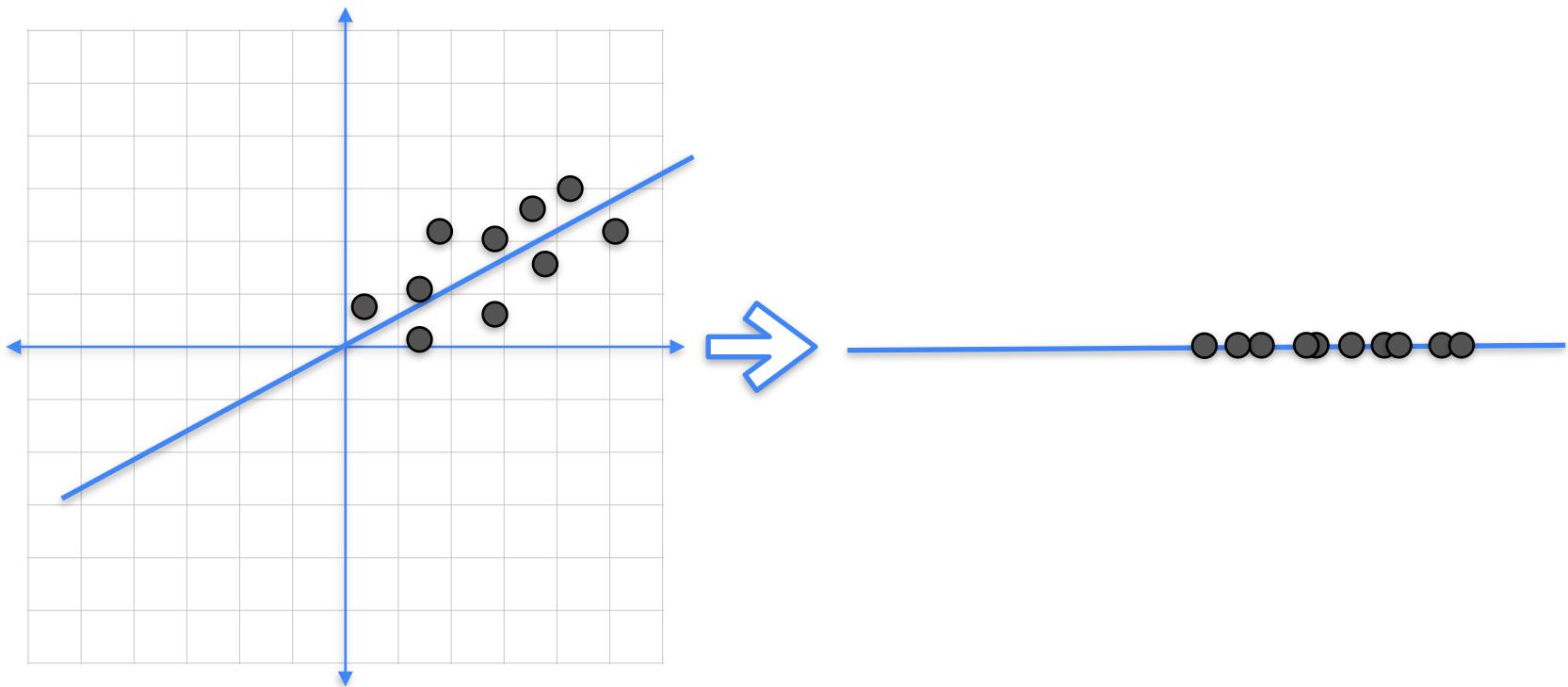
Principal Component Analysis



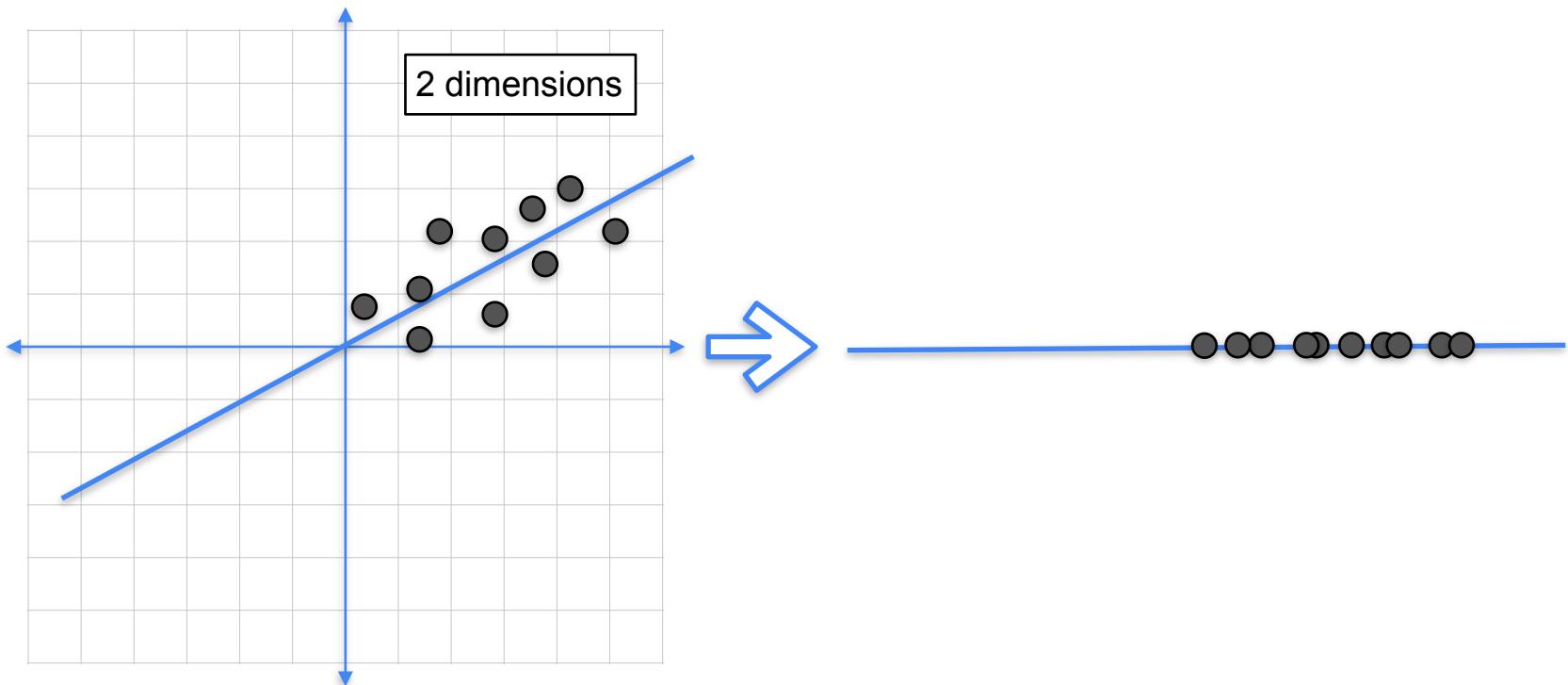
Principal Component Analysis



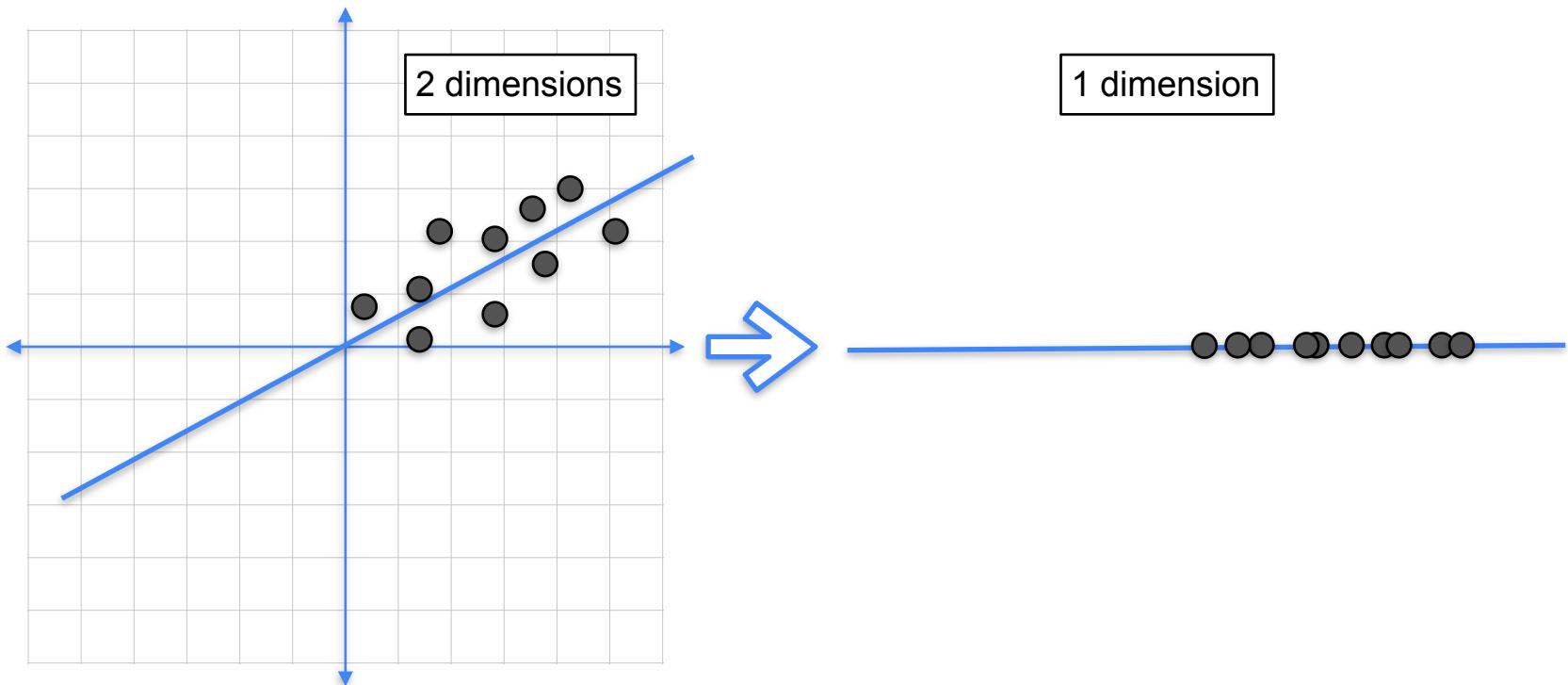
Principal Component Analysis



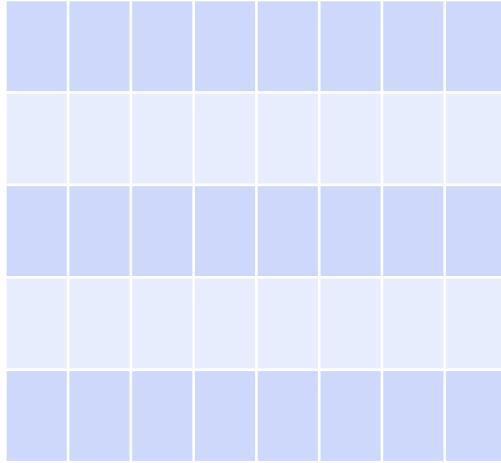
Principal Component Analysis



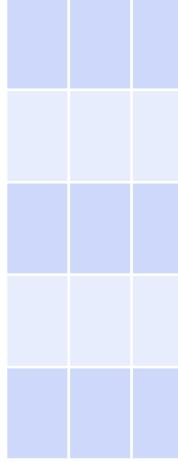
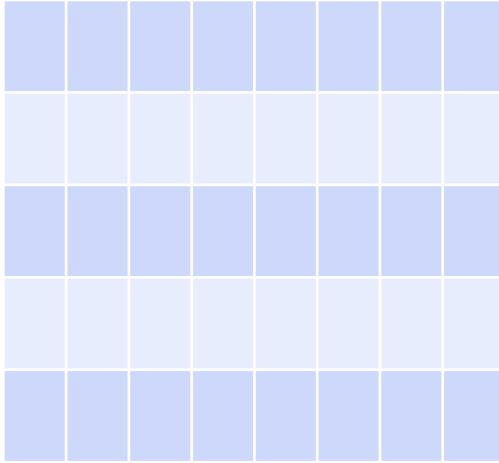
Principal Component Analysis



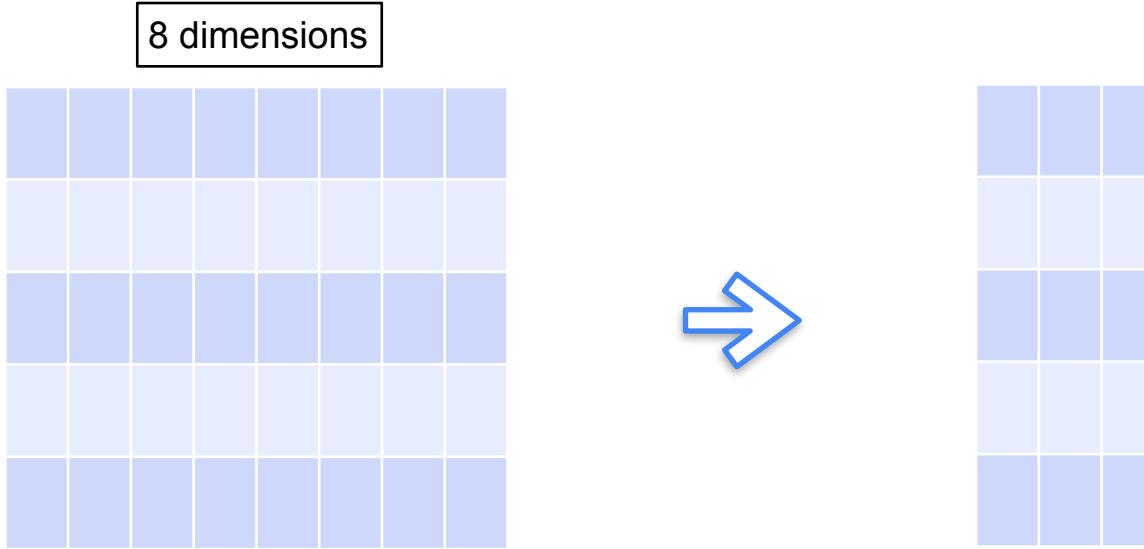
Principal Component Analysis



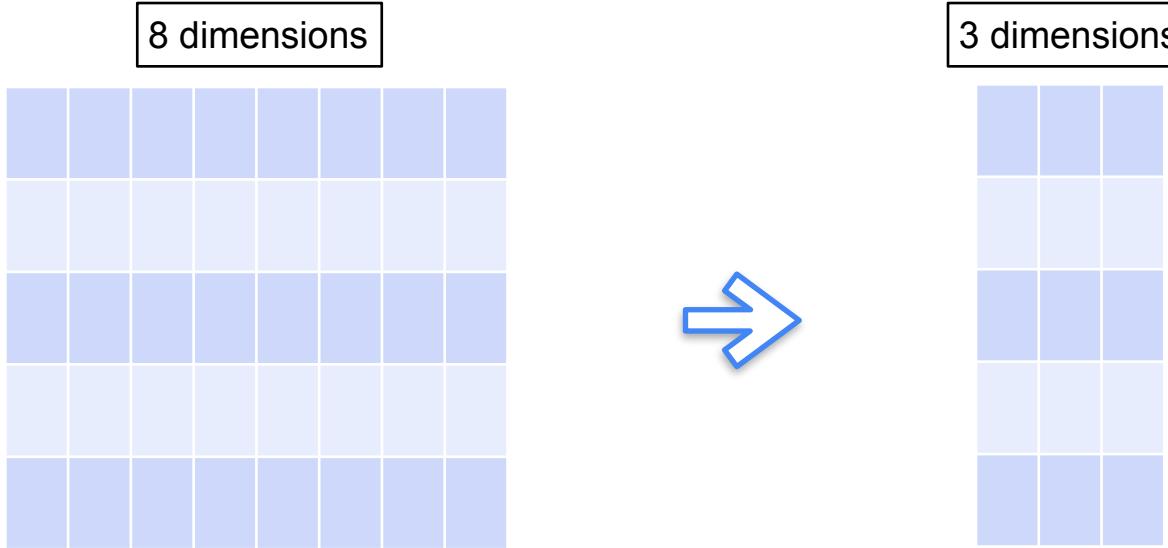
Principal Component Analysis



Principal Component Analysis



Principal Component Analysis



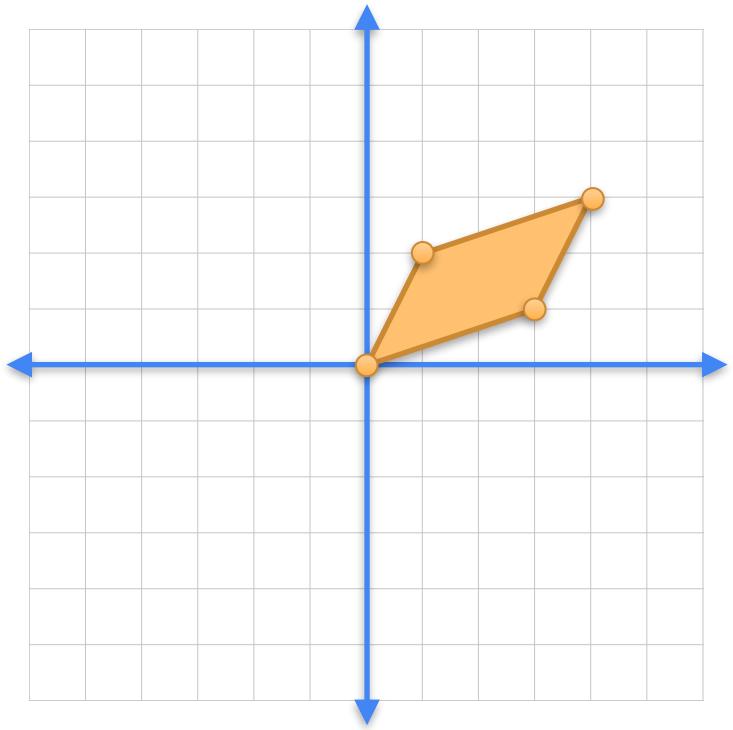
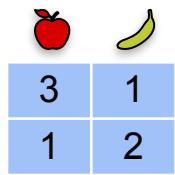
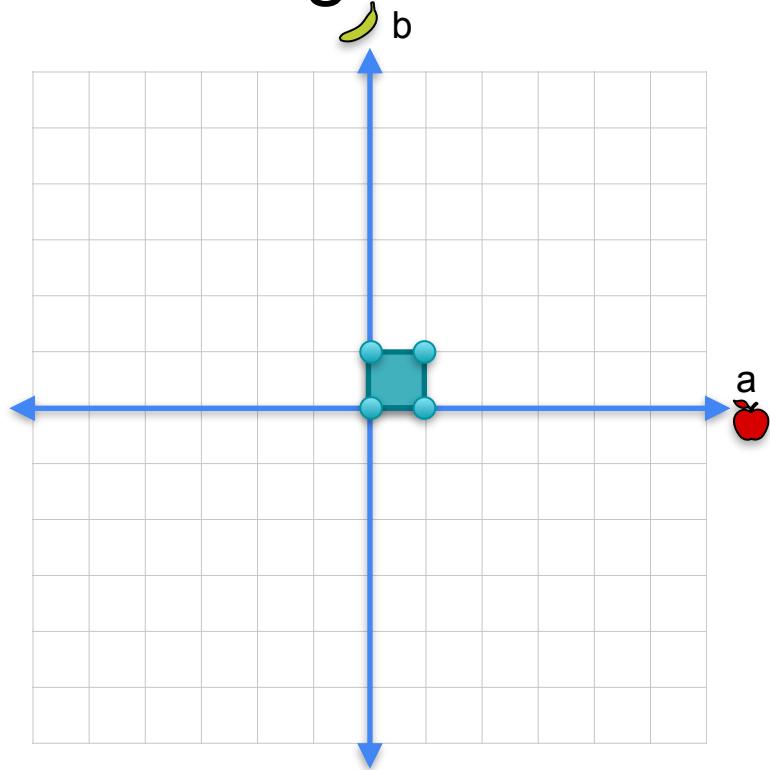


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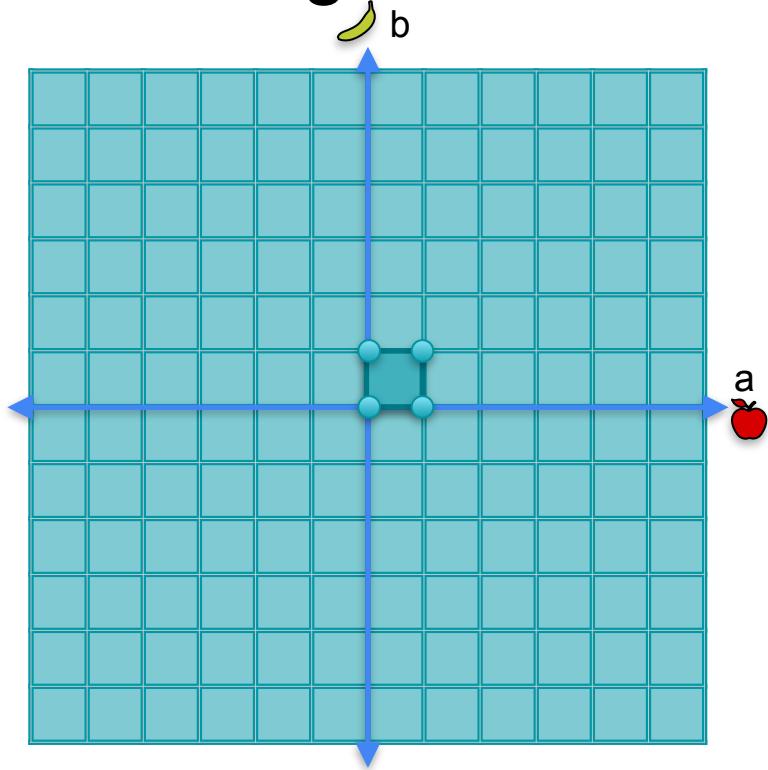
Determinants and Eigenvectors

Singularity and rank of linear transformations

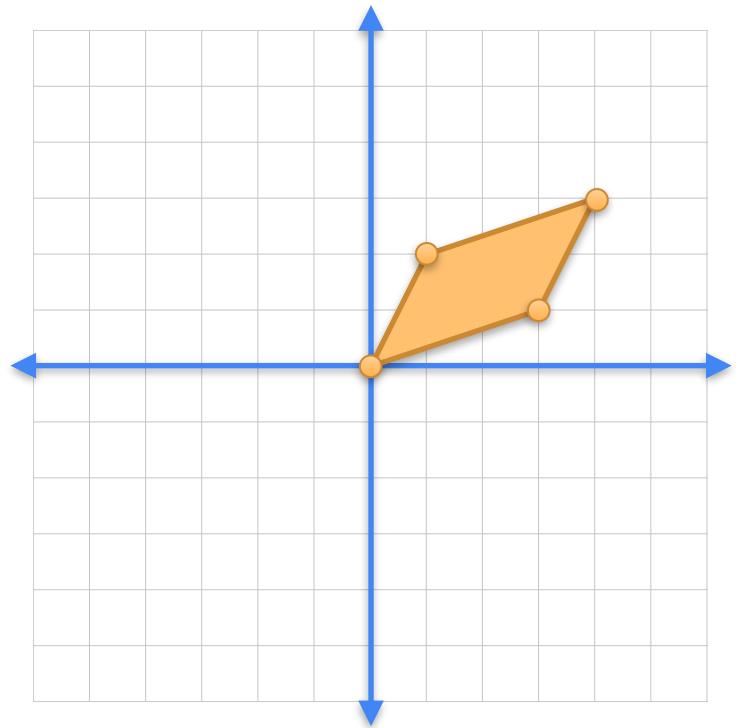
Non-singular transformation



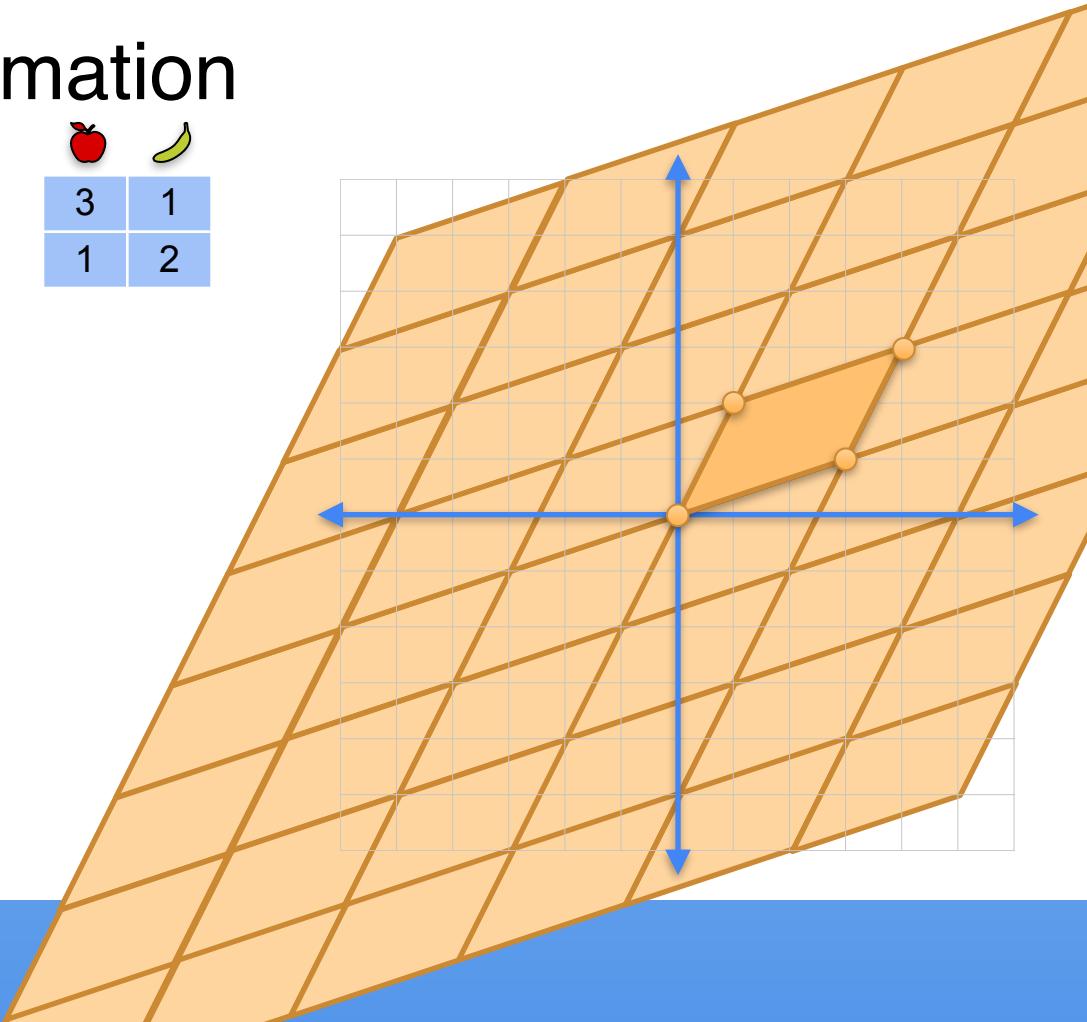
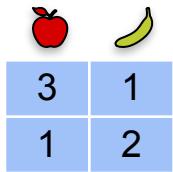
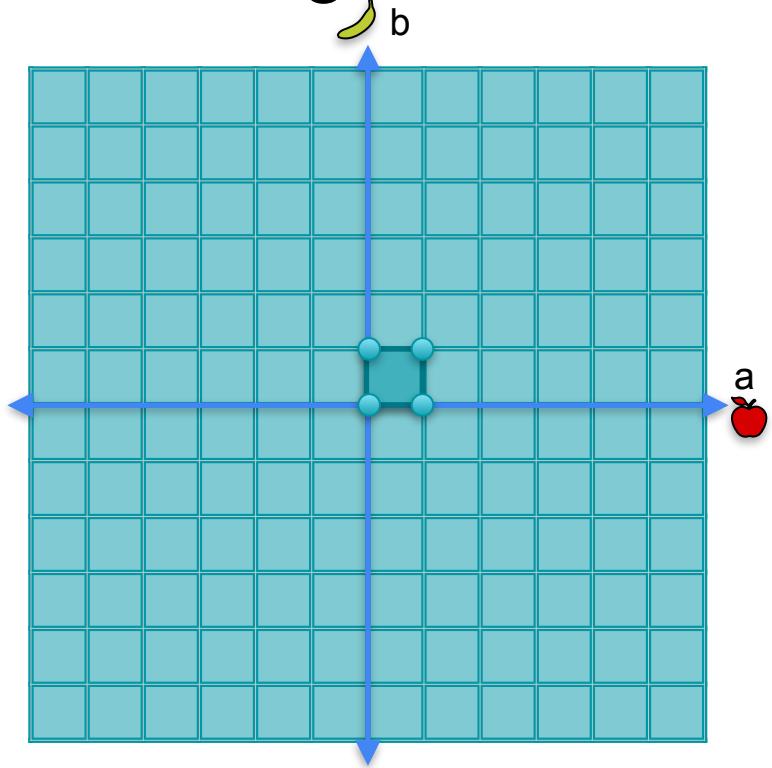
Non-singular transformation



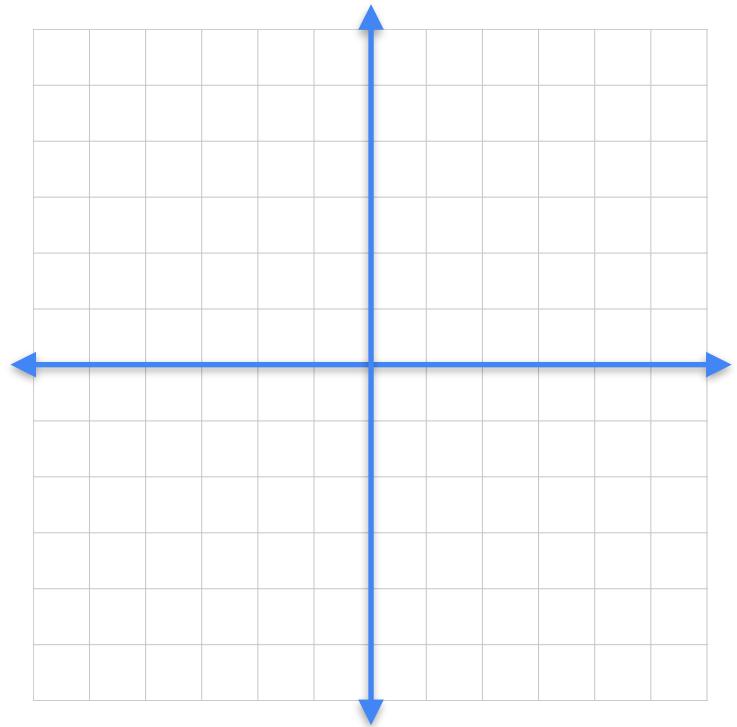
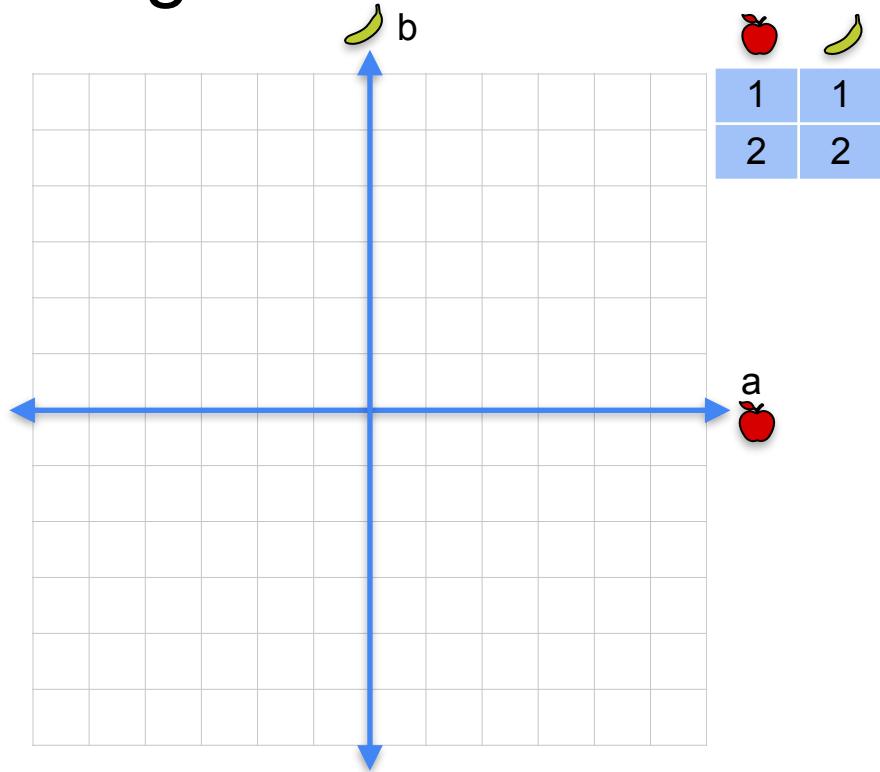
	apple	banana
3	1	
1	2	



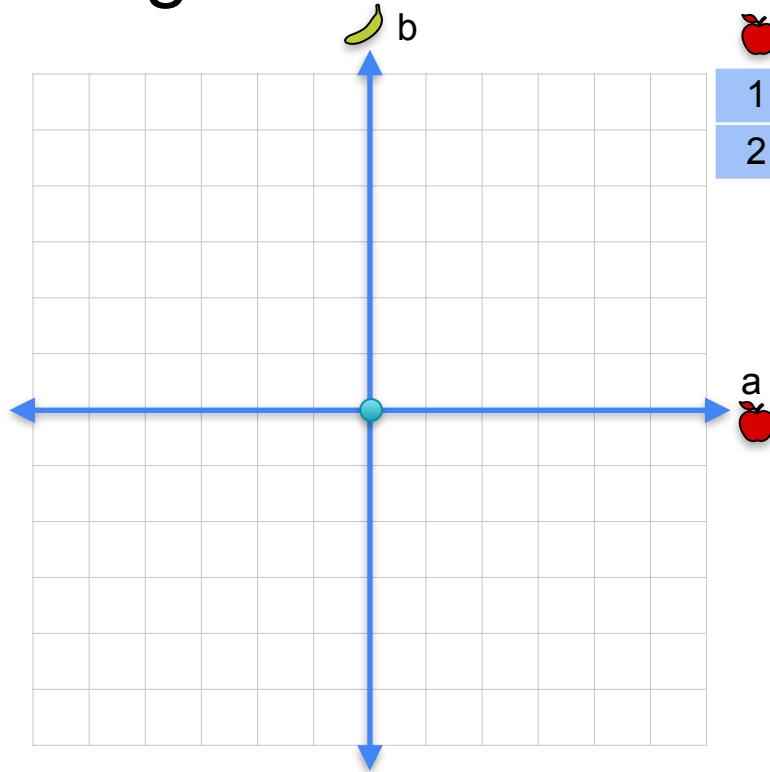
Non-singular transformation



Singular transformation

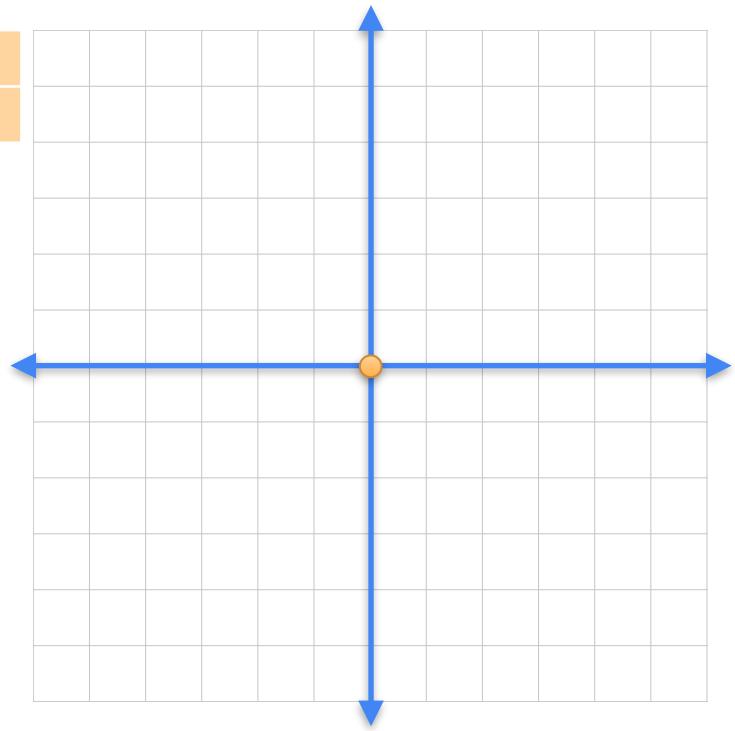


Singular transformation

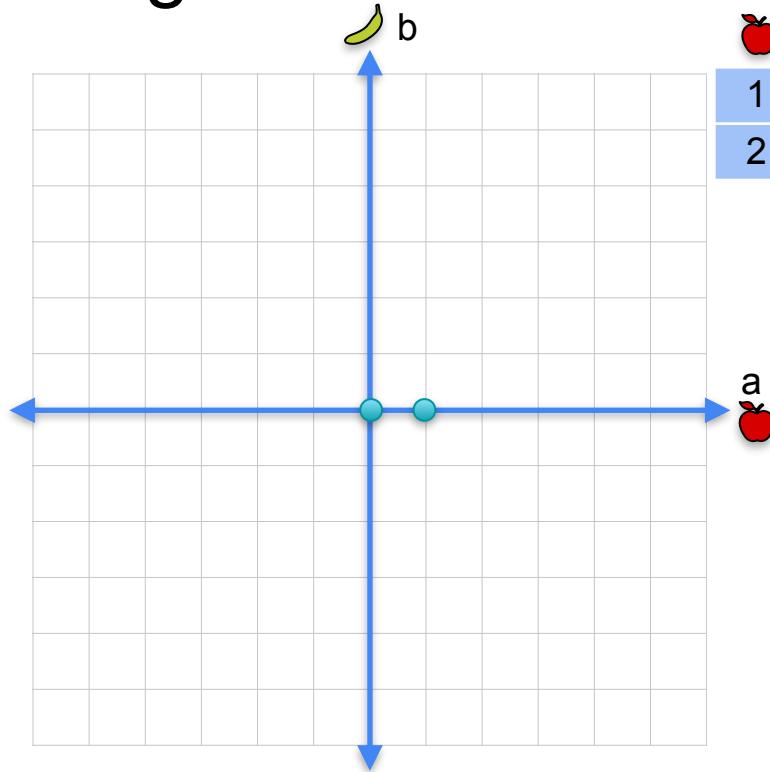


$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} \begin{matrix} 0 \\ 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$

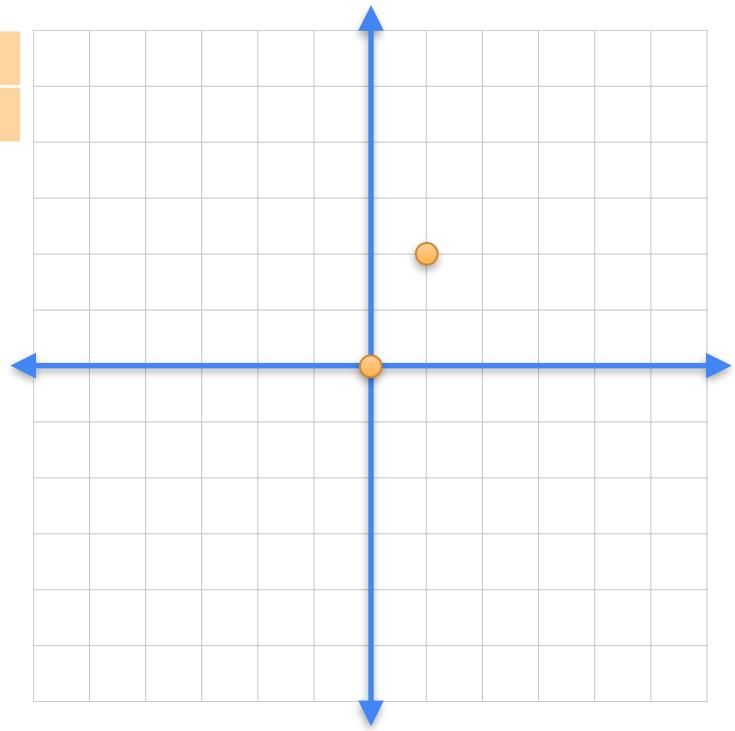


Singular transformation

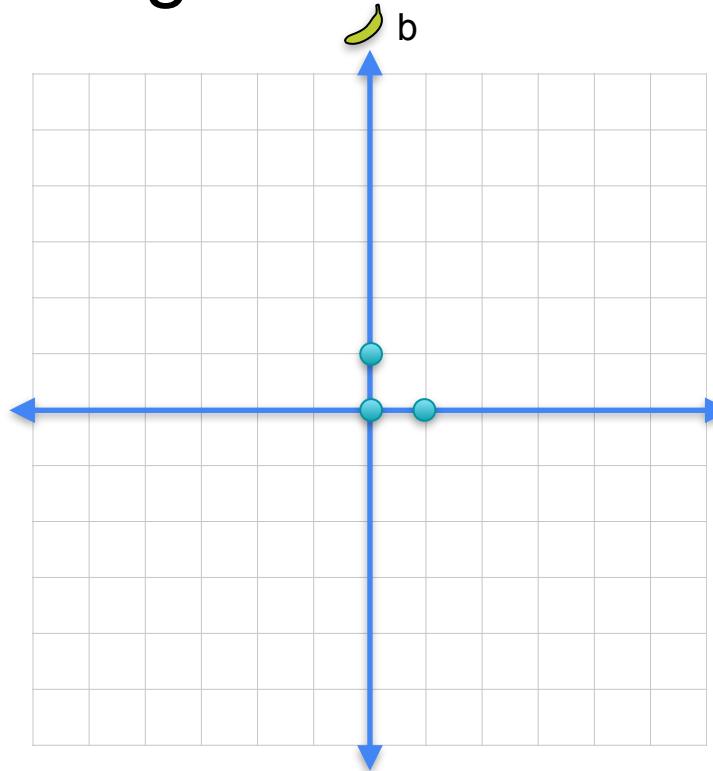


$$\begin{bmatrix} \text{apple} & \text{banana} \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$(0,0) \rightarrow (0,0)$$
$$(1,0) \rightarrow (1,2)$$

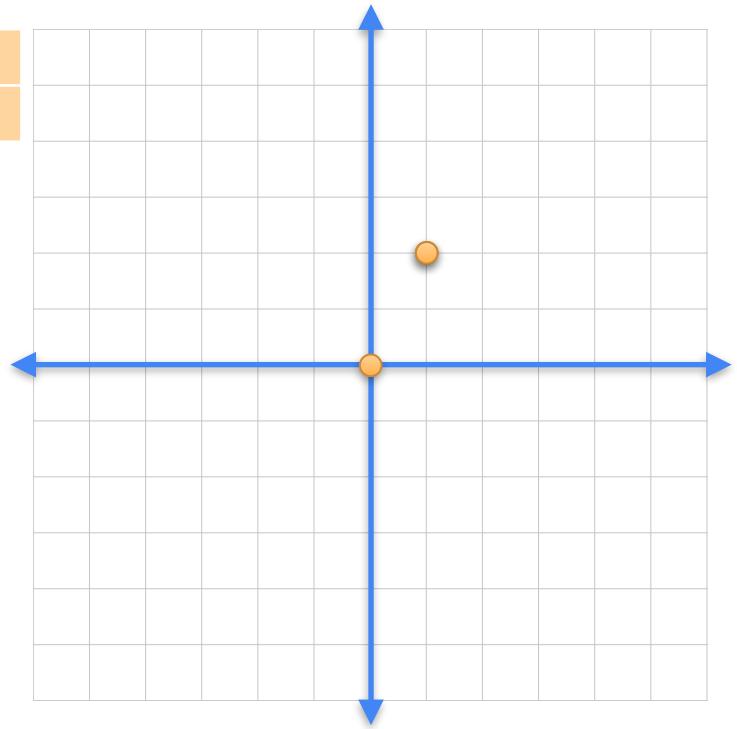


Singular transformation

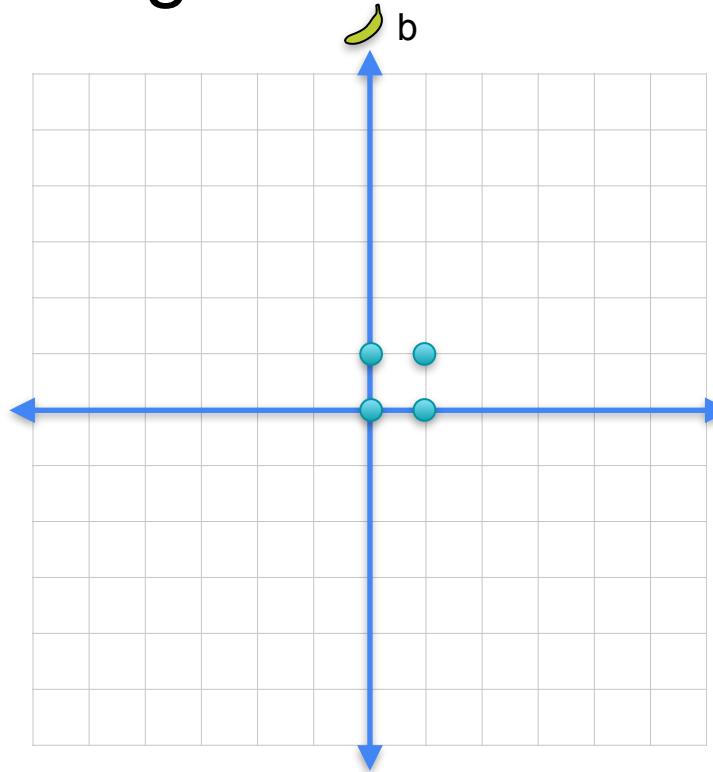


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 1 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (1,2) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$



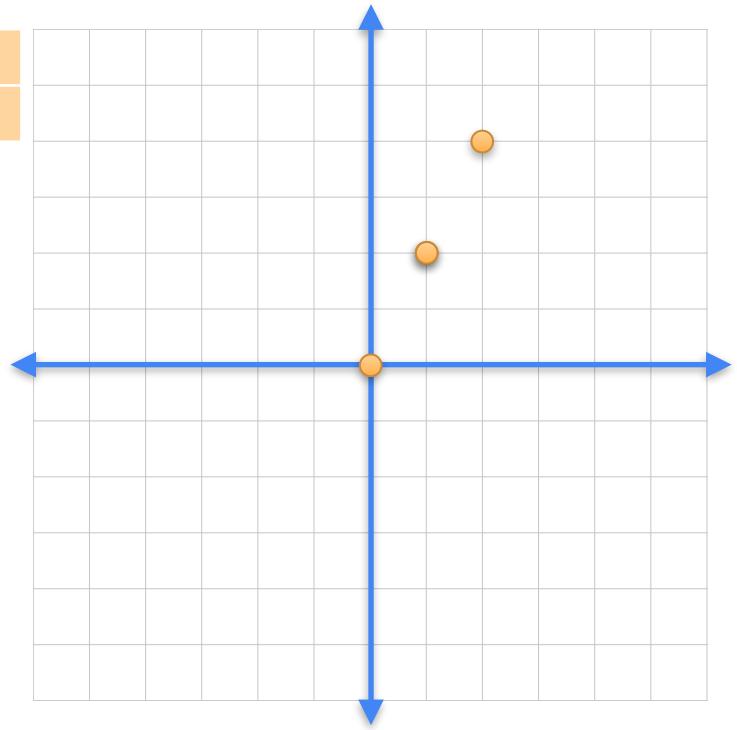
Singular transformation



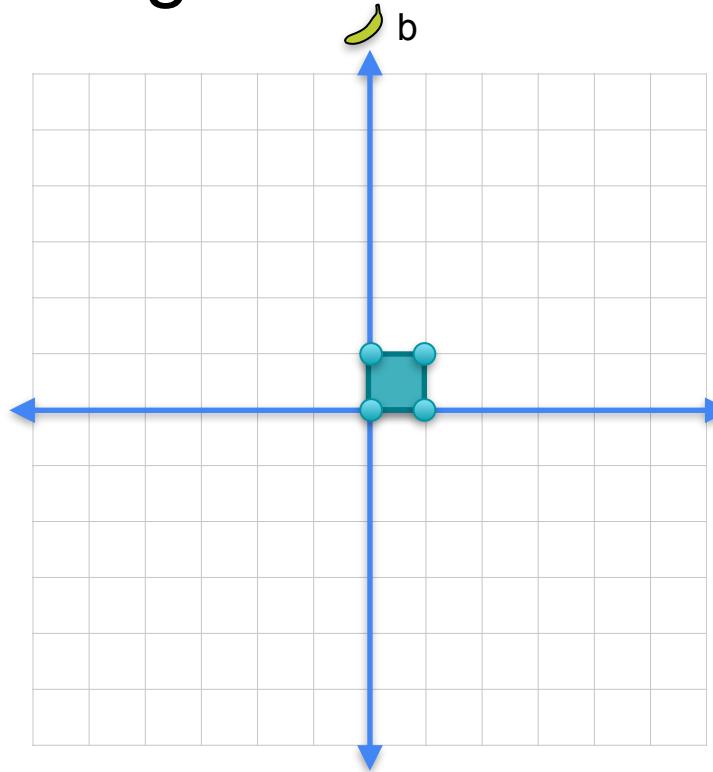
A diagram showing a linear transformation. On the left, there is a 2x2 input matrix with columns [1, 2] and rows [1, 2]. To its right is a multiplication sign (=). To the right of the multiplication sign is a 2x1 output vector with elements [2, 4]. Above the input matrix is a red apple icon, and above the output vector is a banana icon.

$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix}$$

(0,0) \rightarrow (0,0)
(1,0) \rightarrow (1,2)
(0,1) \rightarrow (1,2)
(1,1) \rightarrow (2,4)

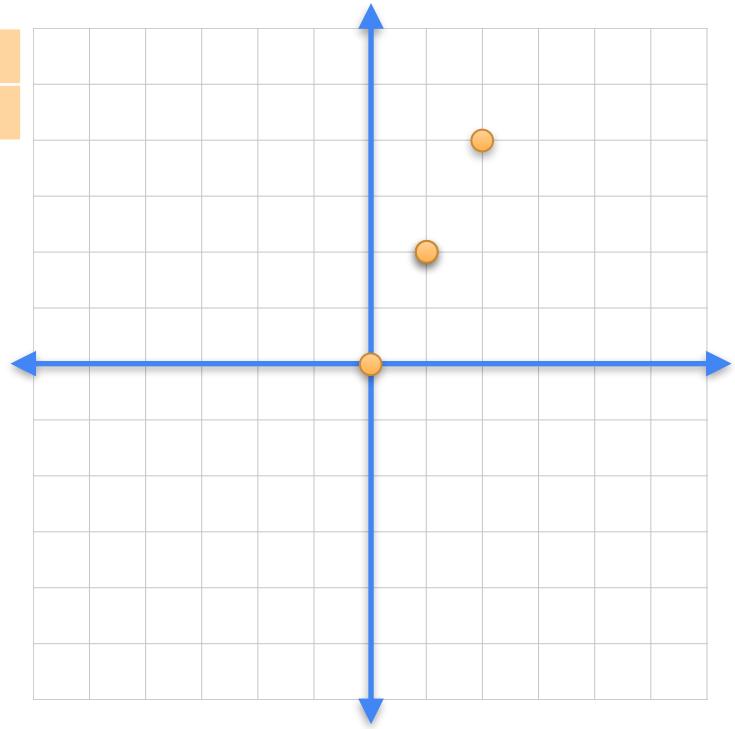


Singular transformation

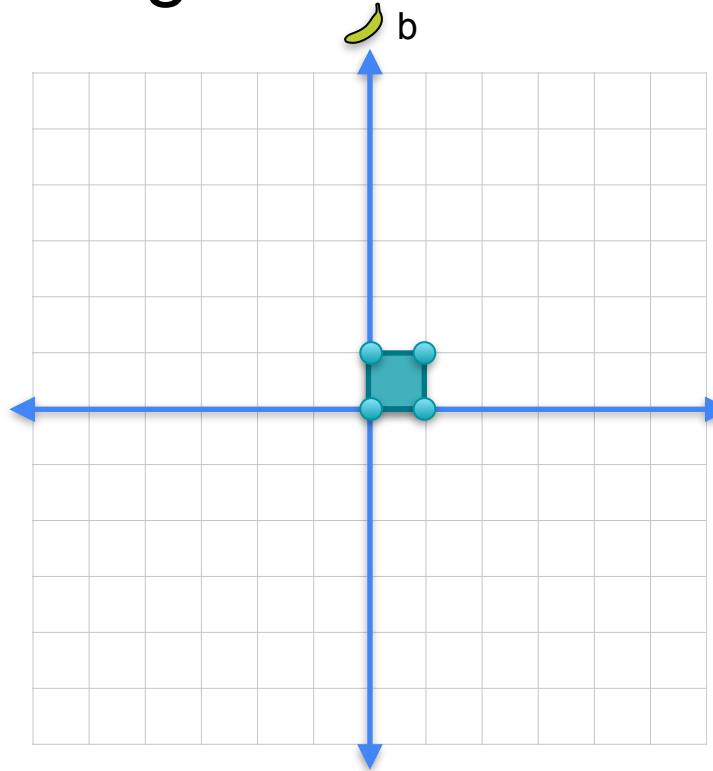


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$

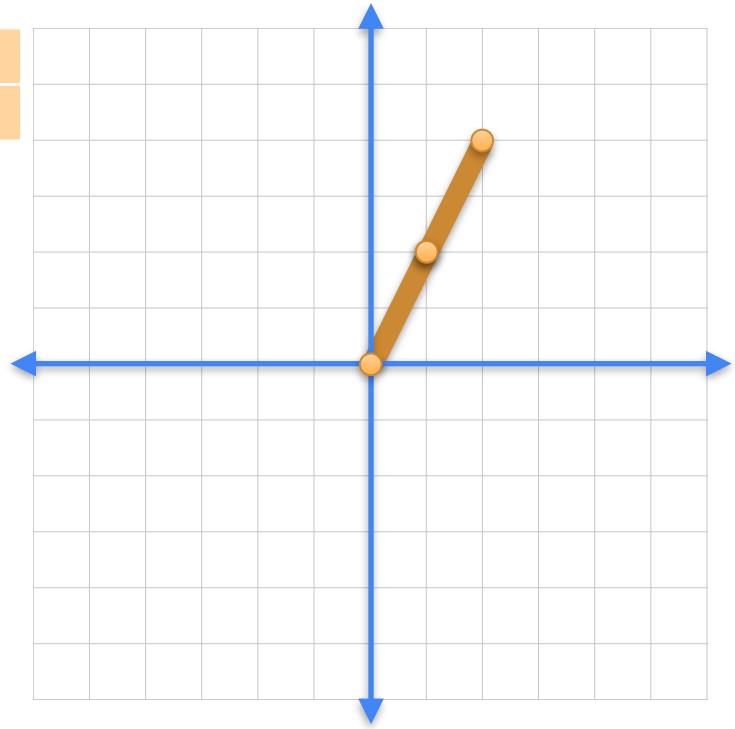


Singular transformation

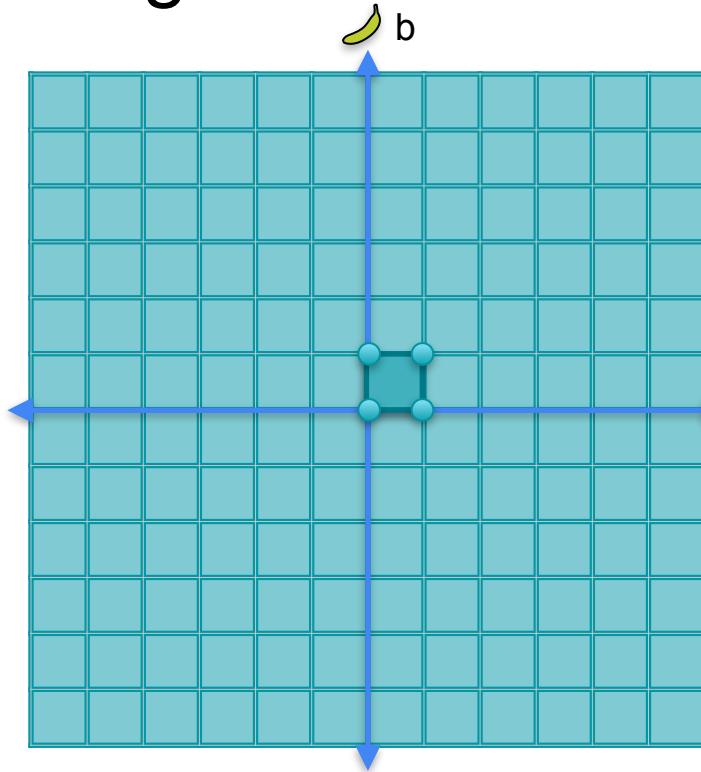


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$

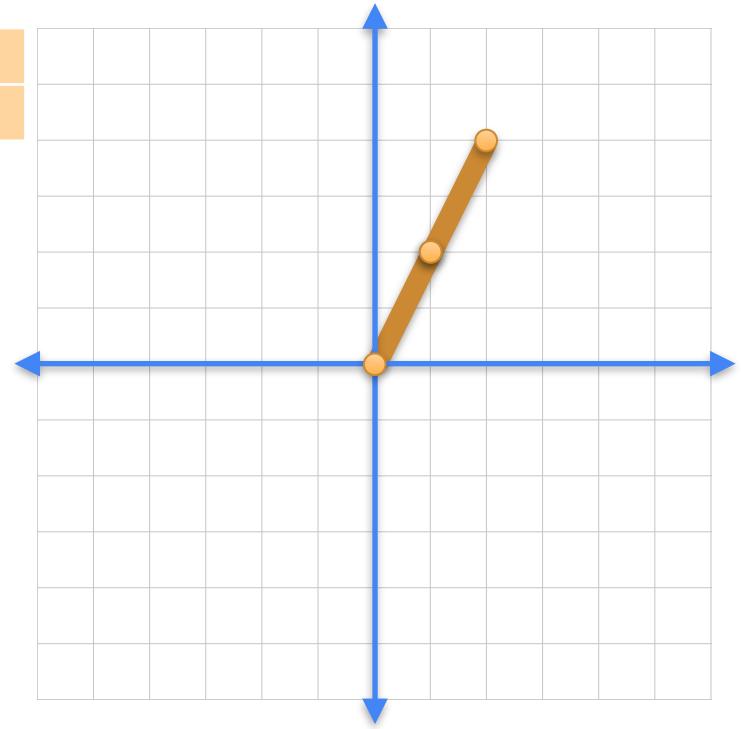


Singular transformation

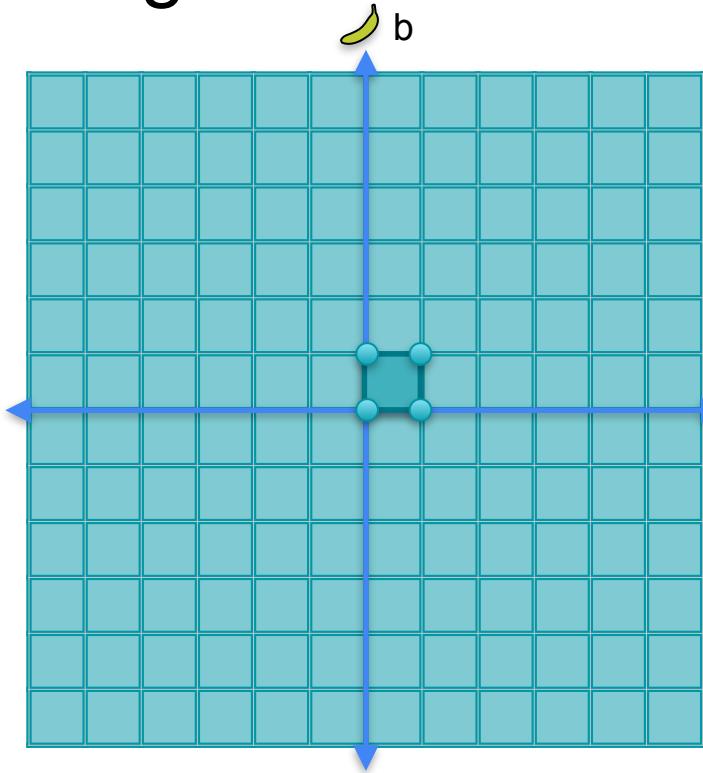


$$\begin{matrix} \text{apple} & \text{banana} \end{matrix}$$
$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{array}{l} (0,0) \rightarrow (0,0) \\ (1,0) \rightarrow (1,2) \\ (0,1) \rightarrow (1,2) \\ (1,1) \rightarrow (2,4) \end{array}$$



Singular transformation

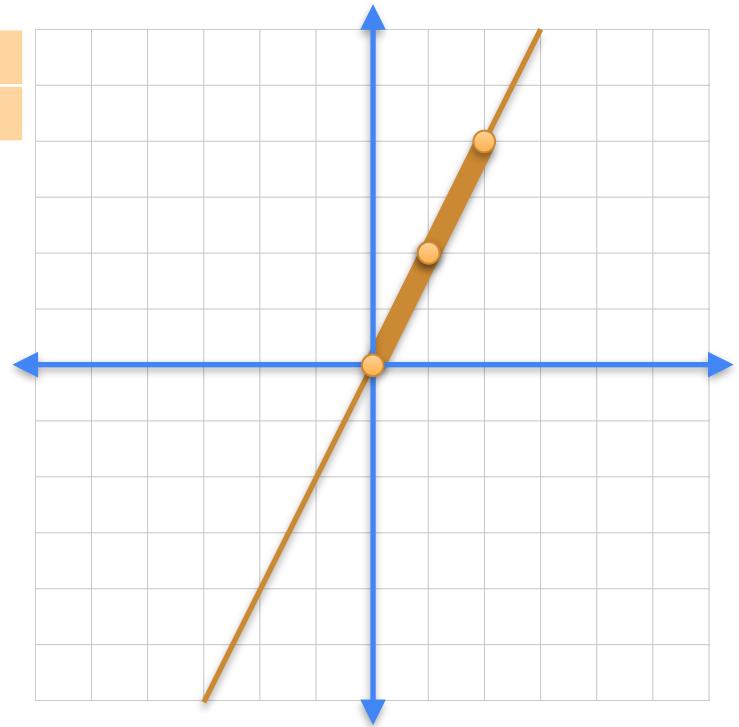


Two red apple icons and two yellow banana icons are shown above a mathematical equation. The equation consists of three matrices: a 2x2 matrix with values 1 and 2, a 1x2 matrix with values 1 and 1, and a 2x1 matrix with values 1 and 1. An equals sign follows these, leading to a 2x1 matrix with values 2 and 4.

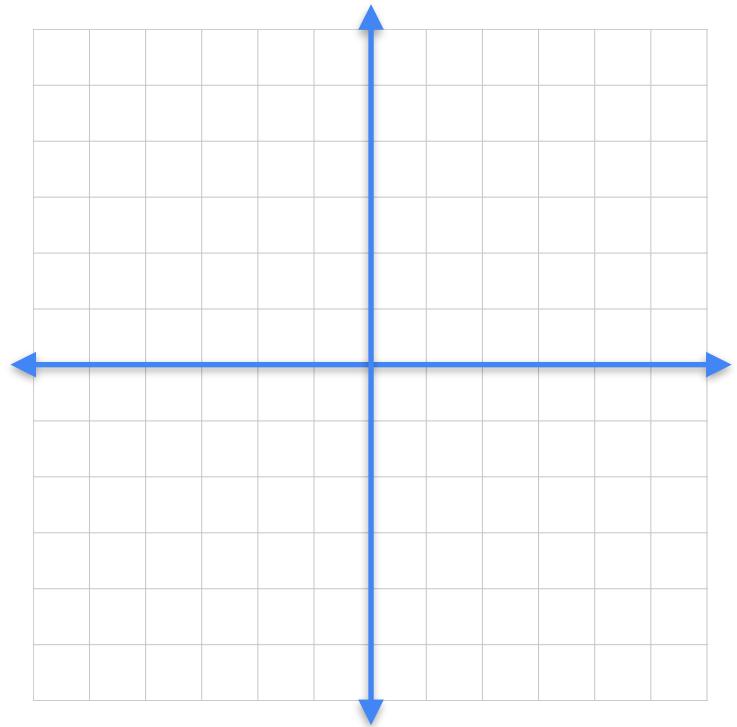
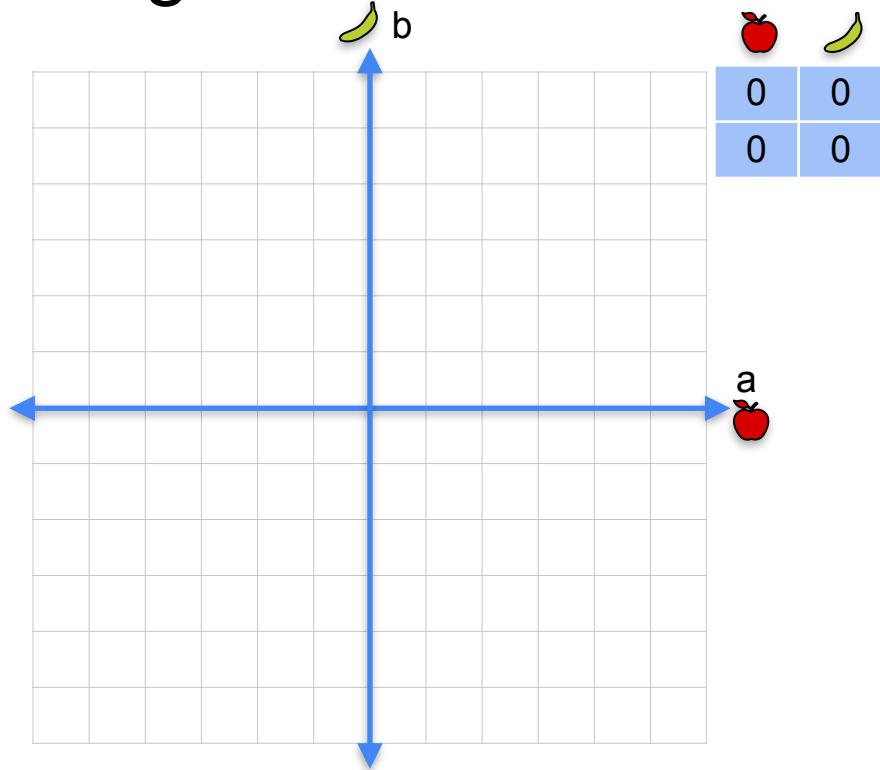
$$\begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} \begin{matrix} 1 \\ 1 \end{matrix} = \begin{matrix} 2 \\ 4 \end{matrix}$$

The following table shows the mapping of input coordinates to output values:

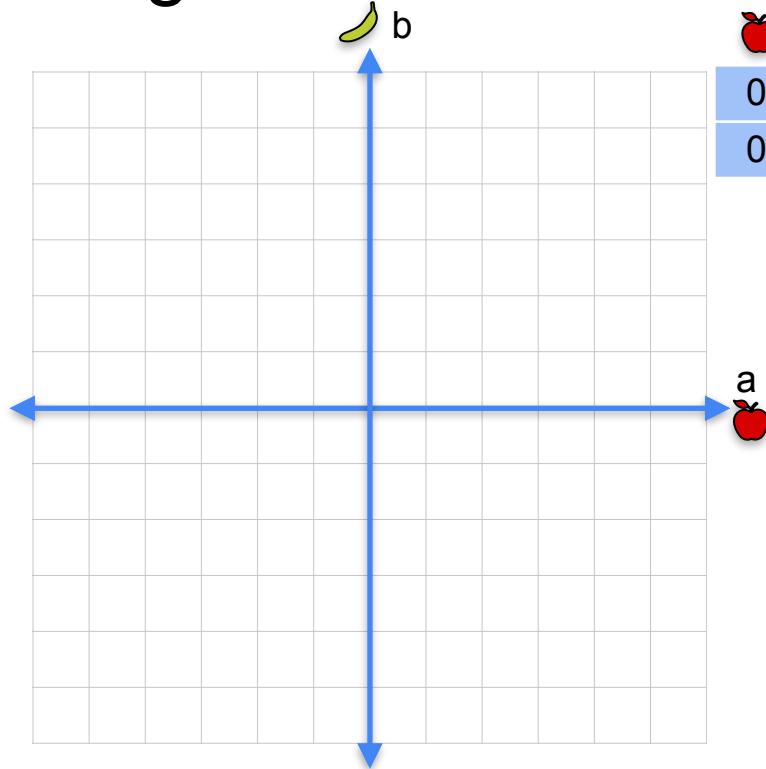
(0,0)	\rightarrow	(0,0)
(1,0)	\rightarrow	(1,2)
(0,1)	\rightarrow	(1,2)
(1,1)	\rightarrow	(2,4)



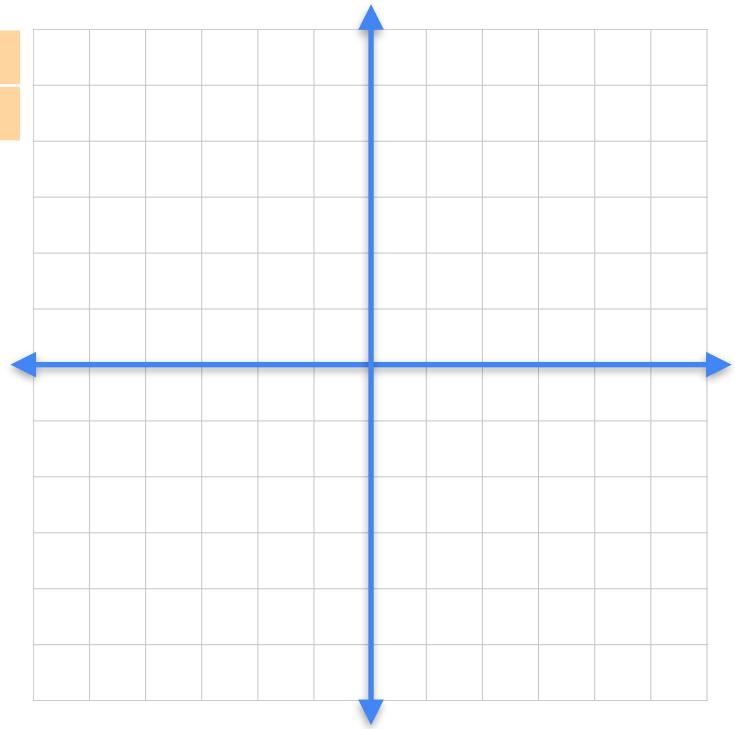
Singular transformation



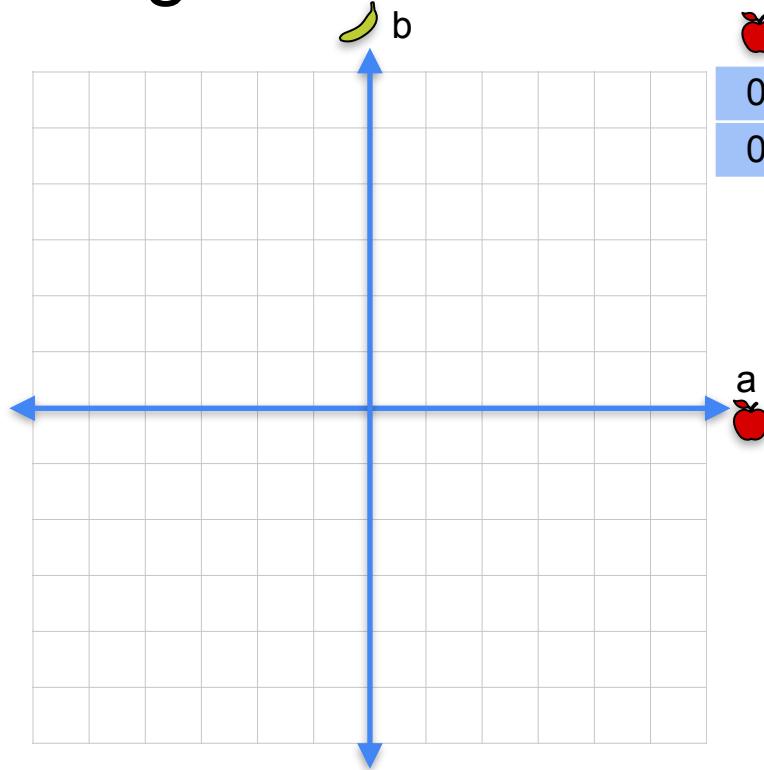
Singular transformation

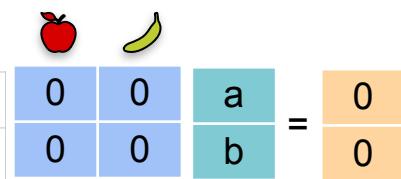


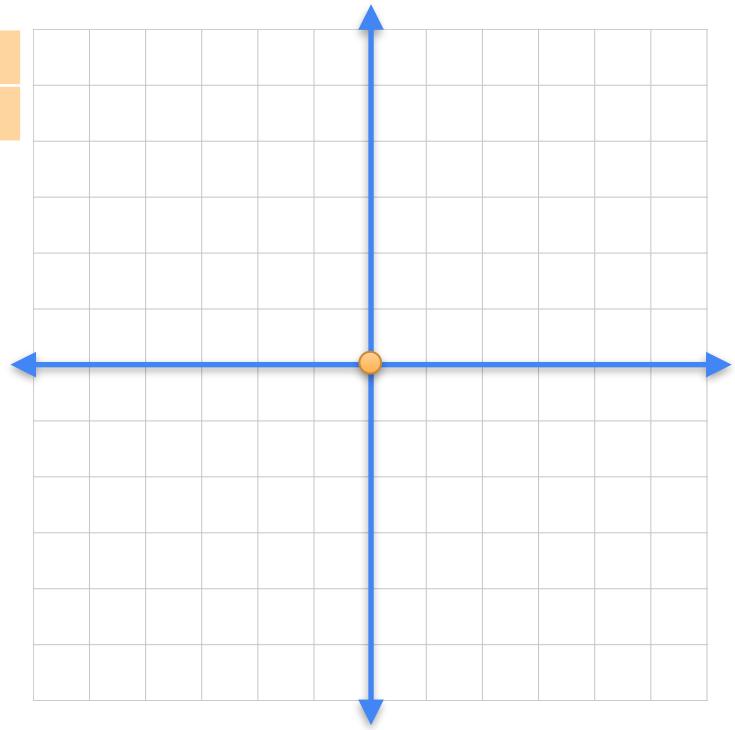
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} a \\ b \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$



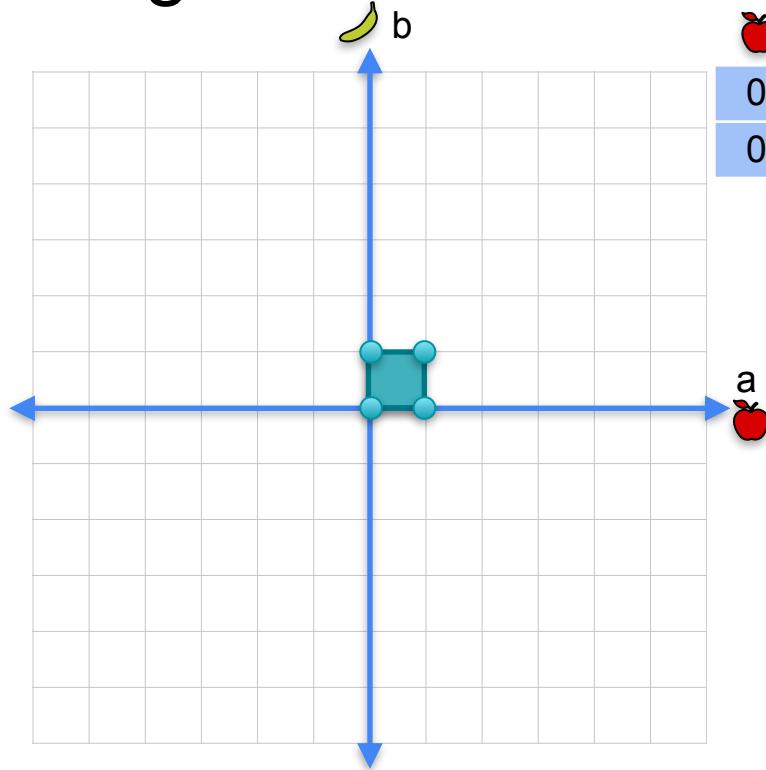
Singular transformation



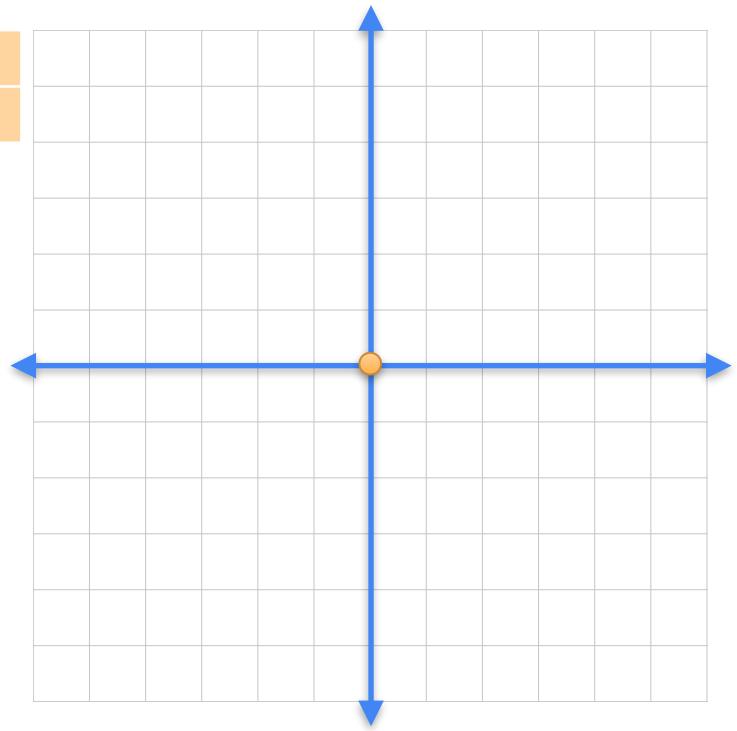

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} a \\ b \end{pmatrix} \end{matrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$



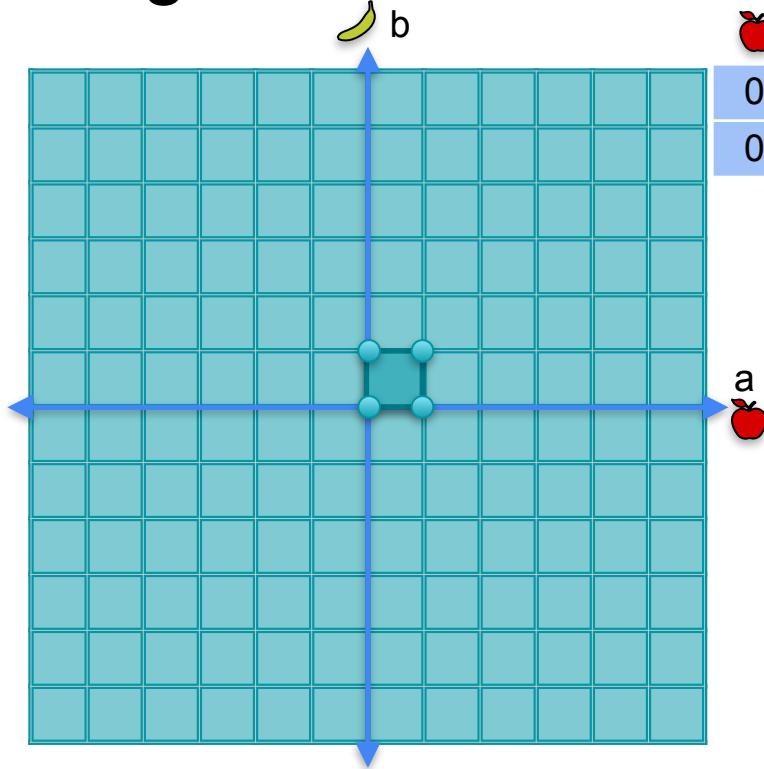
Singular transformation



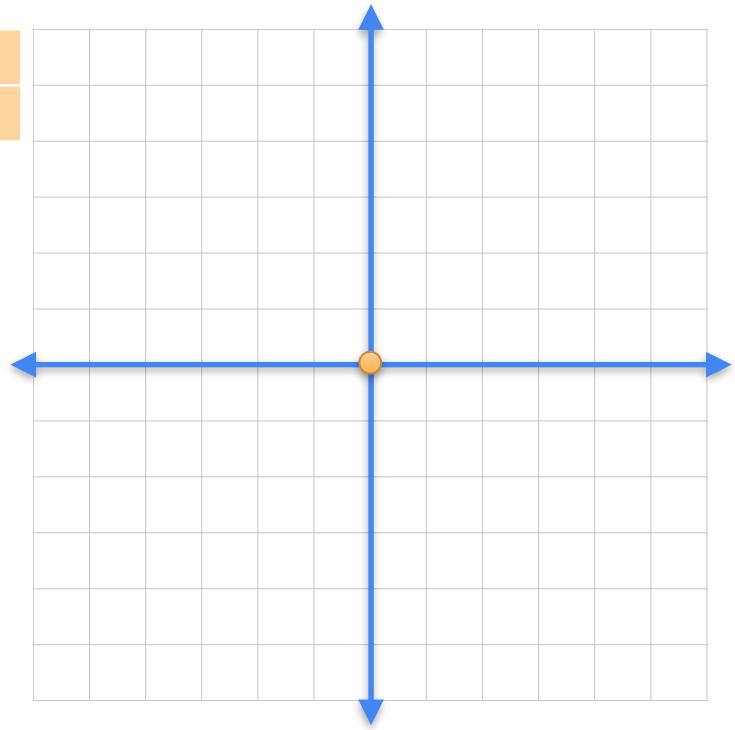
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} a \\ b \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix}$$



Singular transformation



$$\begin{matrix} \text{apple} & \text{banana} \\ 0 & 0 \\ 0 & 0 \end{matrix} \times \begin{matrix} a \\ b \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

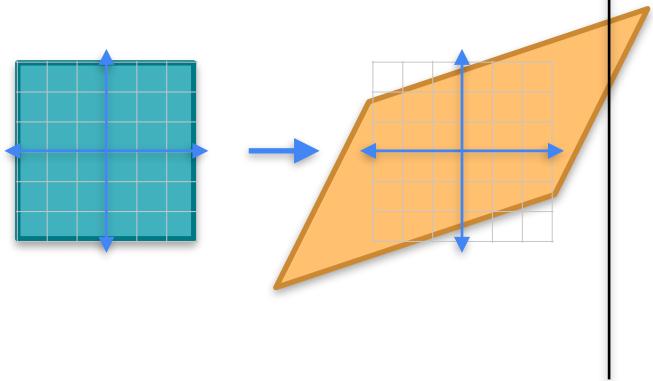


Singular and non-singular transformations

Singular and non-singular transformations

Non-singular

		
3	1	
1	2	



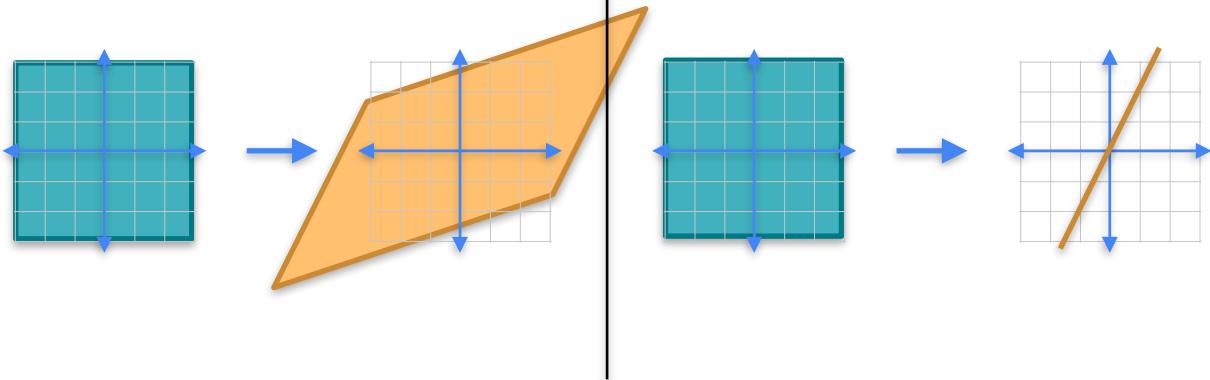
Singular and non-singular transformations

Non-singular

3	1
1	2

Singular

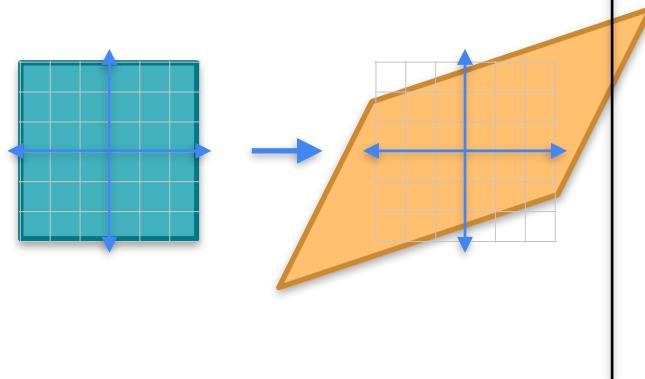
1	1
2	2



Singular and non-singular transformations

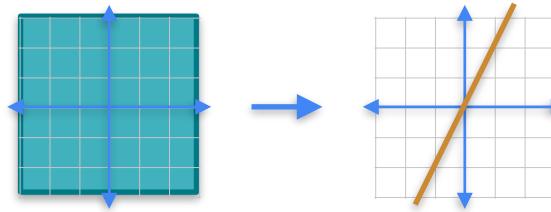
Non-singular

3	1
1	2



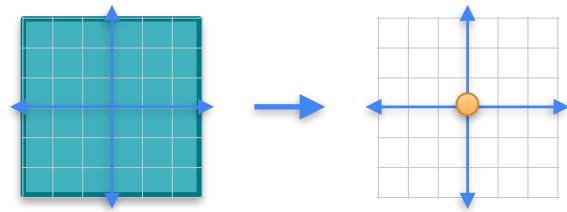
Singular

1	1
2	2



Singular

0	0
0	0

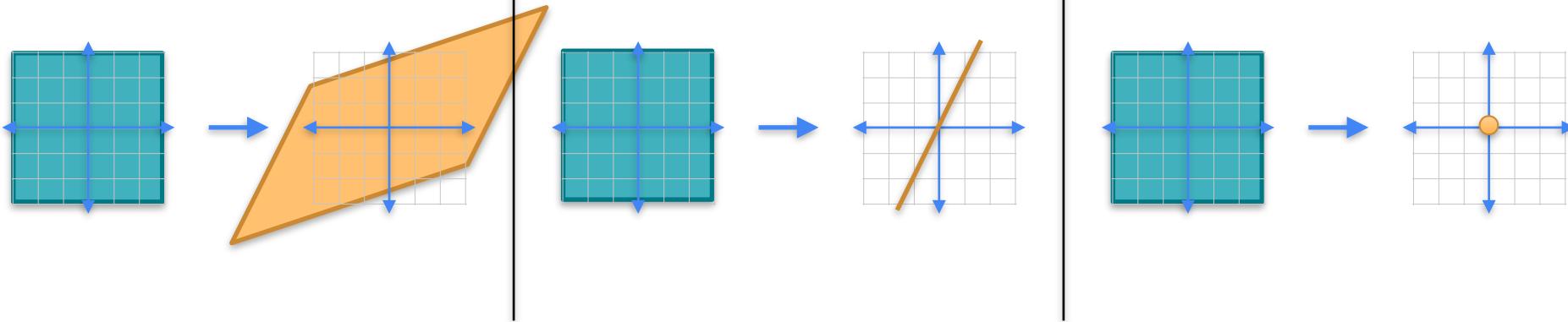


Rank of linear transformations

3	1
1	2

1	1
2	2

0	0
0	0

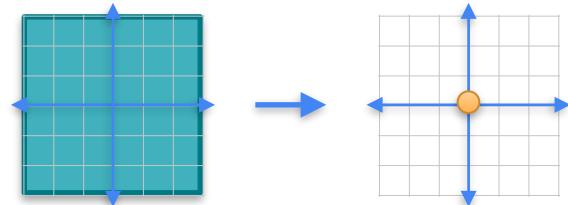
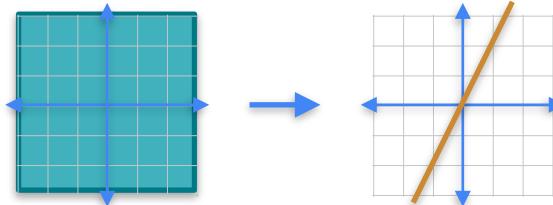
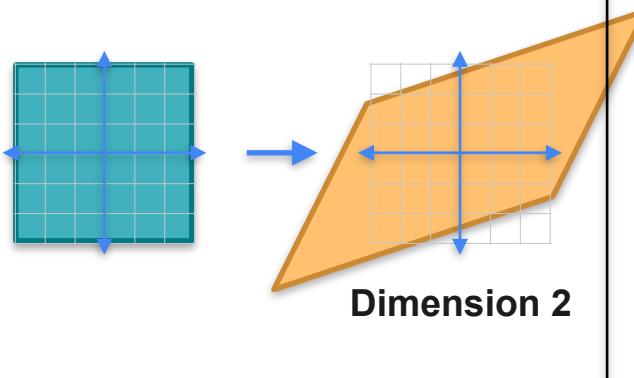


Rank of linear transformations

3	1
1	2

1	1
2	2

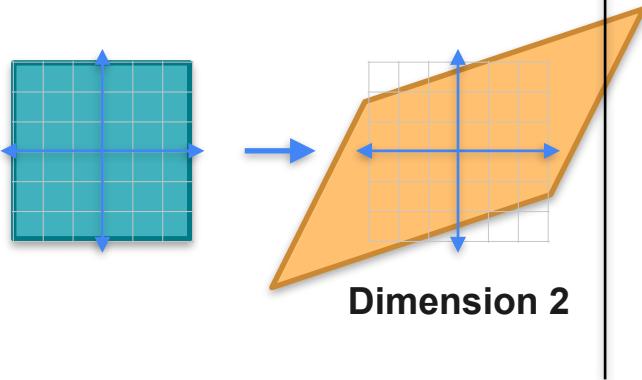
0	0
0	0



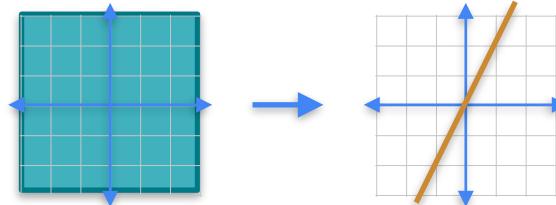
Rank of linear transformations

Rank 2

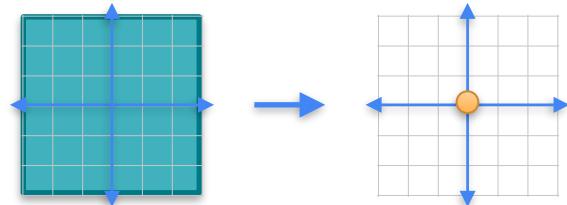
3	1	
1	2	



1	1	
2	2	



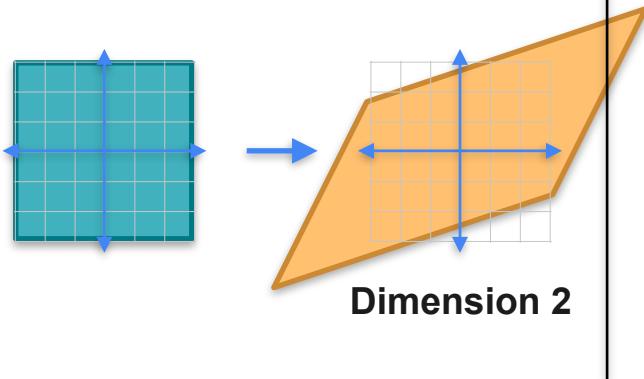
0	0	
0	0	



Rank of linear transformations

Rank 2

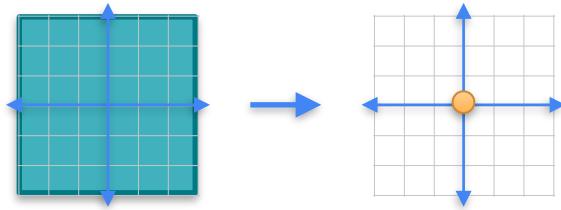
3	1	
1	2	



1	1	
2	2	

Dimension 1

0	0	
0	0	



Rank of linear transformations

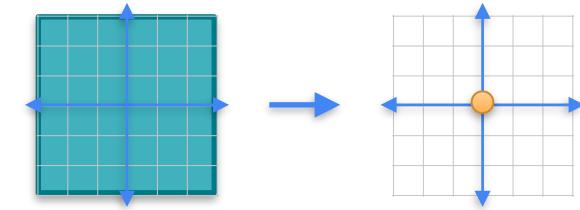
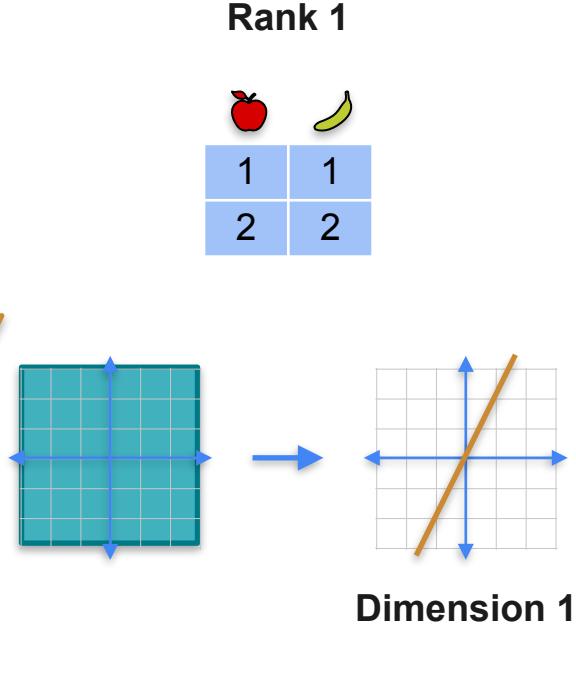
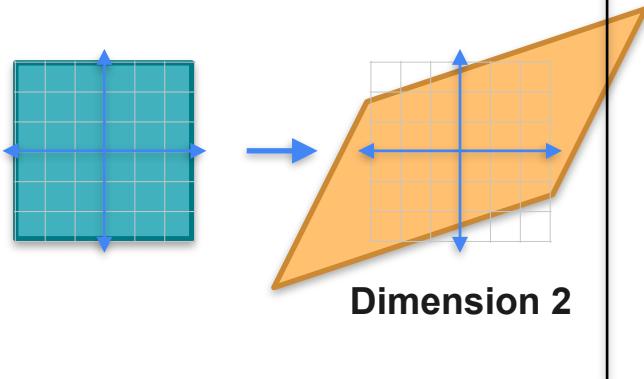
Rank 2

3	1
1	2

Rank 1

1	1
2	2

0	0
0	0



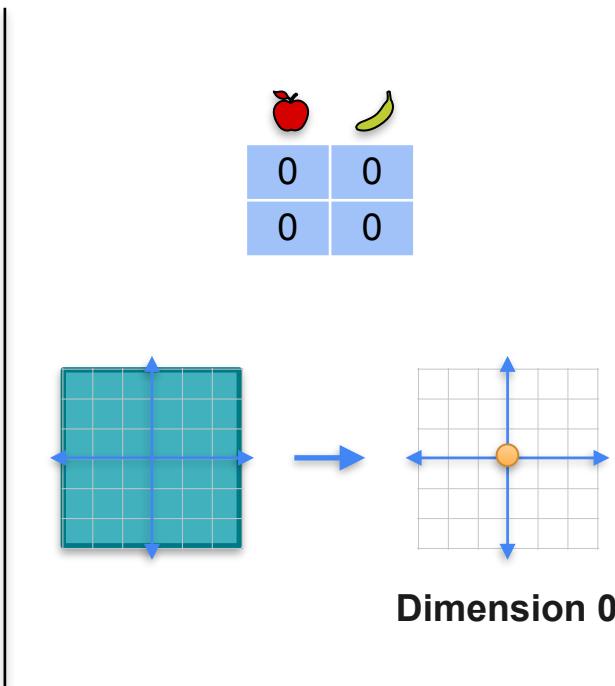
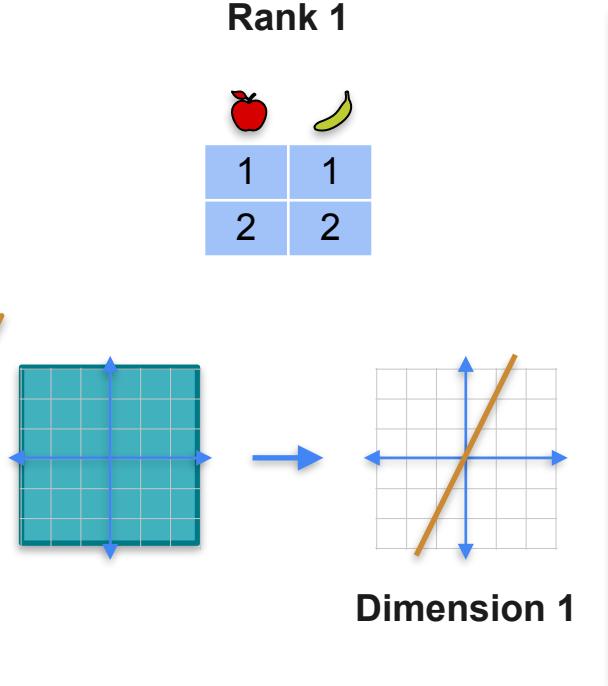
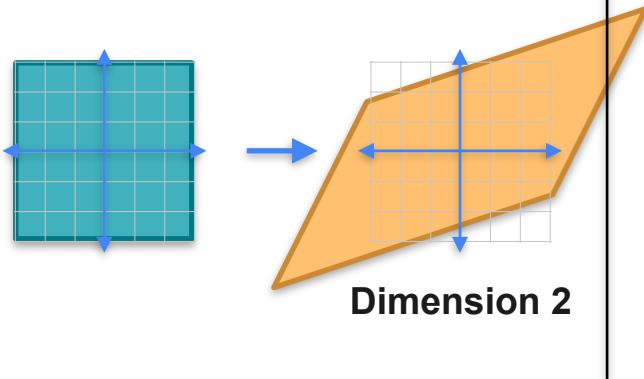
Rank of linear transformations

Rank 2

3	1
1	2

Rank 1

1	1
2	2



Rank of linear transformations

Rank 2

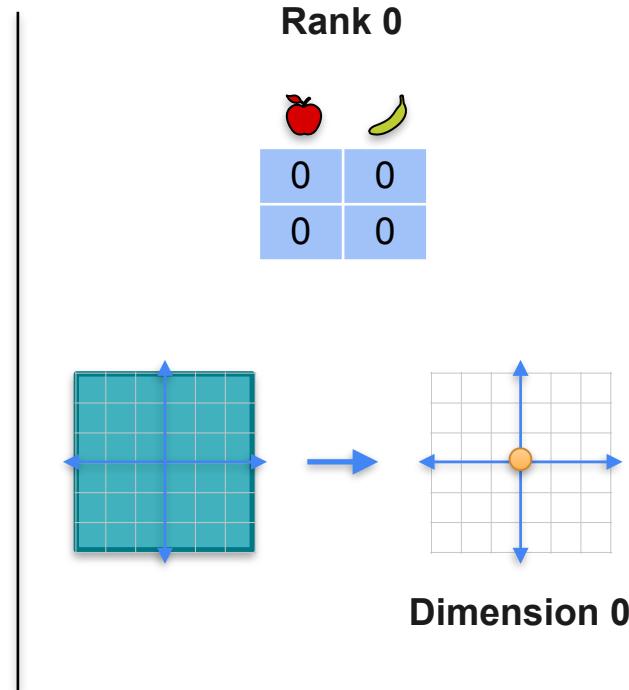
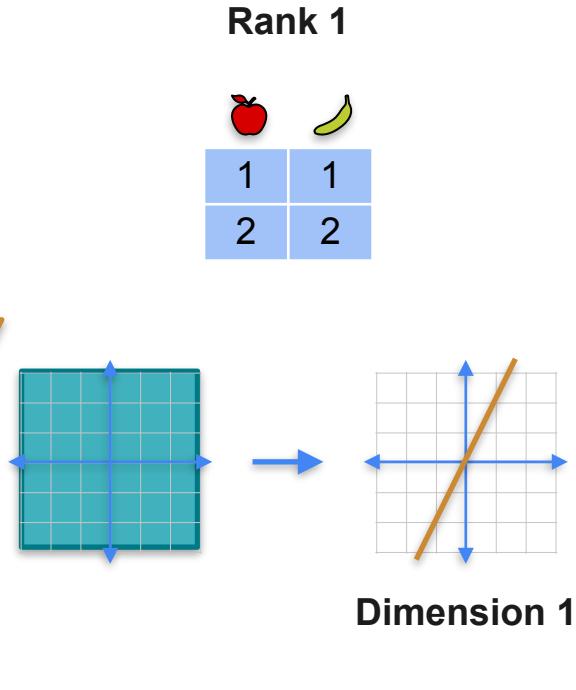
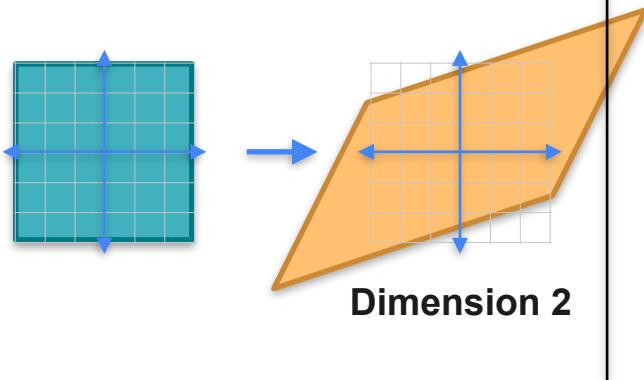
3	1
1	2

Rank 1

1	1
2	2

Rank 0

0	0
0	0



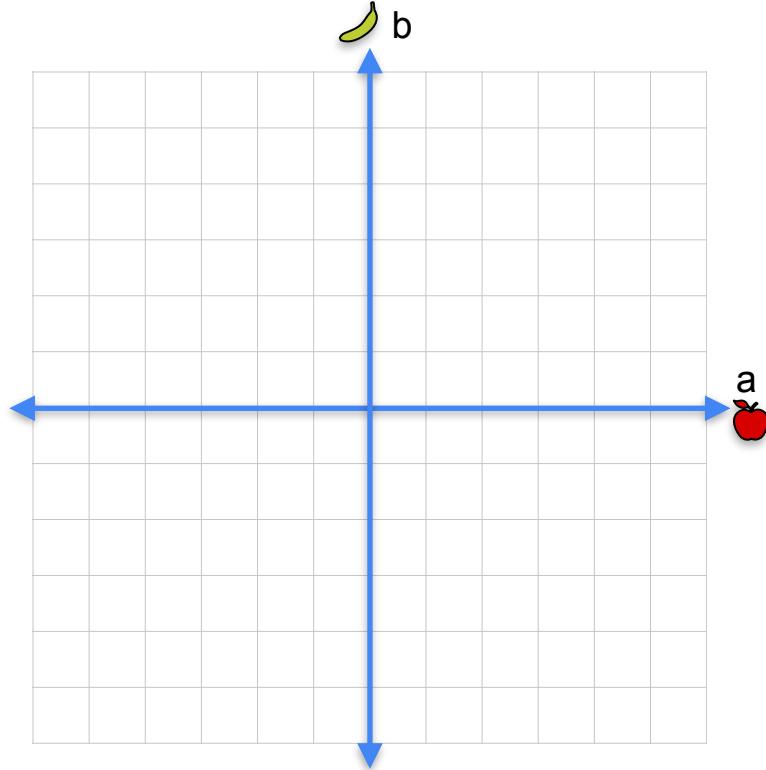


DeepLearning.AI

Determinants and Eigenvectors

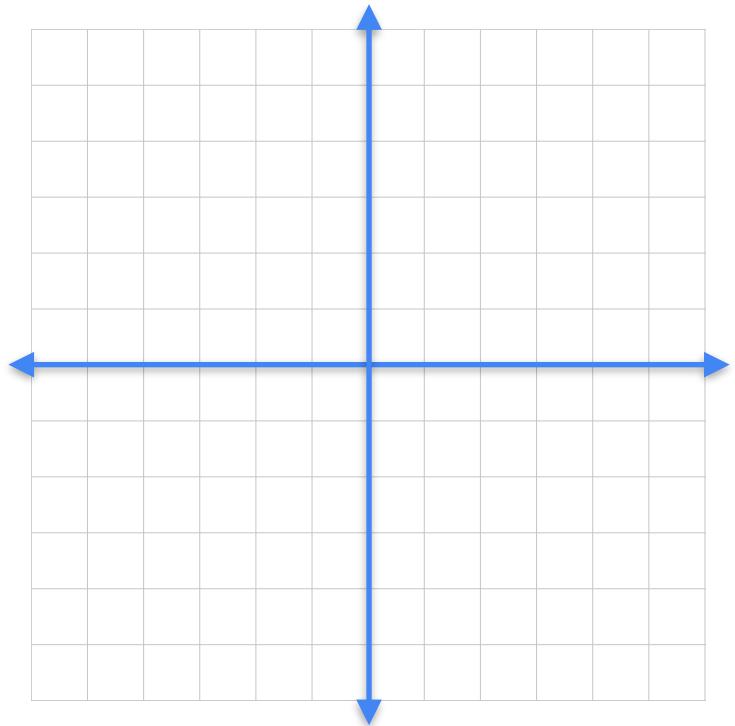
Determinant as an area

Determinant as an area

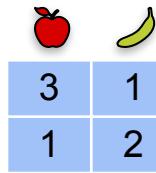
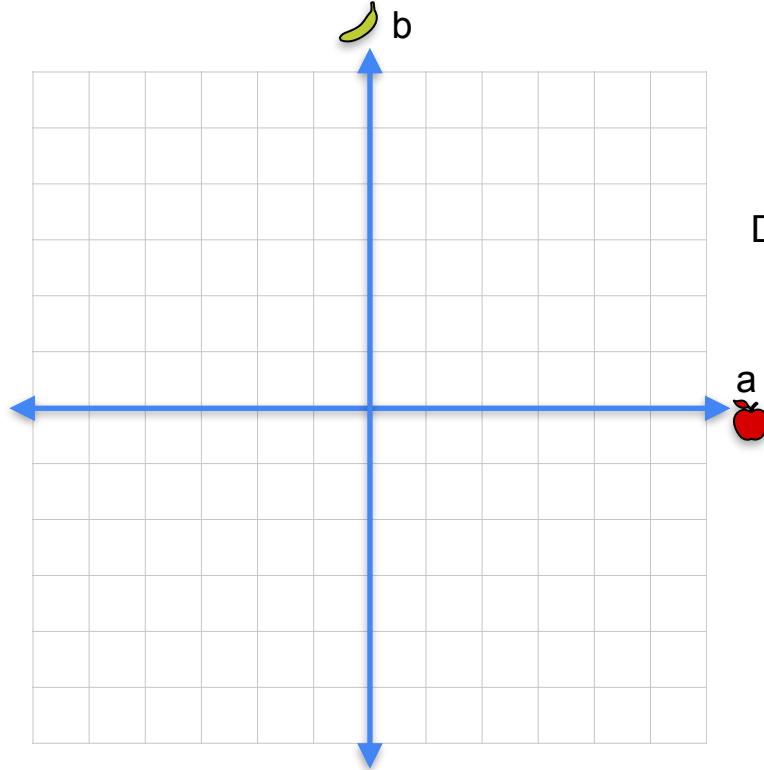


A 2x2 matrix with a red apple icon above it and a yellow banana icon to its right.

3	1
1	2

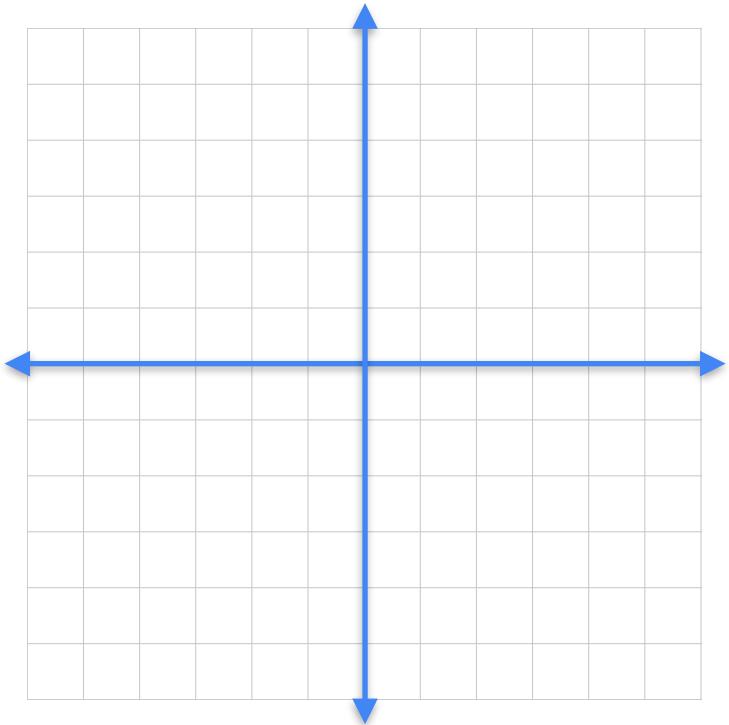


Determinant as an area

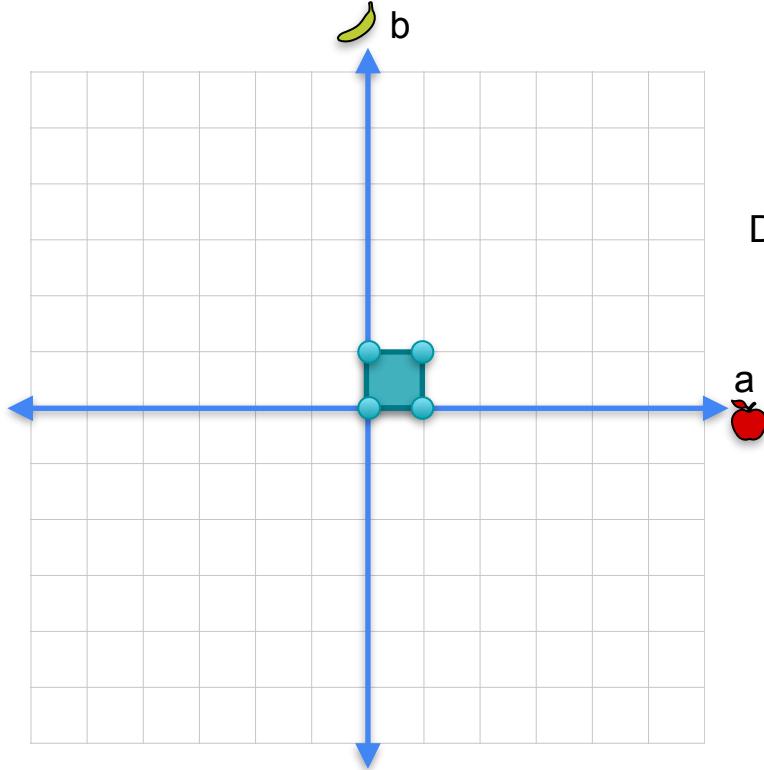


$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$



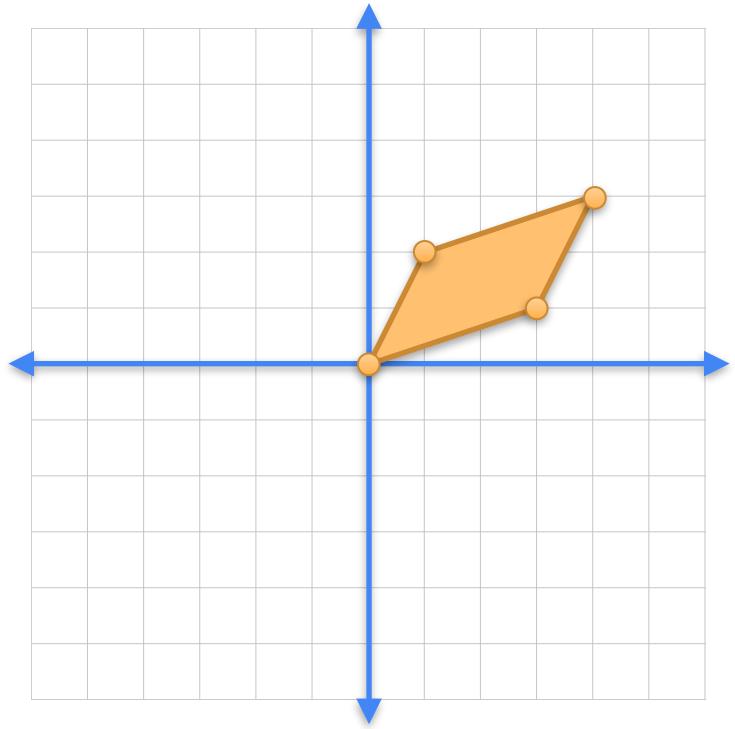
Determinant as an area



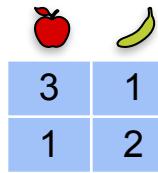
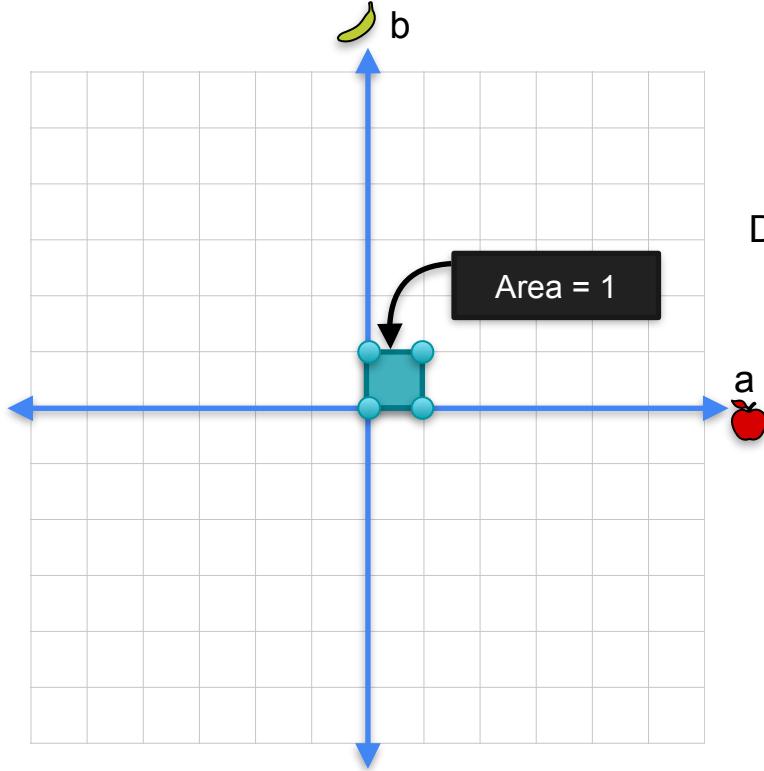
	apple	banana
3	1	
1	2	

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$

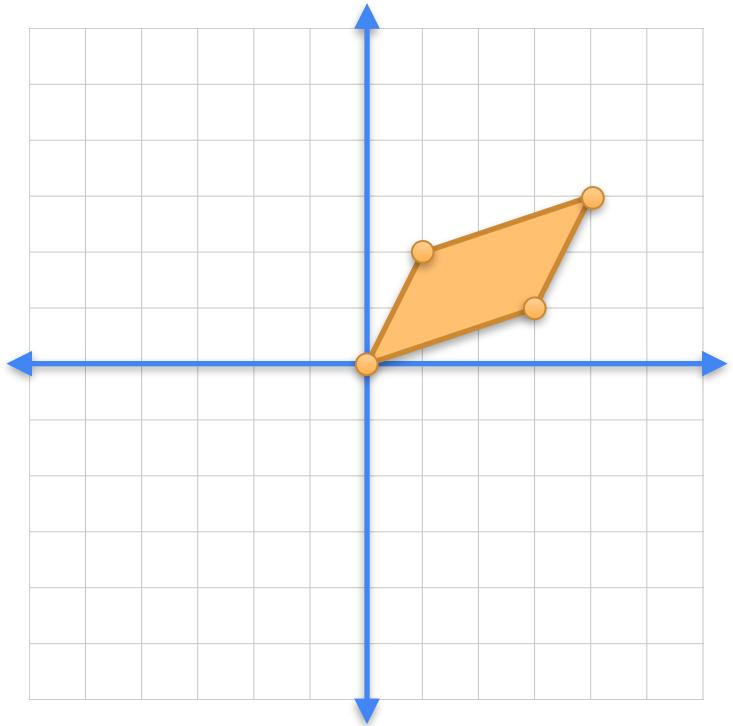


Determinant as an area

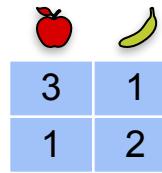
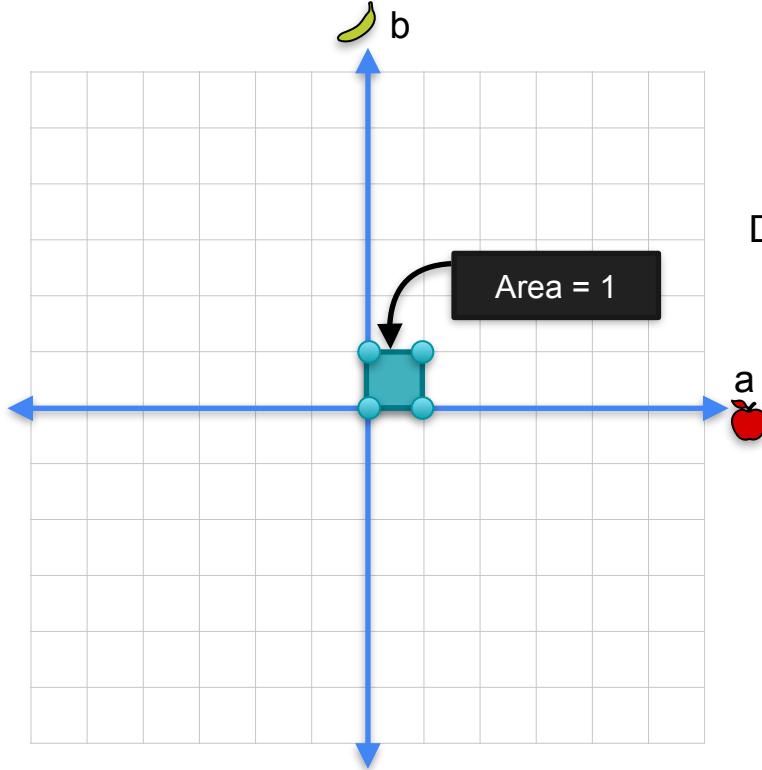


$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$

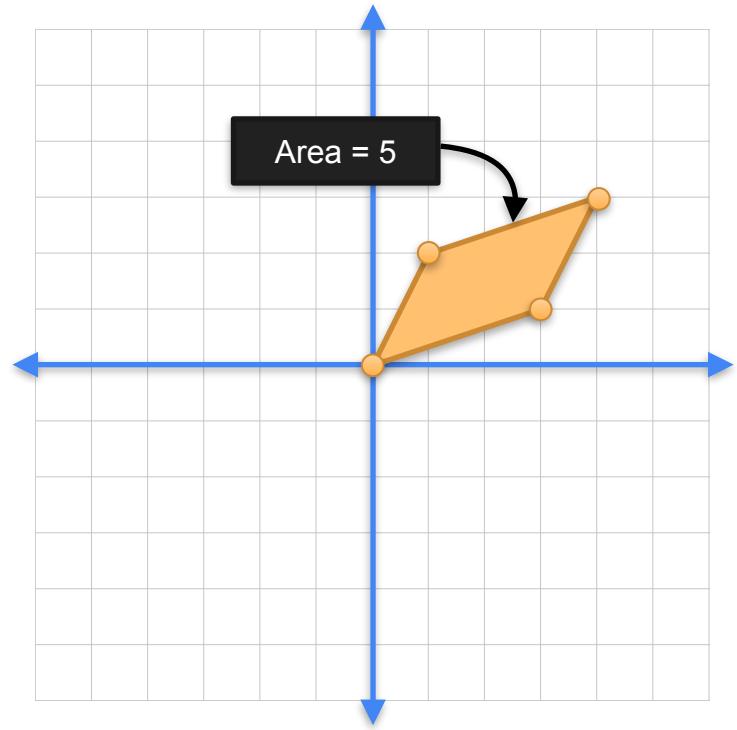


Determinant as an area

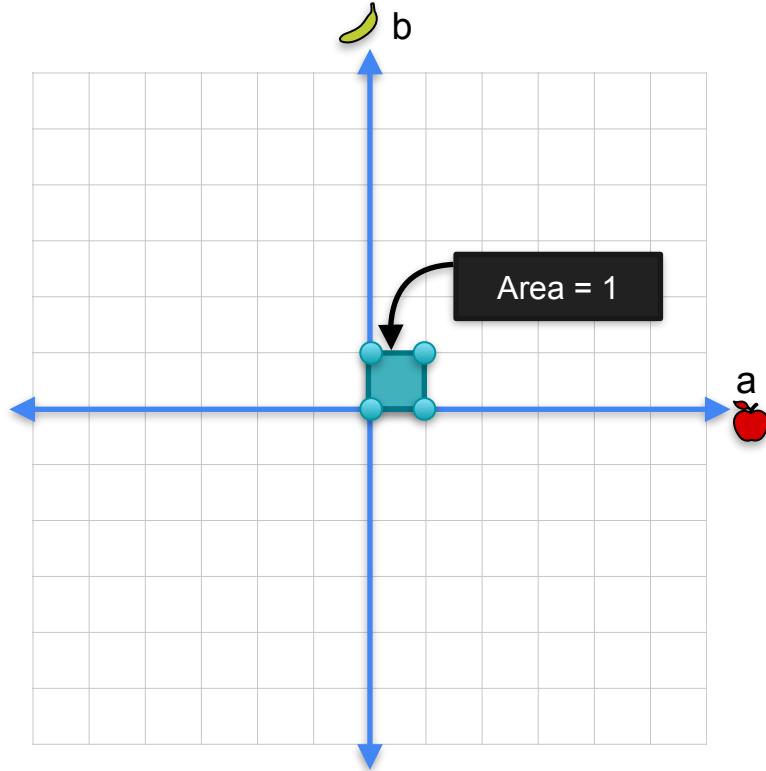


$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

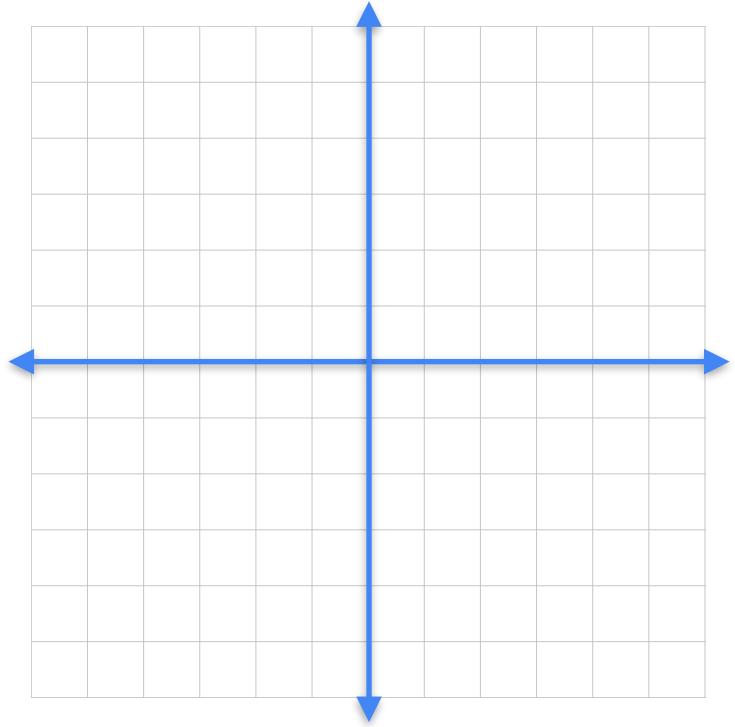
$$\text{Det} = 5$$



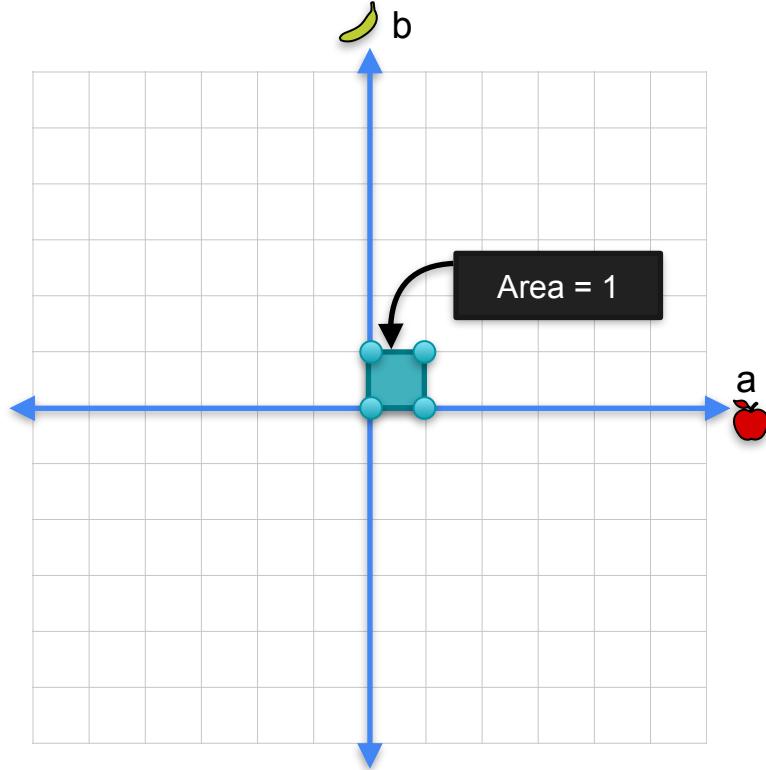
Determinant as an area



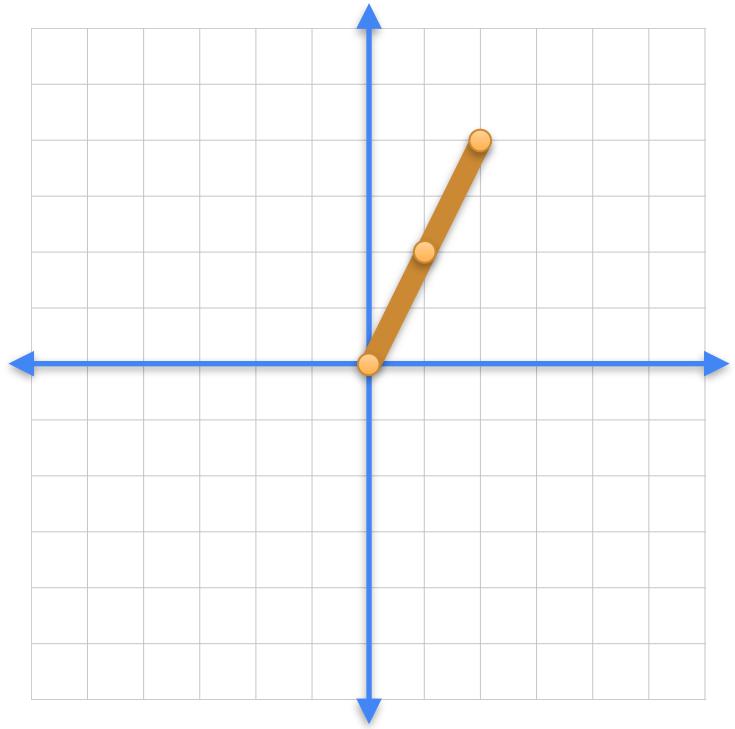
1	1
2	2



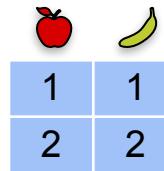
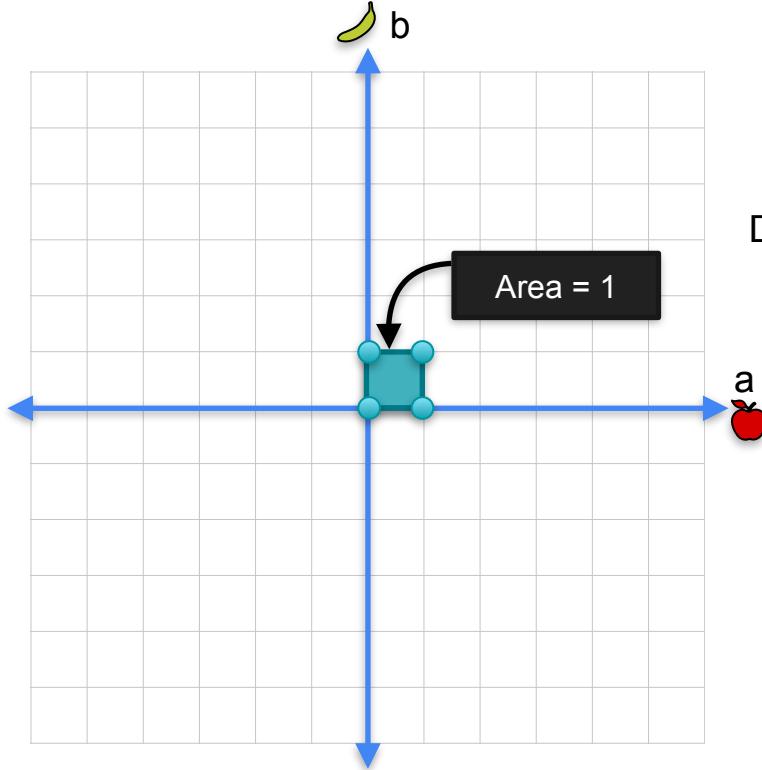
Determinant as an area



1	1
2	2

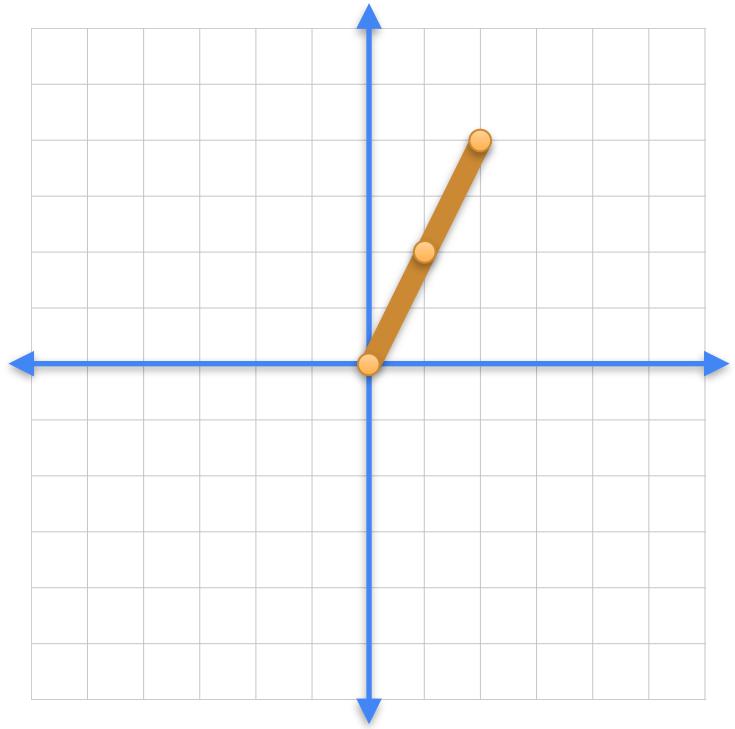


Determinant as an area

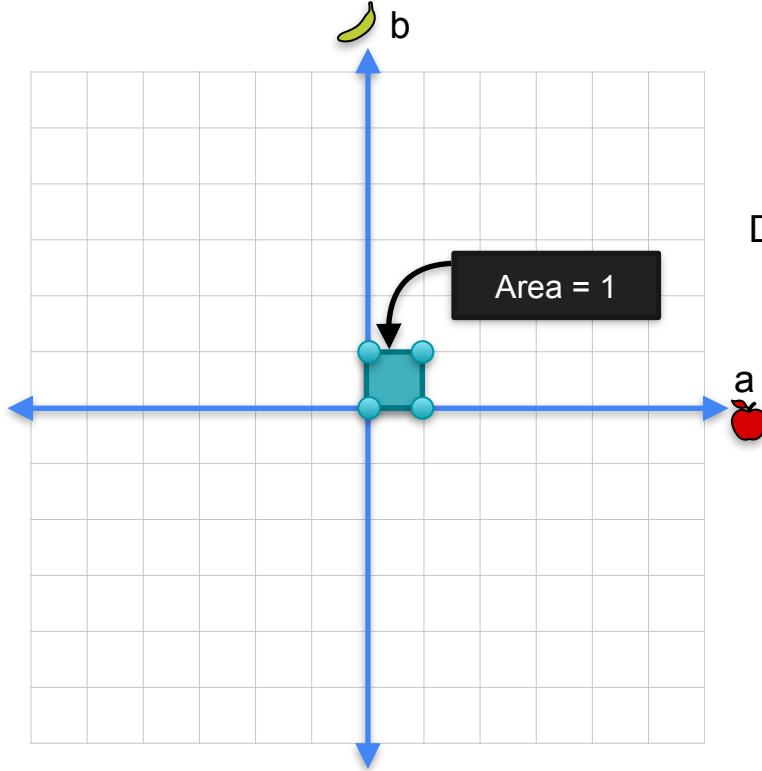


$$\text{Det} = 1 \cdot 2 - 1 \cdot 2$$

$$\text{Det} = 0$$



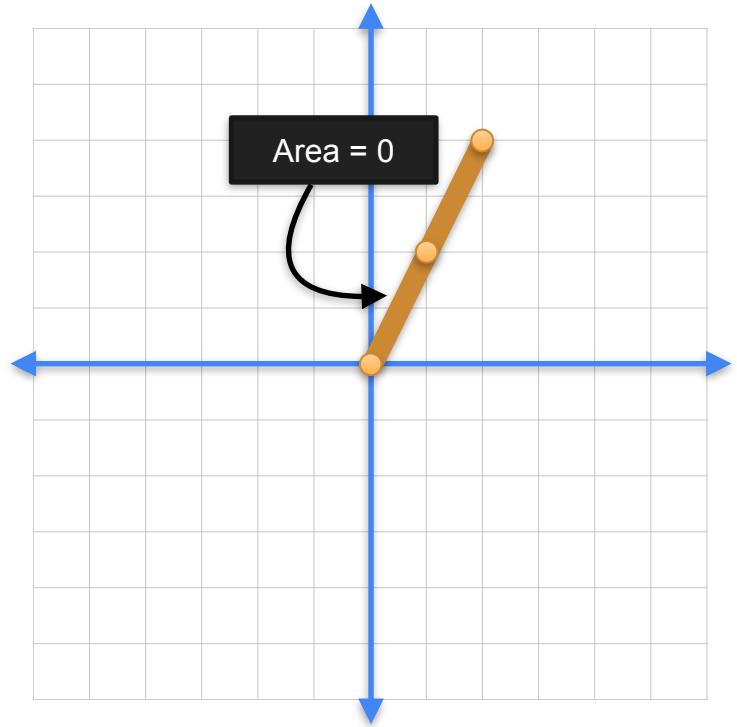
Determinant as an area



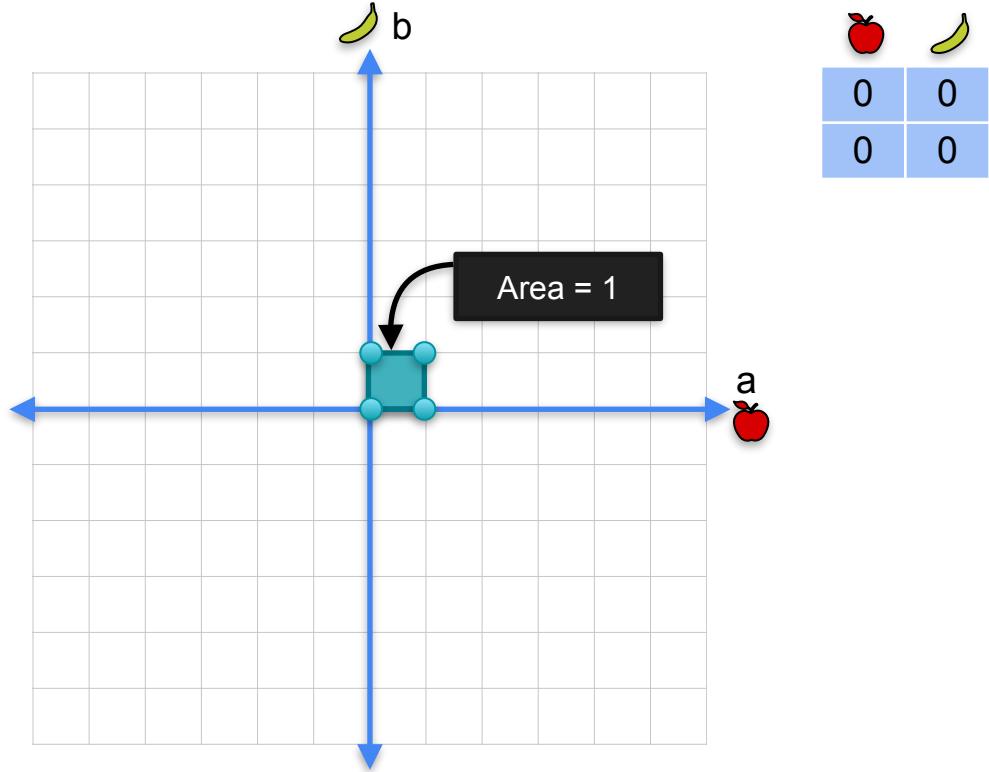
1	1
2	2

$$\text{Det} = 1 \cdot 2 - 1 \cdot 2$$

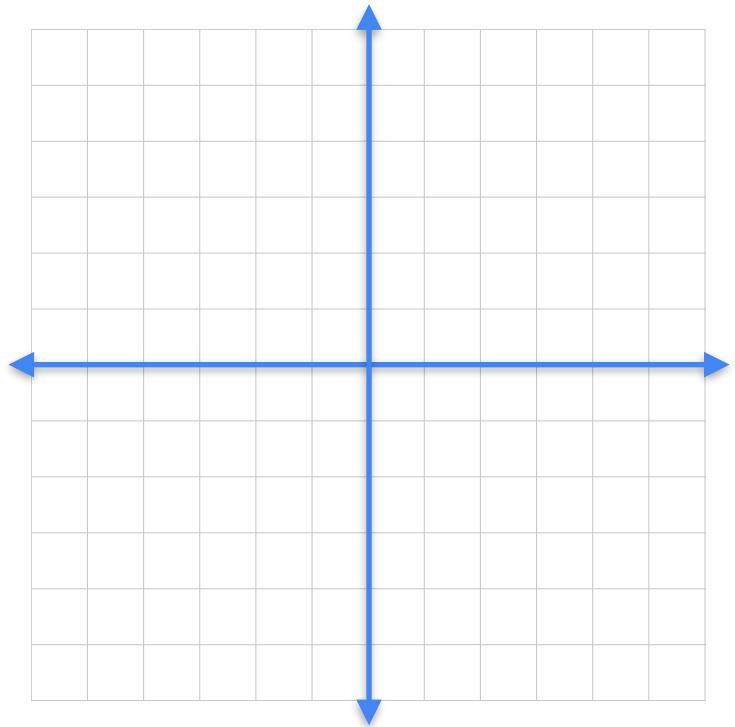
$$\text{Det} = 0$$



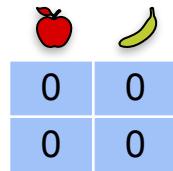
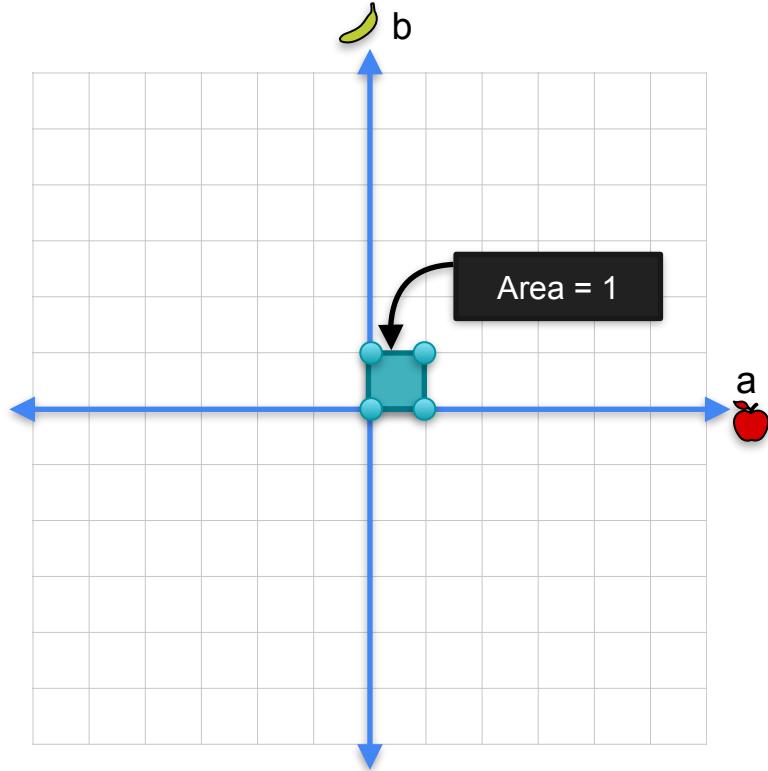
Determinant as an area



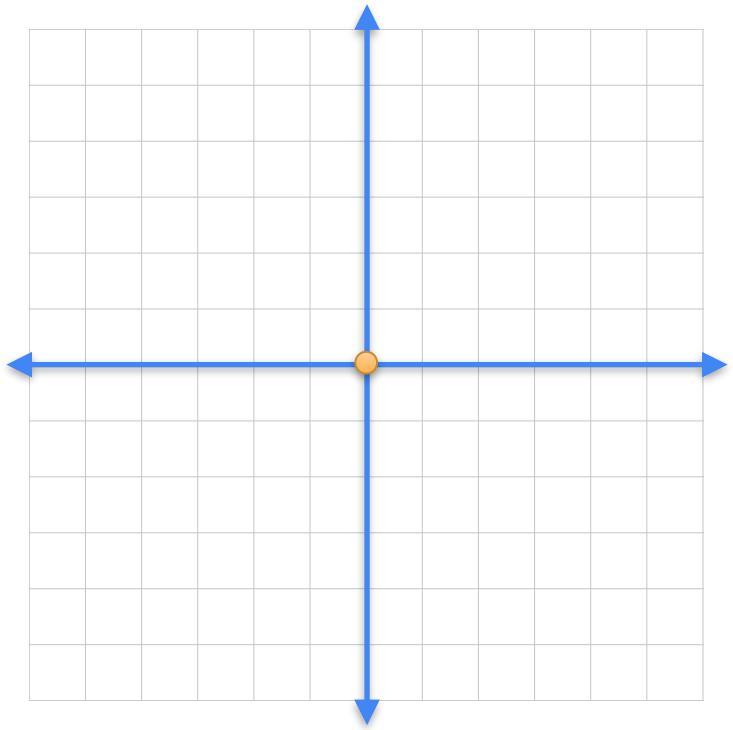
0	0
0	0



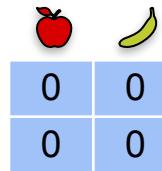
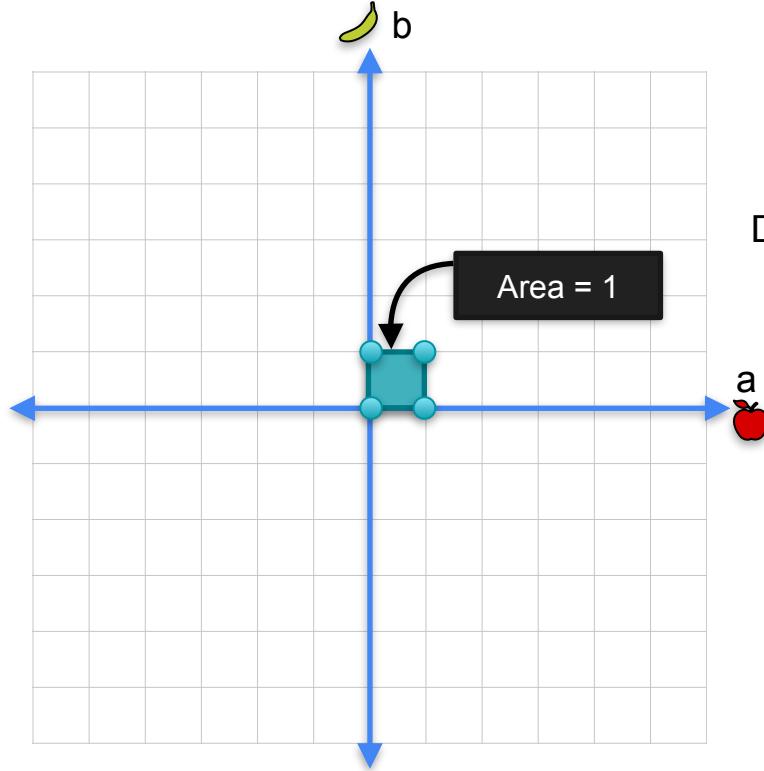
Determinant as an area



	a
0	0
0	0

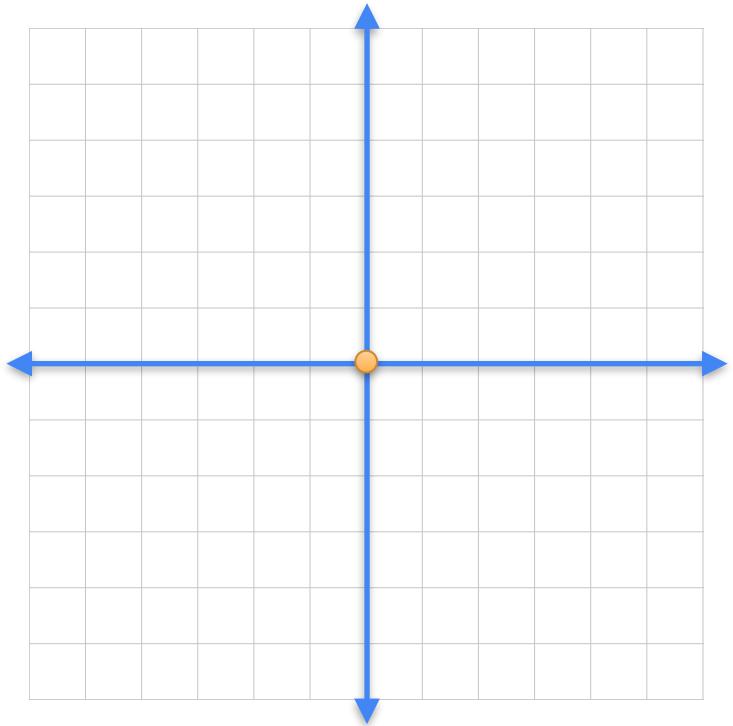


Determinant as an area

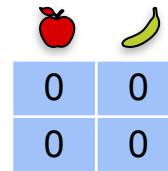
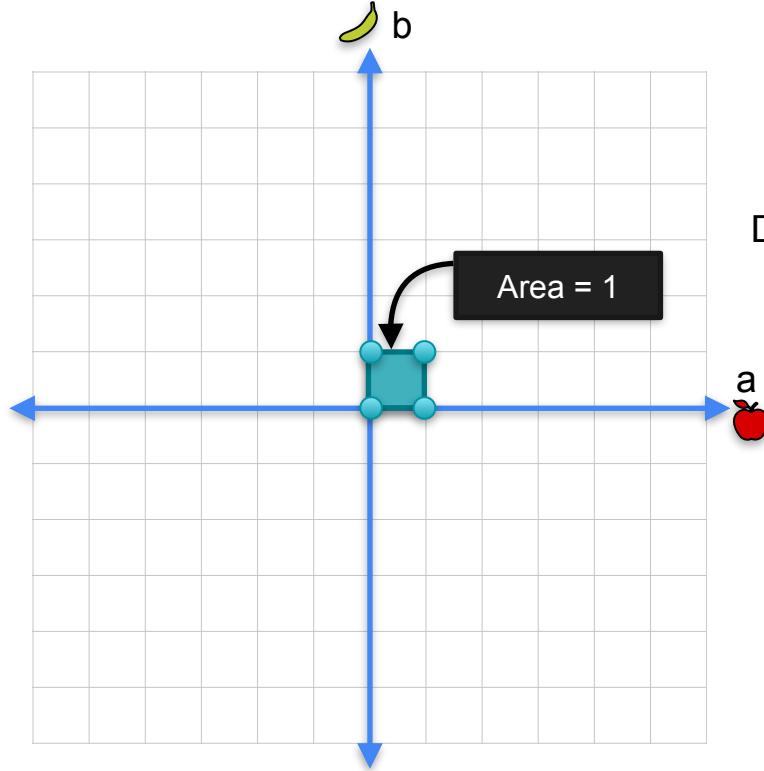


$$\text{Det} = 0 \cdot 0 - 0 \cdot 0$$

$$\text{Det} = 0$$

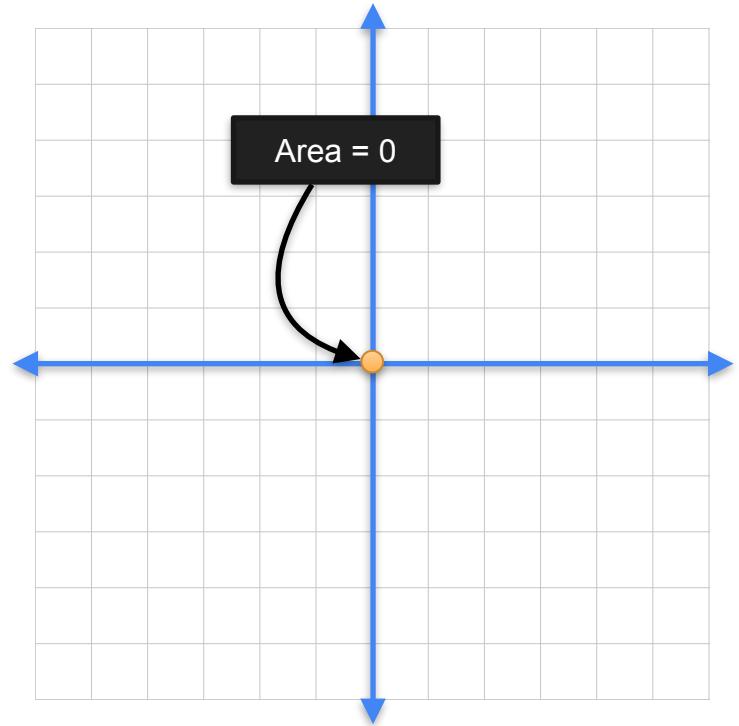


Determinant as an area

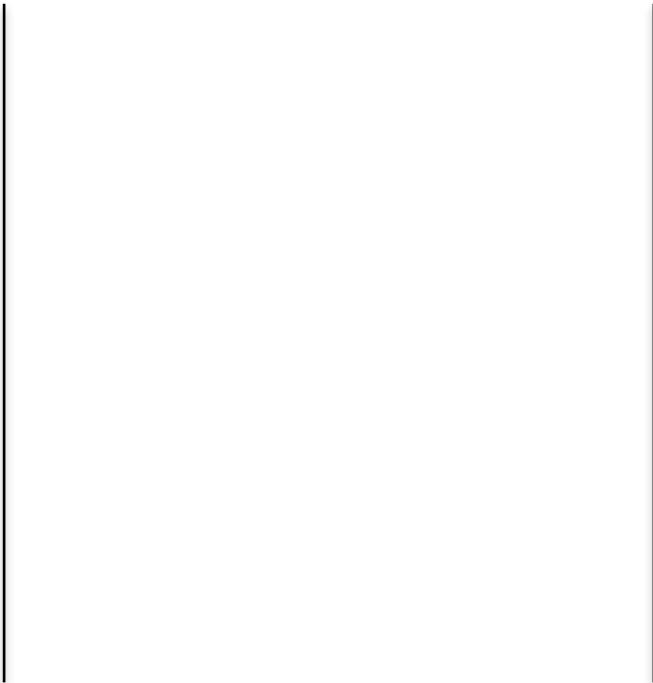


$$\text{Det} = 0 \cdot 0 - 0 \cdot 0$$

$$\text{Det} = 0$$



Determinant as an area



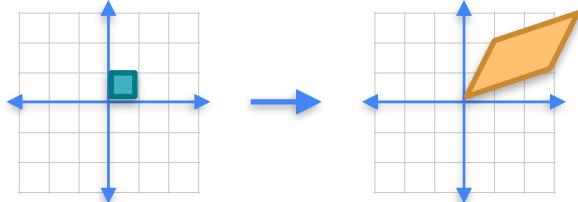
Determinant as an area

Non-singular



3	1
1	2

$$\text{Determinant} = 5$$



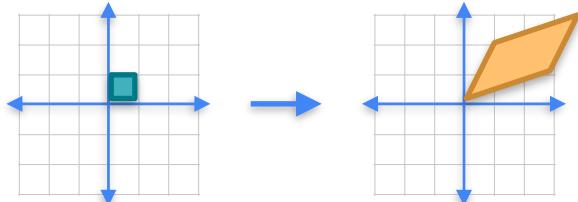
$$\text{Area} = 5$$

Determinant as an area

Non-singular

3	1
1	2

$$\text{Determinant} = 5$$

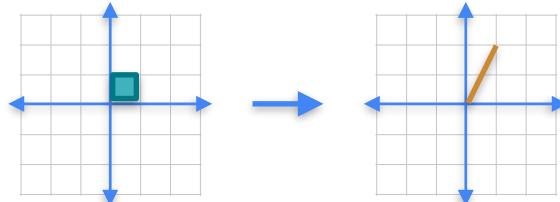


$$\text{Area} = 5$$

Singular

1	1
2	2

$$\text{Determinant} = 0$$



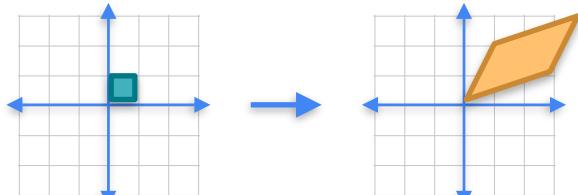
$$\text{Area} = 0$$

Determinant as an area

Non-singular

3	1
1	2

Determinant = 5

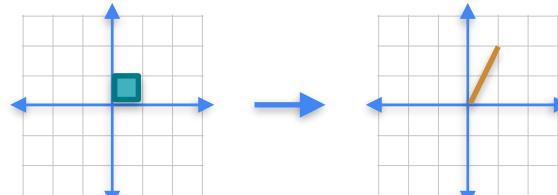


Area = 5

Singular

1	1
2	2

Determinant = 0

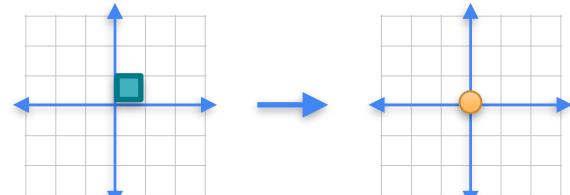


Area = 0

Singular

0	0
0	0

Determinant = 0



Area = 0

Negative determinants?

	
3	1
1	2

	
1	3
2	1

Negative determinants?

	
3	1
1	2

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

$$\text{Det} = 5$$

	
1	3
2	1

Negative determinants?

	
3	1
1	2

$$\text{Det} = 3 \cdot 2 - 1 \cdot 1$$

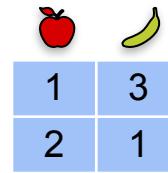
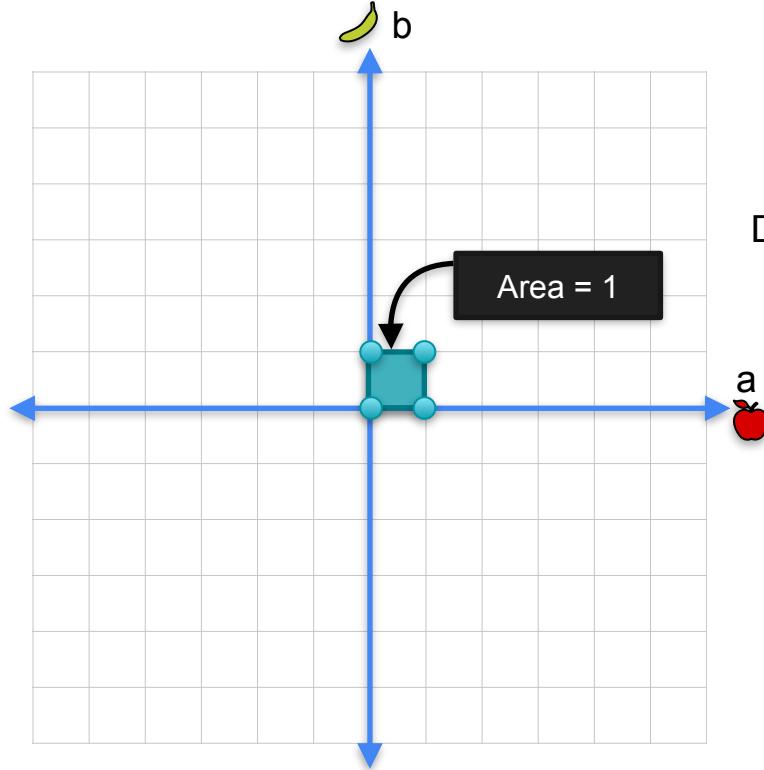
$$\text{Det} = 5$$

	
1	3
2	1

$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

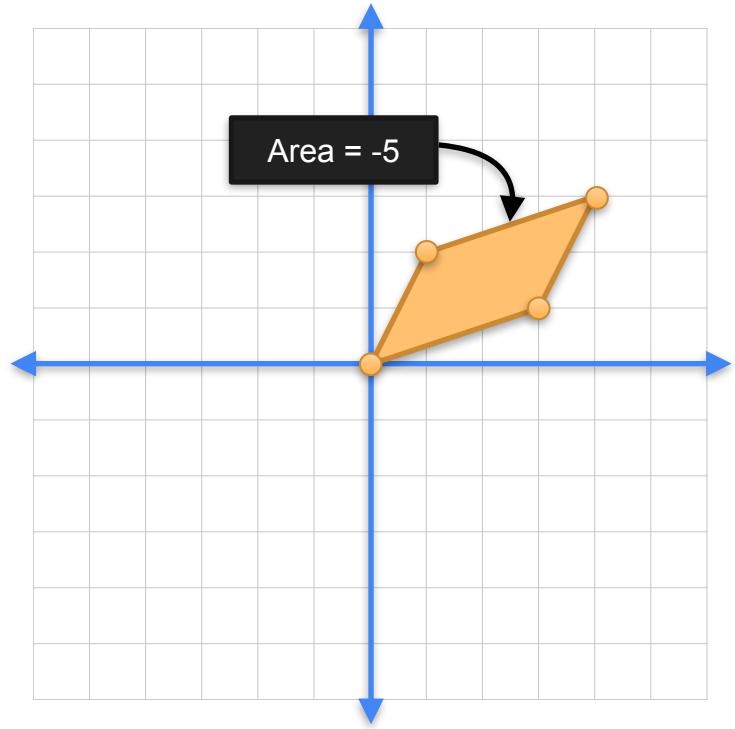
$$\text{Det} = -5$$

Determinant as an area

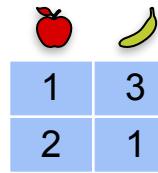
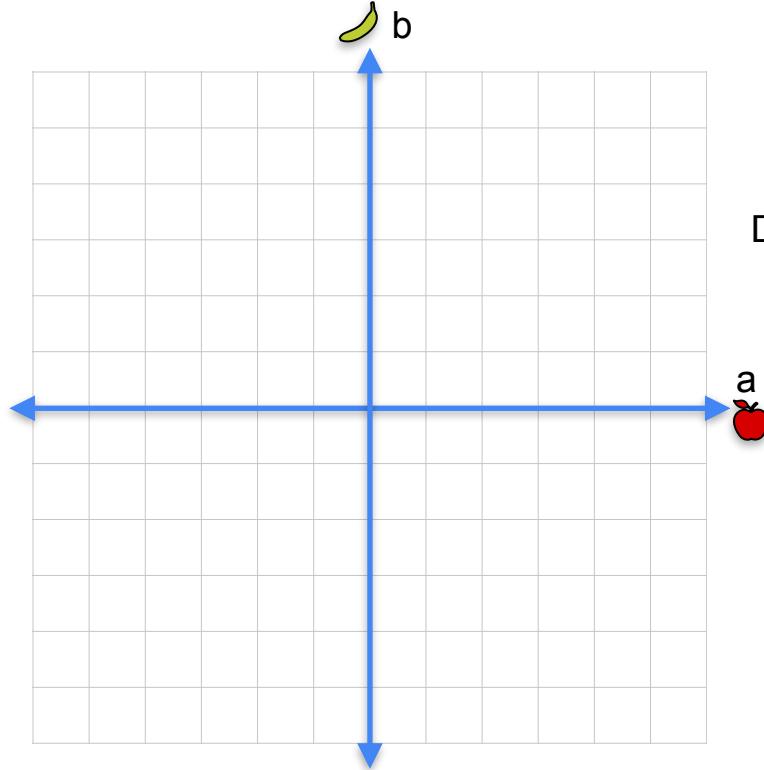


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

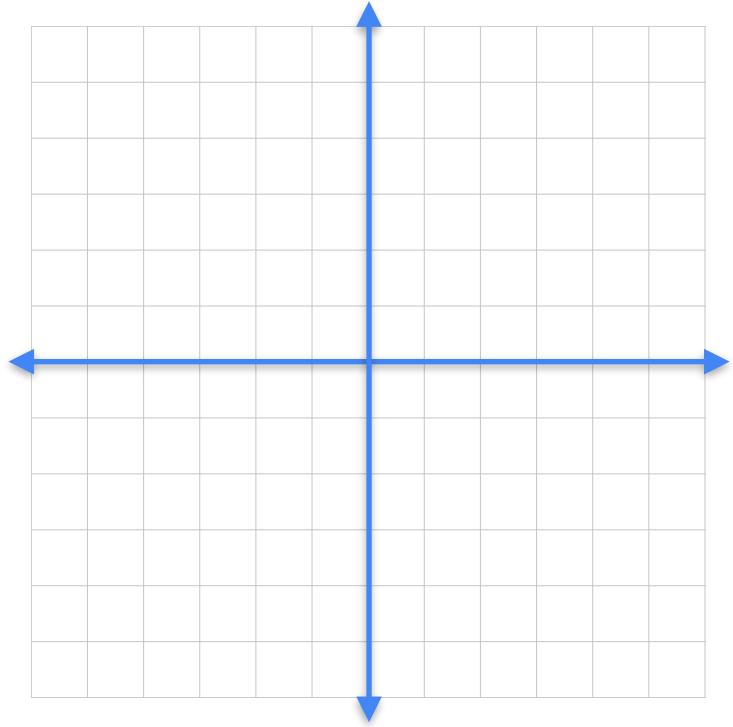


Determinant as an area

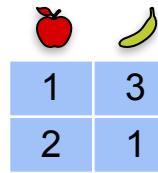
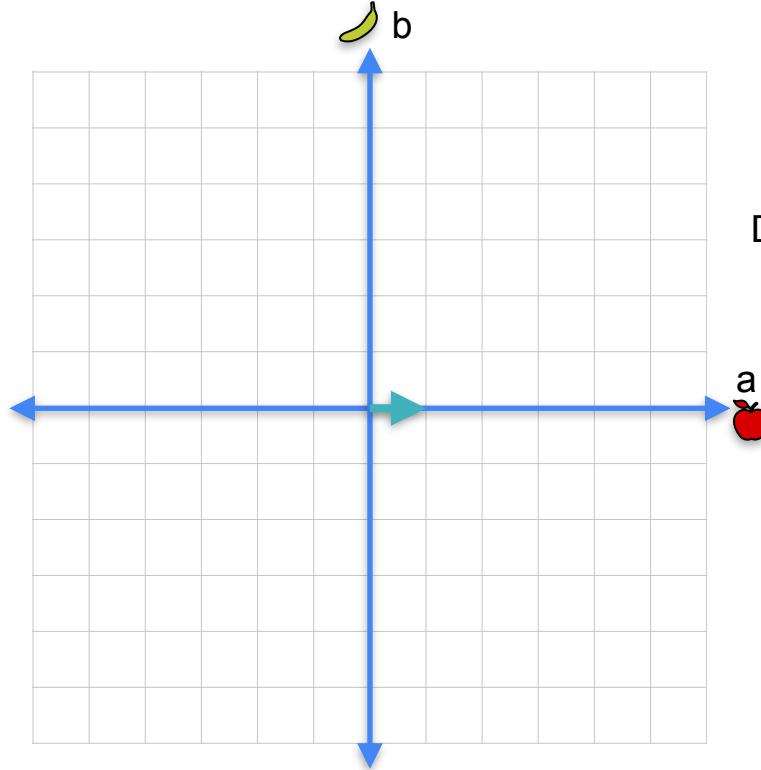


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

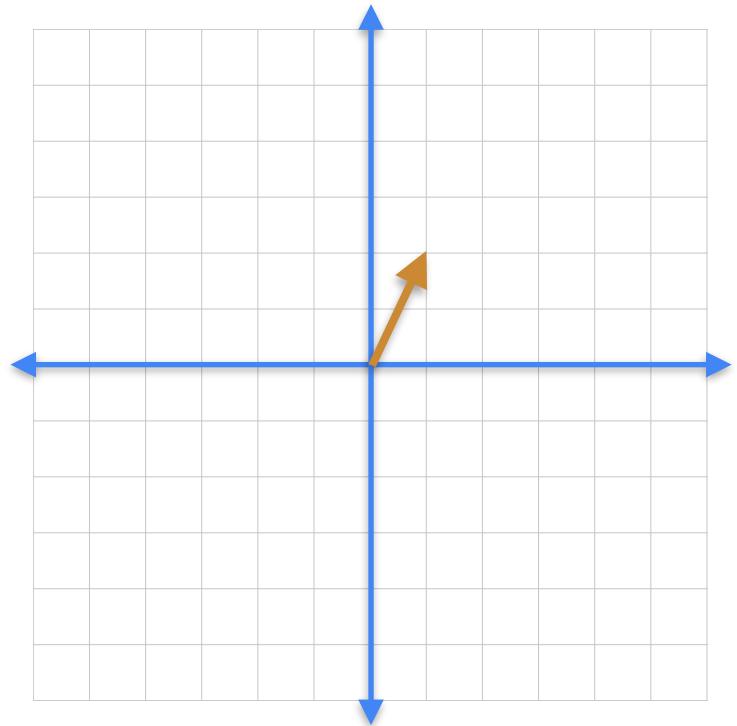


Determinant as an area

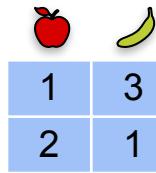
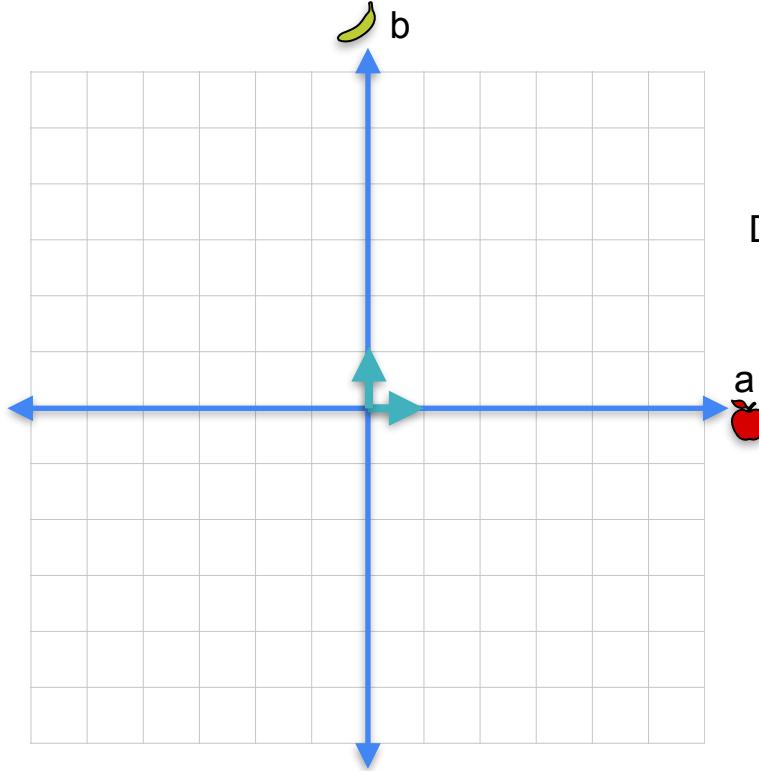


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

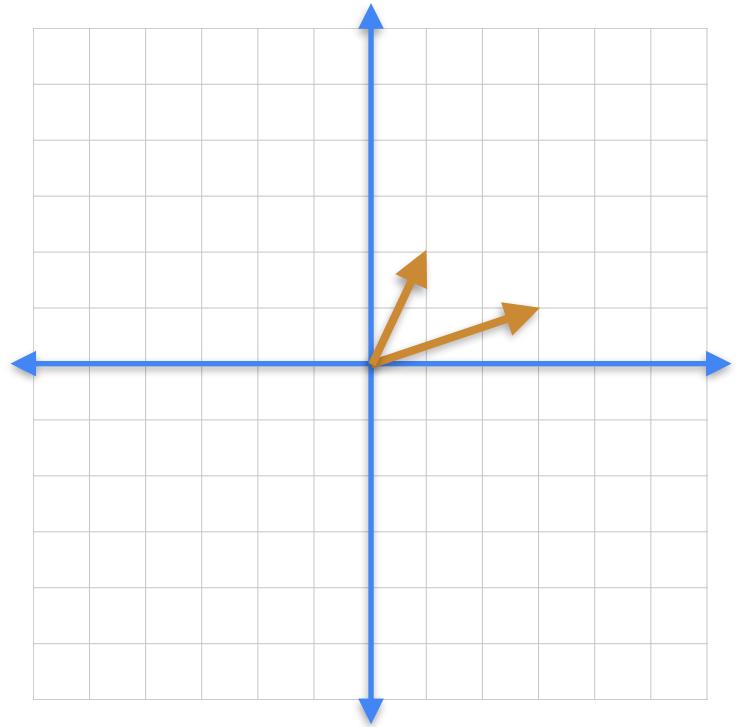


Determinant as an area

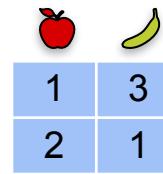
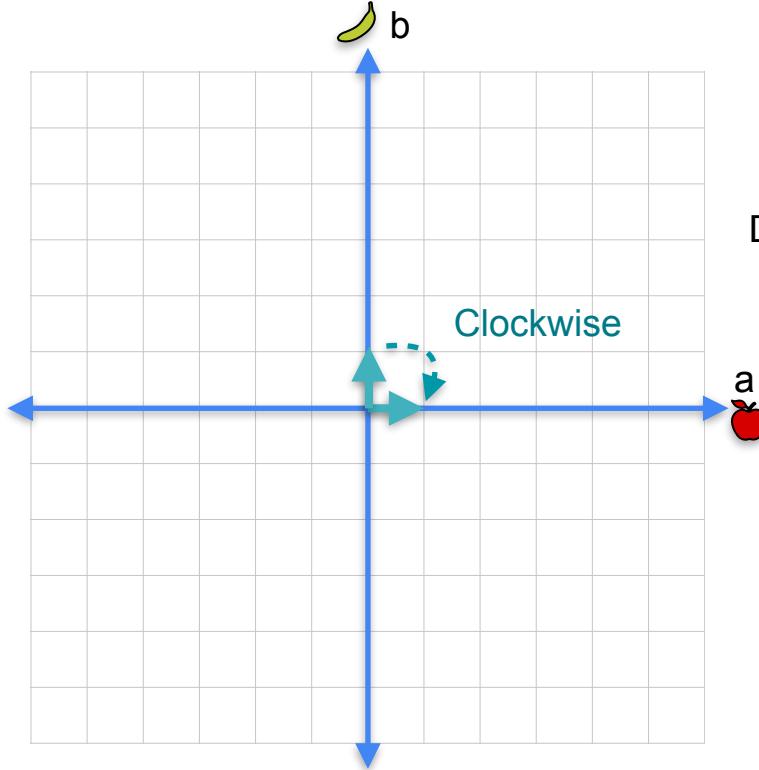


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

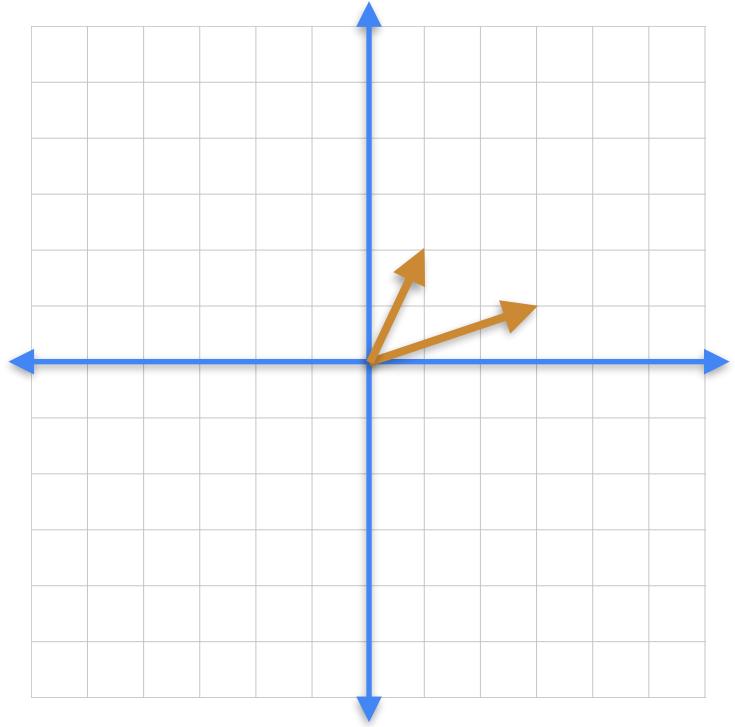


Determinant as an area

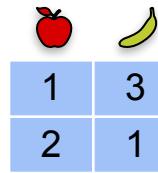
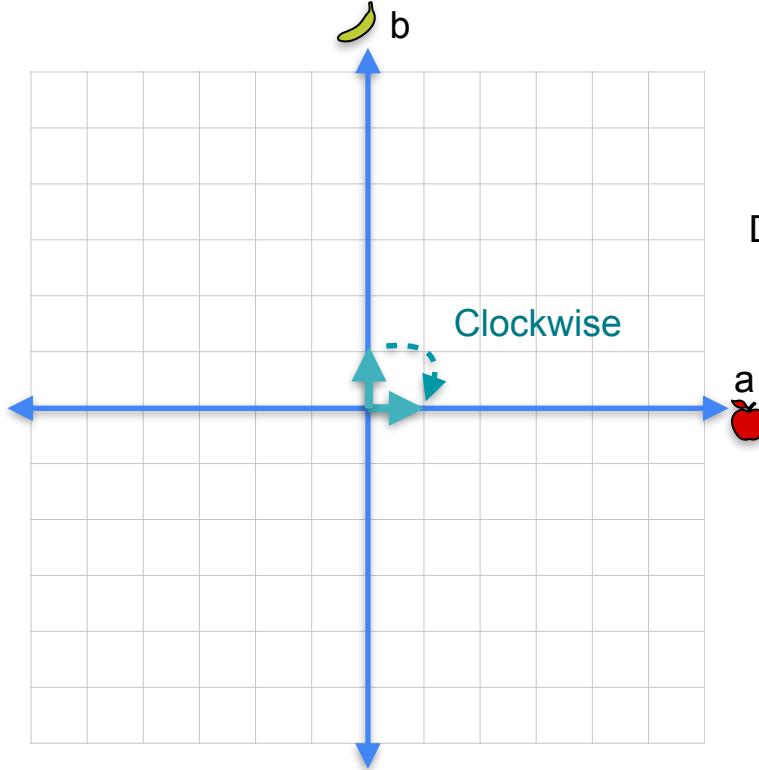


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

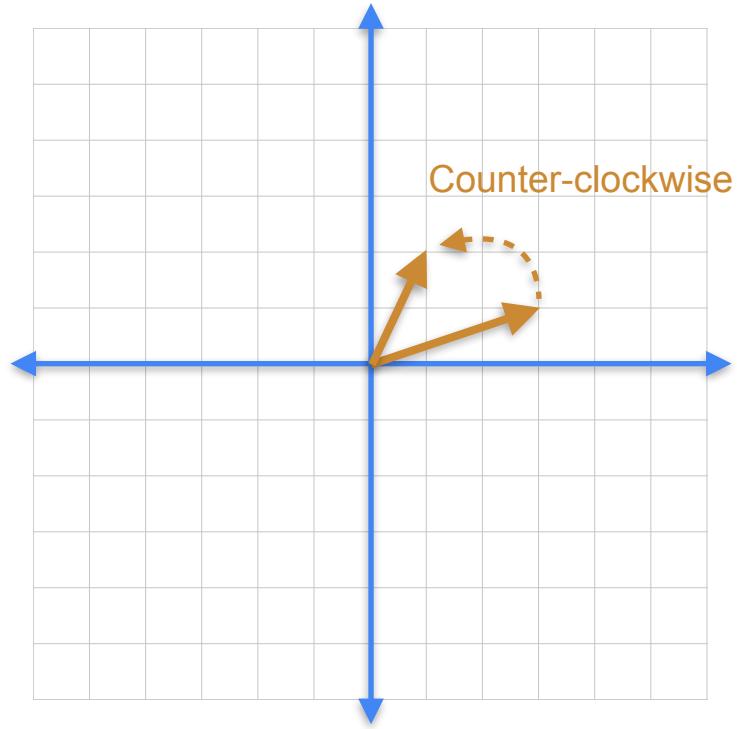


Determinant as an area

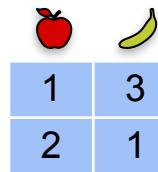
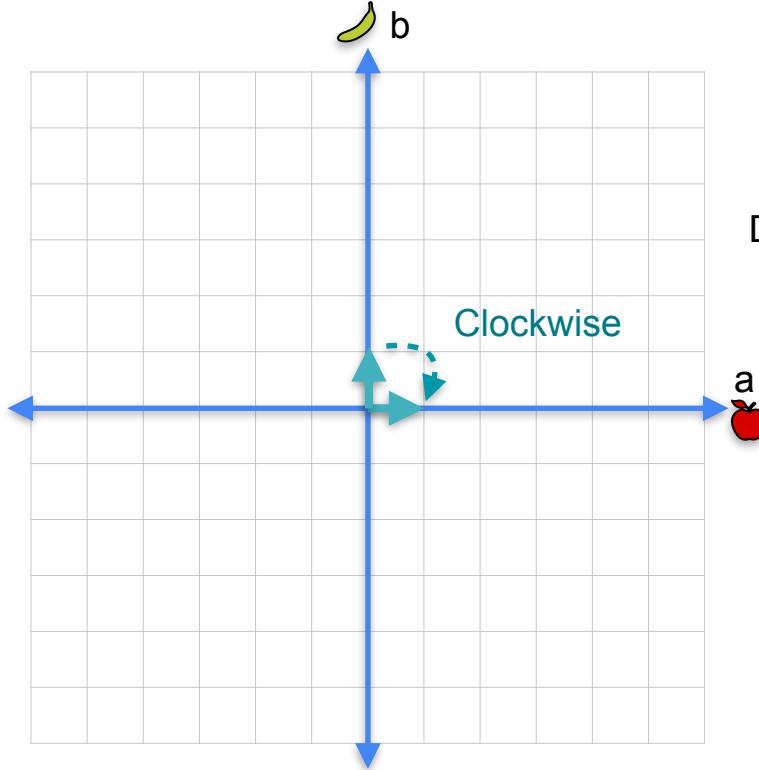


$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$



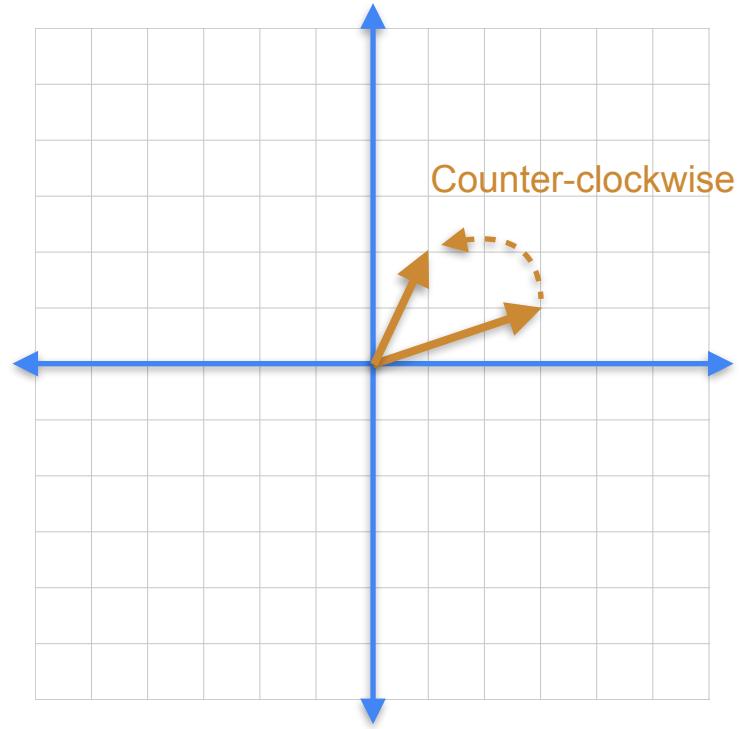
Determinant as an area



$$\text{Det} = 1 \cdot 1 - 3 \cdot 2$$

$$\text{Det} = -5$$

Negative





DeepLearning.AI

Determinants and Eigenvectors

Determinant of a product

Determinant of a product

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 16 & 8 \\ \hline 7 & 6 \\ \hline \end{array}$$

Determinant of a product

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 16 & 8 \\ \hline 7 & 6 \\ \hline \end{array}$$

$$\det = 5$$

$$3 \cdot 2 - 1 \cdot 1$$

Determinant of a product

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 16 & 8 \\ \hline 7 & 6 \\ \hline \end{array}$$

$$\det = 5$$

$$3 \cdot 2 - 1 \cdot 1$$

$$\det = 8$$

$$5 \cdot 2 - 2 \cdot 1$$

Determinant of a product

3	1
1	2

5	2
1	2

=

16	8
7	6

$$\det = 5$$

$$3 \cdot 2 - 1 \cdot 1$$

$$\det = 8$$

$$5 \cdot 2 - 2 \cdot 1$$

$$\det = 40$$

$$16 \cdot 6 - 8 \cdot 7$$

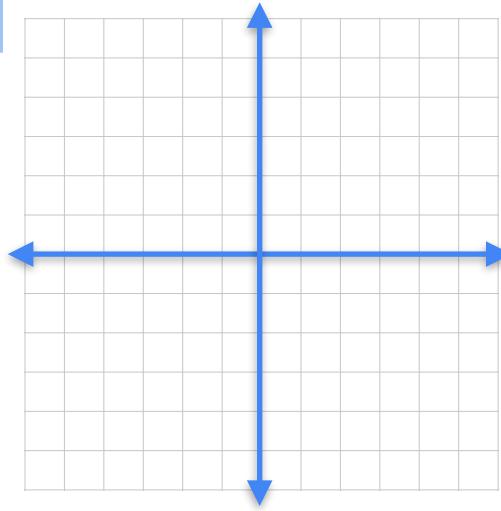
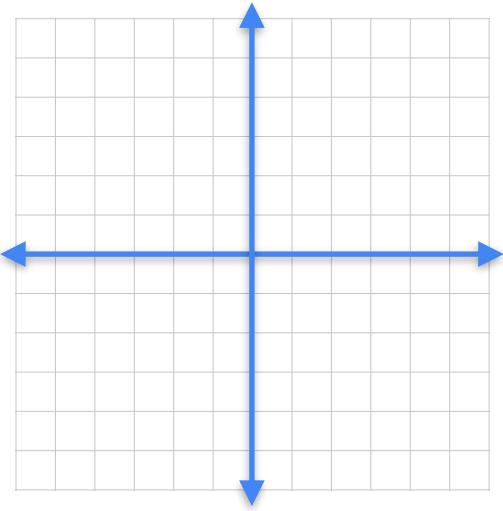
Determinant of a product

$$\det(AB) = \det(A) \det(B)$$

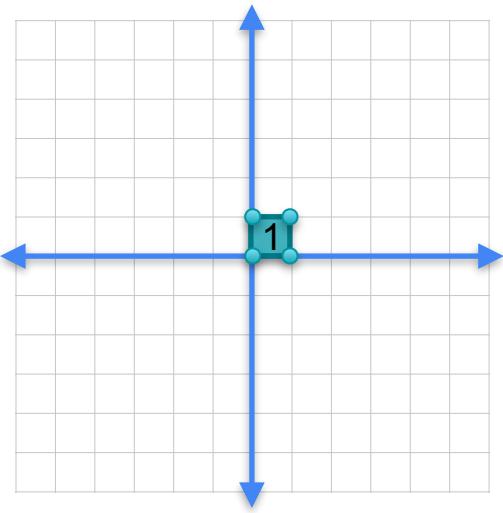
Determinant of a product

3	1
1	2

Det = 5

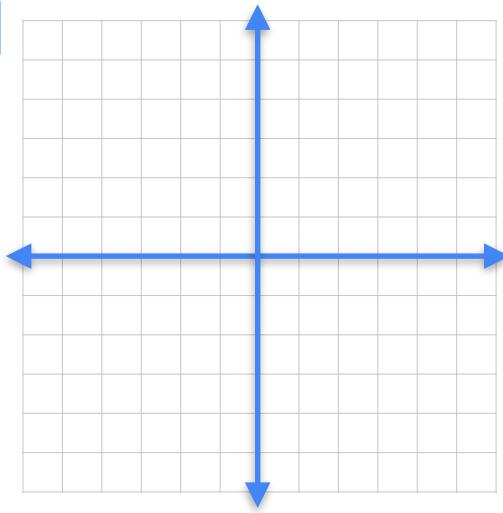


Determinant of a product

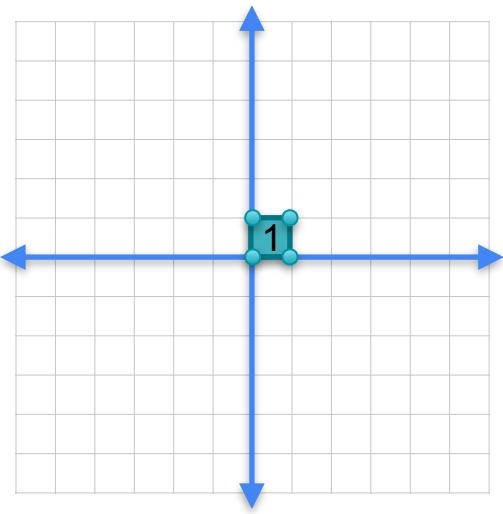


$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}$$

Det = 5

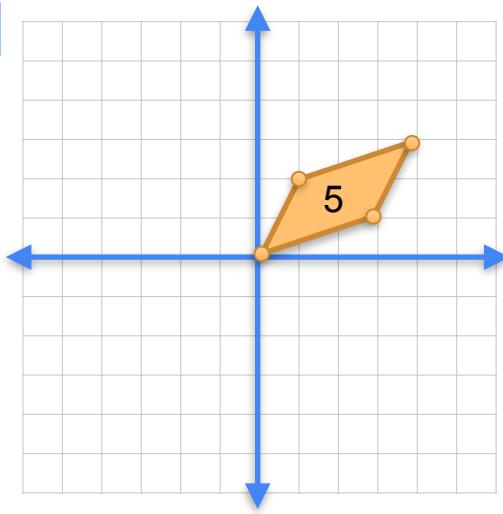


Determinant of a product

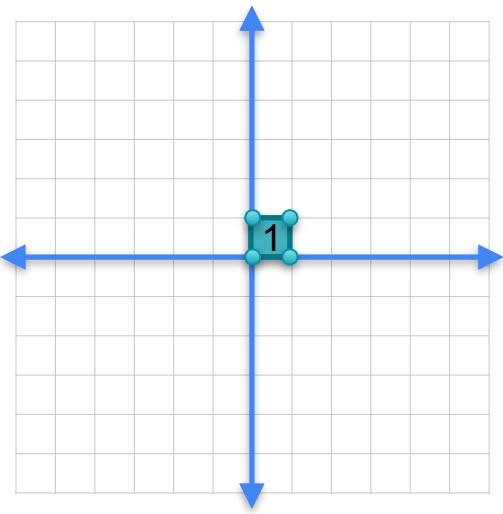


$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}$$

Det = 5

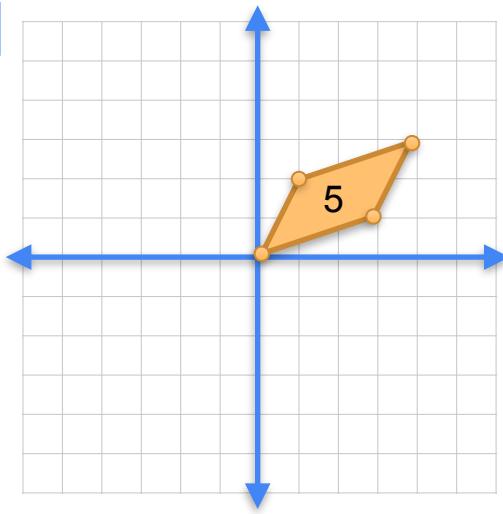


Determinant of a product



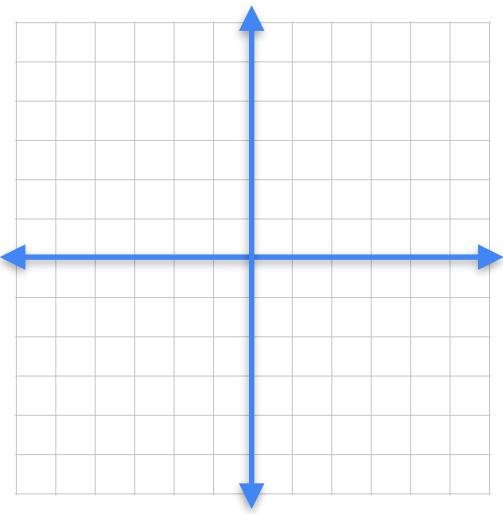
$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array}$$

Det = 5



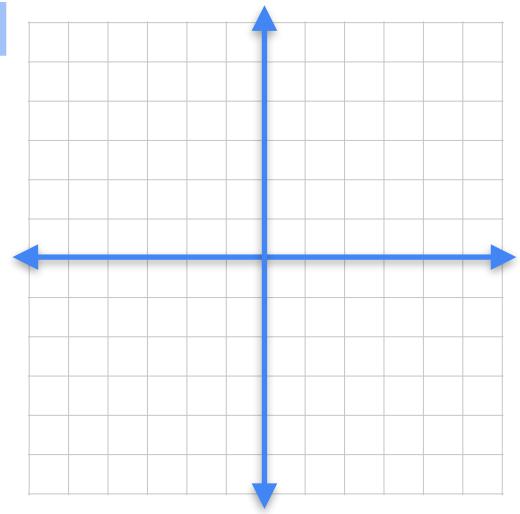
Area blows up by 5

Determinant of a product

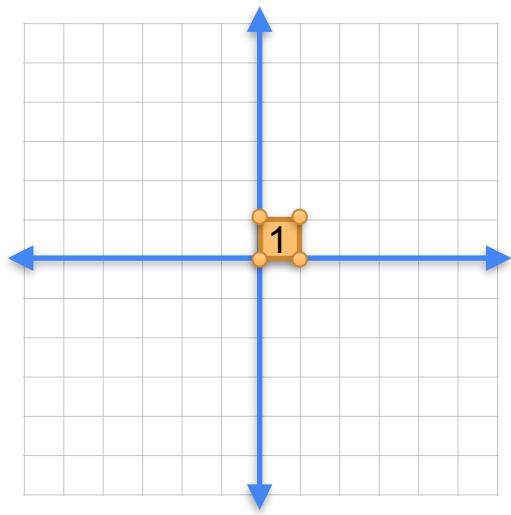


$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3

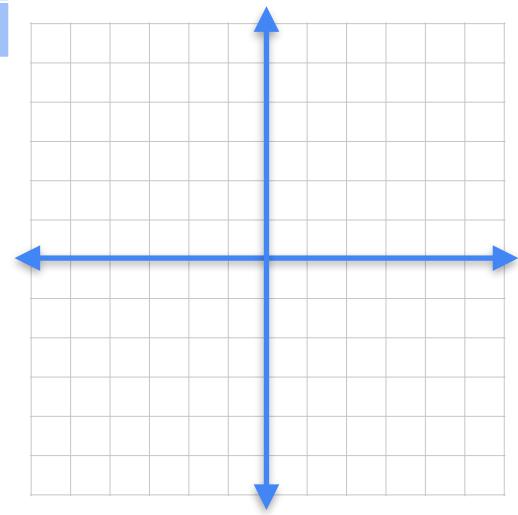


Determinant of a product

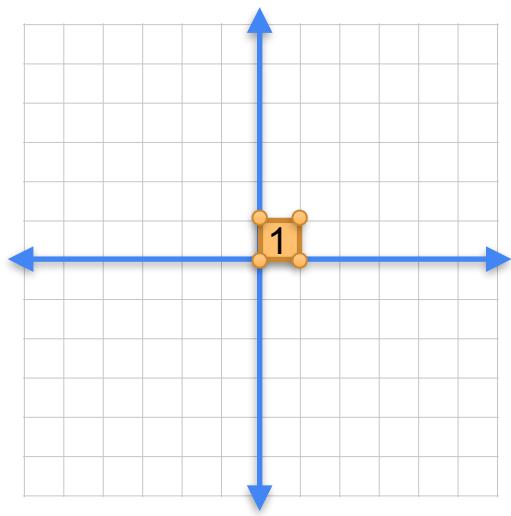


$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3

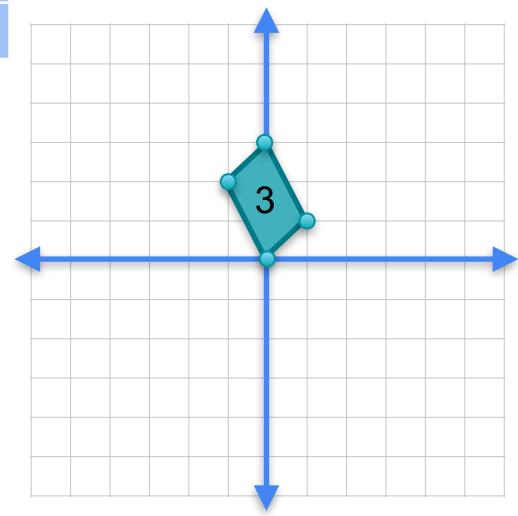


Determinant of a product

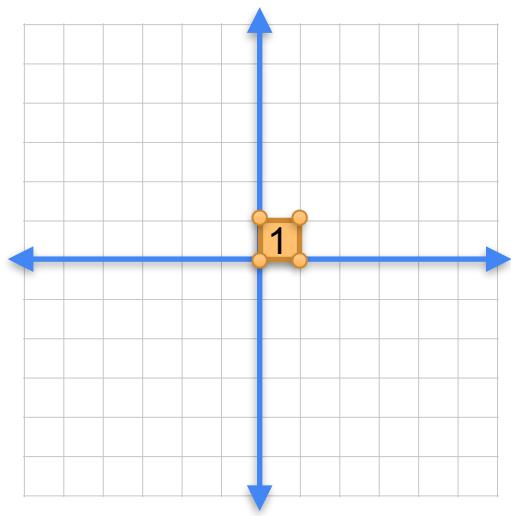


$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3

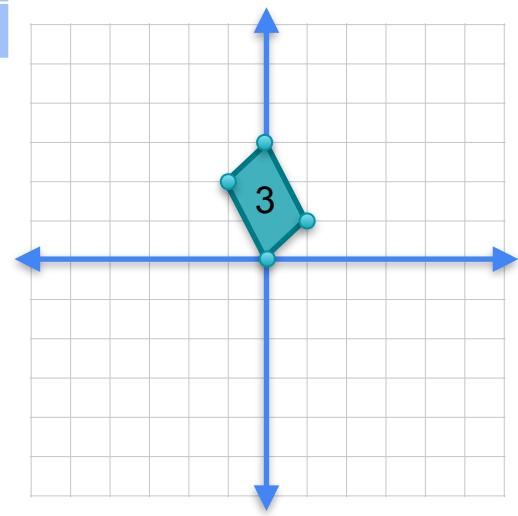


Determinant of a product



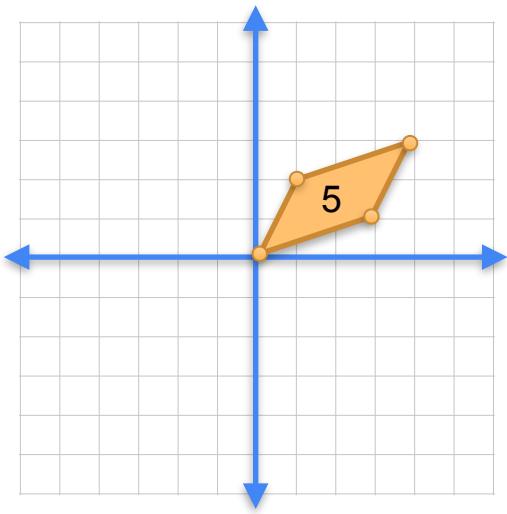
$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3



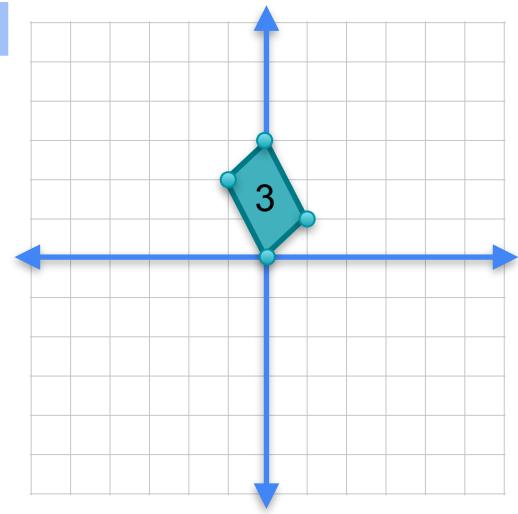
Area blows up by 3

Determinant of a product



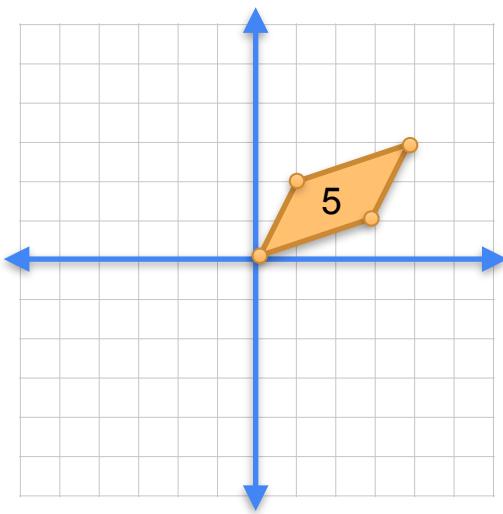
$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline -2 & 1 \\ \hline \end{array}$$

Det = 3



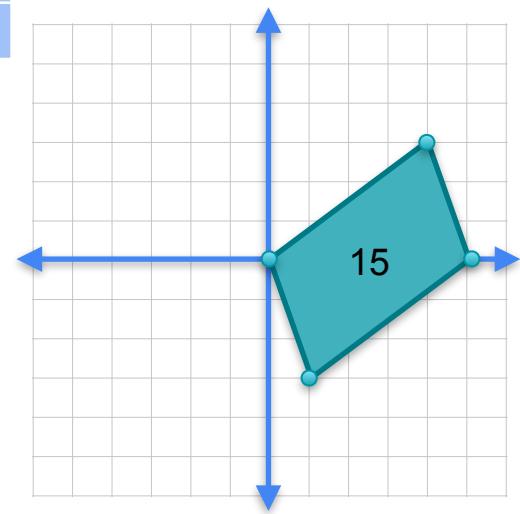
Area blows up by 3

Determinant of a product



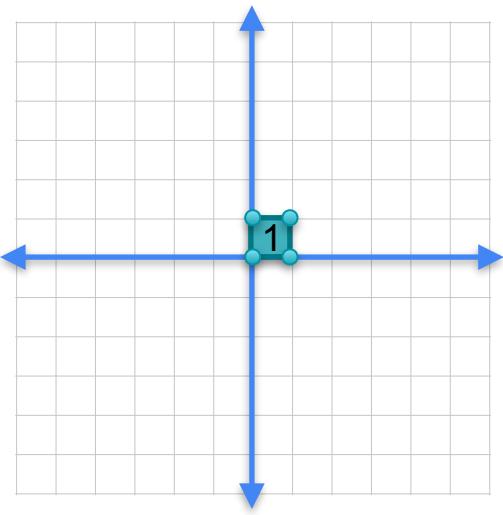
$$\begin{array}{|cc|} \hline 1 & 1 \\ -2 & 1 \\ \hline \end{array}$$

Det = 3



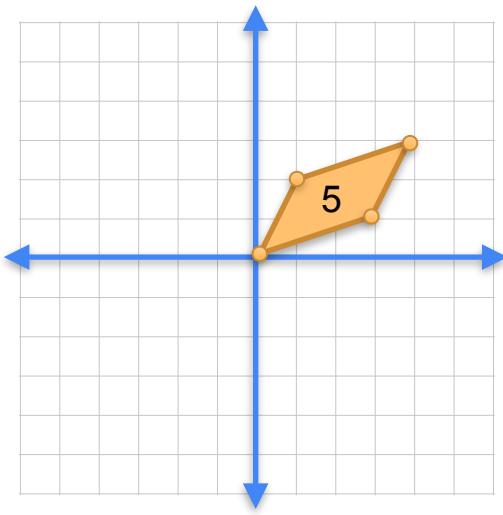
Area blows up by 3

Determinant of a product



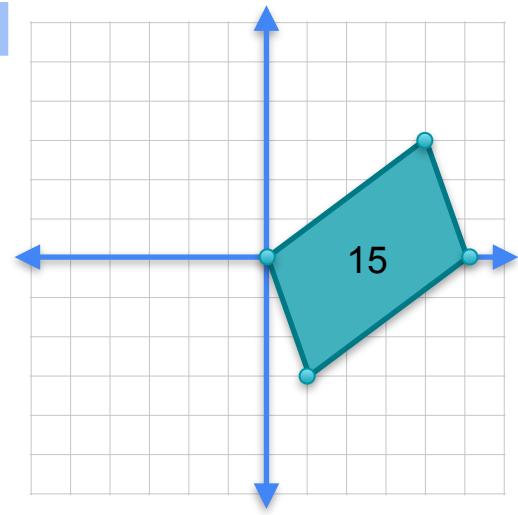
$$\begin{array}{|cc|} \hline 3 & 1 \\ 1 & 2 \\ \hline \end{array}$$

Det = 5

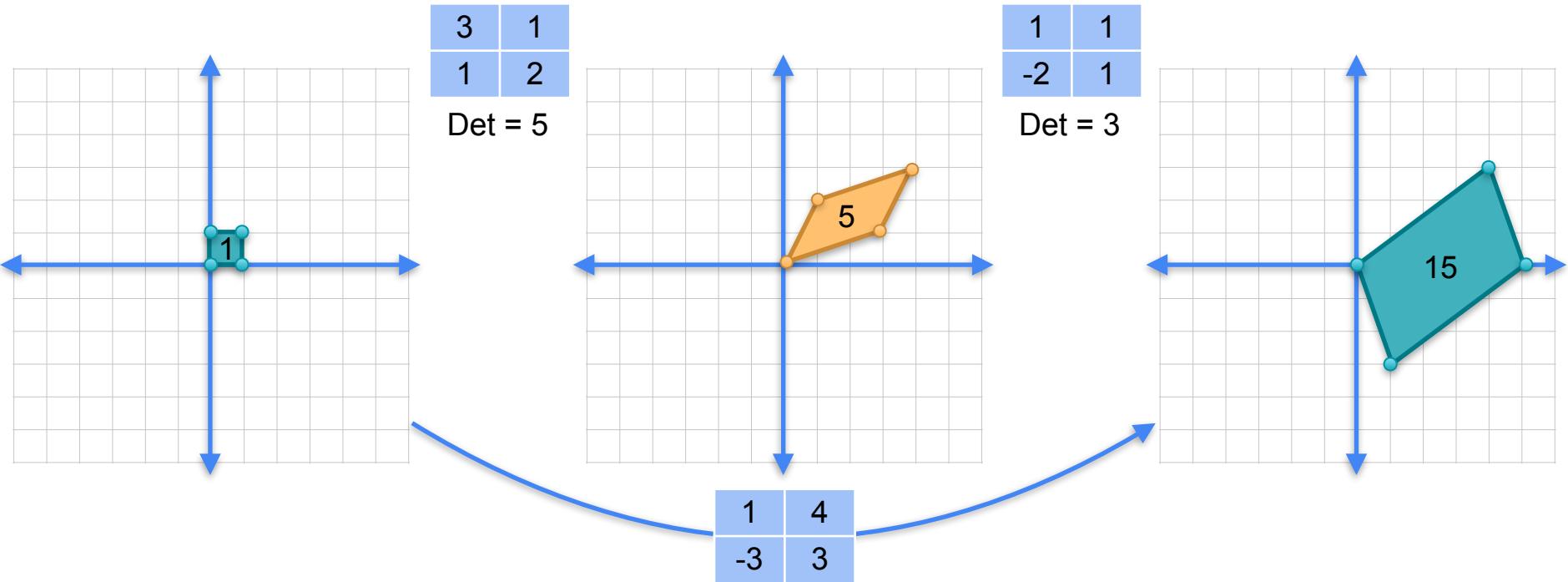


$$\begin{array}{|cc|} \hline 1 & 1 \\ -2 & 1 \\ \hline \end{array}$$

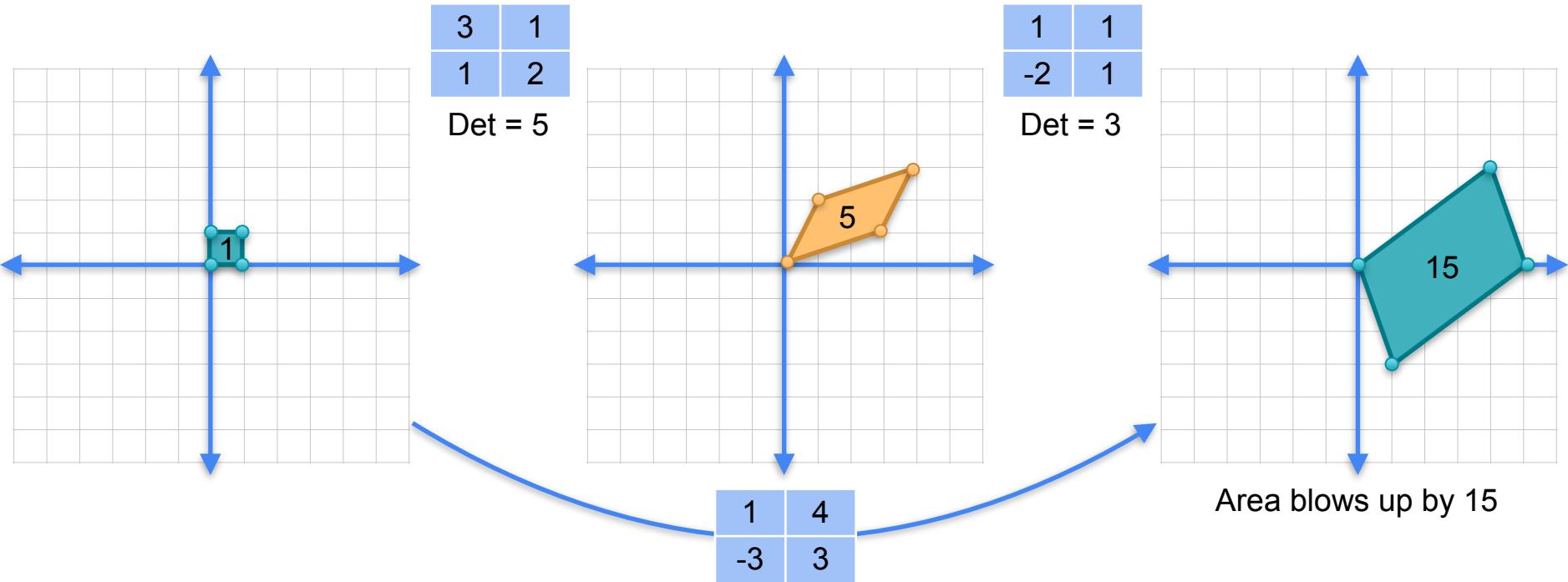
Det = 3



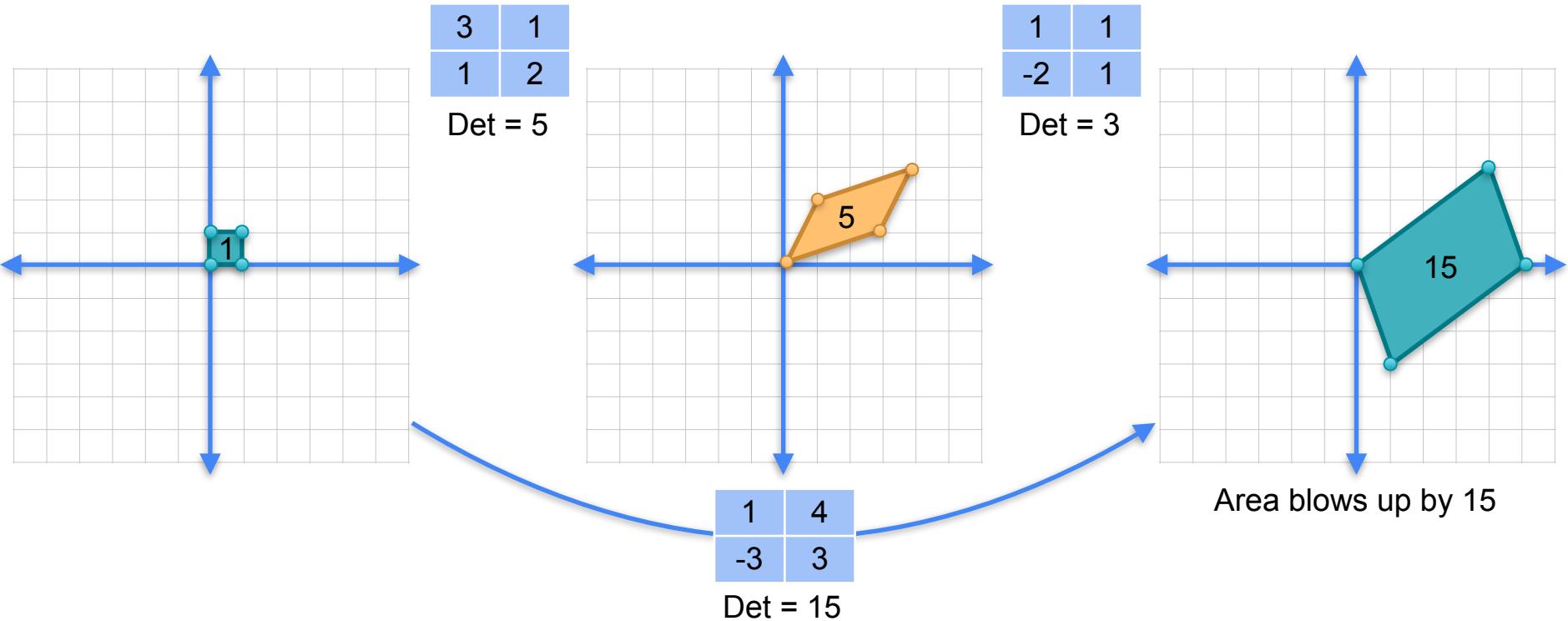
Determinant of a product



Determinant of a product



Determinant of a product



Quiz

- The product of a singular and a non-singular matrix (in any order) is:
 - Singular
 - Non-singular
 - Could be either one

Solution

- If A is non-singular and B is singular, then $\det(AB) = \det(A) \times \det(B) = 0$, since $\det(B) = 0$. Therefore $\det(AB) = 0$, so AB is **singular**.

When one factor is zero

When one factor is zero

5

When one factor is zero

$$5 \cdot 0$$

When one factor is zero

$$5 \cdot 0 = 0$$

When one factor is singular...

Non-singular	Singular	Singular
$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}$	$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix}$	$\begin{matrix} 4 & 8 \\ 3 & 6 \end{matrix}$
$\text{Det} = 5$	$\text{Det} = 0$	$\text{Det} = 0$

If one factor is singular...

3	1
1	2

$$\text{Det} = 5$$

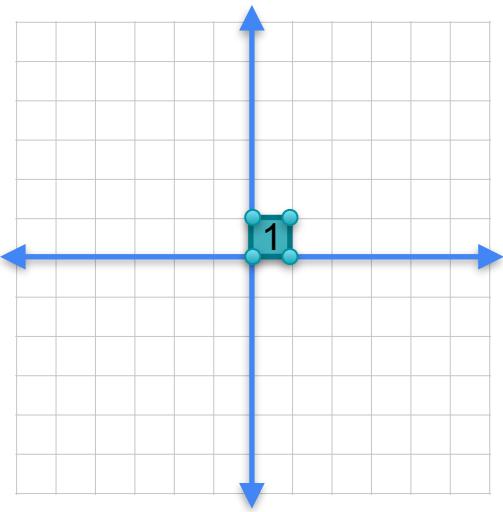
1	2
1	2

$$\text{Det} = 0$$

4	8
3	6

$$\text{Det} = 0$$

If one factor is singular...



3	1
1	2

Det = 5

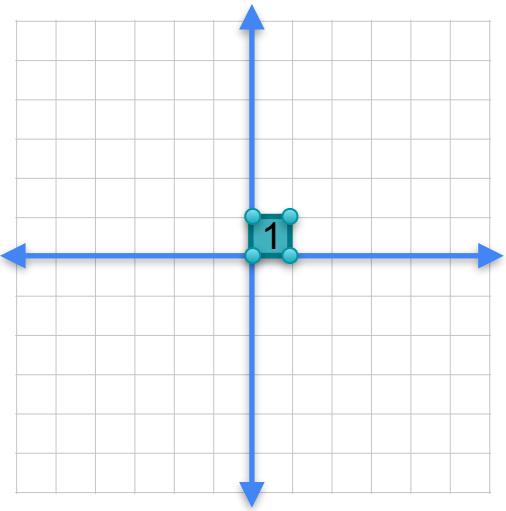
1	2
1	2

Det = 0

4	8
3	6

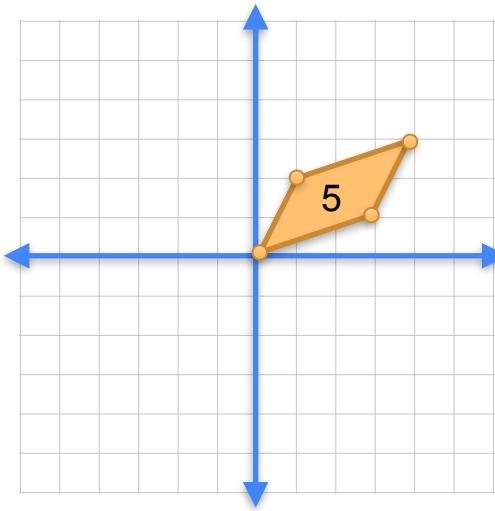
Det = 0

If one factor is singular...



3	1
1	2

Det = 5



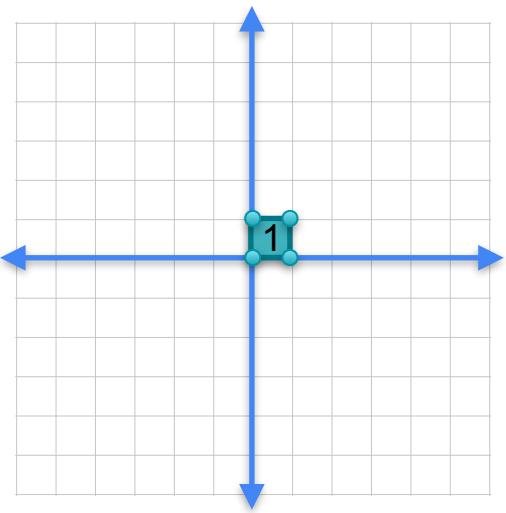
1	2
1	2

Det = 0

4	8
3	6

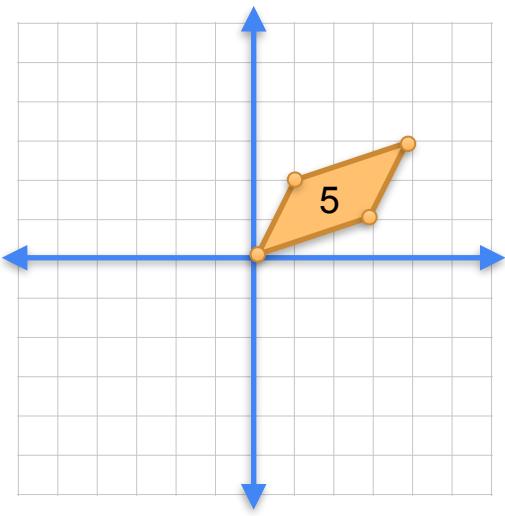
Det = 0

If one factor is singular...



3	1
1	2

$$\text{Det} = 5$$

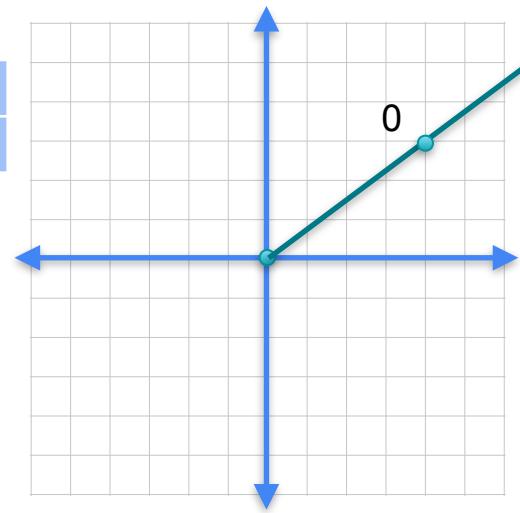


1	2
1	2

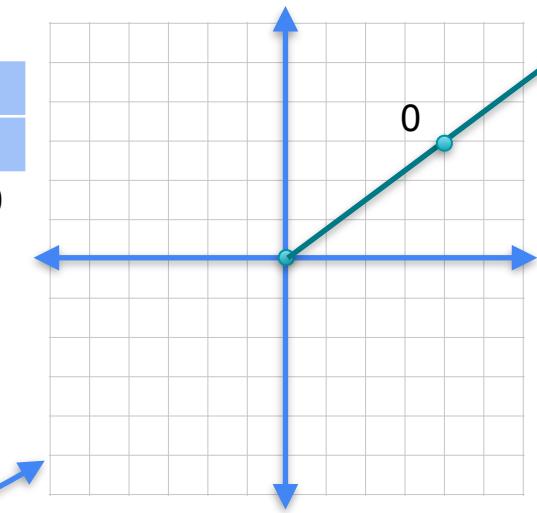
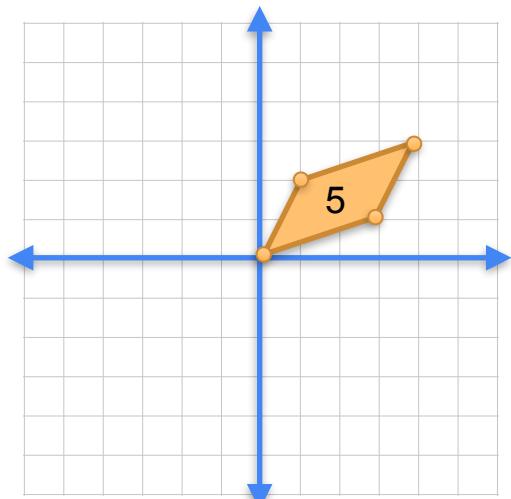
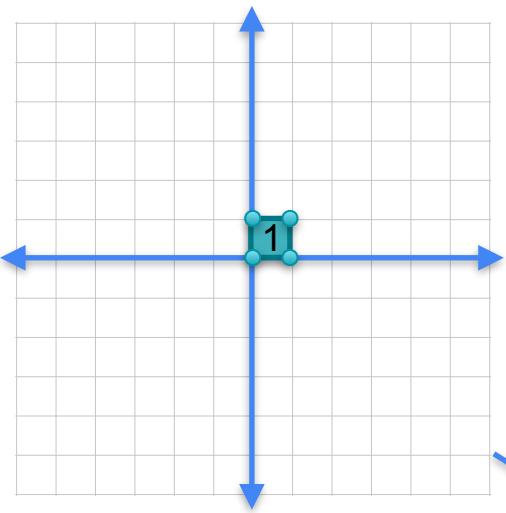
$$\text{Det} = 0$$

4	8
3	6

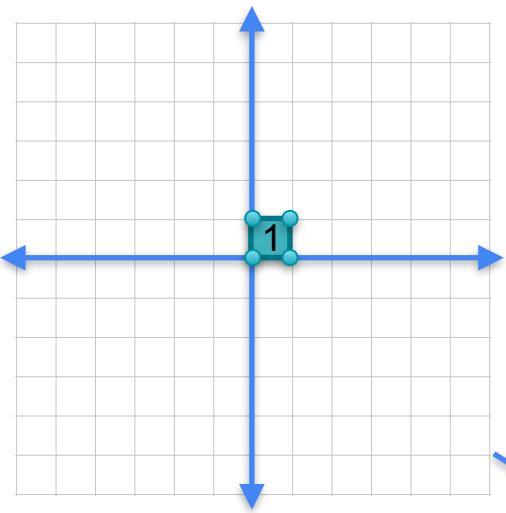
$$\text{Det} = 0$$



If one factor is singular...

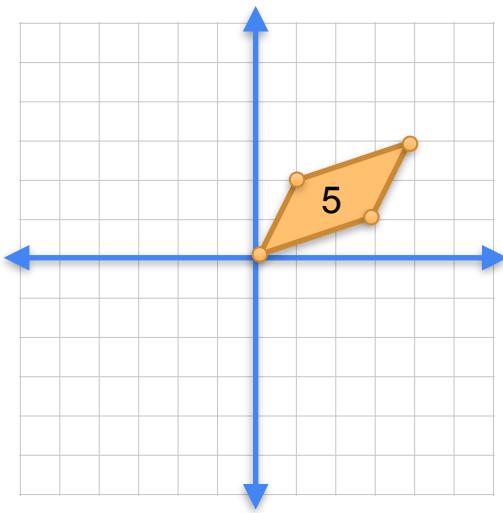


If one factor is singular...



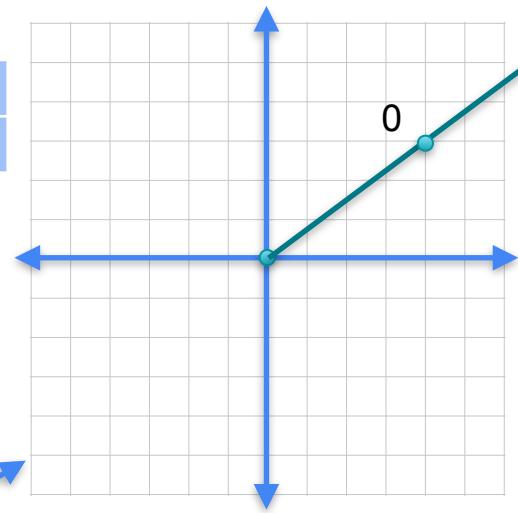
3	1
1	2

Det = 5



1	2
1	2

Det = 0



Area blows up by 0



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Determinants and Eigenvectors

Determinant of inverse

Quiz

- Find the determinants of the following matrices

0.4	-0.2
-0.2	0.6

0.25	-0.25
-0.125	0.625

Solution

$$\text{Det} \begin{array}{|c|c|} \hline 0.4 & -0.2 \\ \hline -0.2 & 0.6 \\ \hline \end{array} = (0.4)(0.6) - (-0.2)(-0.2) = 0.2$$

$$\text{Det} \begin{array}{|c|c|} \hline 0.25 & -0.25 \\ \hline -0.125 & 0.625 \\ \hline \end{array} = (0.25)(0.625) - (-0.125)(-0.25) = 0.125$$

Determinant of an inverse

Determinant of an inverse

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

Determinant of an inverse

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\det = 5$$

Determinant of an inverse

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\det = 5$$

$$\det = 0.2$$

Determinant of an inverse

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{pmatrix}$$

$$\det = 5$$

$$\det = 0.2$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 5$$

$$\det = 0.2$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$\det = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$\det = 0.125$$

$$5^{-1} = 0.2$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$\det = 0.125$$

$$8^{-1} = 0.125$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$8^{-1} = 0.125$$

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0.125$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

$$5^{-1} = 0.2$$

$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$8^{-1} = 0.125$$

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

$$\det = 5$$

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$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

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$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0$$

$$\det = ???$$

Determinant of an inverse

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{vmatrix}$$

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$$\begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix}^{-1} = \begin{vmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{vmatrix}$$

$$\det = 8$$

$$8^{-1} = 0.125$$

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} ? & ? \\ ? & ? \end{vmatrix}$$

$$\det = 0$$

$$0^{-1} = ???$$

Determinant of an inverse

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

Why?

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A) \det(A^{-1})$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(AA^{-1}) = \det(A) \det(A^{-1})$$

$$\det(I) = \det(A) \det(A^{-1})$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\begin{aligned} \det(AA^{-1}) &= \det(A) \det(A^{-1}) \\ \det(I) &= \det(A) \det(A^{-1}) \\ 1 &= \frac{1}{\det(A)} \end{aligned}$$

Why?

$$\det(AB) = \det(A) \det(B)$$

Why is this?

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$\begin{aligned} \det(AA^{-1}) &= \det(A) \det(A^{-1}) \\ \det(I) &= \det(A) \det(A^{-1}) \\ 1 &= \det(A) \det(A^{-1}) \\ 1 &= \frac{1}{\det(A)} \end{aligned}$$

Determinant of the identity matrix

$$\det \begin{array}{|cc|} \hline 1 & 0 \\ 0 & 1 \\ \hline \end{array} = 1 \cdot 1 - 0 \cdot 0 = 1$$

Determinant of the identity matrix

$$\det \begin{array}{|cc|} \hline 1 & 0 \\ 0 & 1 \\ \hline \end{array} = 1 \cdot 1 - 0 \cdot 0 = 1$$

$$\det(I) = 1$$

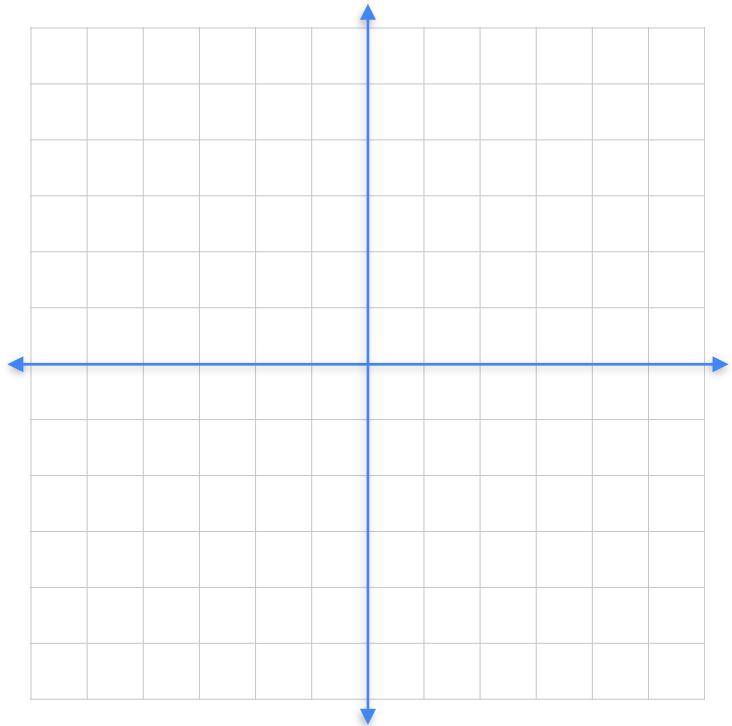


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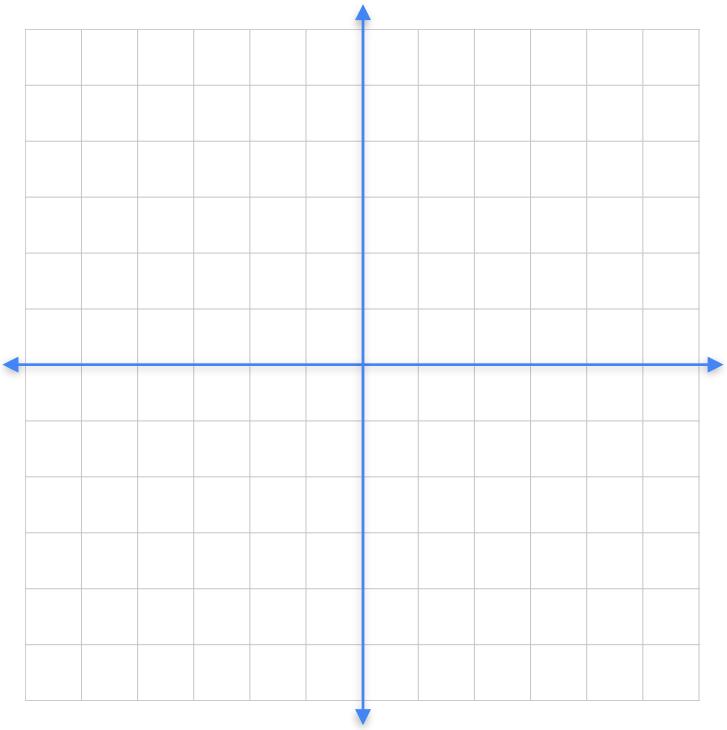
Determinants and Eigenvectors

Bases

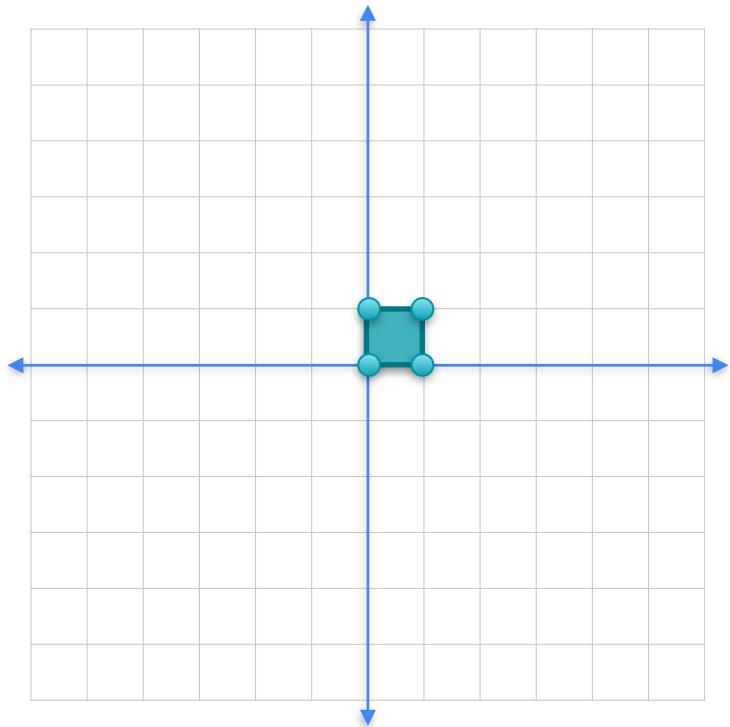
Bases



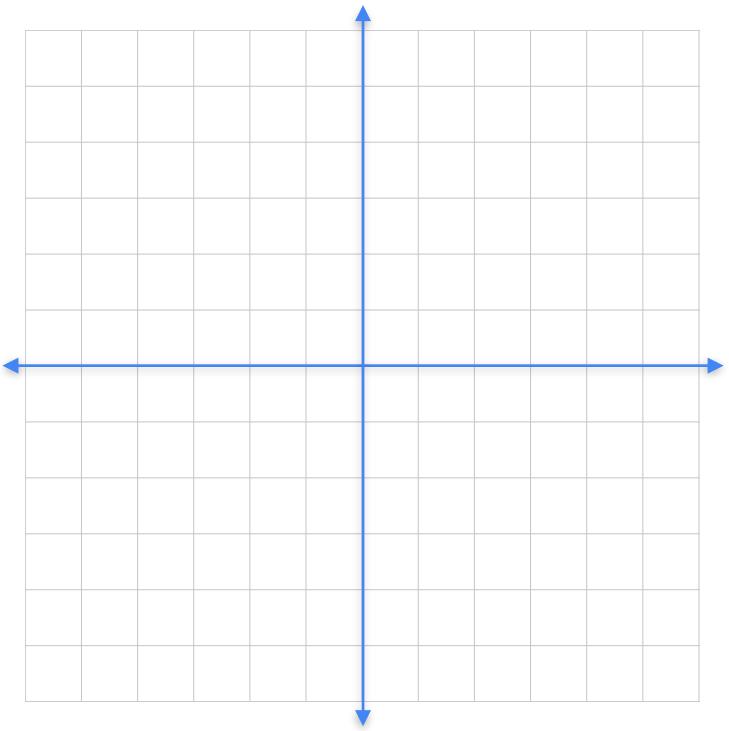
3	1
1	2



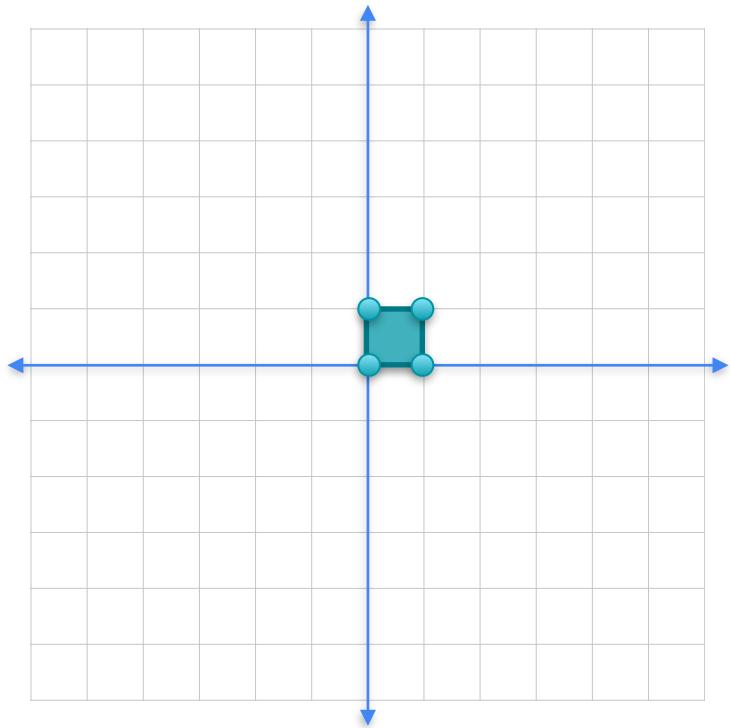
Bases



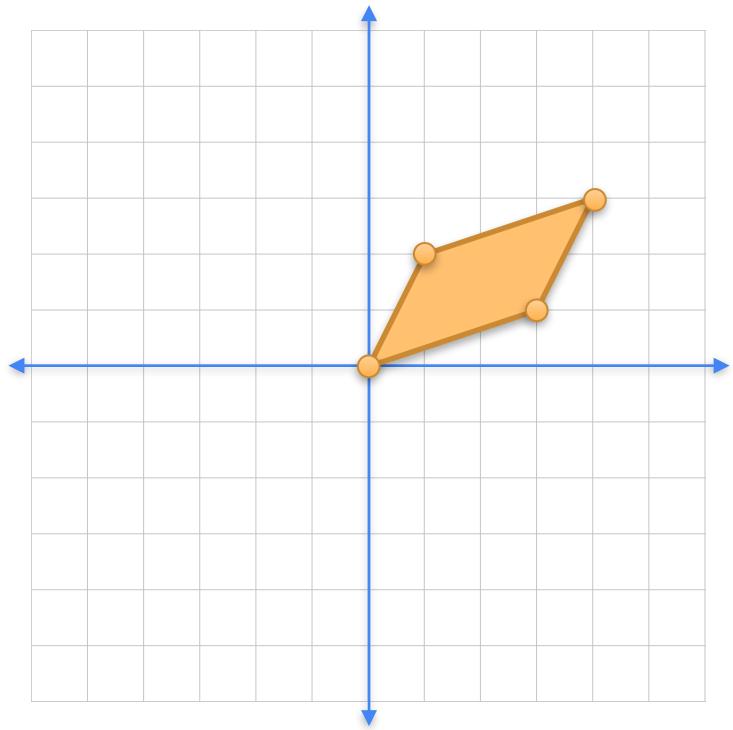
3	1
1	2



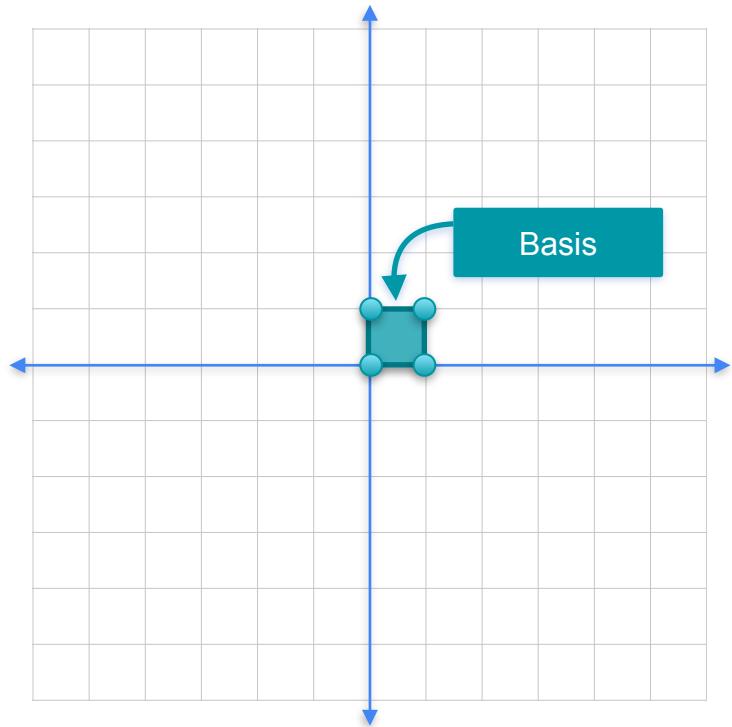
Bases



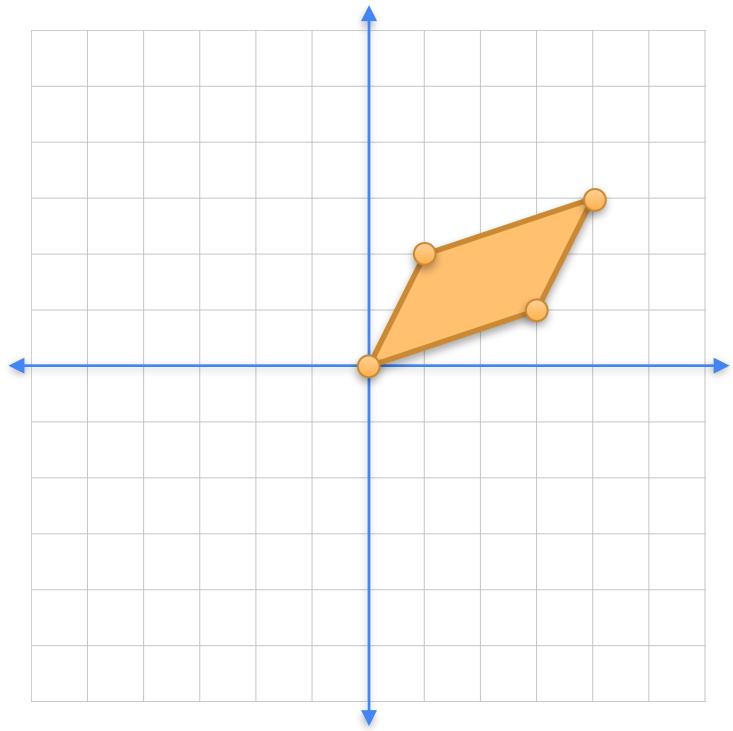
3	1
1	2



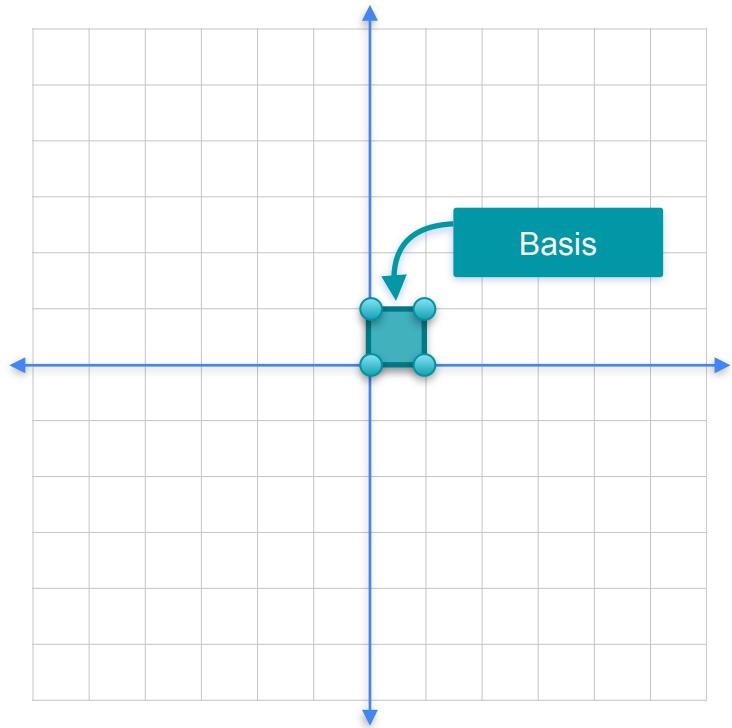
Bases



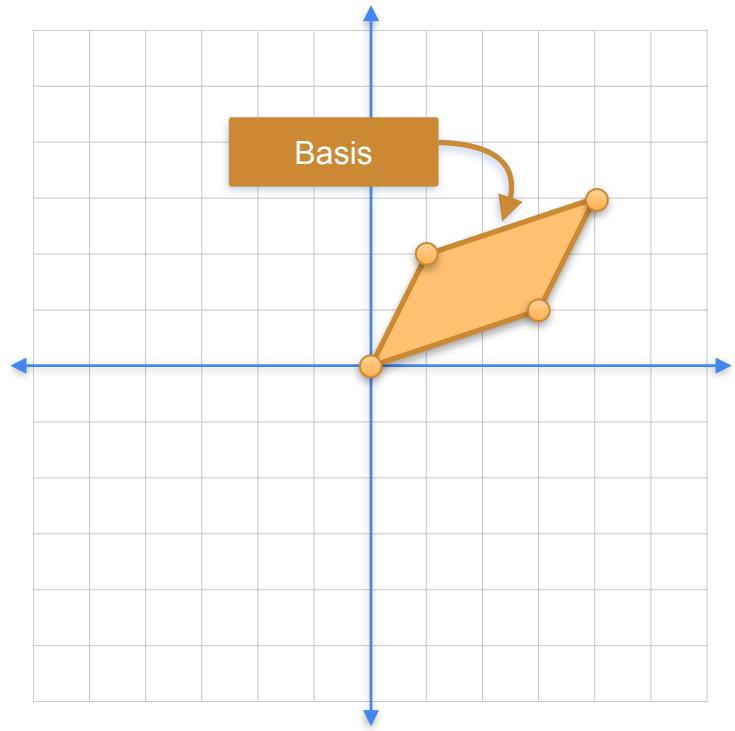
3	1
1	2



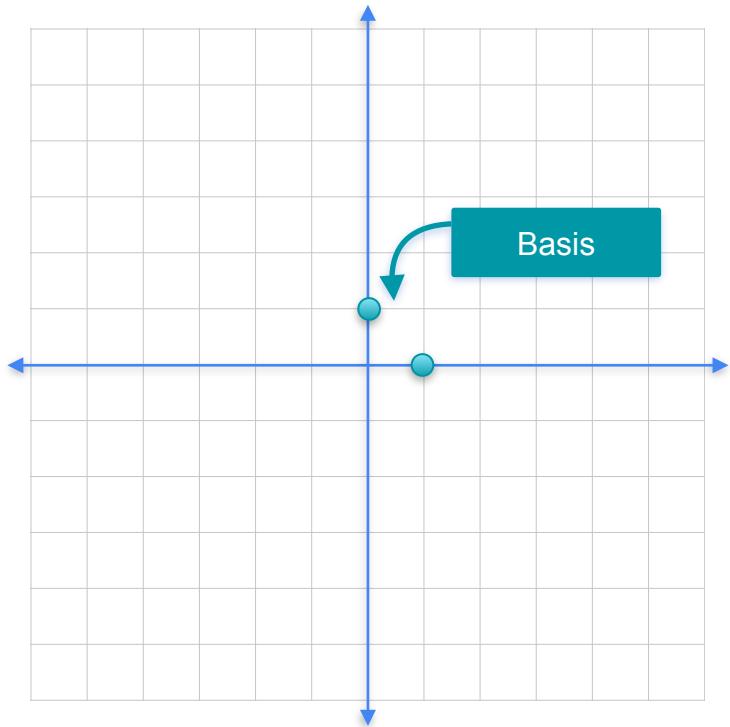
Bases



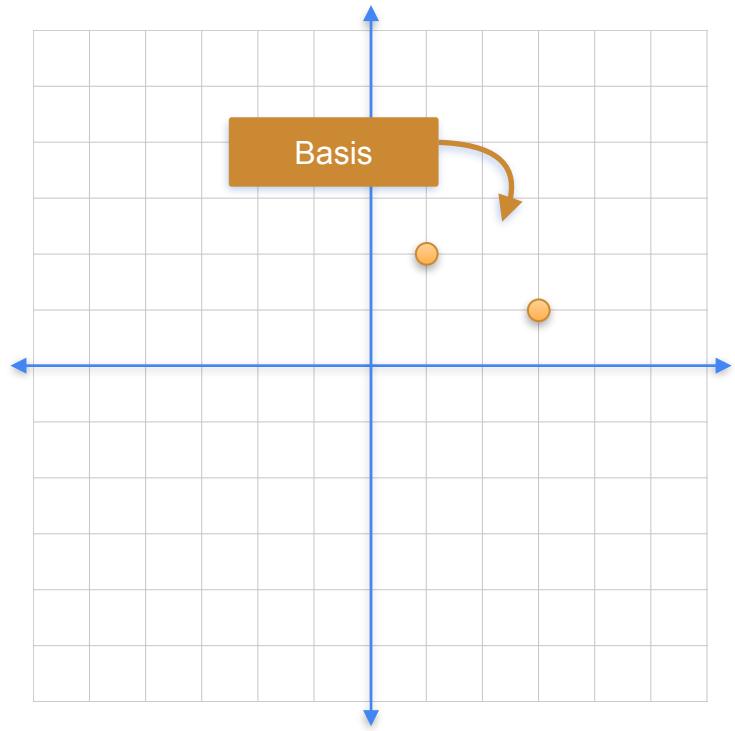
3	1
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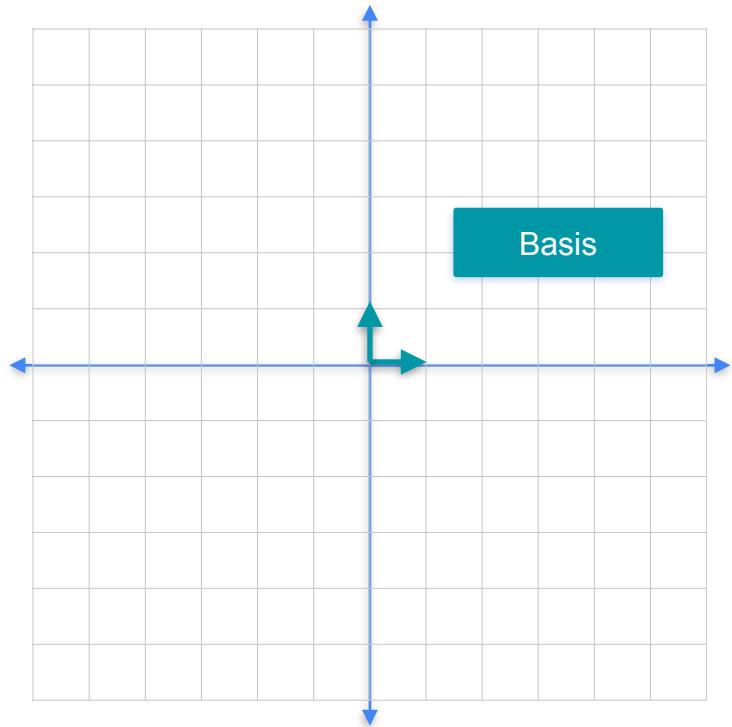
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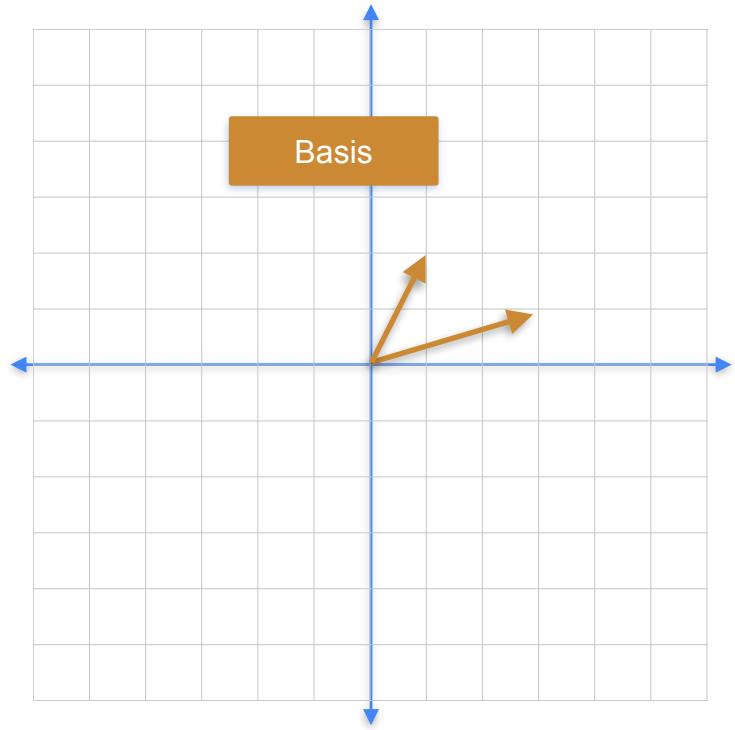
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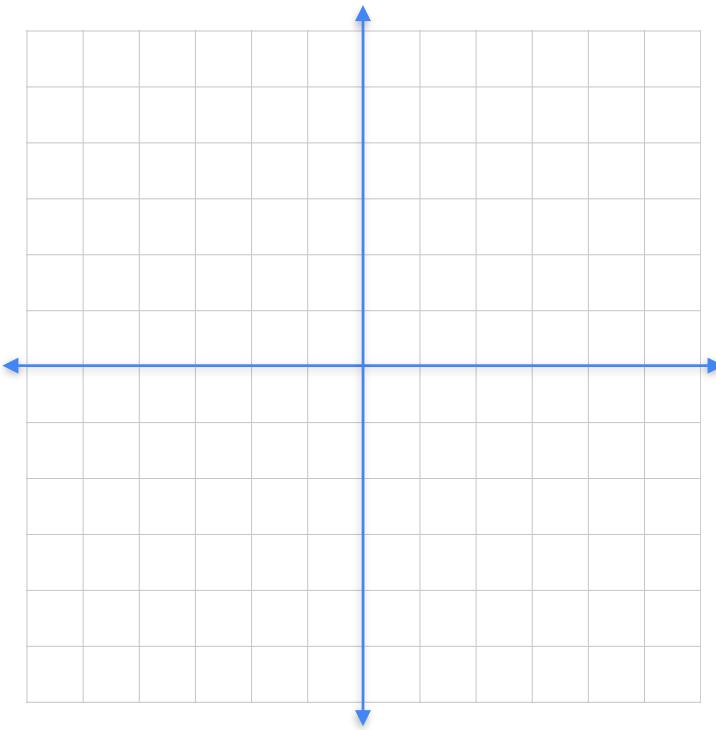
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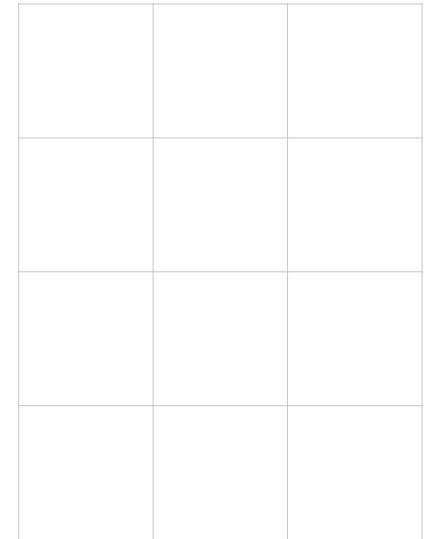
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1	2



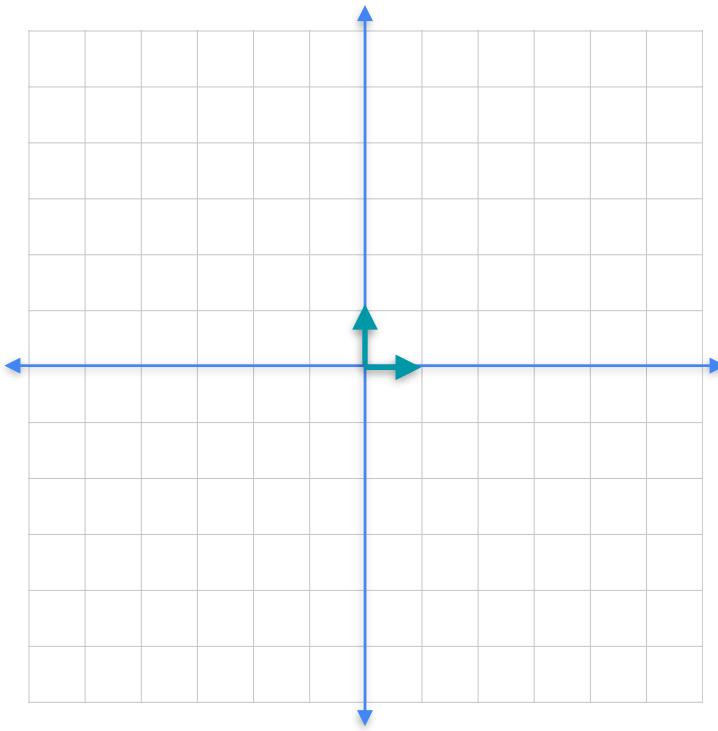
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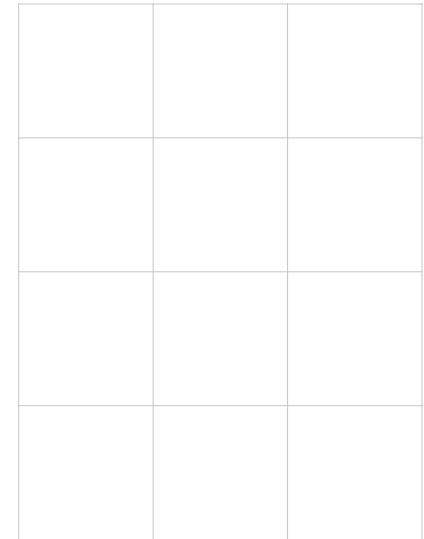
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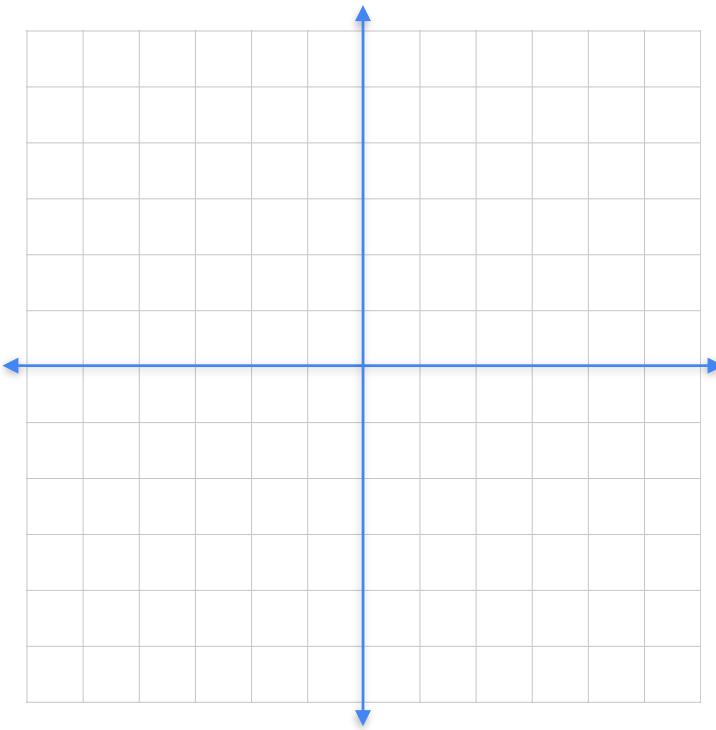
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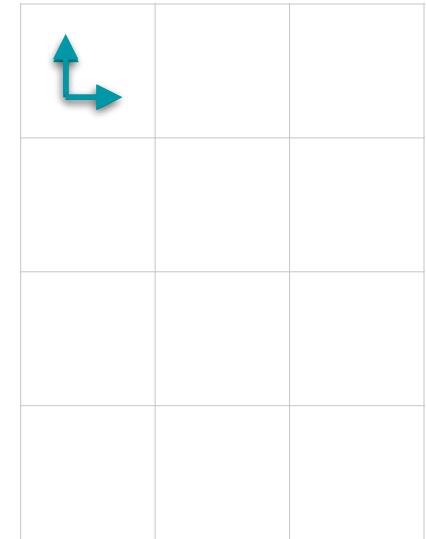
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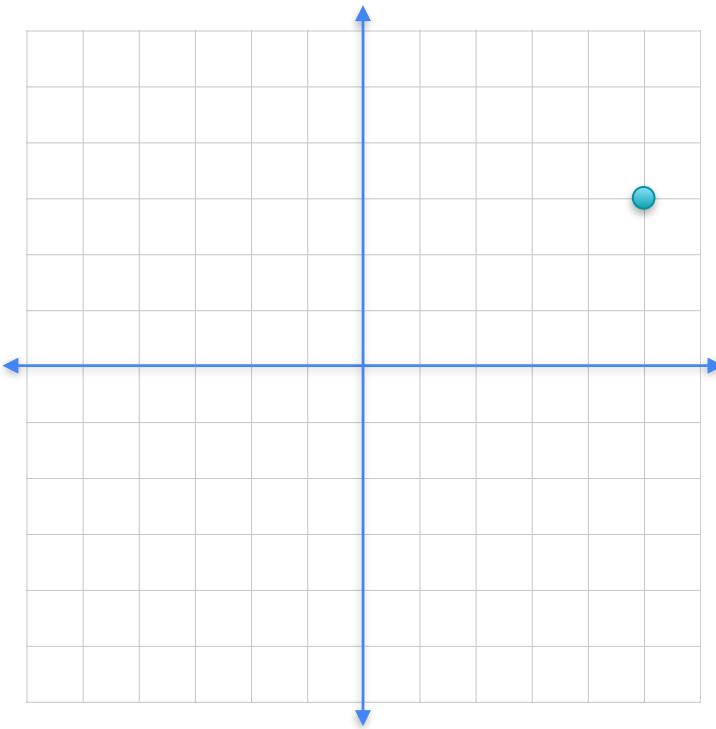
Bases



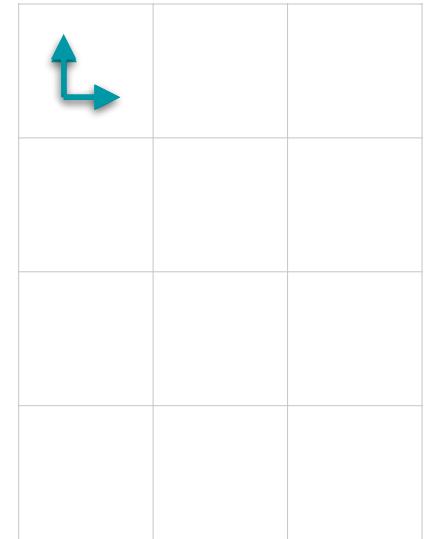
Bases



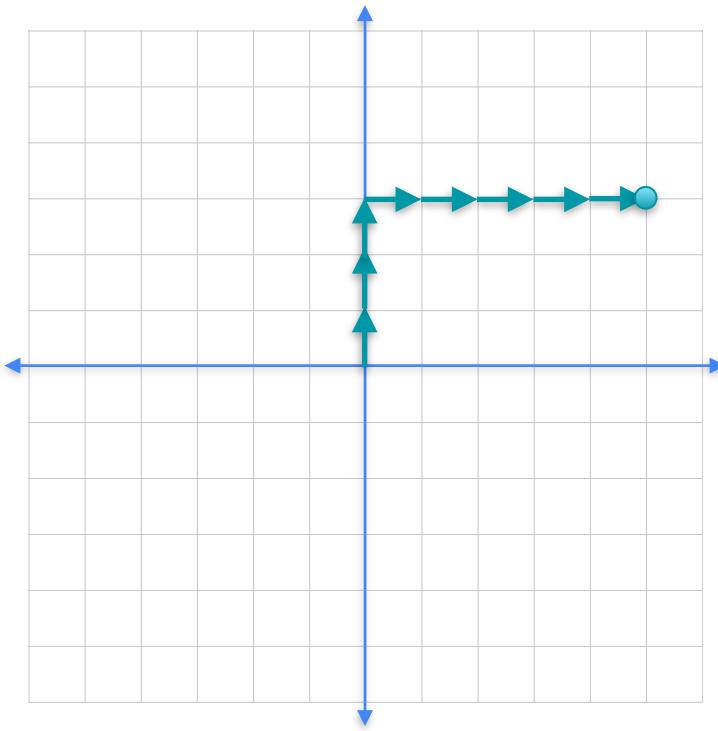
Bases



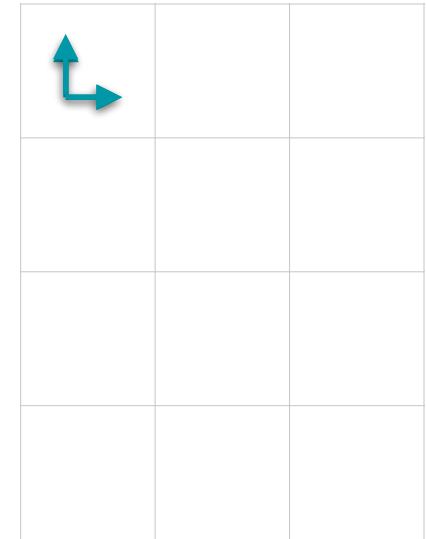
Bases



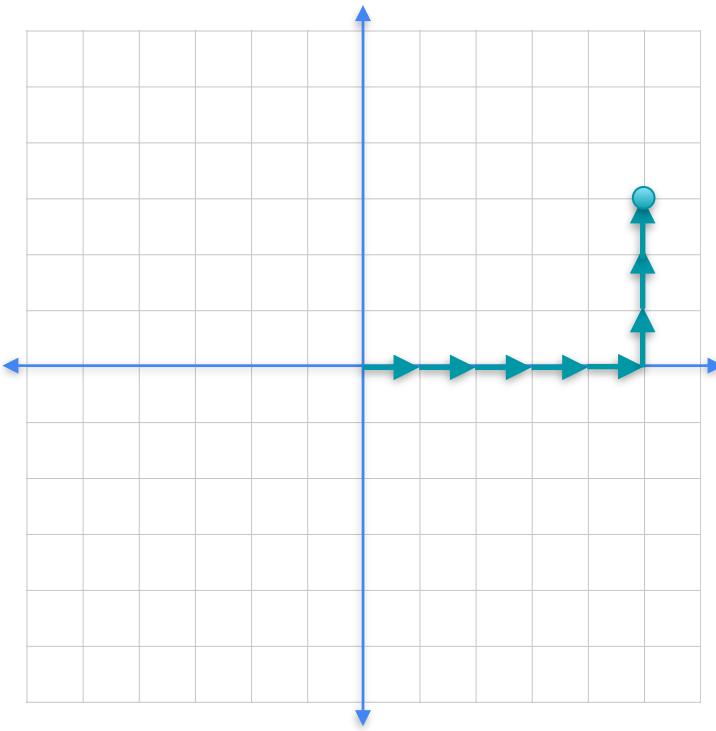
Bases



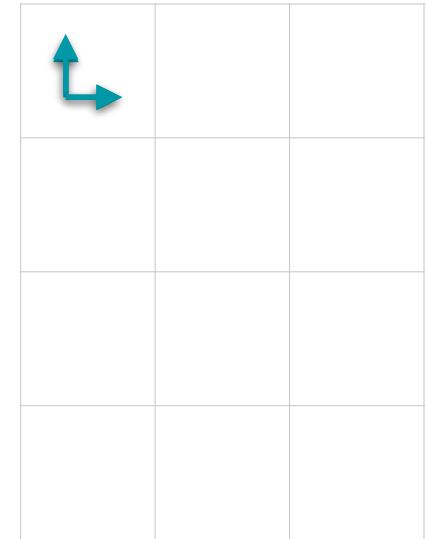
Bases



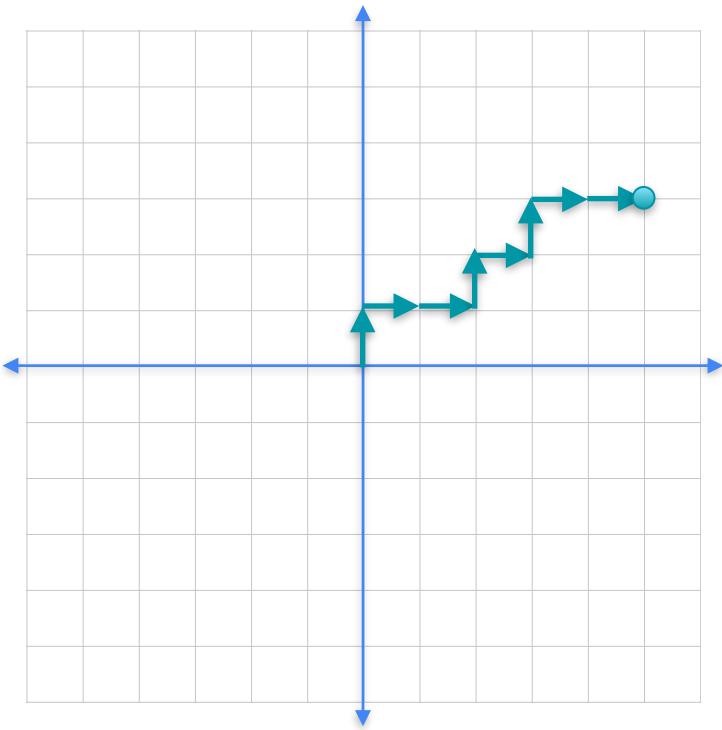
Bases



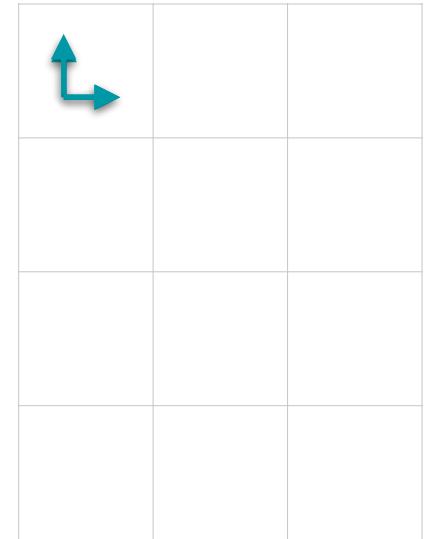
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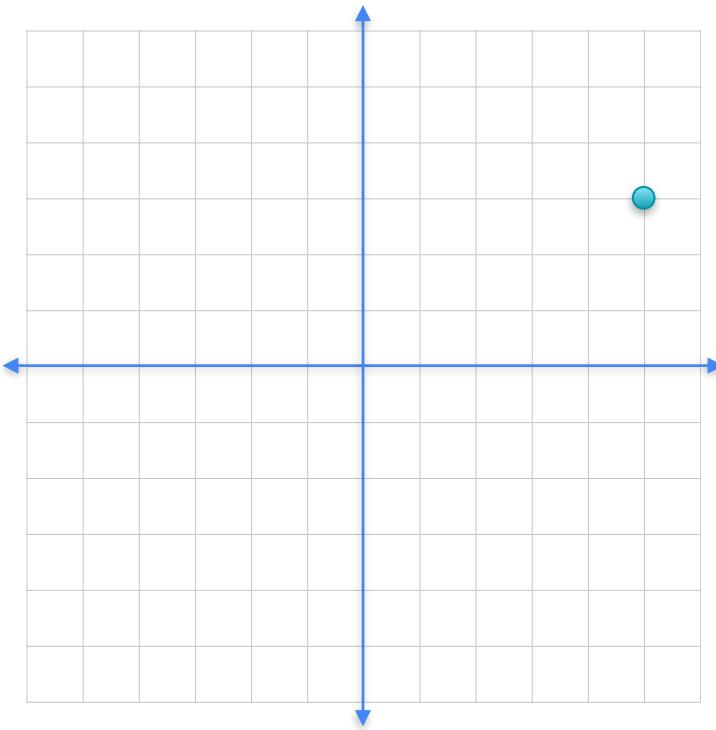
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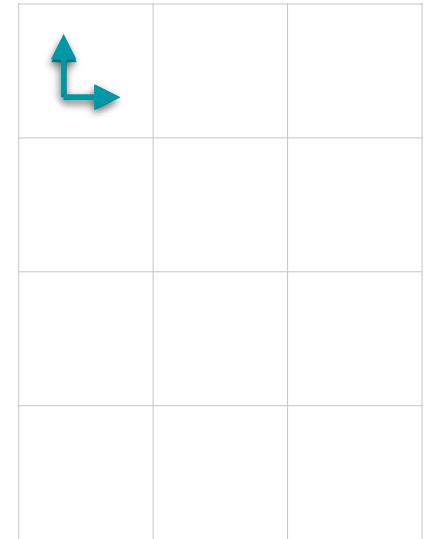
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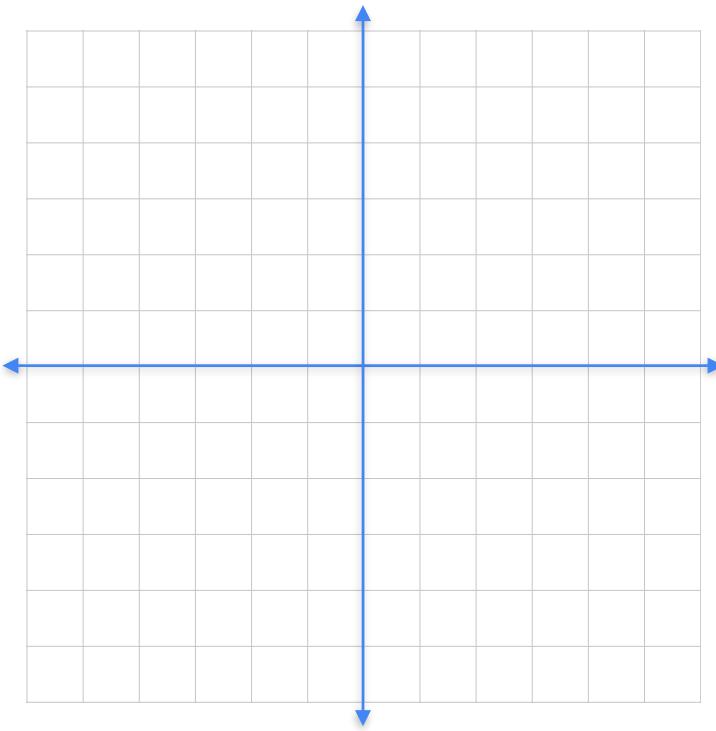
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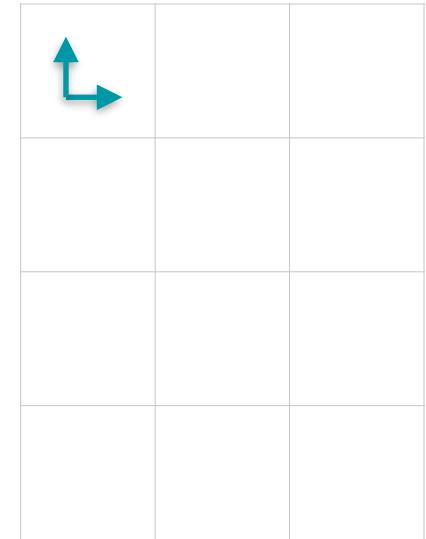
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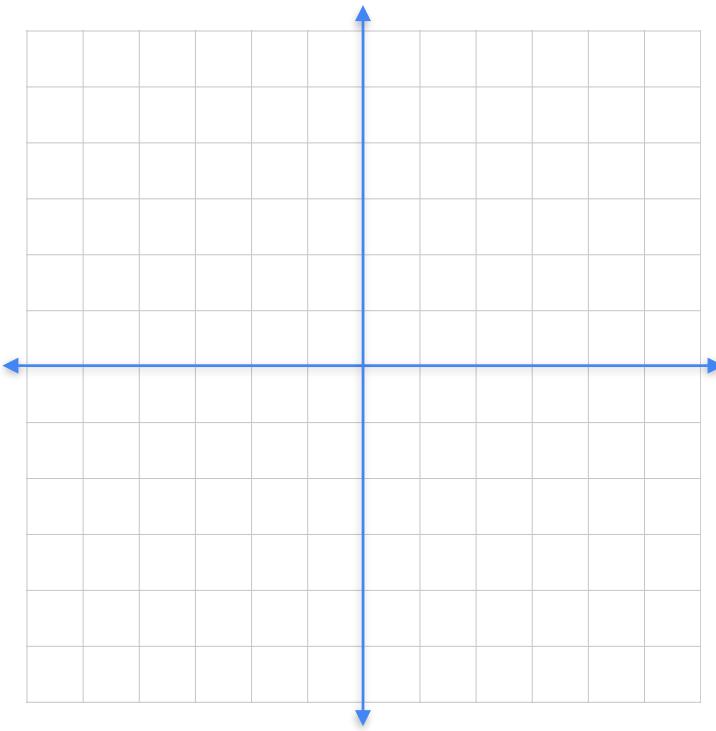
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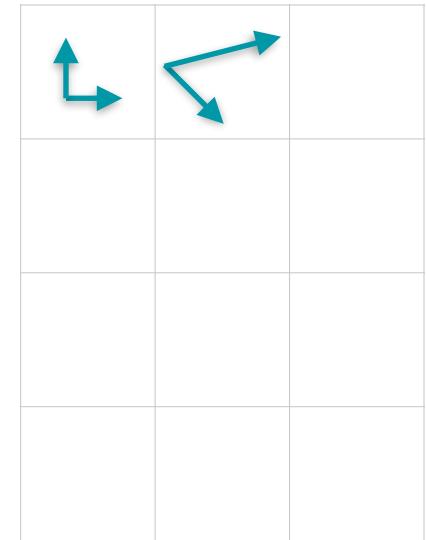
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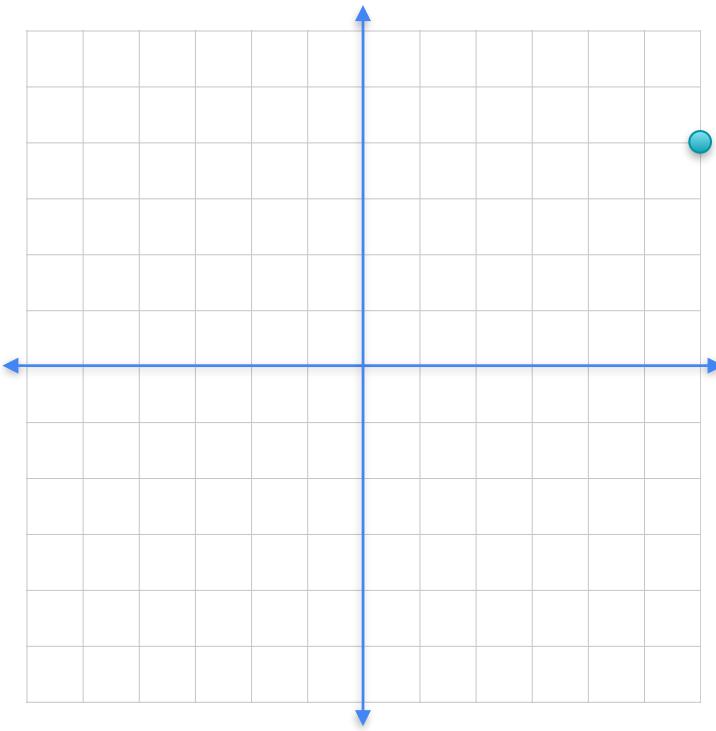
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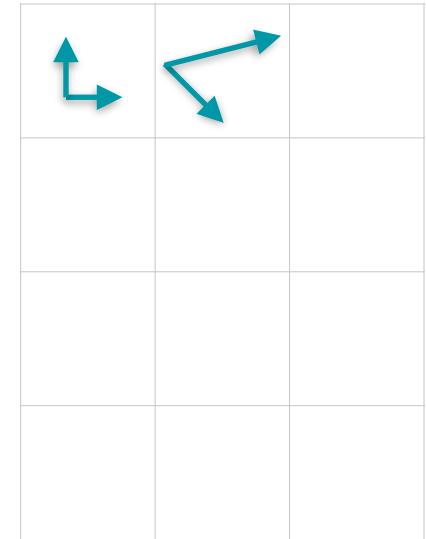
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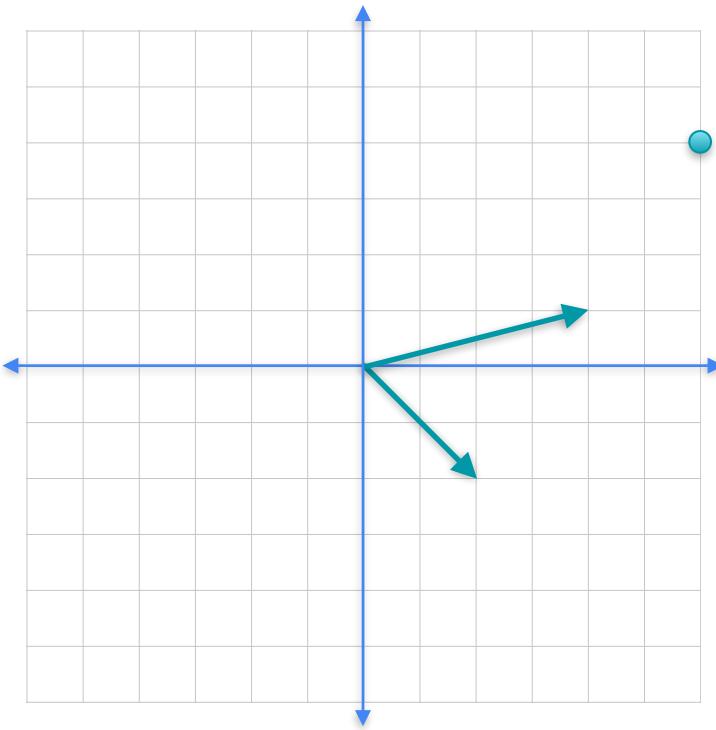
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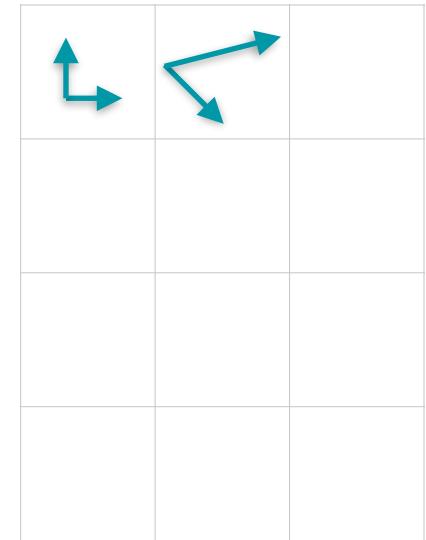
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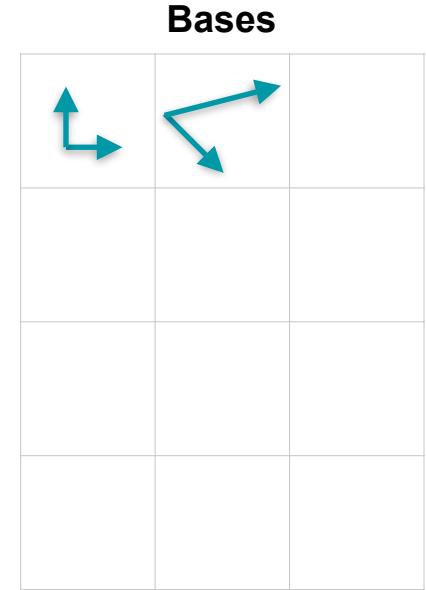
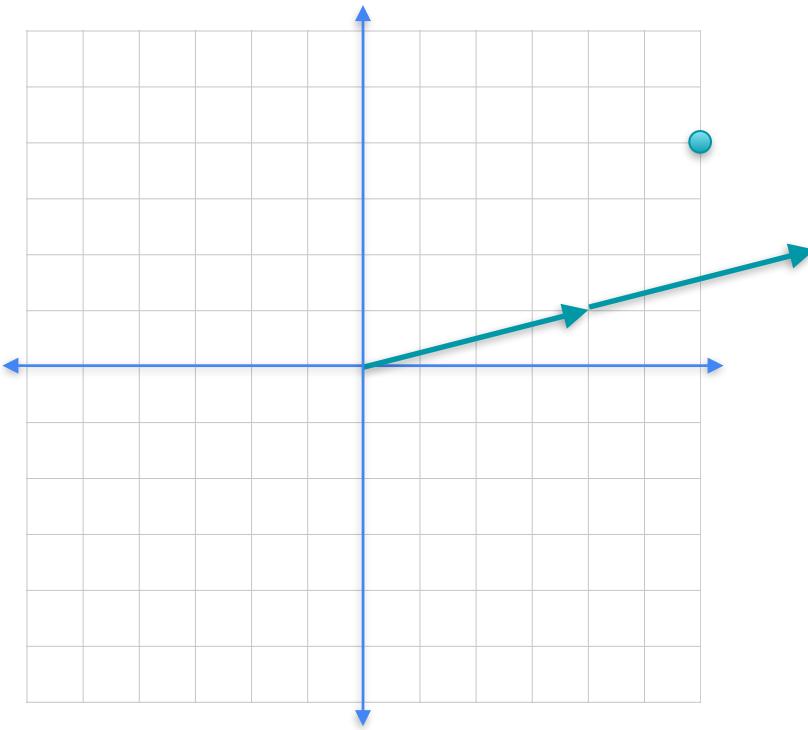
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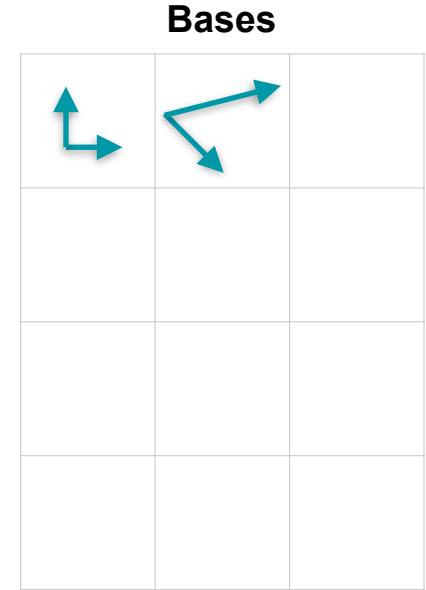
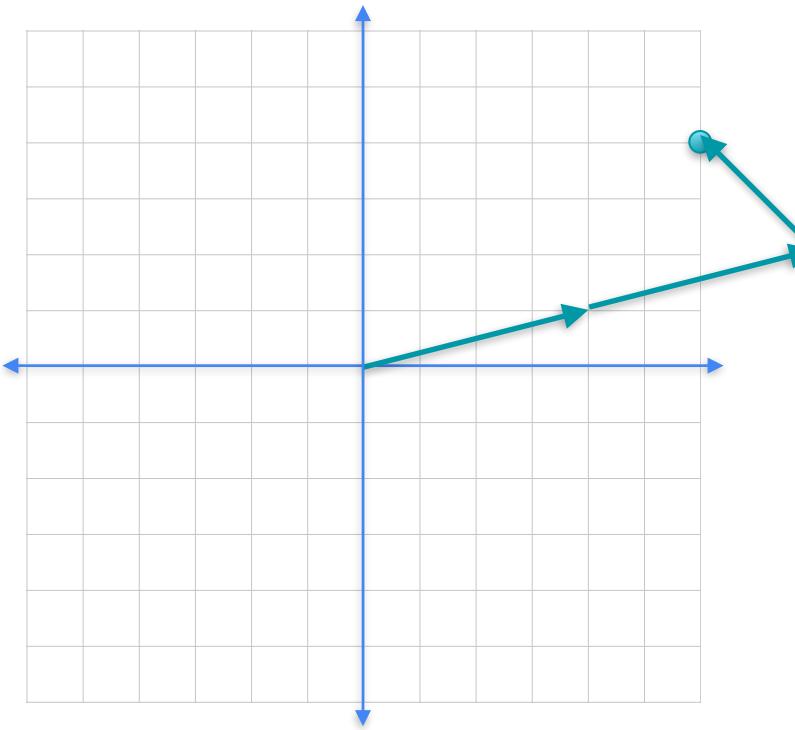
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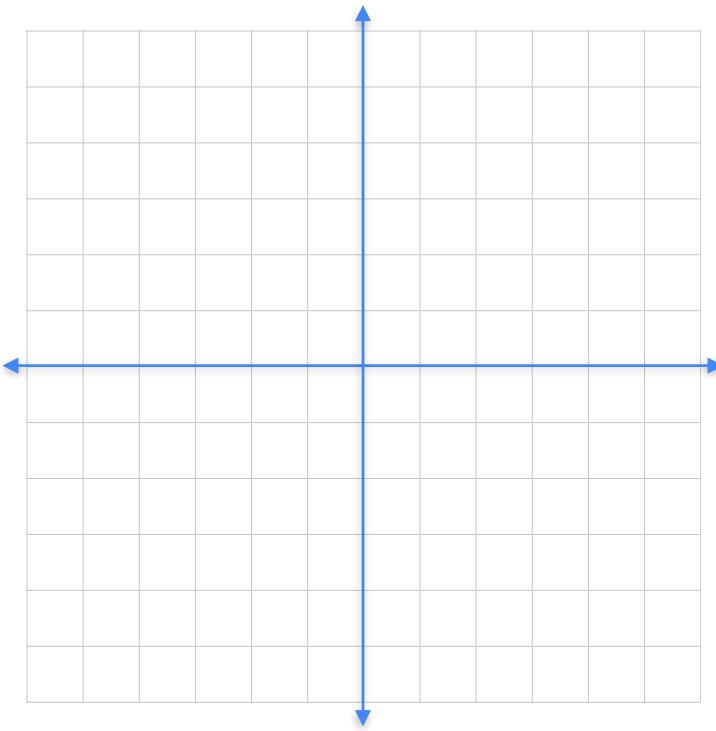
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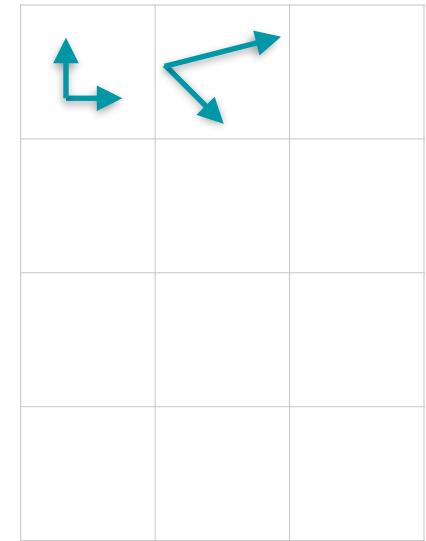
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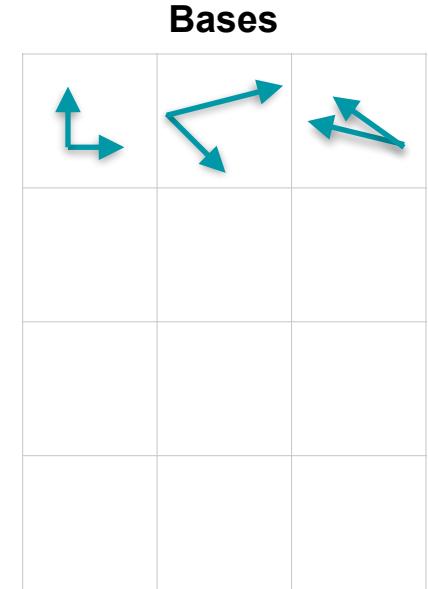
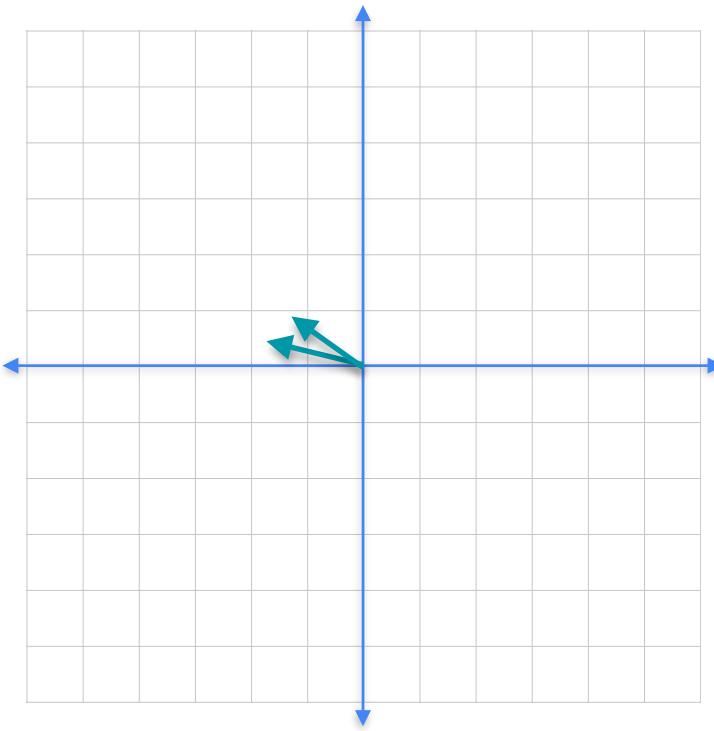
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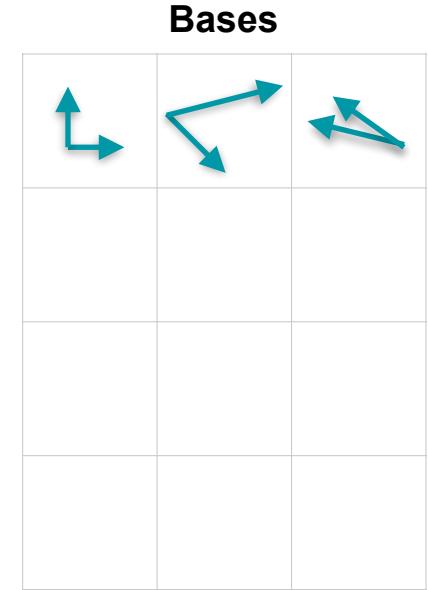
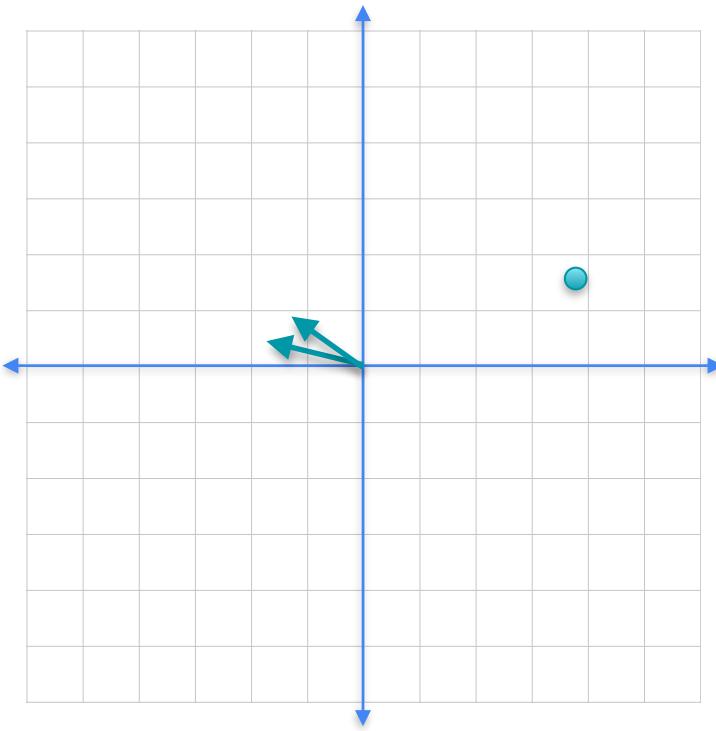
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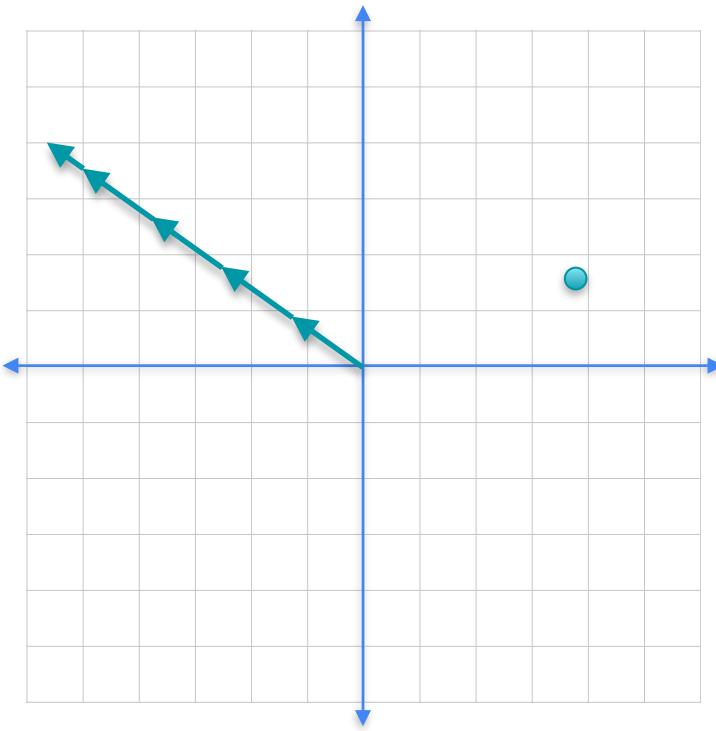
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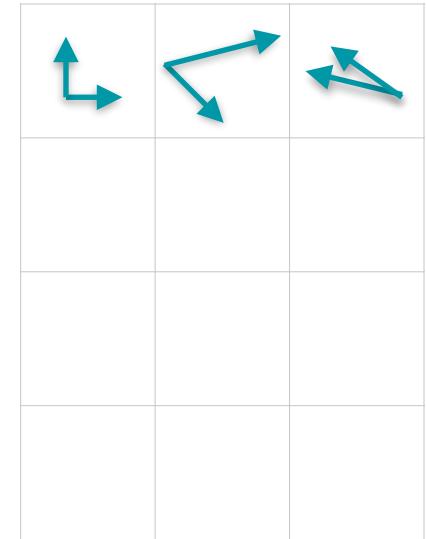
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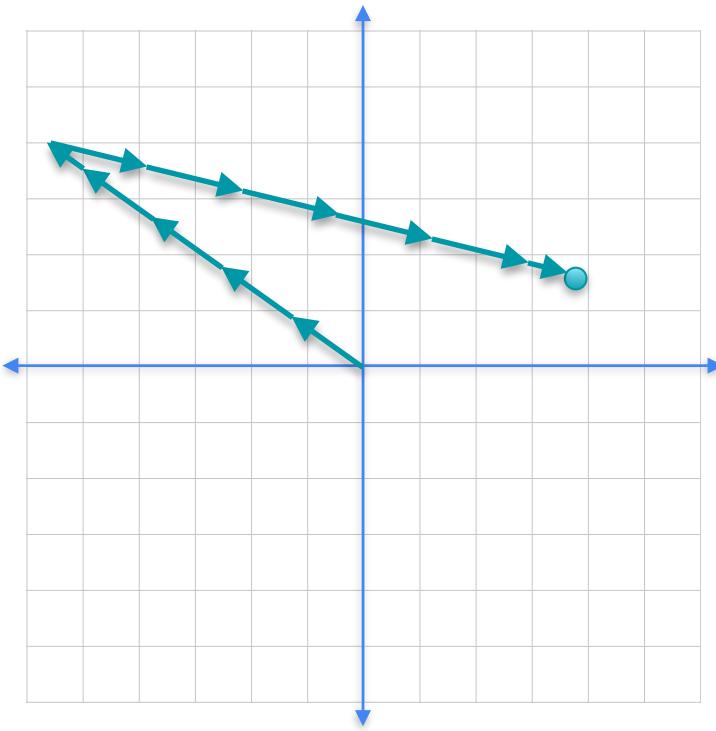
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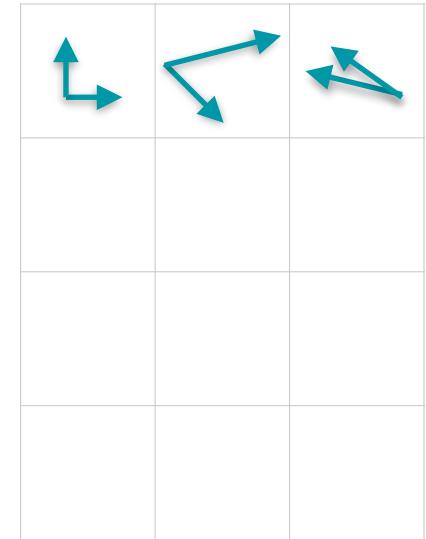
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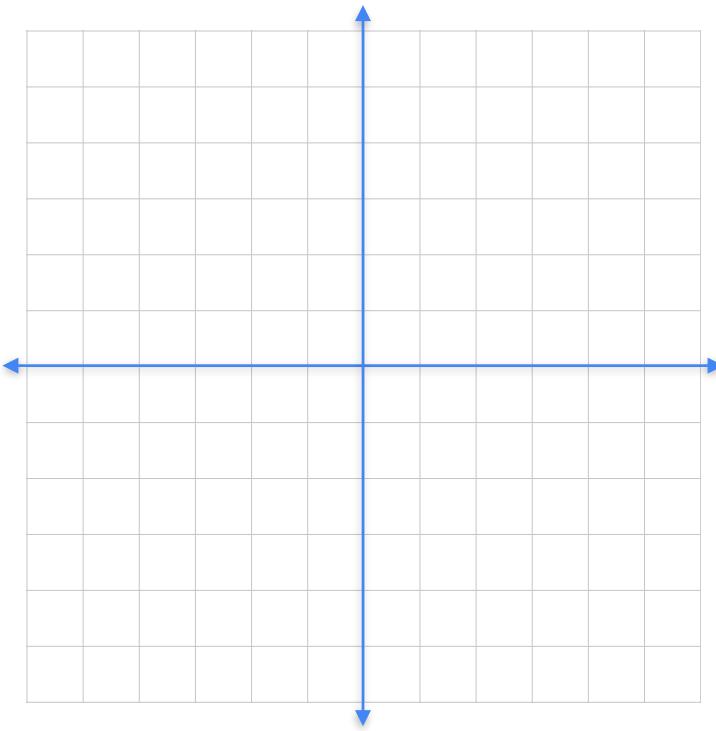
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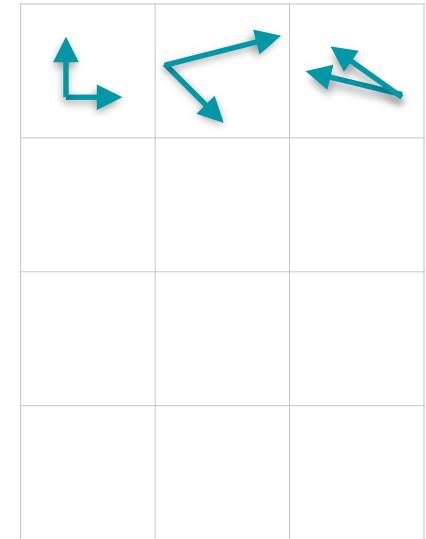
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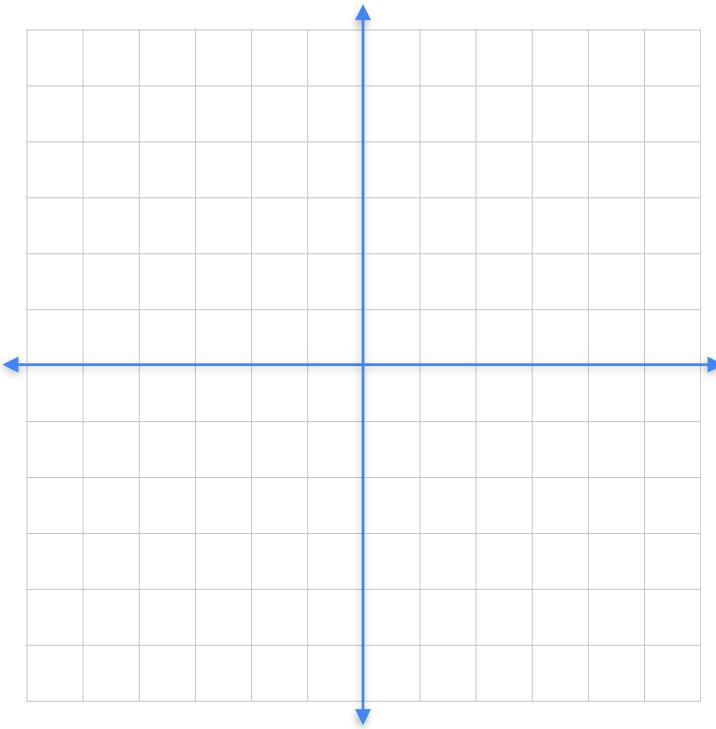
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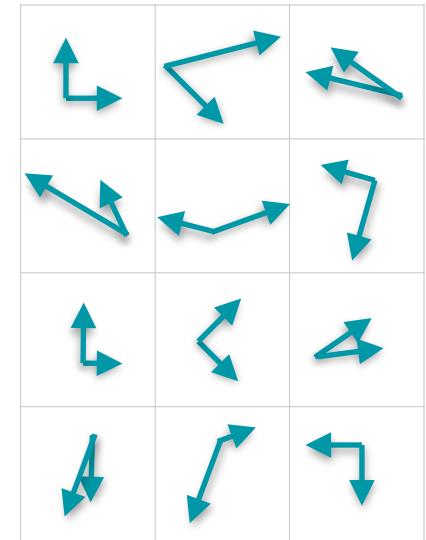
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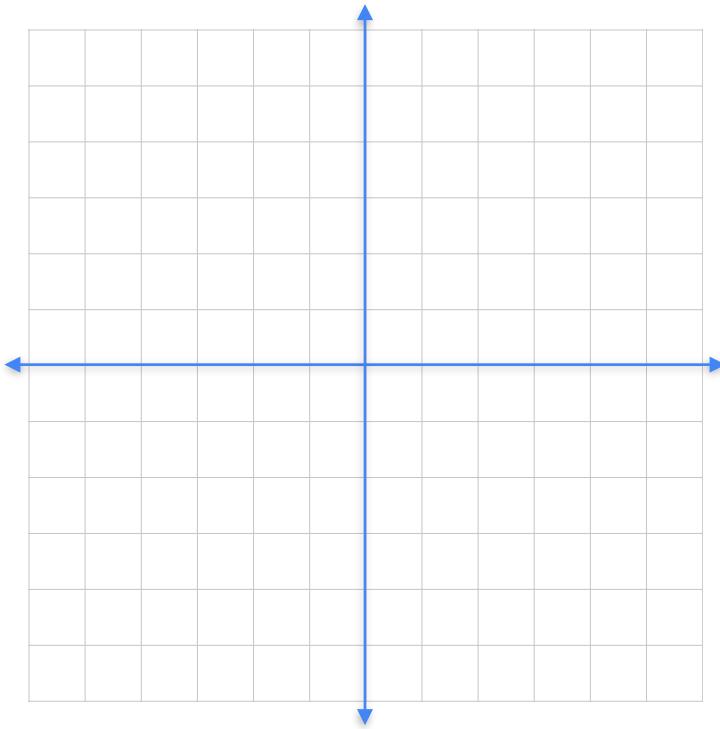
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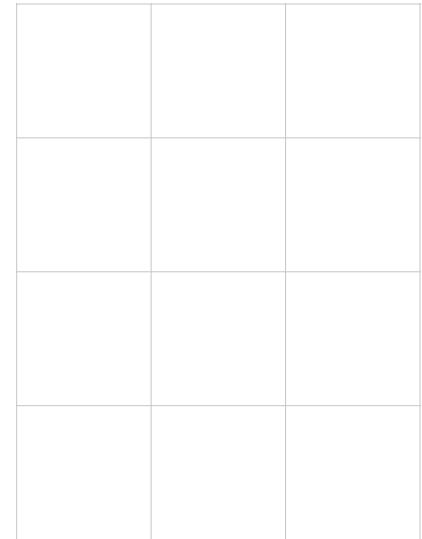
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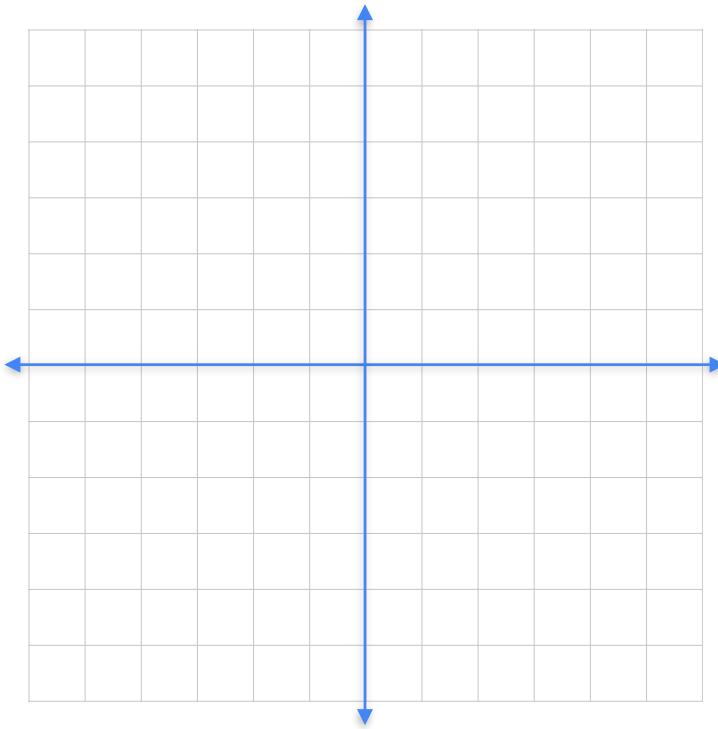
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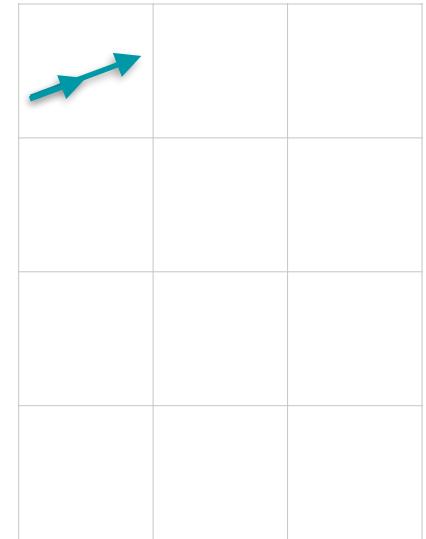
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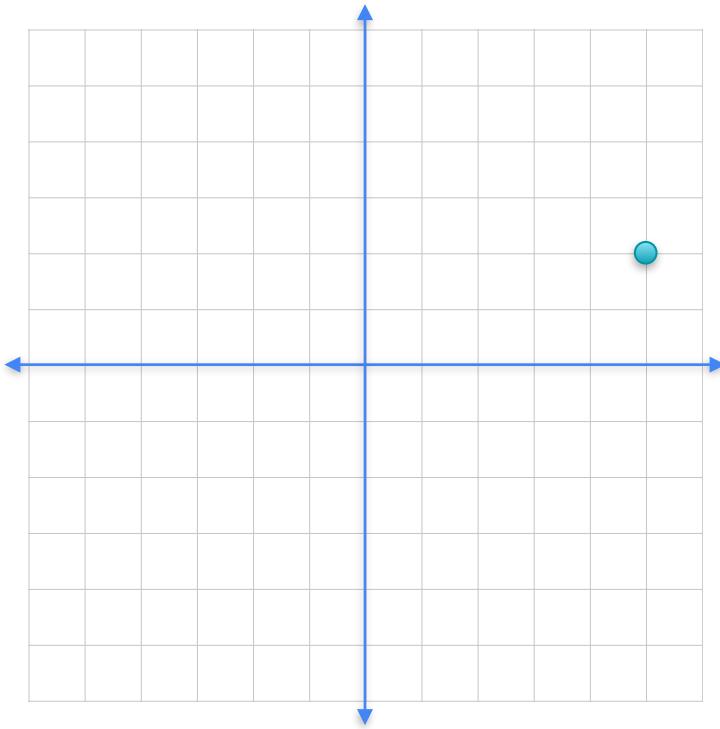
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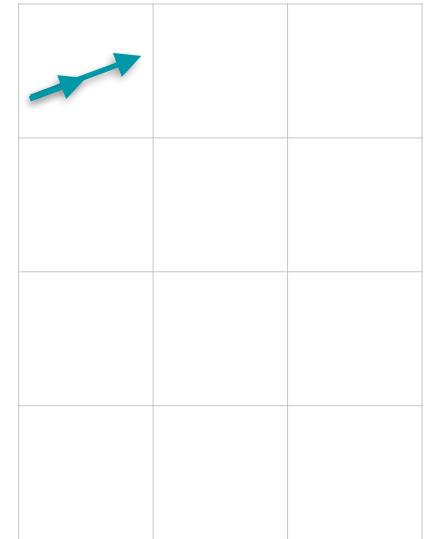
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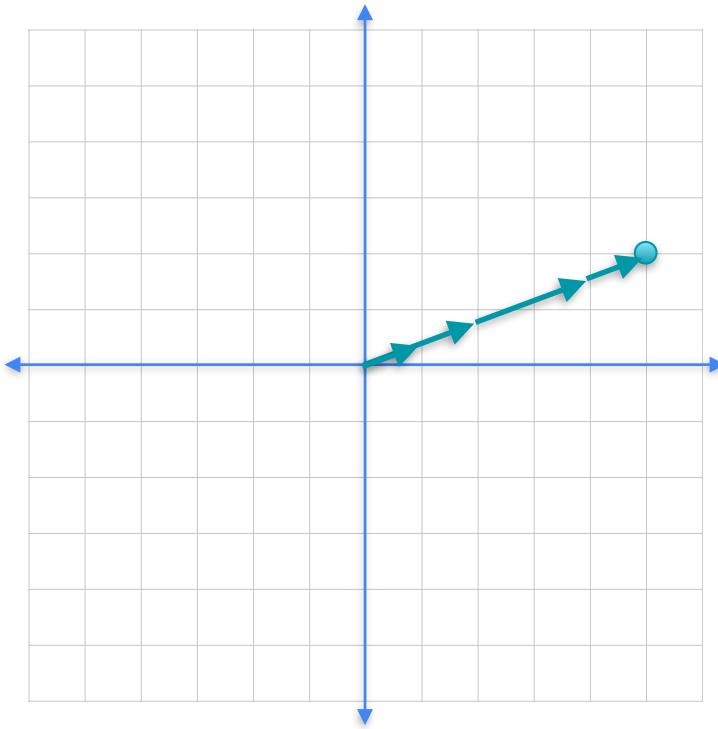
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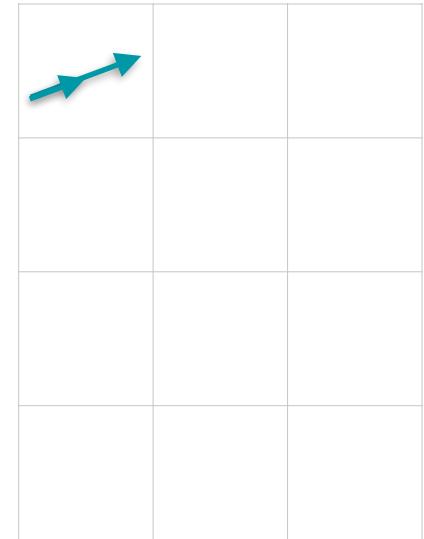
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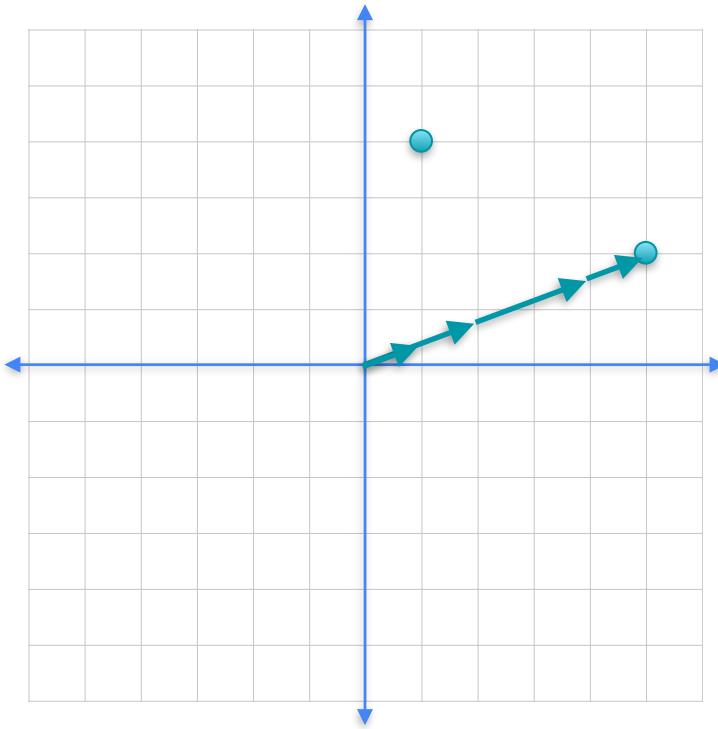
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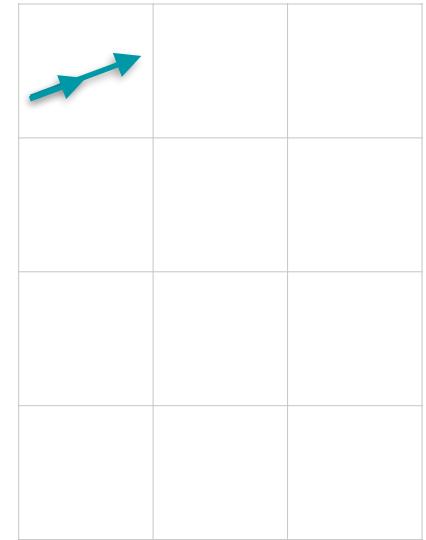
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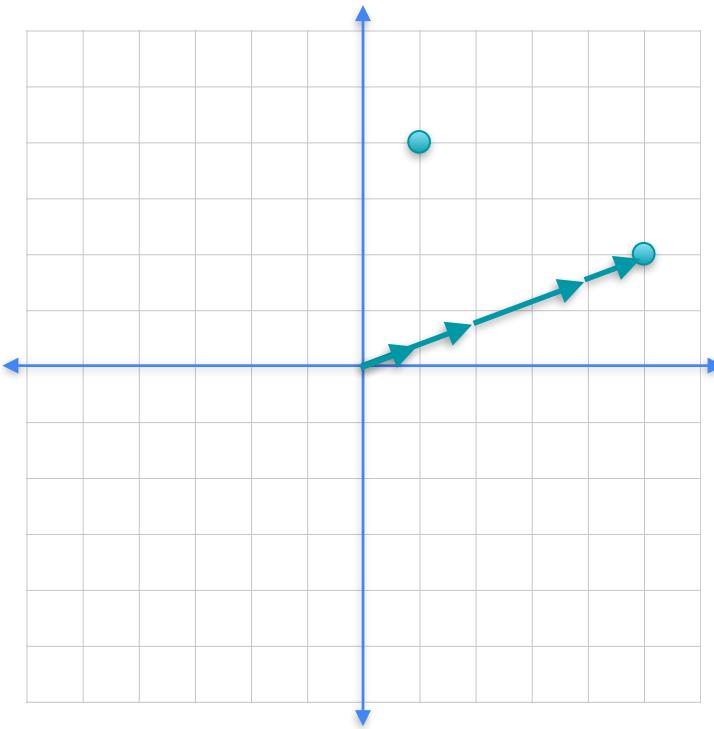
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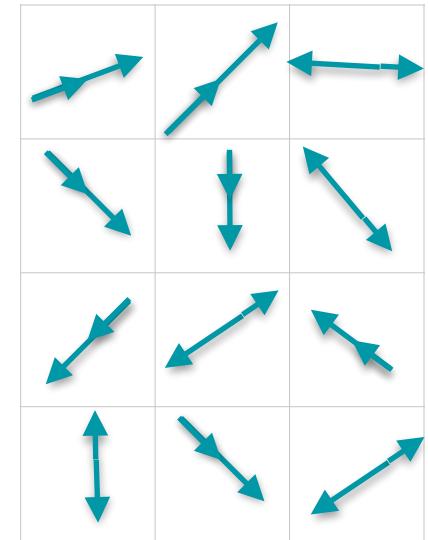
Not bases



What is not a basis?



Not bases



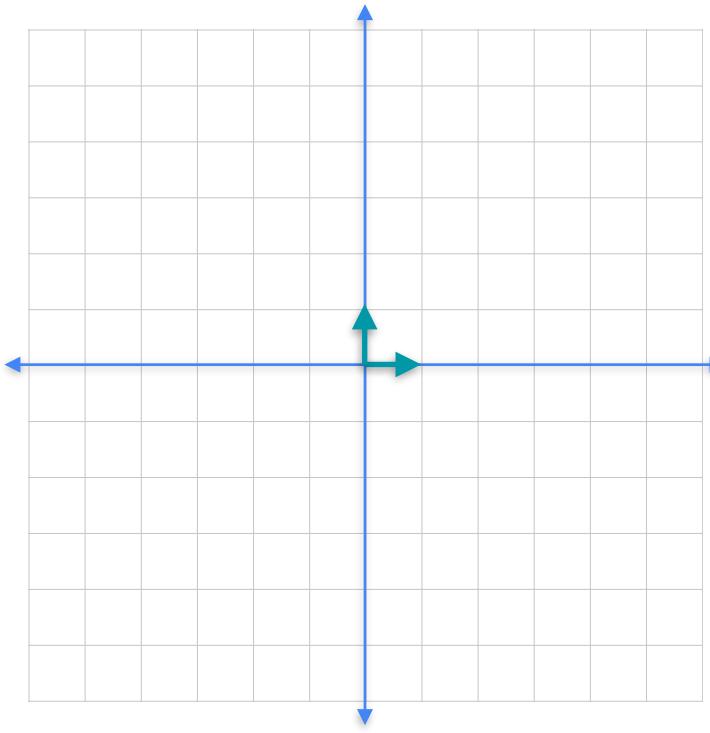


DeepLearning.AI

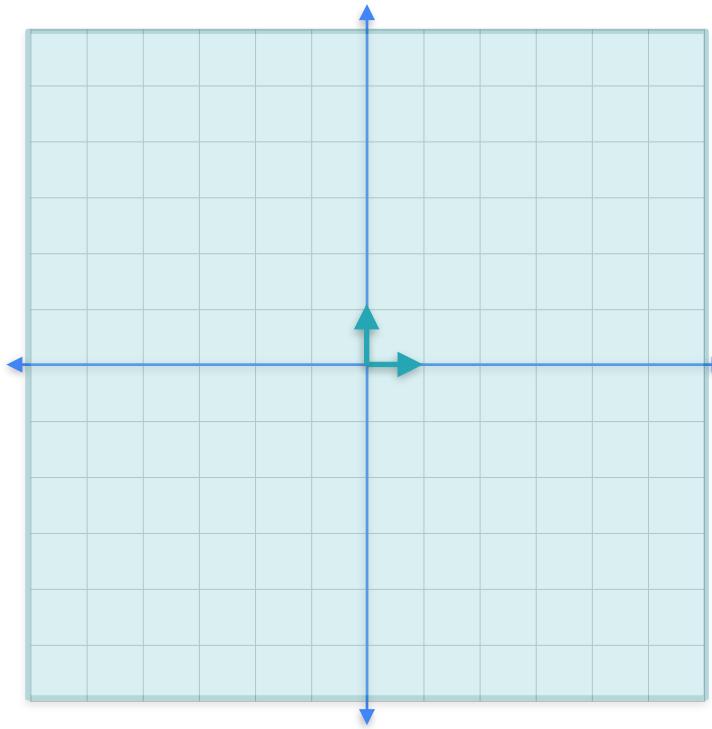
Determinants and Eigenvectors

Span

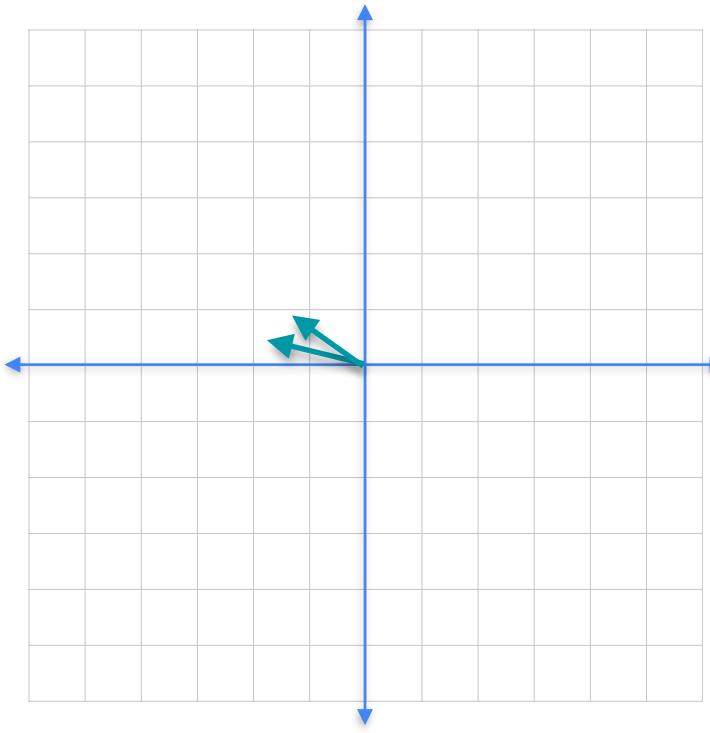
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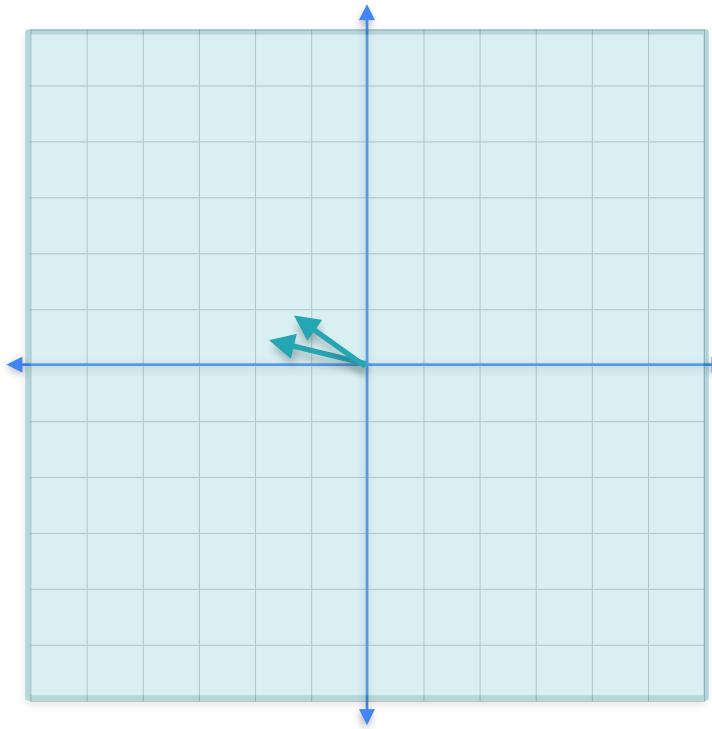
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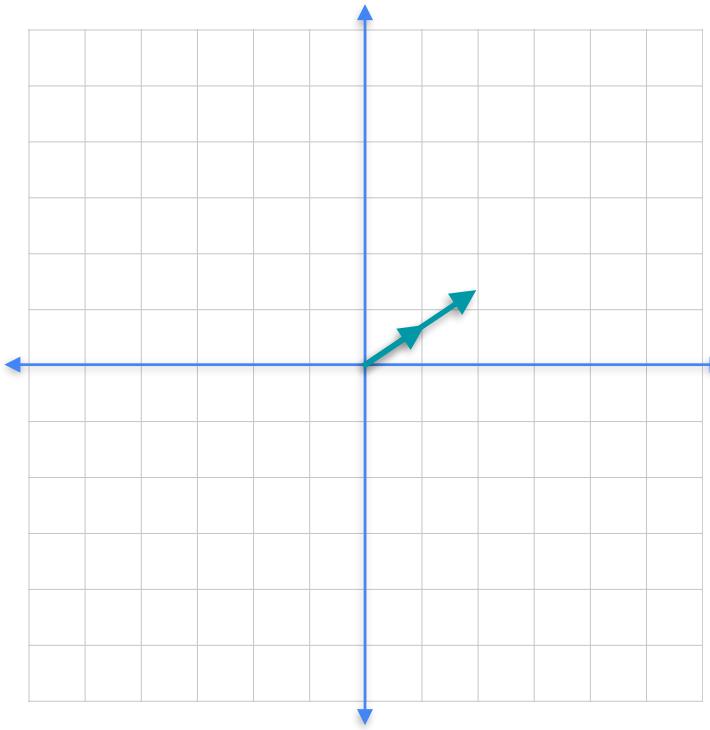
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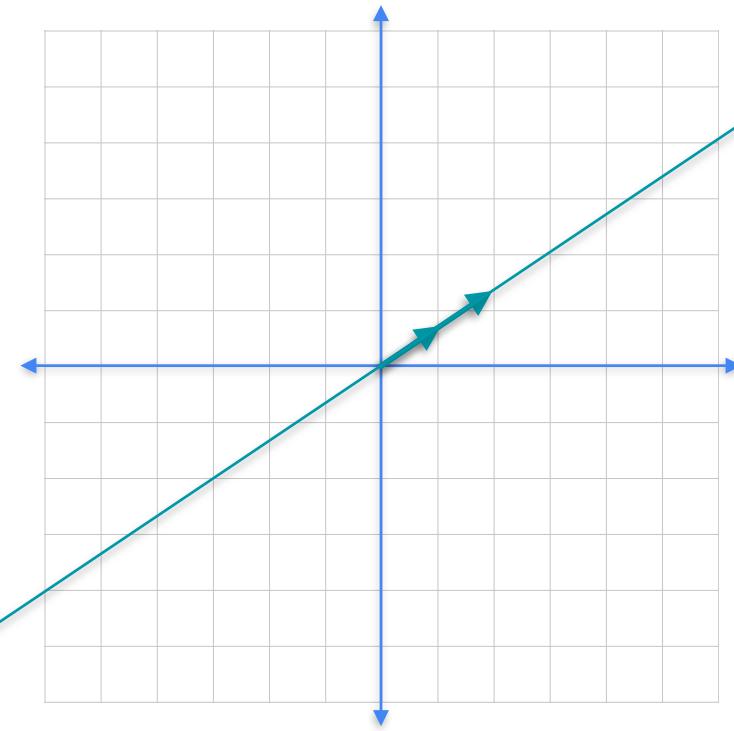
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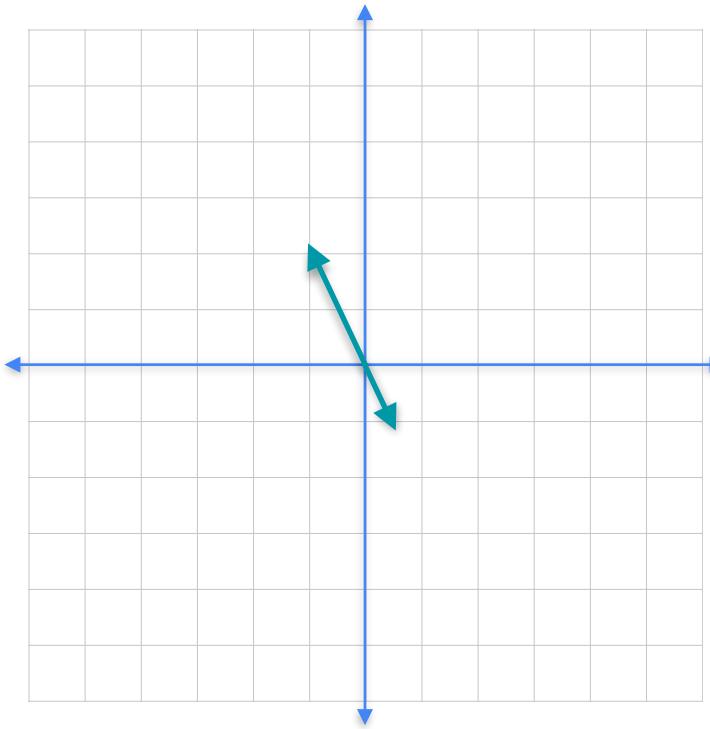
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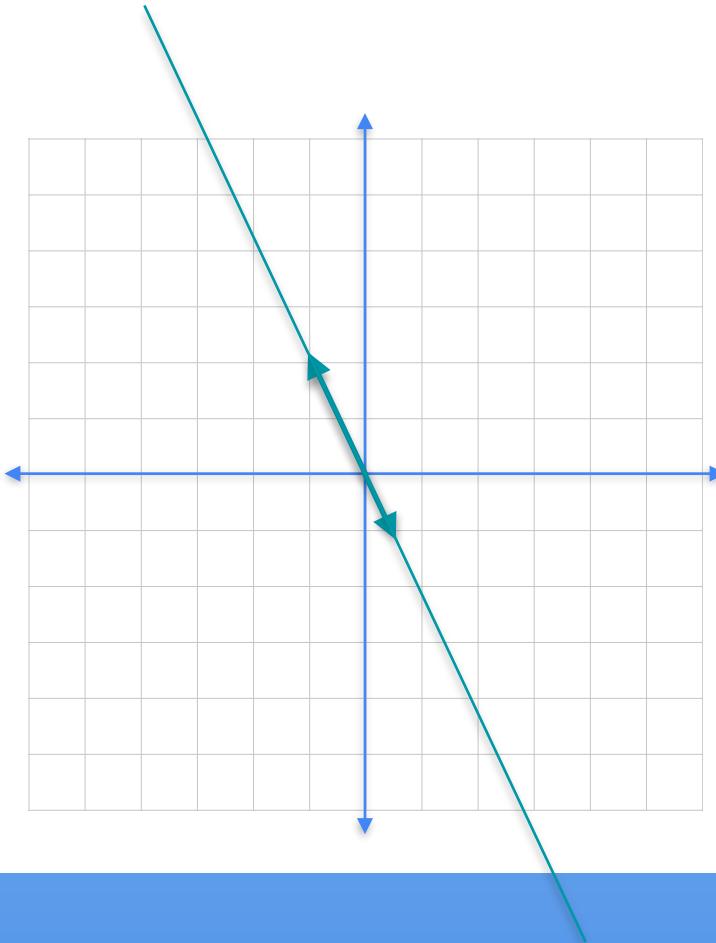
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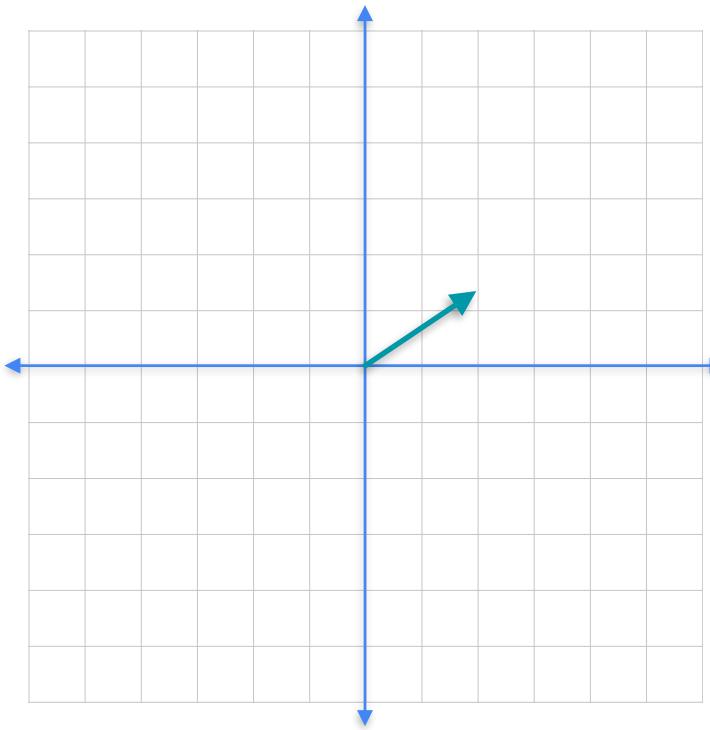
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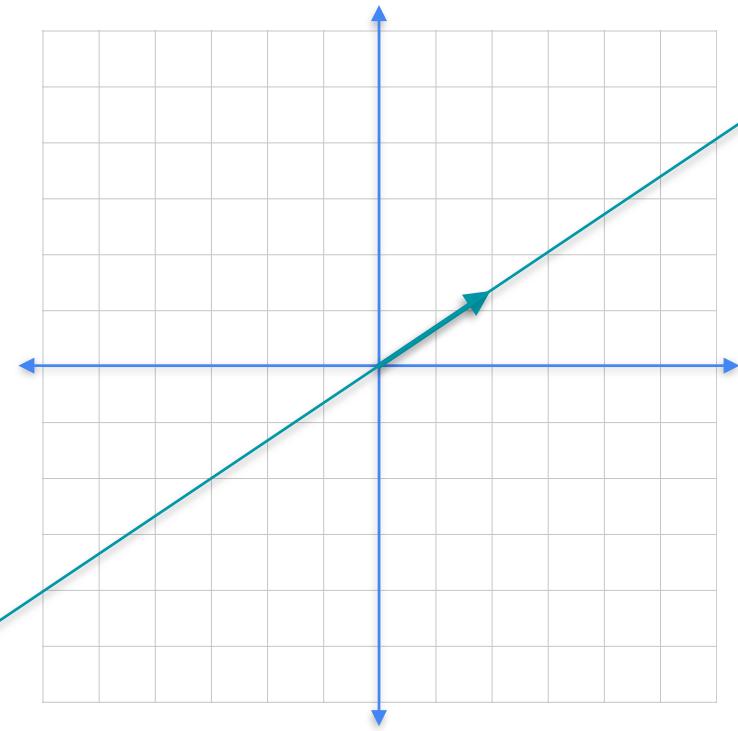
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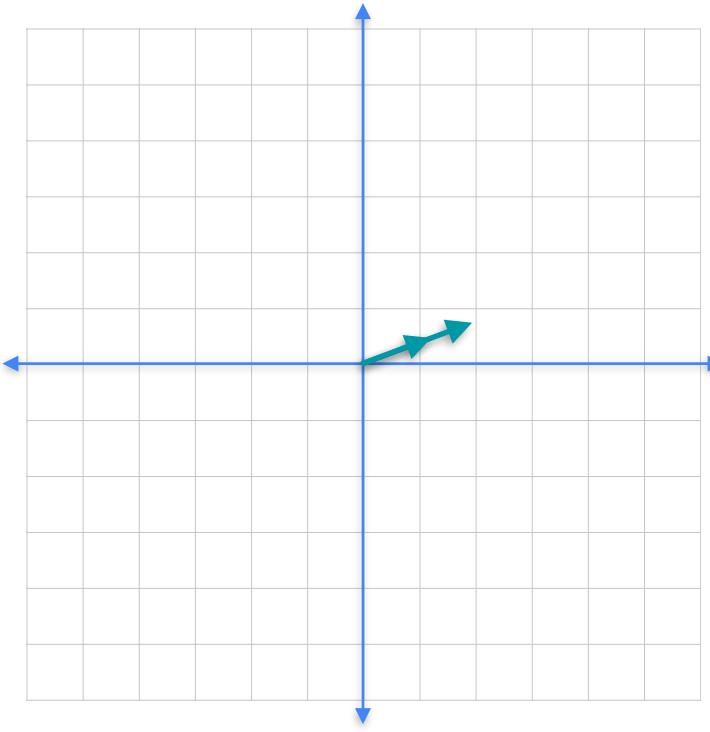
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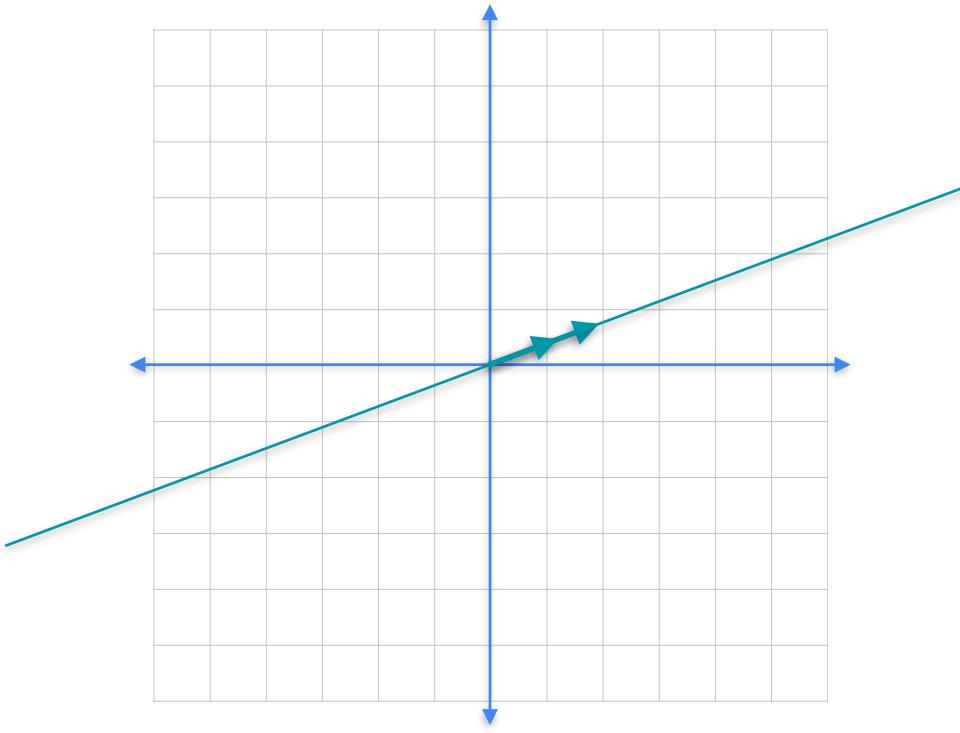
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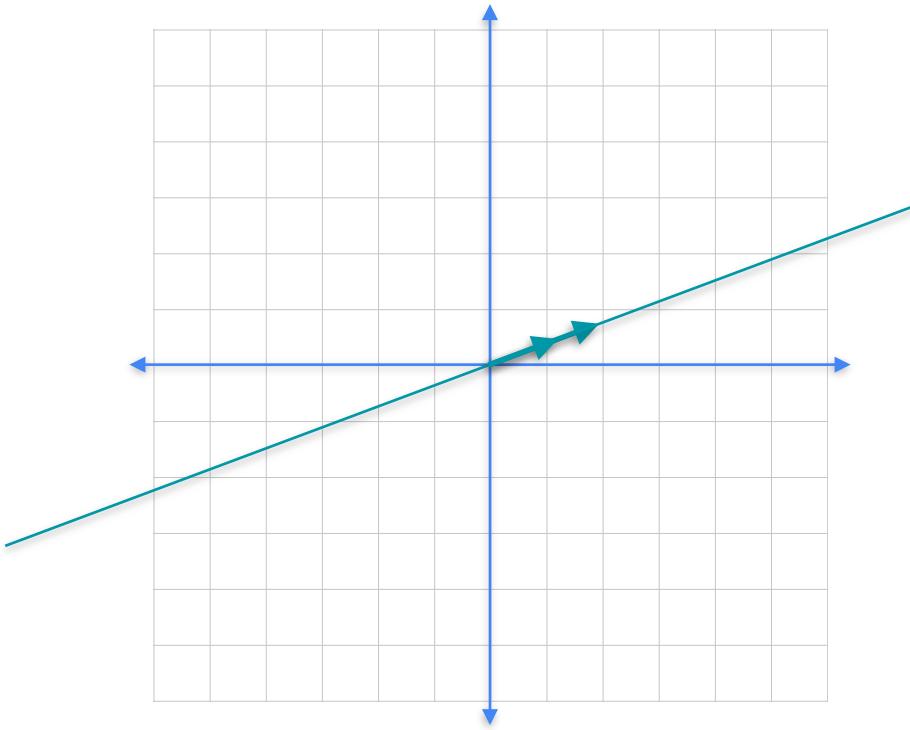
Is this a basis?



Is this a basis?



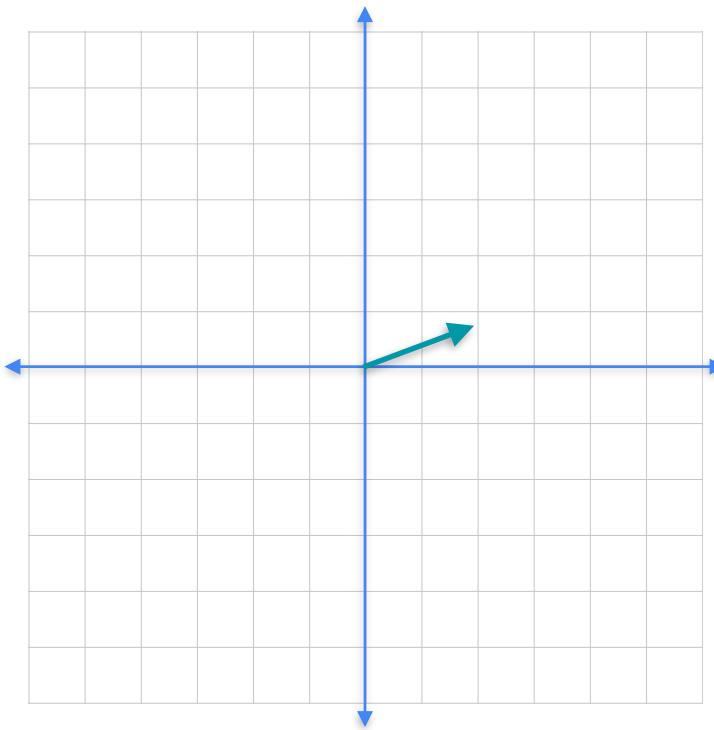
Is this a basis?



No

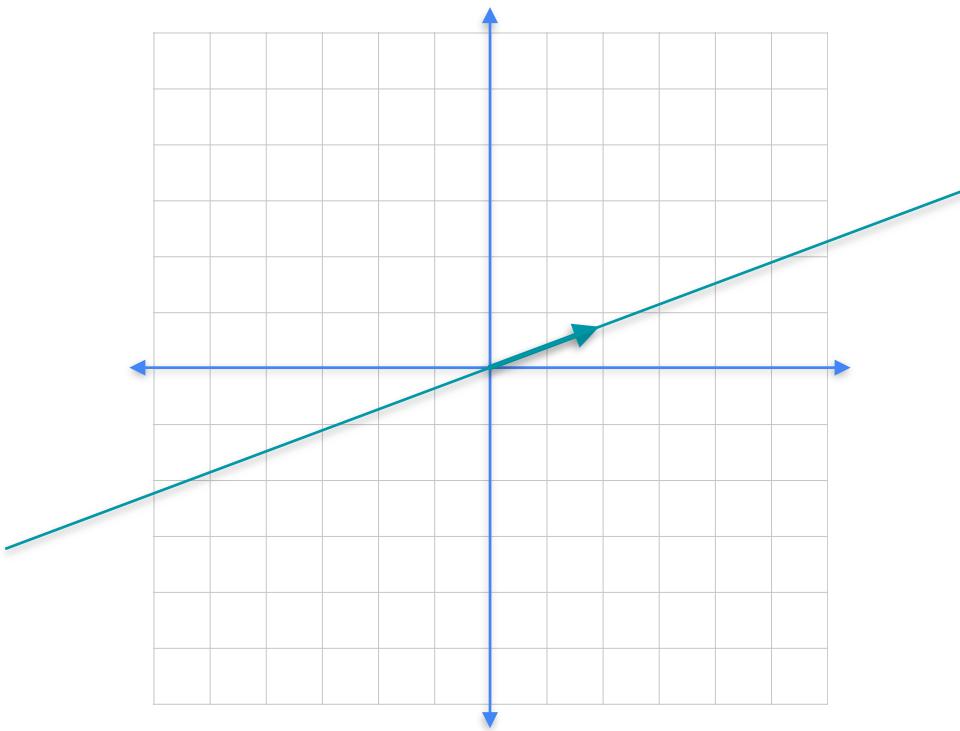
Is this a basis for something?

Bases

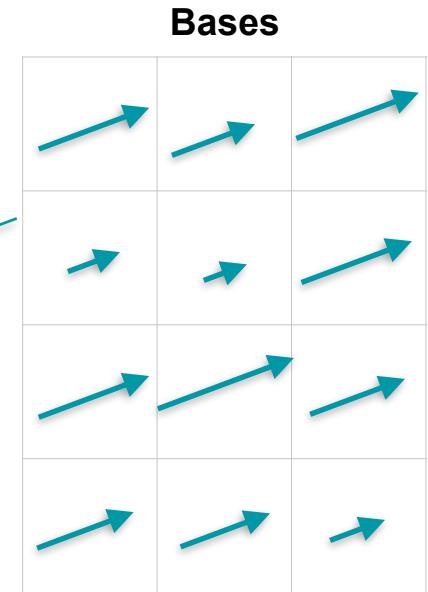
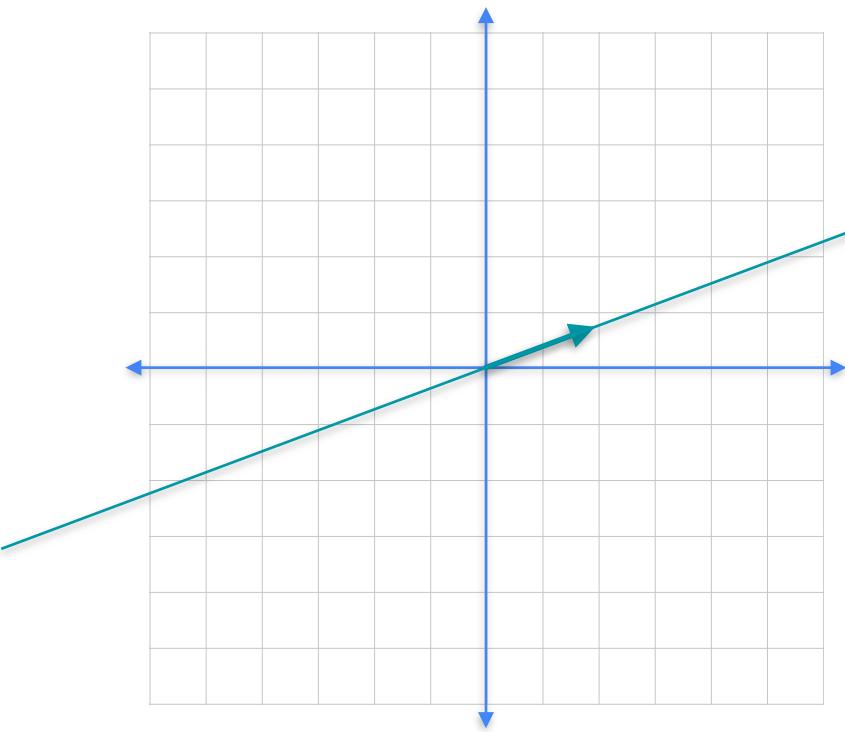


Is this a basis for something?

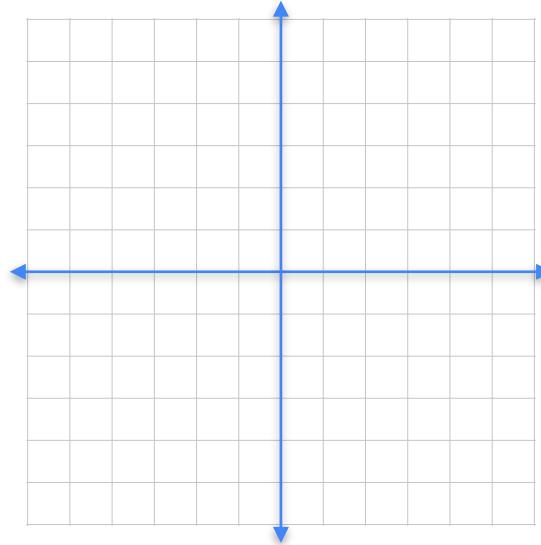
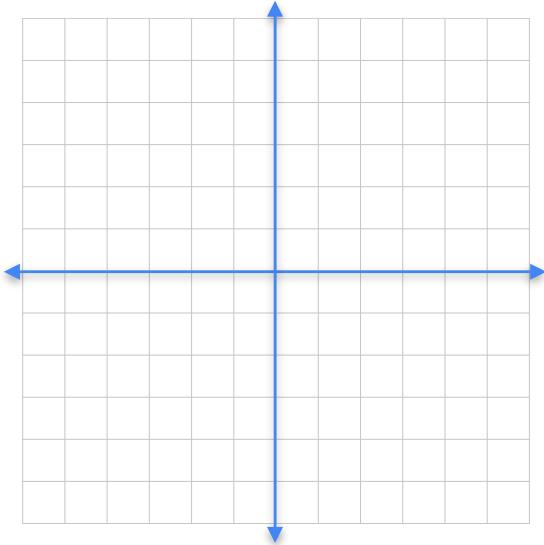
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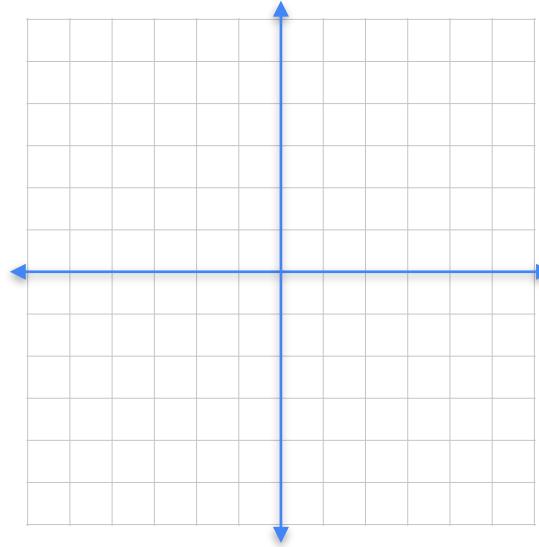
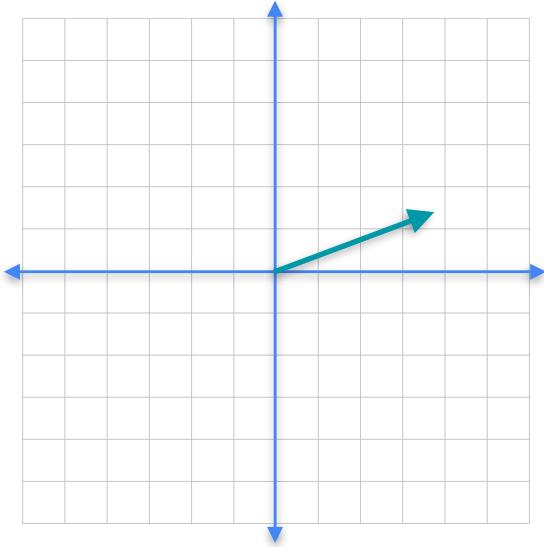
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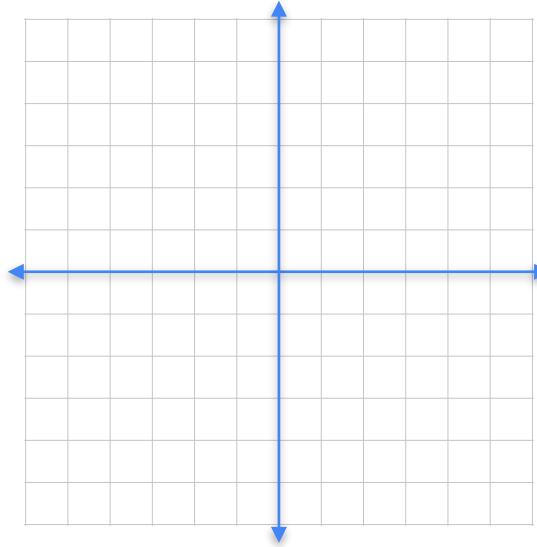
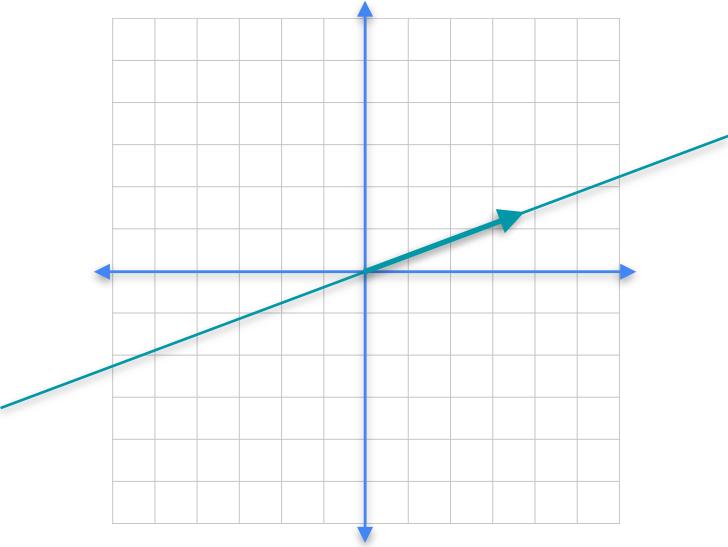
A basis is a minimal spanning set



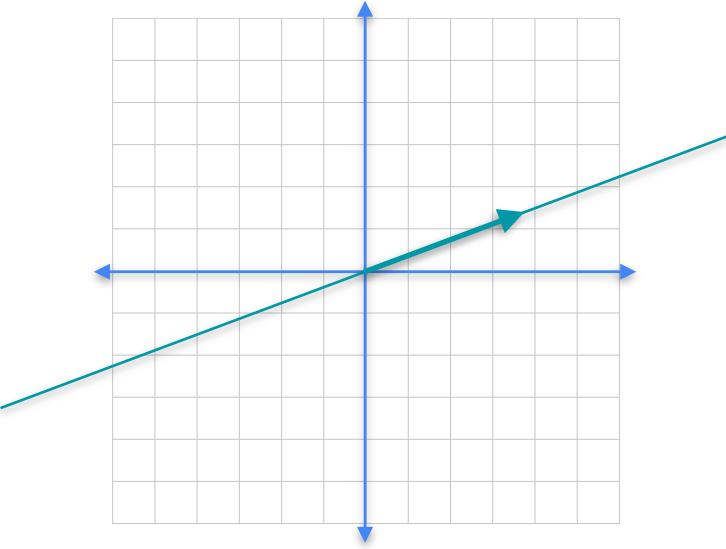
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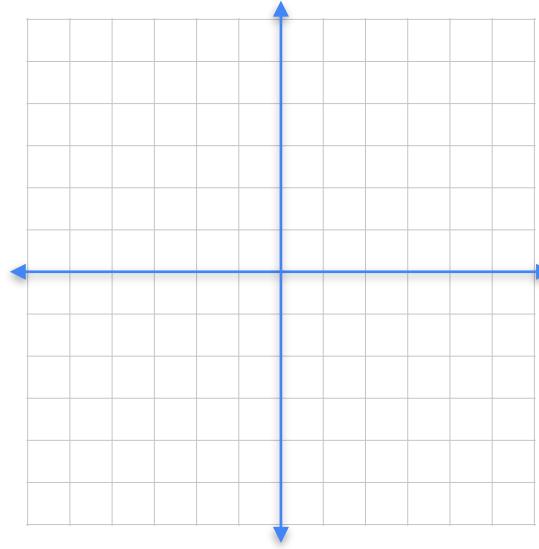
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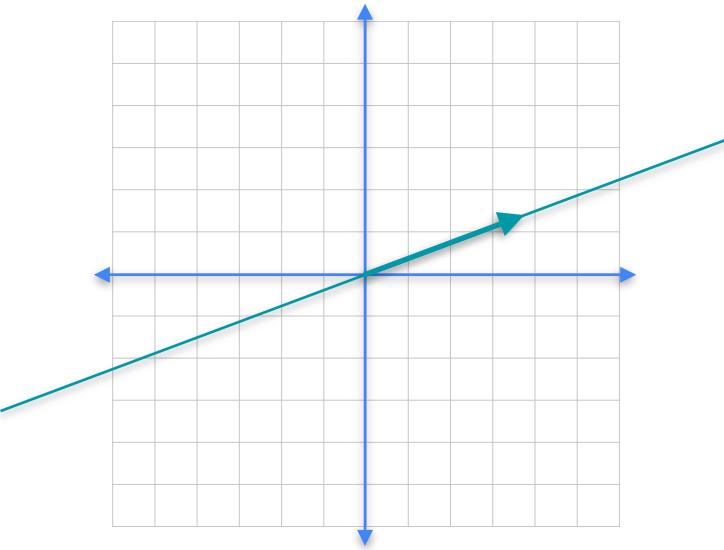
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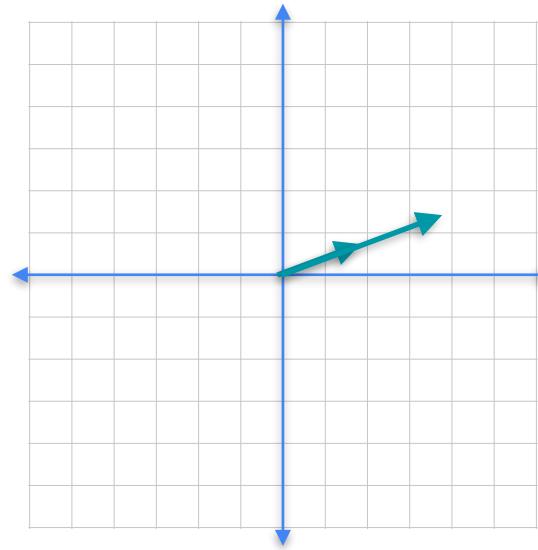
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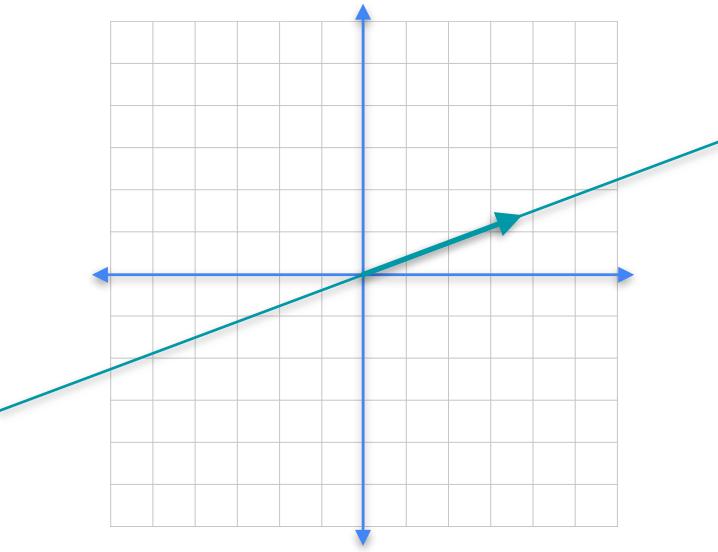
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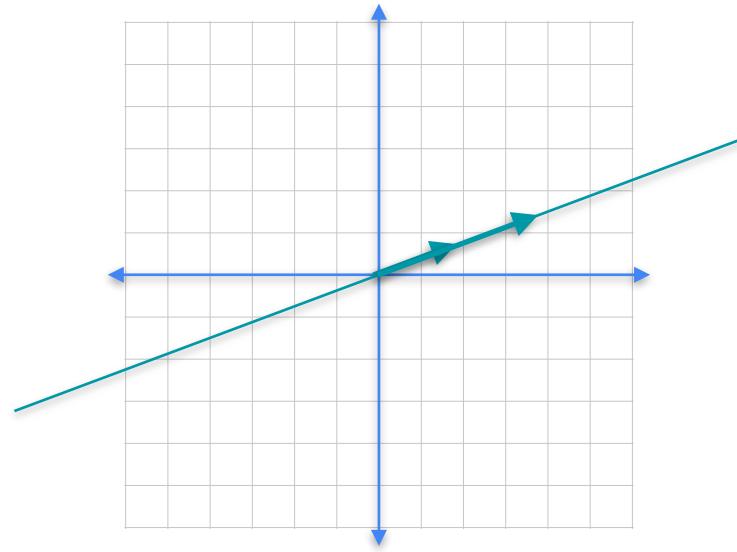
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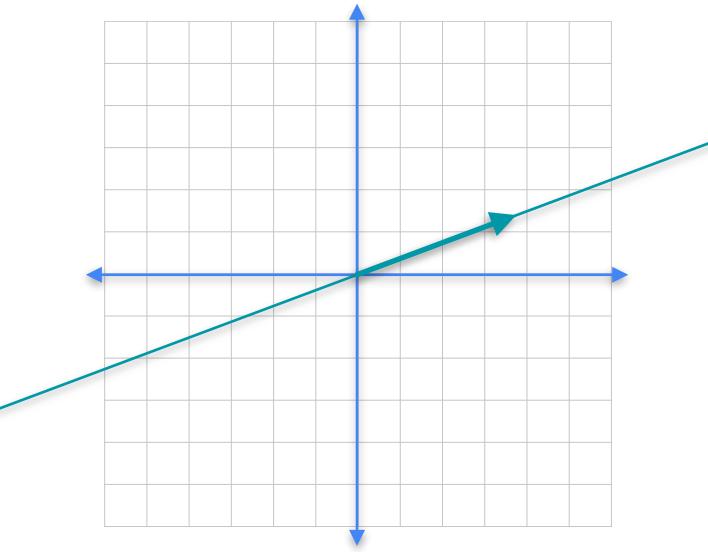
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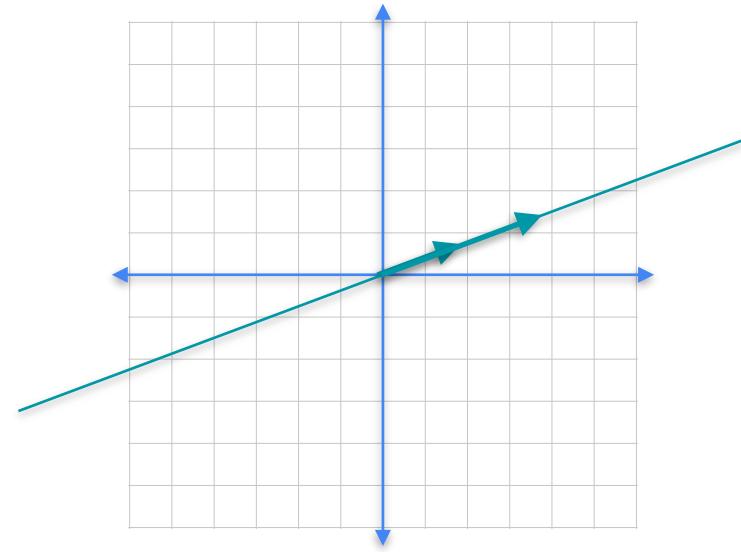
Basis



A basis is a minimal spanning set

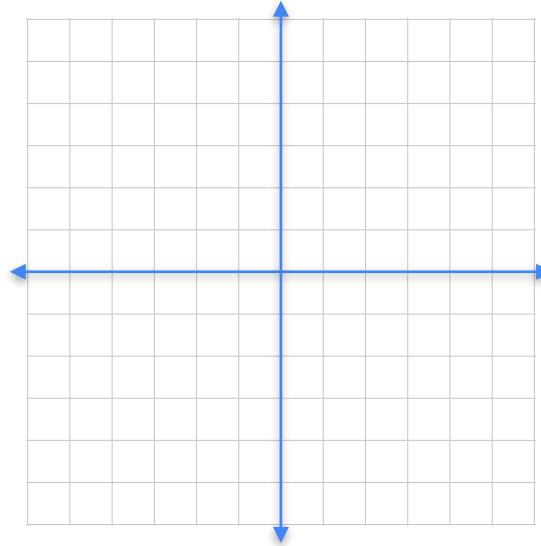
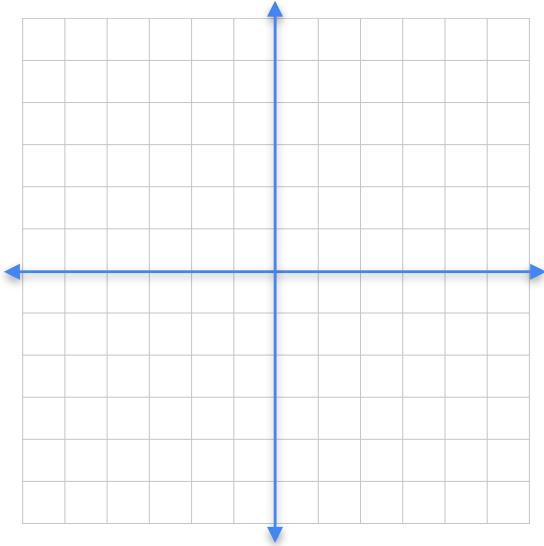


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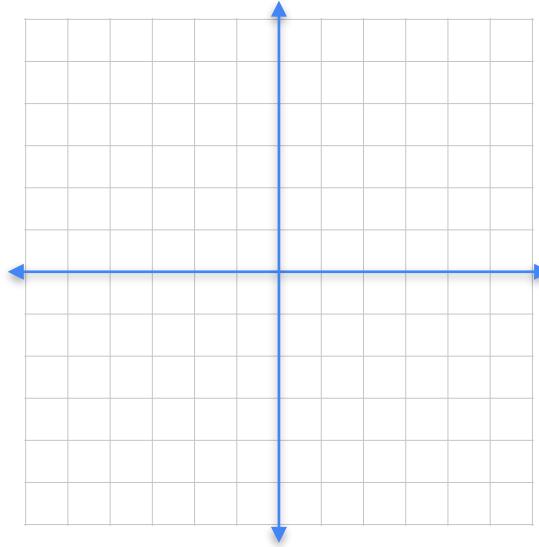
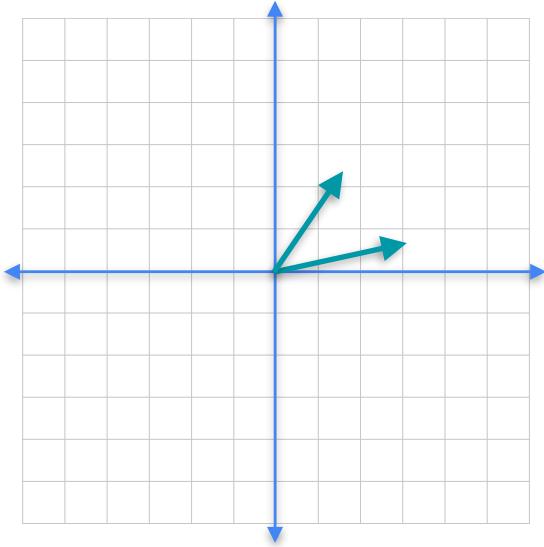


Not a basis

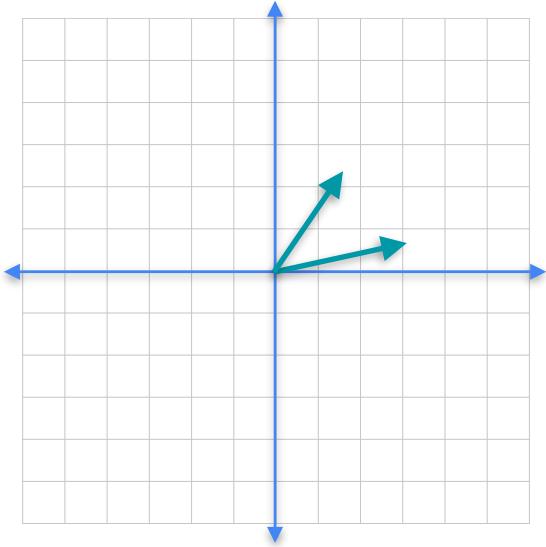
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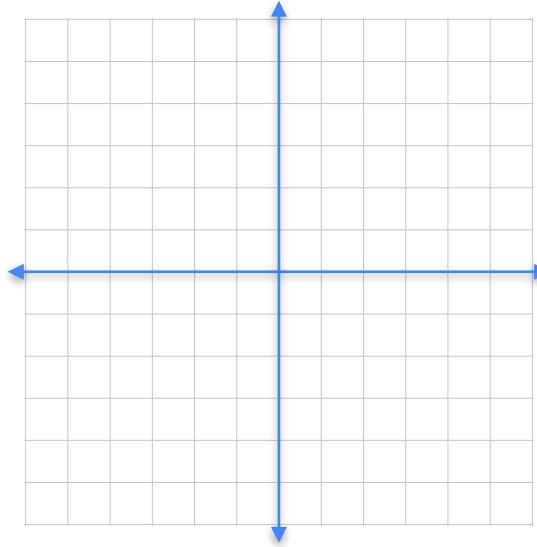
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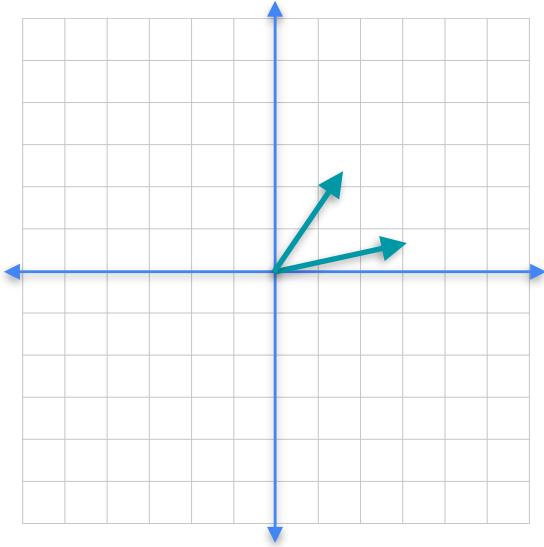
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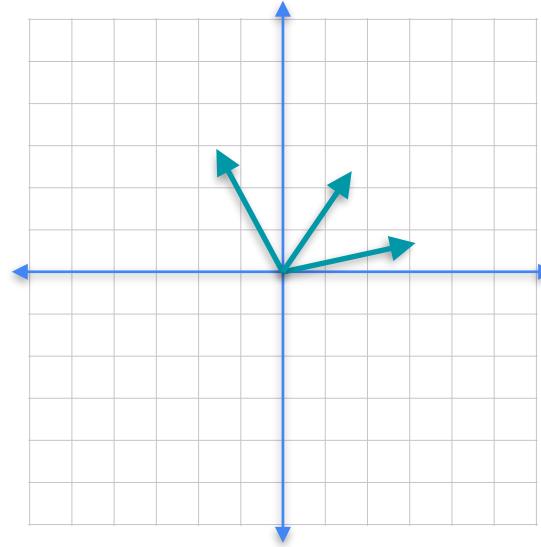
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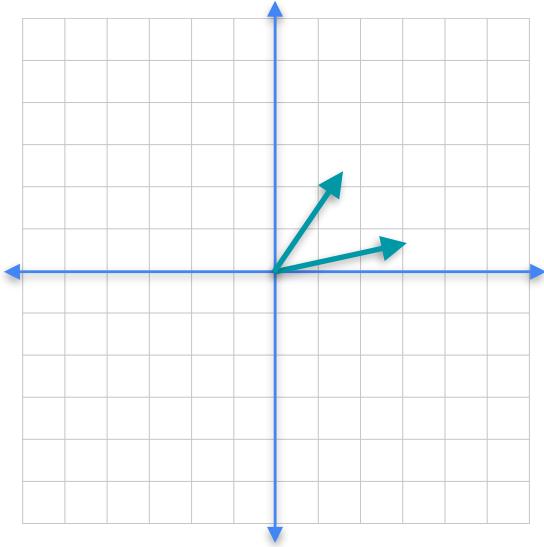
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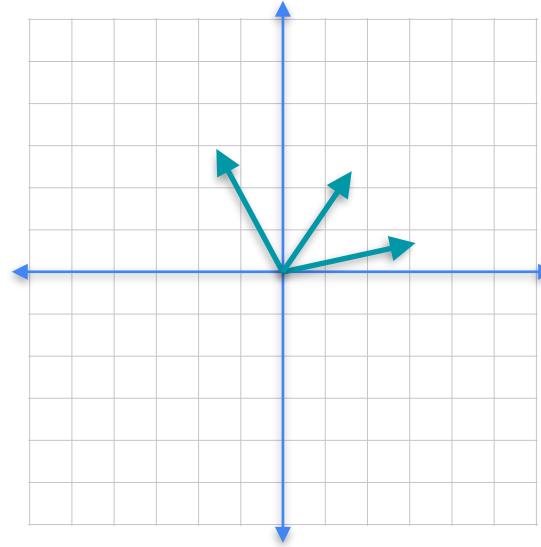
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A basis is a minimal spanning set

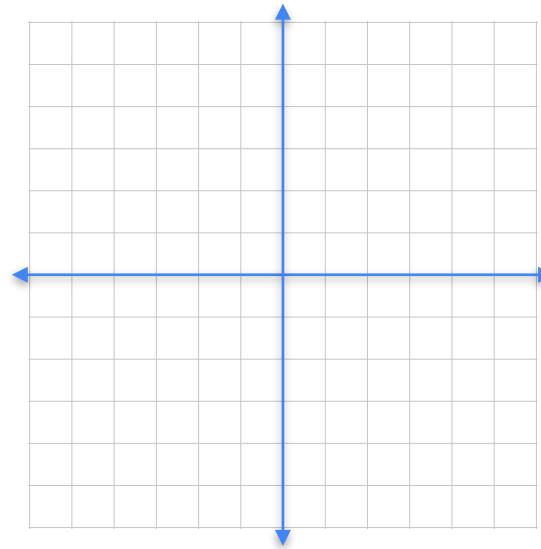
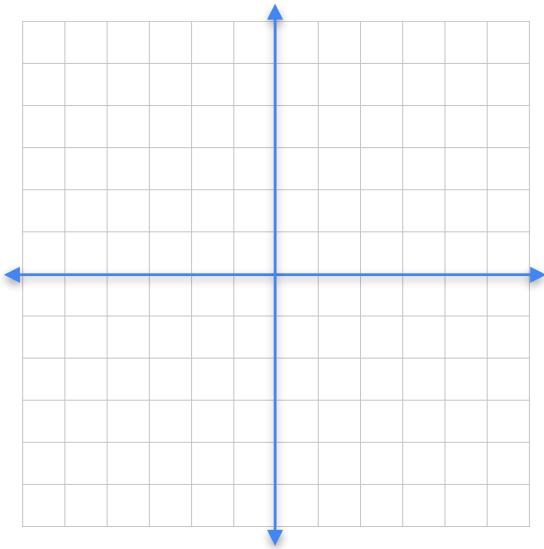


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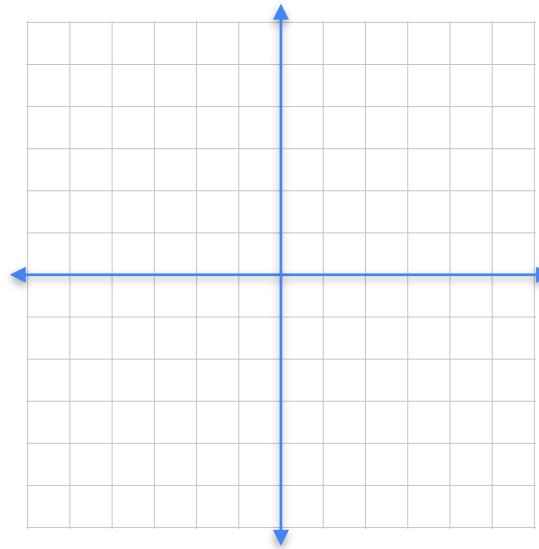
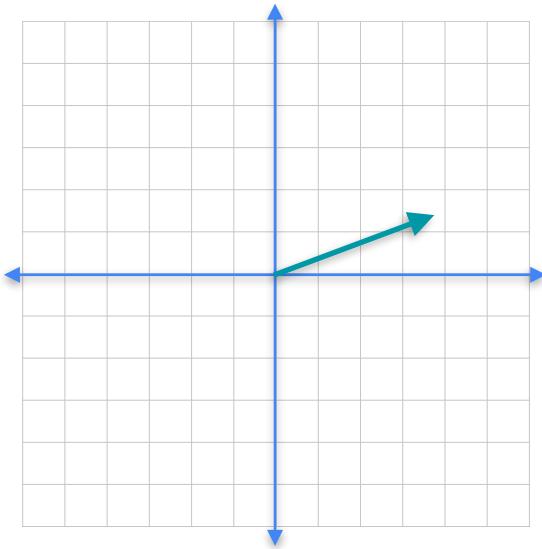


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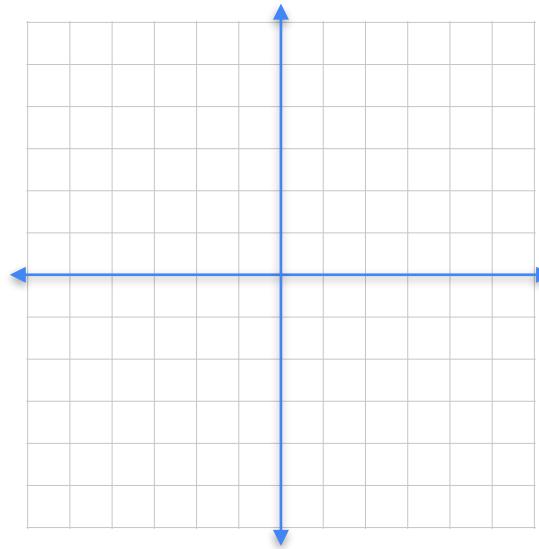
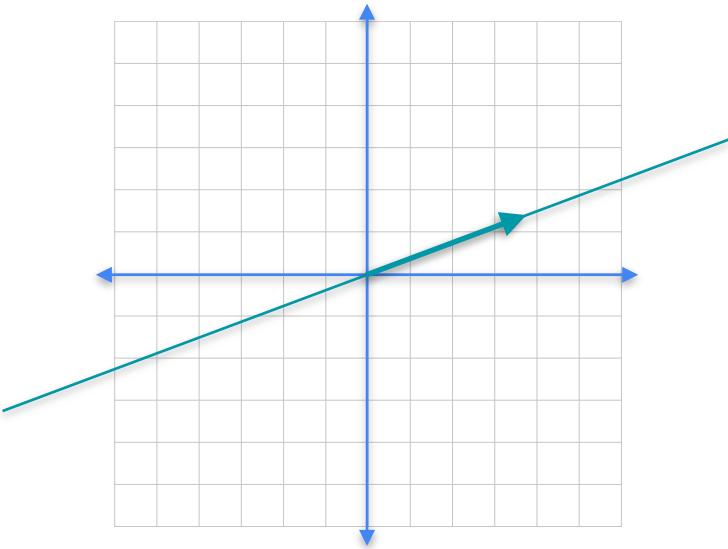
Number of elements in the basis is the dimension



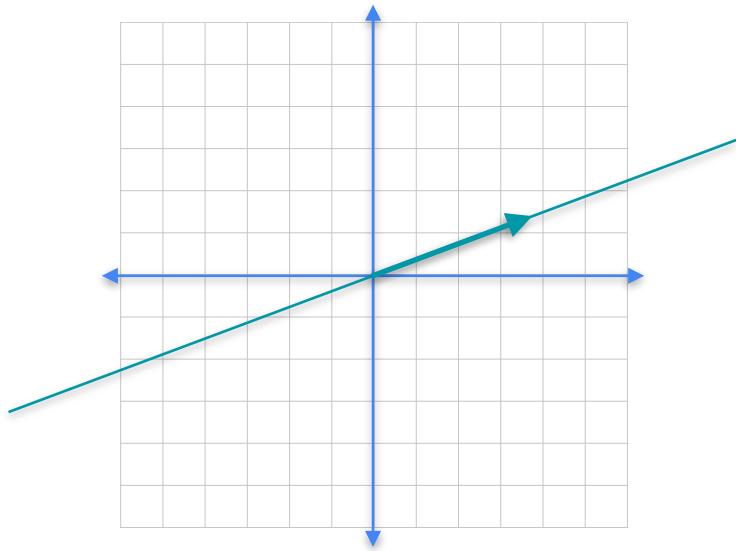
Number of elements in the basis is the dimension



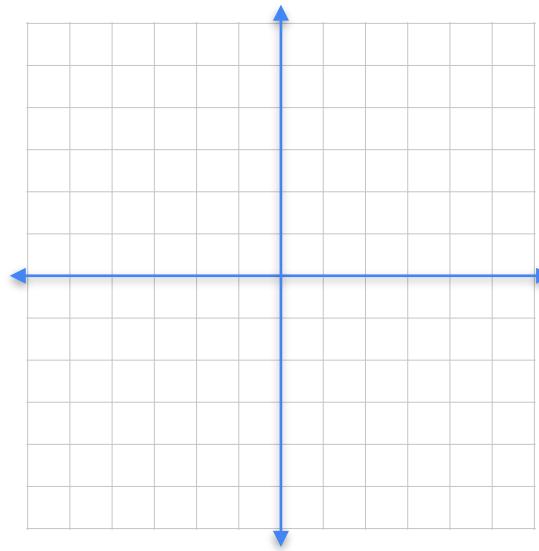
Number of elements in the basis is the dimension



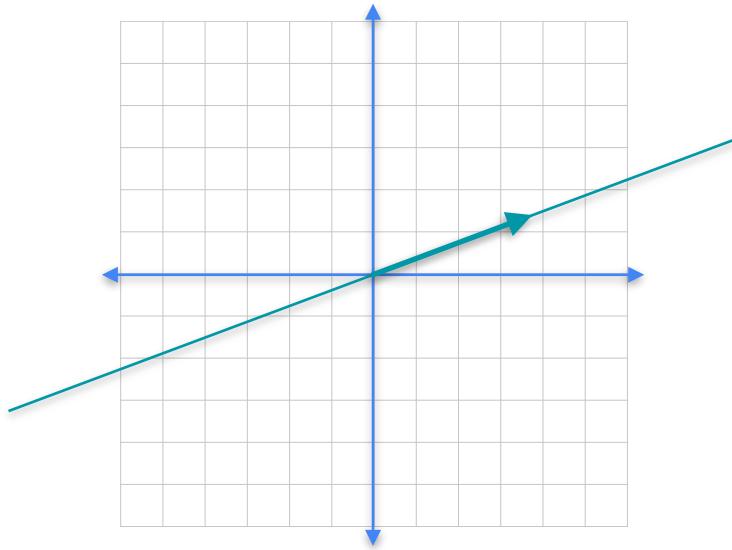
Number of elements in the basis is the dimension



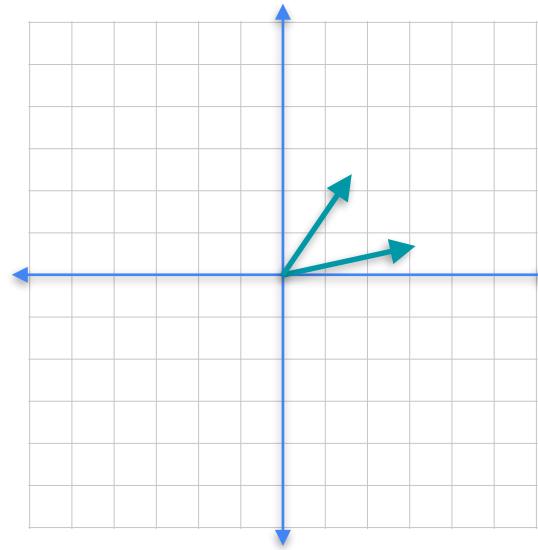
1 element
Dimension = 1



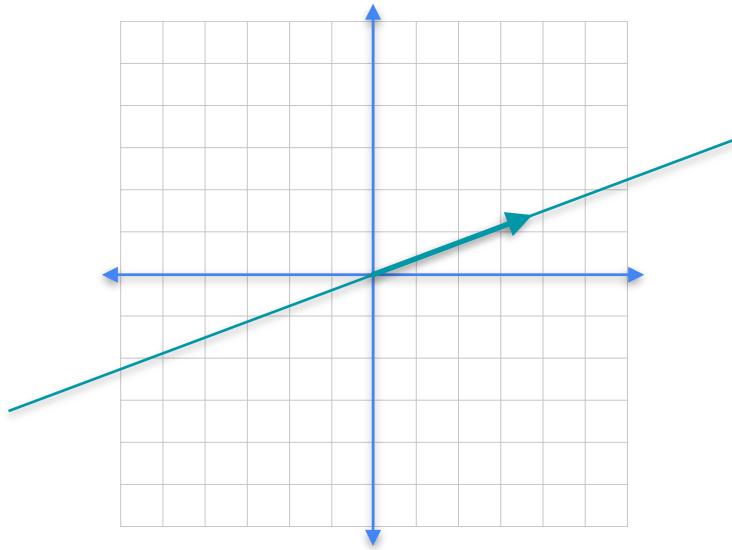
Number of elements in the basis is the dimension



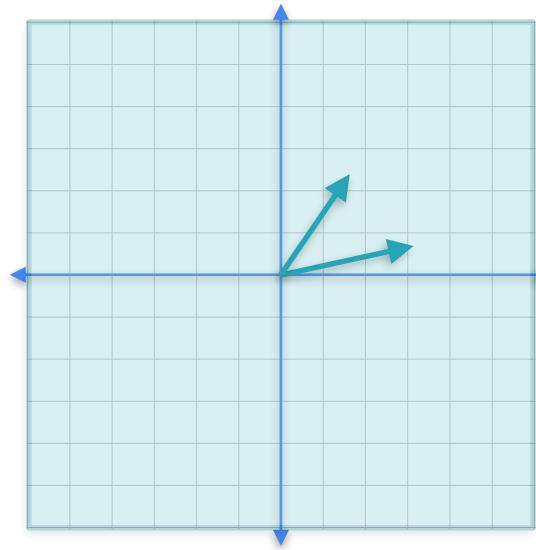
1 element
Dimension = 1



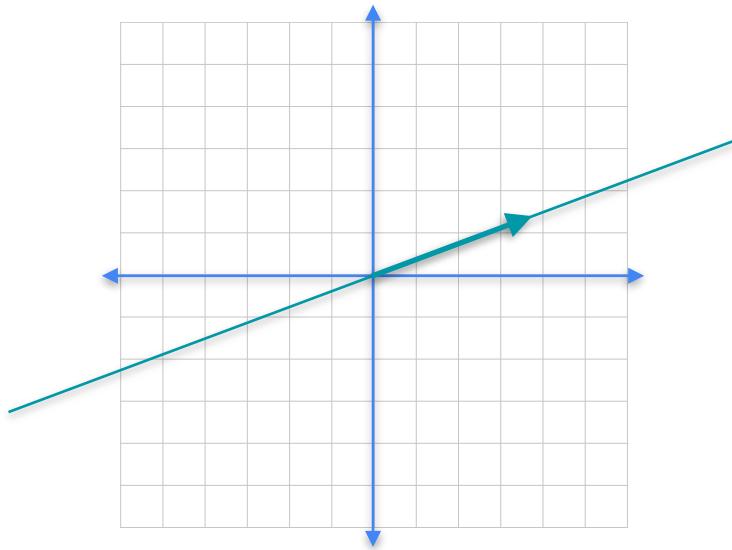
Number of elements in the basis is the dimension



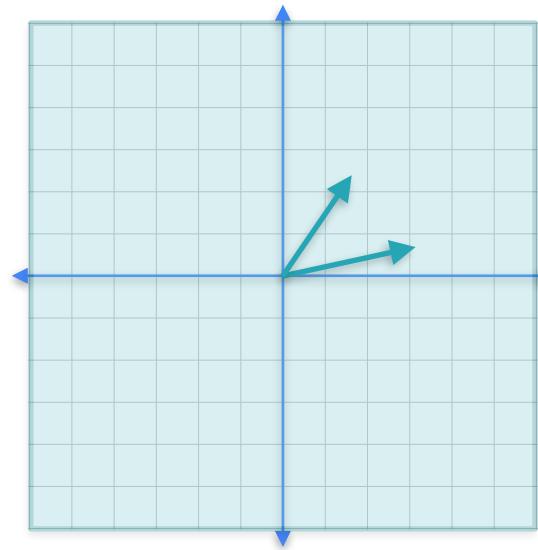
1 element
Dimension = 1



Number of elements in the basis is the dimension



1 element
Dimension = 1



2 elements in the basis
Dimension = 2

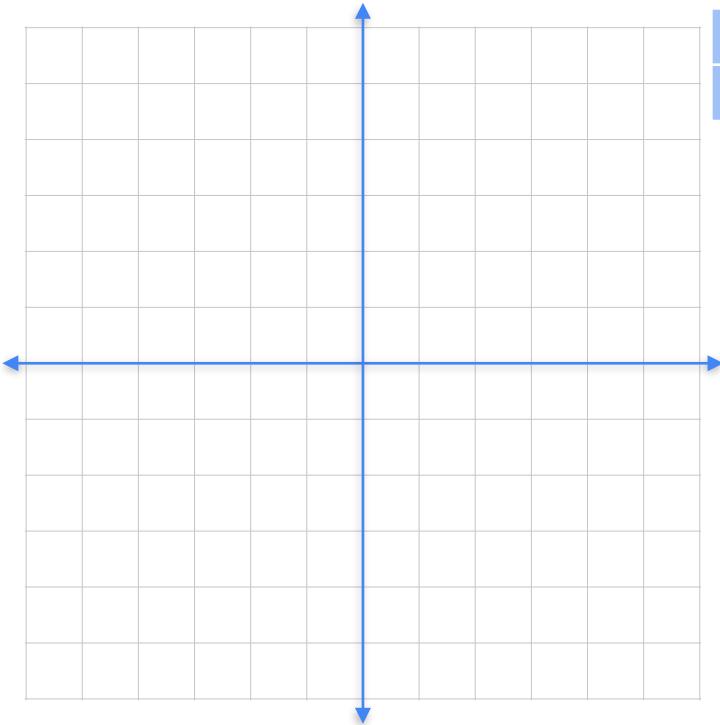


DeepLearning.AI

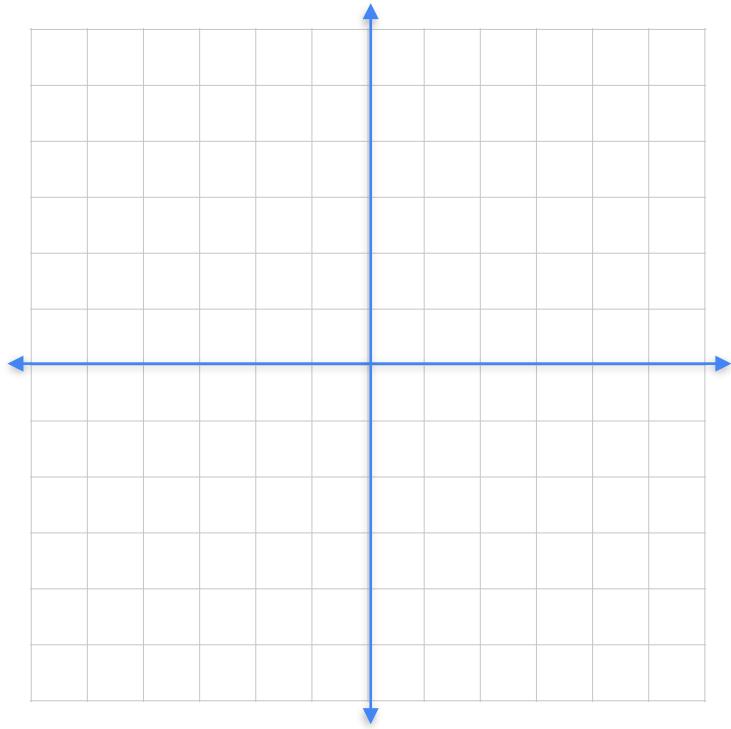
Determinants and Eigenvectors

Eigenbasis

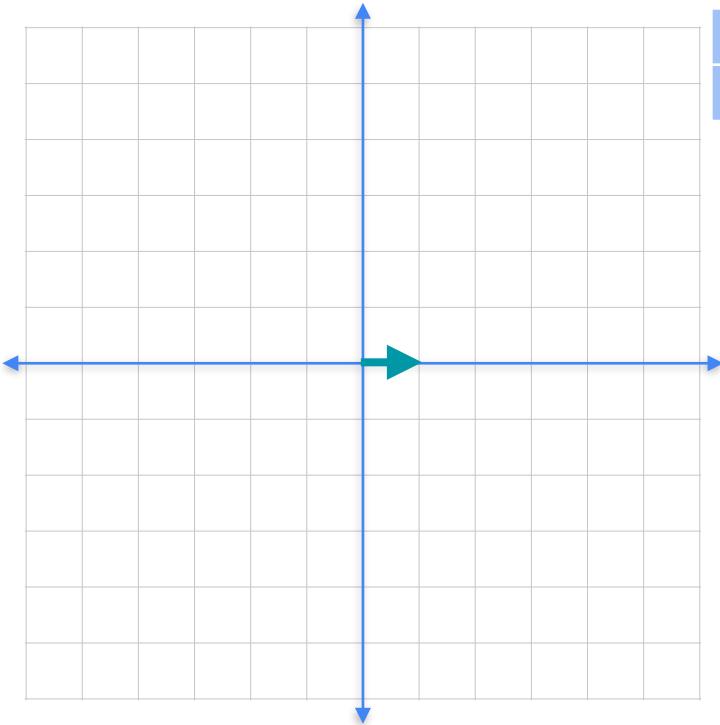
Basis



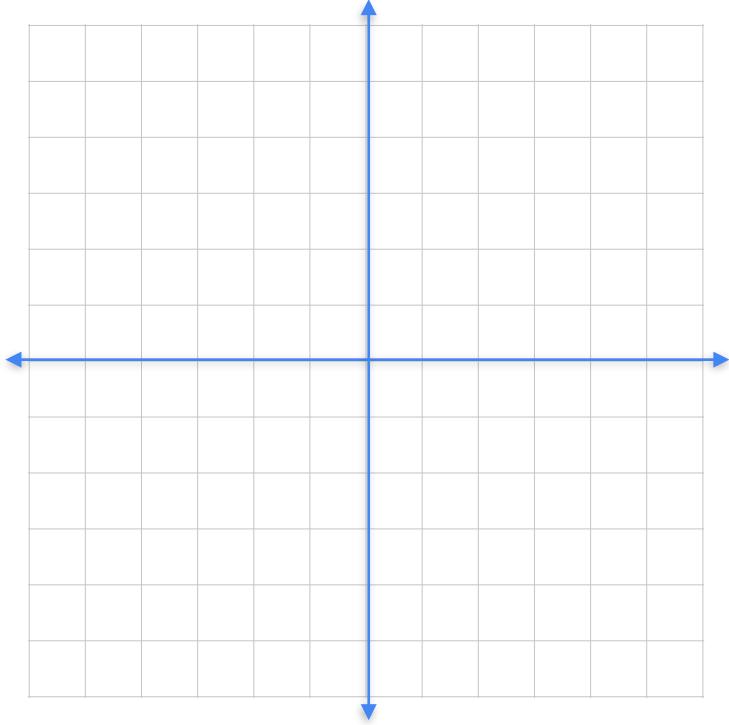
2	1
0	3



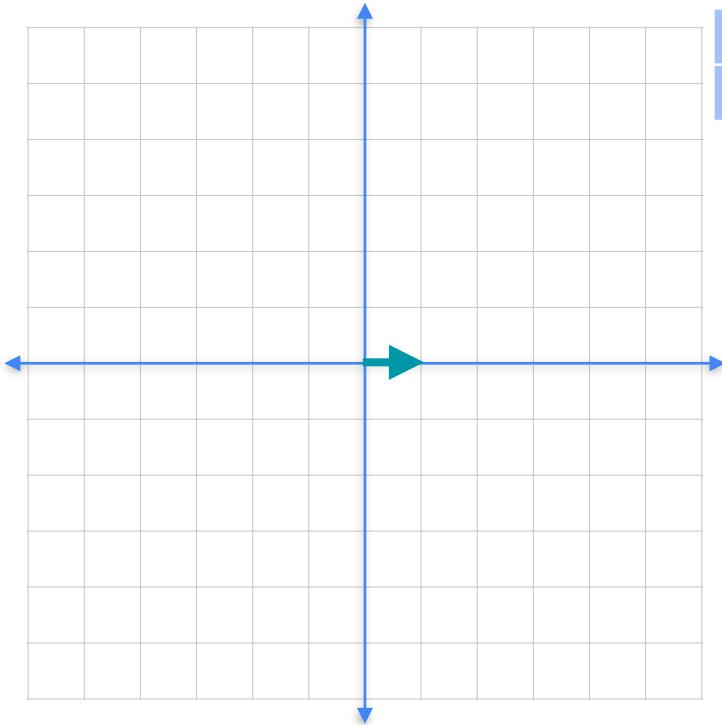
Basis



2	1
0	3

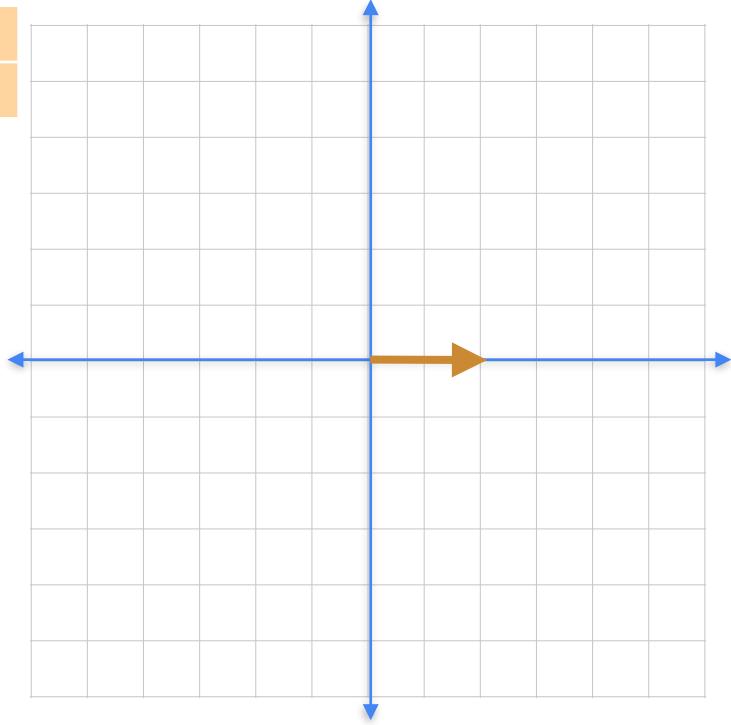


Basis

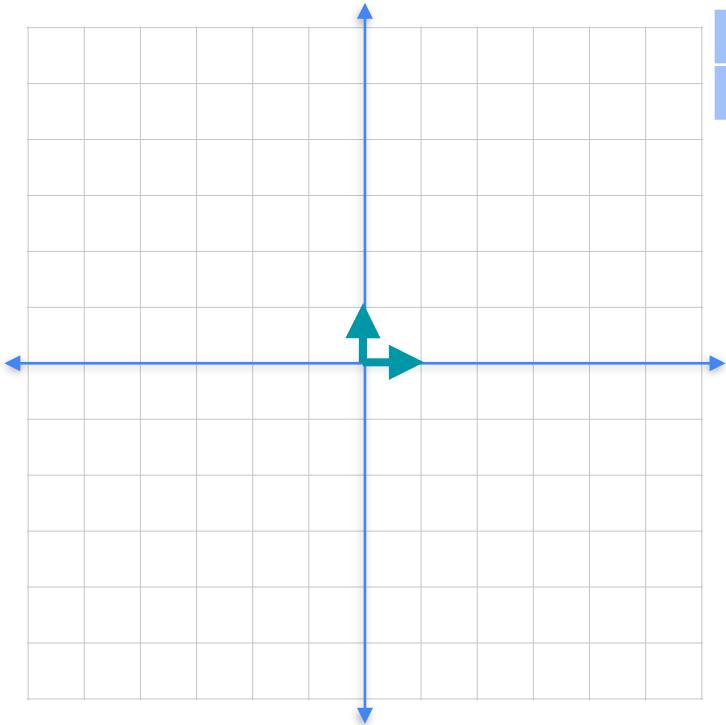


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 0 \end{matrix} = \begin{matrix} 2 \\ 0 \end{matrix}$$

$$(1,0) \rightarrow (2,0)$$

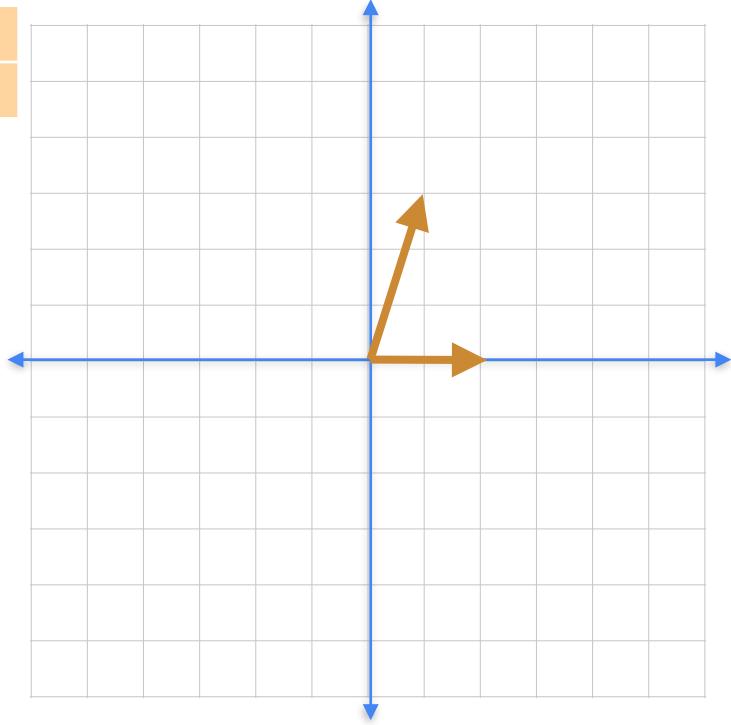


Basis

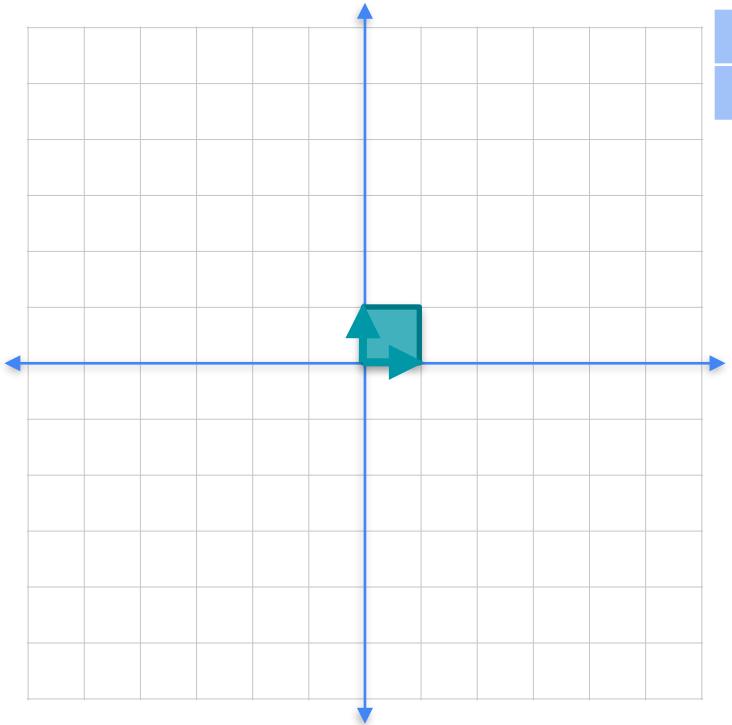


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

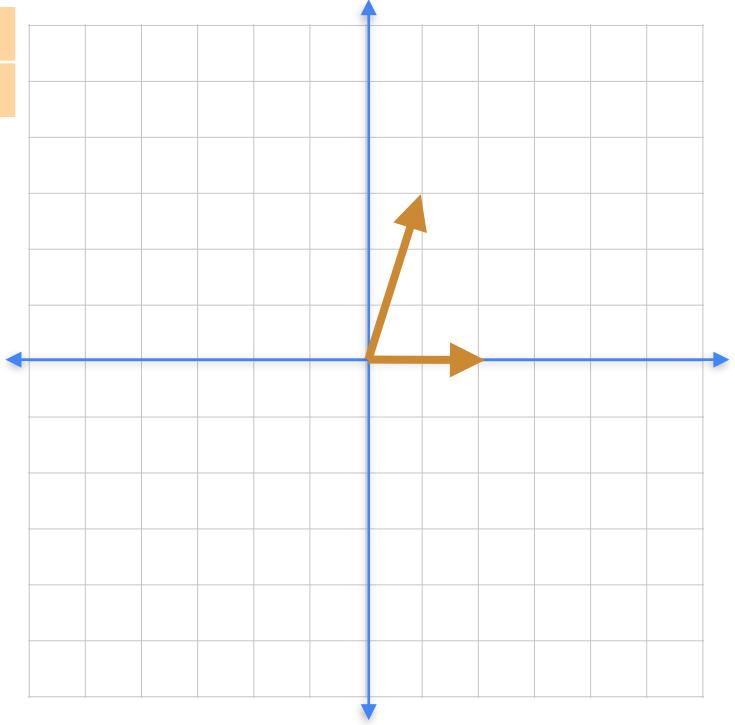


Basis

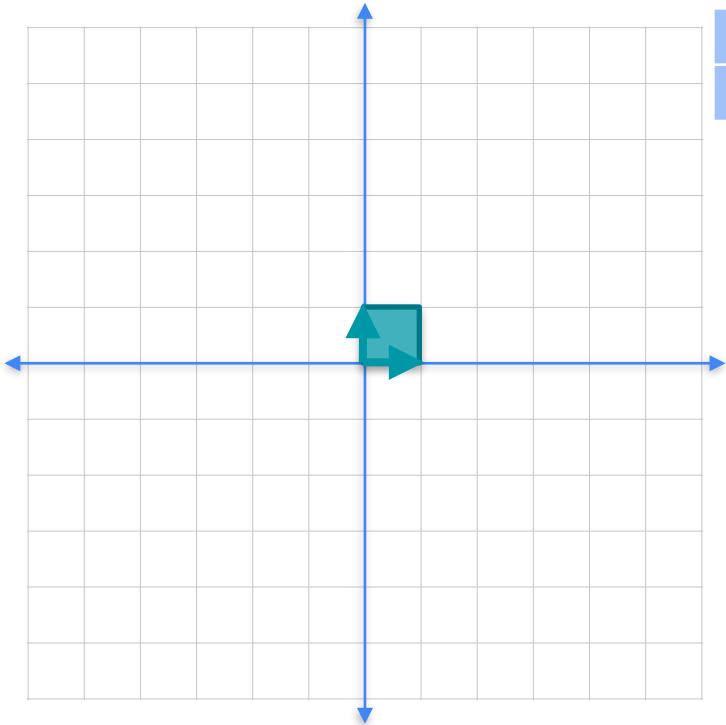


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

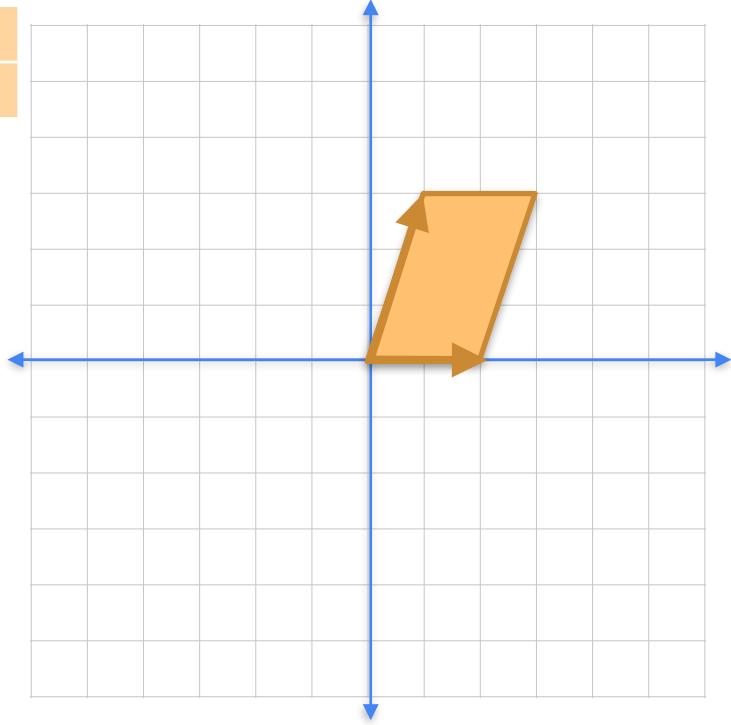


Basis

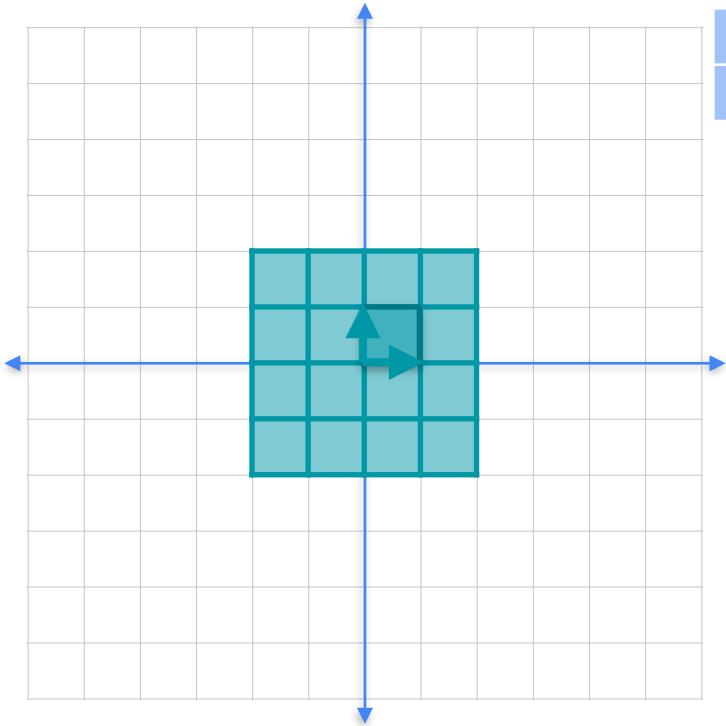


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

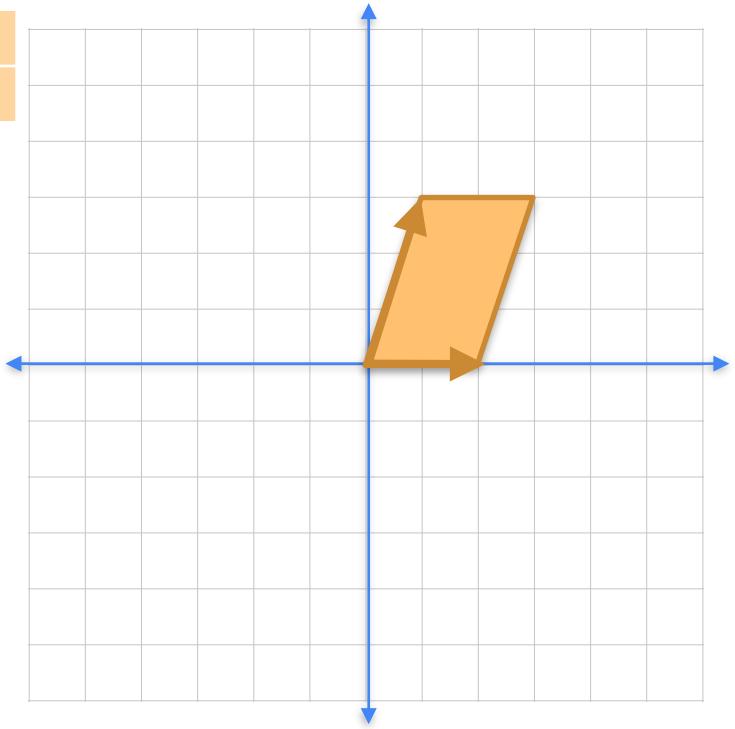


Basis

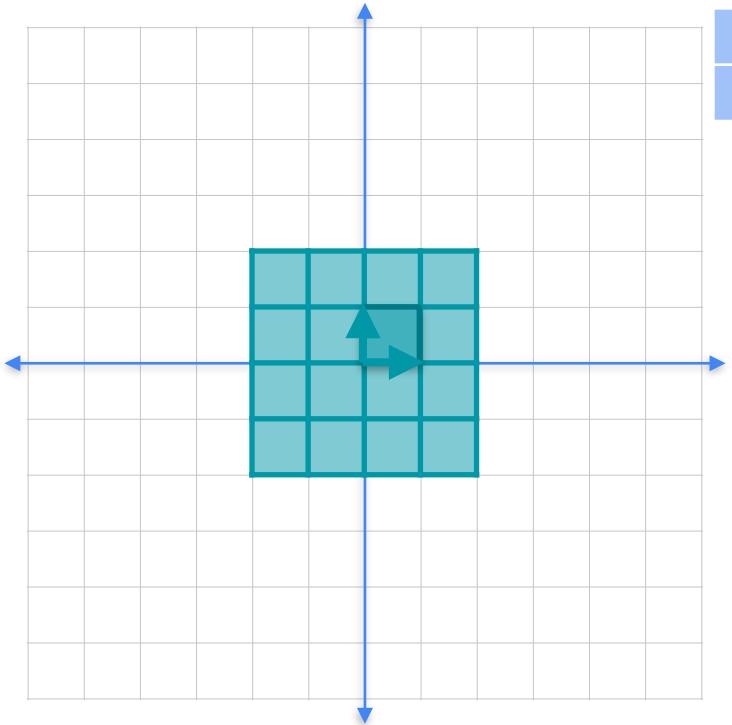


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$

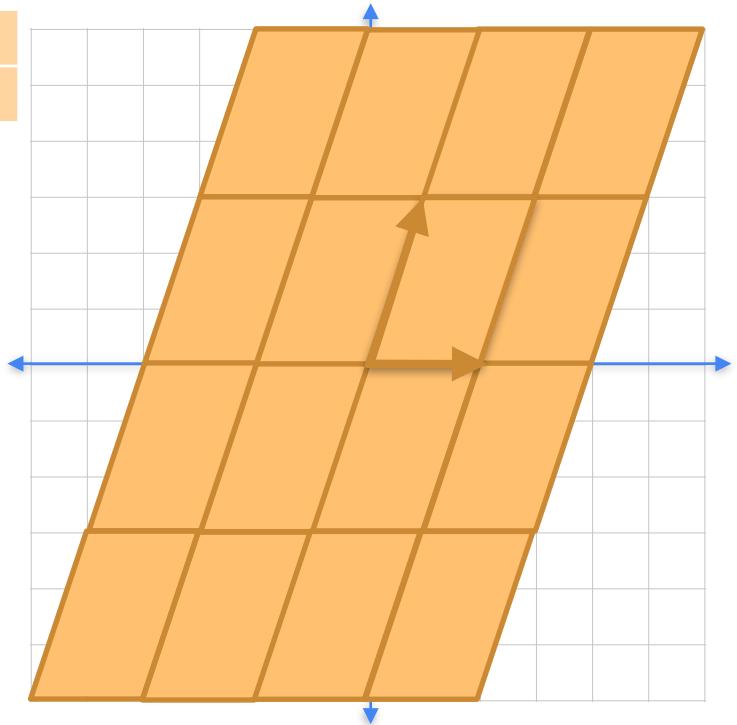


Basis

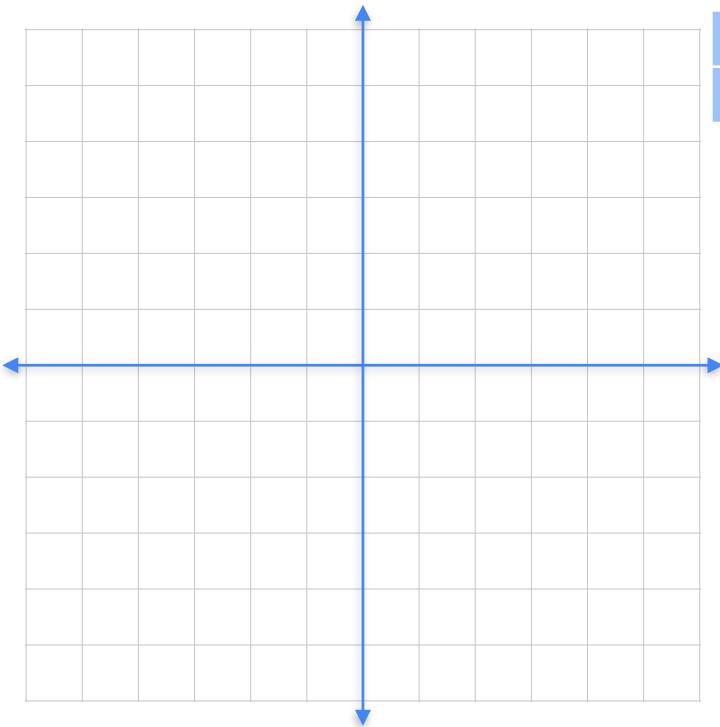


$$\begin{matrix} 2 & 1 & 0 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 1 \\ 3 \end{matrix}$$

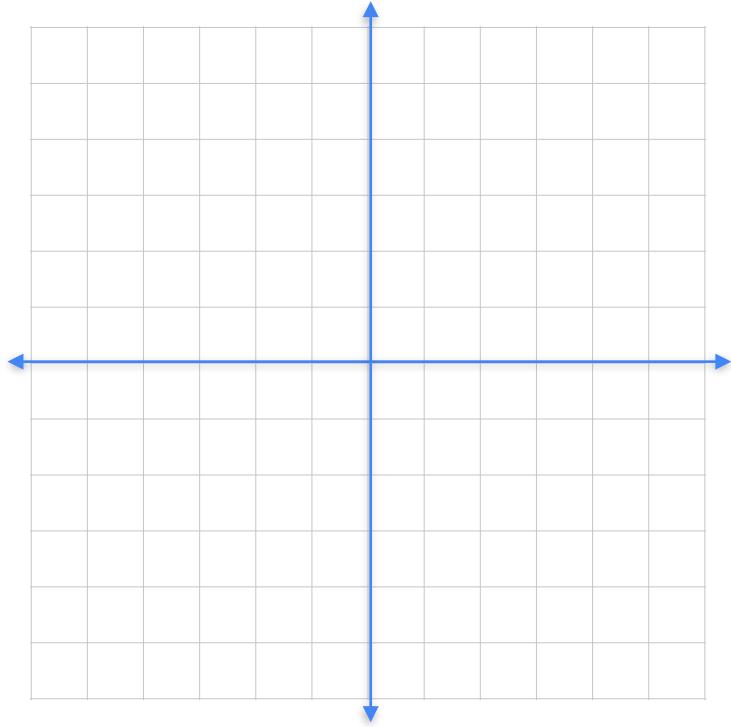
$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (0,1) &\rightarrow (1,3) \end{aligned}$$



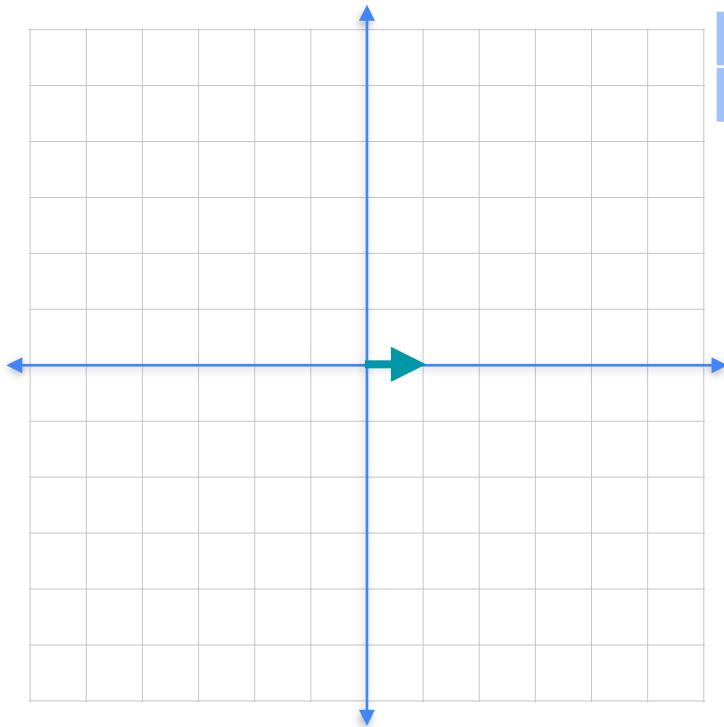
Eigenbasis



2	1
0	3

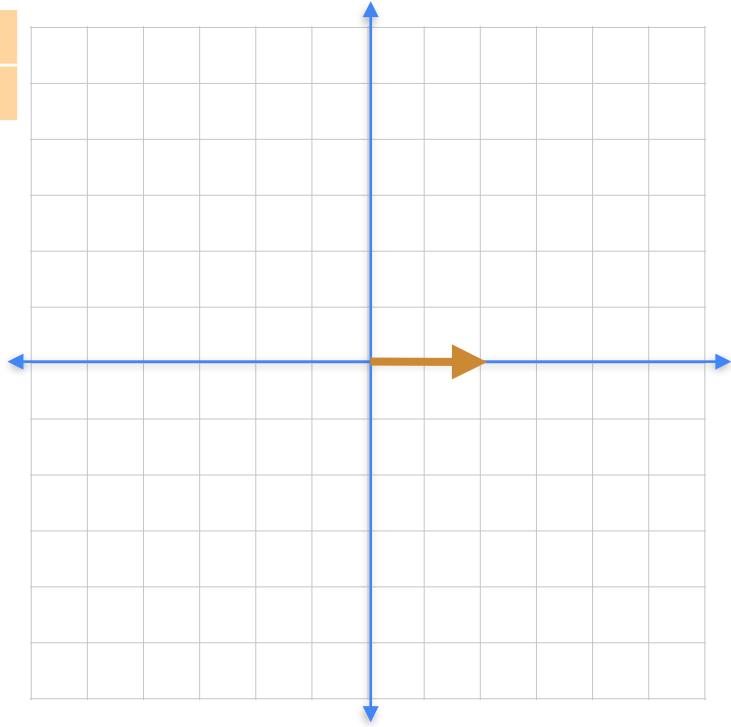


Eigenbasis

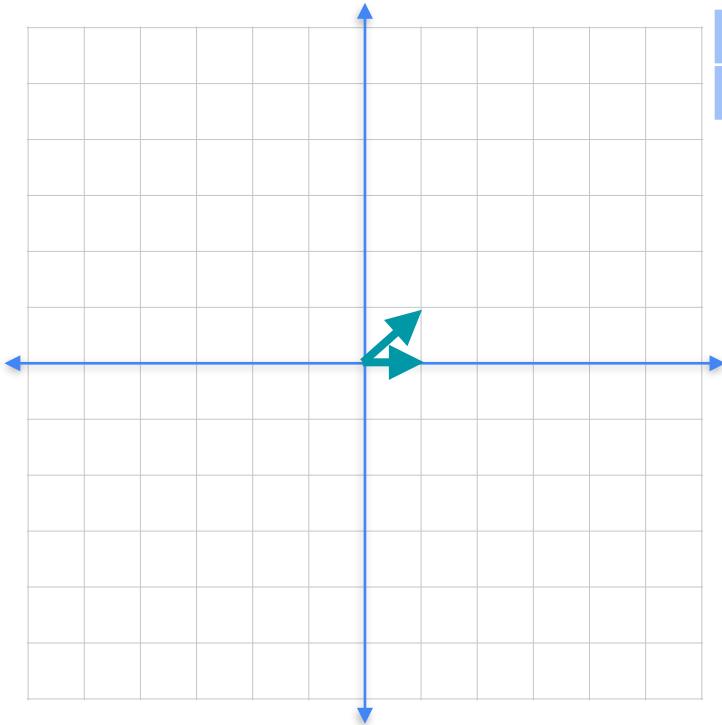


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 0 \end{matrix} = \begin{matrix} 2 \\ 0 \end{matrix}$$

$(1,0) \rightarrow (2,0)$

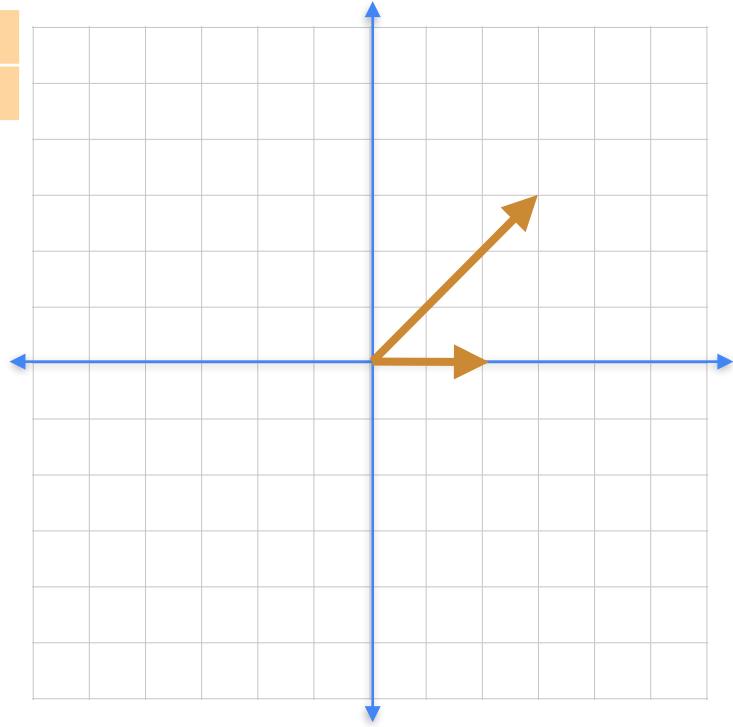


Eigenbasis

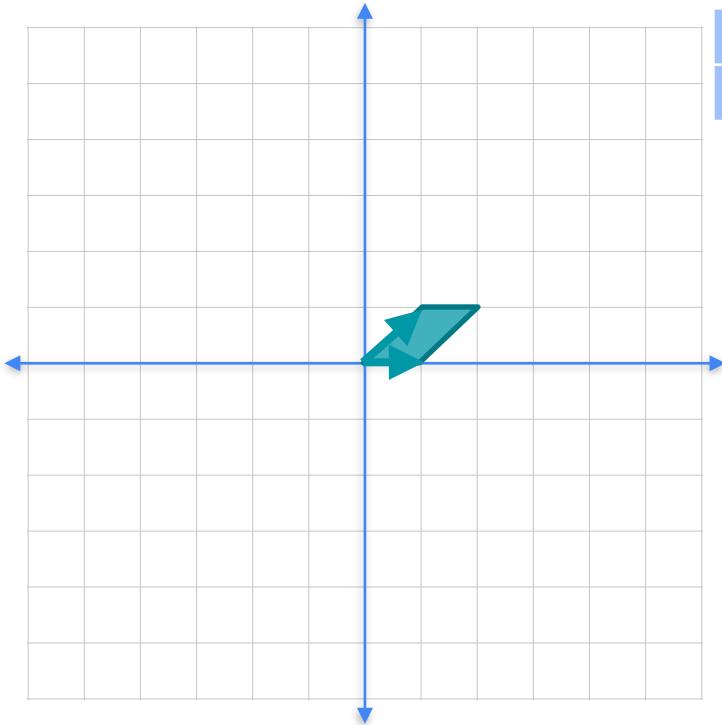


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

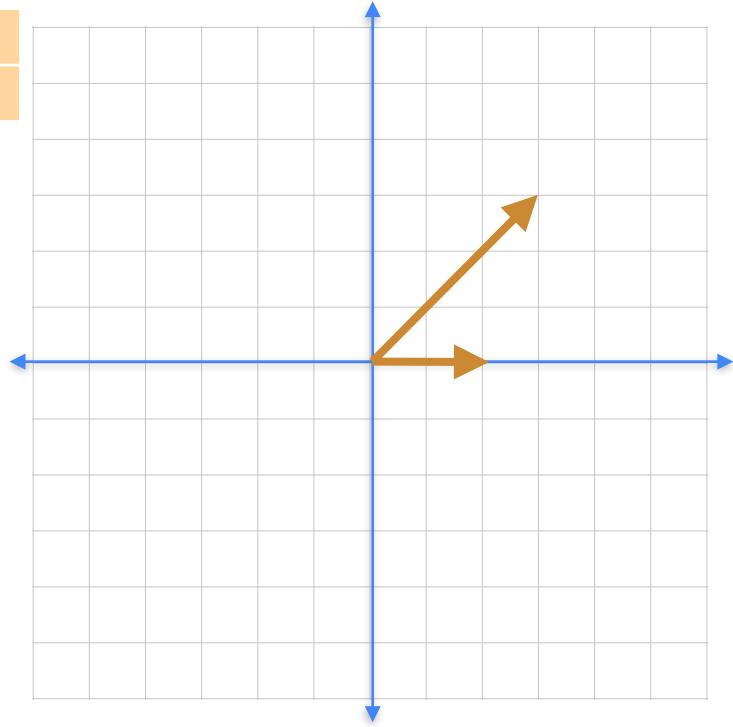


Eigenbasis

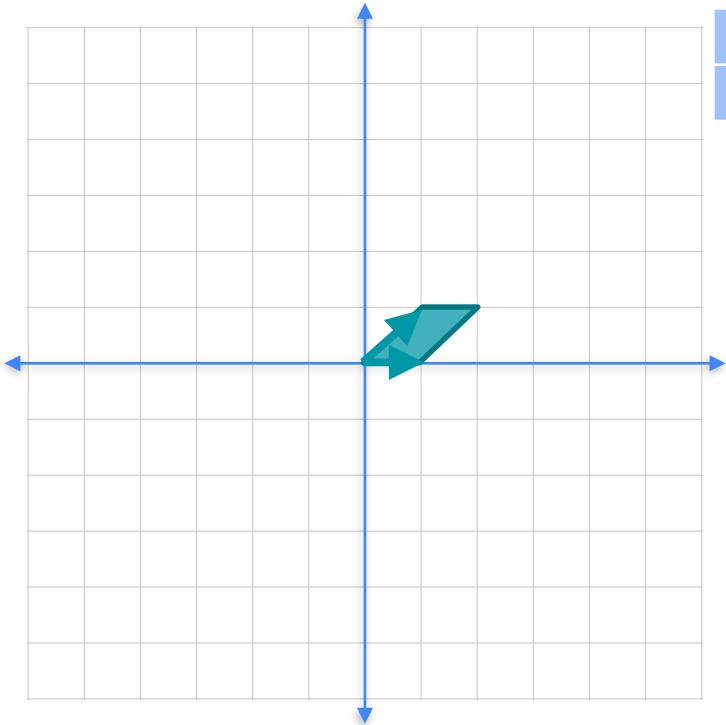


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

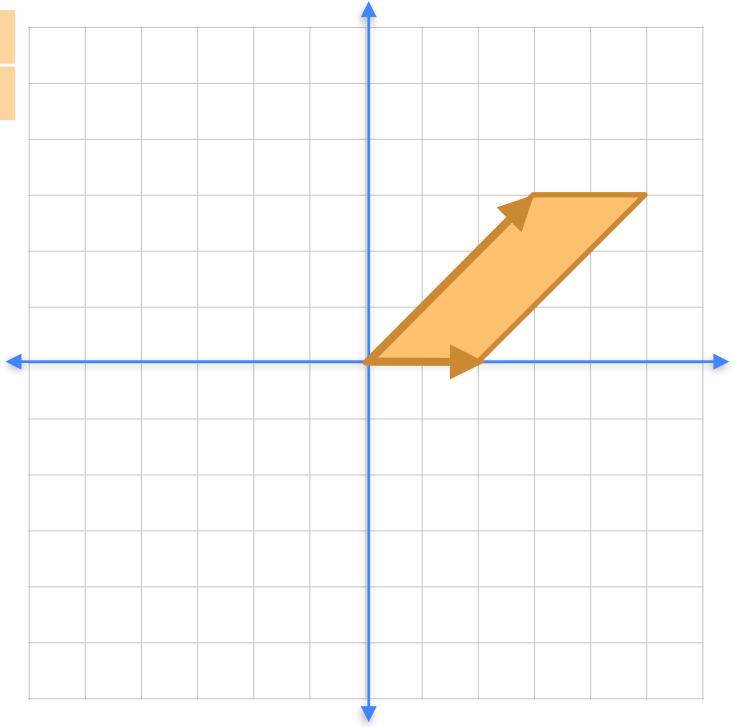


Eigenbasis

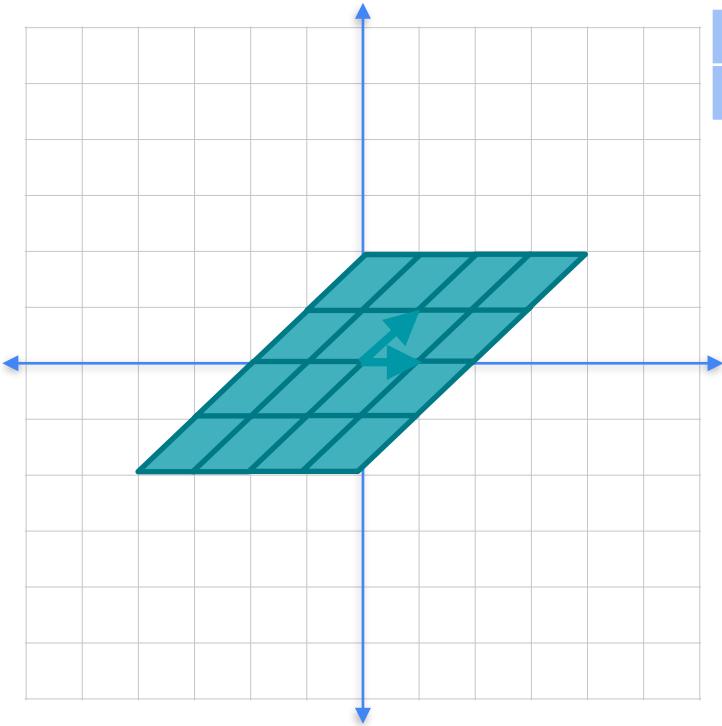


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned}(1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3)\end{aligned}$$

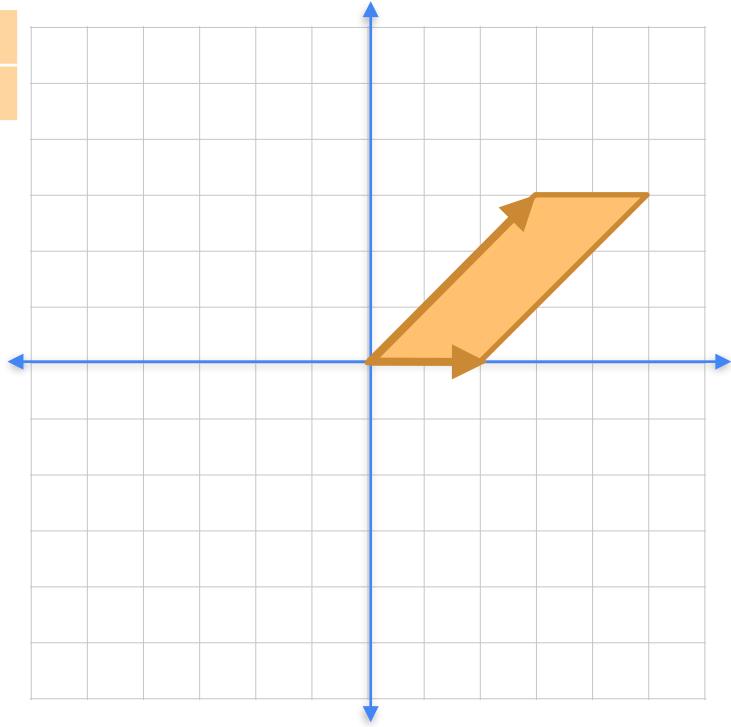


Eigenbasis

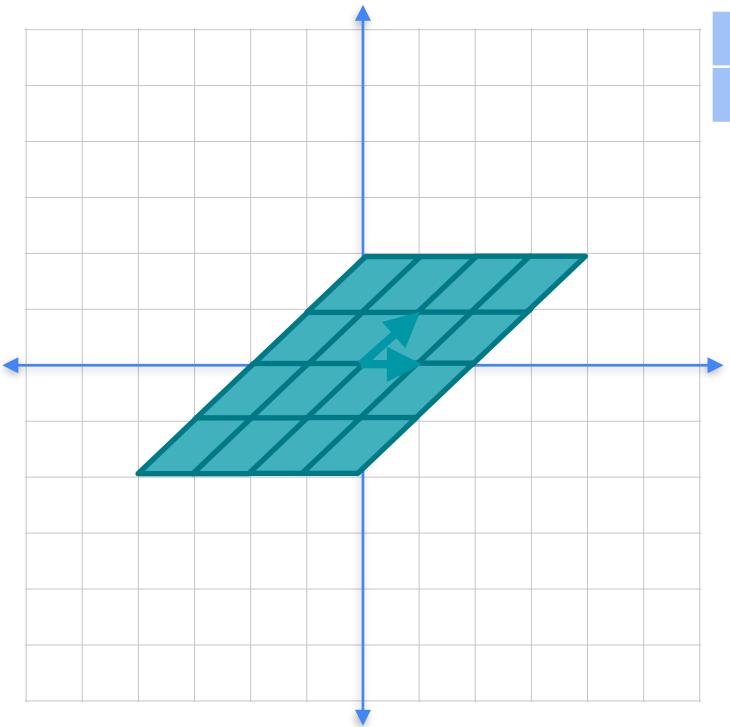


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

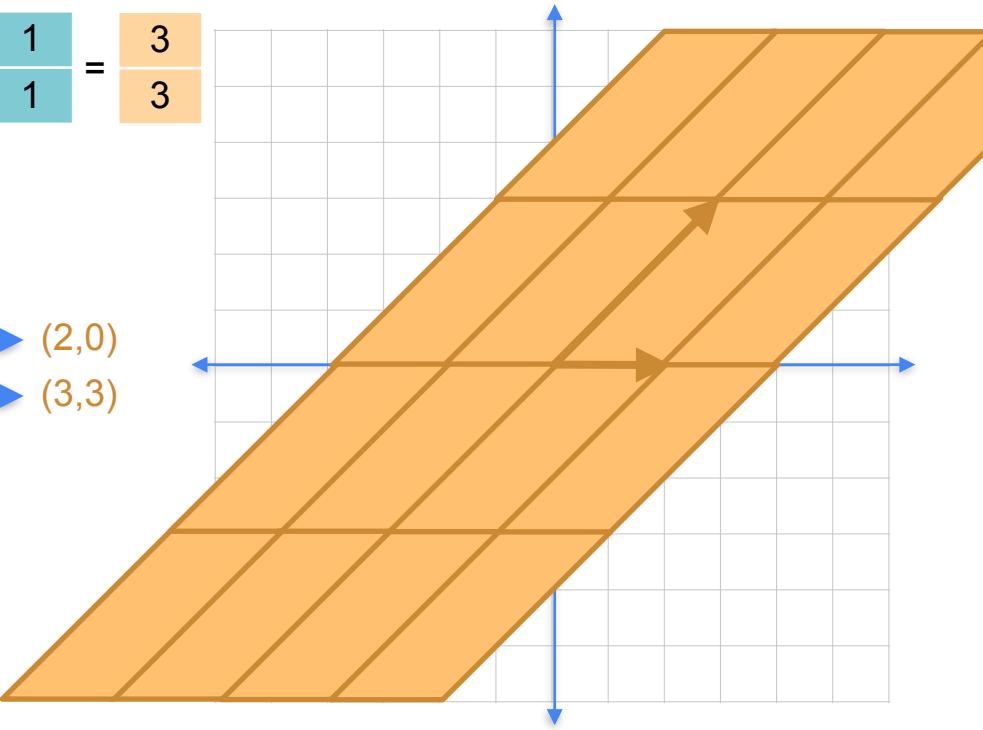


Eigenbasis

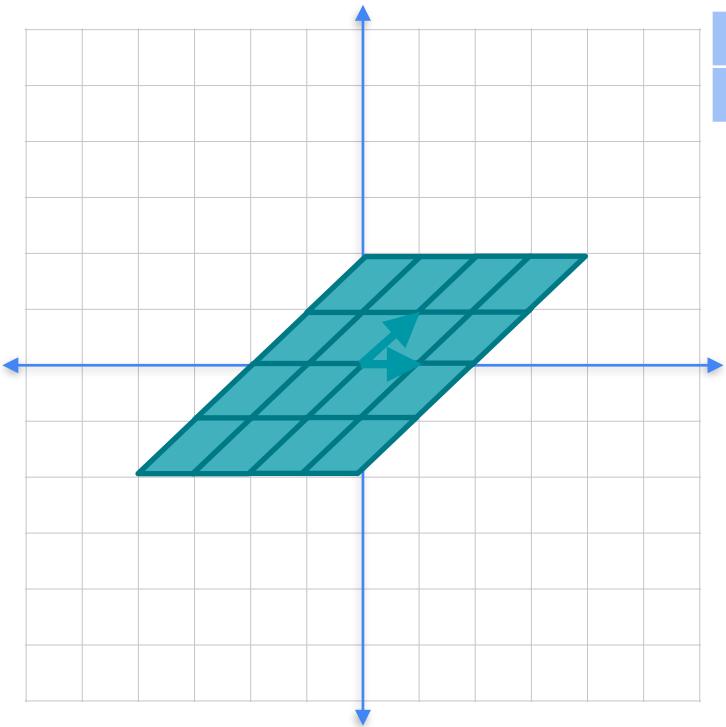


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$

$$\begin{aligned} (1,0) &\rightarrow (2,0) \\ (1,1) &\rightarrow (3,3) \end{aligned}$$

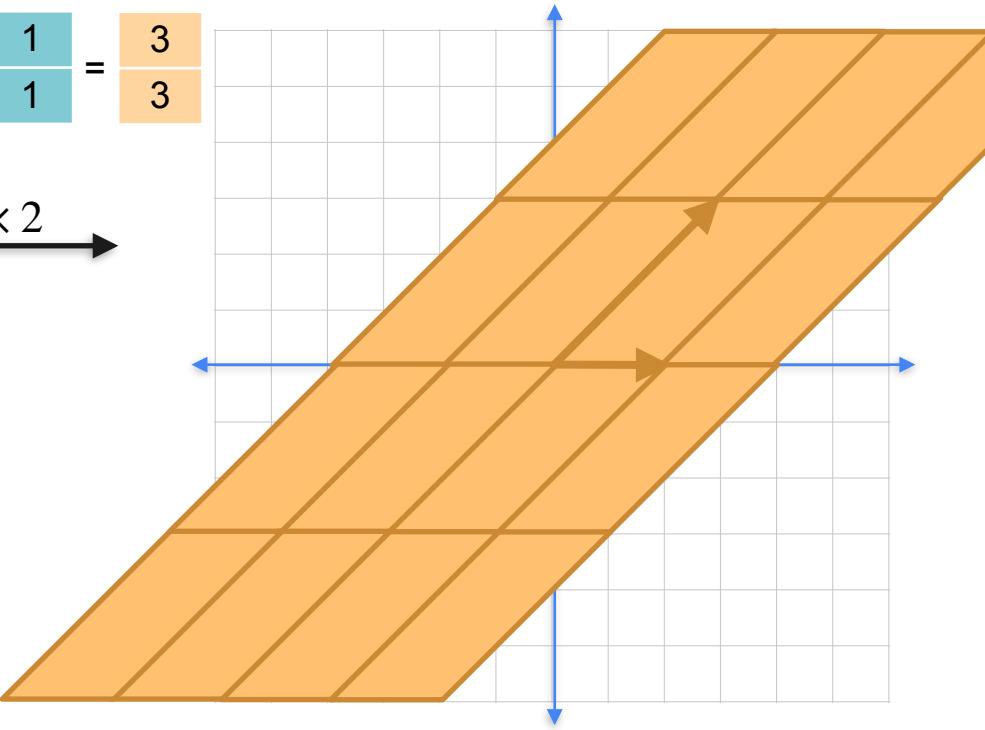


Eigenbasis

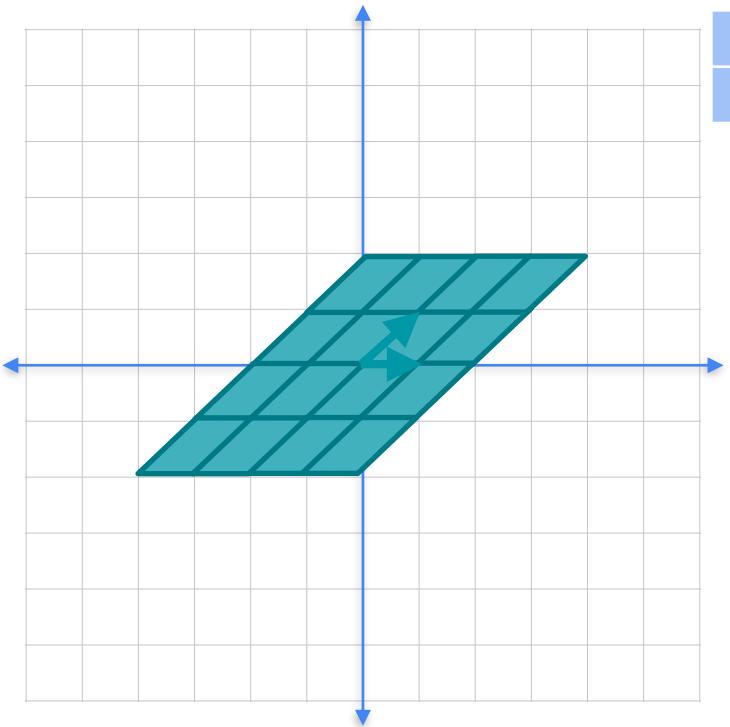


$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 & \\ 3 & \end{matrix}$$

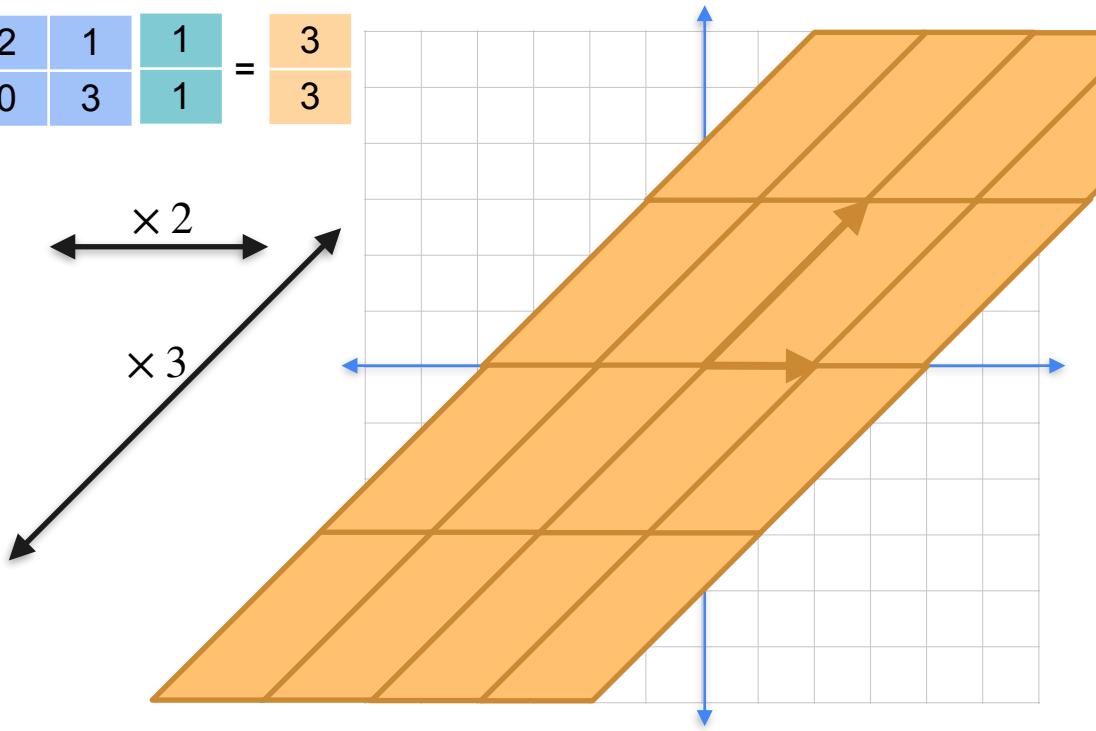
$\times 2$



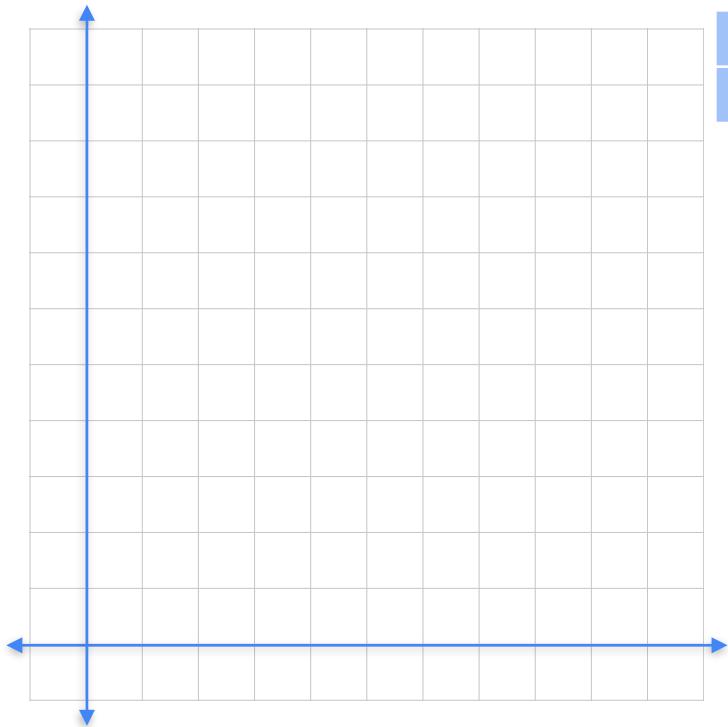
Eigenbasis



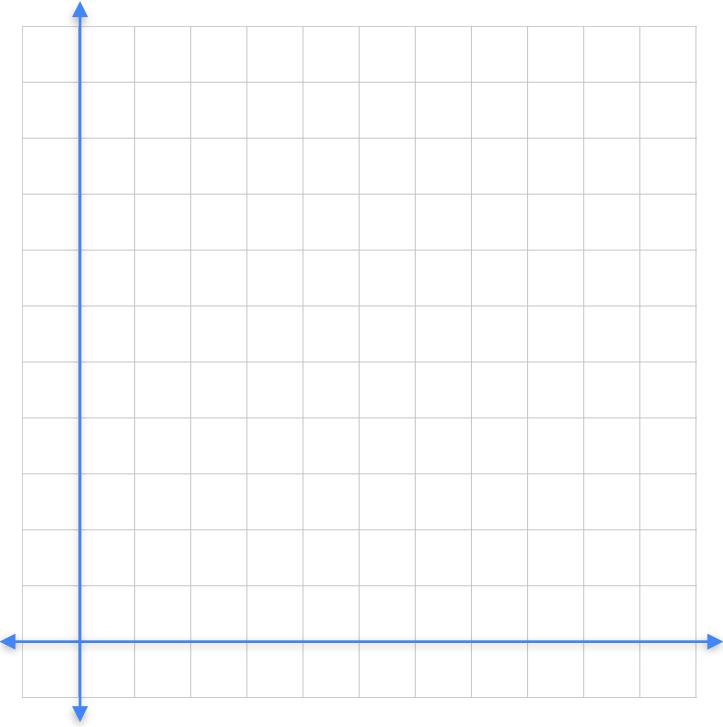
$$\begin{matrix} 2 & 1 & 1 \\ 0 & 3 & 1 \end{matrix} = \begin{matrix} 3 \\ 3 \end{matrix}$$



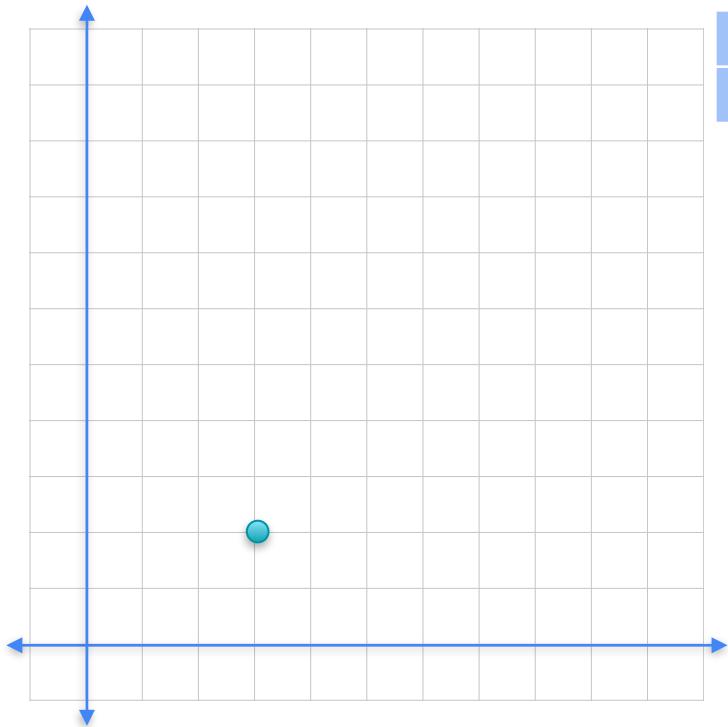
Eigenbasis



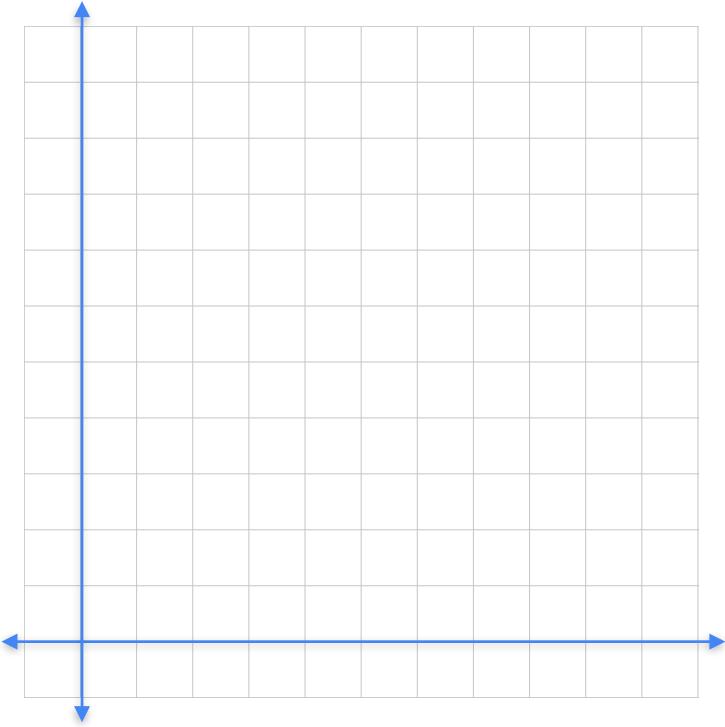
2	1
0	3



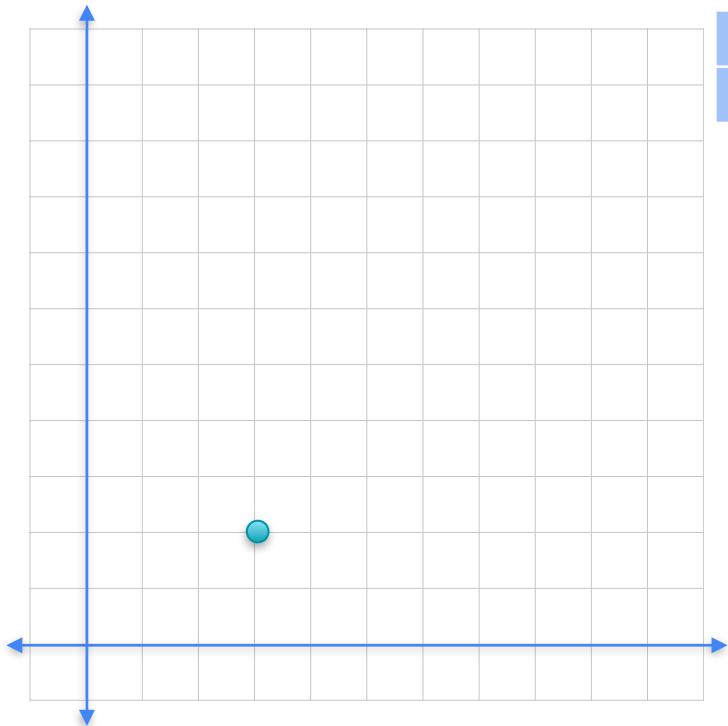
Eigenbasis



2	1
0	3

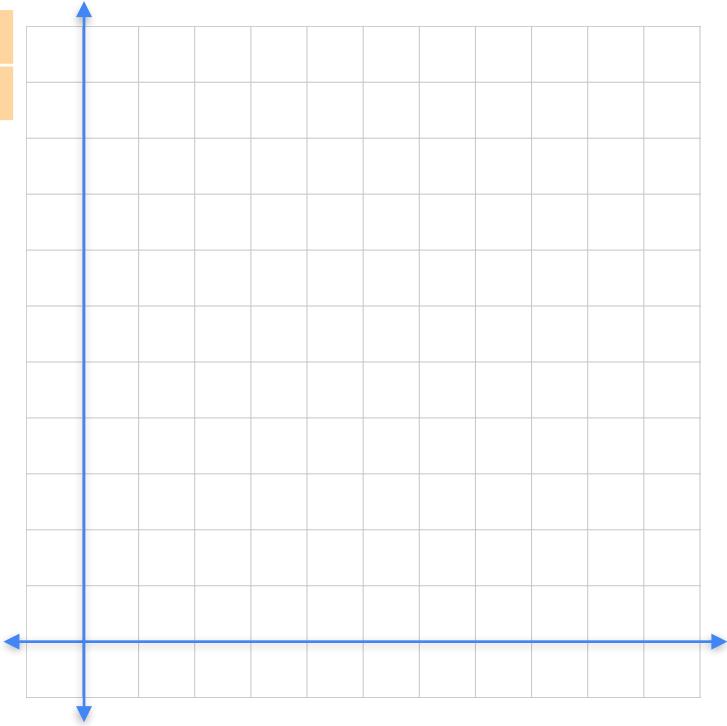


Eigenbasis

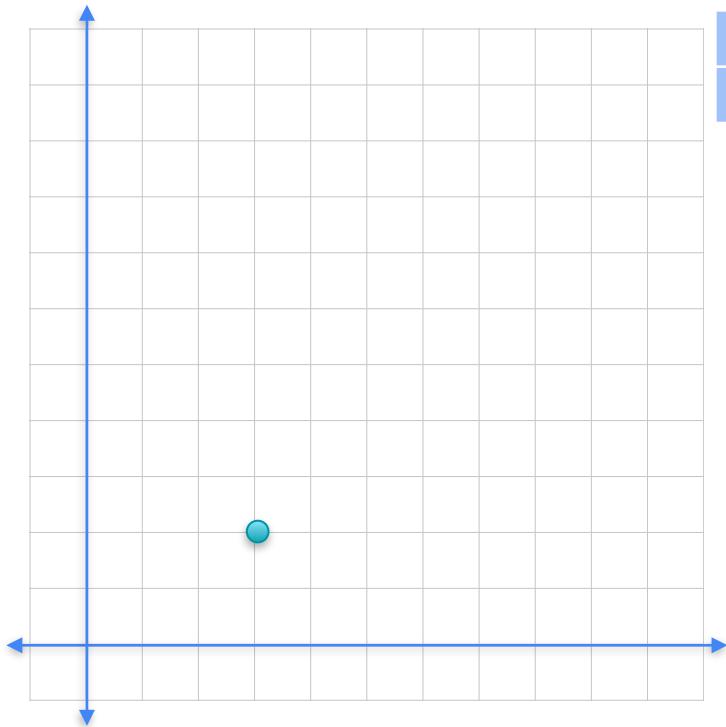


$$\begin{matrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{matrix} = \begin{matrix} 8 \\ 6 \end{matrix}$$

$(3,2) \rightarrow (8,6)$

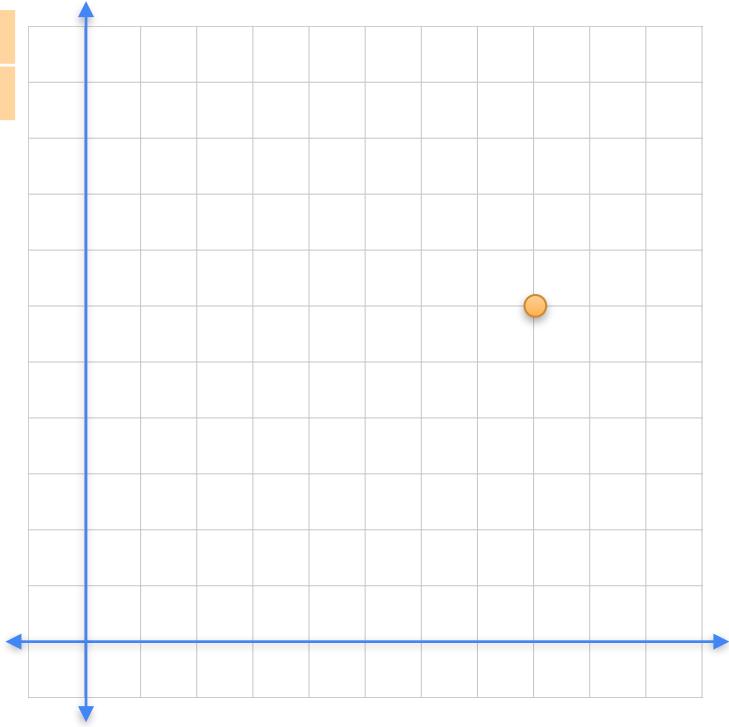


Eigenbasis

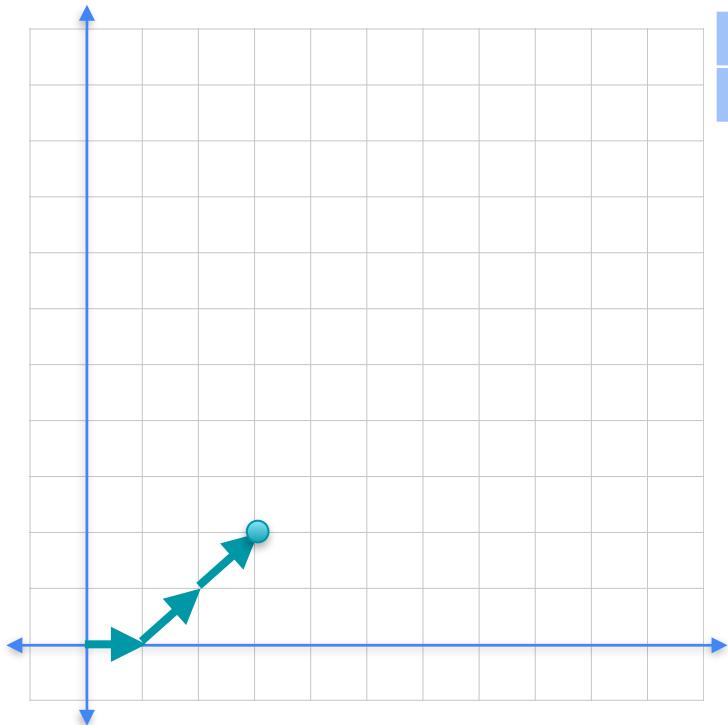


$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

$(3,2) \rightarrow (8,6)$

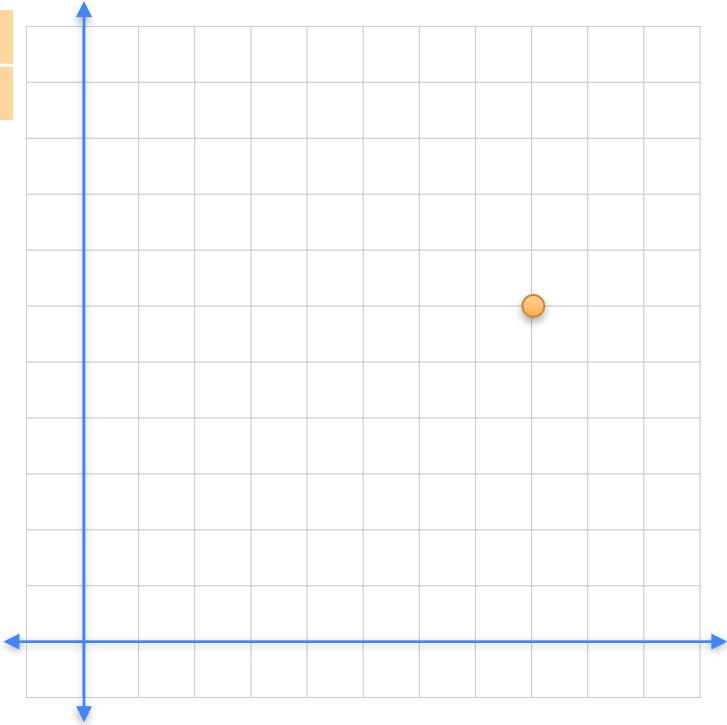


Eigenbasis

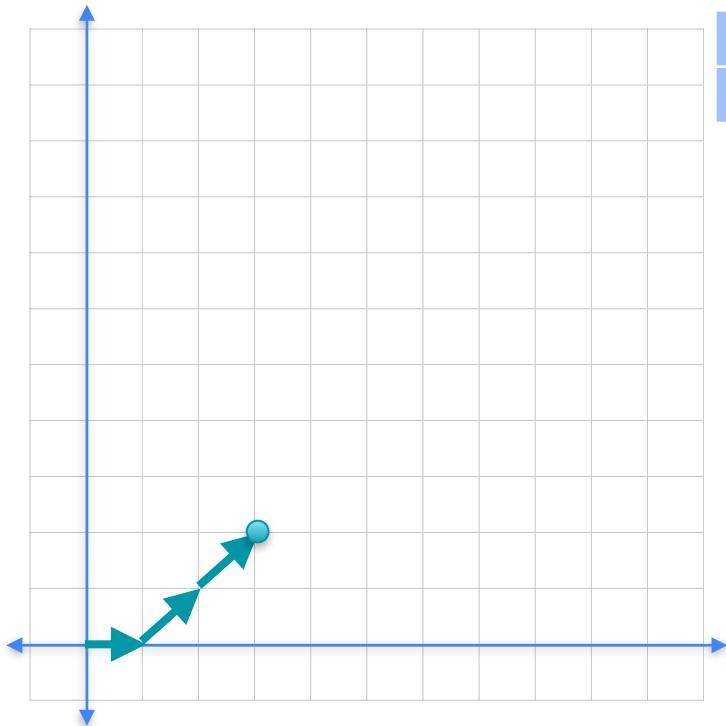


$$\begin{matrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{matrix} = \begin{matrix} 8 \\ 6 \end{matrix}$$

$(3,2) \rightarrow (8,6)$

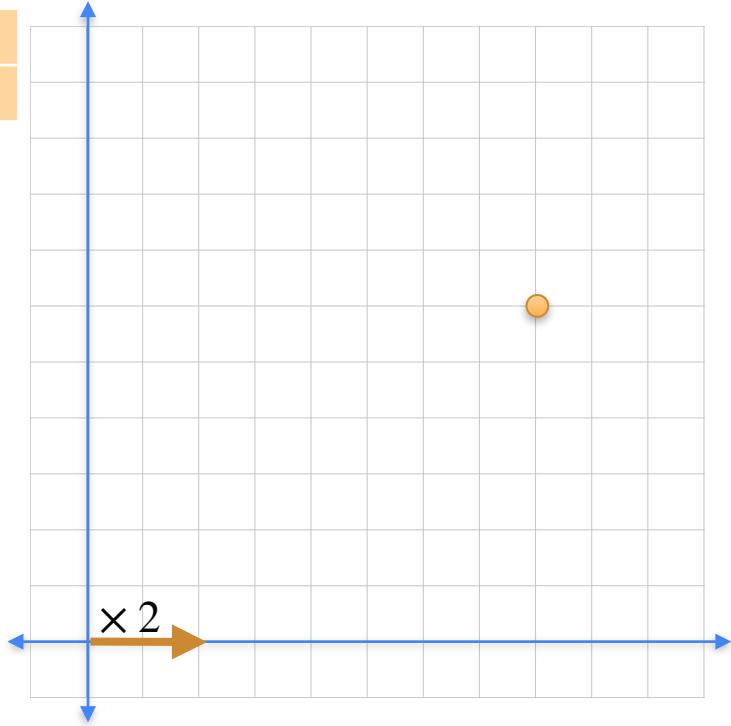


Eigenbasis

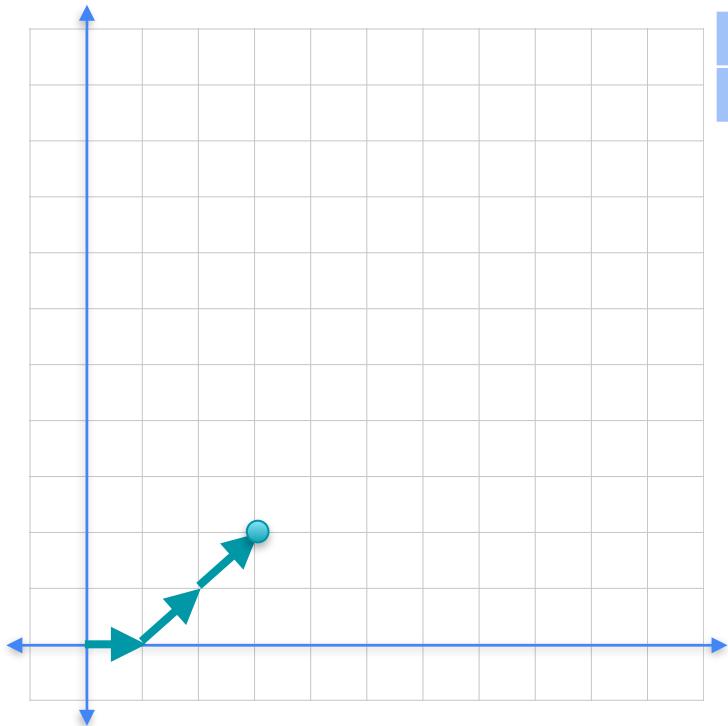


$$\begin{matrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{matrix} = \begin{matrix} 8 \\ 6 \end{matrix}$$

$$(3,2) \rightarrow (8,6)$$

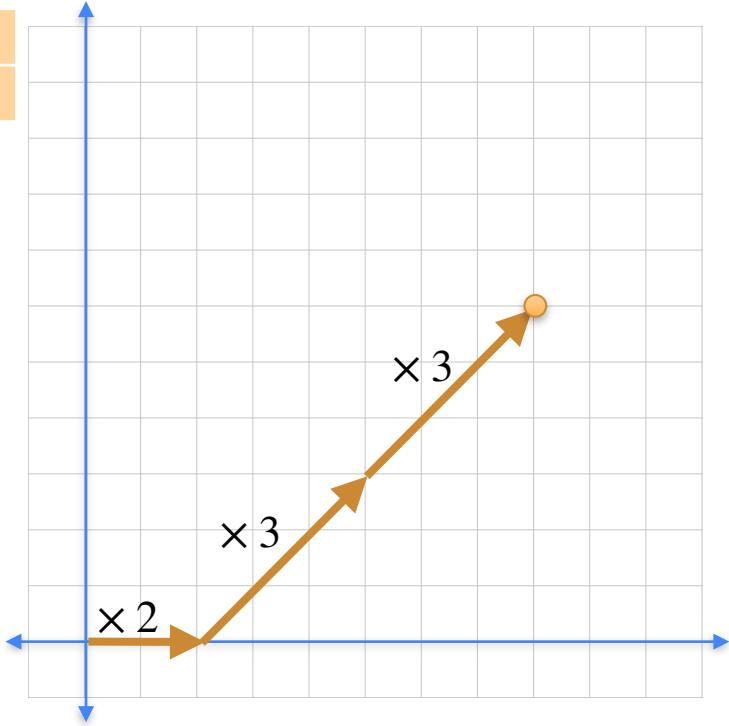


Eigenbasis



$$\begin{matrix} 2 & 1 & 3 \\ 0 & 3 & 2 \end{matrix} = \begin{matrix} 8 \\ 6 \end{matrix}$$

$(3,2) \rightarrow (8,6)$



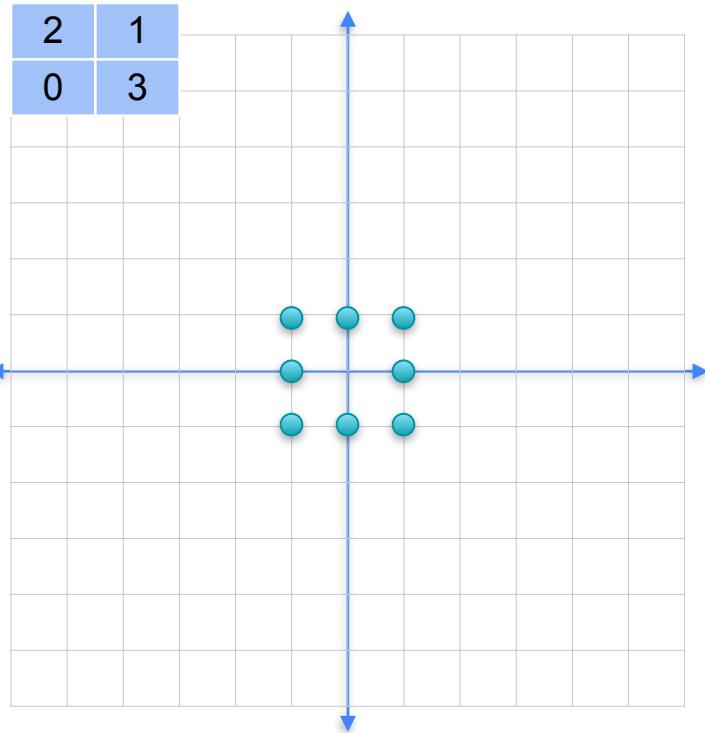


DeepLearning.AI

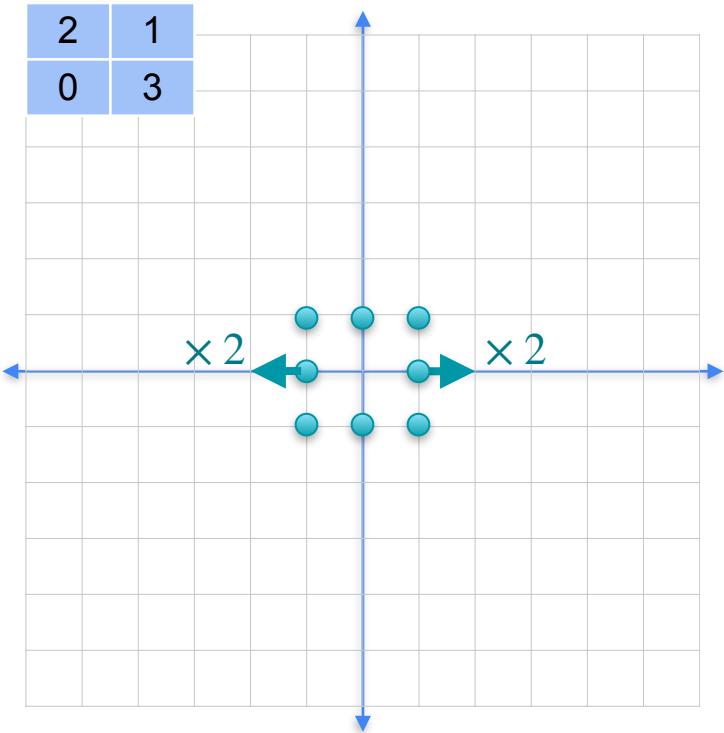
Determinants and Eigenvectors

Eigenvalues and eigenvectors

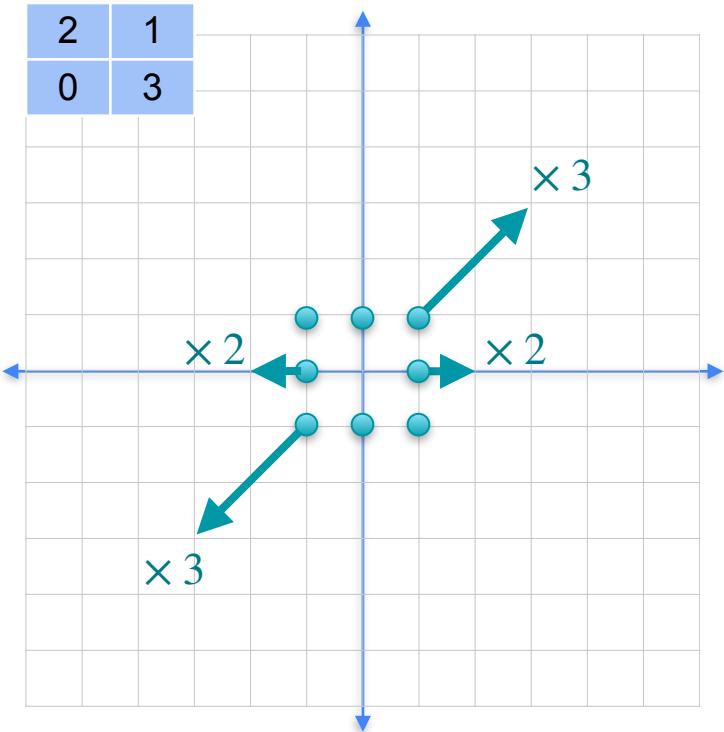
Finding eigenvalues



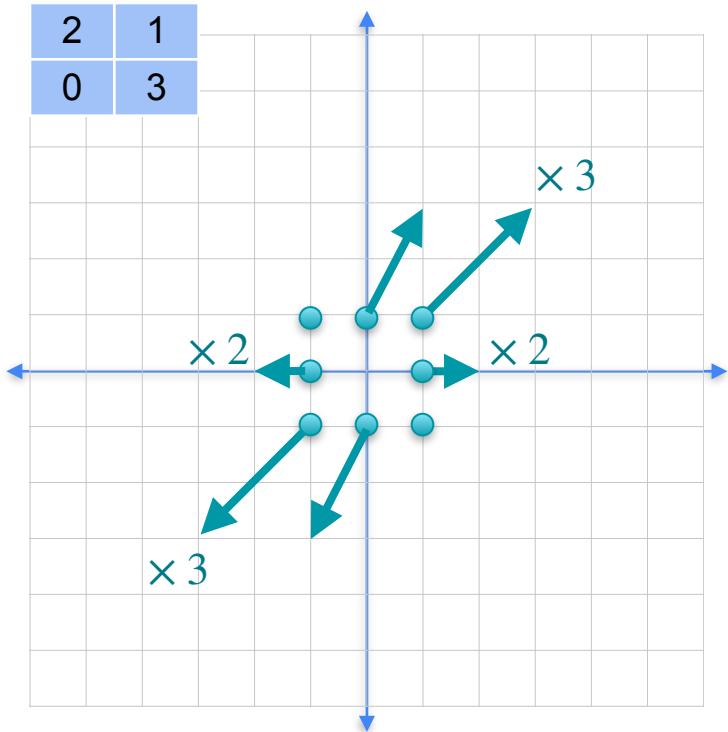
Finding eigenvalues



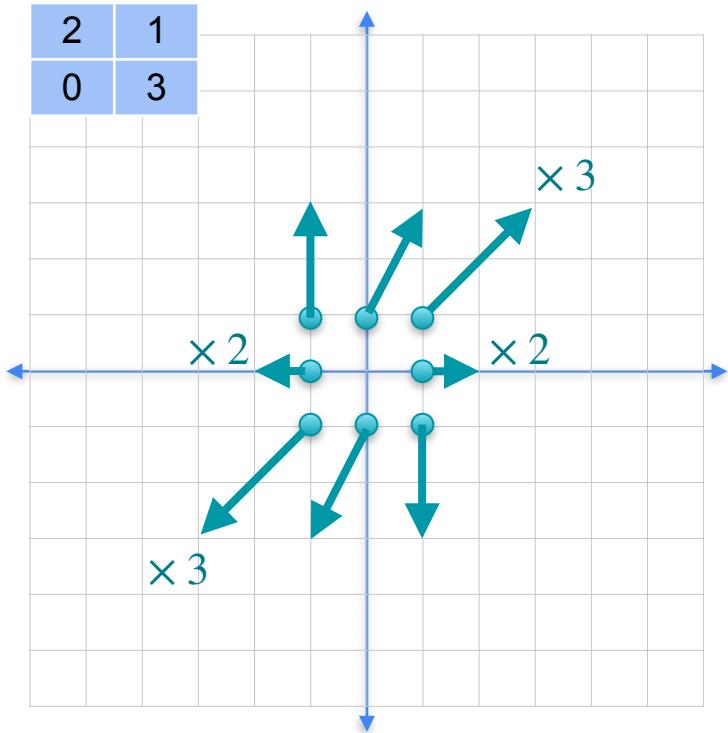
Finding eigenvalues



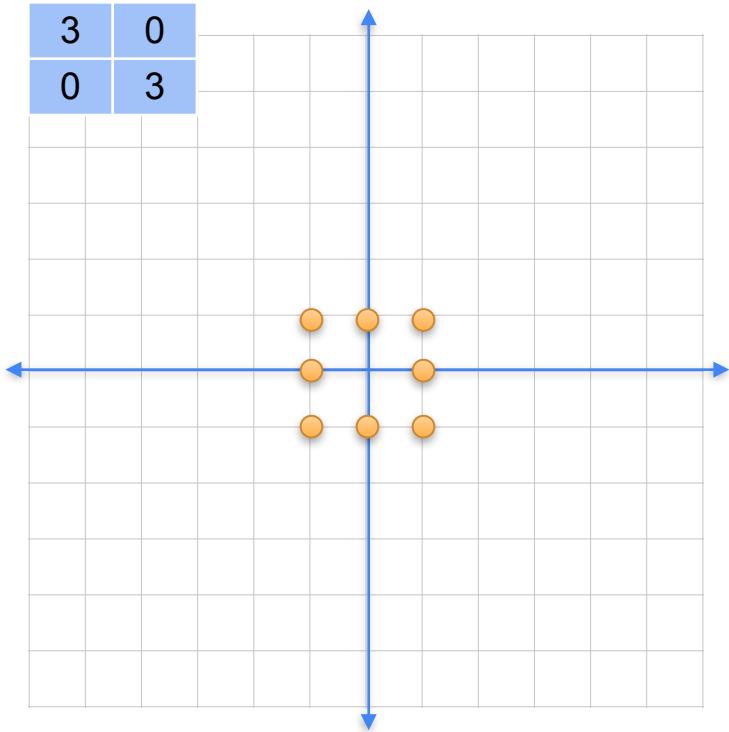
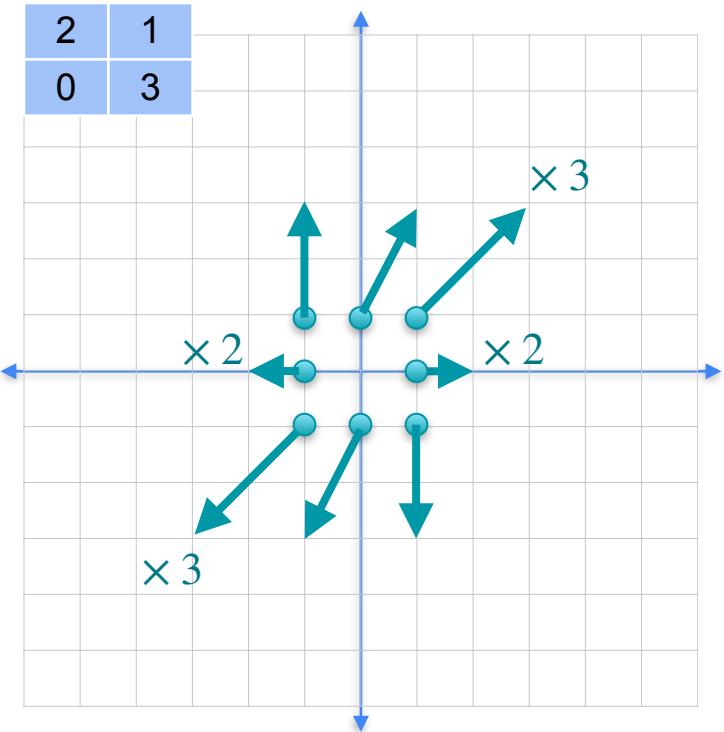
Finding eigenvalues



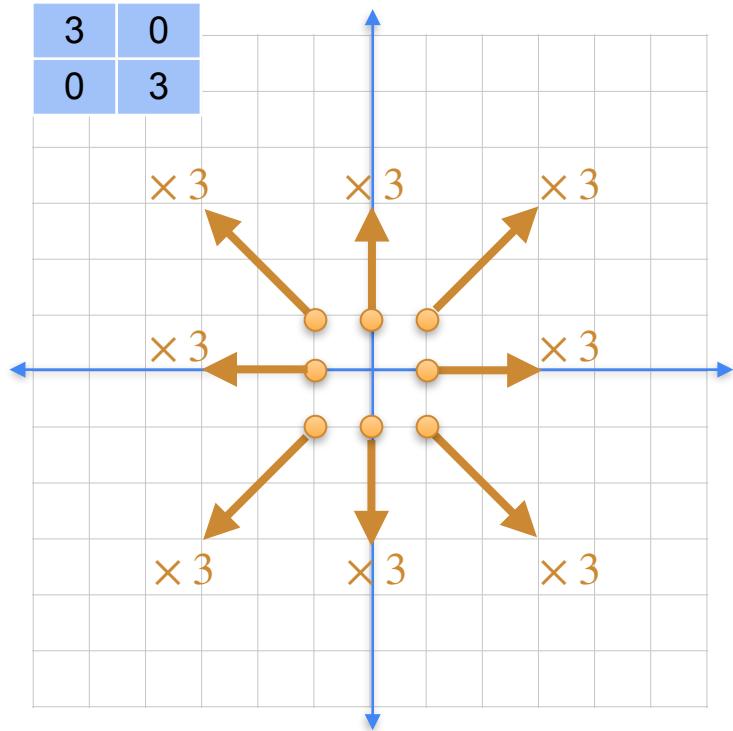
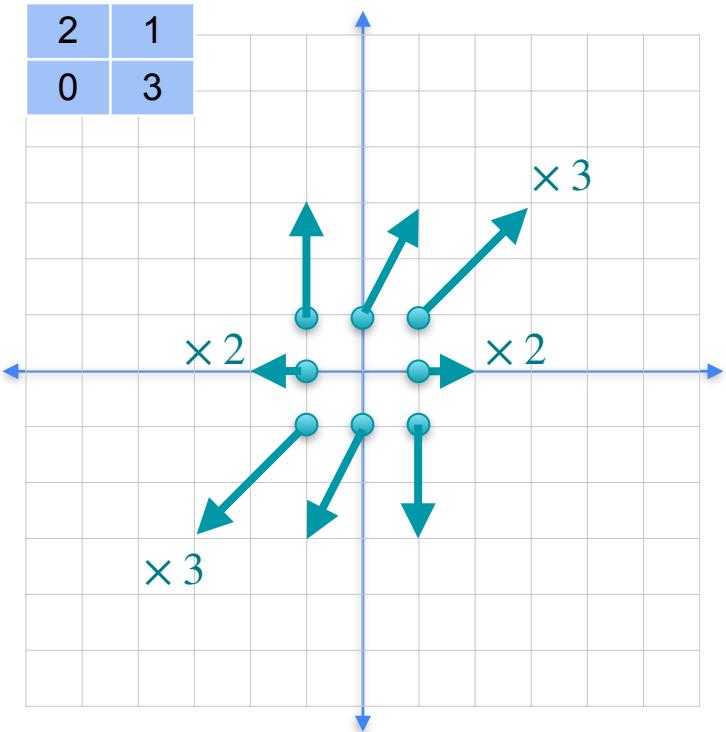
Finding eigenvalues



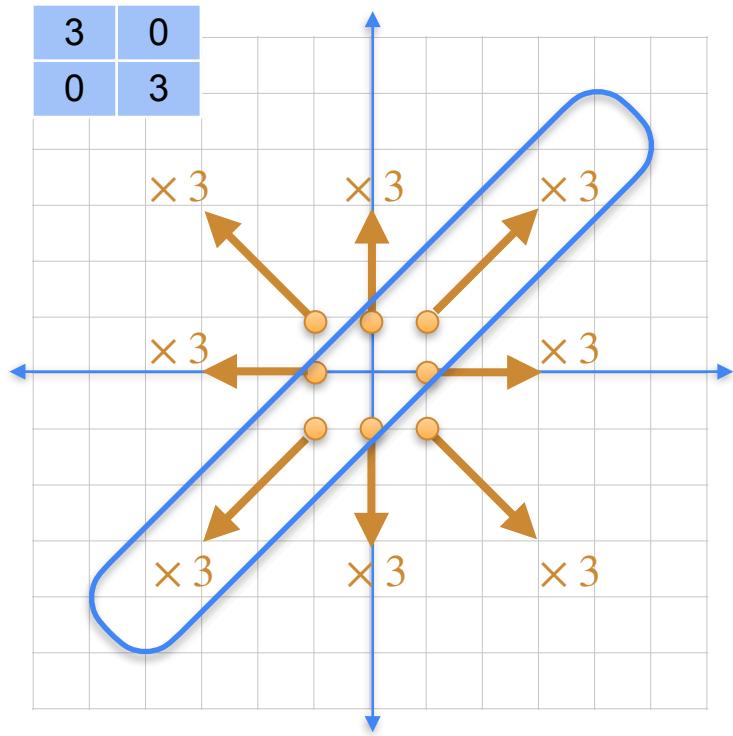
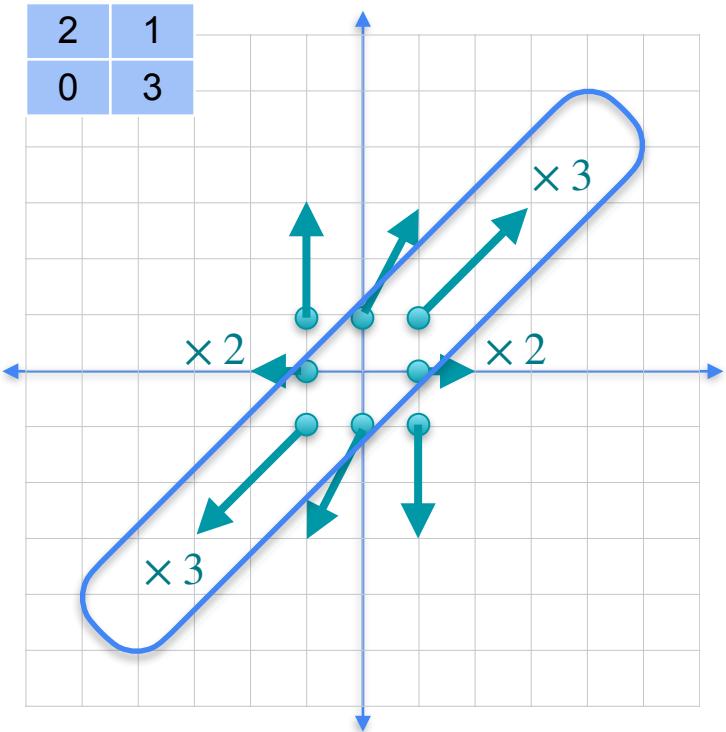
Finding eigenvalues



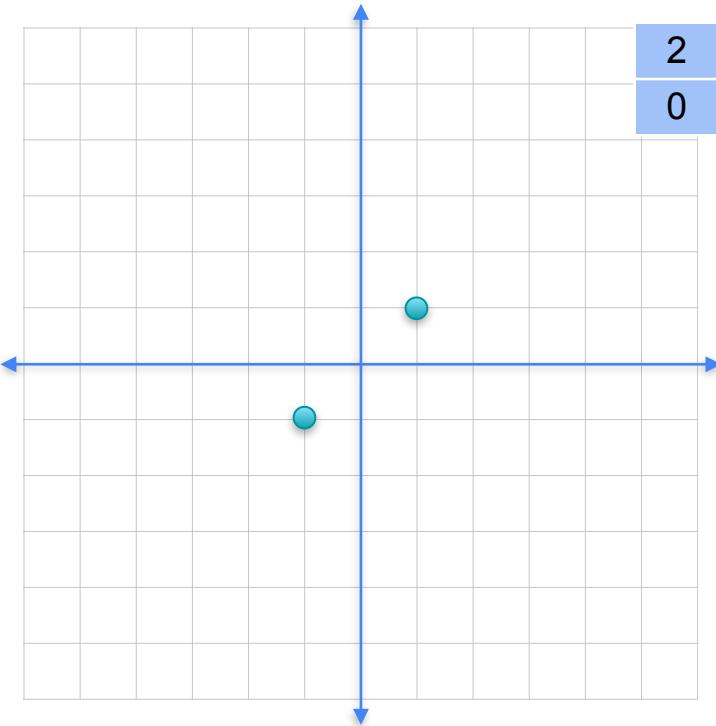
Finding eigenvalues



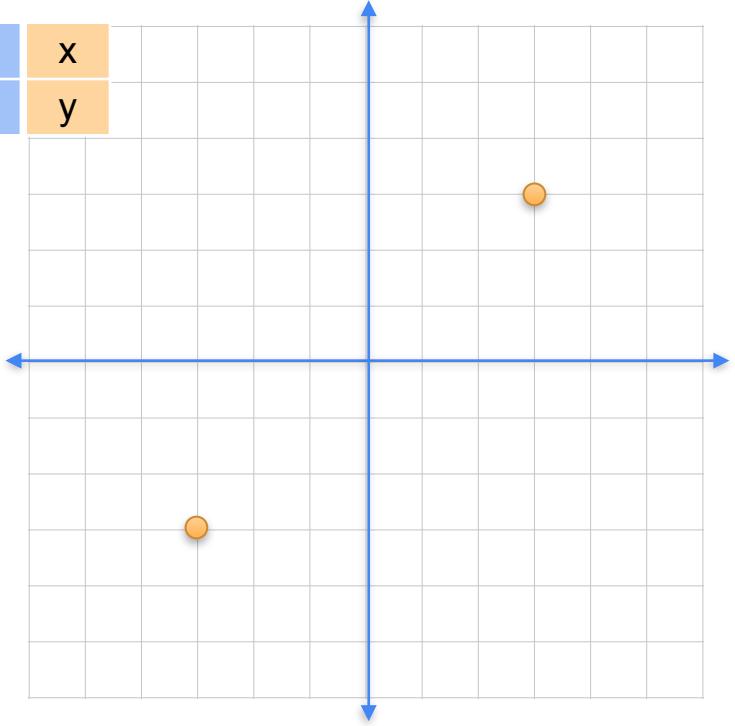
Finding eigenvalues



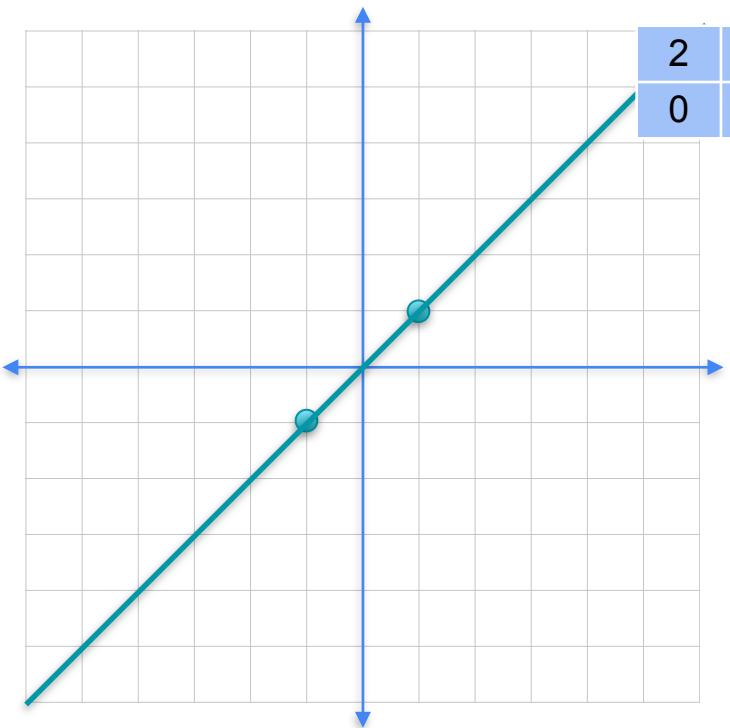
Finding eigenvalues



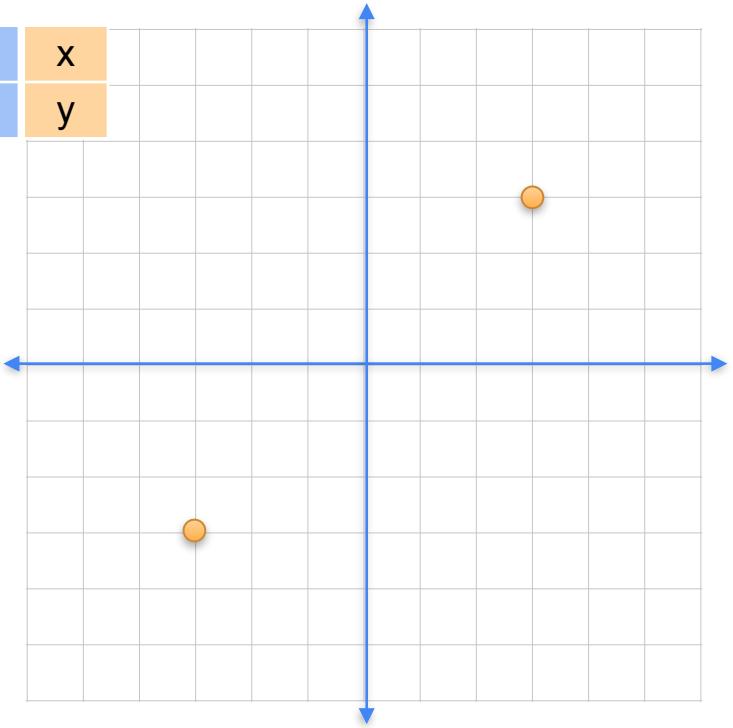
$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$



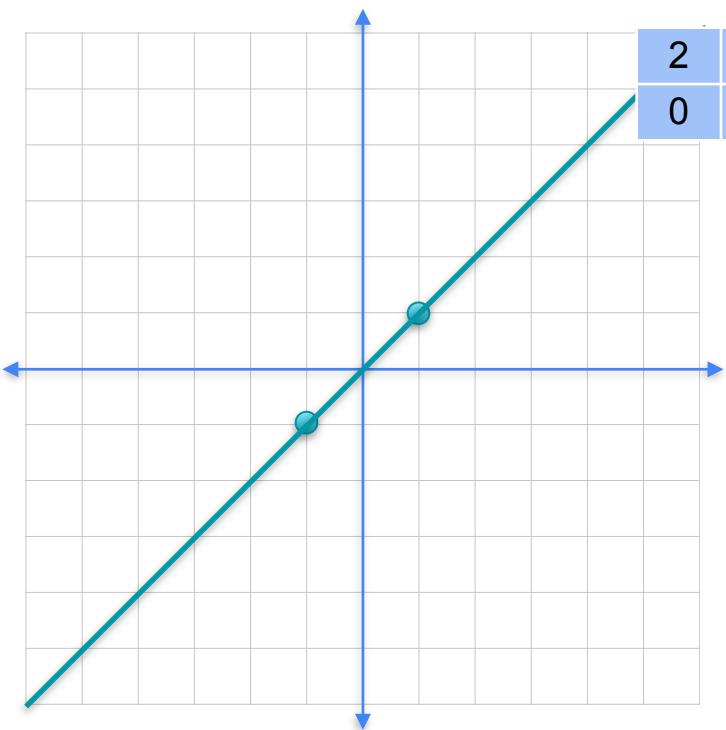
Finding eigenvalues



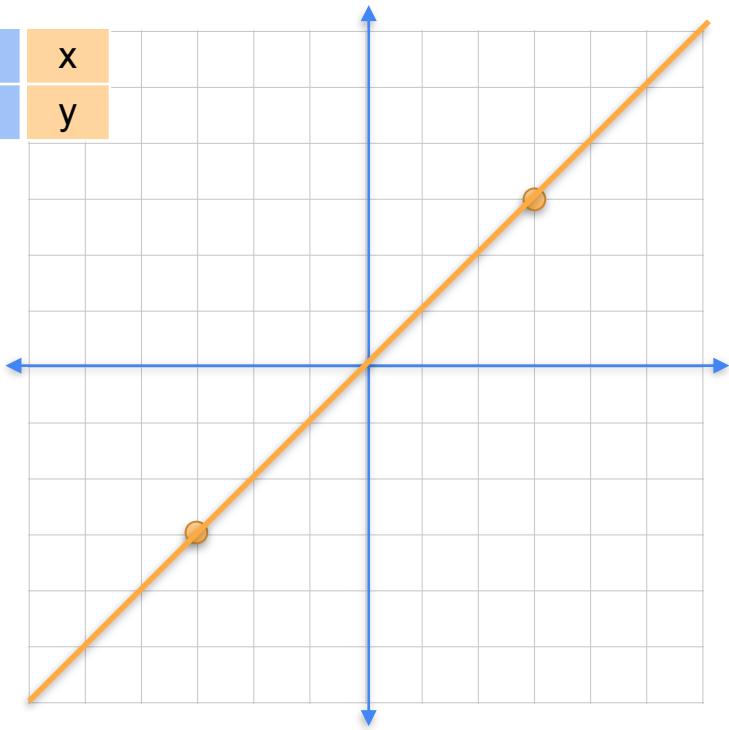
$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



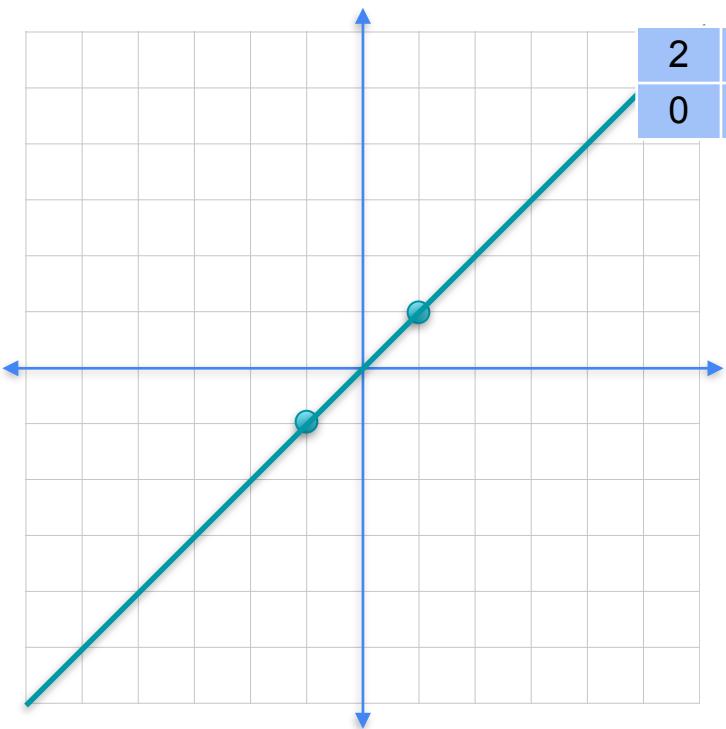
Finding eigenvalues



$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

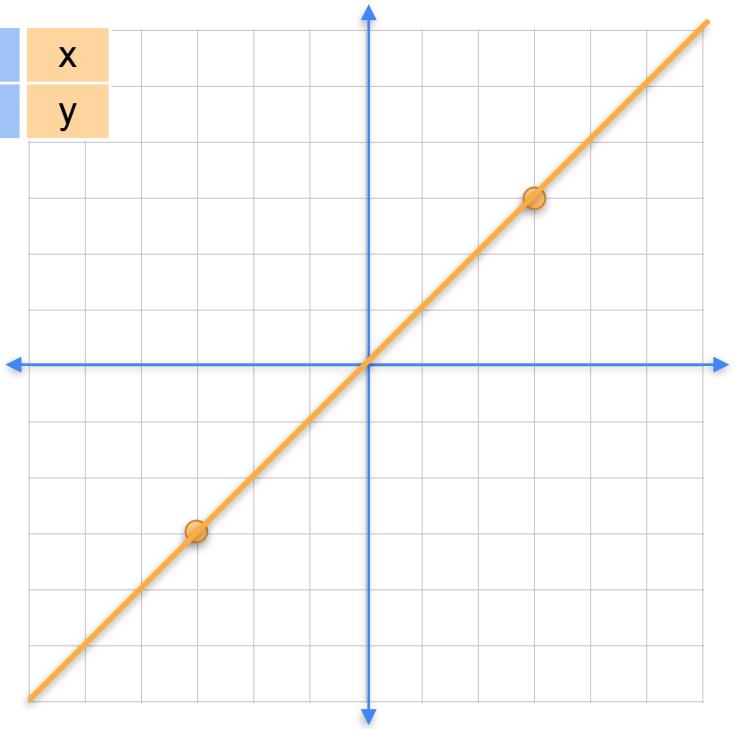


Finding eigenvalues

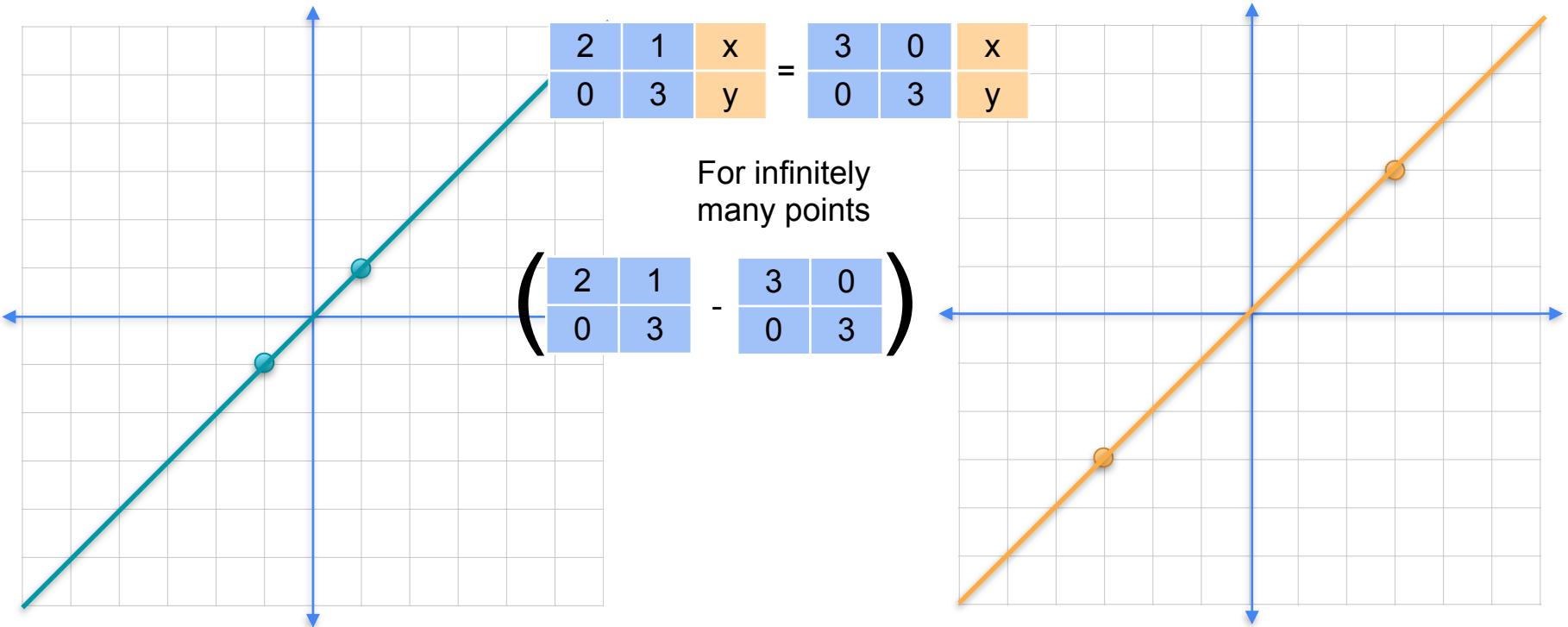


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

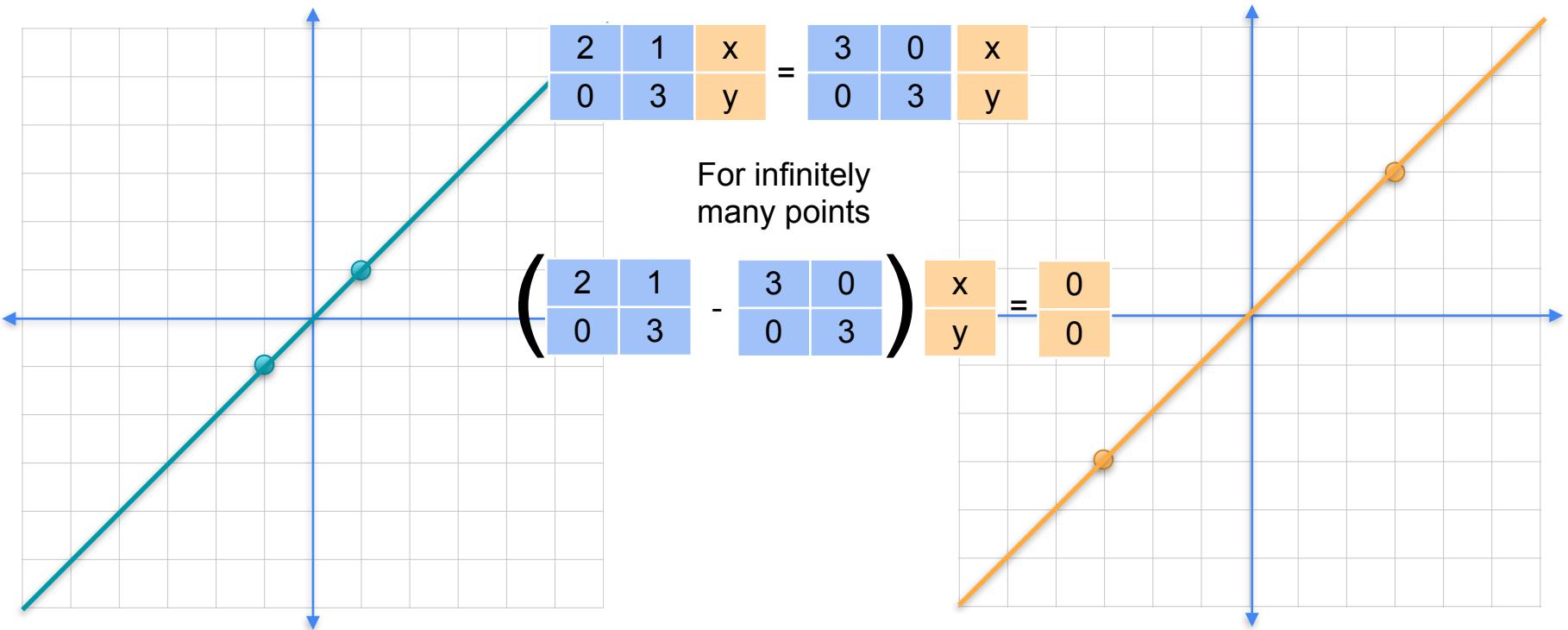
For infinitely
many points



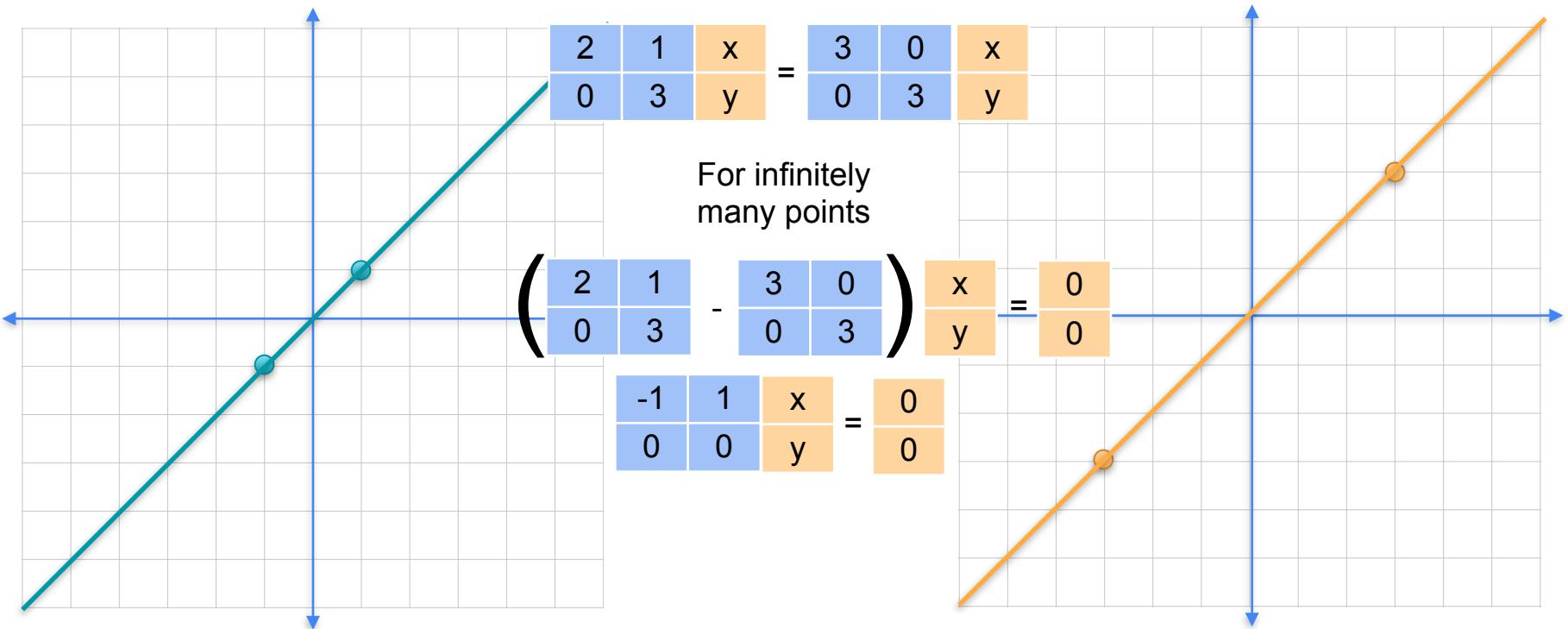
Finding eigenvalues



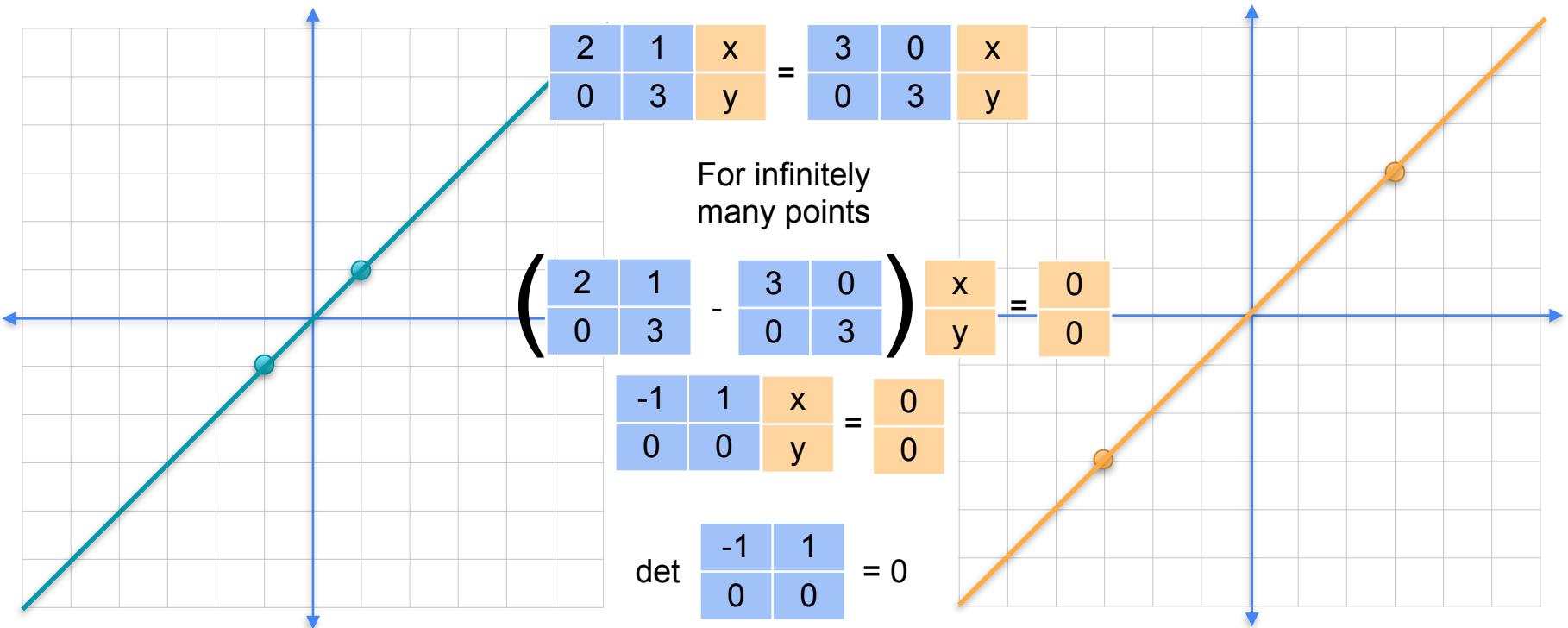
Finding eigenvalues



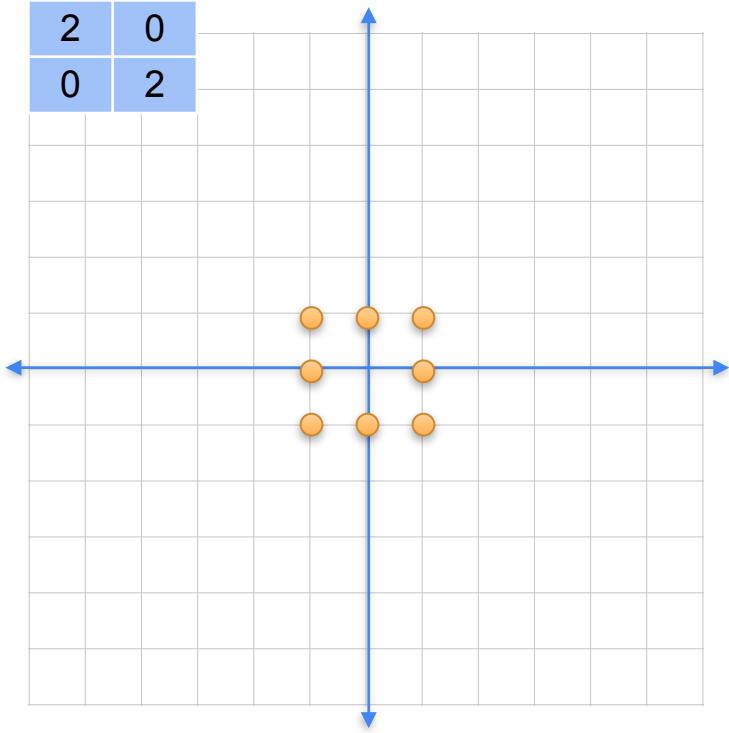
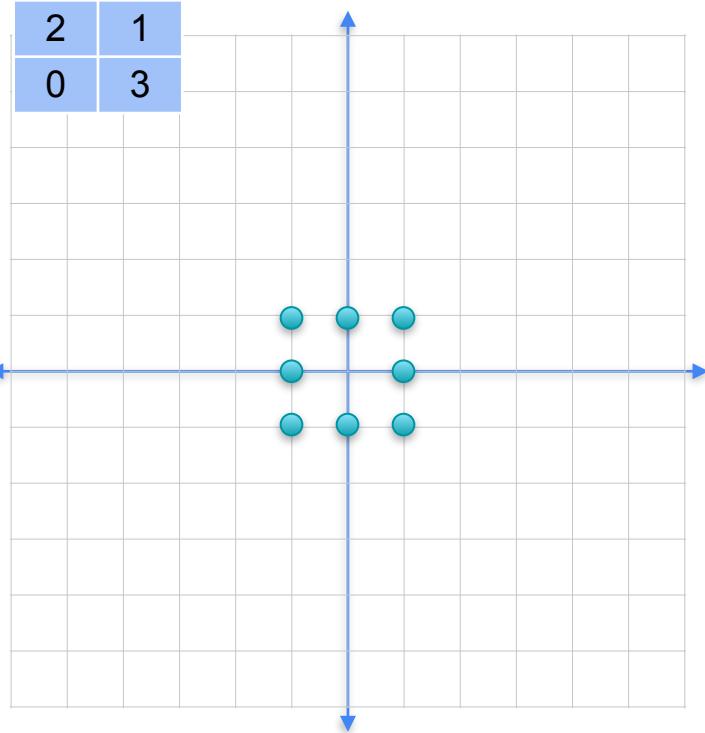
Finding eigenvalues



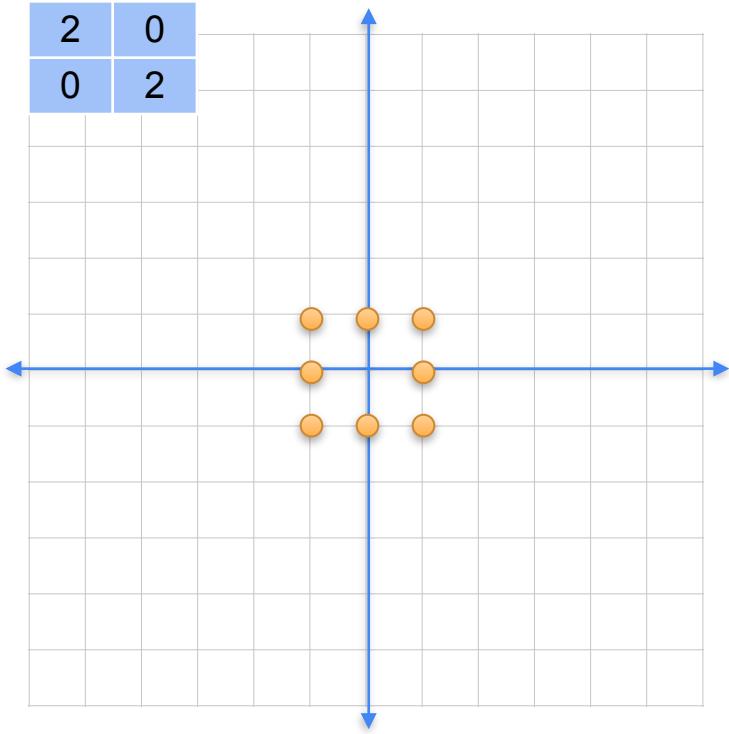
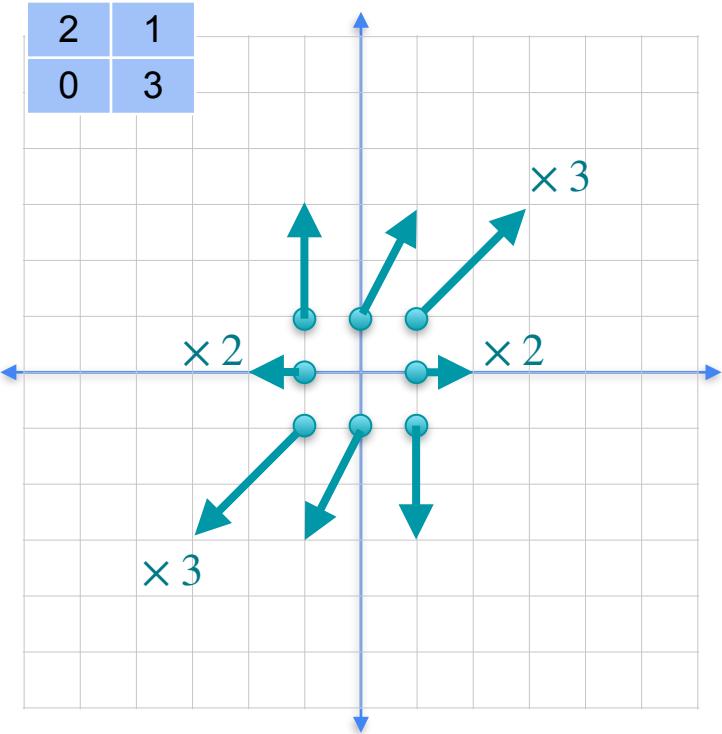
Finding eigenvalues



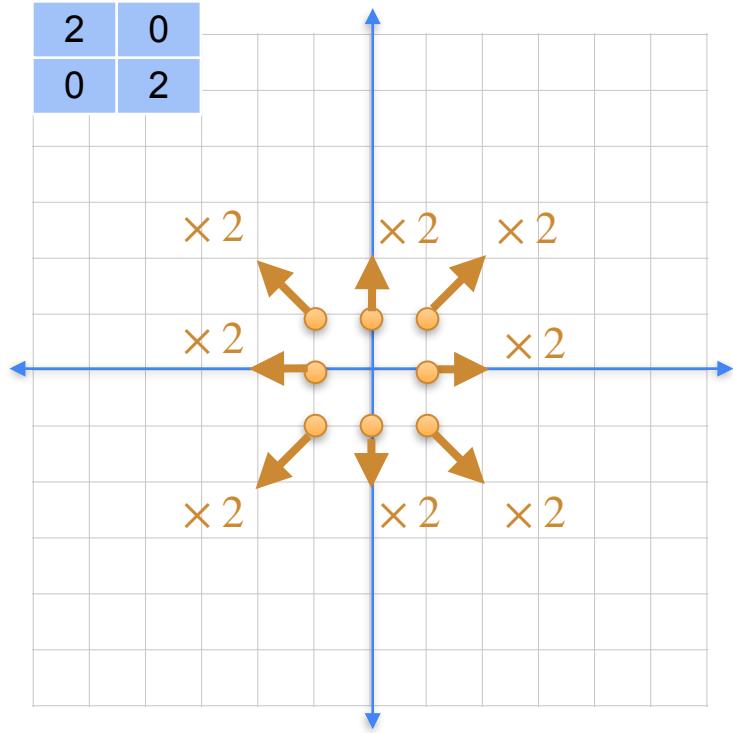
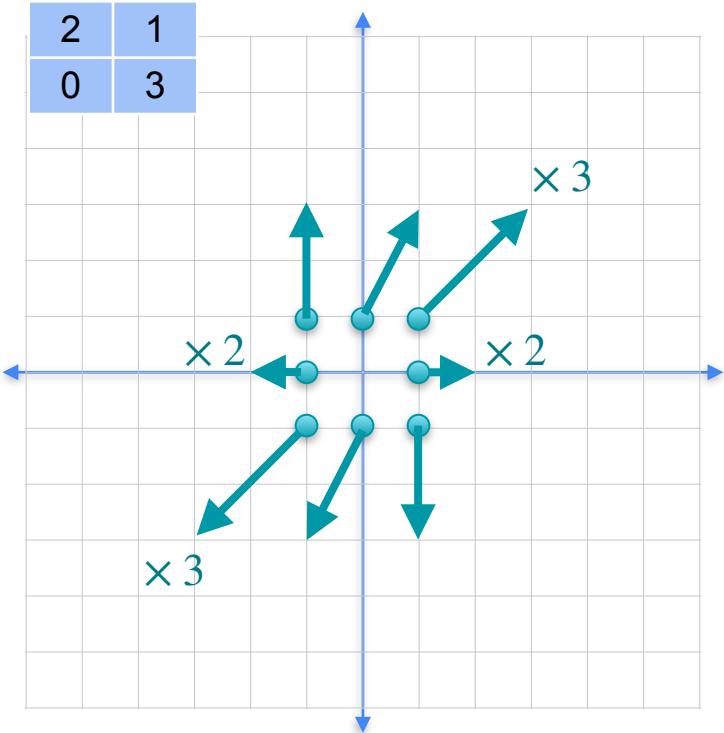
Finding eigenvalues



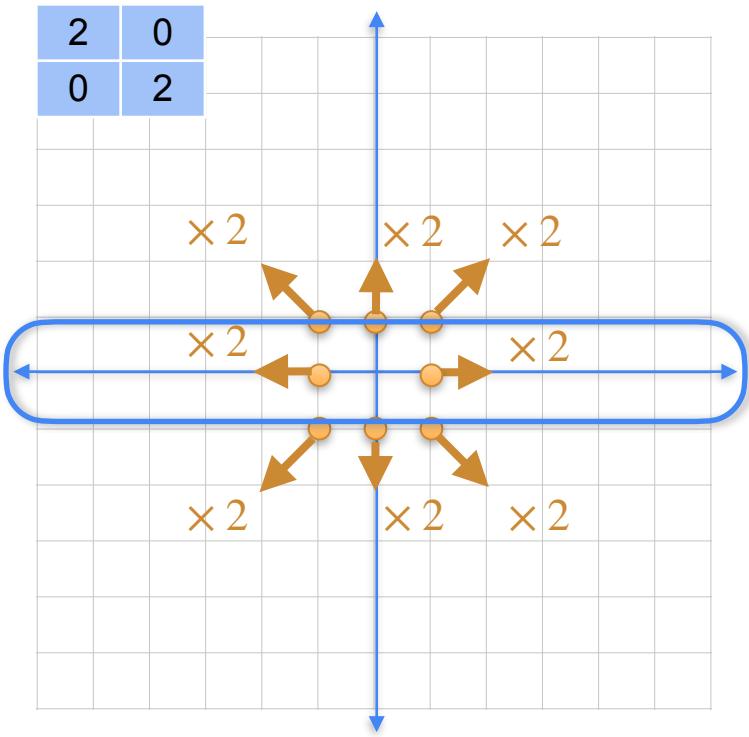
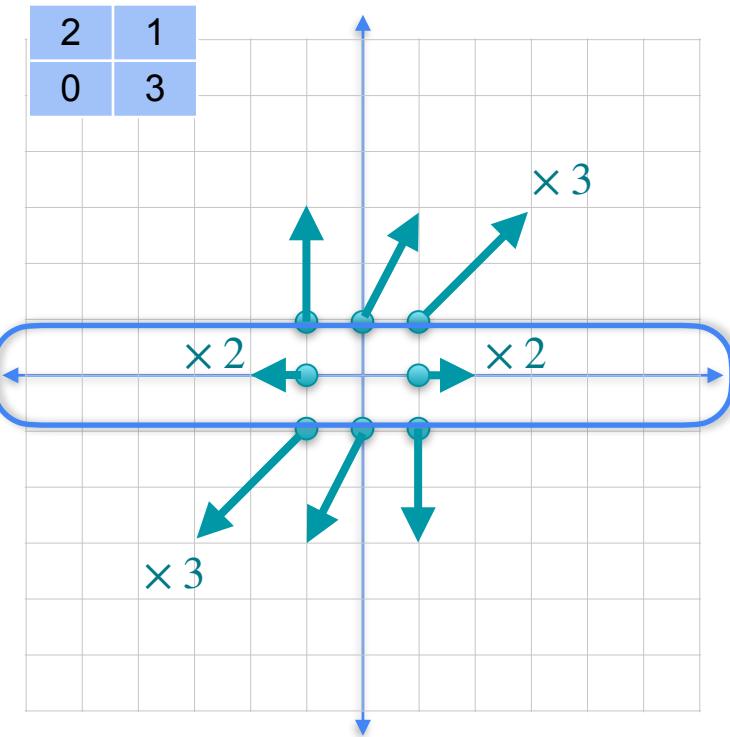
Finding eigenvalues



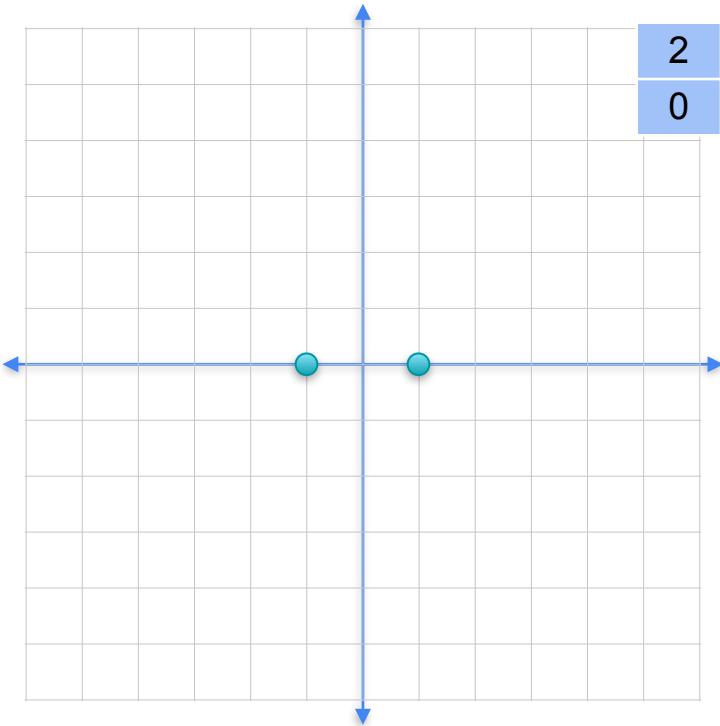
Finding eigenvalues



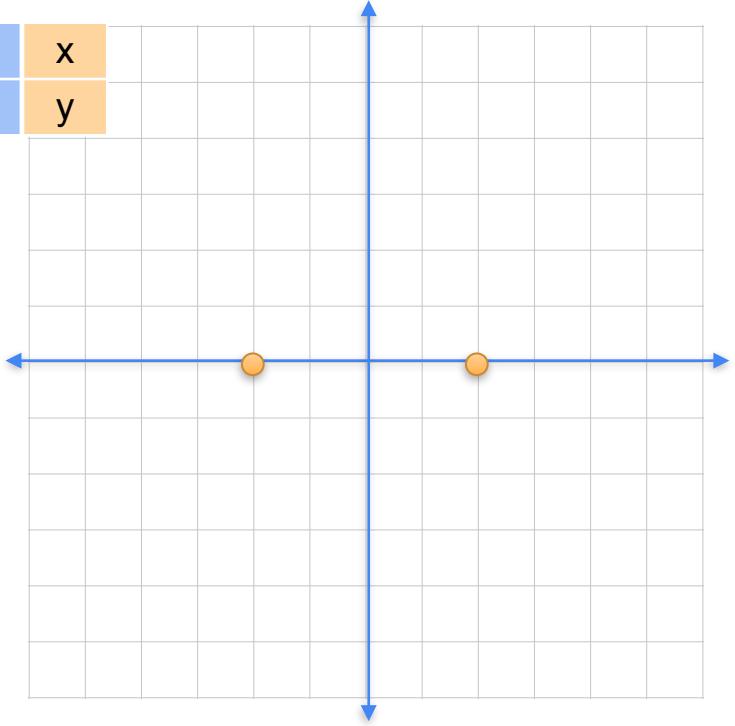
Finding eigenvalues



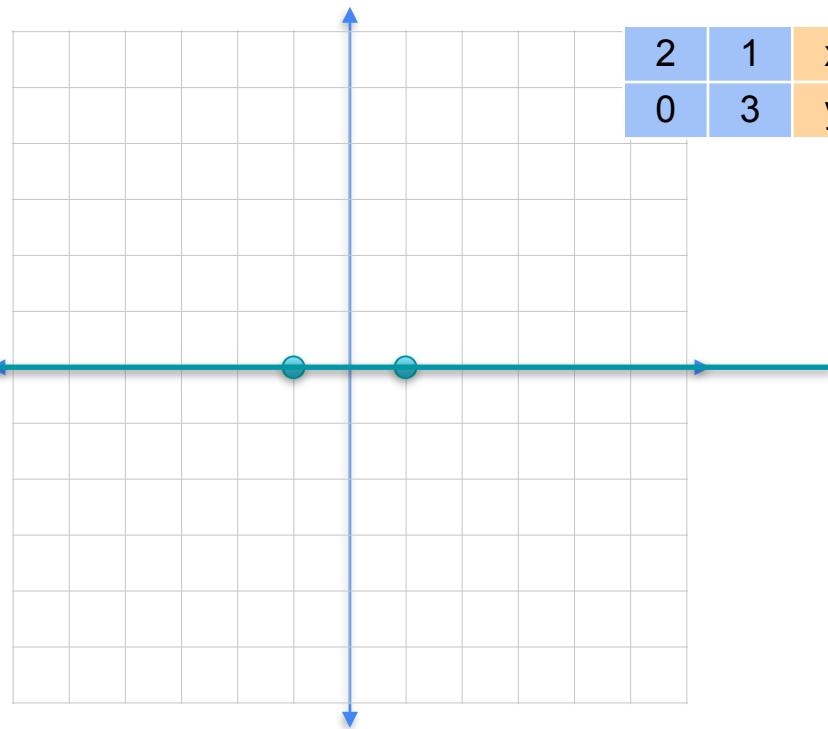
Finding eigenvalues



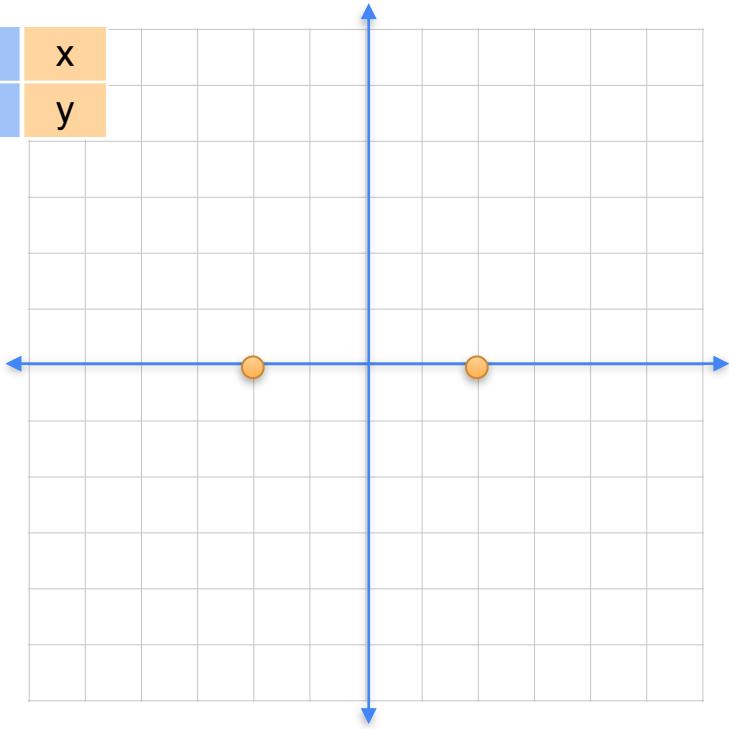
$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 2 & 0 & x \\ 0 & 2 & y \end{bmatrix}$$



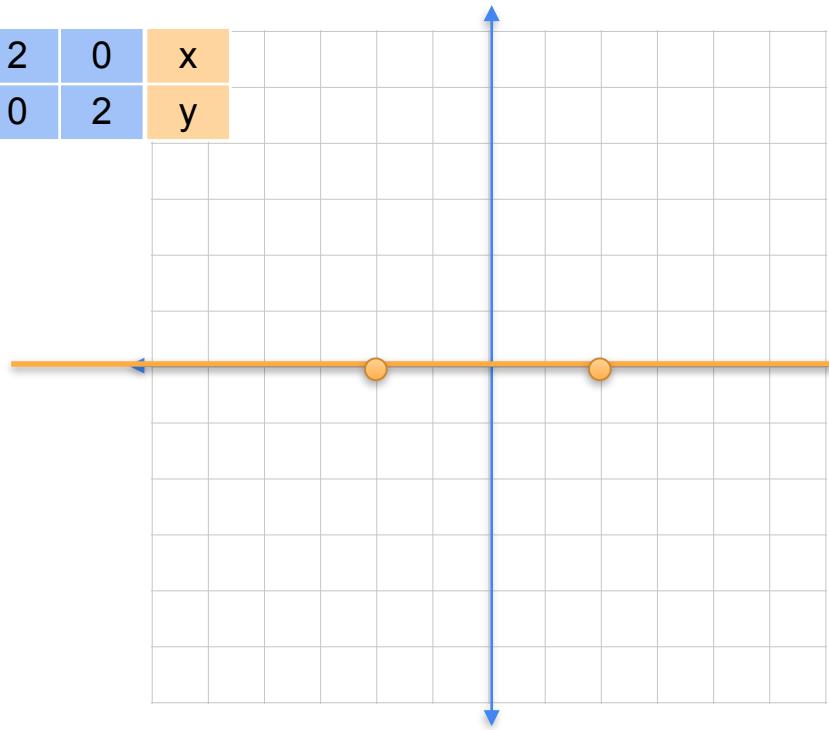
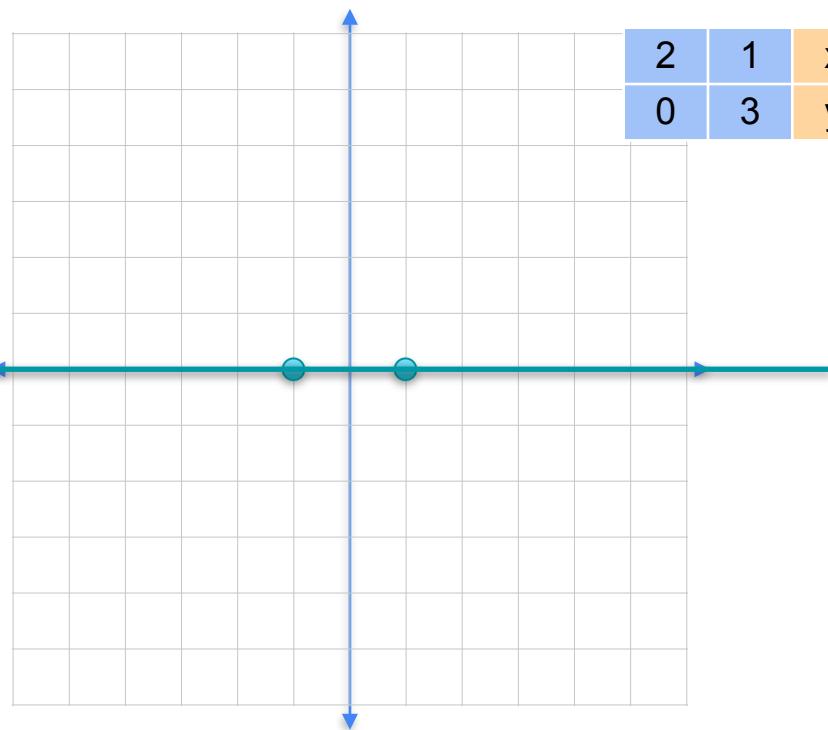
Finding eigenvalues



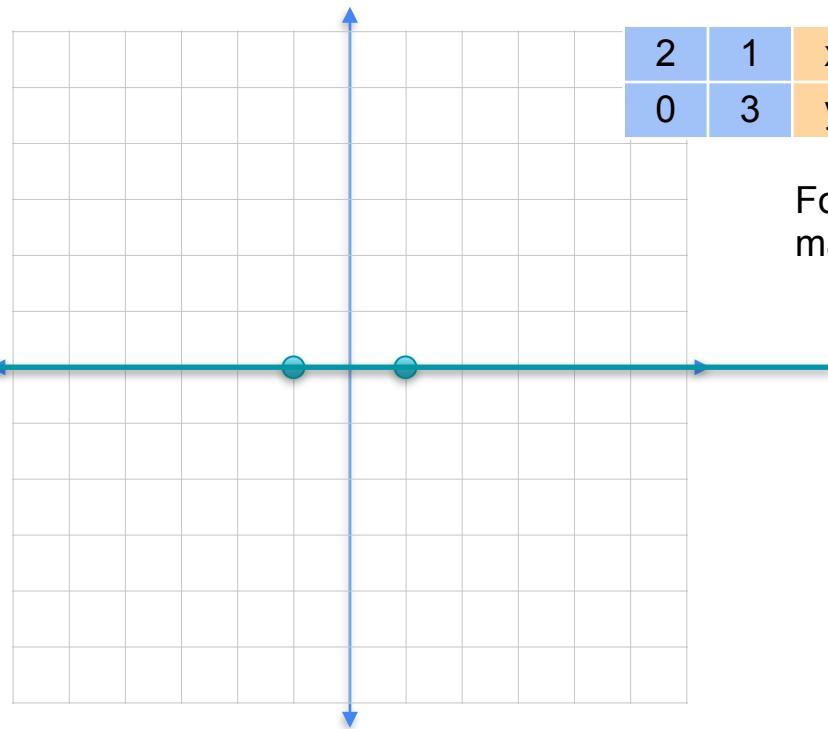
$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 2 & 0 & x \\ 0 & 2 & y \end{bmatrix}$$



Finding eigenvalues

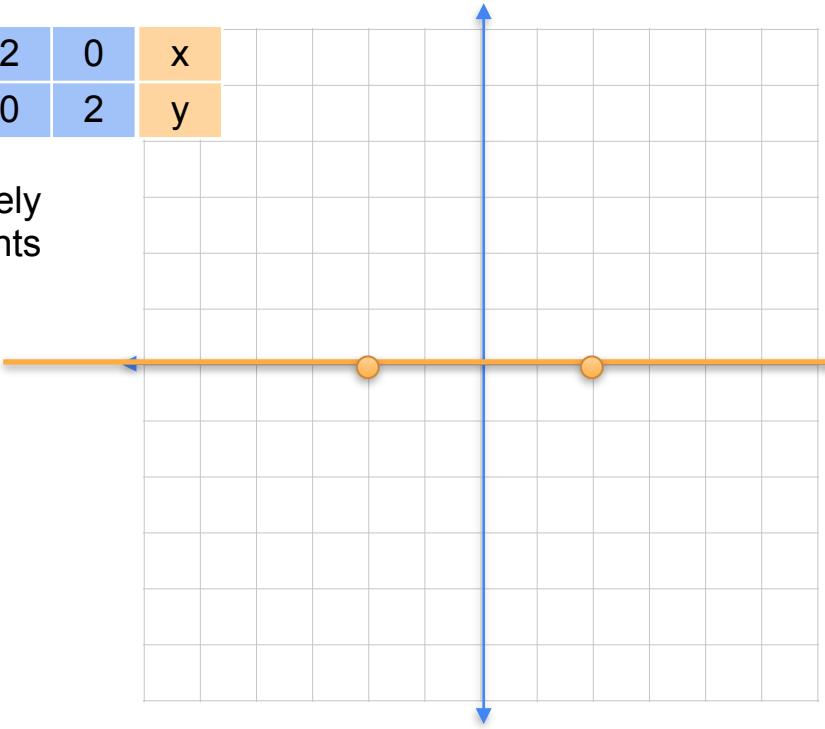


Finding eigenvalues

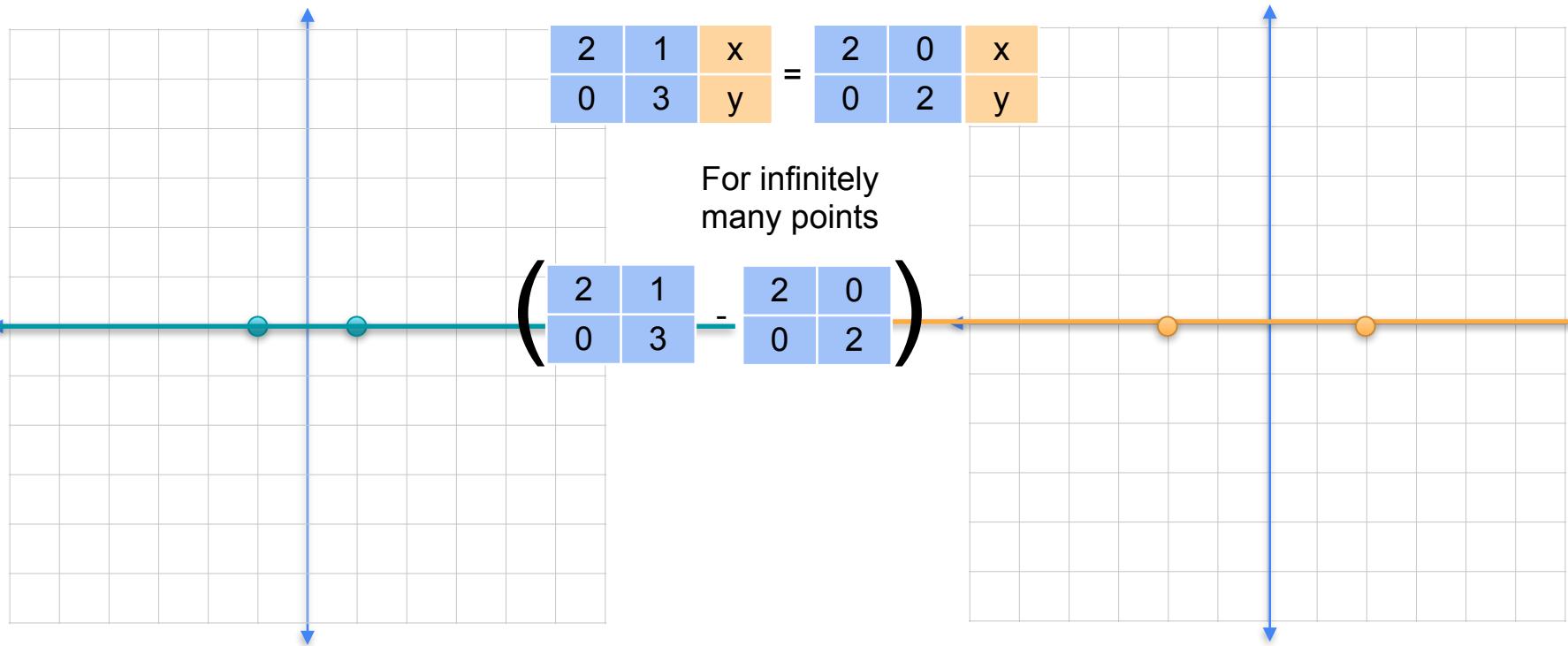


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 2 & 0 & x \\ 0 & 2 & y \end{bmatrix}$$

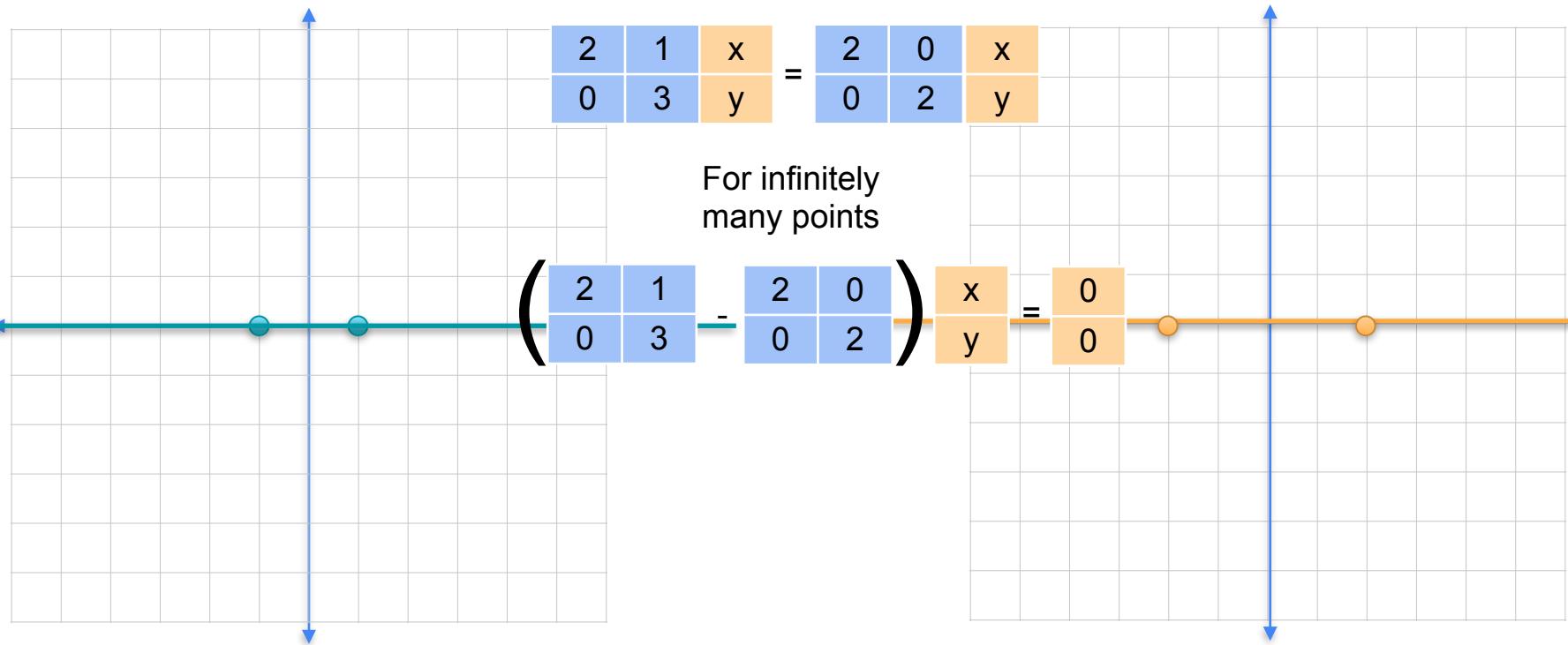
For infinitely
many points



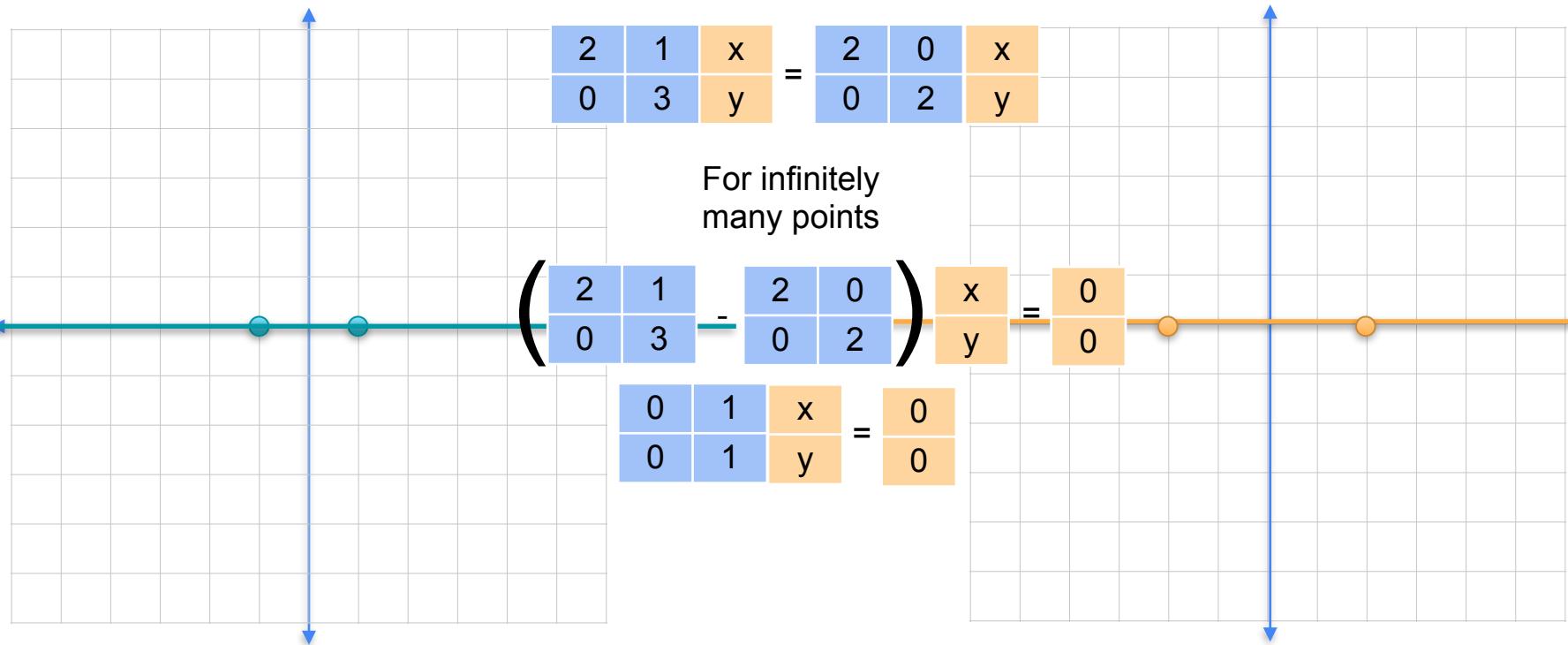
Finding eigenvalues



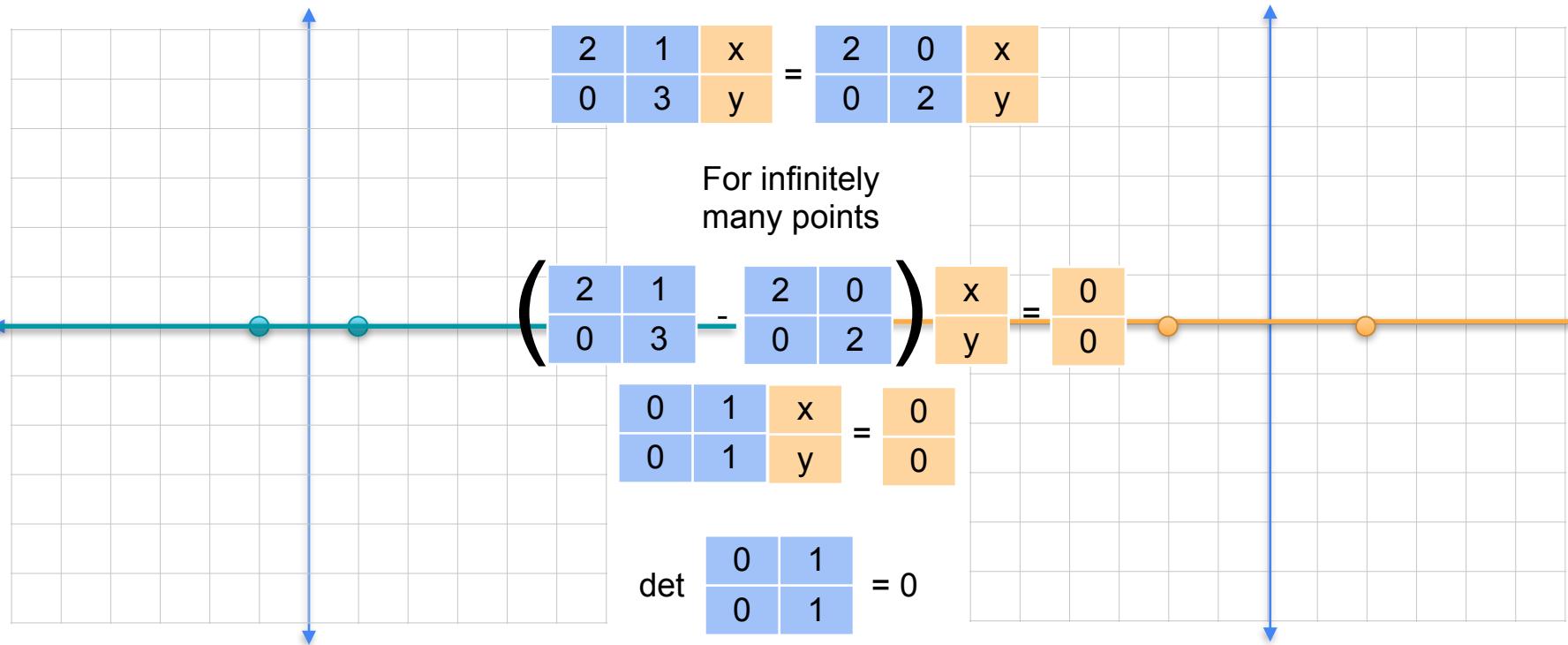
Finding eigenvalues



Finding eigenvalues



Finding eigenvalues



Finding eigenvalues

2	1
0	3

Finding eigenvalues

If λ is an eigenvalue:

2	1
0	3

Finding eigenvalues

If λ is an eigenvalue:

2	1
0	3

λ	0
0	λ

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = \begin{matrix} \lambda & 0 & x \\ 0 & \lambda & y \end{matrix}$$

For infinitely many (x,y)

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

$$\lambda = 2$$

$$\lambda = 3$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = 2 \begin{matrix} x \\ y \end{matrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

$$x = 1$$

$$0x + 3y = 2y$$

$$y = 0$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

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$$y = 0$$

$$\begin{array}{|c|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
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$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
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$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$
$$\begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \end{array}$$

Quiz

- Find the eigenvalues and eigenvectors of this matrix:

9	4
4	3

Solution

- Eigenvalues: 11, 1
- Eigenvectors: (2,1), (-1,2)

9	4
4	3

- The characteristic polynomial is

$$\det \begin{vmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{vmatrix} = (9 - \lambda)(3 - \lambda) - 4 \cdot 4 = 0$$

- Which factors as $\lambda^2 - 12\lambda + 11 = (\lambda - 11)(\lambda - 1)$

- The solutions are $\lambda = 11$
 $\lambda = 1$



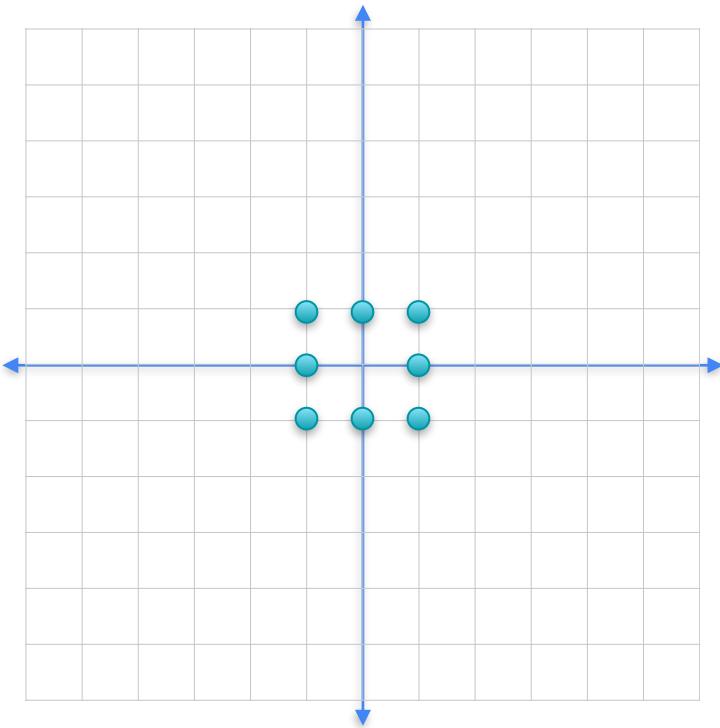
DeepLearning.AI

Determinants and Eigenvectors

Conclusion

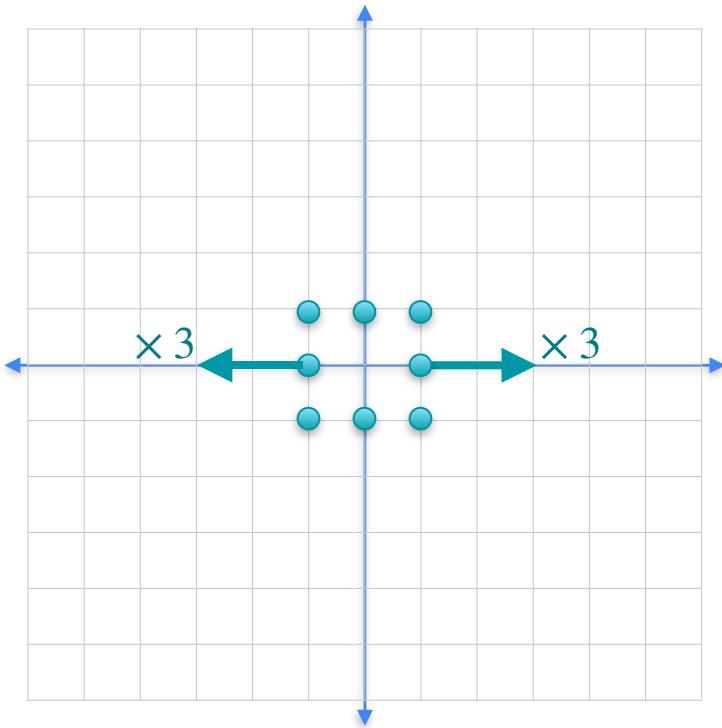
Find eigenvalues

2	1
0	3



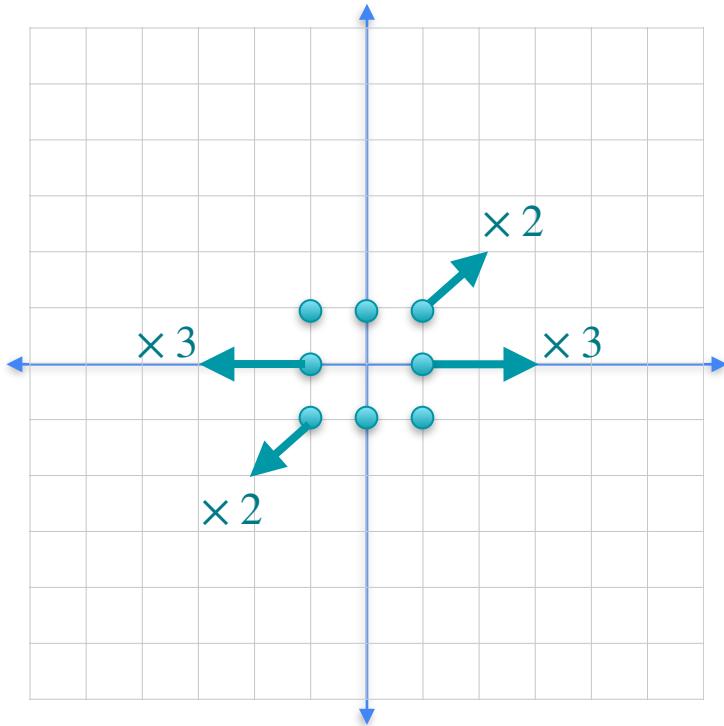
Find eigenvalues

2	1
0	3



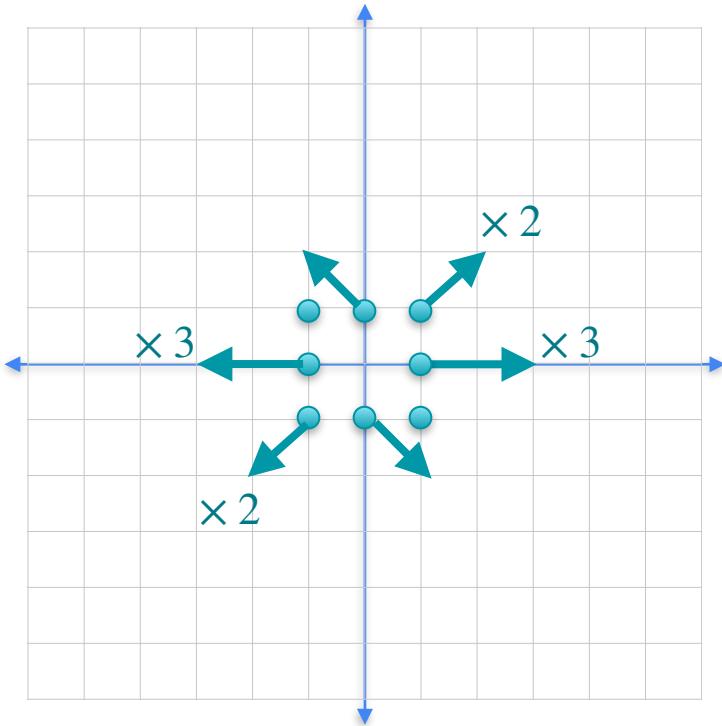
Find eigenvalues

2	1
0	3



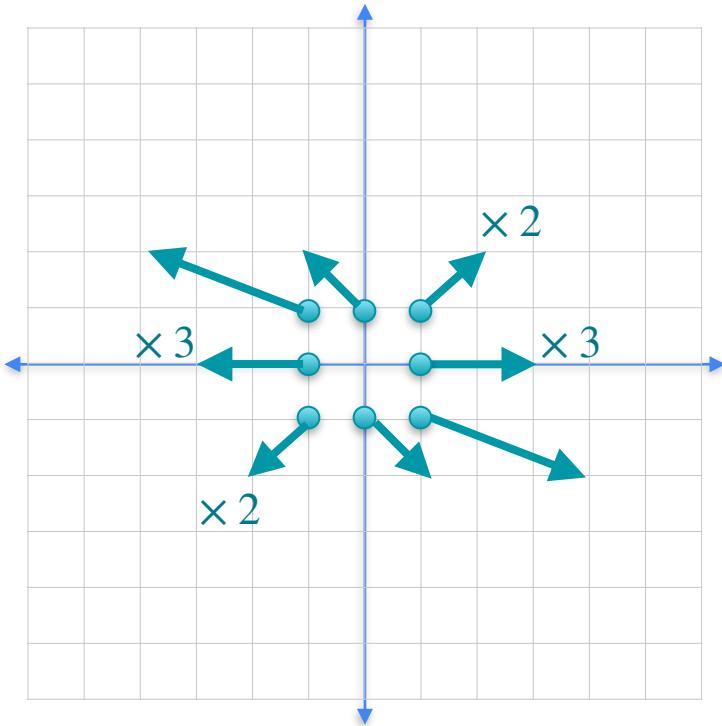
Find eigenvalues

2	1
0	3



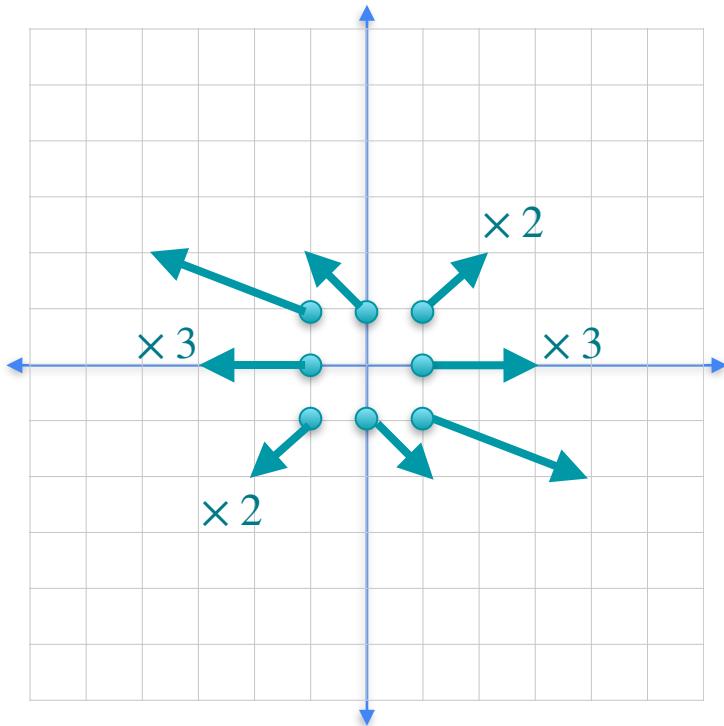
Find eigenvalues

2	1
0	3

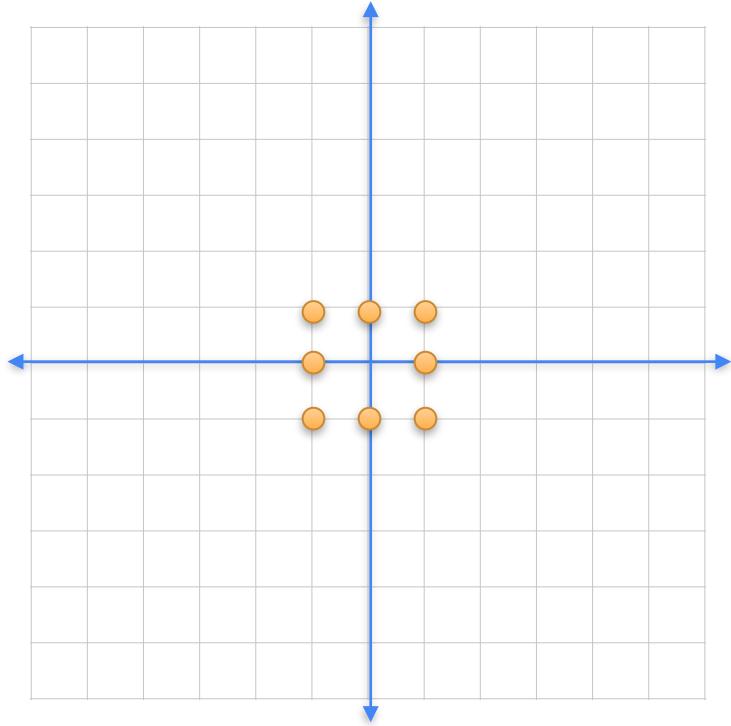


Find eigenvalues

2	1
0	3

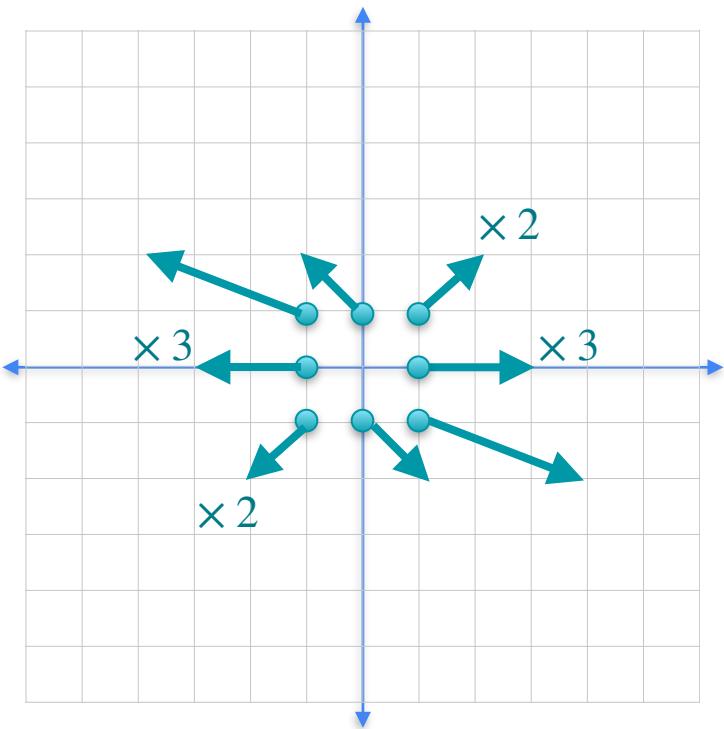


3	0
0	3

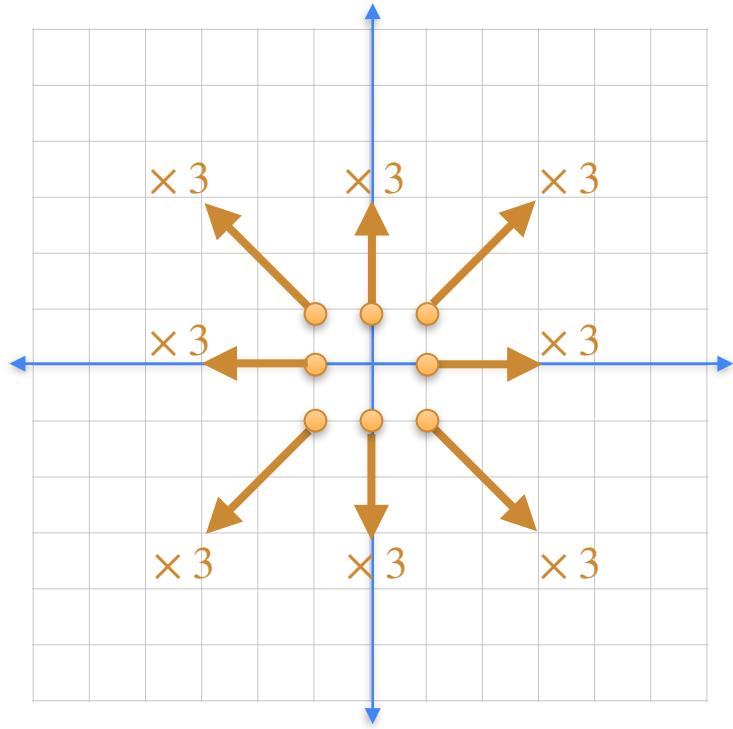


Find eigenvalues

2	1
0	3

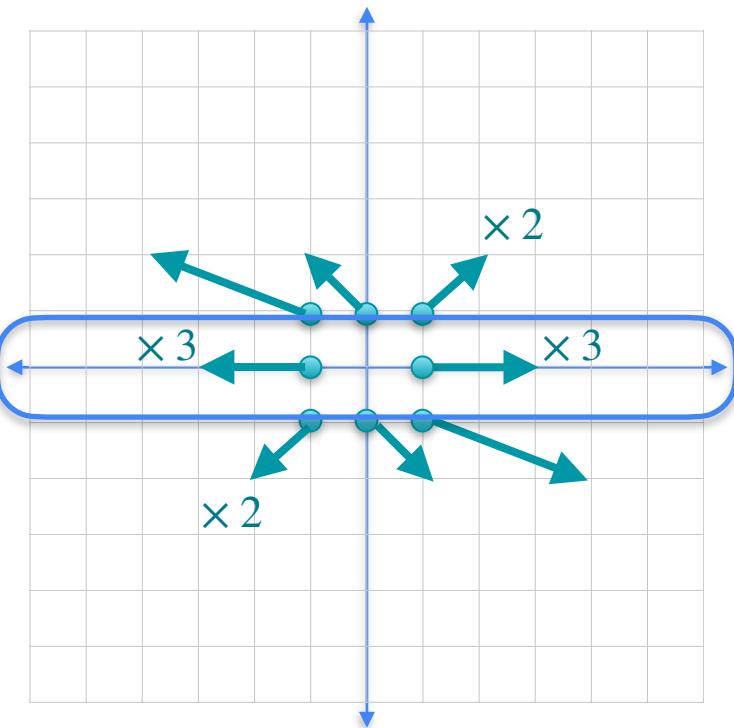


3	0
0	3

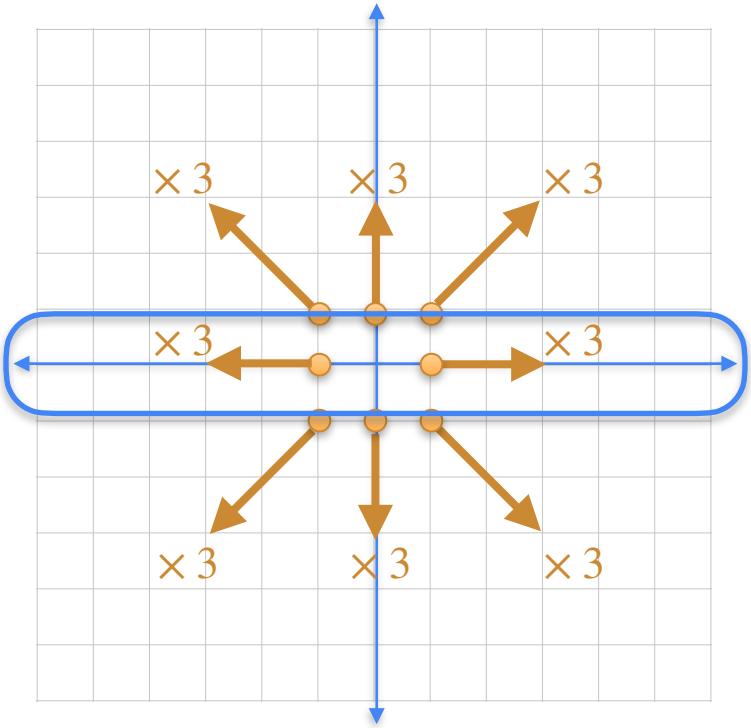


Find eigenvalues

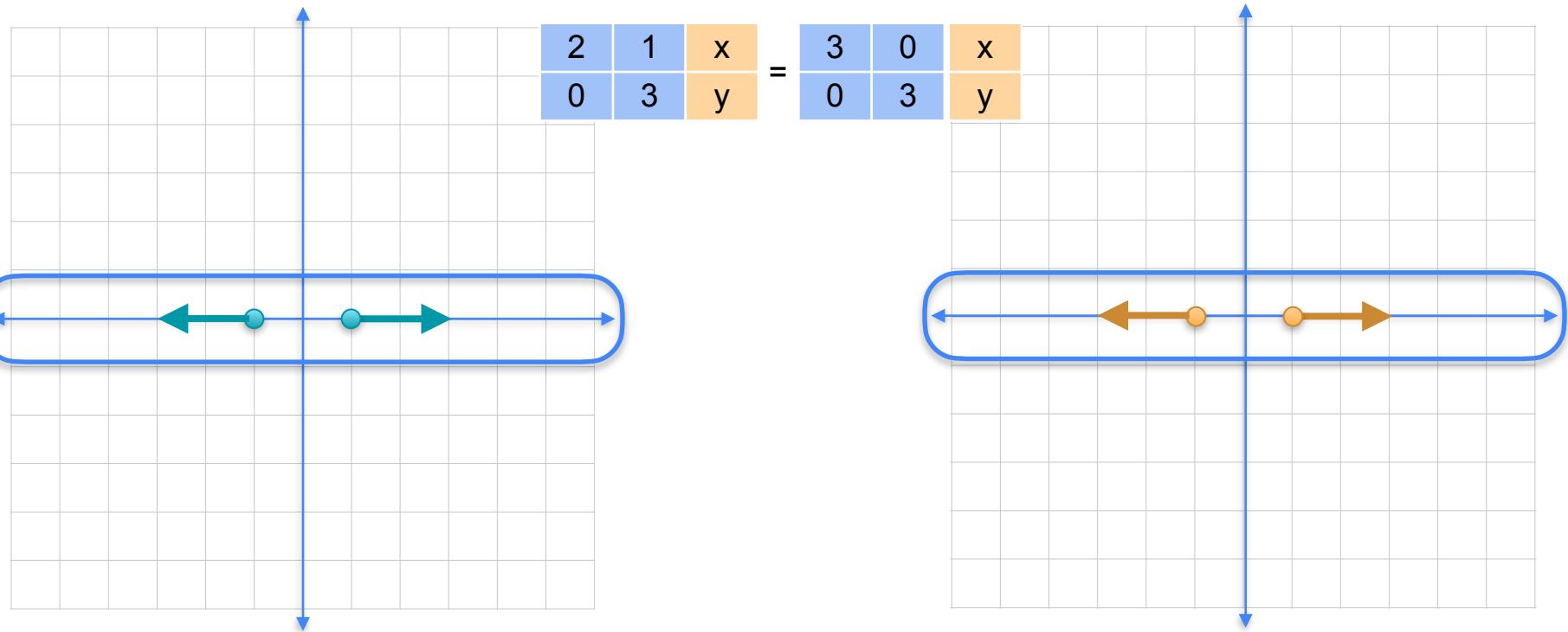
2	1
0	3



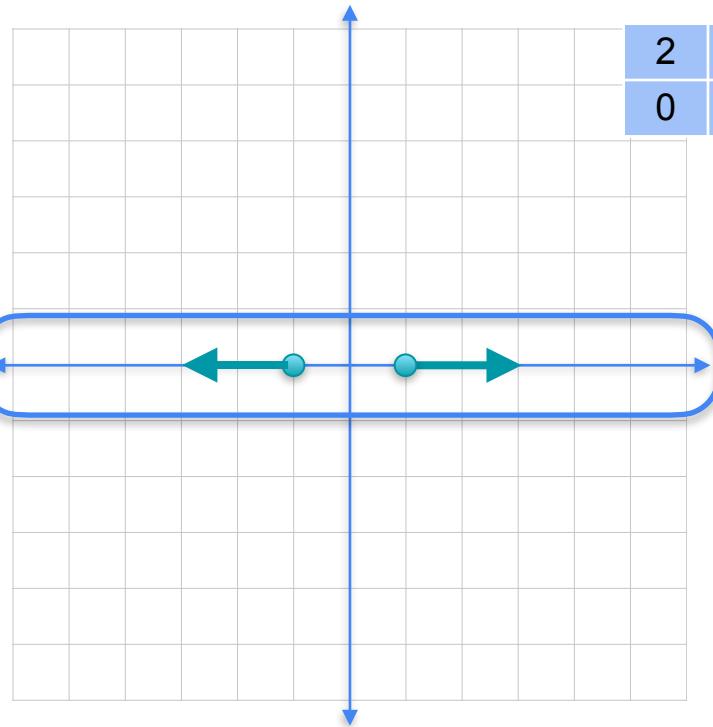
3	0
0	3



Finding eigenvalues

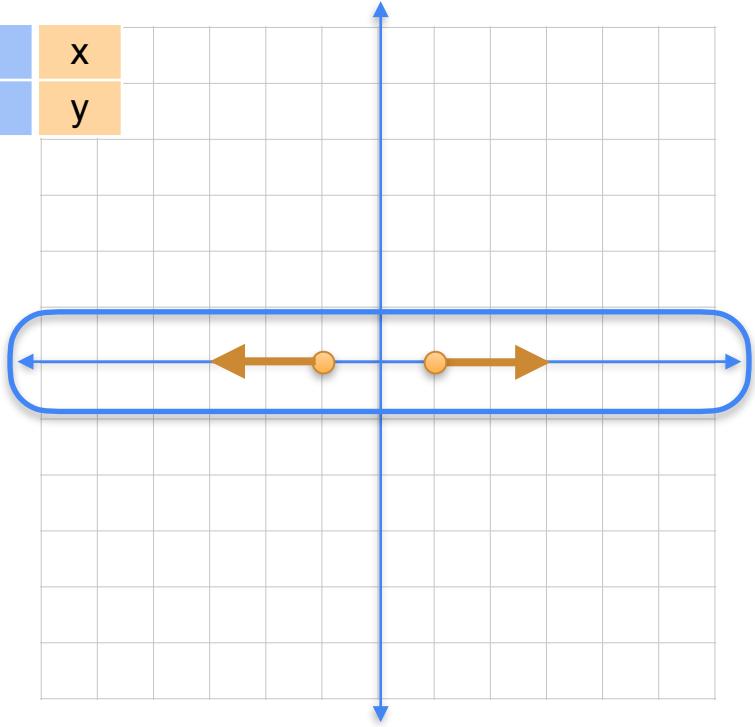


Finding eigenvalues

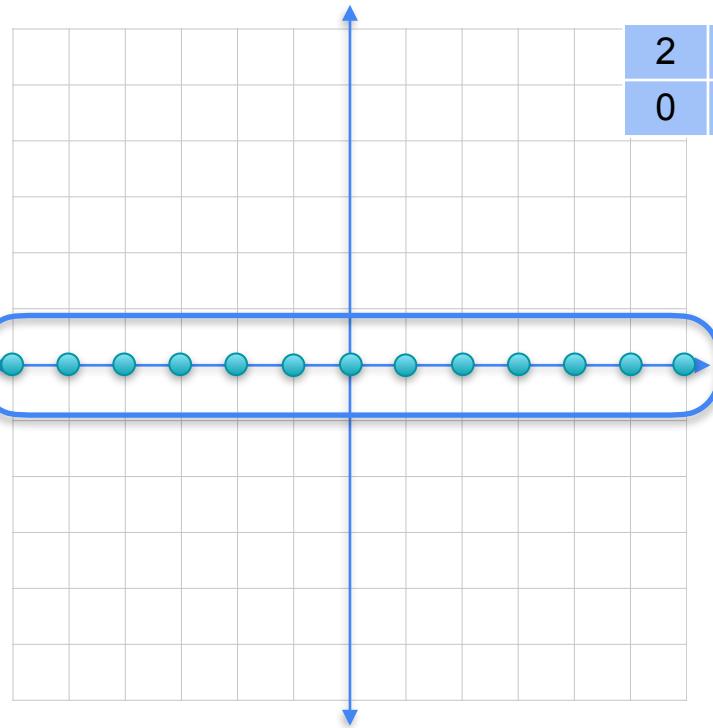


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

For infinitely
many points

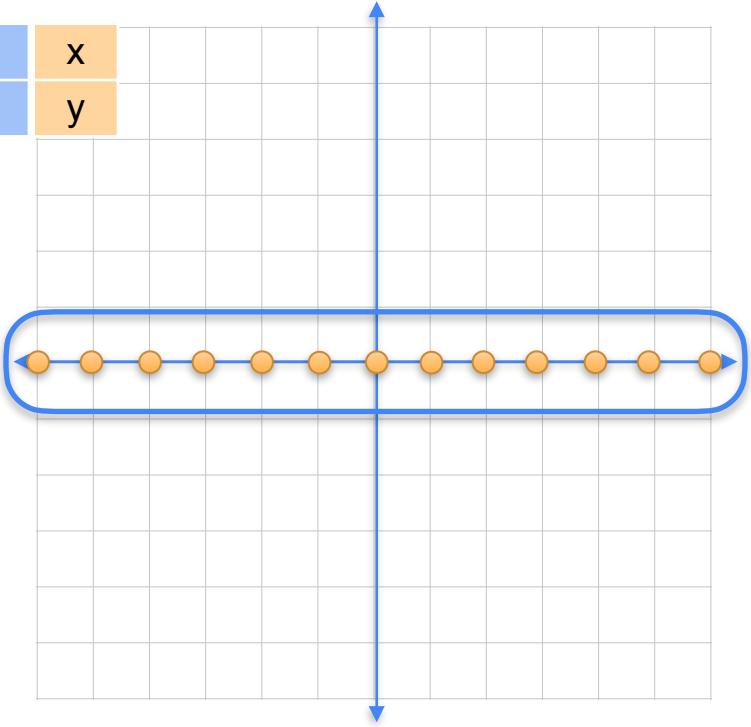


Finding eigenvalues

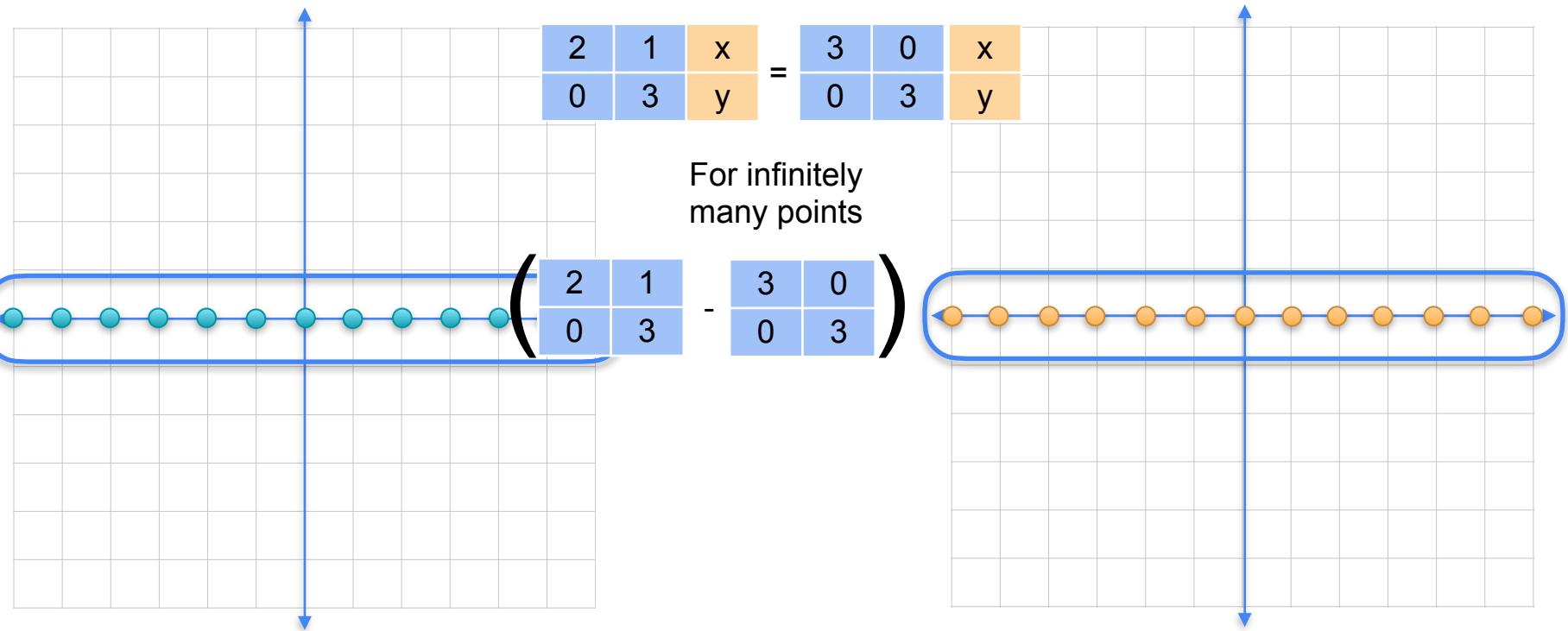


$$\begin{bmatrix} 2 & 1 & x \\ 0 & 3 & y \end{bmatrix} = \begin{bmatrix} 3 & 0 & x \\ 0 & 3 & y \end{bmatrix}$$

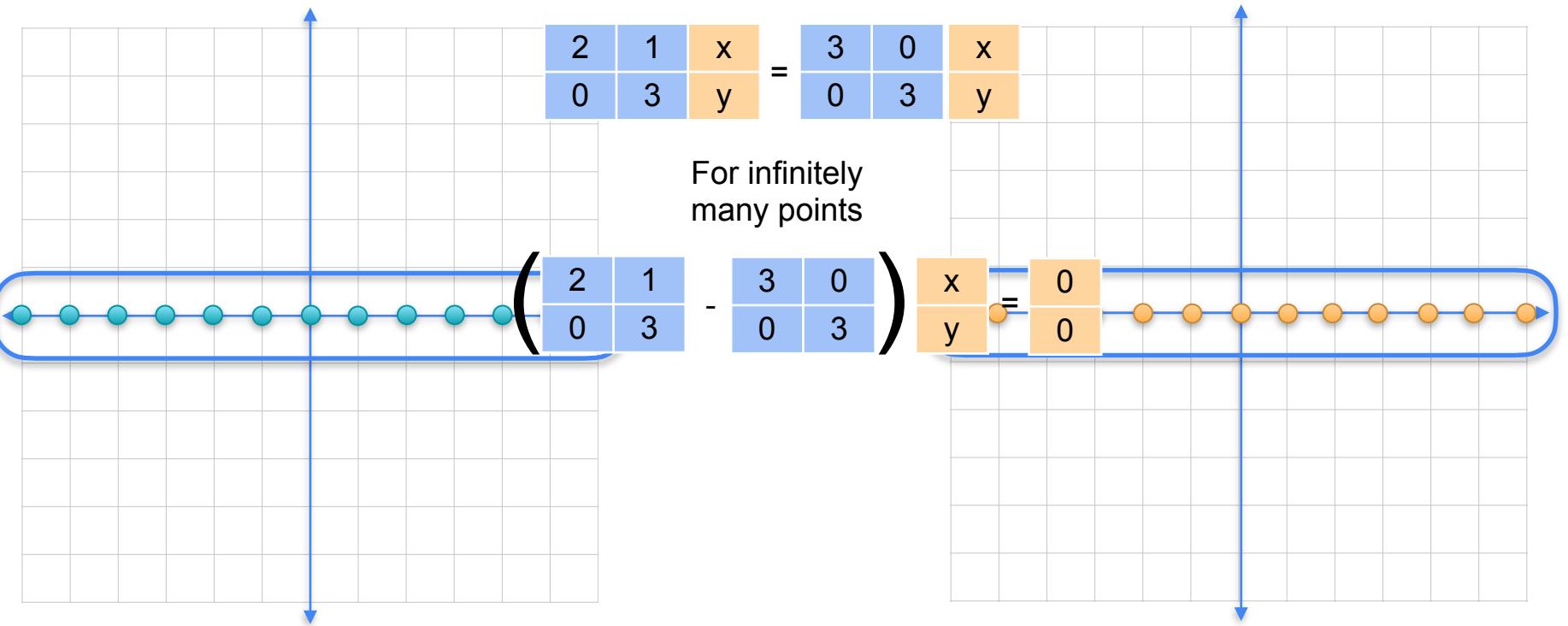
For infinitely
many points



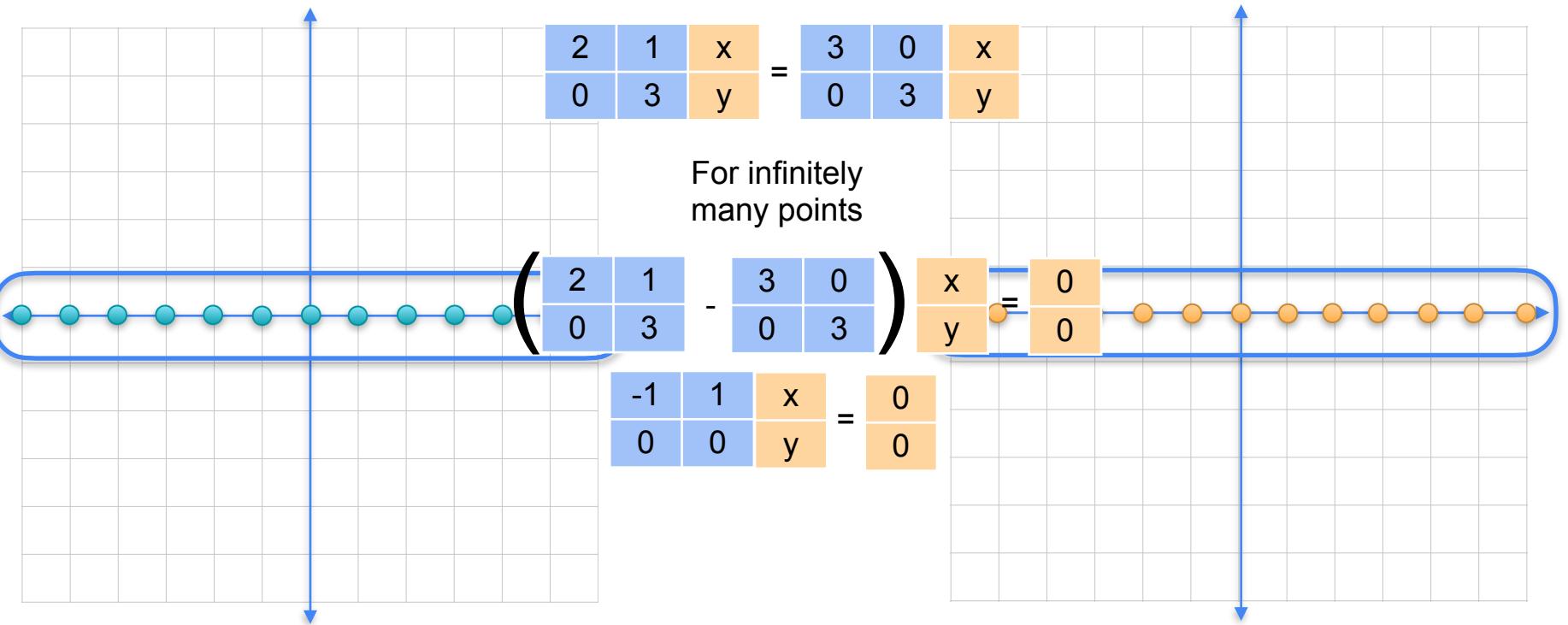
Finding eigenvalues



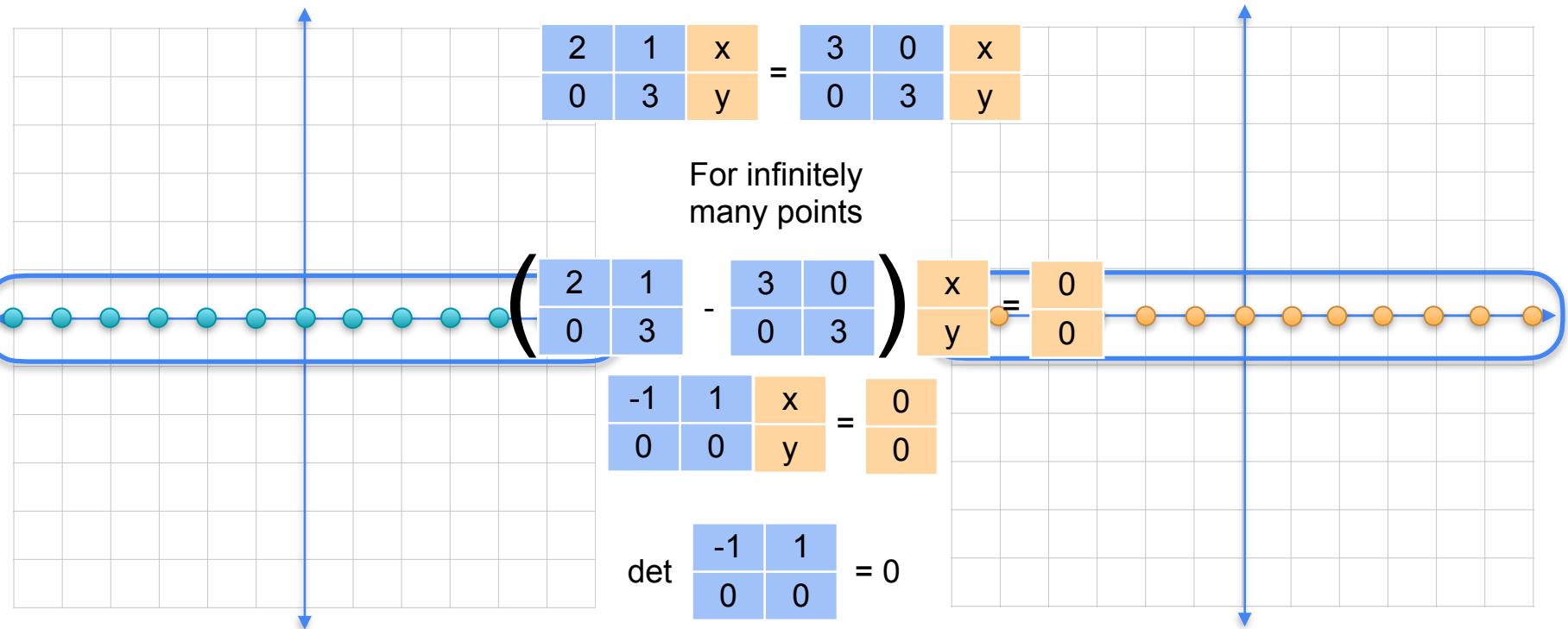
Finding eigenvalues



Finding eigenvalues

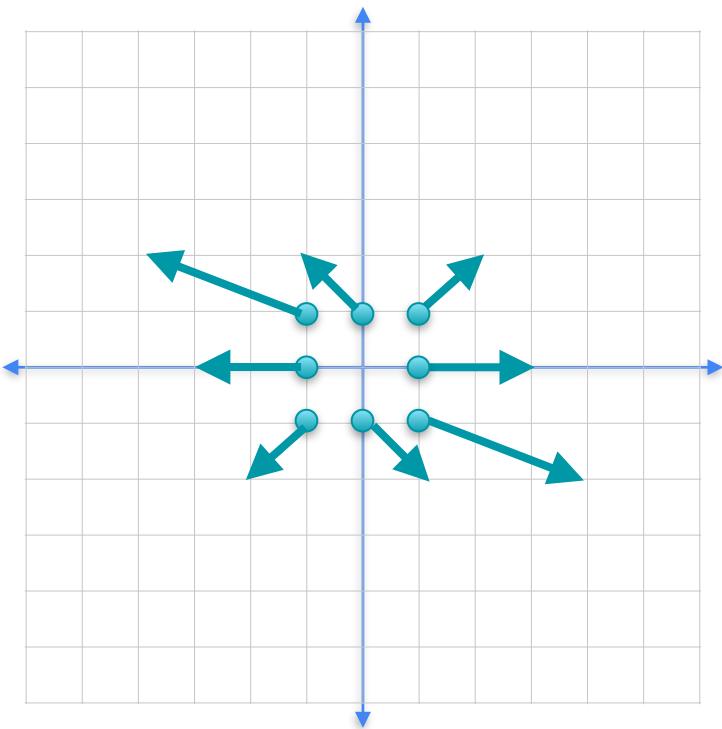


Finding eigenvalues

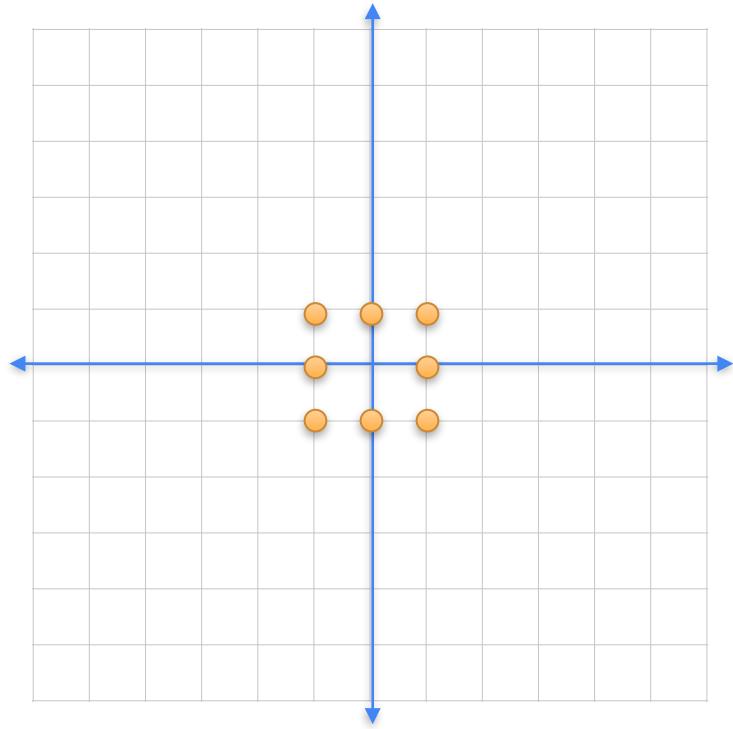


Find eigenvalues

2	1
0	3

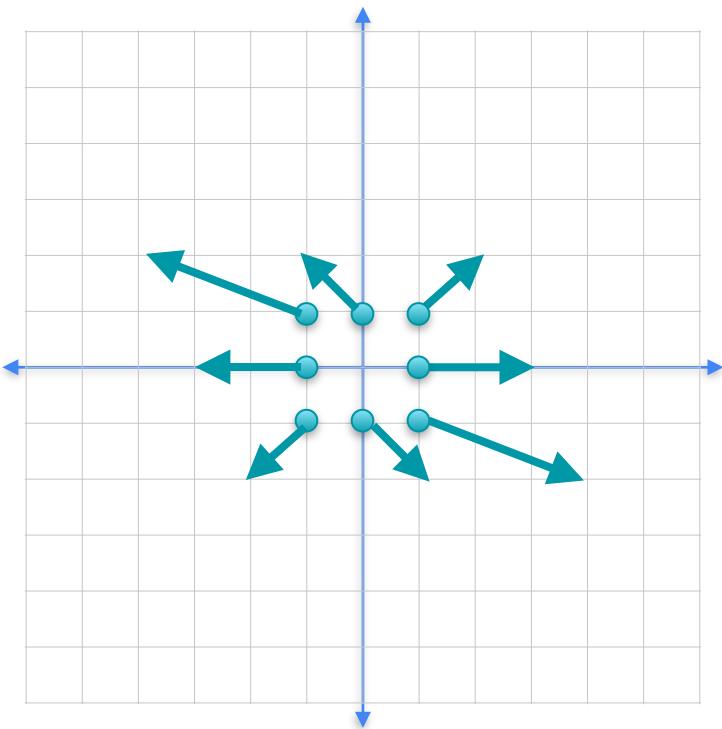


2	0
0	2

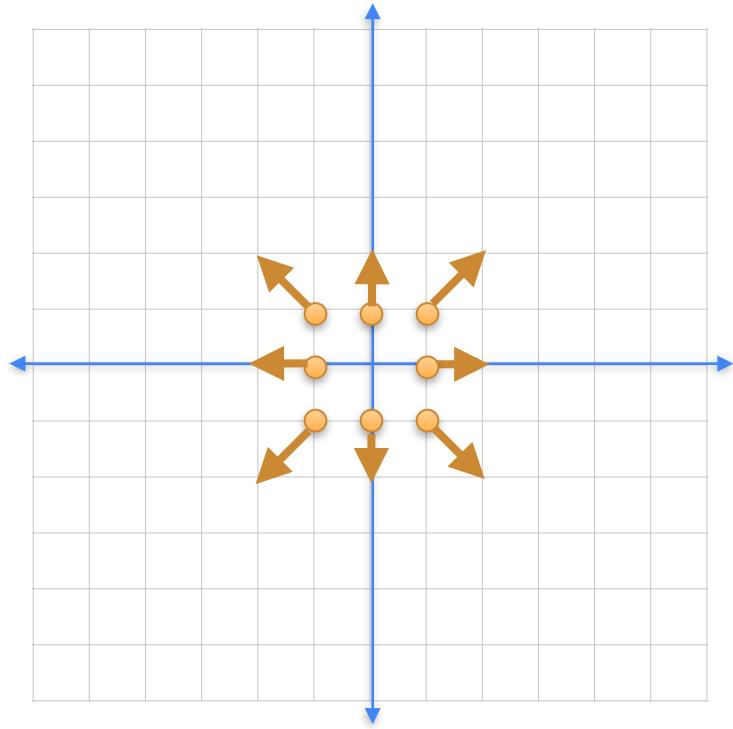


Finding eigenvalues

2	1
0	3

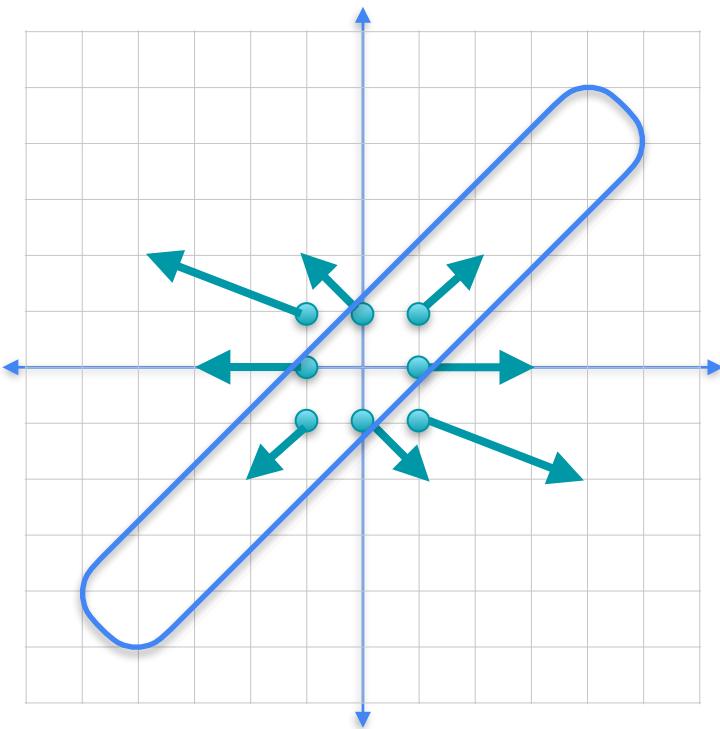


2	0
0	2

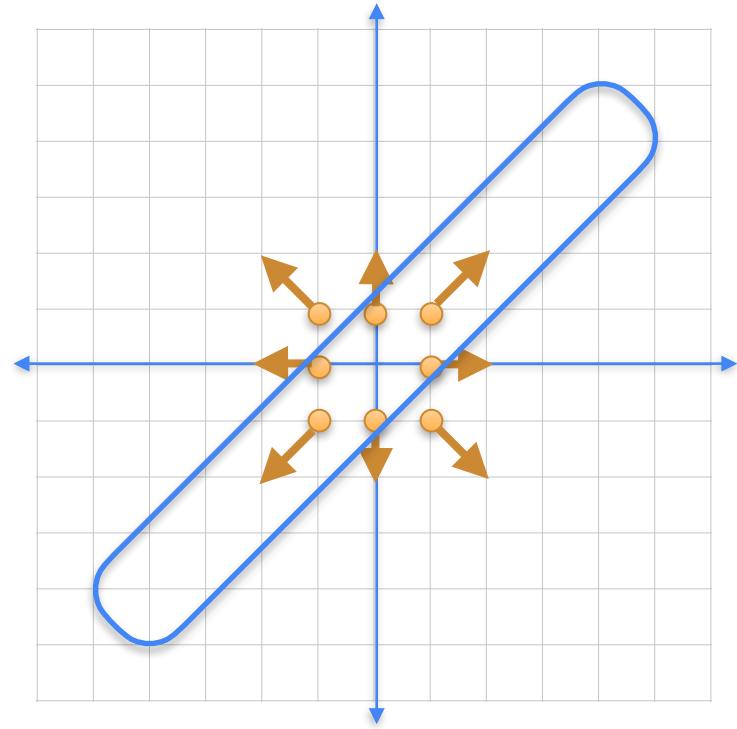


Find eigenvalues

2	1
0	3

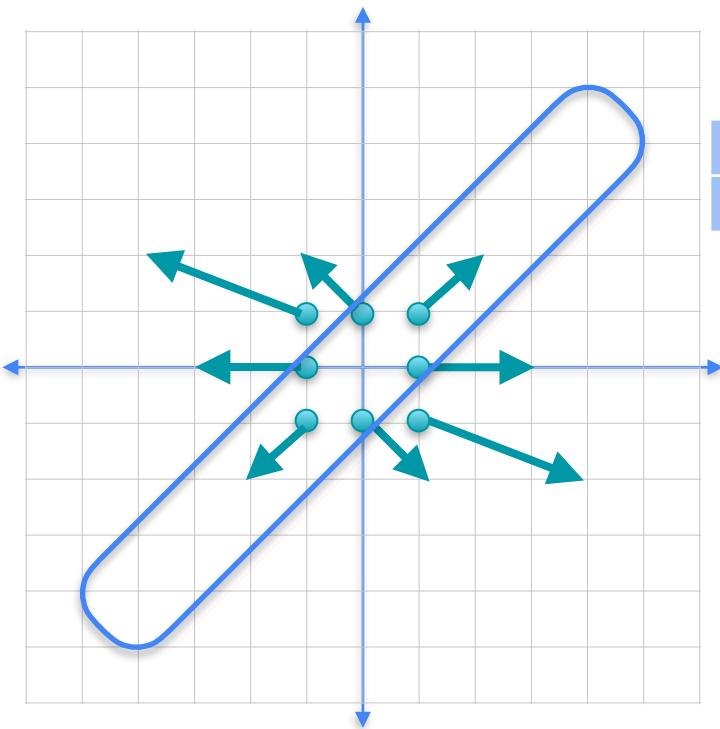


2	0
0	2



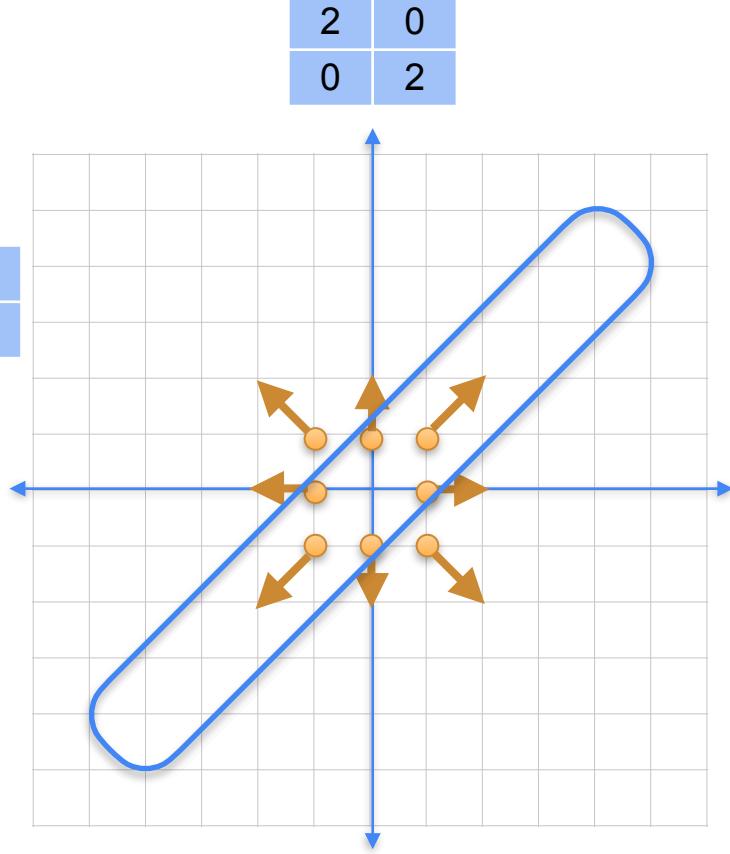
Finding eigenvalues

2	1
0	3



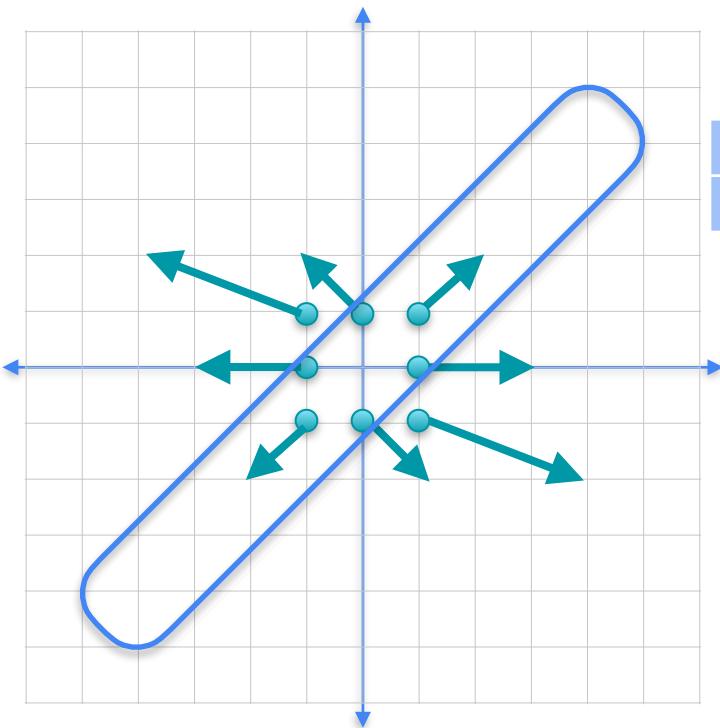
2	1
0	3

2	0
0	2



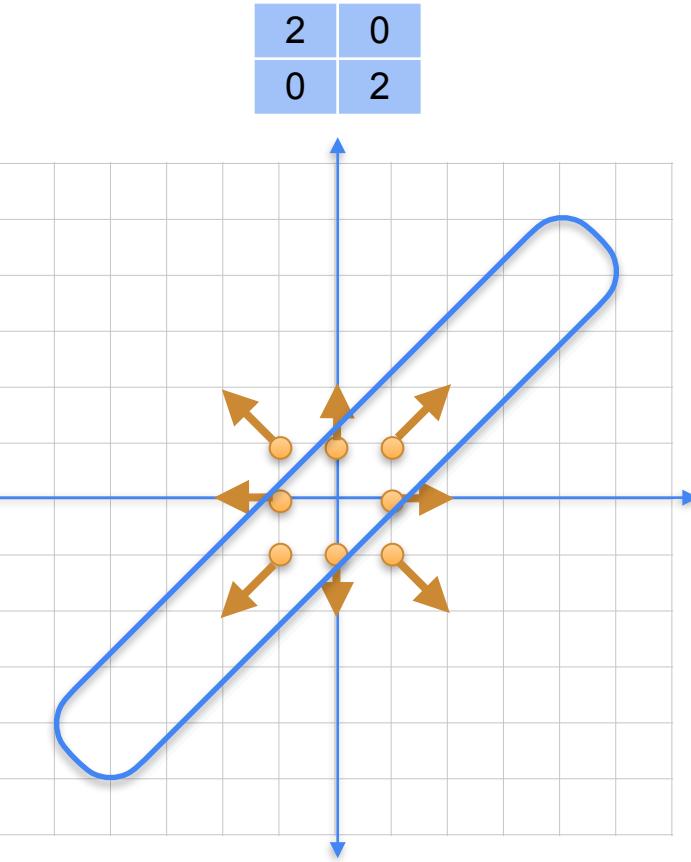
Finding eigenvalues

2	1
0	3



2	1
0	3

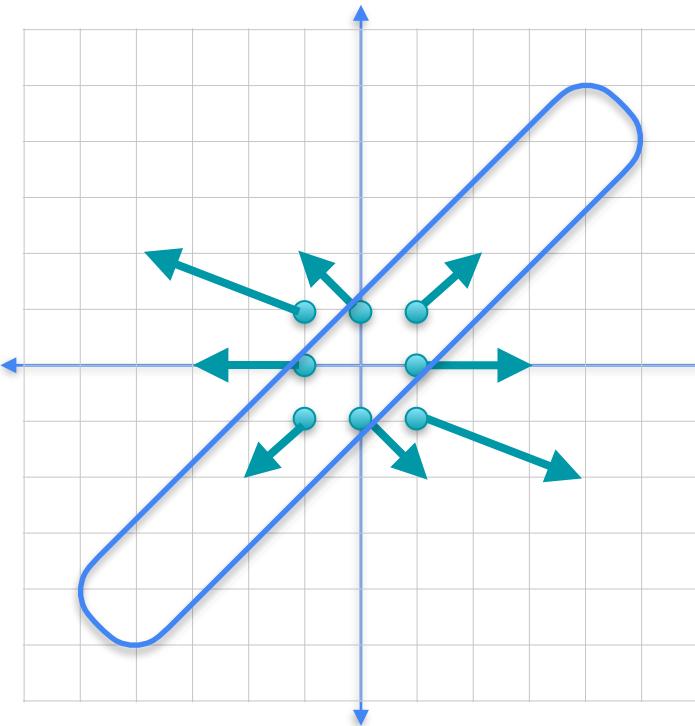
0	1
0	1



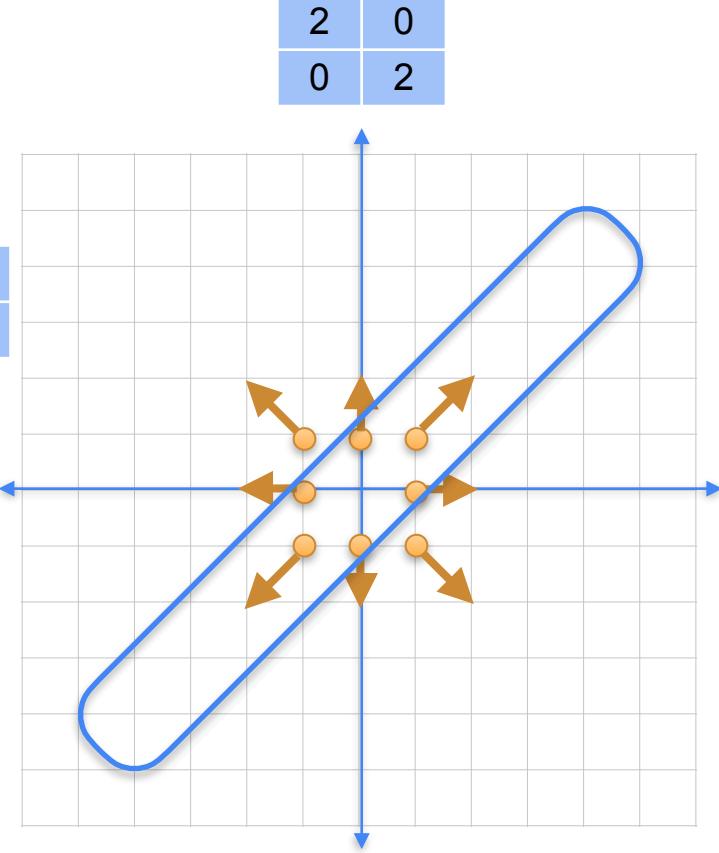
2	0
0	2

Finding eigenvalues

2	1
0	3

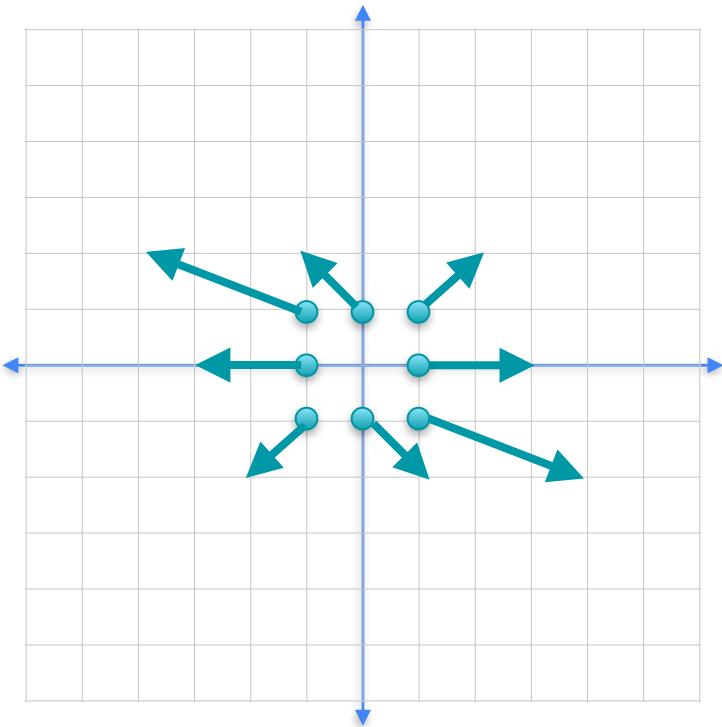


$$\begin{matrix} 2 & 1 \\ 0 & 3 \end{matrix} - \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix} = \det \begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix} = 0$$

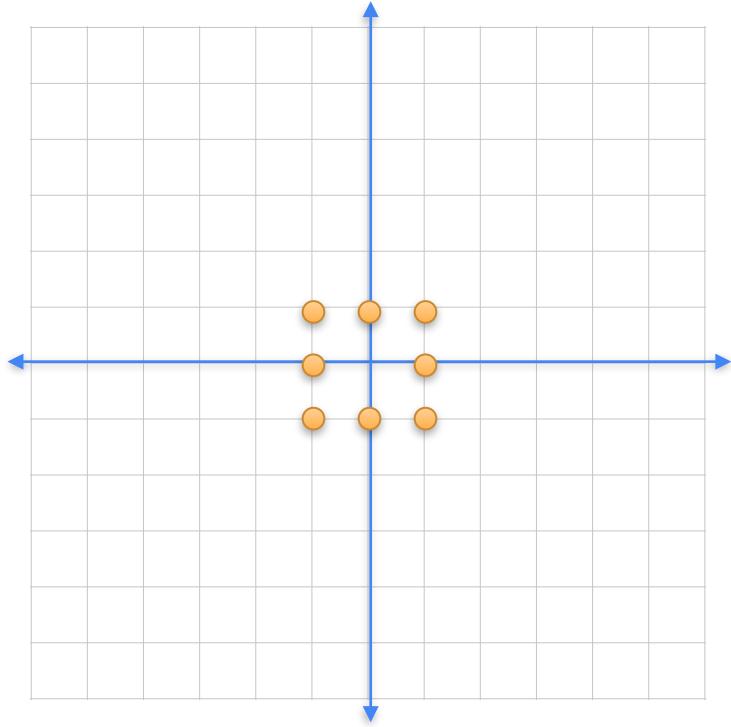


Find eigenvalues

2	1
0	3

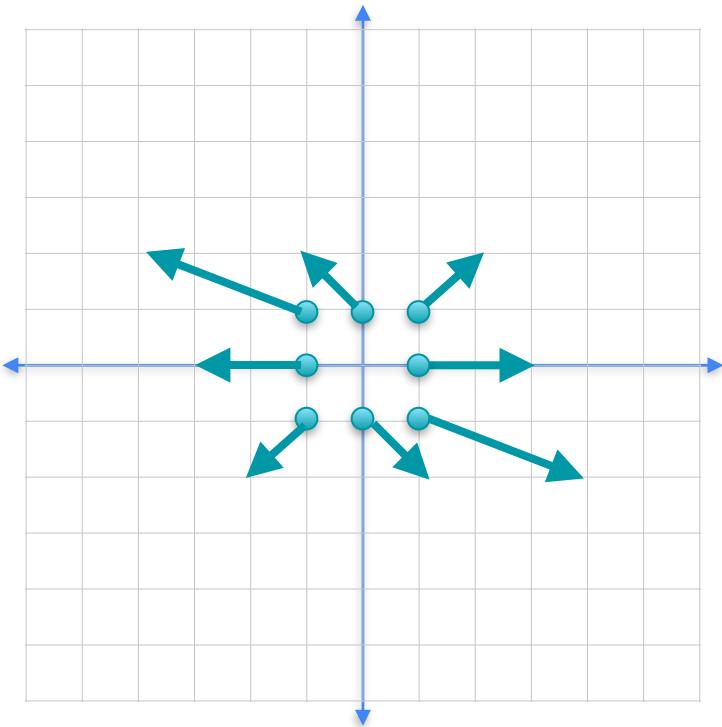


4	0
0	4

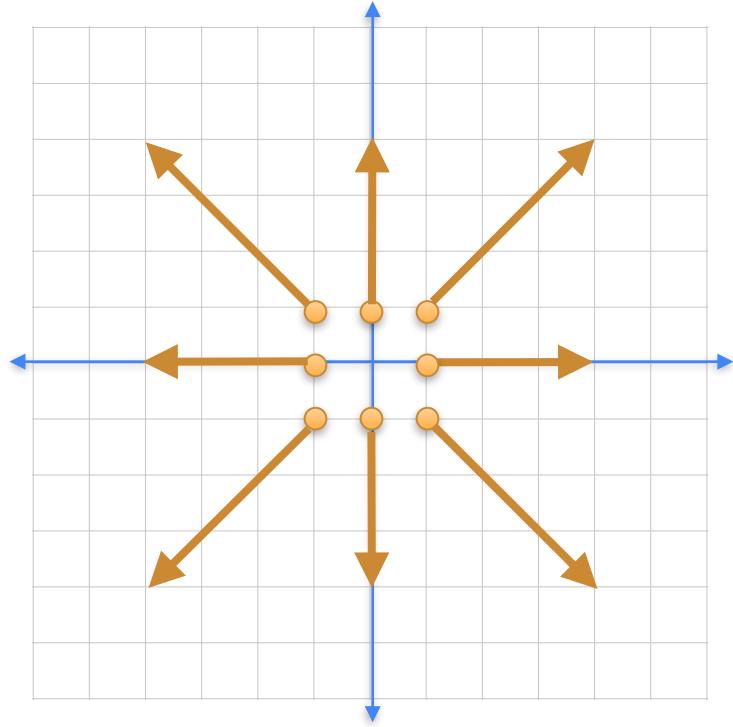


Find eigenvalues

2	1
0	3

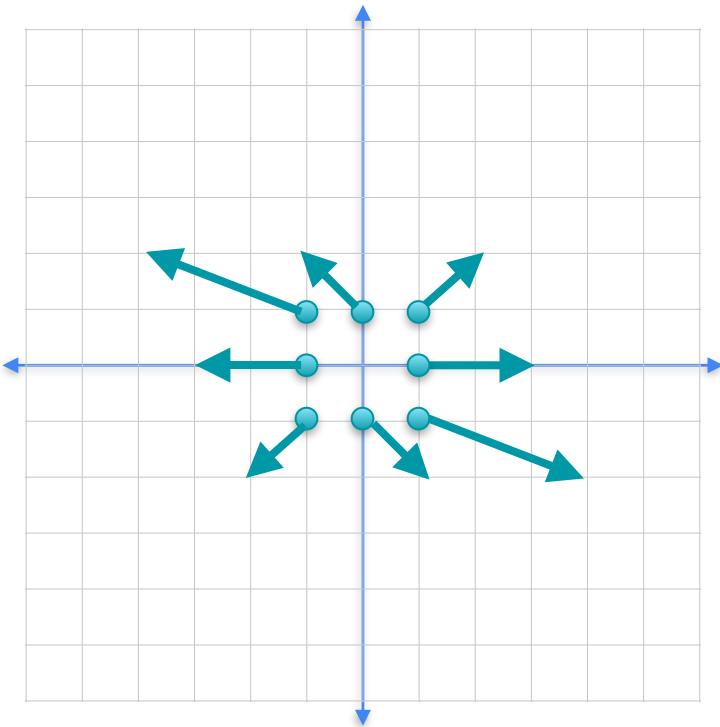


4	0
0	4

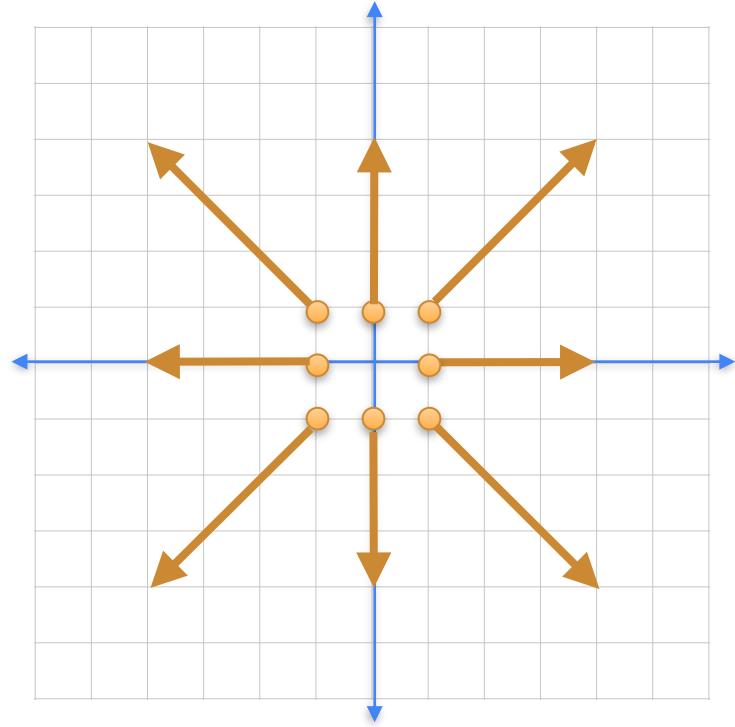


Find eigenvalues

2	1
0	3

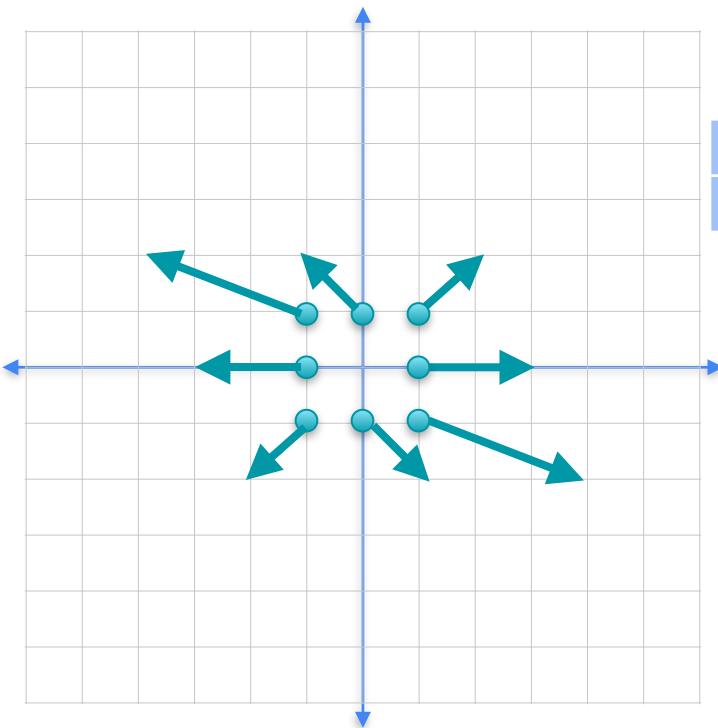


4	0
0	4



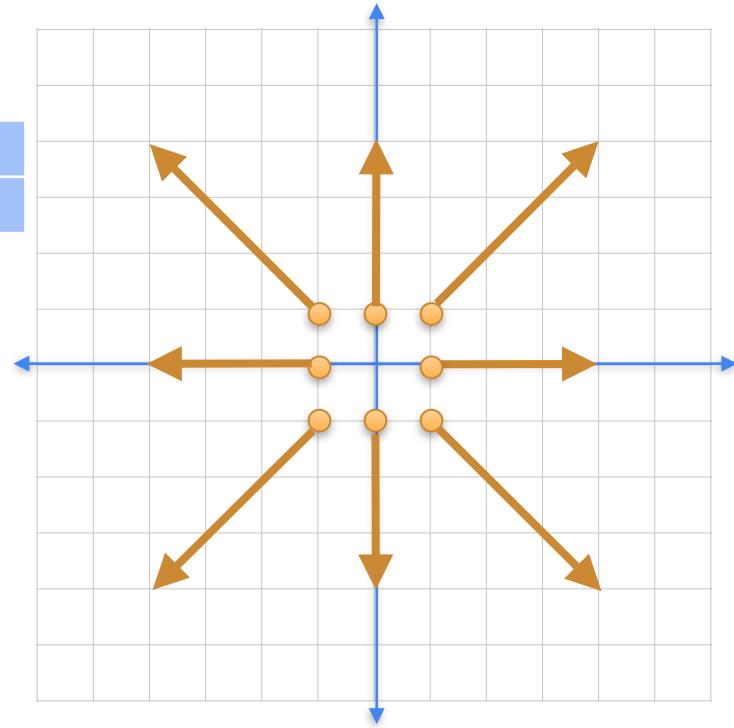
Finding eigenvalues

2	1
0	3



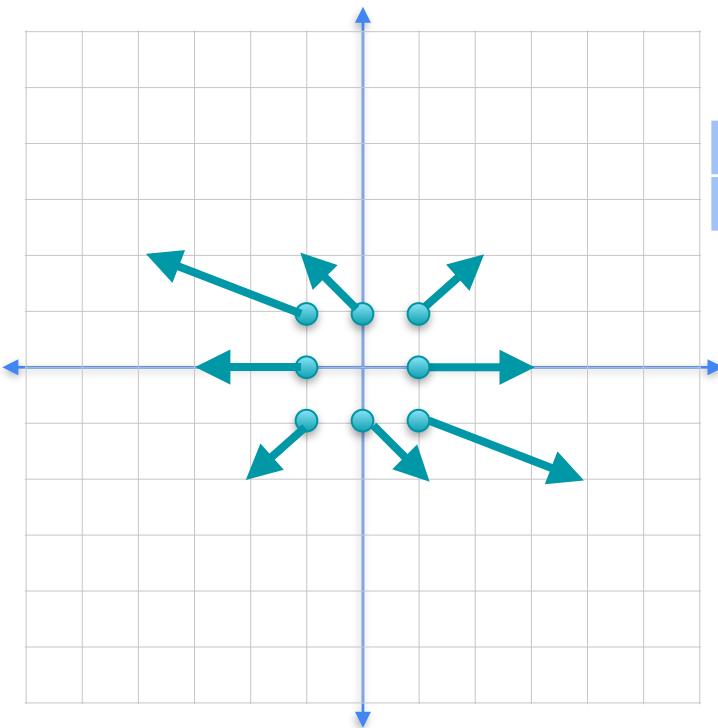
2	1
0	3

4	0
0	4



Finding eigenvalues

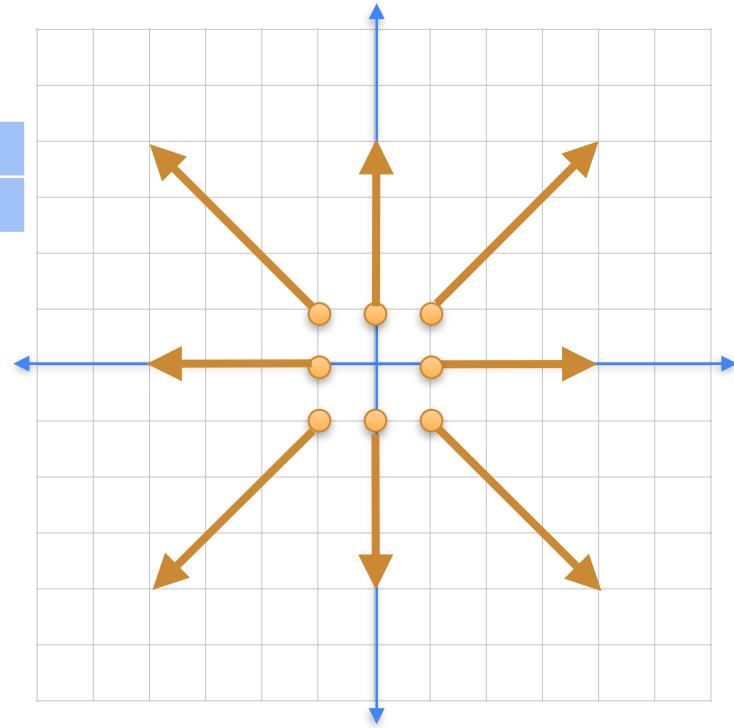
2	1
0	3



2	1
0	3

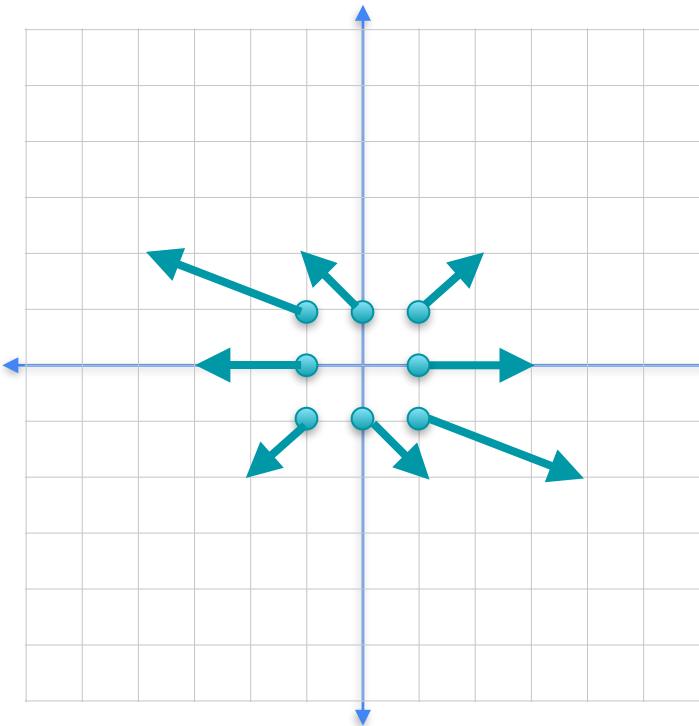
-2	1
0	-1

4	0
0	4



Find eigenvalues

2	1
0	3



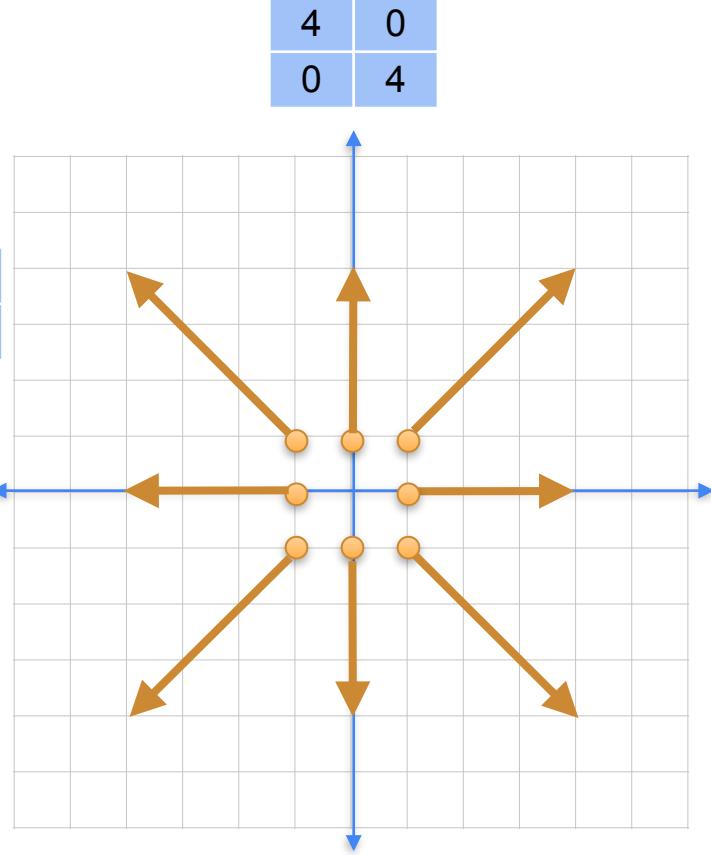
2	1
0	3

4	0
0	4

det

-2	1
0	-1

$\neq 0$



Finding eigenvalues

2	1
0	3

Finding eigenvalues

2	1
0	3

λ	0
0	λ

Finding eigenvalues

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = \begin{matrix} \lambda & 0 & x \\ 0 & \lambda & y \end{matrix}$$

Finding eigenvalues

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = \begin{matrix} \lambda & 0 & x \\ 0 & \lambda & y \end{matrix}$$

$$\begin{matrix} 2-\lambda & 1 & x \\ 0 & 3-\lambda & y \end{matrix} = \begin{matrix} 0 & 0 \end{matrix}$$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial $(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$

Finding eigenvalues

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0 \quad \begin{array}{l} \lambda = 2 \\ \lambda = 3 \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0 \quad \begin{array}{l} \lambda = 2 \\ \lambda = 3 \end{array}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

$$\begin{aligned}\lambda &= 2 \\ \lambda &= 3\end{aligned}$$

Finding eigenvalues

If λ is an eigenvalue:

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \lambda & 0 & x \\ \hline 0 & \lambda & y \\ \hline \end{array}$$

For infinitely many (x,y)

$$\begin{array}{|c|c|c|} \hline 2-\lambda & 1 & x \\ \hline 0 & 3-\lambda & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Has infinitely many solutions

$$\det \begin{array}{|c|c|} \hline 2-\lambda & 1 \\ \hline 0 & 3-\lambda \\ \hline \end{array} = 0$$

Characteristic polynomial

$$(2 - \lambda)(3 - \lambda) - 1 \cdot 0 = 0$$

$$\lambda = 2$$

$$\lambda = 3$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = 2 \begin{matrix} x \\ y \end{matrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

$$x = 1$$

$$0x + 3y = 2y$$

$$y = 0$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{matrix} 2 & 1 & x \\ 0 & 3 & y \end{matrix} = 2 \begin{matrix} x \\ y \end{matrix}$$

$$2x + y = 2x$$

$$x = 1$$

$$0x + 3y = 2y$$

$$y = 0$$

$$\begin{matrix} 1 \\ 0 \end{matrix}$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

Finding eigenvectors

Eigenvalues:

$$\lambda = 2$$

$$\lambda = 3$$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 2x$$

$$x = 1$$

$$\begin{array}{|c|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$0x + 3y = 2y$$

$$y = 0$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$

$$2x + y = 3x$$

$$0x + 3y = 3y$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$

Finding eigenvectors

Eigenvalues: $\lambda = 2$
 $\lambda = 3$

Solve the equations

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 2 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 2x$$
$$0x + 3y = 2y$$
$$x = 1$$
$$y = 0$$
$$\begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & 1 & x \\ \hline 0 & 3 & y \\ \hline \end{array} = 3 \begin{array}{|c|c|} \hline x \\ \hline y \\ \hline \end{array}$$
$$2x + y = 3x$$
$$0x + 3y = 3y$$
$$x = 1$$
$$y = 1$$
$$\begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \end{array}$$

Quiz

- Find the eigenvalues and eigenvectors of this matrix:

9	4
4	3

Solution

- Eigenvalues: 11, 1
- Eigenvectors: (2,1), (-1,2)

9	4
4	3

- The characteristic polynomial is

$$\det \begin{array}{|cc|} \hline 9-\lambda & 4 \\ 4 & 3-\lambda \\ \hline \end{array} = (9 - \lambda)(3 - \lambda) - 4 \cdot 4 = 0$$

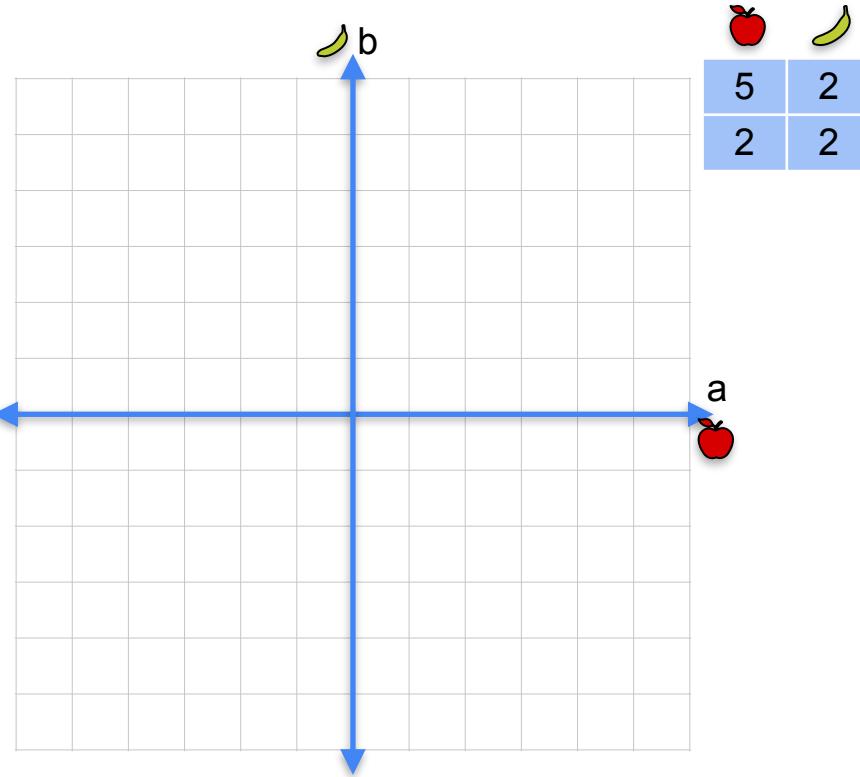
- Which factors as $\lambda^2 - 12\lambda + 11 = (\lambda - 11)(\lambda - 1)$

- The solutions are $\lambda = 11$
 $\lambda = 1$

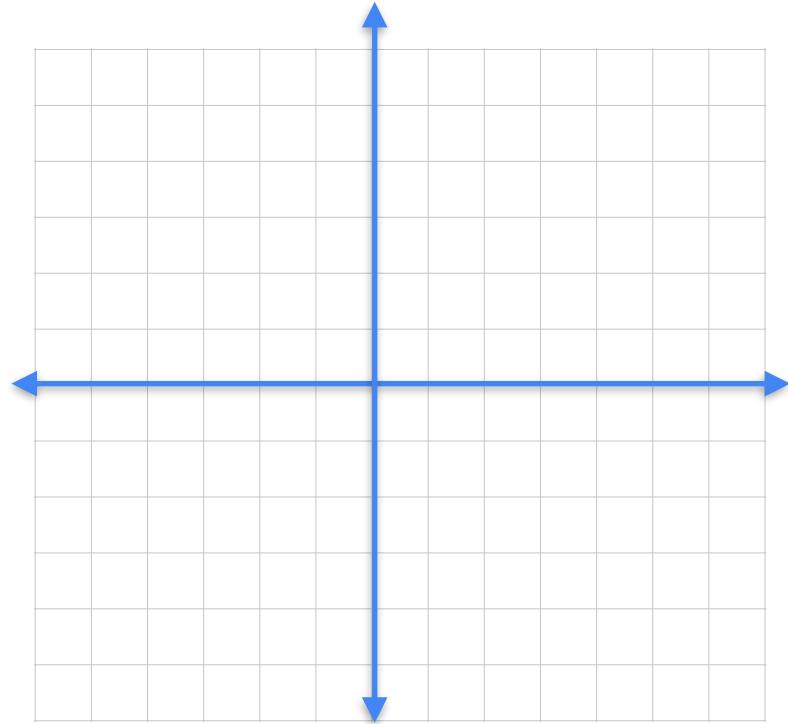
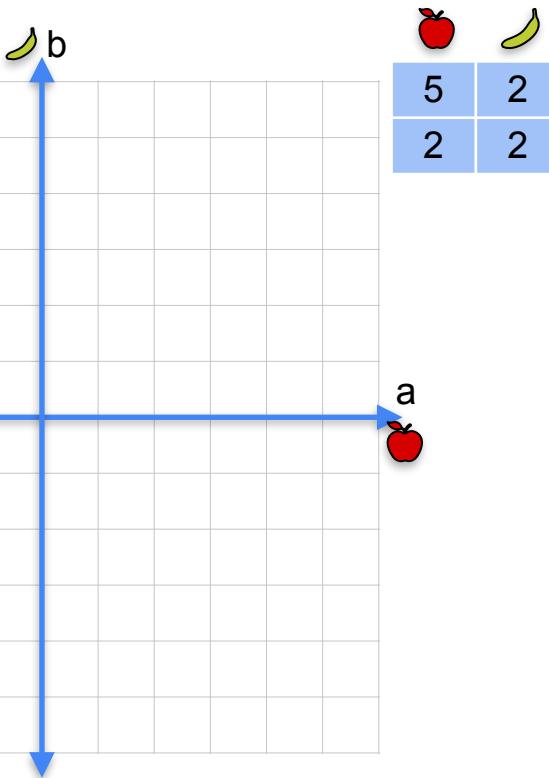
Matrices as linear transformations

	
5	2
2	2

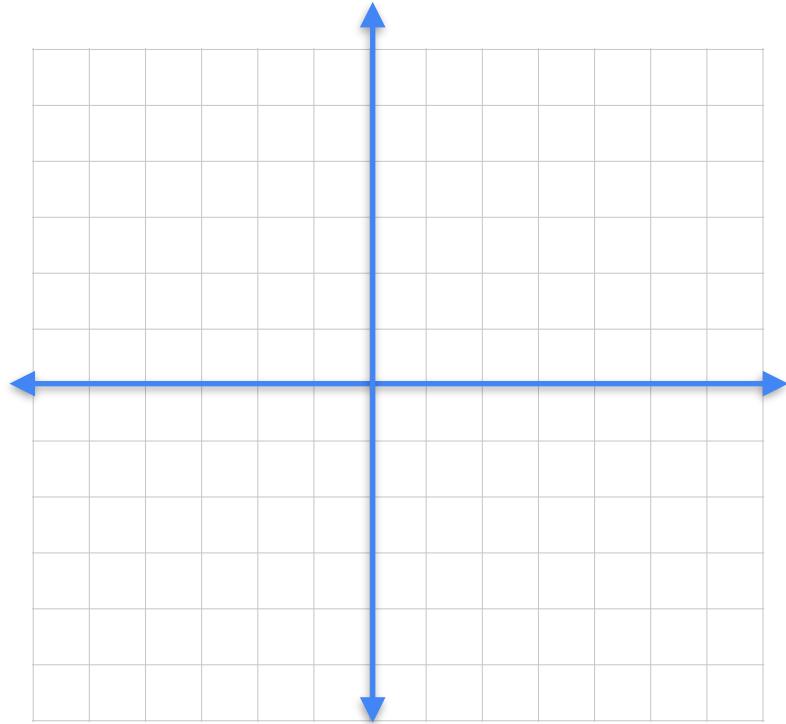
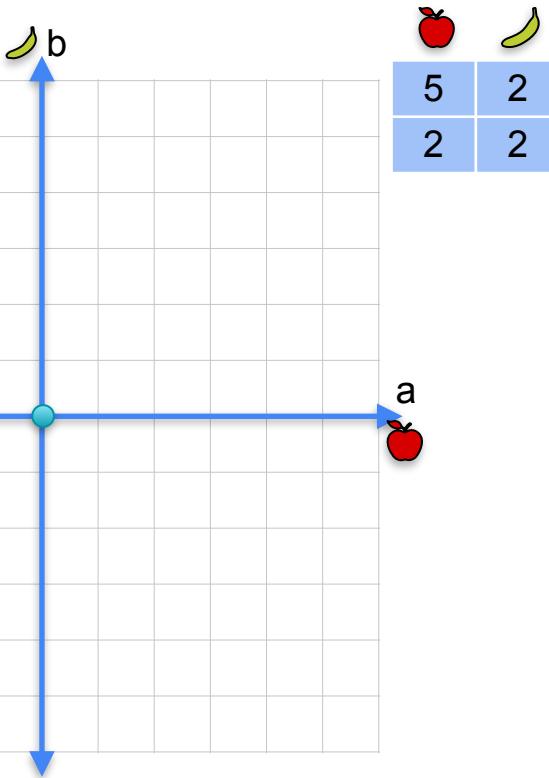
Matrices as linear transformations



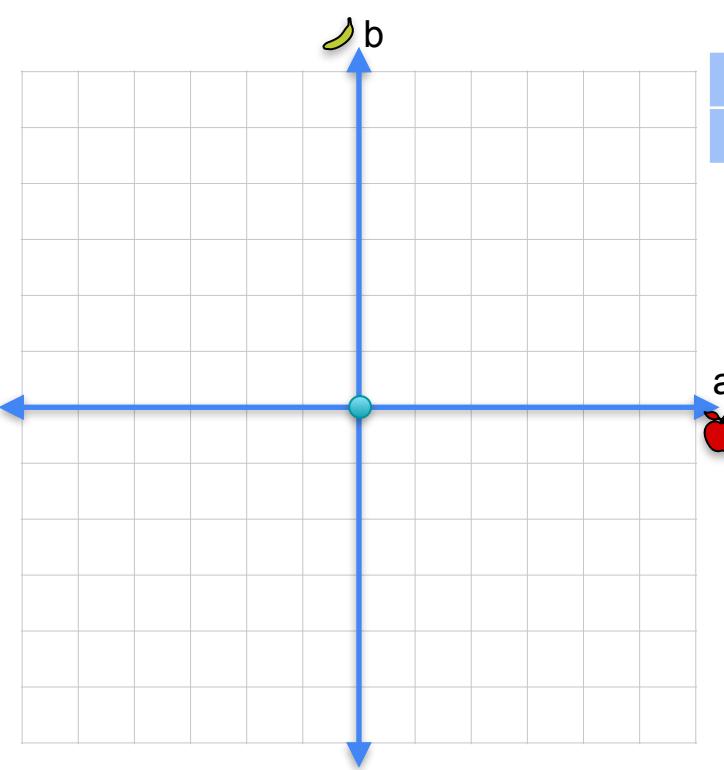
Matrices as linear transformations



Matrices as linear transformations

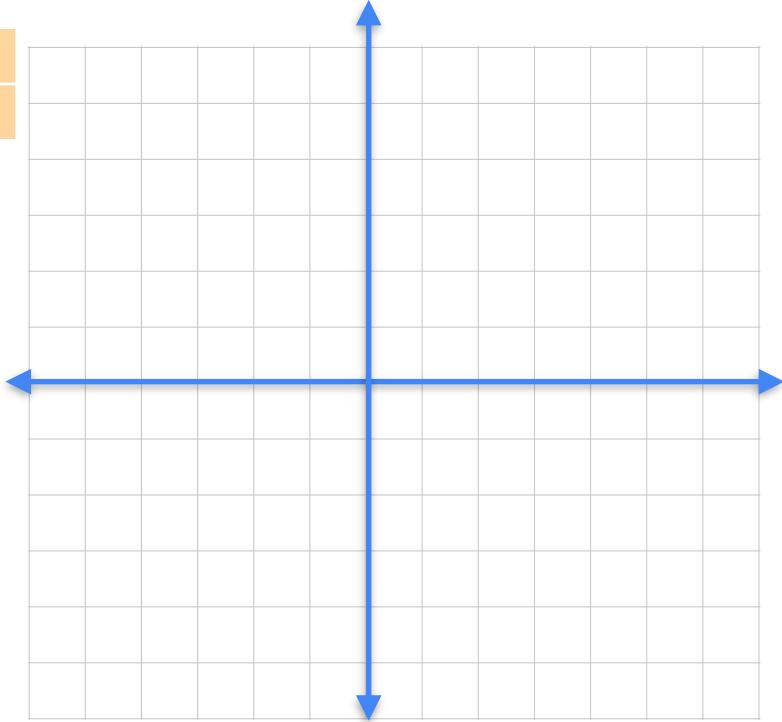


Matrices as linear transformations



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$



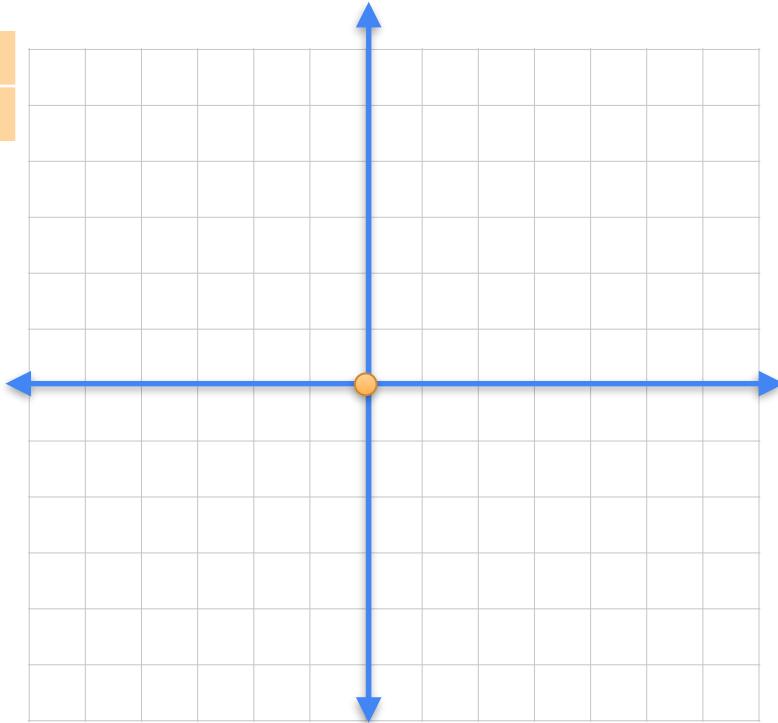
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ 5 & 2 & 0 \\ 2 & 2 & 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$



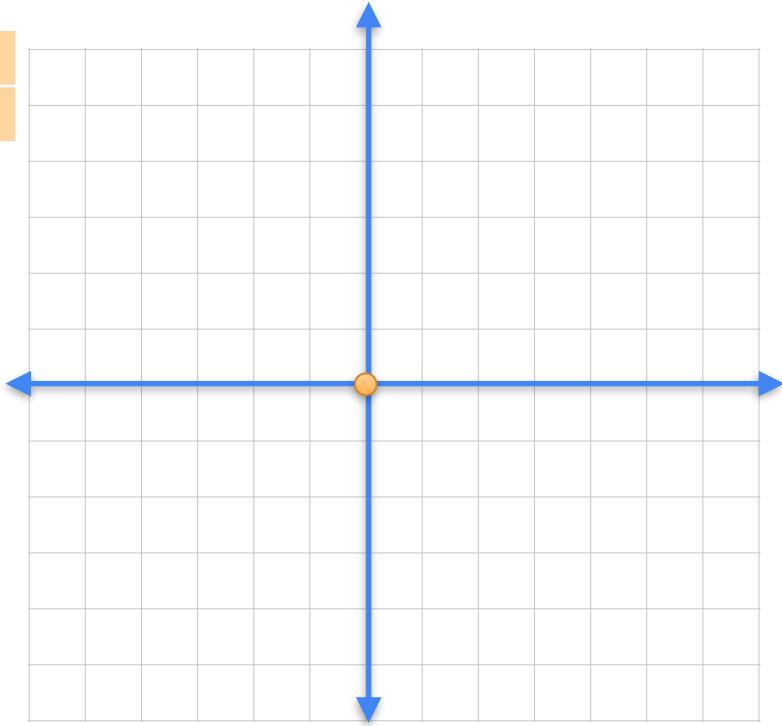
Matrices as linear transformations

b

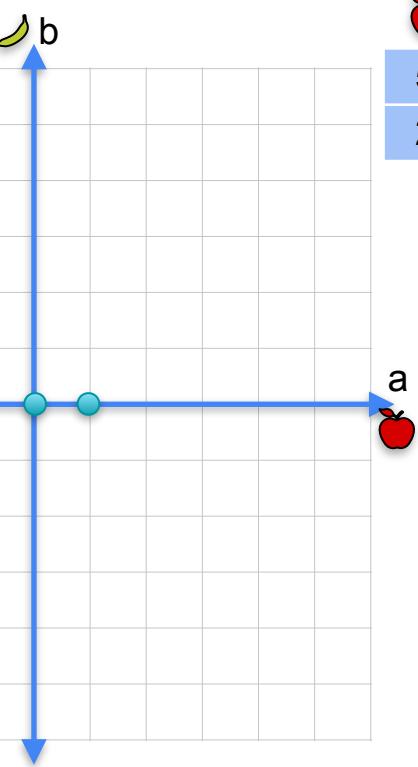
a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$

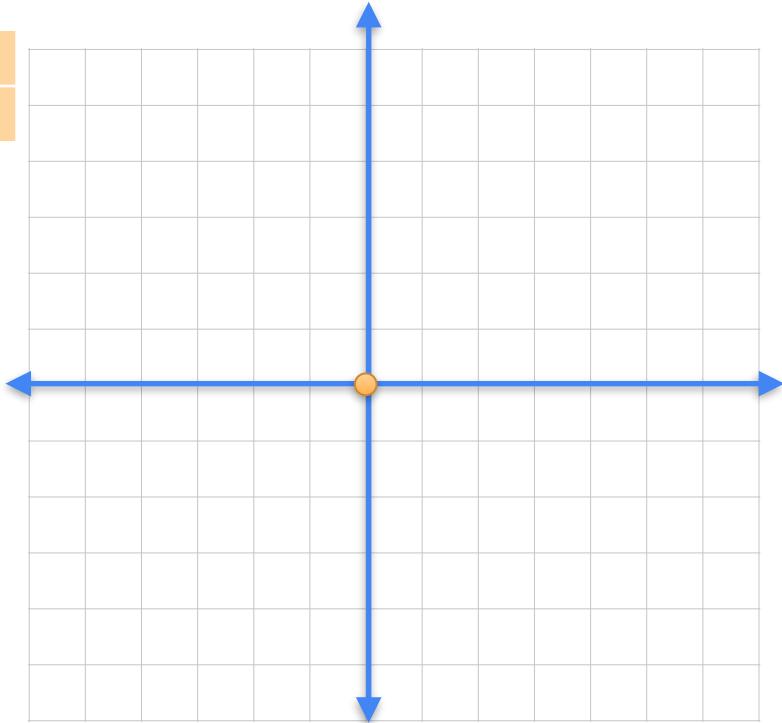


Matrices as linear transformations

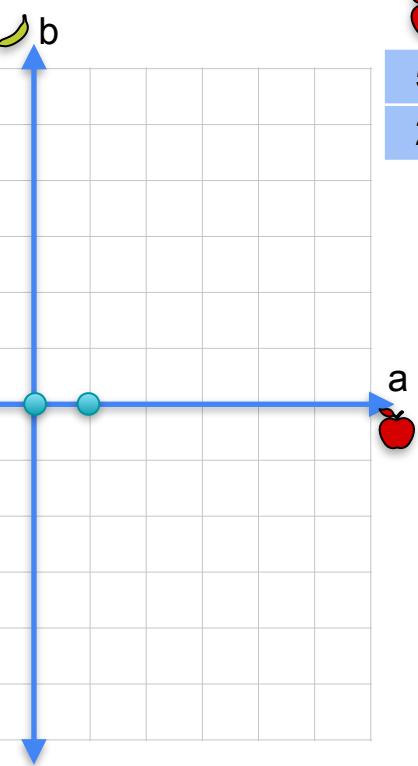


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

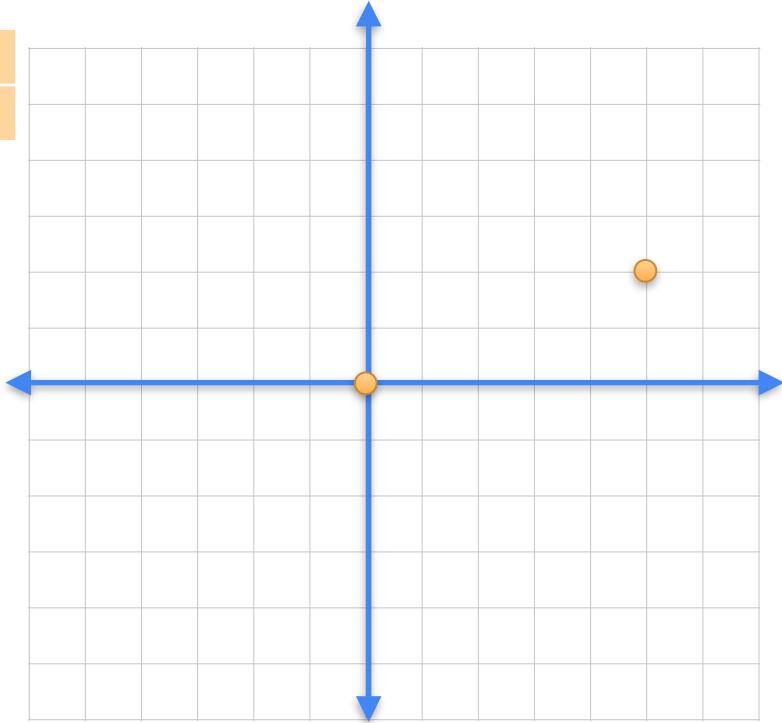


Matrices as linear transformations

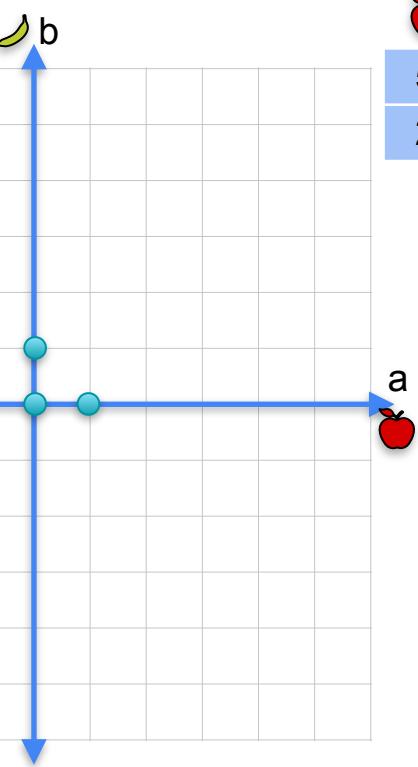


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$

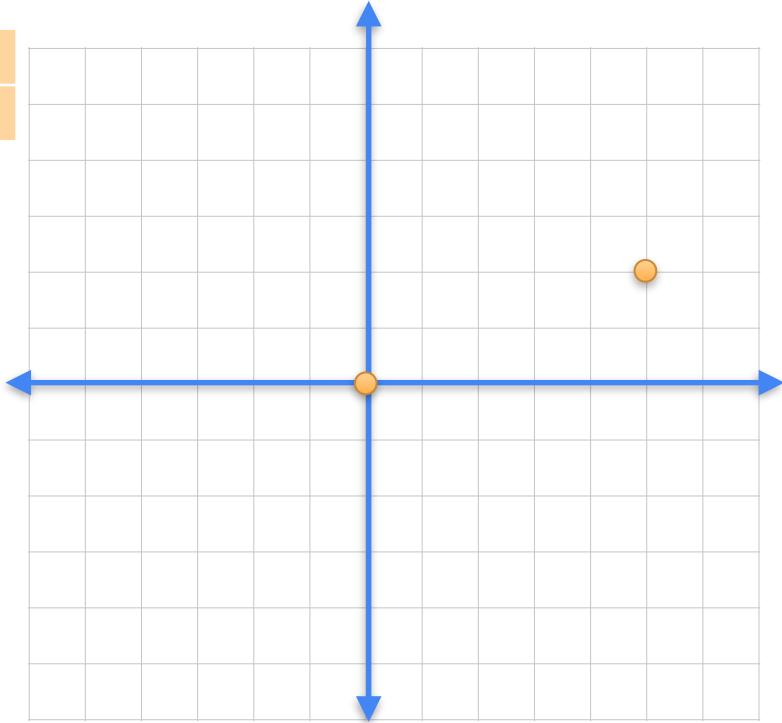


Matrices as linear transformations



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \end{aligned}$$



Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 0 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 2 & 2 \\ 2 & 2 \end{matrix} \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$

o

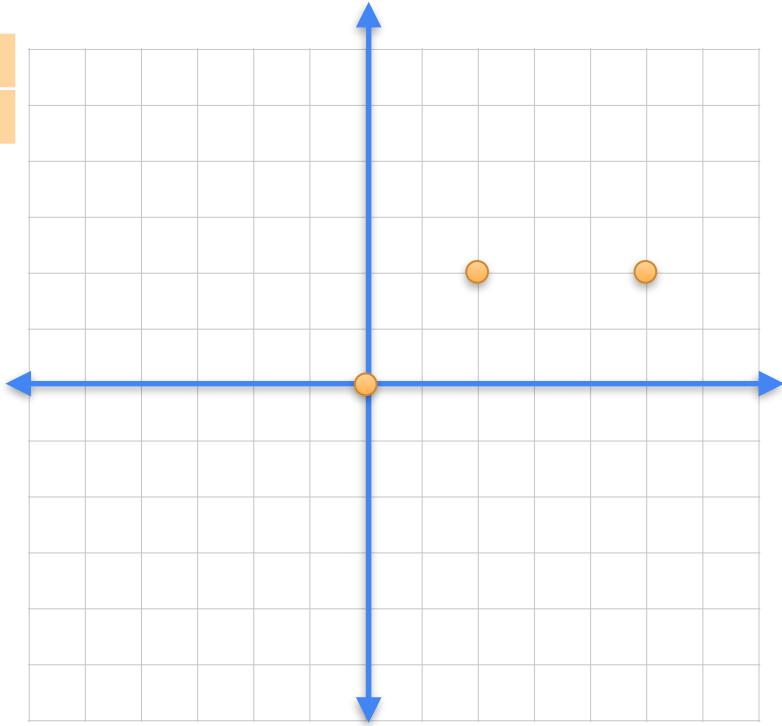
Matrices as linear transformations

b

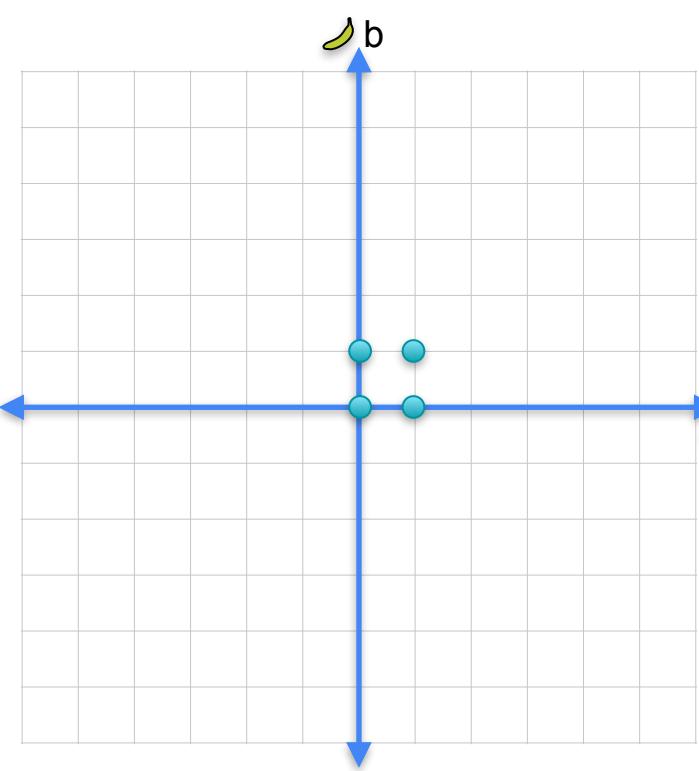
a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 0 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 2 & 2 \\ 2 & 2 \end{matrix} \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \end{aligned}$$

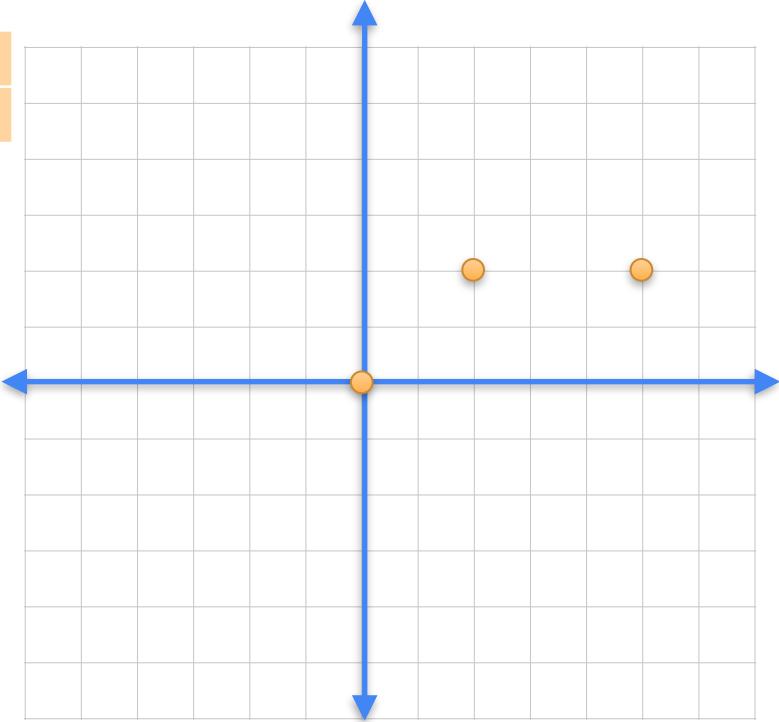


Matrices as linear transformations

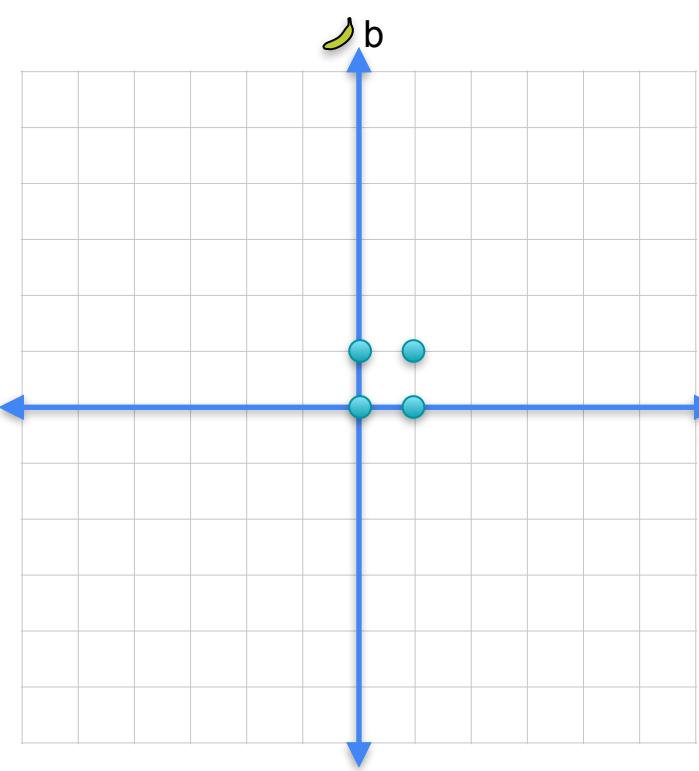


A diagram illustrating matrix multiplication. On the left, there is a 2x3 matrix with columns labeled 'apple' and 'banana'. The matrix has entries 5, 2, 0 in the first row and 2, 2, 1 in the second row. To the right of the matrix is an equals sign followed by a 2x1 column vector with entries 2 and 2. This represents the multiplication of the matrix by a column vector.

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$

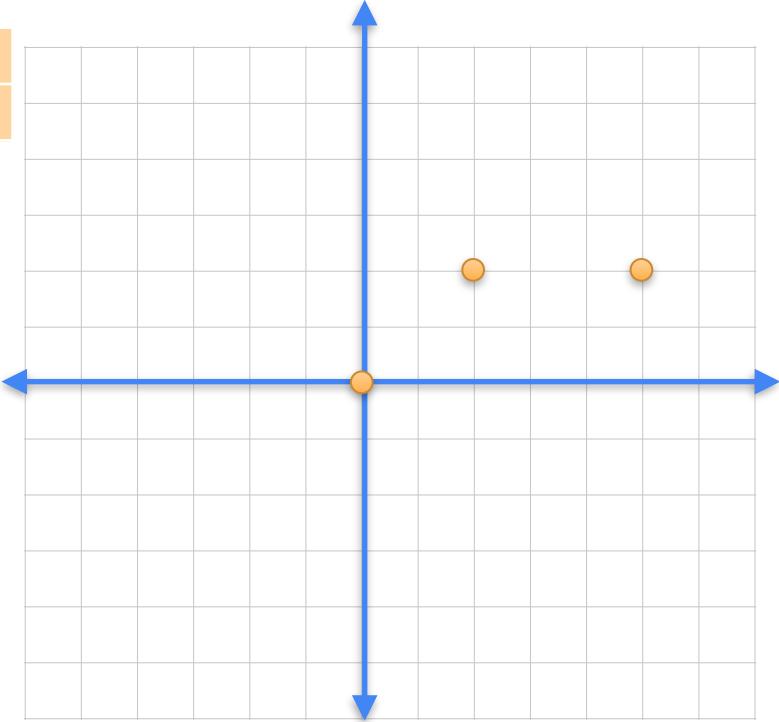


Matrices as linear transformations



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 1 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} 7 \\ 2 \end{matrix} \end{matrix}$$

$$\begin{aligned} (0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (3,1) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (4,3) \end{aligned}$$



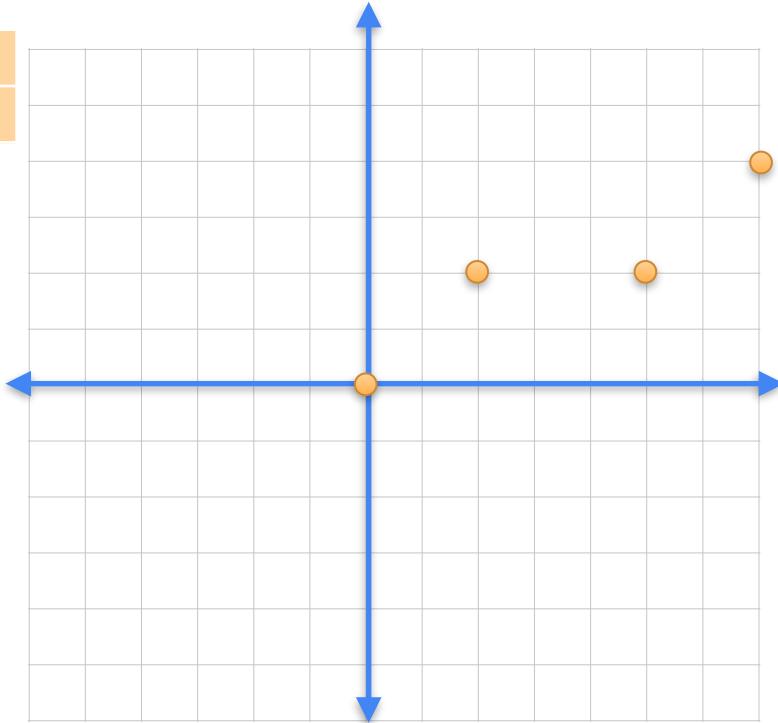
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 7 & 2 \\ 2 & \end{matrix} \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$



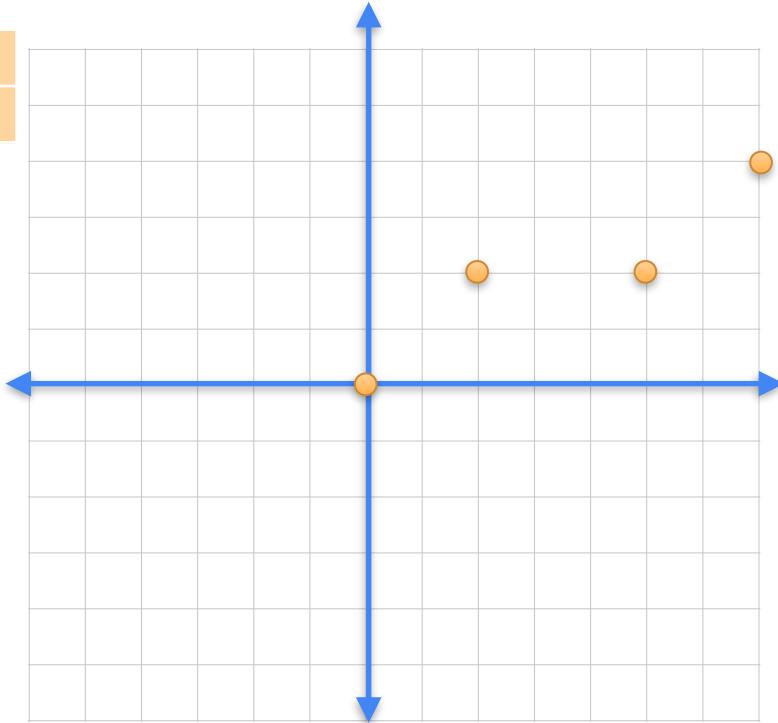
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 7 & 2 \\ 2 & \end{matrix} \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$



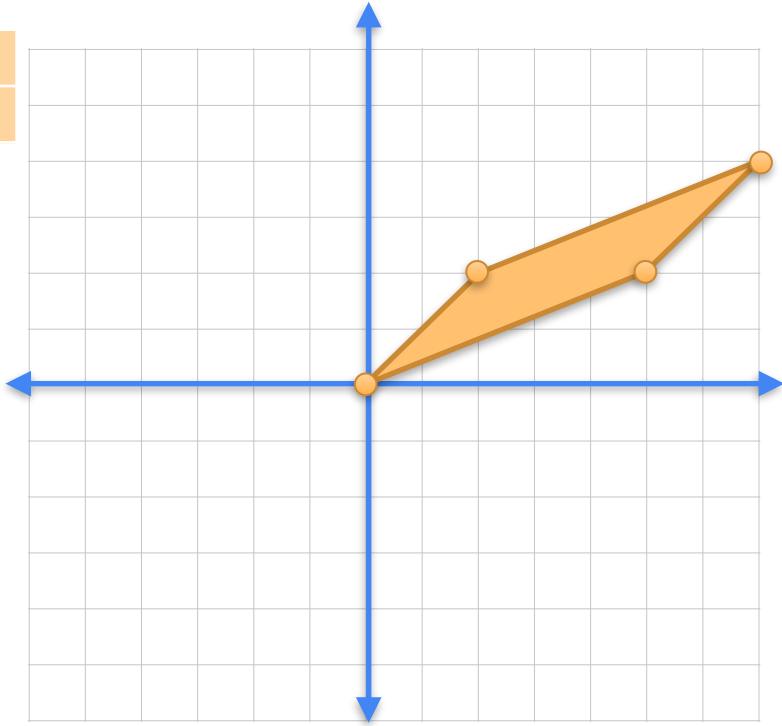
Matrices as linear transformations

b

a

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 5 & 2 & 1 \\ 2 & 2 & 1 \end{matrix} & = \begin{matrix} 7 & 2 \\ \text{apple} & \text{banana} \end{matrix} \end{matrix}$$

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$

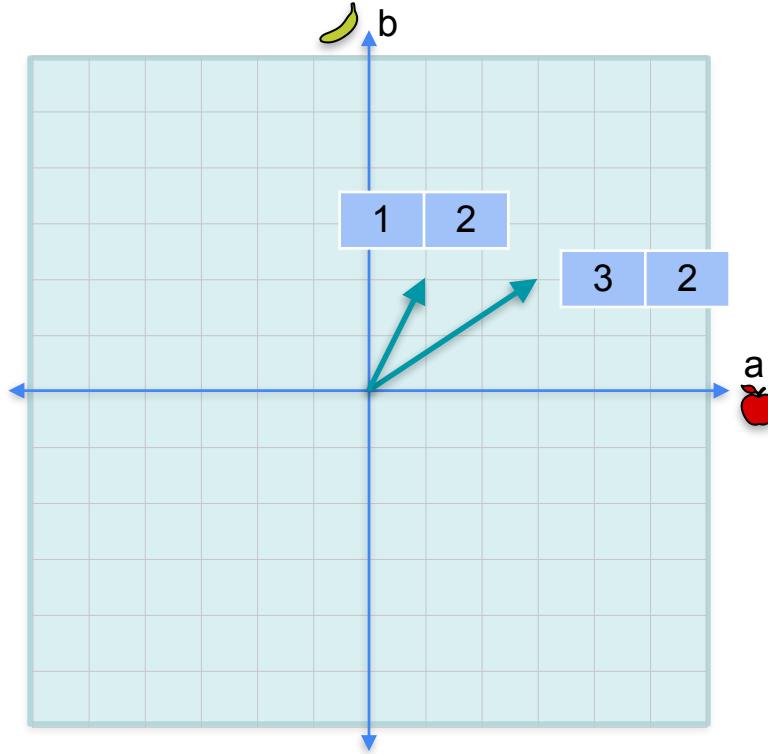


Row span of a matrix

	
3	2
1	2

Rows

3	2
1	2

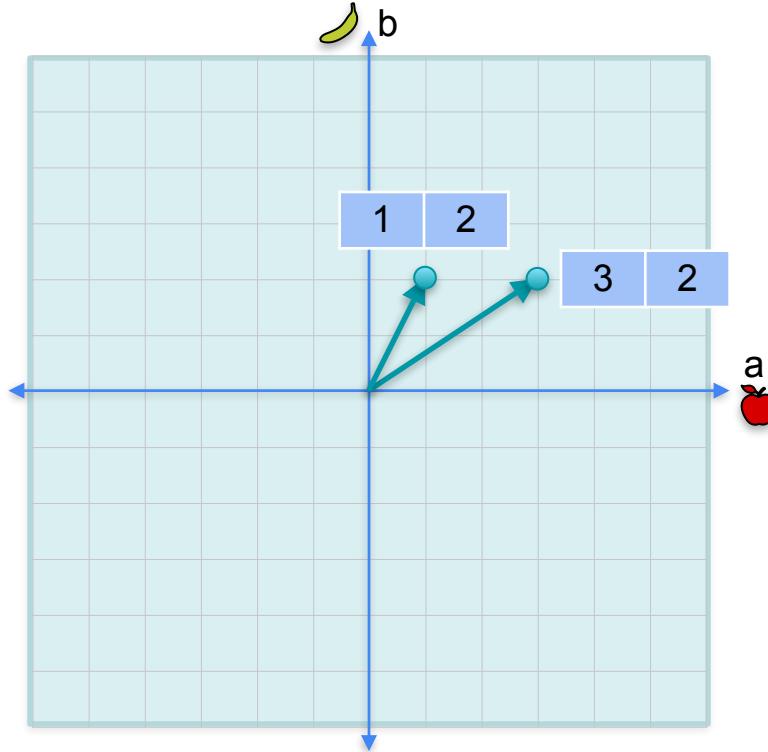


Row span of a matrix

	
3	2
1	2

Rows

3	2
1	2

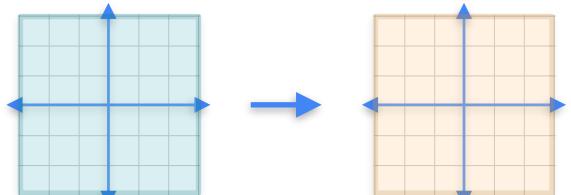


Span of the rows

Non-singular

3	1
1	2

Rank = 2

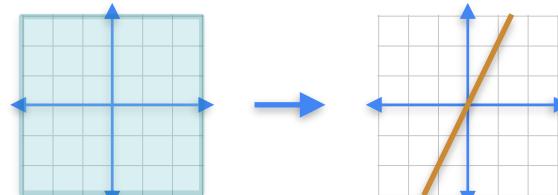


The whole plane

Singular

1	1
2	2

Rank = 1

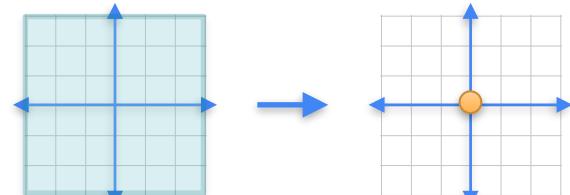


A line

Singular

0	0
0	0

Rank = 0



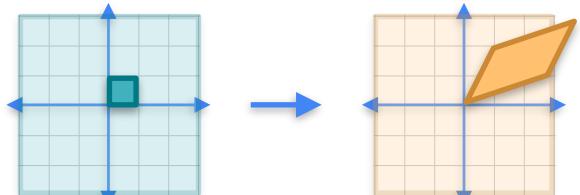
A point

Basis vectors

Non-singular

3	1
1	2

Rank = 2

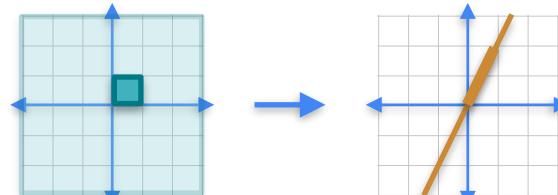


The whole plane

Singular

1	1
2	2

Rank = 1

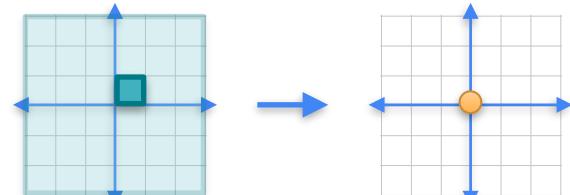


A line

Singular

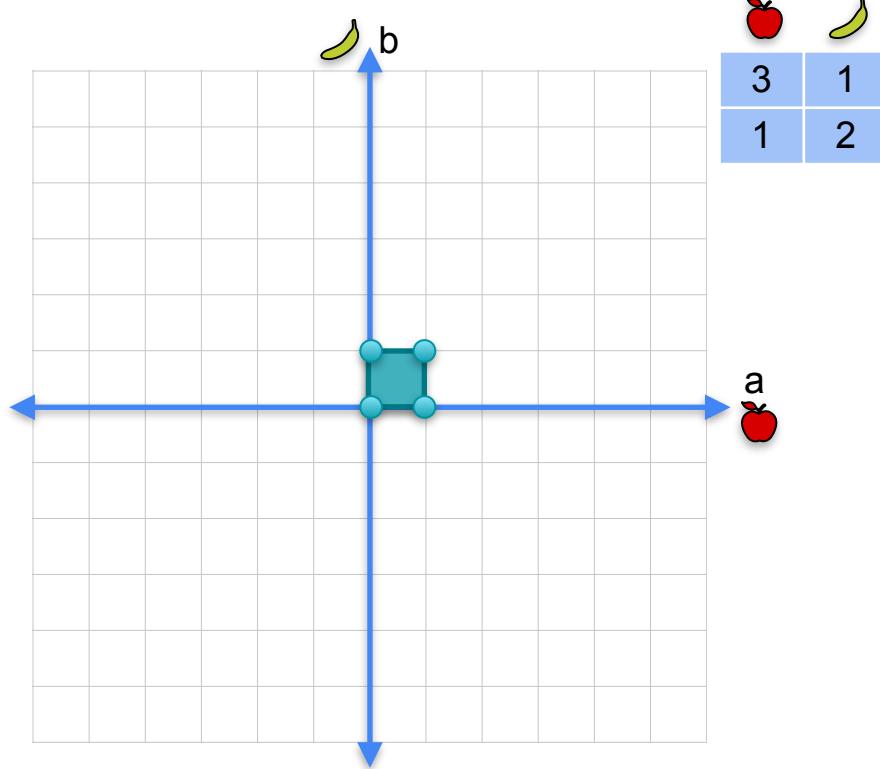
0	0
0	0

Rank = 0

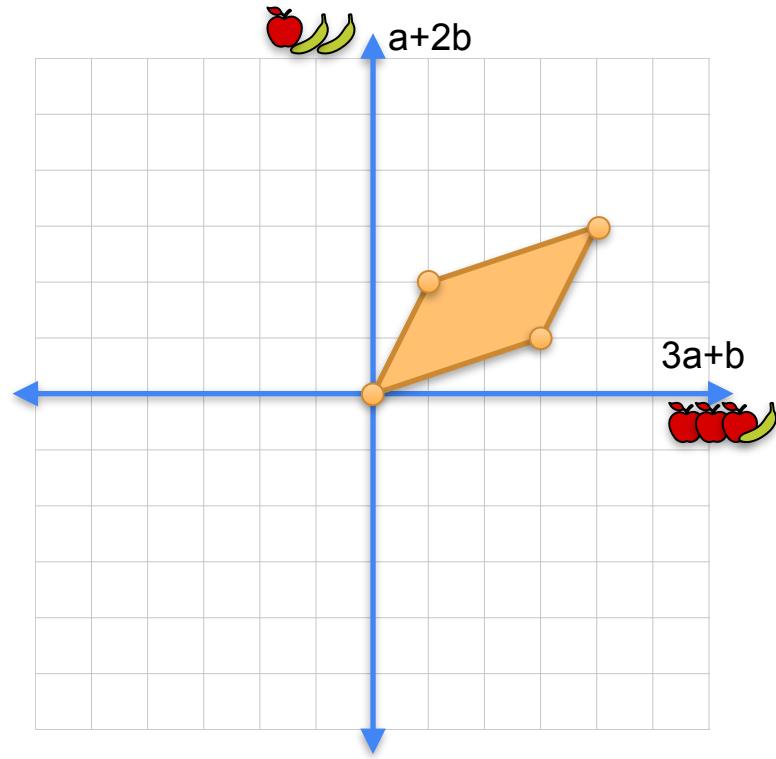


A point

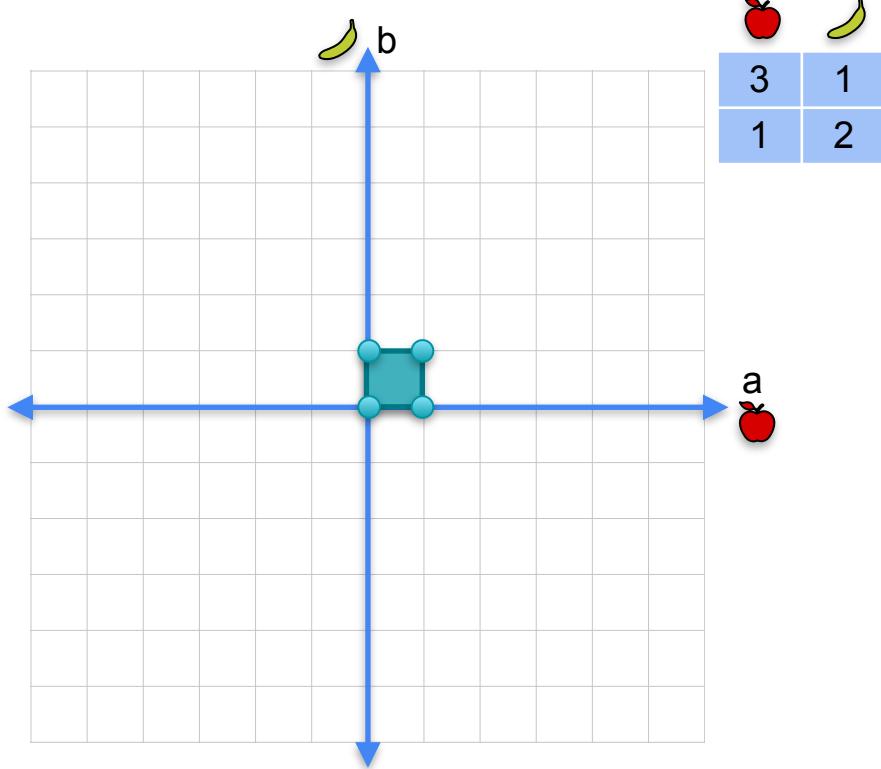
Linear transformation



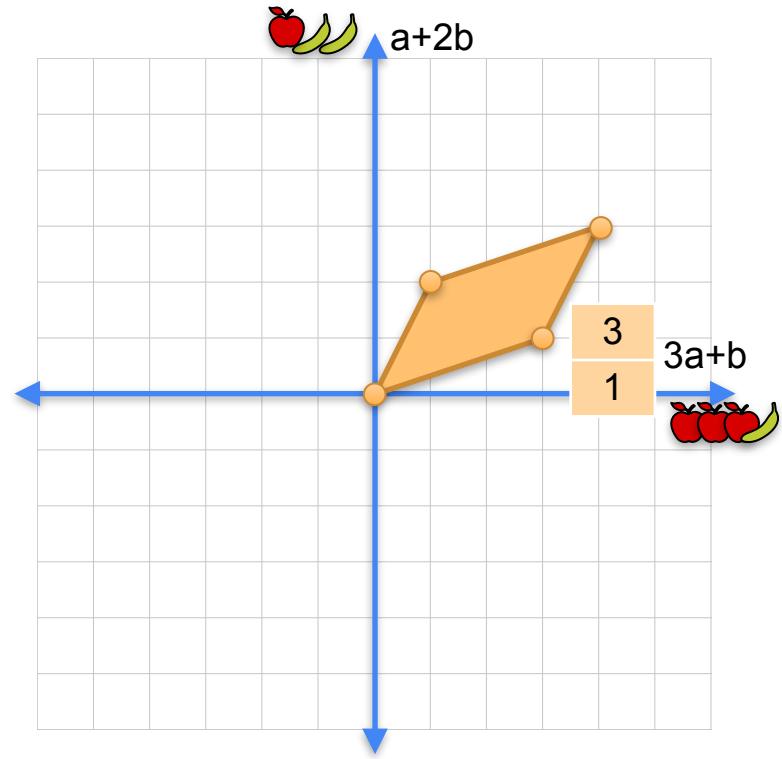
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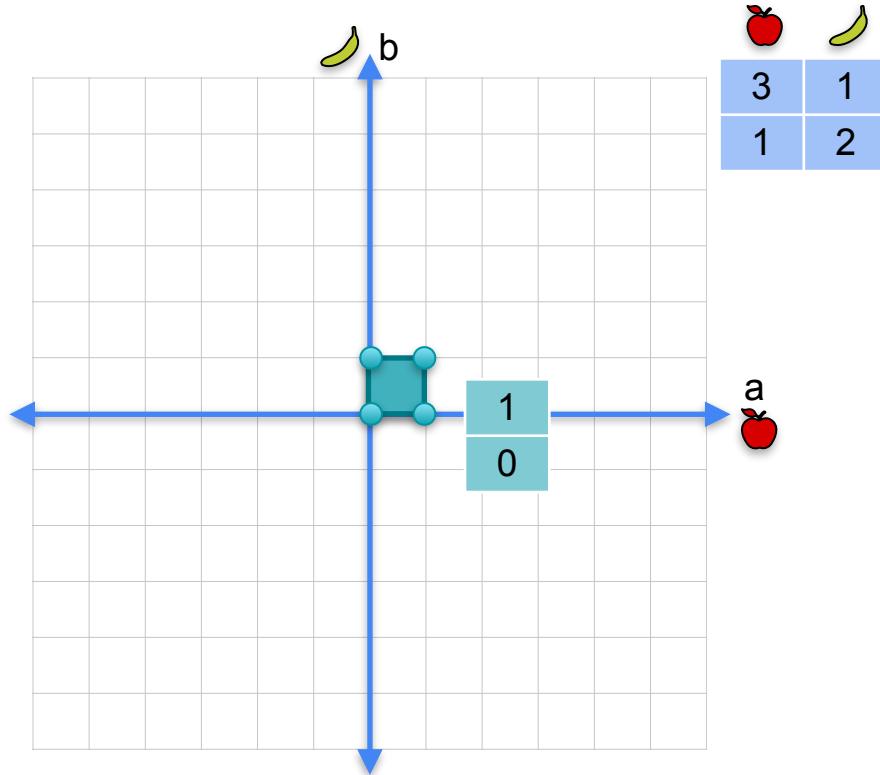
Linear transformation



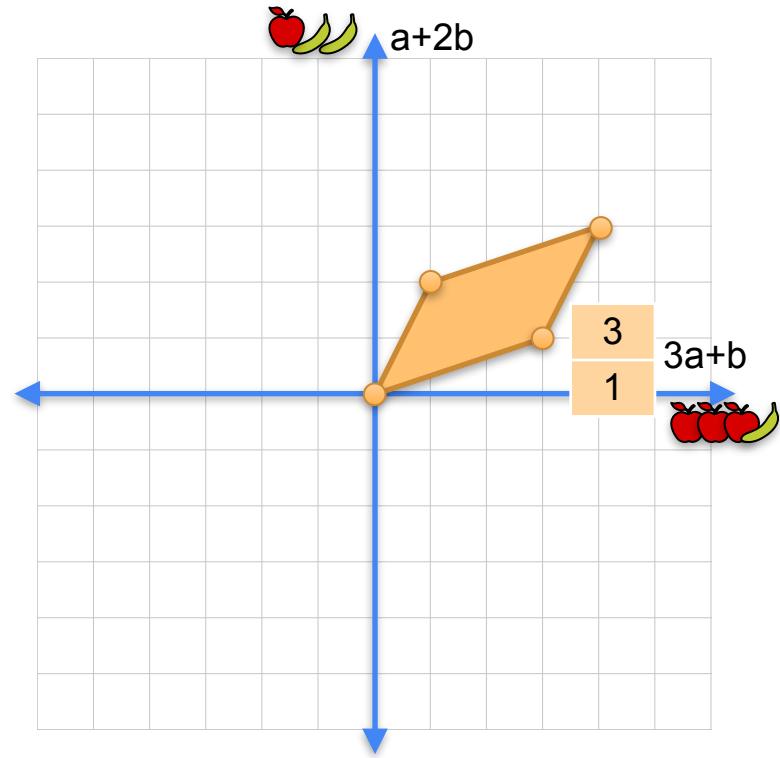
=



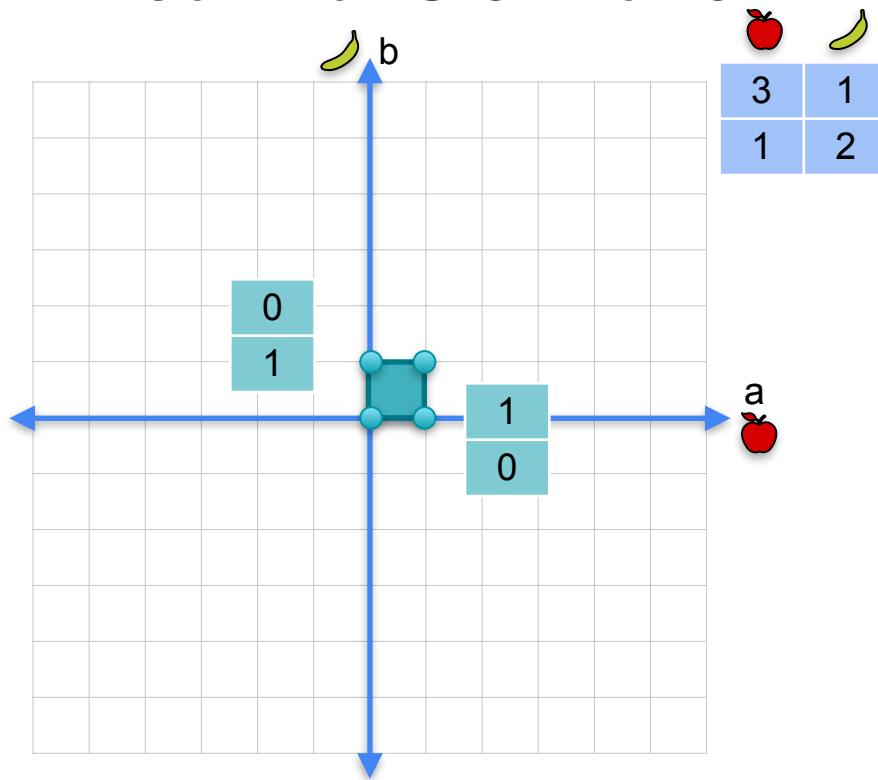
Linear transformation



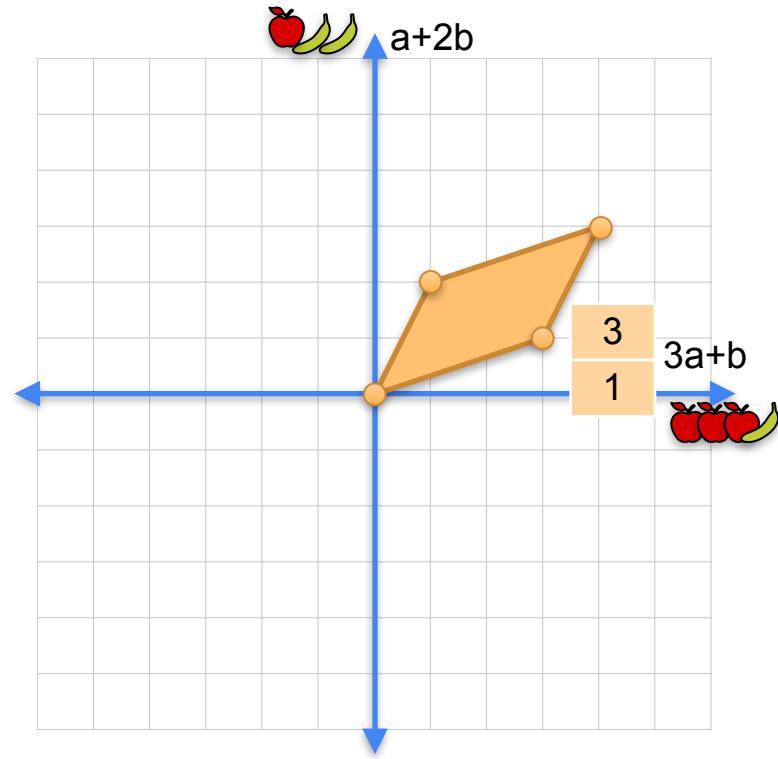
=



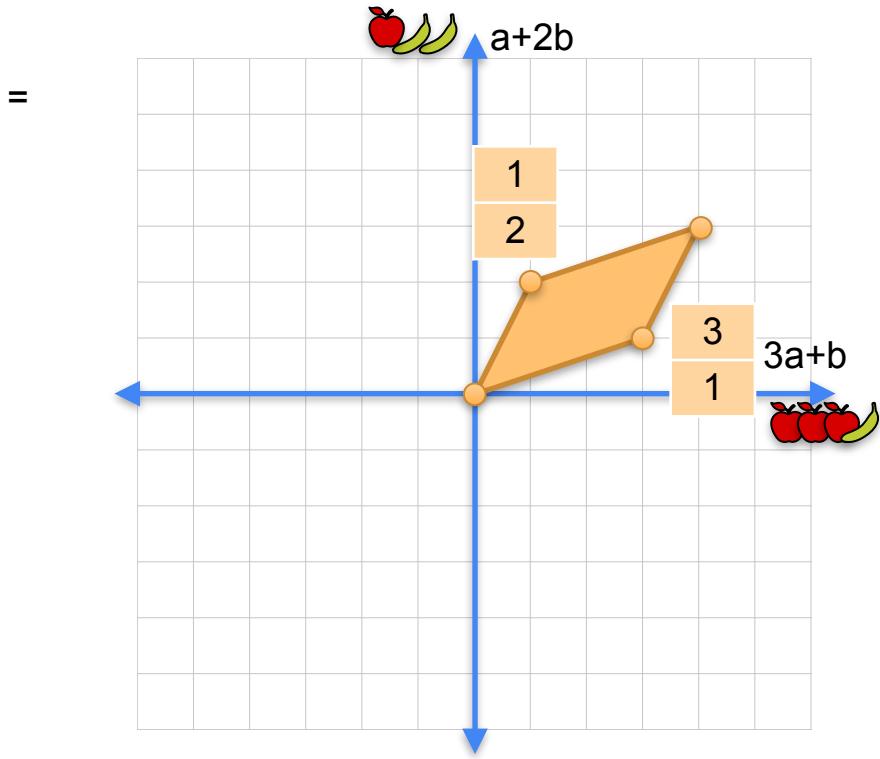
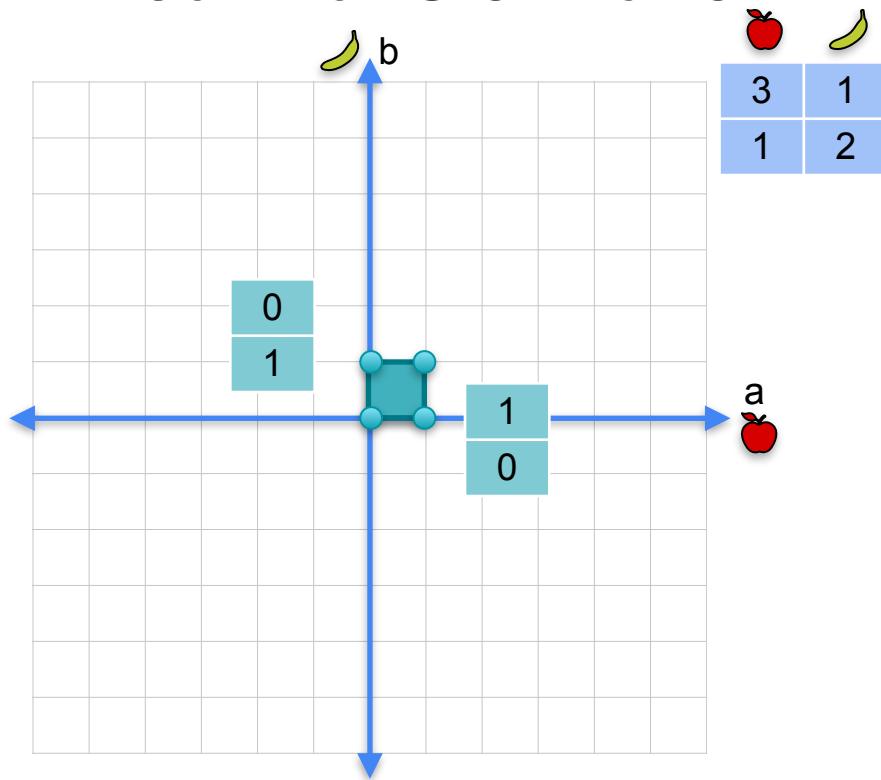
Linear transformation



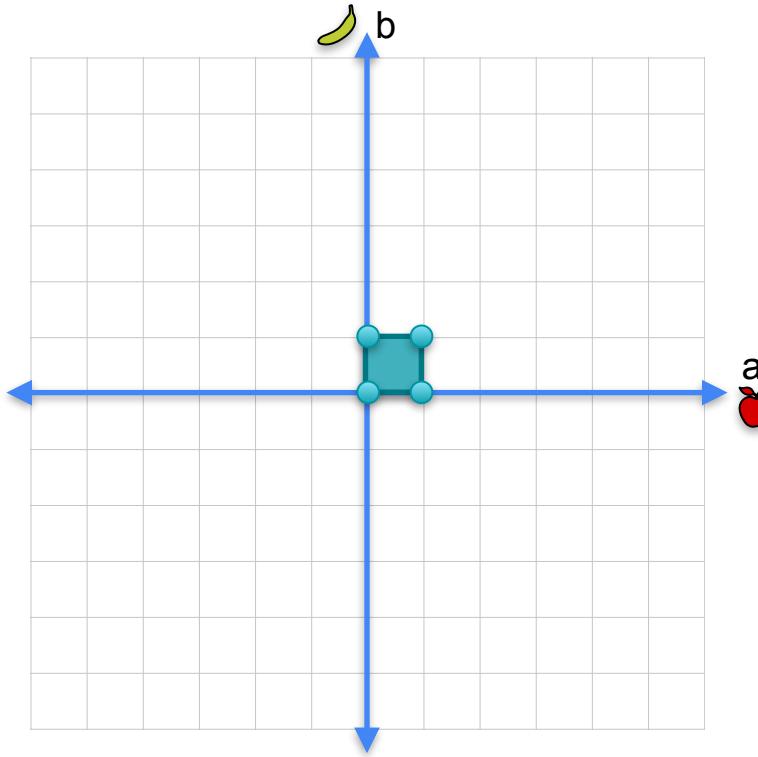
=



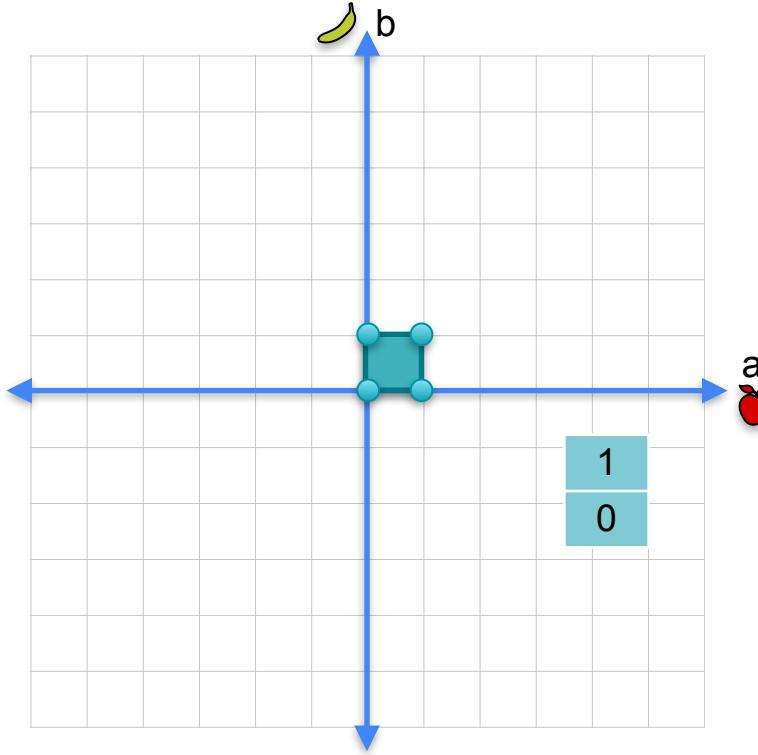
Linear transformation



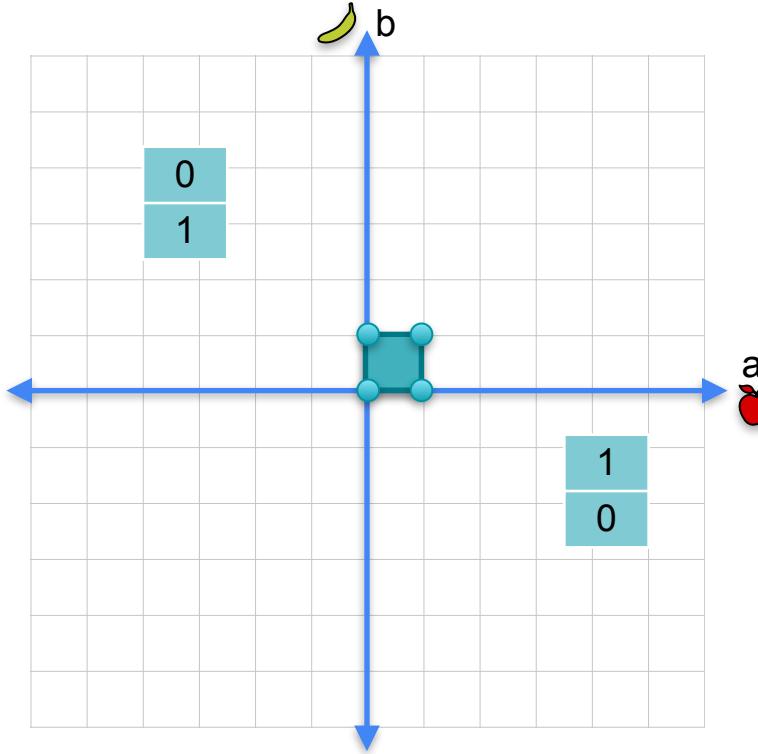
Linear transformation



Linear transformation



Linear transformation



3D



DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 4

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations

A matrix and its corresponding system of equations

A matrix and its corresponding system of equations

	
1	1
1	2

A matrix and its corresponding system of equations

	
1	1
1	2

	
1	1
2	2

A matrix and its corresponding system of equations

	
1	1
1	2

	
1	1
2	2

	
0	0
0	0

A matrix and its corresponding system of equations

System 1

	1	1
	1	2

- $a + b = 0$
- $a + 2b = 0$

1	1
2	2

0	0
0	0

A matrix and its corresponding system of equations

System 1

	1	1
	1	2

$$\bullet \quad a + b = 0$$

$$\bullet \quad a + 2b = 0$$

System 2

	1	1
	2	2

$$\bullet \quad a + b = 0$$

$$\bullet \quad 2a + 2b = 0$$

	0	0
	0	0

A matrix and its corresponding system of equations

System 1

• + = 0	1	1
• + 2 = 0	1	2

System 2

• + = 0	1	1
• 2 + 2 = 0	2	2

System 3

• 0 + 0 = 0	0	0
• 0 + 0 = 0	0	0

A matrix and its corresponding system of equations

System 1

	1	1
	1	2

System 2

	1	1
	2	2

System 3

	0	0
	0	0

The only two numbers a,
b, such that

- $a+b = 0$
 - and
 - $a+2b = 0$
- are:
 $a=0$ and $b=0$

A matrix and its corresponding system of equations

System 1

• $a + b = 0$		
• $a + 2b = 0$		

System 2

• $a + b = 0$		
• $2a + 2b = 0$		

System 3

• $0a + 0b = 0$		
• $0a + 0b = 0$		

The only two numbers a,
b, such that

- $a+b = 0$
 - and
 - $a+2b = 0$
- are:
 $a=0$ and $b=0$

Any pair $(x, -x)$ satisfies that

- $a+b = 0$
 - and
 - $a+2b = 0$
- For example:
 $(1, -1), (2, -2), (-8, 8)$, etc.

A matrix and its corresponding system of equations

System 1

• $a + b = 0$	1	1
• $a + 2b = 0$	1	2

System 2

• $a + b = 0$	1	1
• $2a + 2b = 0$	2	2

System 3

• $0a + 0b = 0$	0	0
• $0a + 0b = 0$	0	0

The only two numbers a,
b, such that

- $a+b = 0$
 - and
 - $a+2b = 0$
- are:
 $a=0$ and $b=0$

Any pair $(x, -x)$ satisfies that

- $a+b = 0$
- and
- $a+2b = 0$

For example:
 $(1,-1), (2,-2), (-8,8)$, etc.

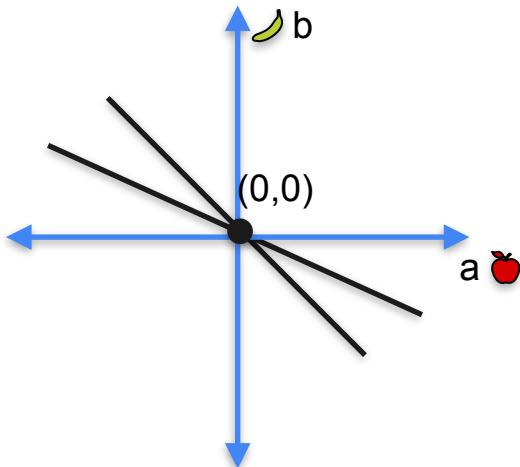
Any pair of numbers satisfies that

- $0a+0b = 0$
 - and
 - $0a+0b = 0$
- For example:
 $(1,2), (3,-9), (-90,8.34)$, etc.

The set of solutions of a system of equations

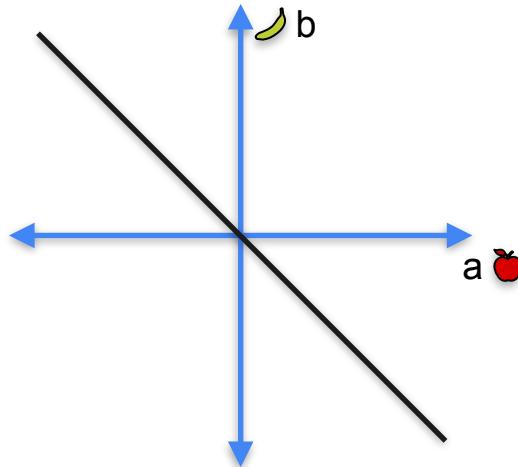
System 1

- $a + b = 0$
 
- $a + 2b = 0$
 



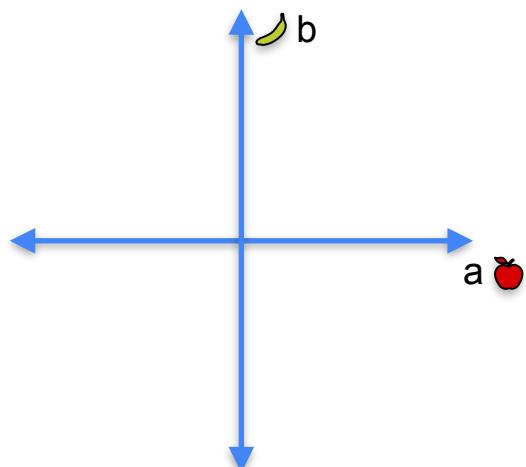
System 2

- $a + b = 0$
 
- $2a + 2b = 0$
 



System 3

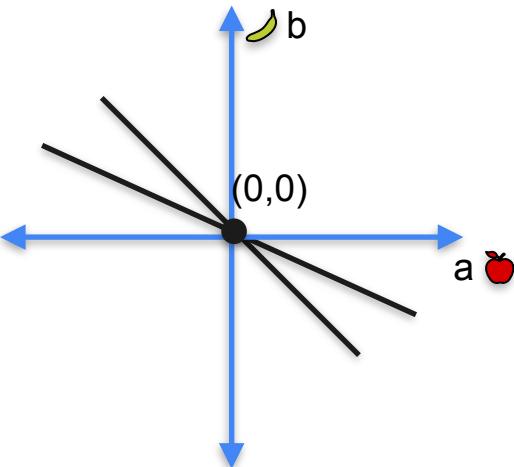
- $0a + 0b = 0$
 
- $0a + 0b = 0$
 



The set of solutions of a system of equations

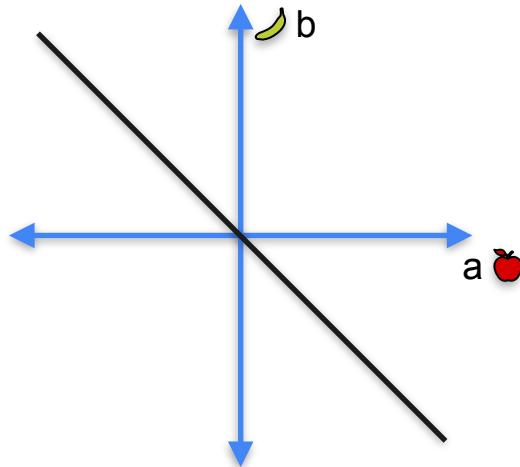
System 1

- $a + b = 0$
 - $a + 2b = 0$
- Solution**
- $a = 0$
 - $b = 0$



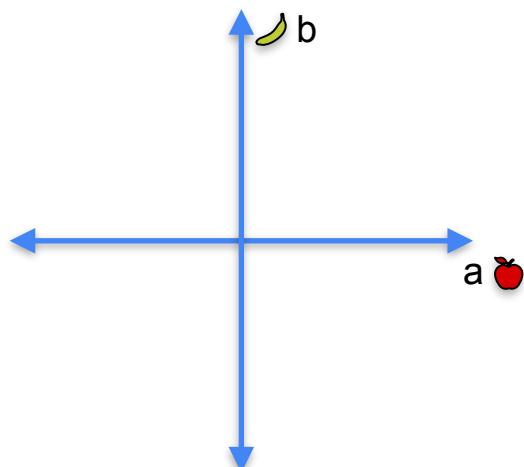
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

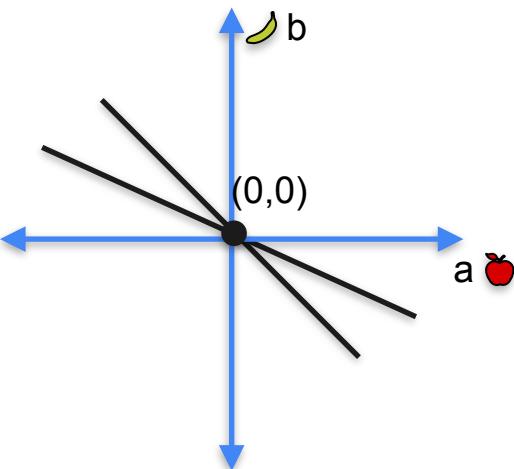
- $0a + 0b = 0$
- $0a + 0b = 0$



The set of solutions of a system of equations

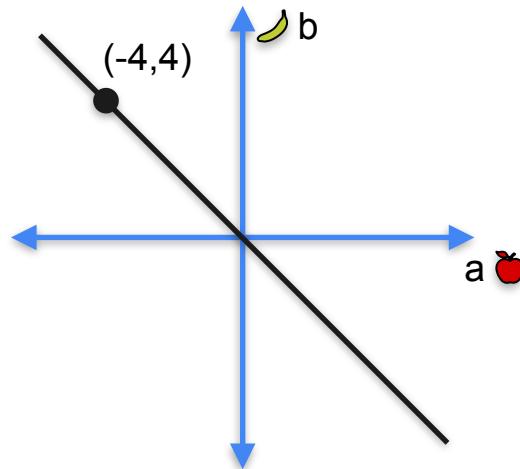
System 1

- $a + b = 0$
 - $a + 2b = 0$
- Solution**
- $a = 0$
 - $b = 0$



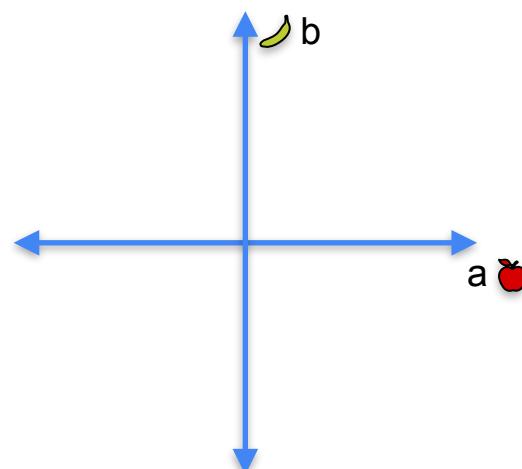
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

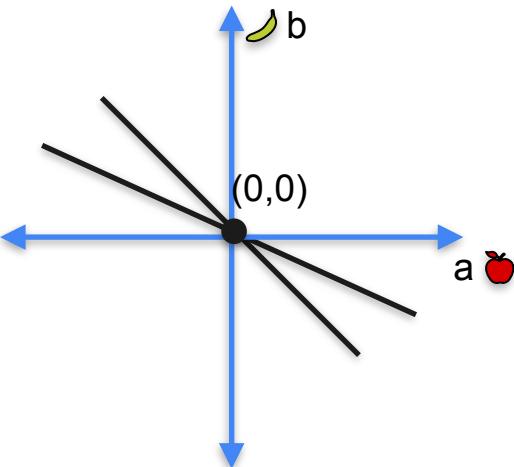


The set of solutions of a system of equations

System 1

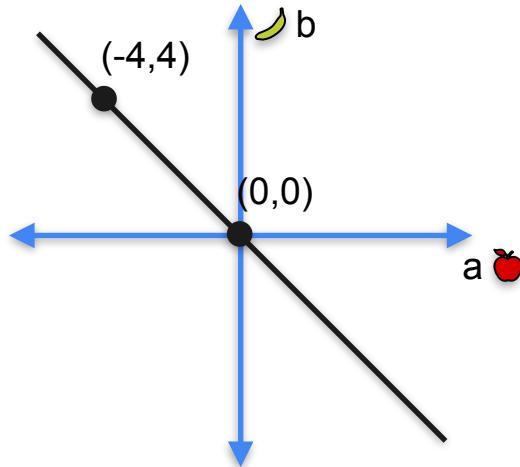
- + = **0**
- + 2 = **0**

Solution
• $a = 0$
• $b = 0$



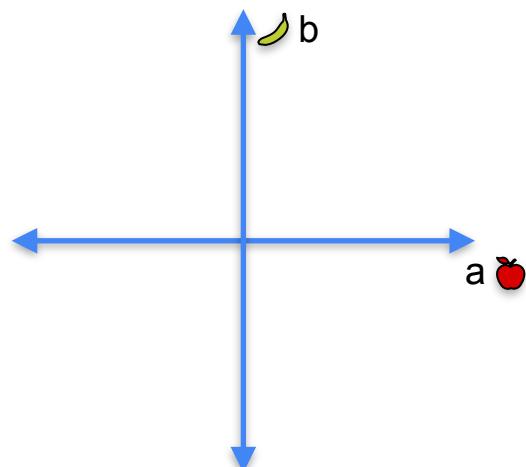
System 2

- + = **0**
- 2 + 2 = **0**



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

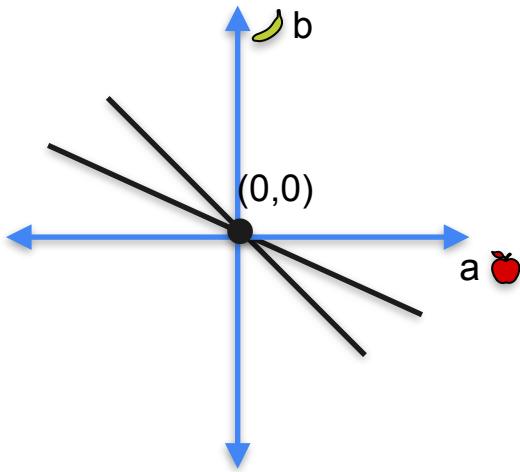


The set of solutions of a system of equations

System 1

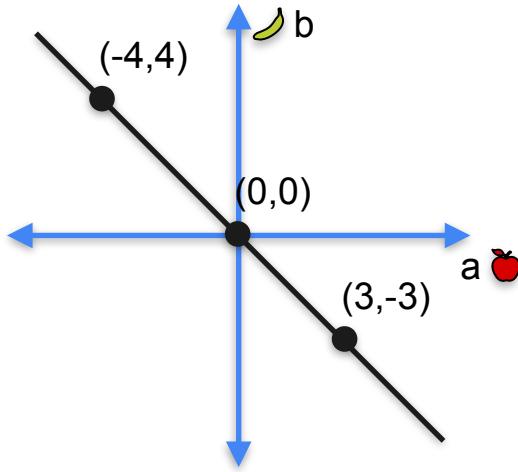
- + = **0**
- + 2 = **0**

Solution
• $a = 0$
• $b = 0$



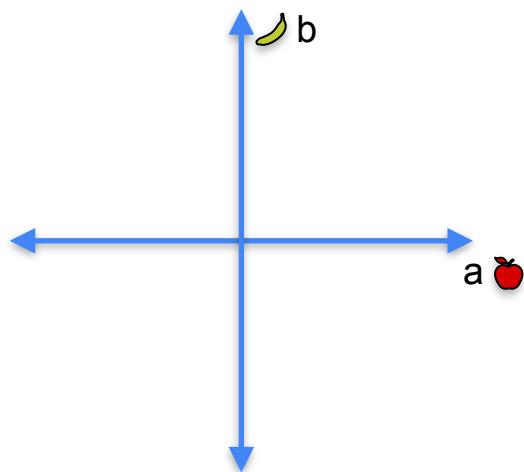
System 2

- + = **0**
- 2 + 2 = **0**



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

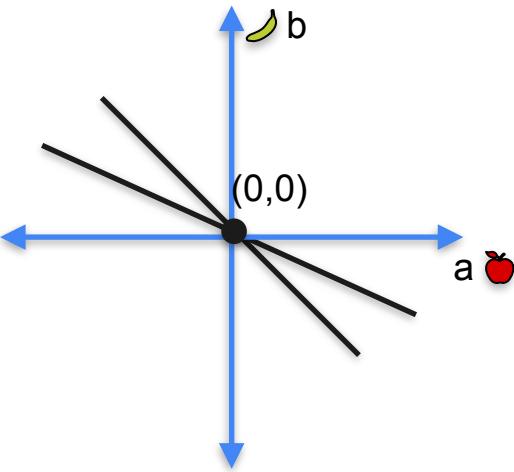


The set of solutions of a system of equations

System 1

- + = **0**
- + 2bananas = **0**

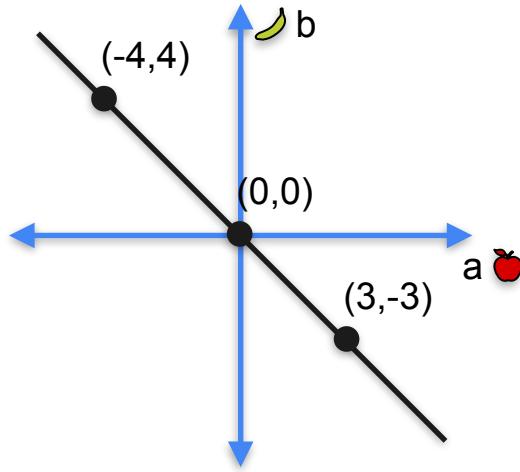
Solution
• $a = 0$
• $b = 0$



System 2

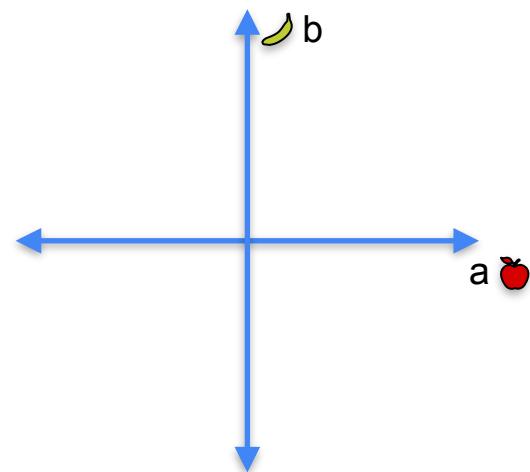
- + = **0**
- $2\text{apples} + 2\text{bananas} = \mathbf{0}$

Solutions
• any a
• $b = -a$



System 3

- $0a + 0b = \mathbf{0}$
- $0a + 0b = \mathbf{0}$

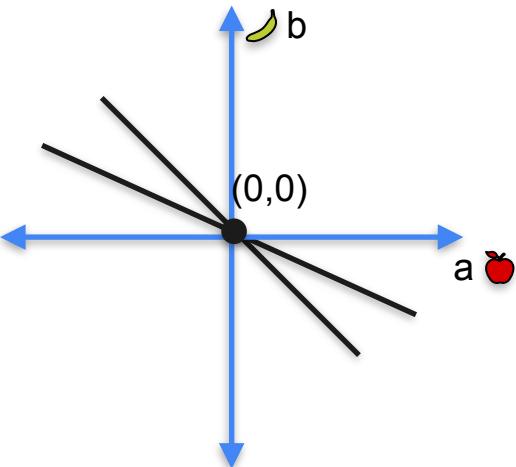


The set of solutions of a system of equations

System 1

- + = **0**
- + 2 = **0**

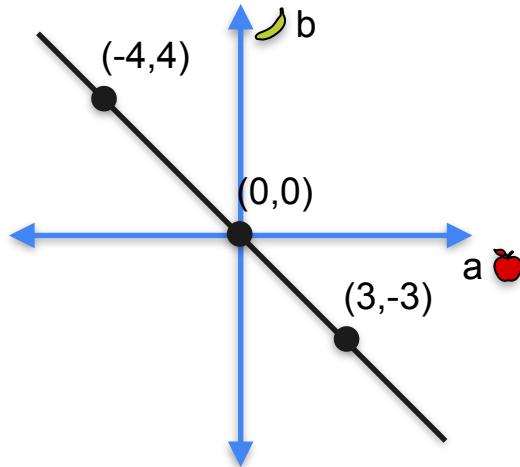
Solution
• $a = 0$
• $b = 0$



System 2

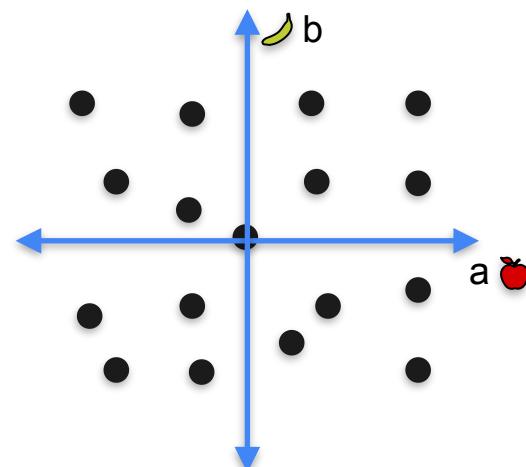
- + = **0**
- 2 + 2 = **0**

Solutions
• any a
• $b = -a$



System 3

- $0a + 0b = 0$
- $0a + 0b = 0$

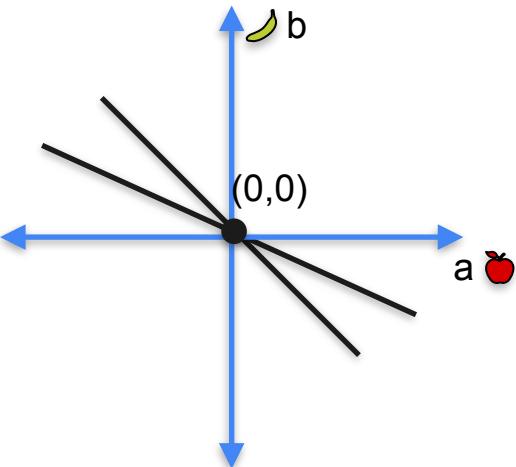


The set of solutions of a system of equations

System 1

- + = **0**
- + 2 = **0**

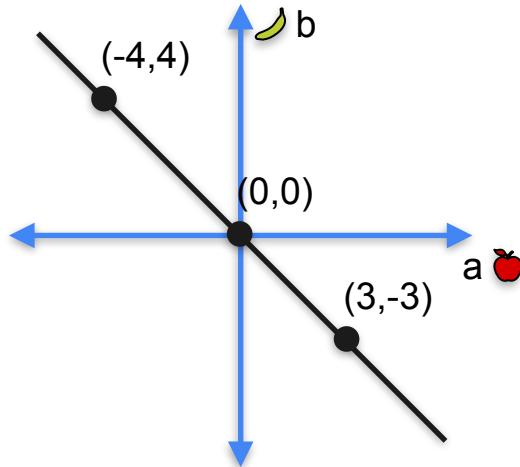
Solution
• $a = 0$
• $b = 0$



System 2

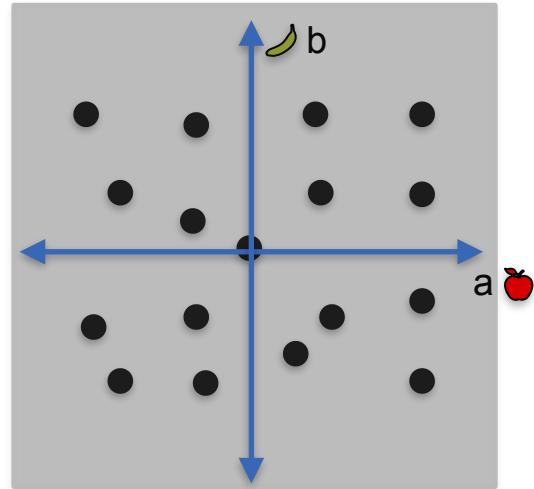
- + = **0**
- 2 + 2 = **0**

Solutions
• any a
• $b = -a$



System 3

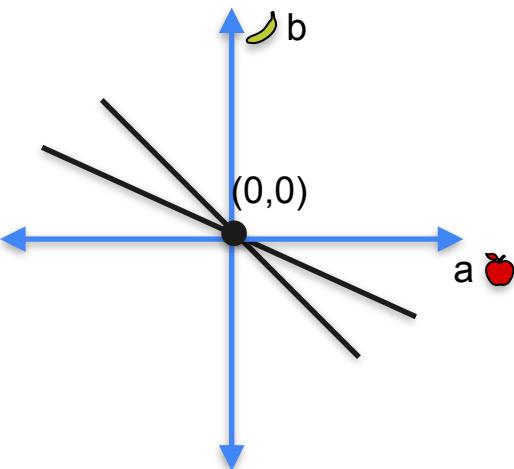
- $0a + 0b = 0$
- $0a + 0b = 0$



The set of solutions of a system of equations

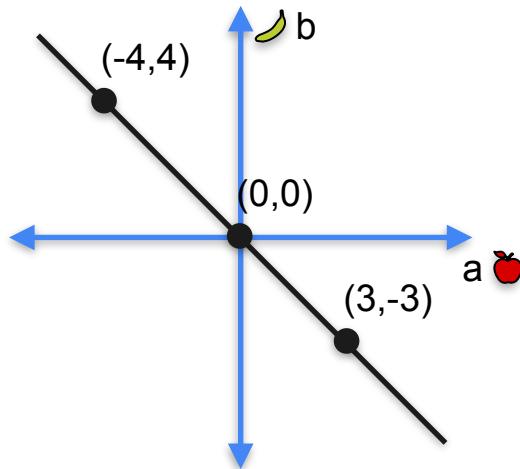
System 1

- + = **0**
 - + 2 = **0**
- Solution**
- $a = 0$
 - $b = 0$



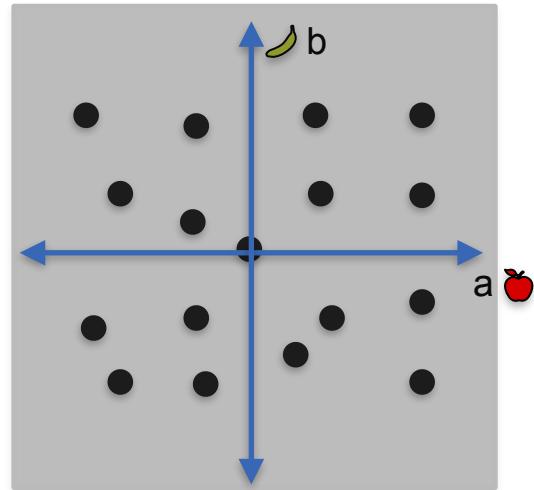
System 2

- + = **0**
 - 2 + 2 = **0**
- Solutions**
- any a
 - $b = -a$



System 3

- $0a + 0b = \mathbf{0}$
 - $0a + 0b = \mathbf{0}$
- Solutions**
- any a
 - any b



The null space of a matrix

1	1
1	2

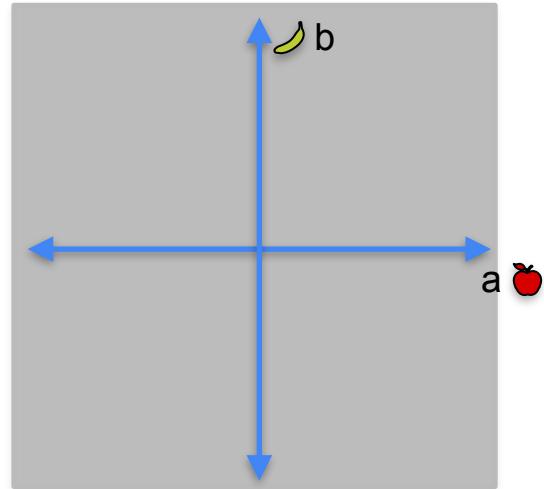
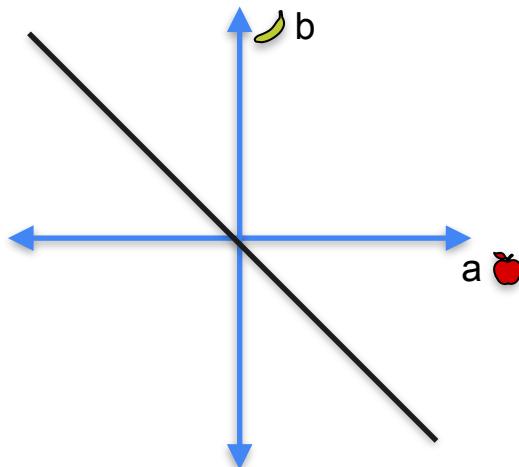
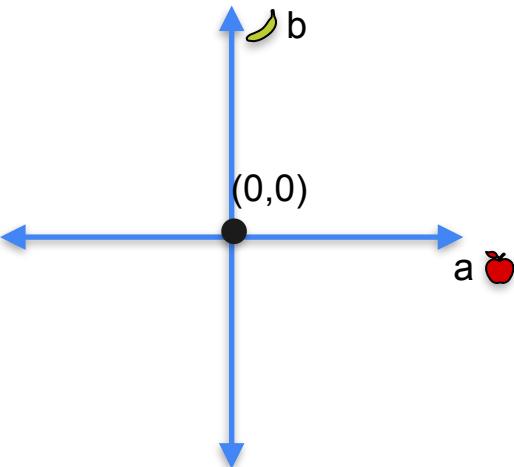
- Null space**
- $a = 0$
 - $b = 0$

1	1
2	2

- Null space**
- any a
 - $b = -a$

0	0
0	0

- Null space**
- any a
 - any b

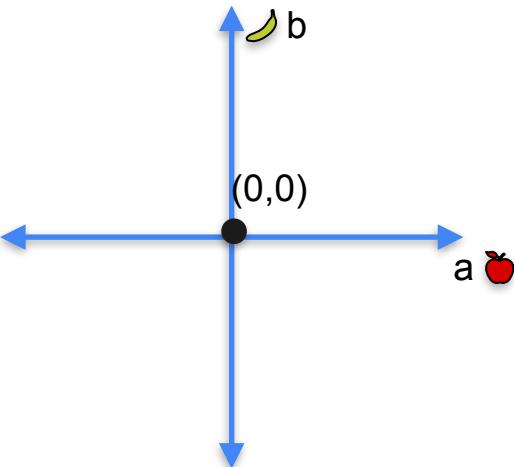


The null space of a matrix

1	1
1	2

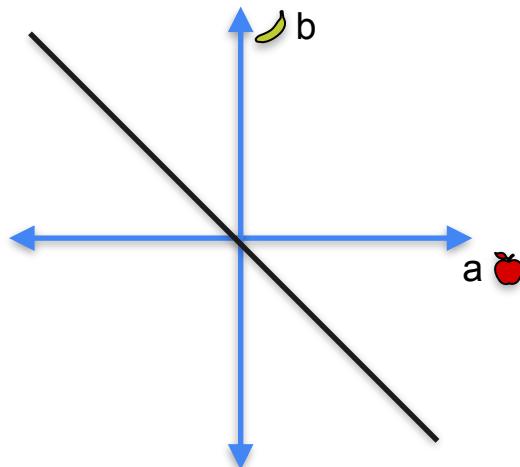
- Null space**
- $a = 0$
 - $b = 0$

Dimension = 0



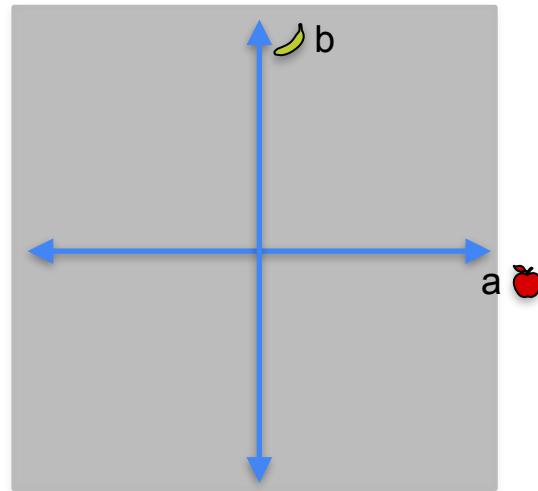
1	1
2	2

- Null space**
- any a
 - $b = -a$



0	0
0	0

- Null space**
- any a
 - any b



The null space of a matrix

1	1
1	2

- Null space
- $a = 0$
 - $b = 0$

Dimension = 0

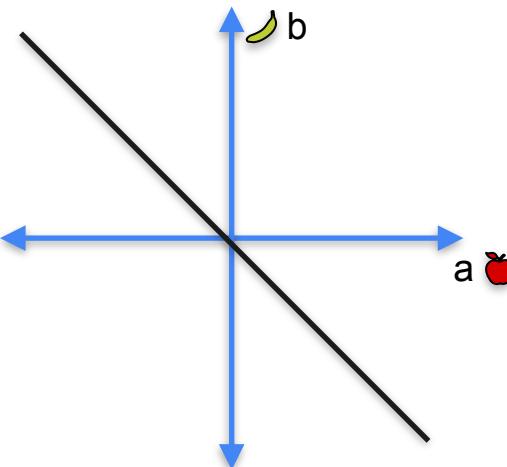
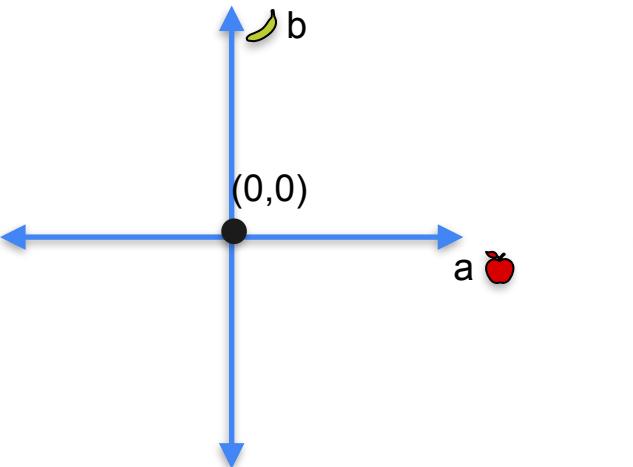
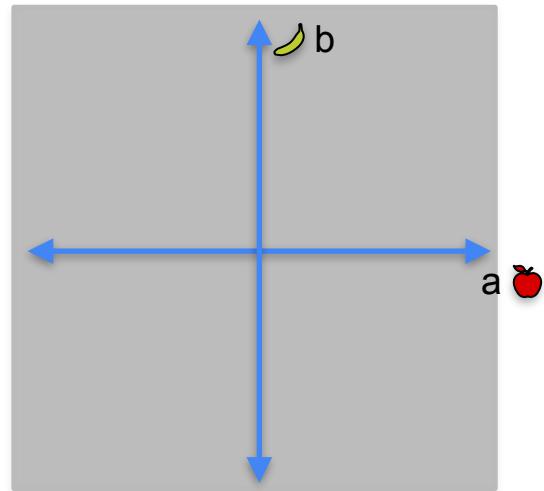
1	1
2	2

- Null space
- any a
 - $b = -a$

Dimension = 1

0	0
0	0

- Null space
- any a
 - any b

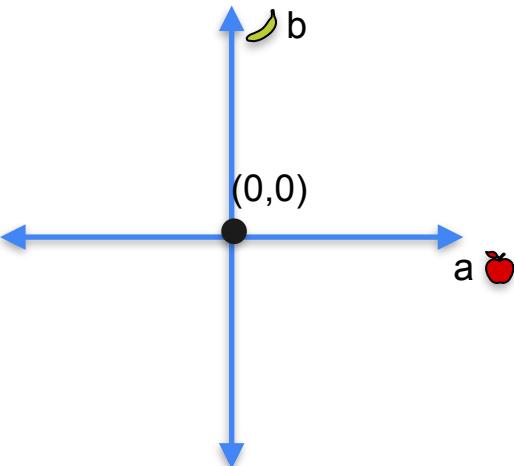


The null space of a matrix

1	1
1	2

Null space
• $a = 0$
• $b = 0$

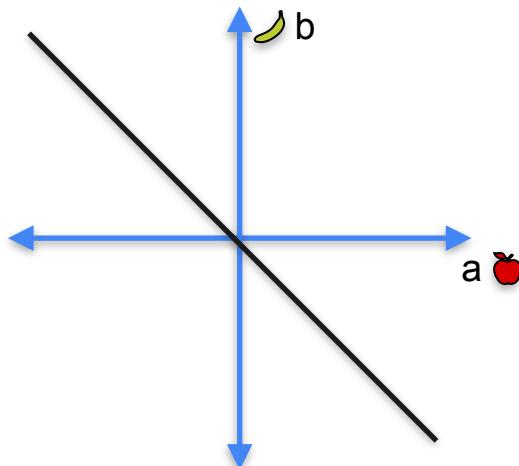
Dimension = 0



1	1
2	2

Null space
• any a
• $b = -a$

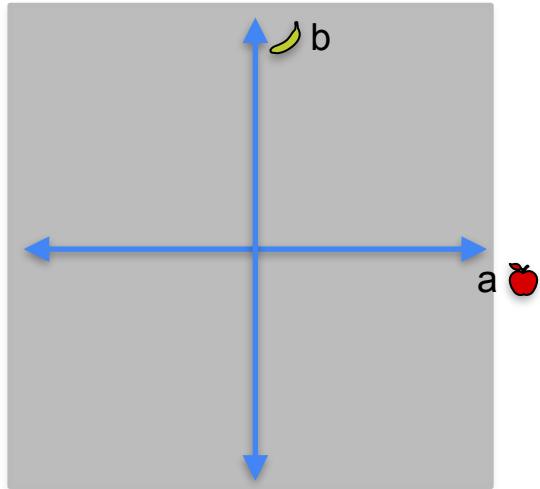
Dimension = 1



0	0
0	0

Null space
• any a
• any b

Dimension = 2



The null space of a matrix

1	1
1	2

Null space
• $a = 0$
• $b = 0$

Dimension = 0

1	1
2	2

Null space
• any a
• $b = -a$

Dimension = 1

0	0
0	0

Null space
• any a
• any b

Dimension = 2

The null space of a matrix

1	1
1	2

Null space
• $a = 0$
• $b = 0$

Dimension = 0

1	1
2	2

Null space
• any a
• $b = -a$

Dimension = 1

0	0
0	0

Null space
• any a
• any b

Dimension = 2



Non-singular

The null space of a matrix

1	1
1	2

Null space
• $a = 0$
• $b = 0$

Dimension = 0

1	1
2	2

Null space
• any a
• $b = -a$

Dimension = 1

0	0
0	0

Null space
• any a
• any b

Dimension = 2



Non-singular



Singular



The null space of a matrix

1	1
1	2

Null space
• $a = 0$
• $b = 0$

Dimension = 0

1	1
2	2

Null space
• any a
• $b = -a$

Dimension = 1

0	0
0	0

Null space
• any a
• any b

Dimension = 2



Non-singular



Singular



Singular

The null space of a matrix

1	1
1	2

- Null space**
- $a = 0$
 - $b = 0$

Dimension = 0



Non-singular

1	1
2	2

Dimension = 1



Singular

0	0
0	0

Dimension = 2



Singular

More conceptual explanation of the null space

- Elaborate here

Quiz: Null space of a matrix

Problem: Determine the dimension of the null space of the following two matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Solutions: Null space of a matrix

Matrix 1: Notice that this is a non-singular matrix, since the determinant is 16. Therefore, the null space is only the point (0,0). The dimension is 0.

5	1
-1	3

Matrix 2: The corresponding system of equation has the equations $2a - b = 0$ and $-6a + 3b = 0$. Some inspection shows that the first equation has the points (1,2), (2,4), (3,6), etc. as solutions. All of them are also solutions to the second equation, $-6a + 3b = 0$. Therefore the null space is all the points of the form $(x, 2x)$. The dimension of this null space is 1, and the matrix is singular.

2	-1
-6	3

Systems of linear equations

Systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

1	1	1
1	1	2
1	1	3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

1	1	1
1	1	2
1	1	3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

1	1	1
2	2	2
3	3	3

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

1	1	1
1	2	1
1	1	2

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

1	1	1
1	1	2
1	1	3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

1	1	1
2	2	2
3	3	3

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

0	0	0
0	0	0
0	0	0

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Null space for systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

Solution space



System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

Solution space



System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Null space for systems of linear equations

System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$

Solution space



System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$

Solution space



System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Solution space



System 4

- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$
- $0a + 0b + 0c = 0$

Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

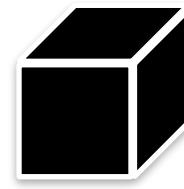
Solution space



System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

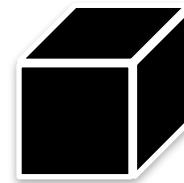
Solution space



System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



Dimension = 1

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

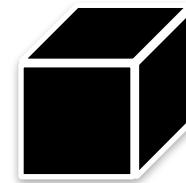
Solution space



System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



Dimension = 1

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

Solution space

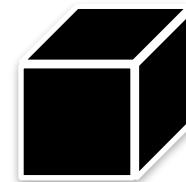


Dimension = 2

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Null space for systems of linear equations

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$

Solution space



Dimension = 0

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$

Solution space



Dimension = 1

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$

Solution space

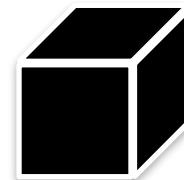


Dimension = 2

System 4

- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$
- $0a + 0b + 0c = \mathbf{0}$

Solution space



Dimension = 3

Null space for matrices

Matrix 1

1	1	1
1	2	1
1	1	2

Null space



Dimension = 0

Matrix 2

1	1	1
1	1	2
1	1	3

Null space



Dimension = 1

Matrix 3

1	1	1
2	2	2
3	3	3

Null space

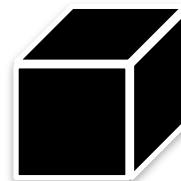


Dimension = 2

Matrix 4

0	0	0
0	0	0
0	0	0

Null space



Dimension = 3

Quiz: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$

Dimension = 1

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

All points of the form
 $(x, -x, 0)$

1	1	1
0	2	2
0	0	3

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

All points of the form
 $(x, -x, 0)$
Dimension = 1

1	1	1
0	2	2
0	0	3

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

All points of the form
 $(x, -x, 0)$
Dimension = 1

1	1	1
0	2	2
0	0	3

- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

The point
 $(0,0,0)$

Solution: Null space

Problem: Determine the dimension of the null space for the following matrices.

1	0	1
0	1	0
3	2	3

- $a + c = \mathbf{0}$
- $b = \mathbf{0}$
- $3a + 2b + 3c = \mathbf{0}$

All points of the form
 $(x, 0, -x)$
Dimension = 1

1	1	1
1	1	2
0	0	-1

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $c = \mathbf{0}$

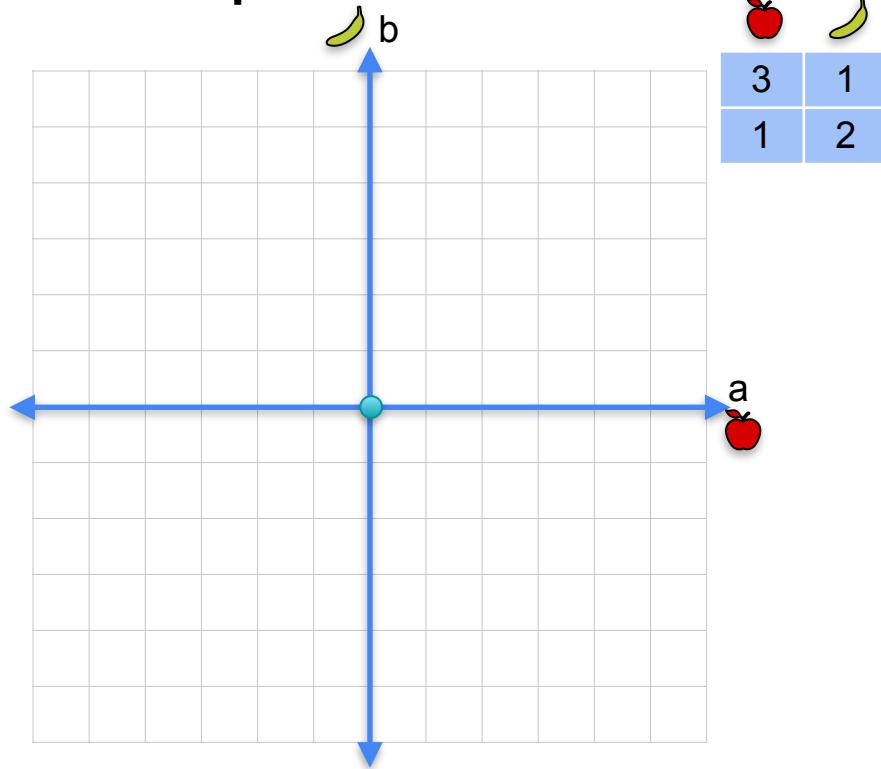
All points of the form
 $(x, -x, 0)$
Dimension = 1

1	1	1
0	2	2
0	0	3

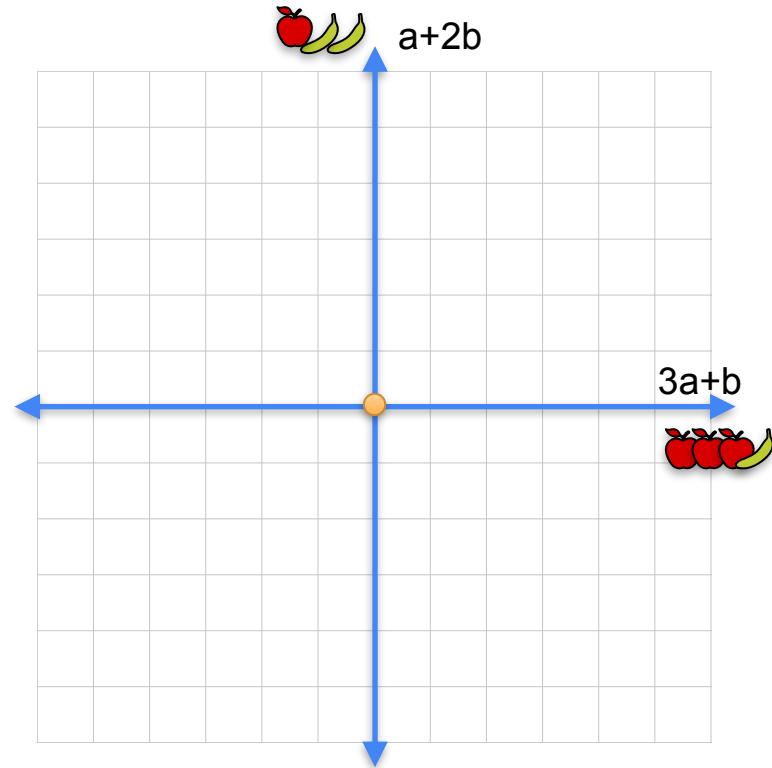
- $a + b + c = \mathbf{0}$
- $2b + 2c = \mathbf{0}$
- $3c = \mathbf{0}$

The point
 $(0,0,0)$
Dimension = 0

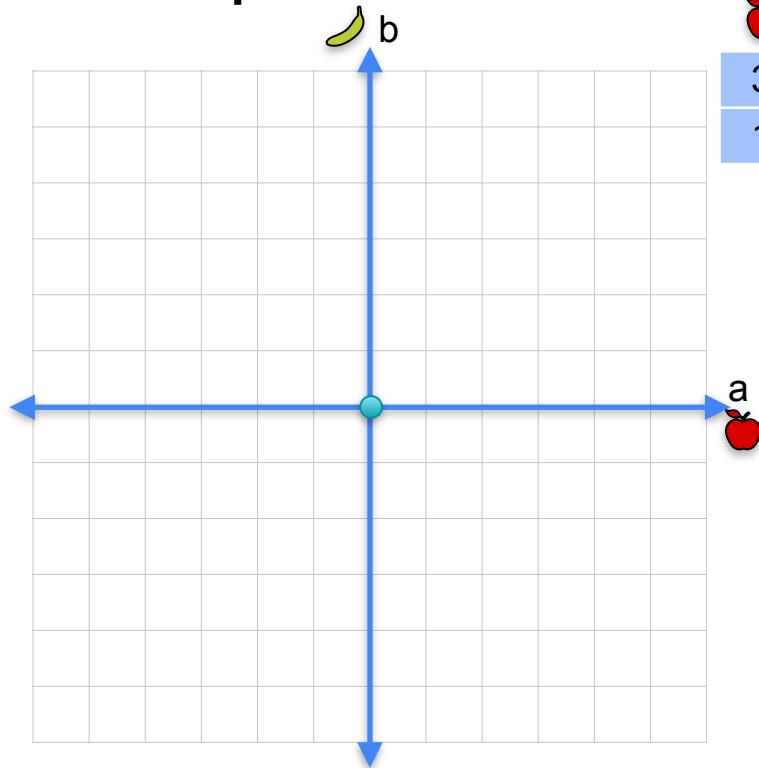
Null space



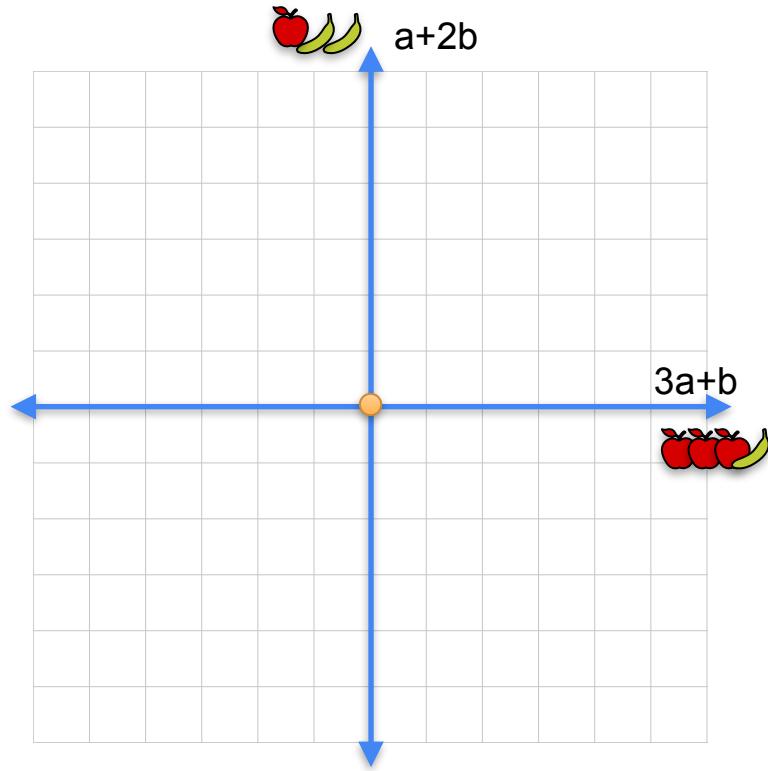
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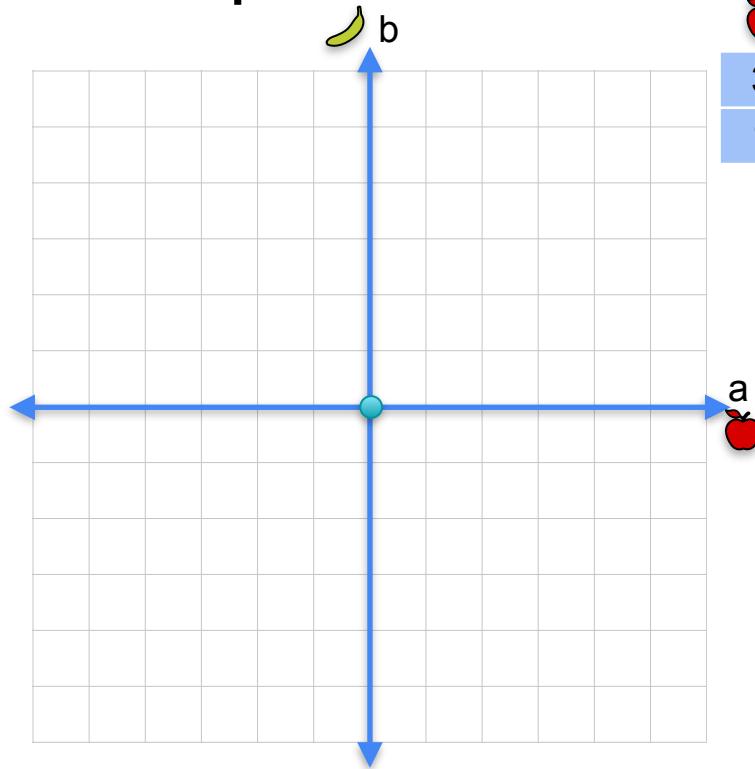
Null space



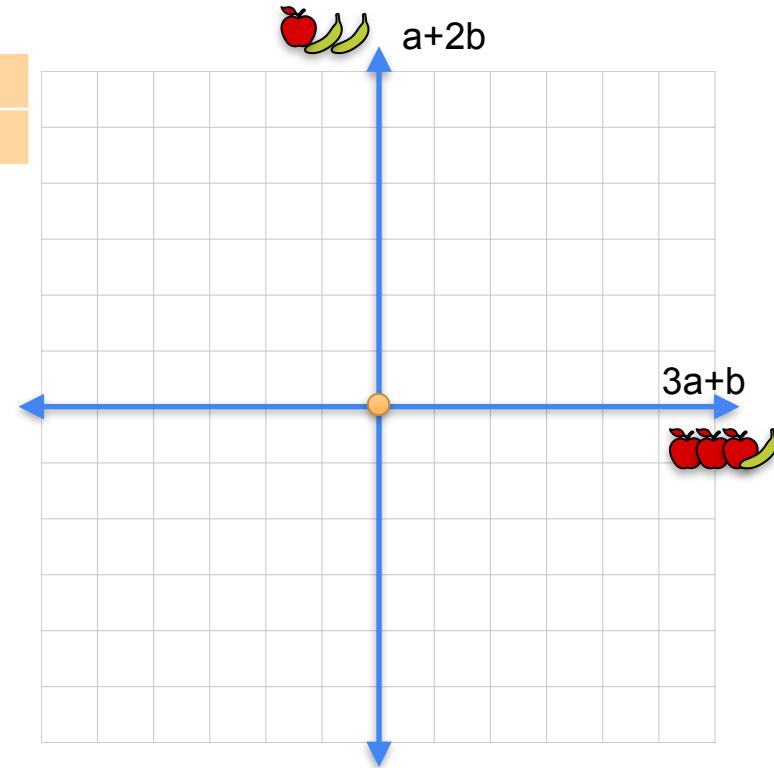
$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} a \\ b \end{matrix} \end{array} =$$



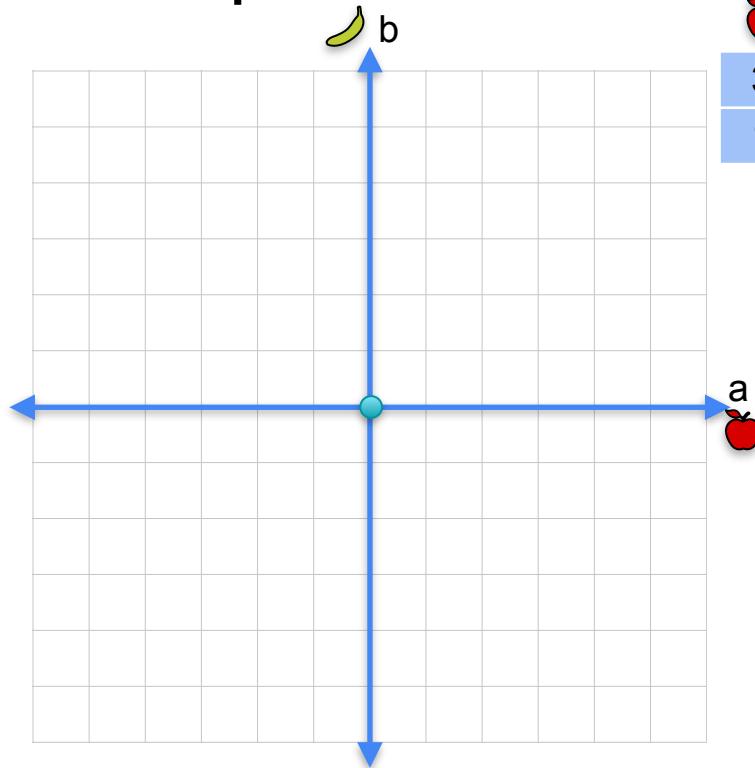
Null space



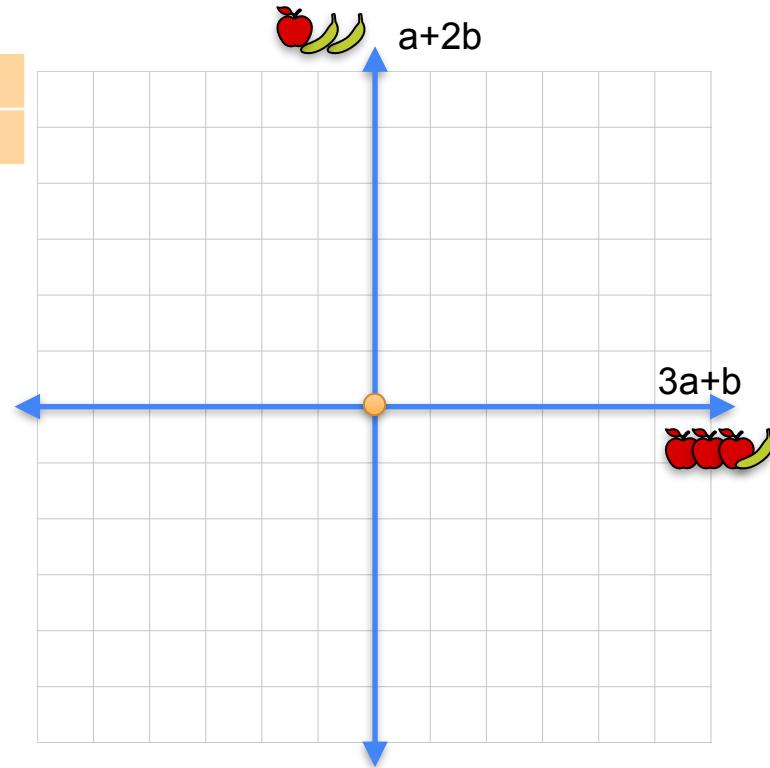
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 3 & 1 & a \\ 1 & 2 & b \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$



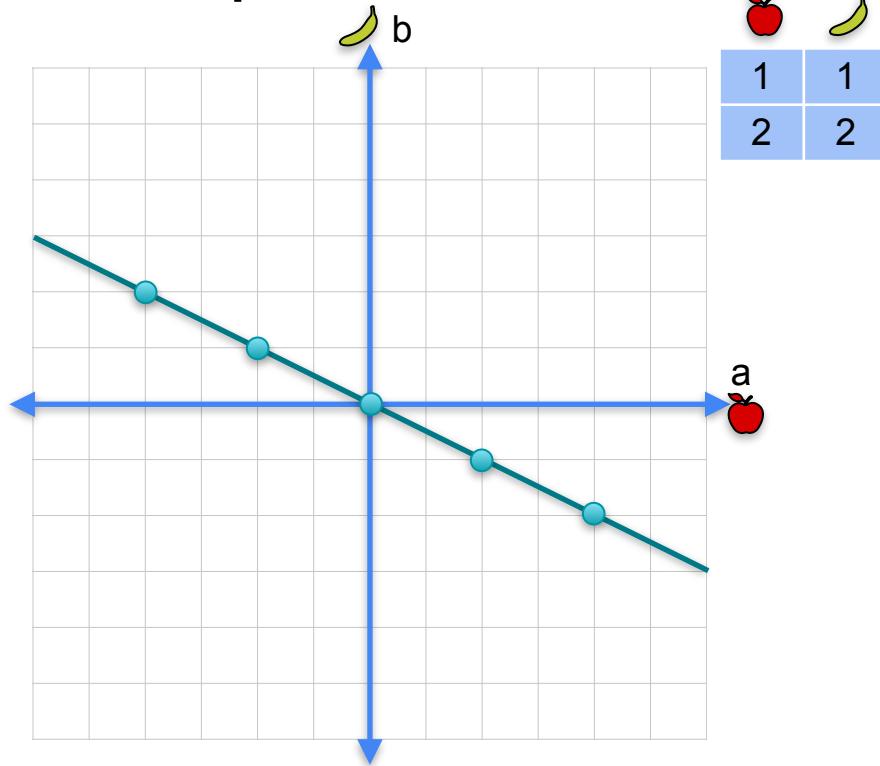
Null space



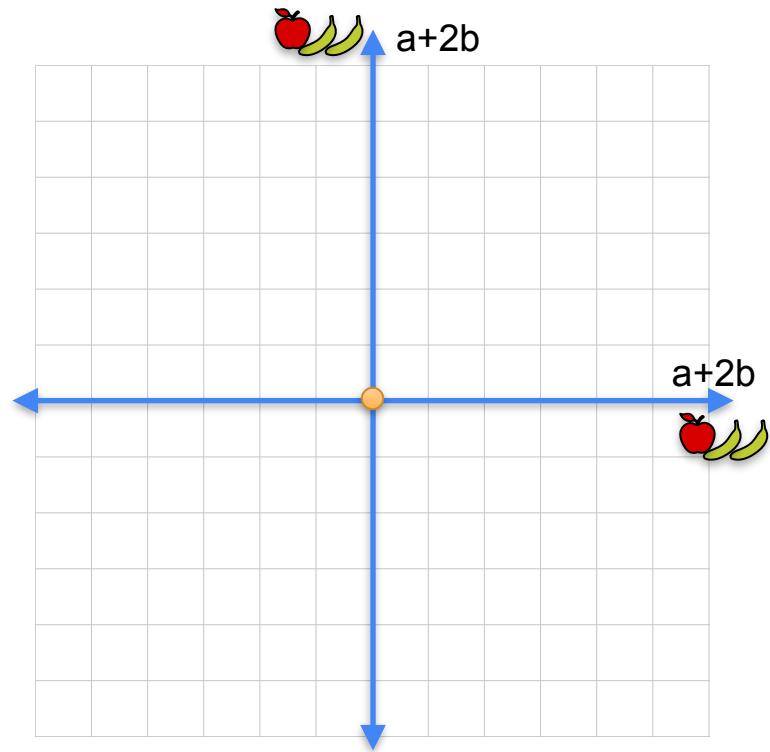
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 3 & 1 & 0 \\ 1 & 2 & 0 \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$



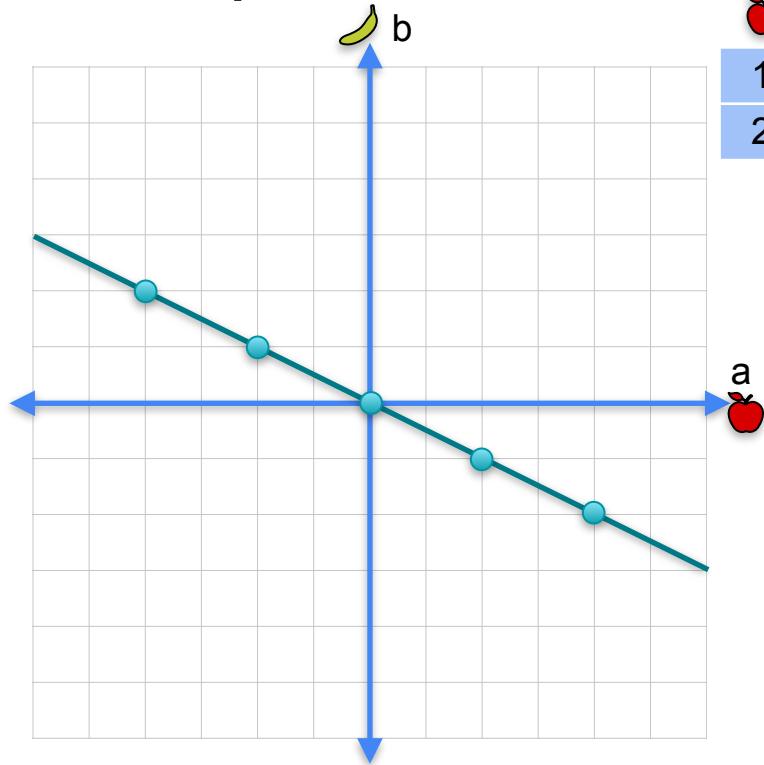
Null space



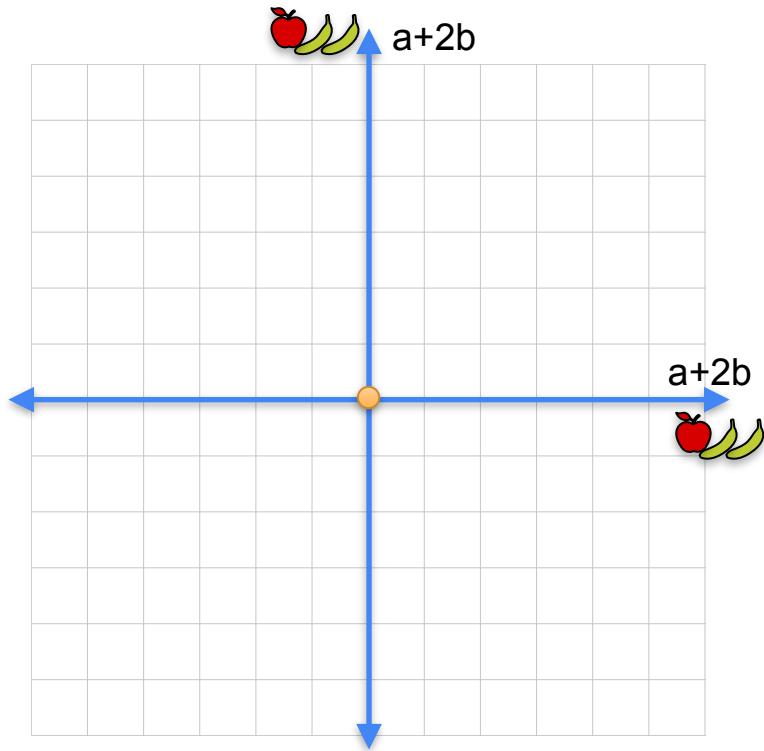
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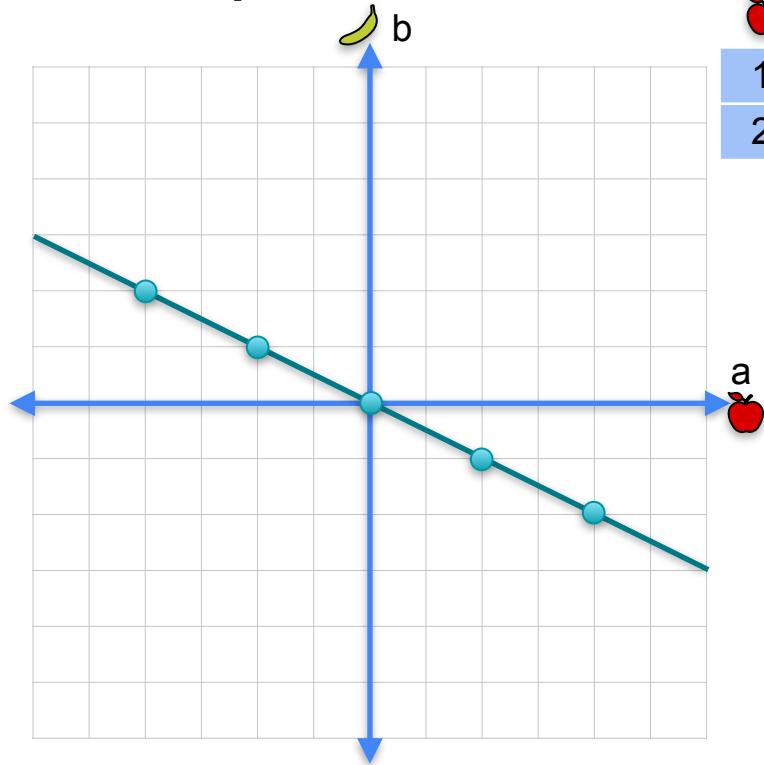
Null space



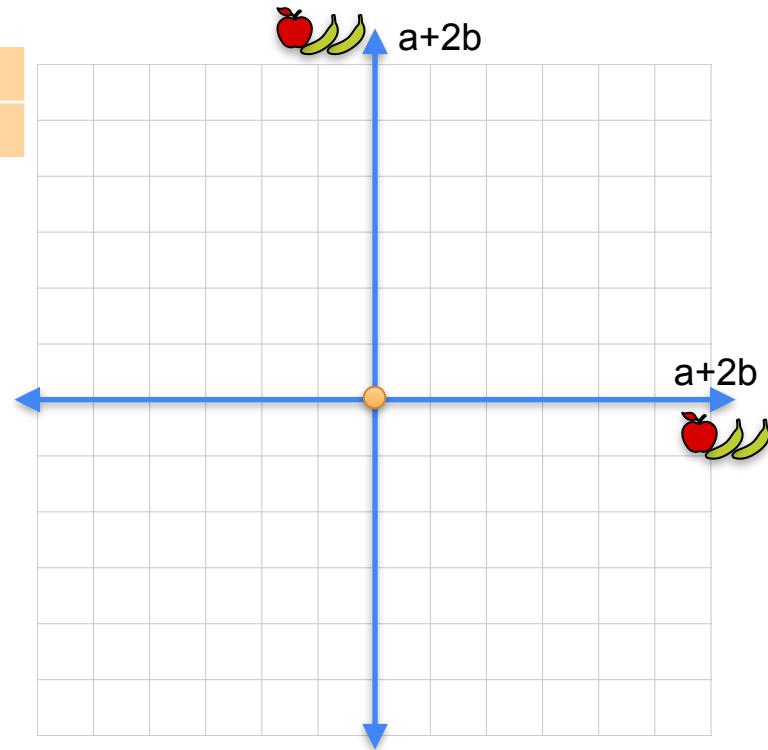
$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} a \\ b \end{matrix} \end{array} =$$



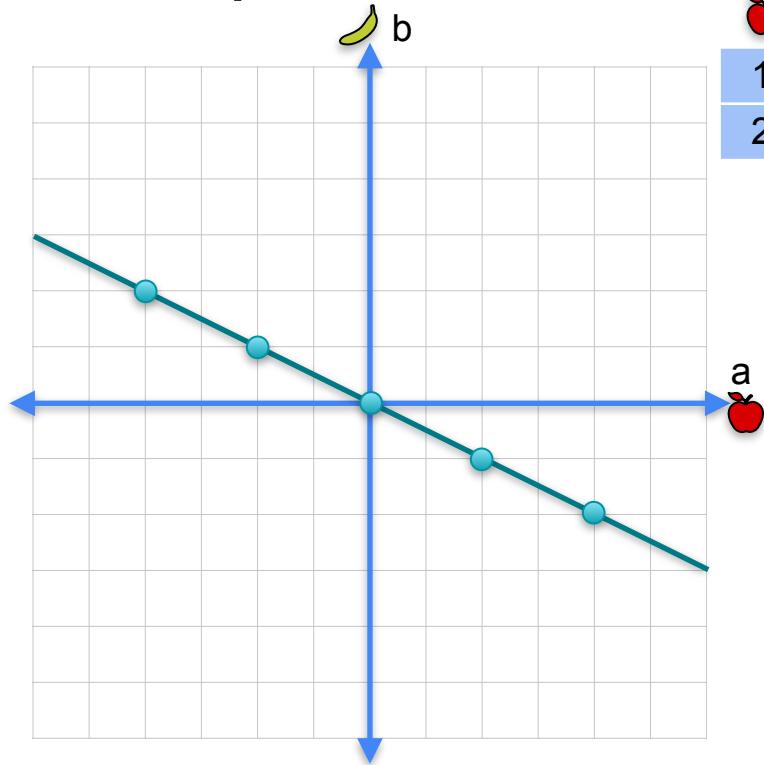
Null space



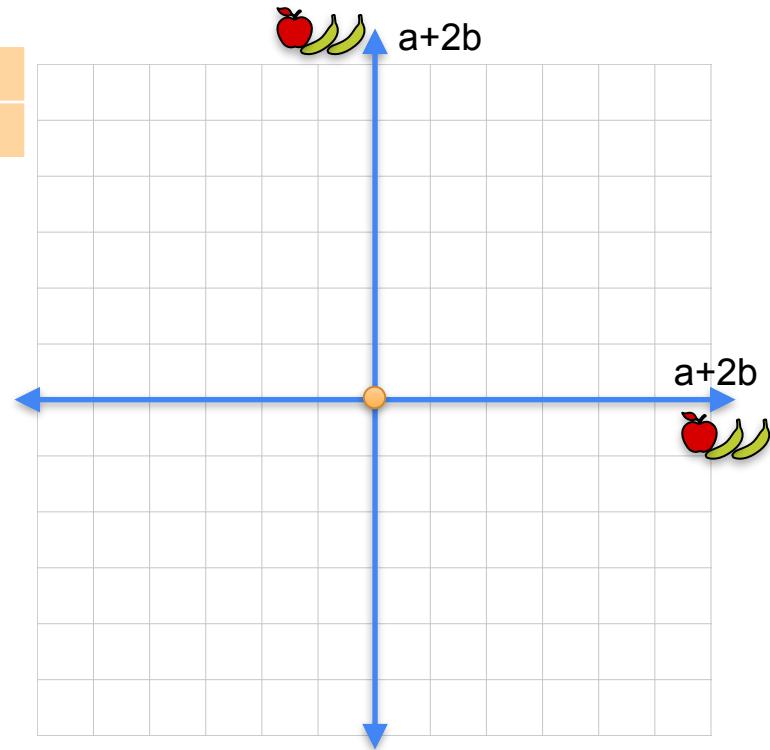
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 1 & 1 & a \\ 2 & 2 & b \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$



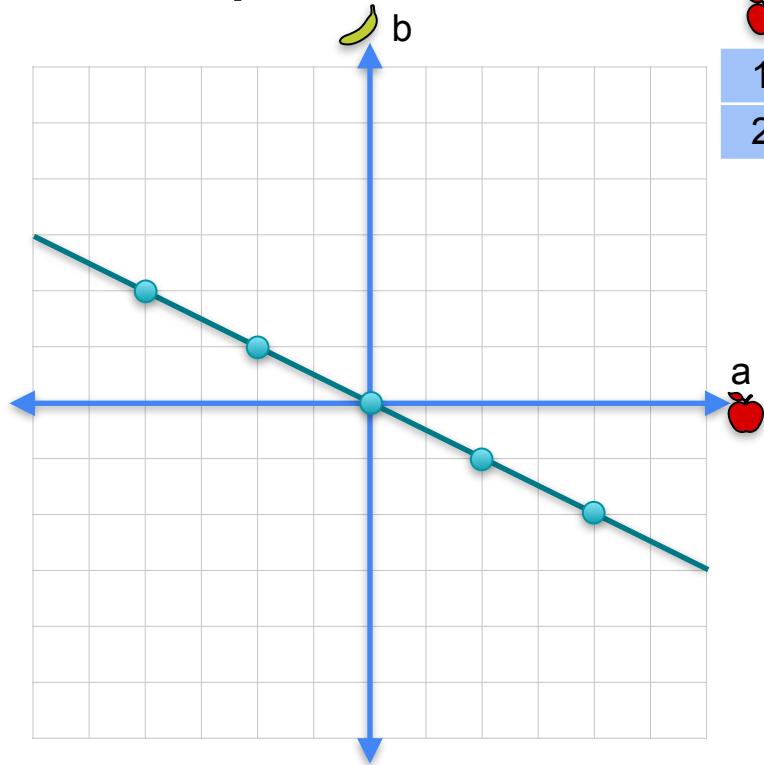
Null space



$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 1 & 1 & 0 \\ 2 & 2 & 0 \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$



Null space

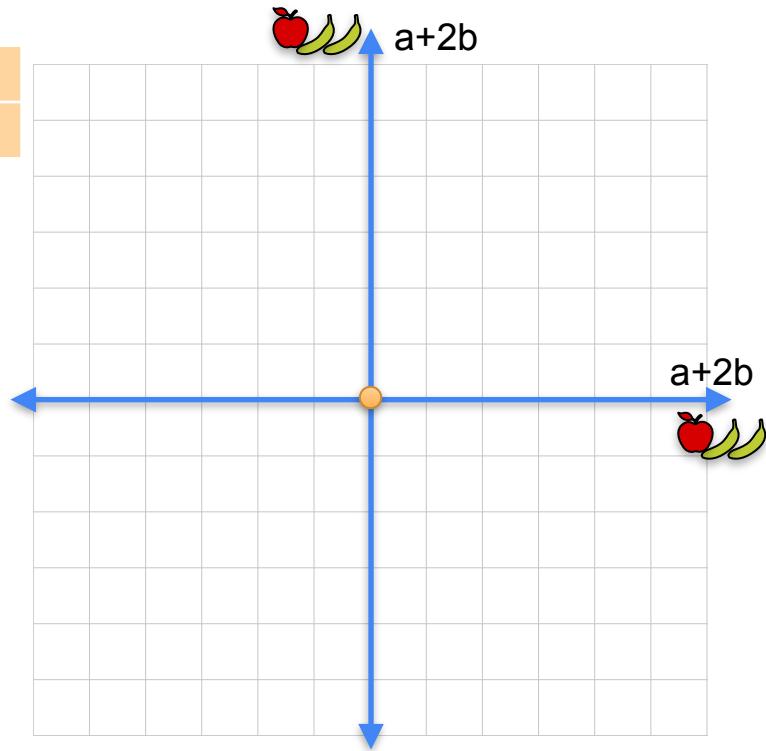


1	1	0
2	2	0

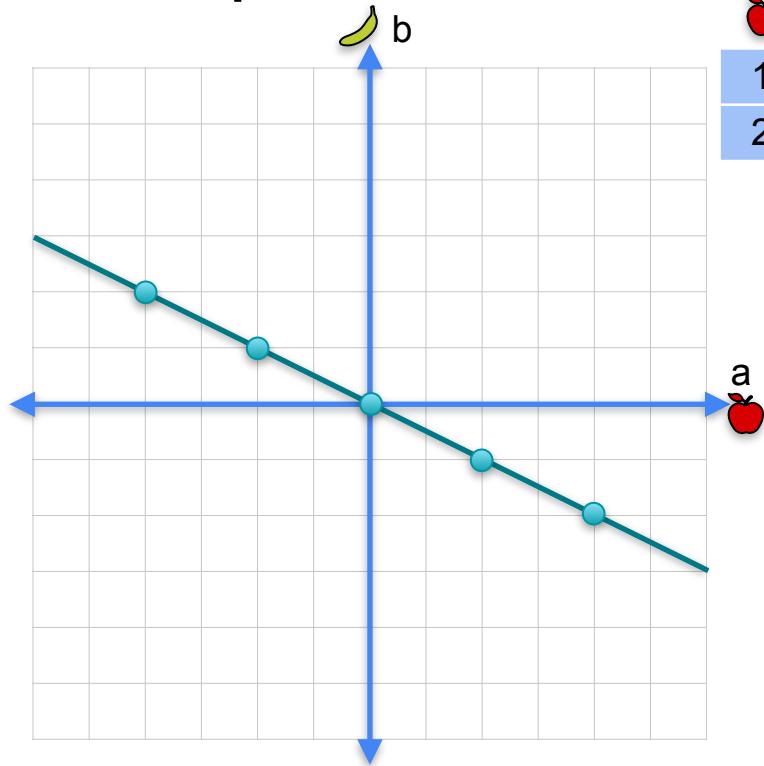
 $=$

0
0

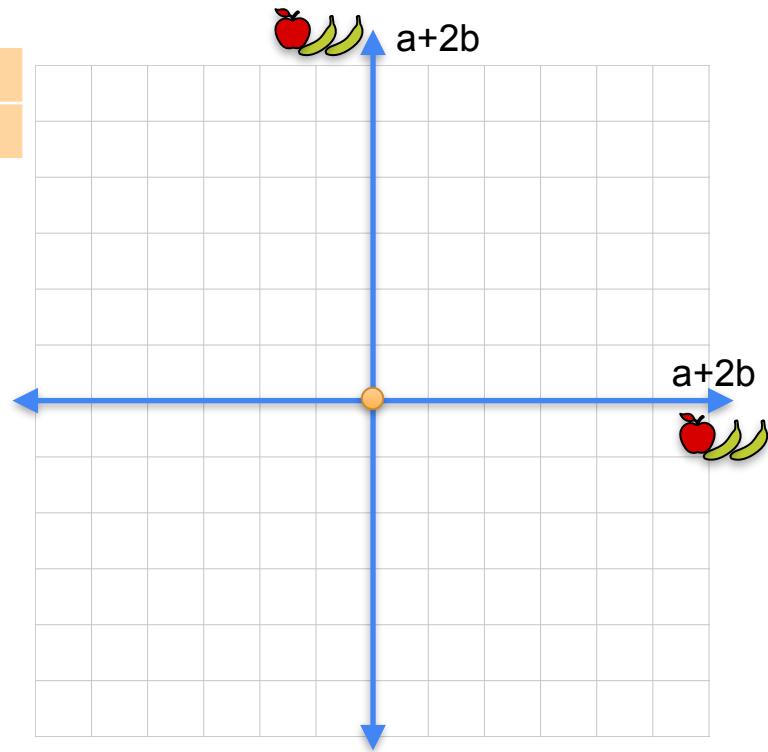
2
-1



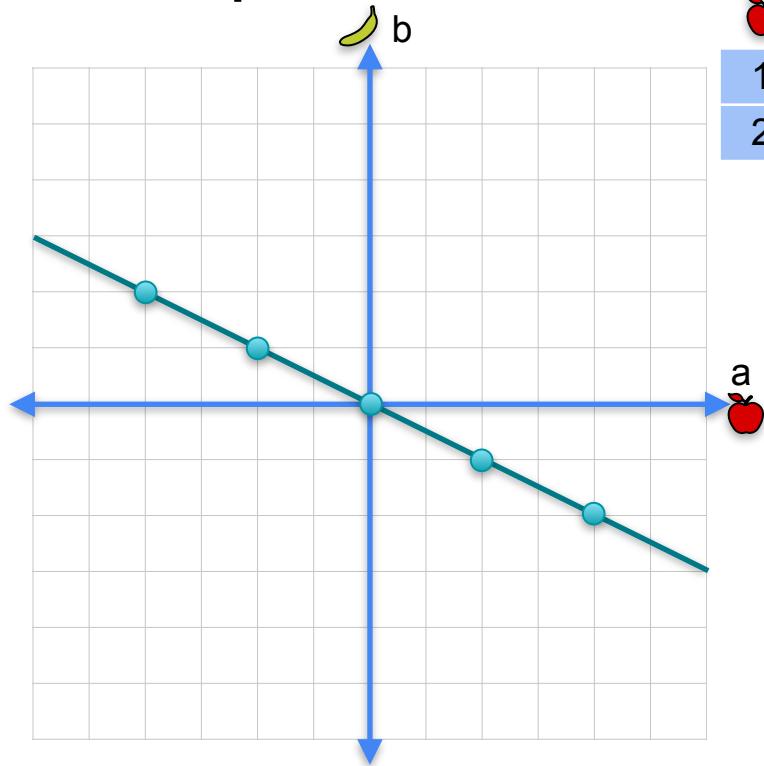
Null space



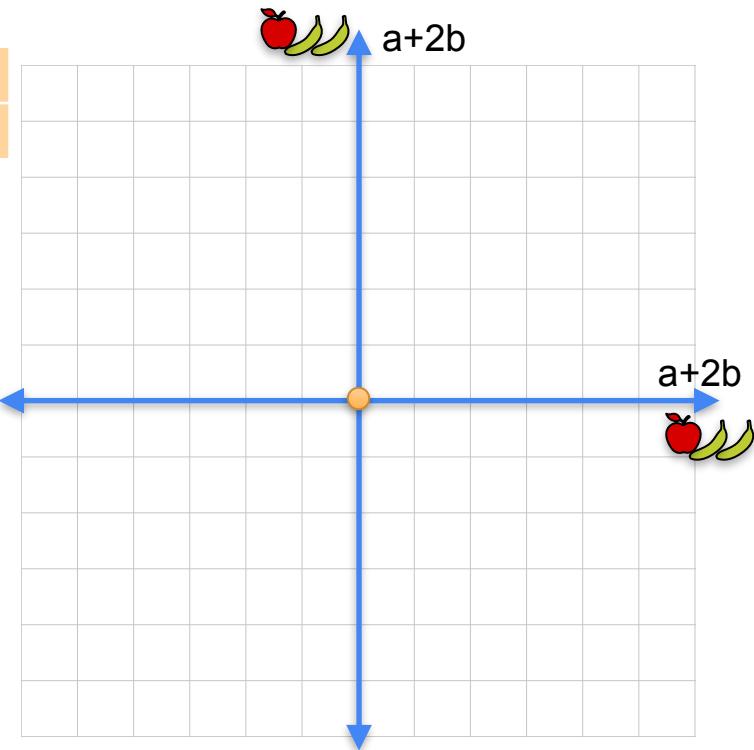
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|c|} \hline 1 & 1 & 0 \\ \hline 2 & 2 & 0 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline \end{array} \\[10pt] \begin{array}{|c|c|} \hline 2 & 4 \\ \hline -1 & -2 \\ \hline \end{array} \end{array}$$



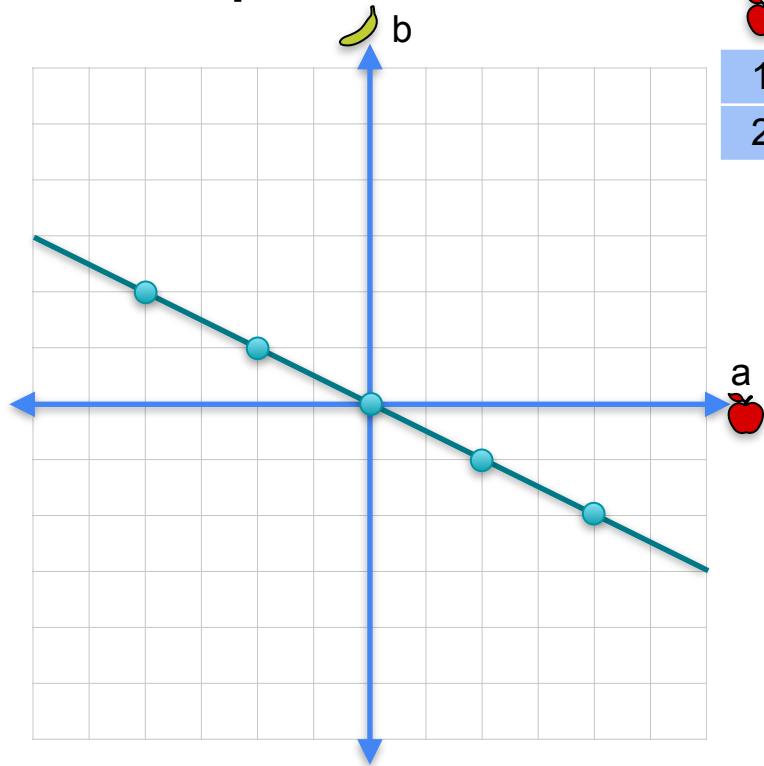
Null space



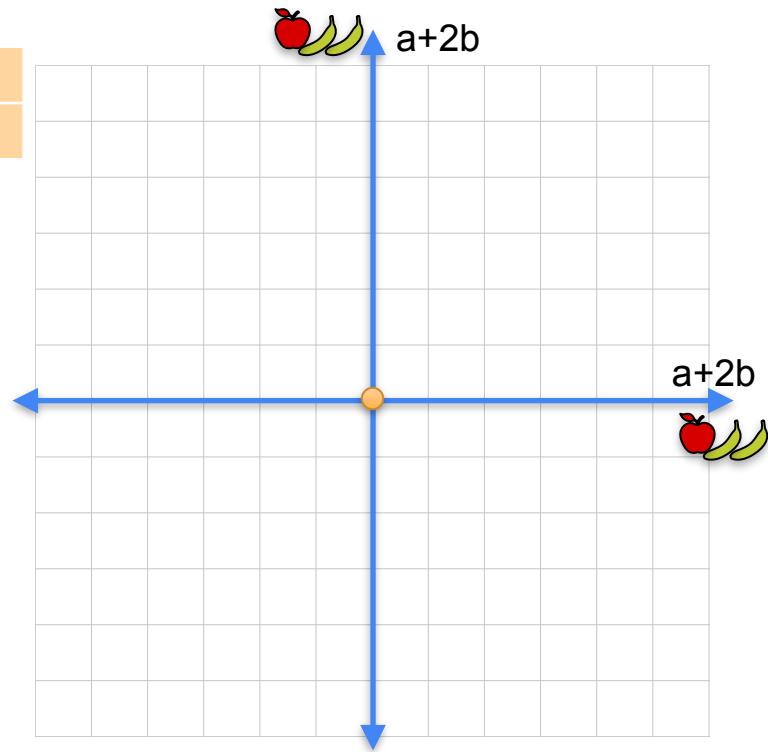
$$\begin{array}{cc|c} \text{apple} & \text{banana} & \\ \hline 1 & 1 & 0 \\ 2 & 2 & 0 \end{array} = \begin{array}{cc} 0 \\ 0 \end{array}$$
$$\begin{array}{c} 2 \\ -1 \\ -2 \\ 1 \end{array}$$



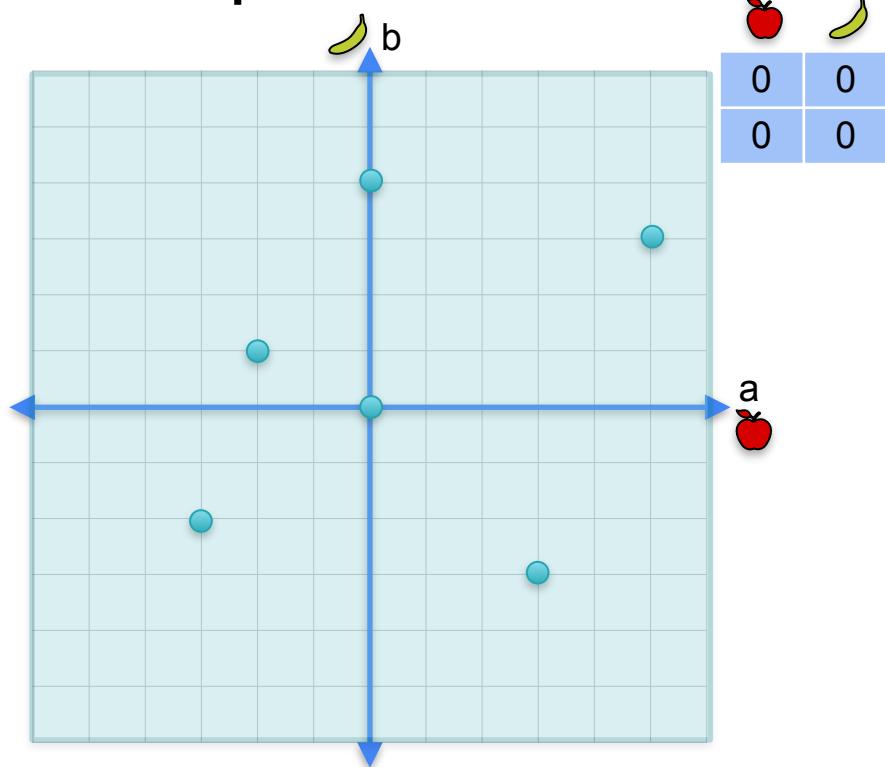
Null space



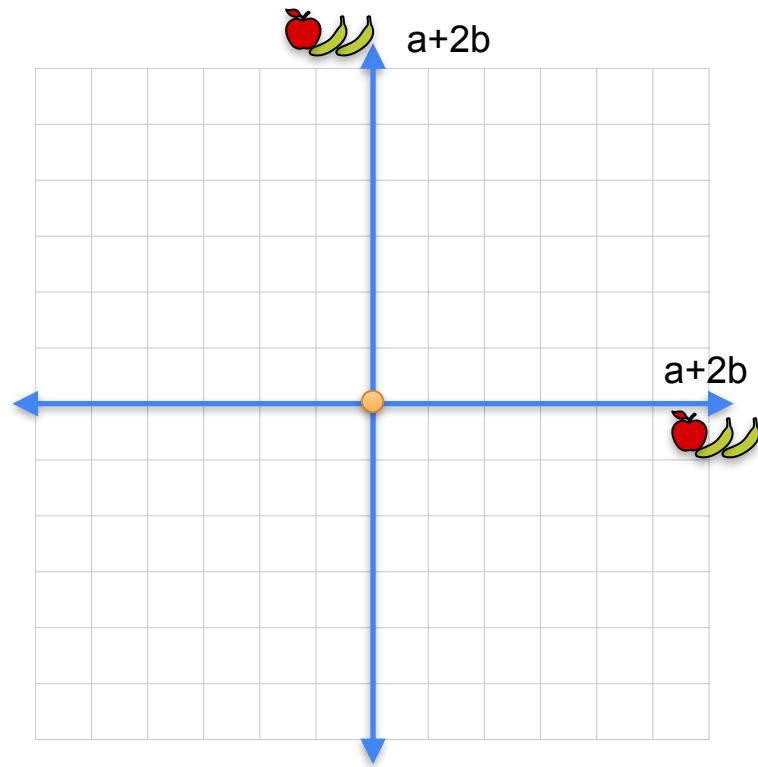
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$
$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 2 & -1 \\ -2 & 1 \end{matrix} & \begin{matrix} 4 \\ -2 \\ 2 \end{matrix} \end{matrix}$$



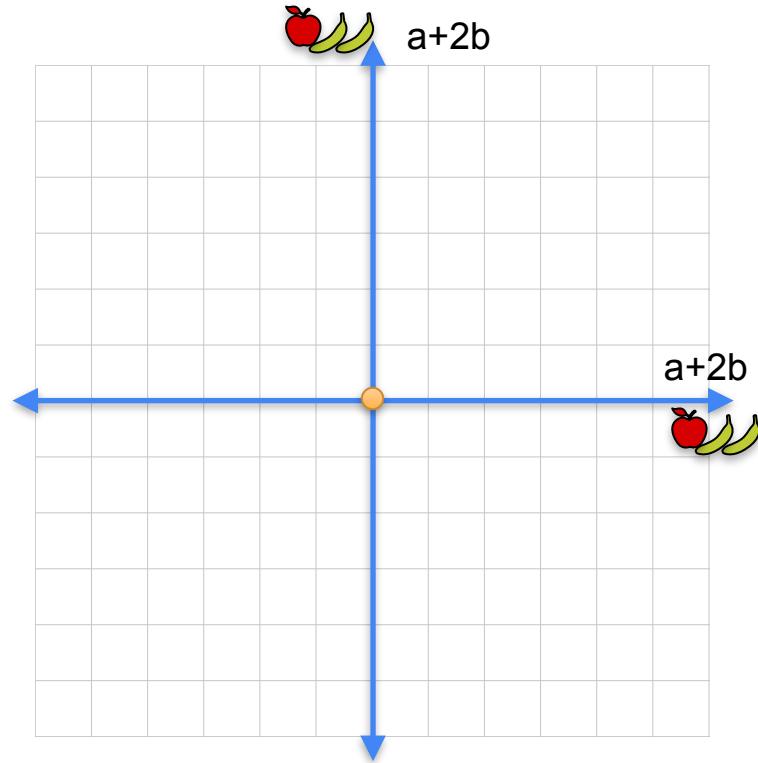
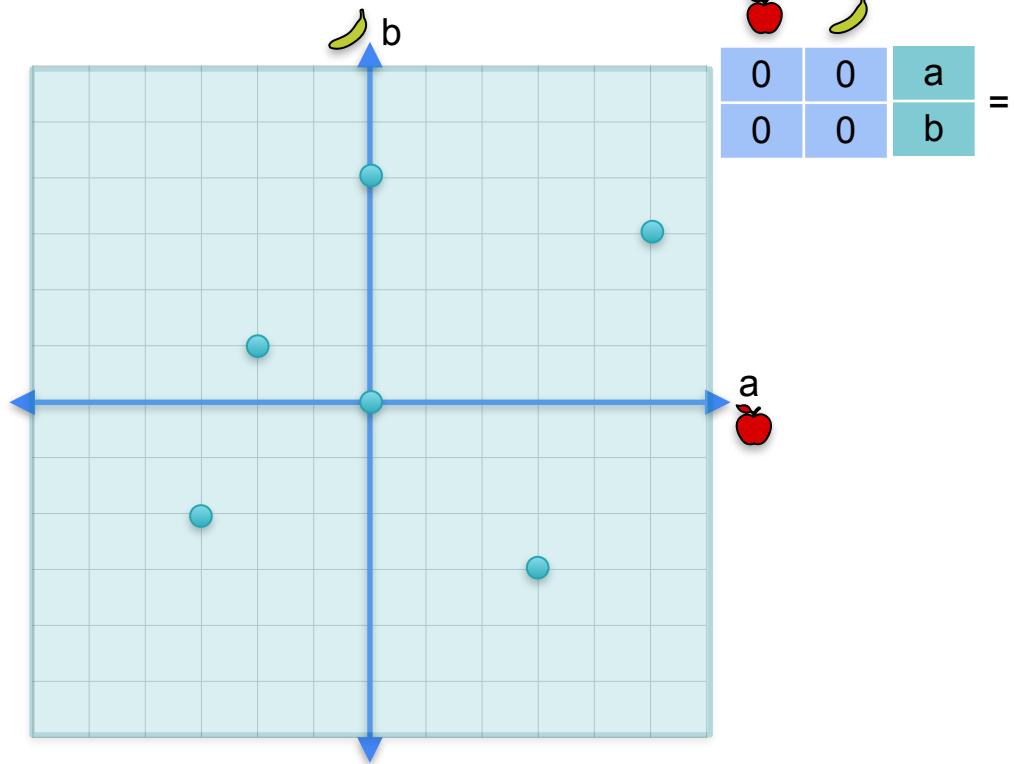
Null space



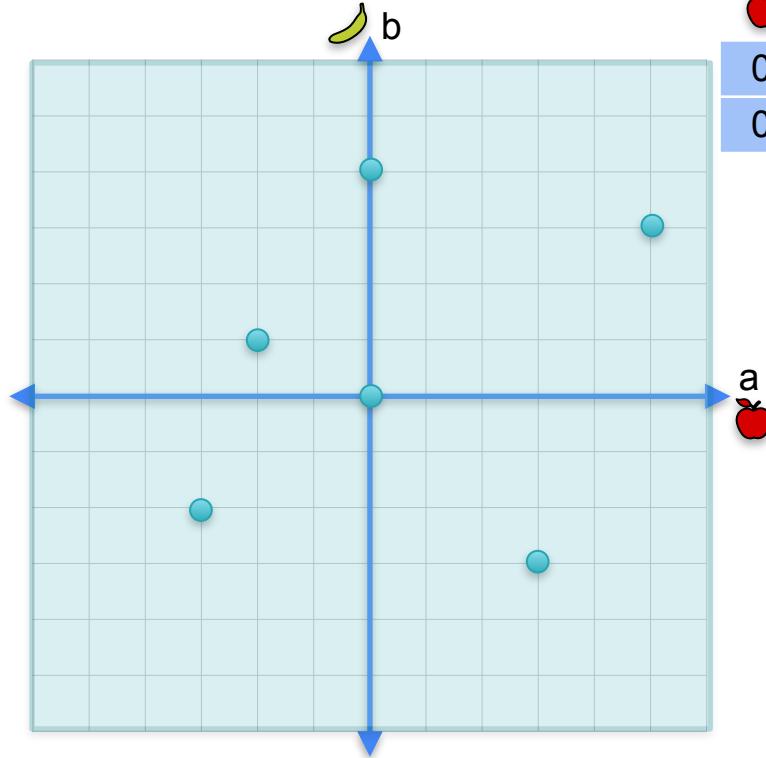
=



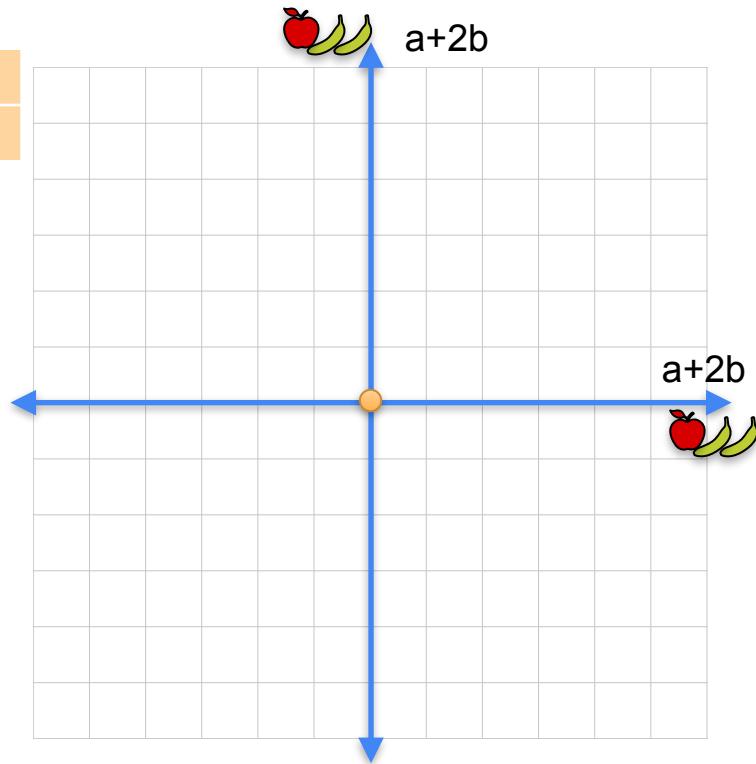
Null space



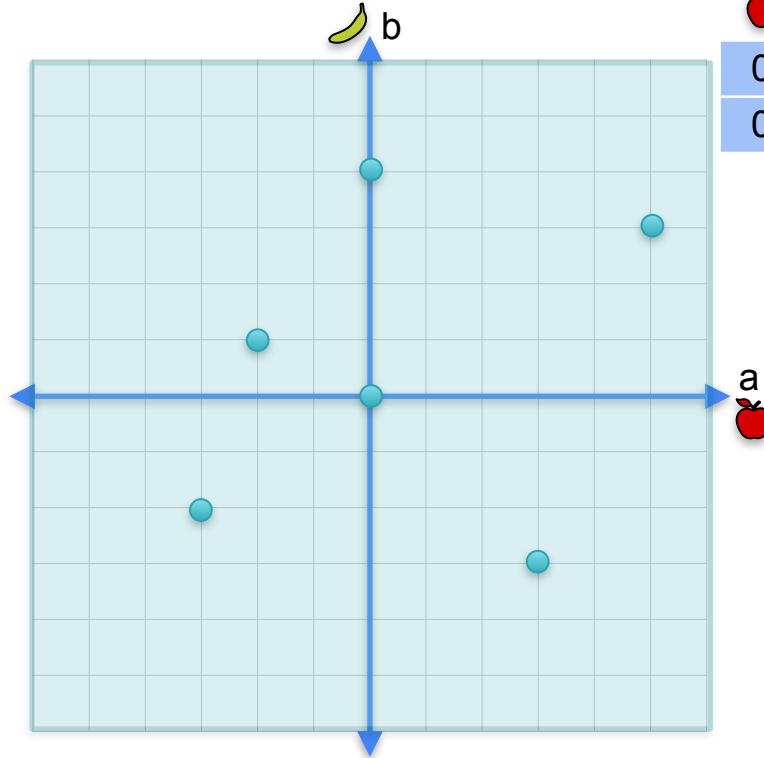
Null space



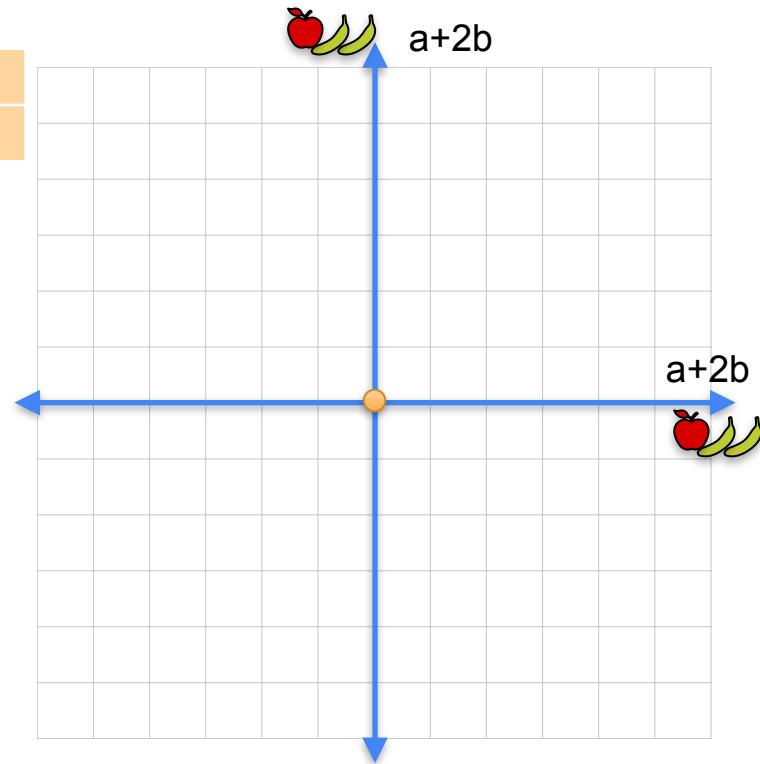
$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} a \\ b \end{matrix} \end{array} = \begin{array}{c} ? \\ ? \end{array}$$



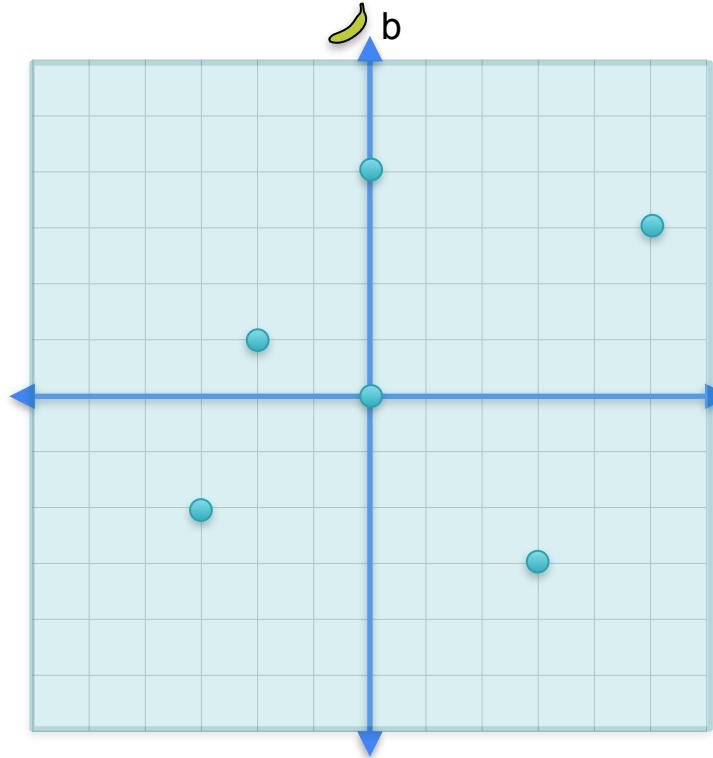
Null space



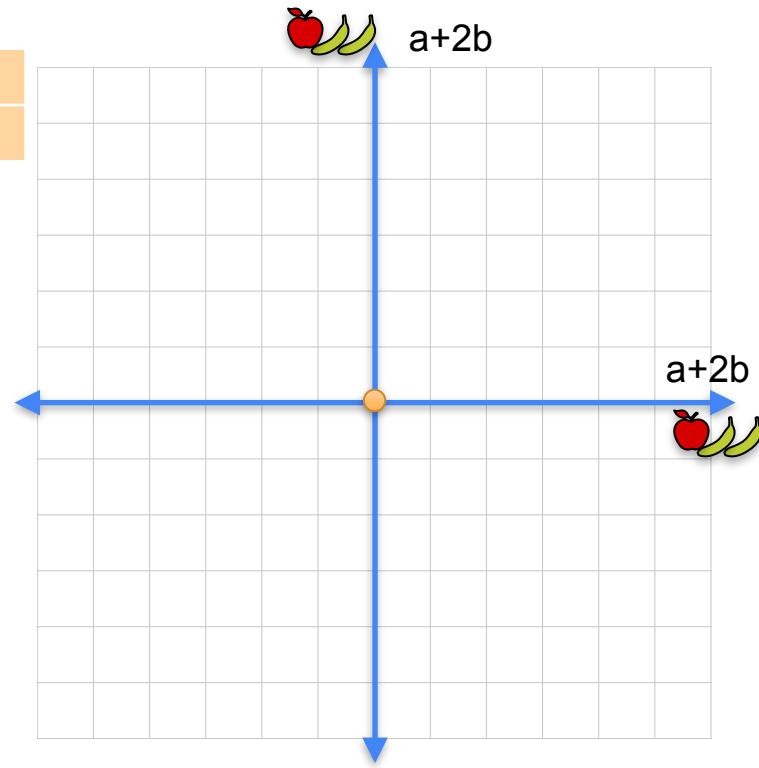
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} ? \\ ? \end{matrix} \end{array}$$



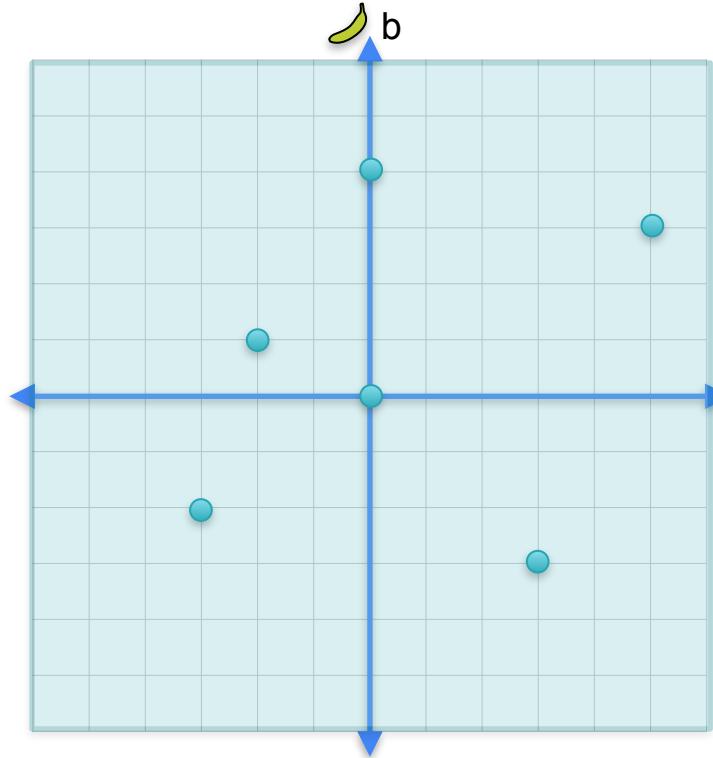
Null space



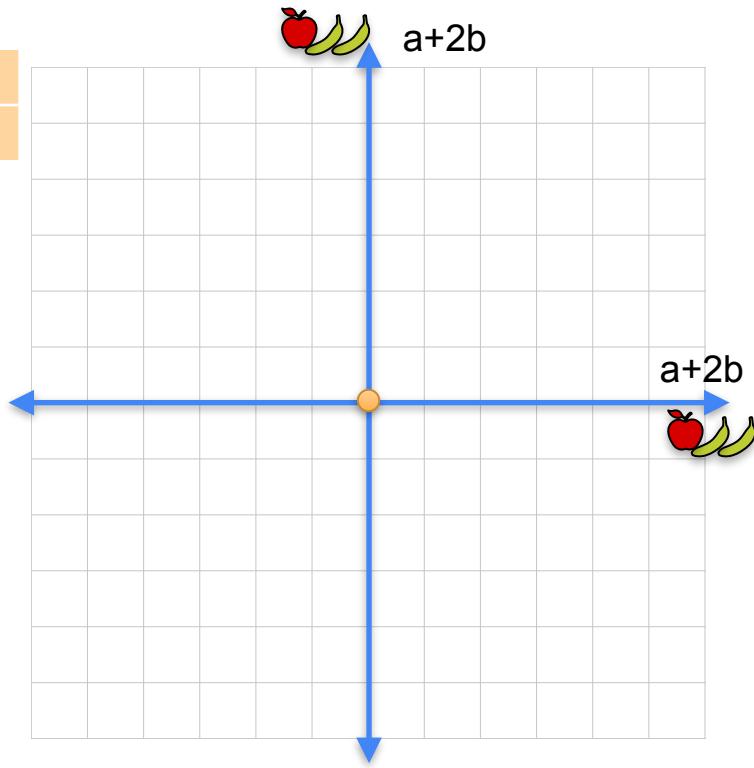
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} ? \\ ? \end{matrix} \\ \begin{matrix} -1 \\ 2 \end{matrix} \end{array}$$



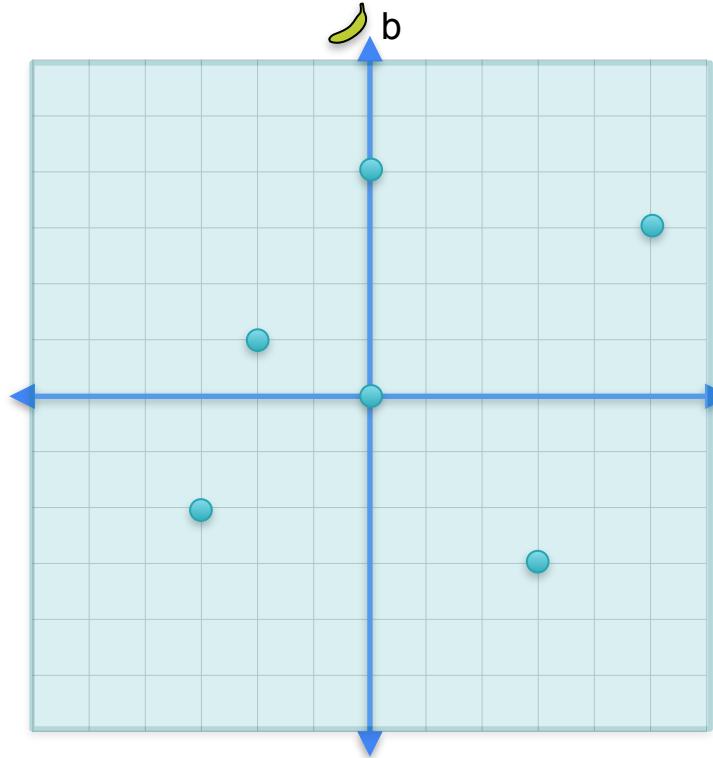
Null space



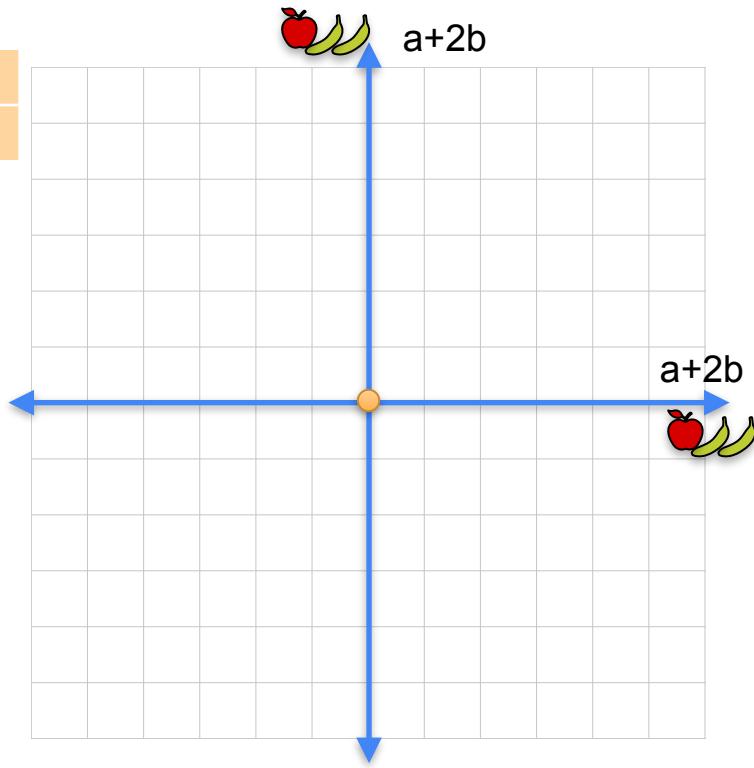
$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} ? \\ ? \end{matrix} \\ \begin{matrix} -1 \\ 2 \end{matrix} \quad \begin{matrix} 5 \\ 3 \end{matrix} \end{array}$$



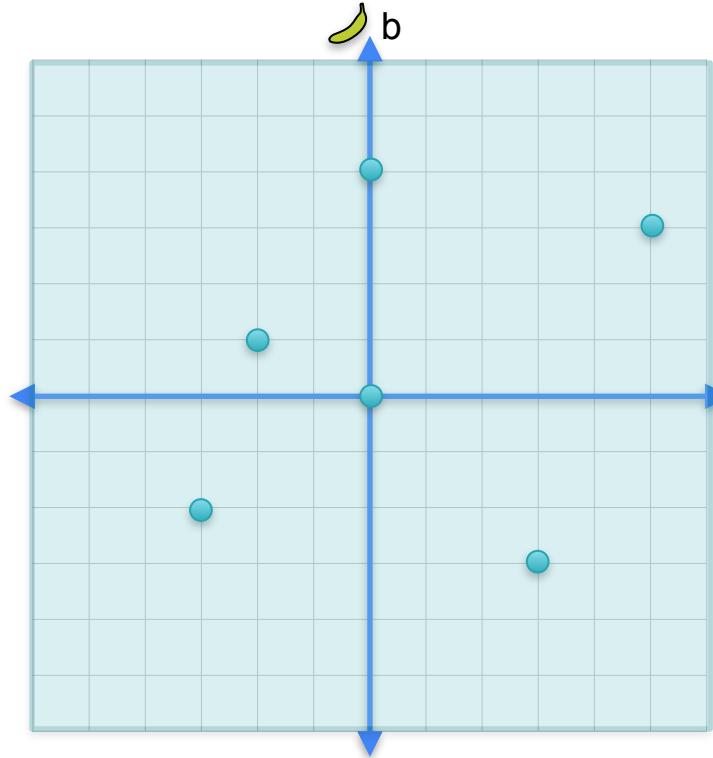
Null space



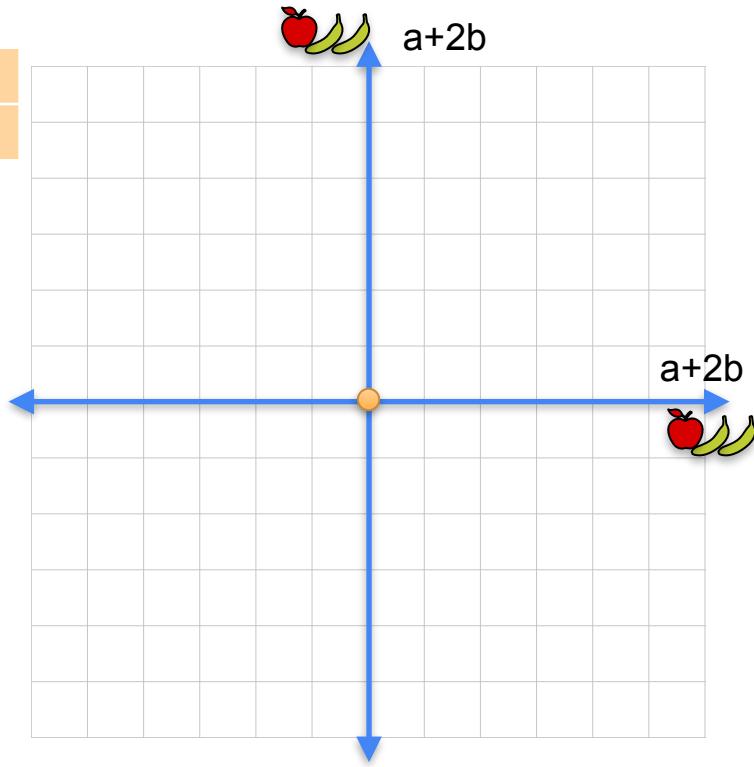
$$\begin{array}{ccc} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & = & \begin{matrix} ? \\ ? \end{matrix} \\ \begin{matrix} -1 \\ 2 \end{matrix} & & \begin{matrix} 5 \\ 3 \end{matrix} \\ \begin{matrix} 0 \\ 4 \end{matrix} & & \end{array}$$



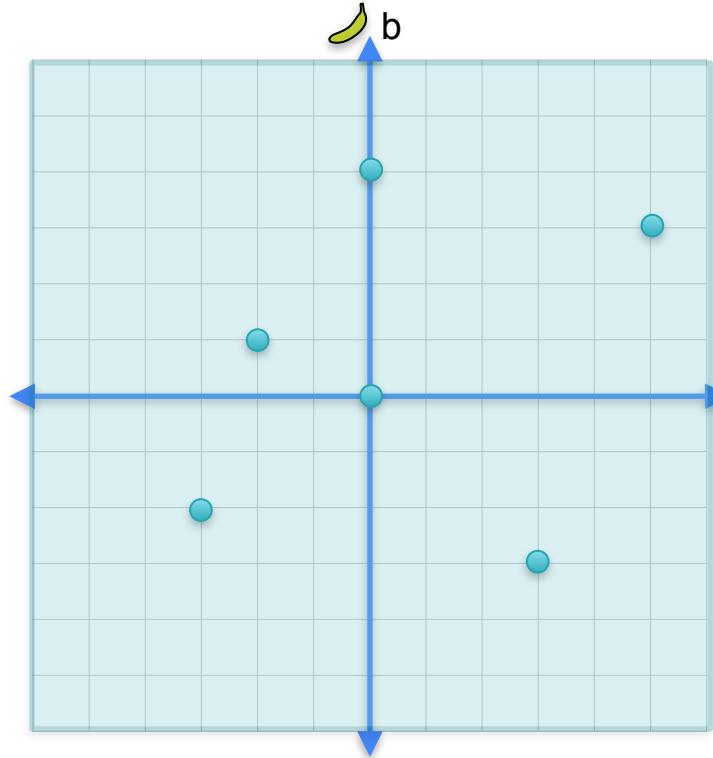
Null space



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} = \begin{matrix} ? \\ ? \end{matrix} \\ \begin{matrix} -1 & 5 \\ 2 & 3 \end{matrix} \\ \begin{matrix} 0 & -1 \\ 4 & -3 \end{matrix} \end{array}$$

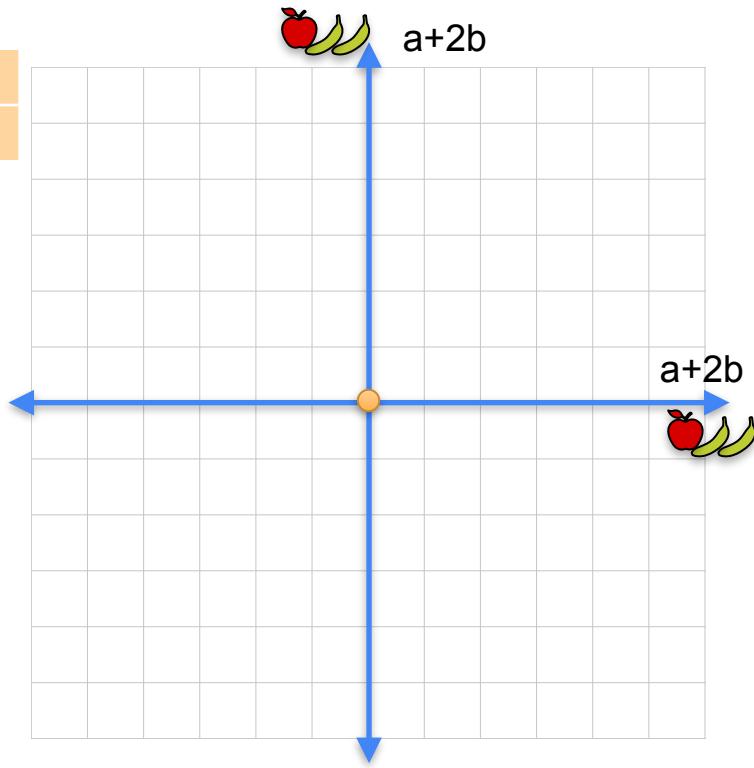


Null space



$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & = \begin{matrix} ? \\ ? \end{matrix} \end{array}$$

$\begin{matrix} -1 & 5 \\ 2 & 3 \\ 0 & -1 \\ 4 & -3 \\ 3 & -3 \end{matrix}$

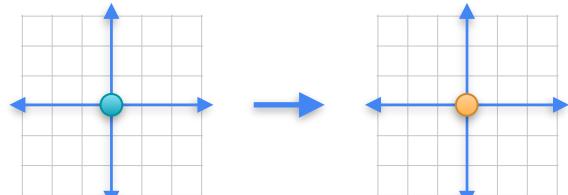


Null space

Non-singular

3	1
1	2

Rank = 2

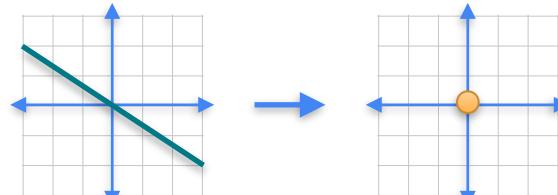


Dimension = 0

Singular

1	1
2	2

Rank = 1

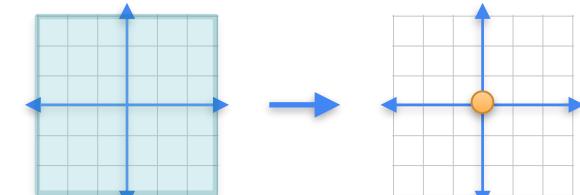


Dimension = 1

Singular

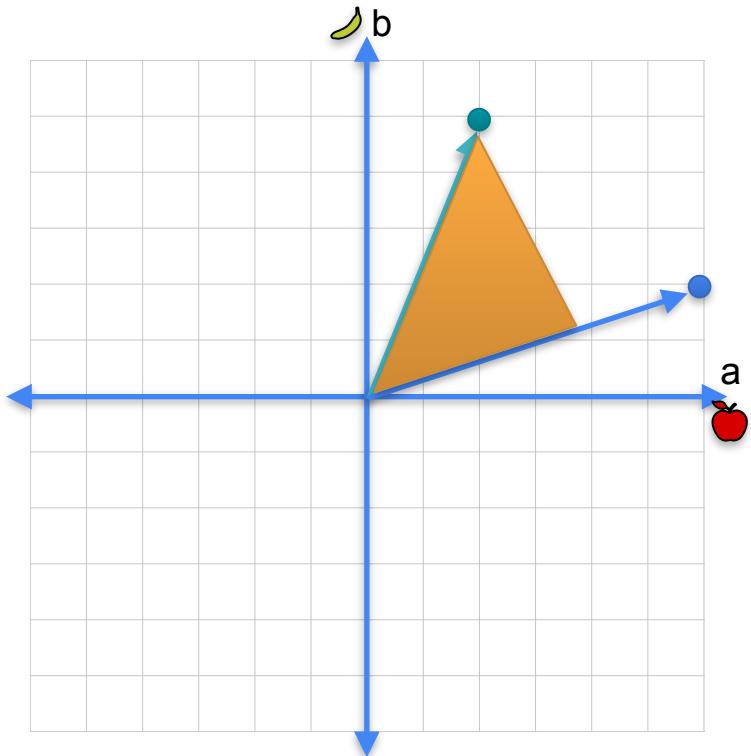
0	0
0	0

Rank = 0



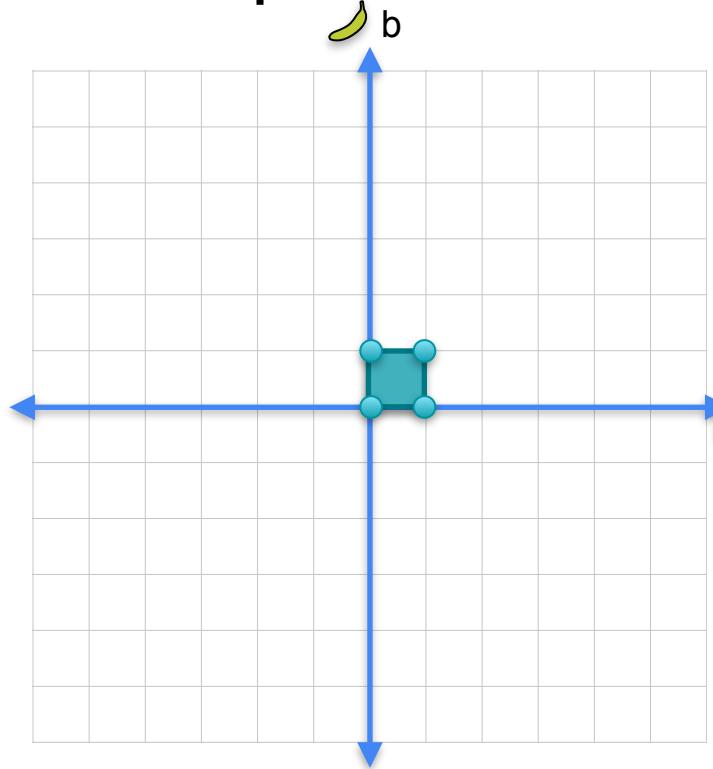
Dimension = 2

Dot product as an area

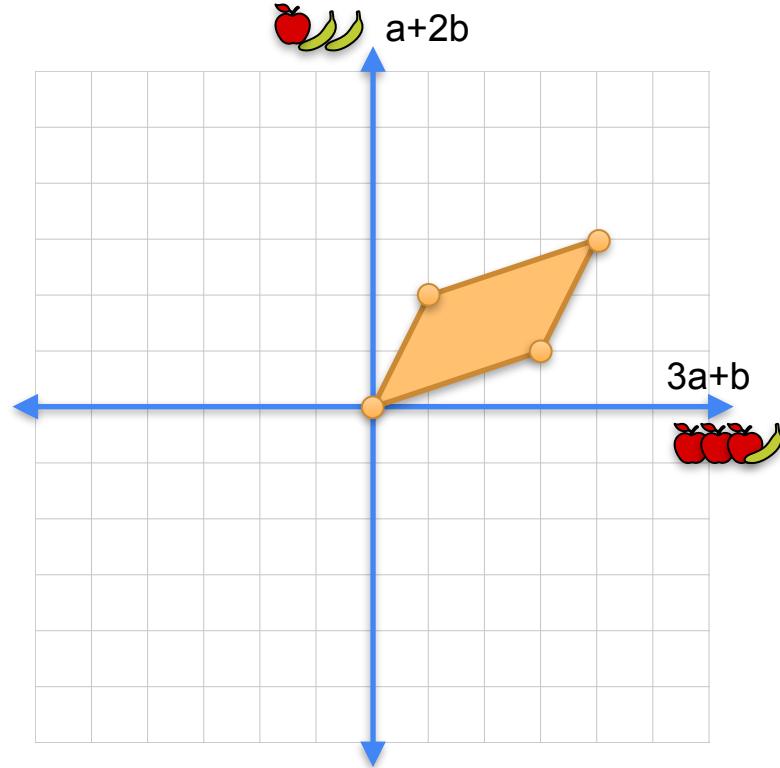


$$\begin{matrix} \text{apple} \\ 6 \end{matrix} \quad \begin{matrix} \text{banana} \\ 2 \end{matrix} \cdot \begin{matrix} \$\text{apple} \\ \$\text{banana} \end{matrix} \begin{matrix} 2 \\ 5 \end{matrix} = \$ \quad 22$$

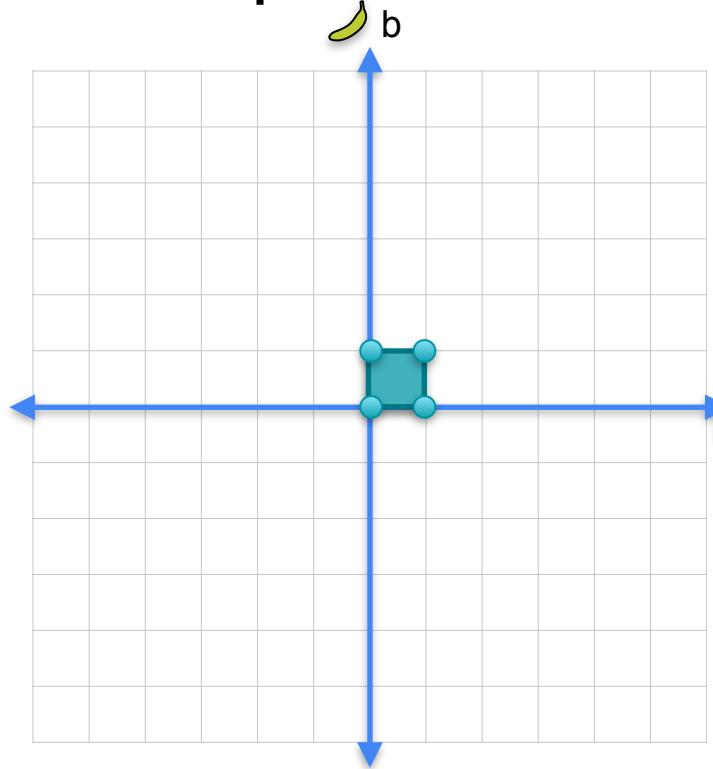
Row space



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{l} (0,0) \rightarrow (0,0) \\ (1,0) \rightarrow (3,1) \\ (0,1) \rightarrow (1,2) \\ (1,1) \rightarrow (4,3) \end{array}$$

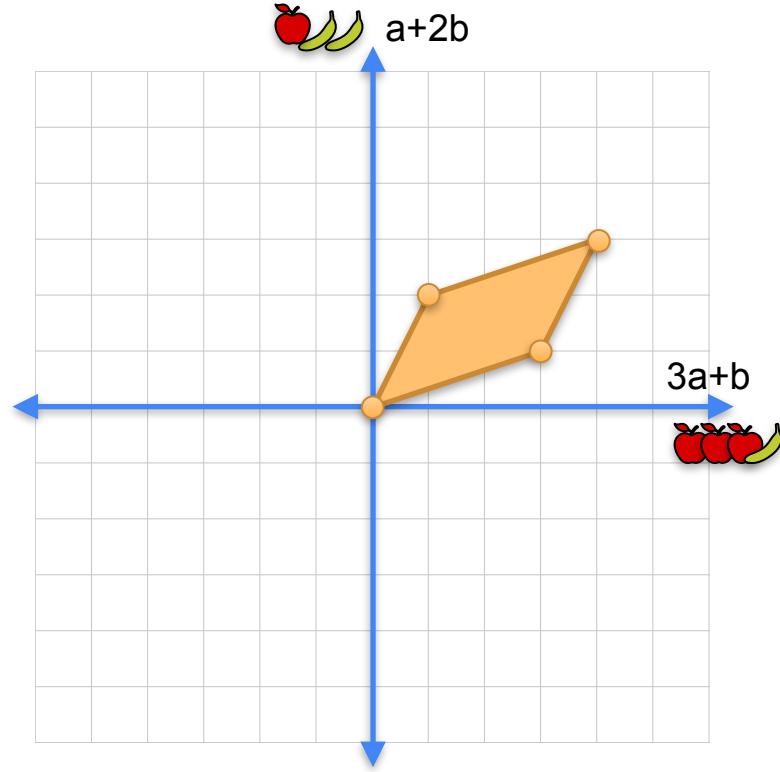


Row space

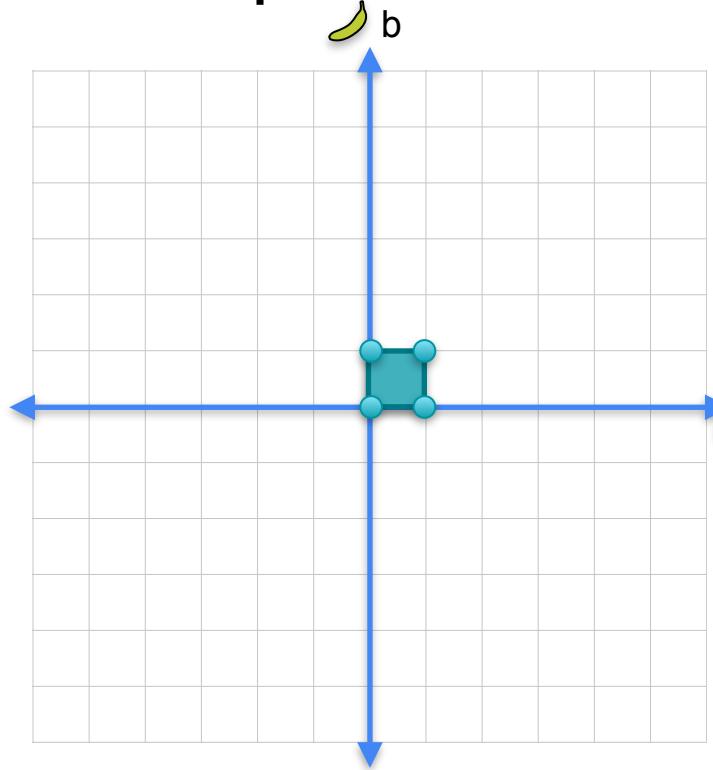


$$\begin{array}{|c|c|c|} \hline 3 & 1 & 0 \\ \hline 1 & 2 & 0 \\ \hline \end{array} =$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

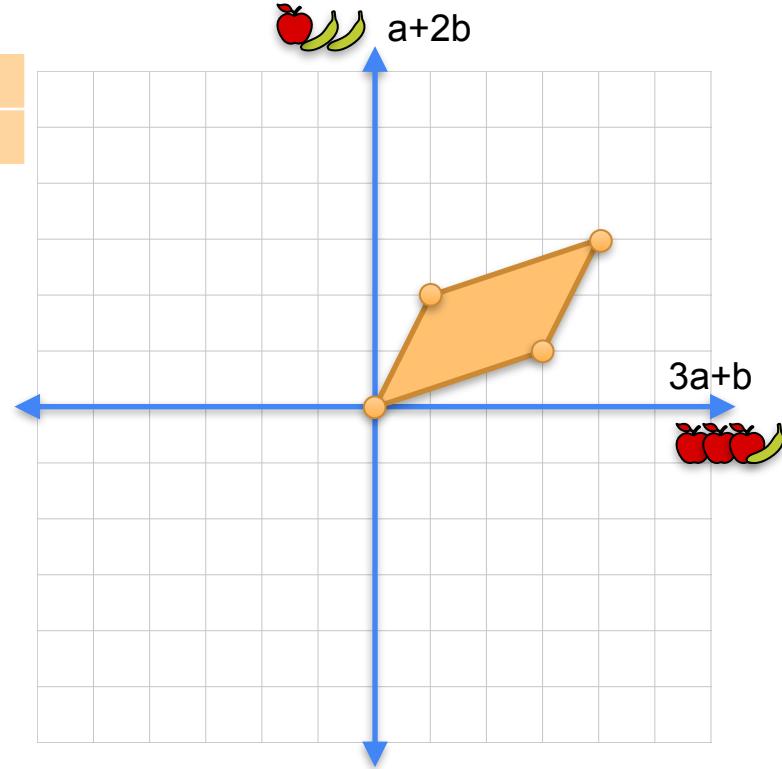


Row space

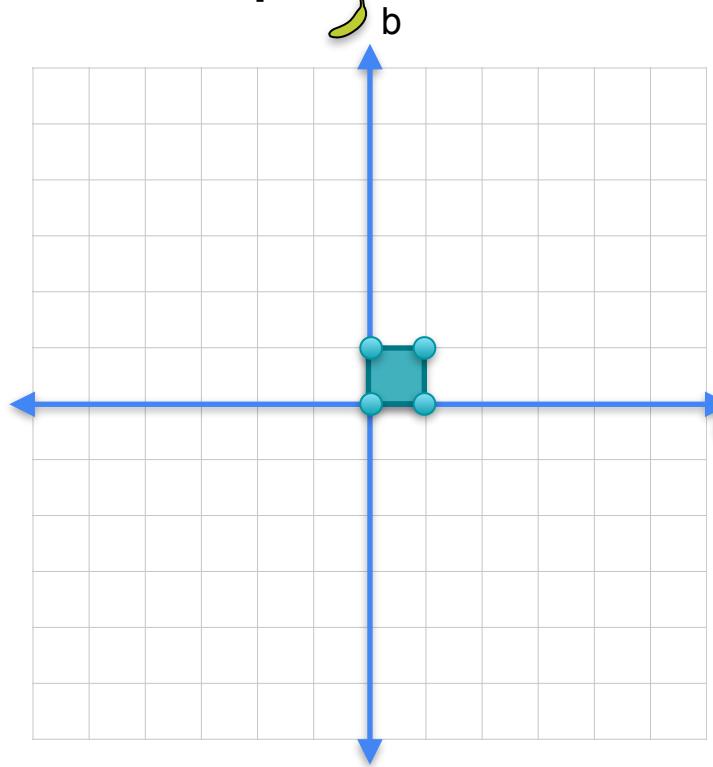


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ = & \begin{matrix} 0 \\ 0 \end{matrix} \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$



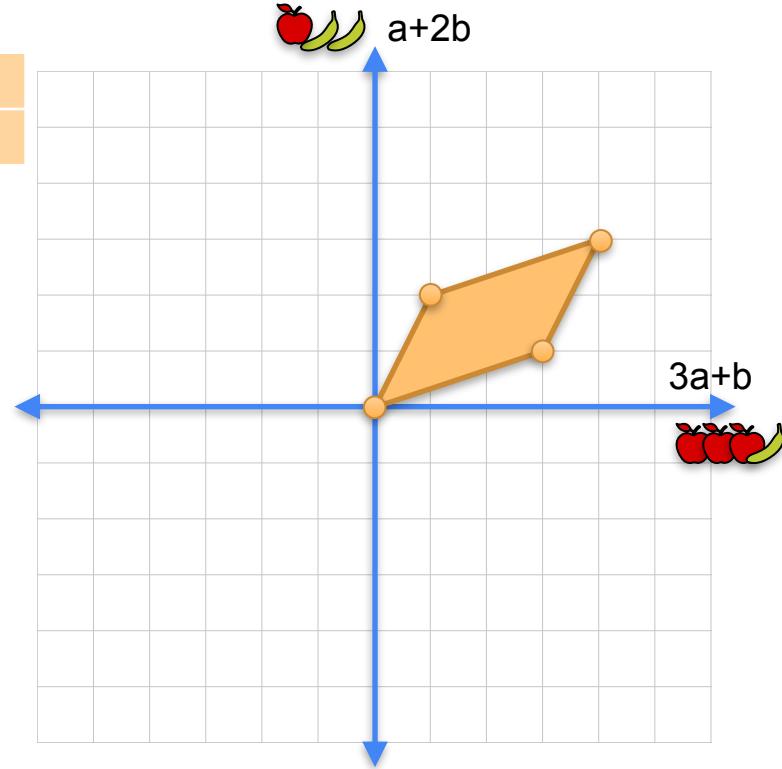
Row space



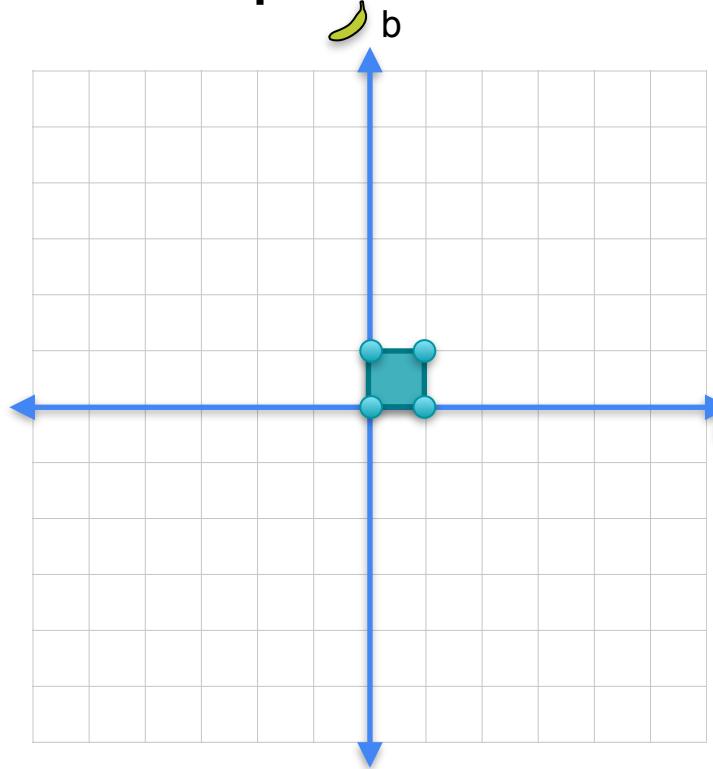
A 2x2 matrix equation. The left side shows a matrix with columns labeled 'apple' and 'banana'. The matrix has rows: [3, 1] and [1, 2]. To the right of the equals sign is another matrix with columns labeled 'apple' and 'banana', and rows [1, 0] and [0, 0].

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & = \begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

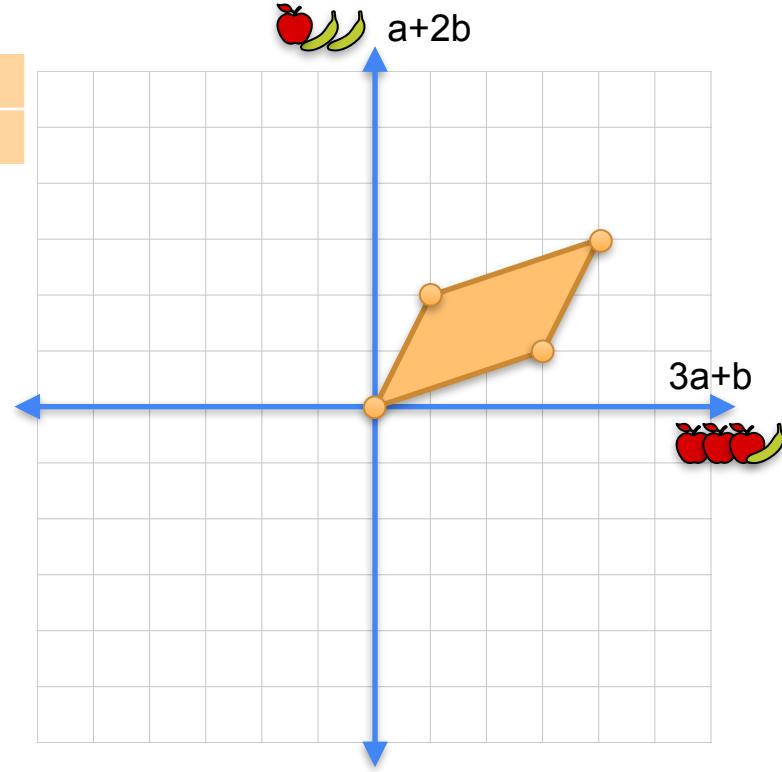


Row space

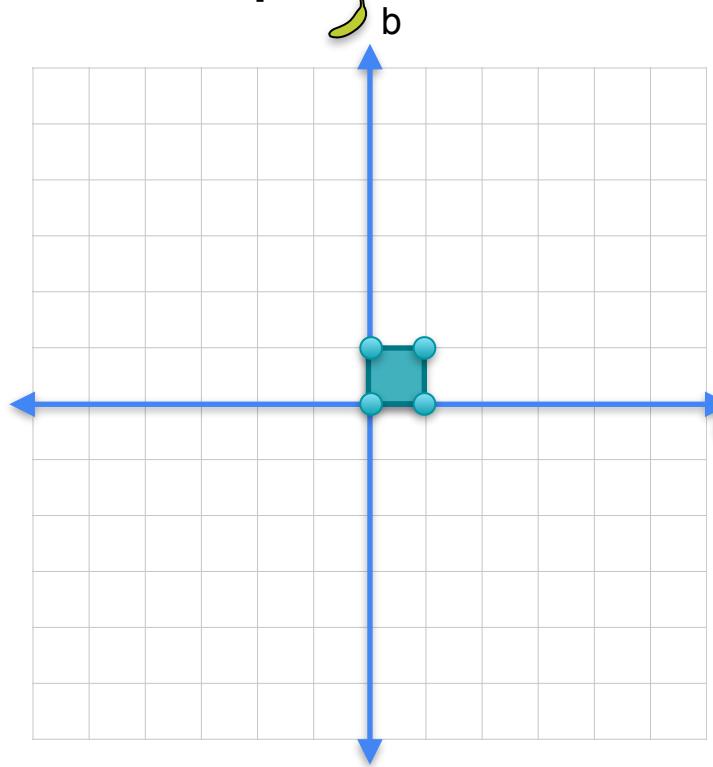


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} \end{array} = \begin{array}{c} 3 \\ 1 \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

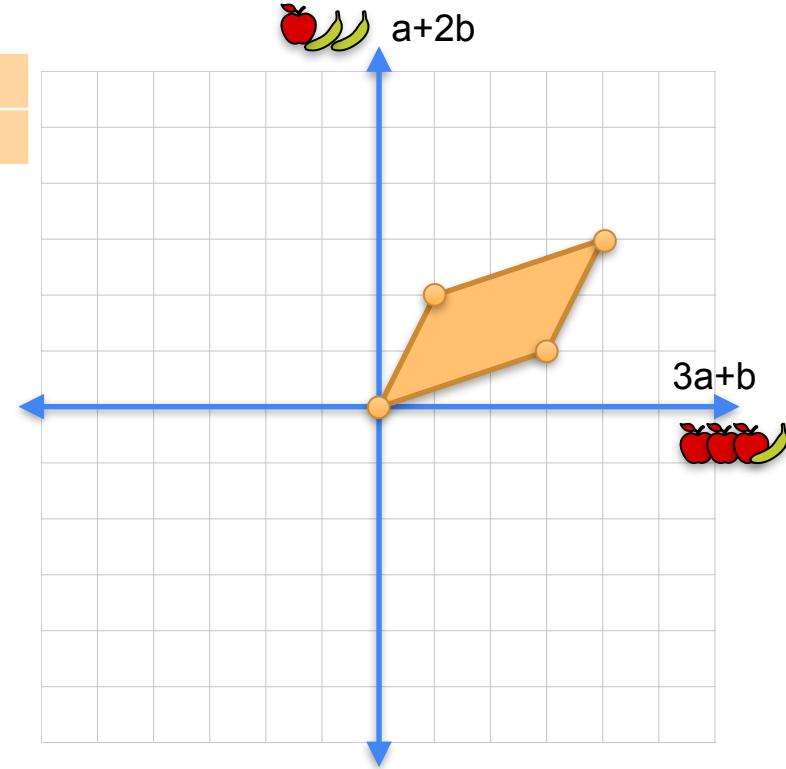


Row space

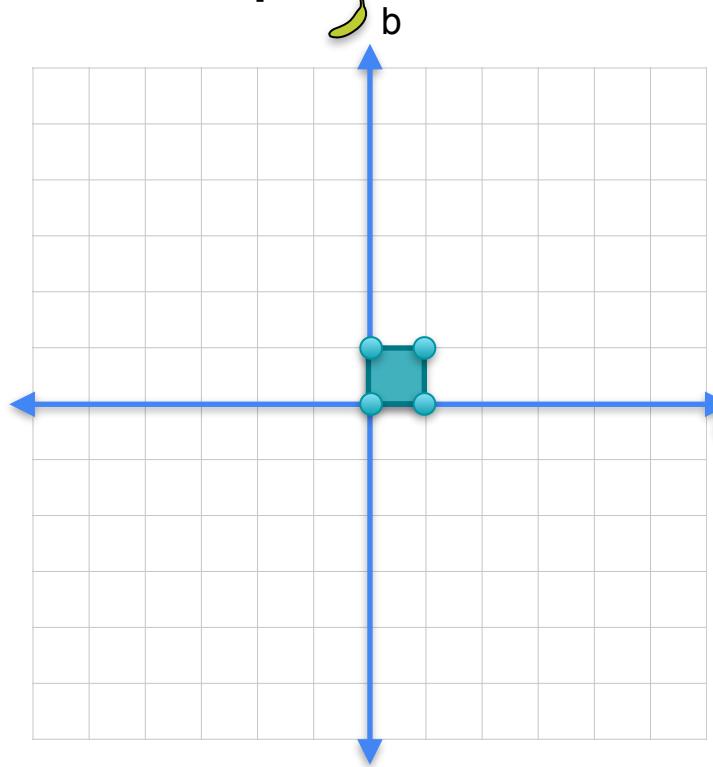


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 0 \\ 1 \end{matrix} \end{array} = \begin{array}{c} 3 \\ 1 \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

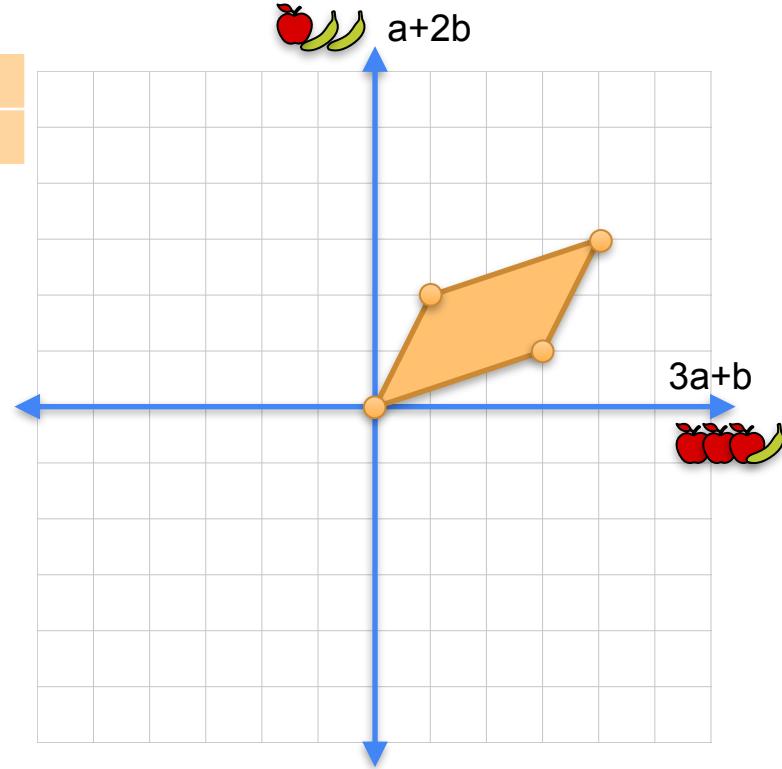


Row space

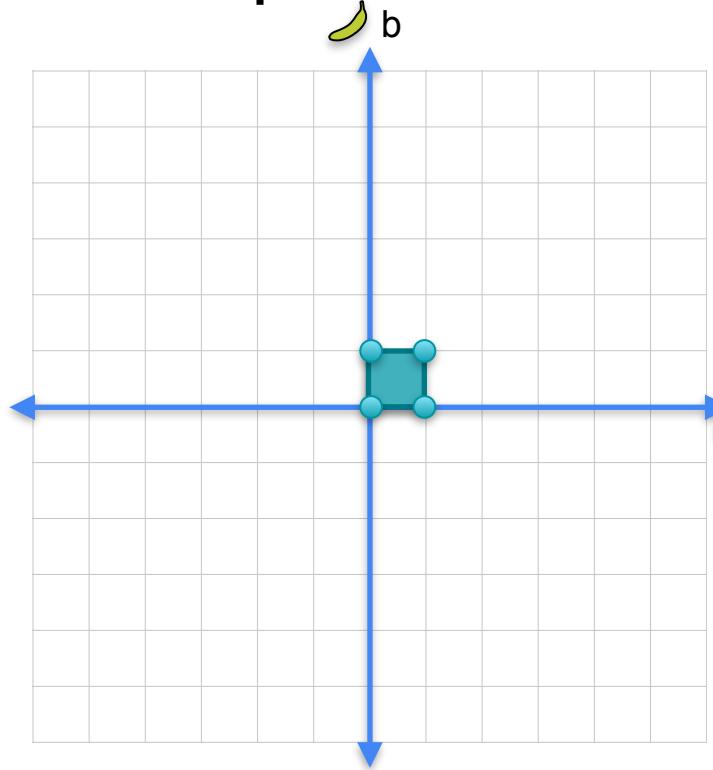


$$\begin{array}{cc|c} \text{apple} & \text{banana} \\ \hline 3 & 1 & 0 \\ 1 & 2 & 1 \end{array} = \begin{array}{cc} 1 & \\ & 2 \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$

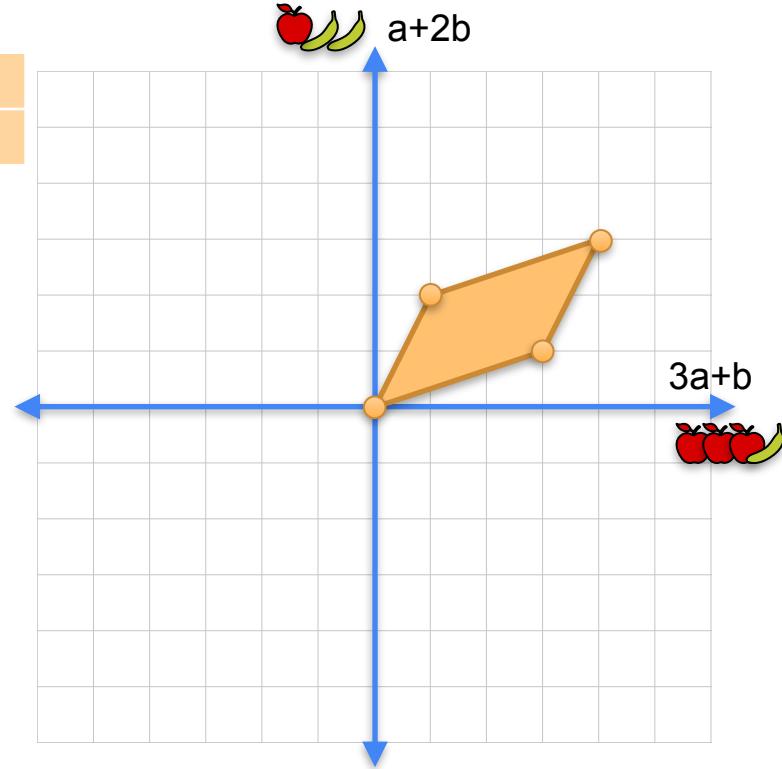


Row space

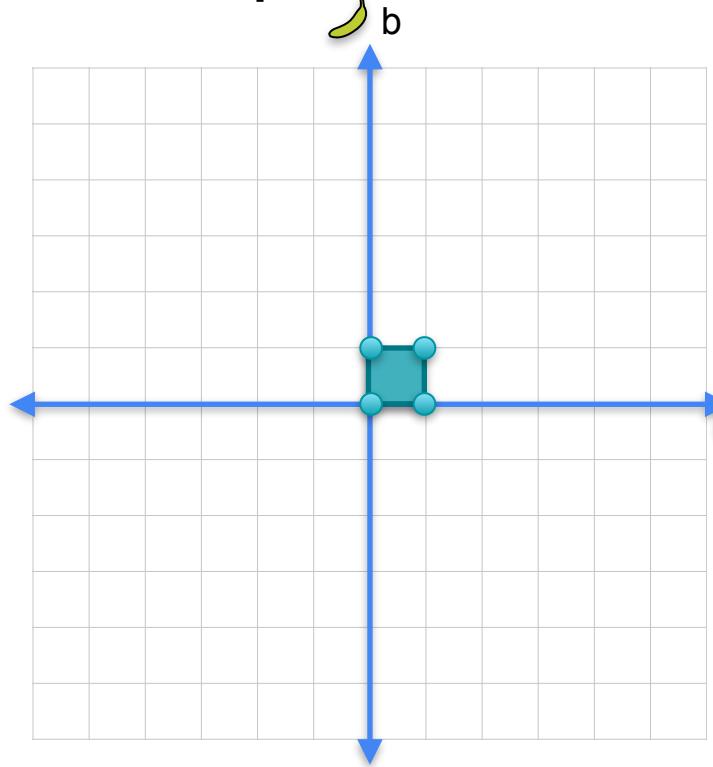


$$\begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{array} = \begin{array}{cc} \text{apple} & \text{banana} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \end{array}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$



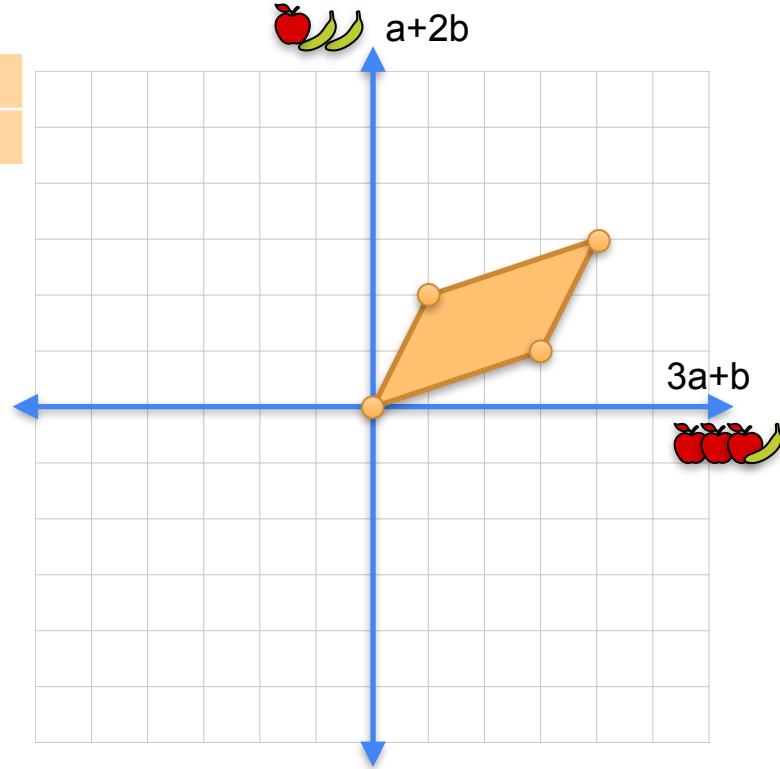
Row space



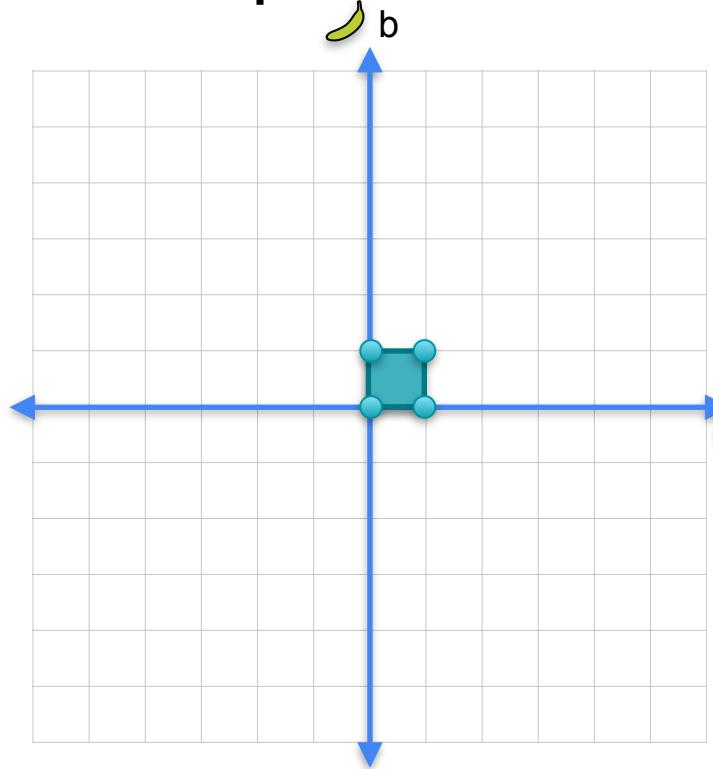
A 2x3 matrix equation. The left side shows a 2x2 matrix with columns labeled 'apple' and 'banana'. The first column has entries 3 and 1, the second column has entries 1 and 2. To the right of the matrix is an equals sign. To the right of the equals sign is a 2x1 matrix with entries 4 and 3. Above the matrix, there is a small diagram of an apple and a banana.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & = \begin{matrix} 4 \\ 3 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (3,1)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (4,3)$



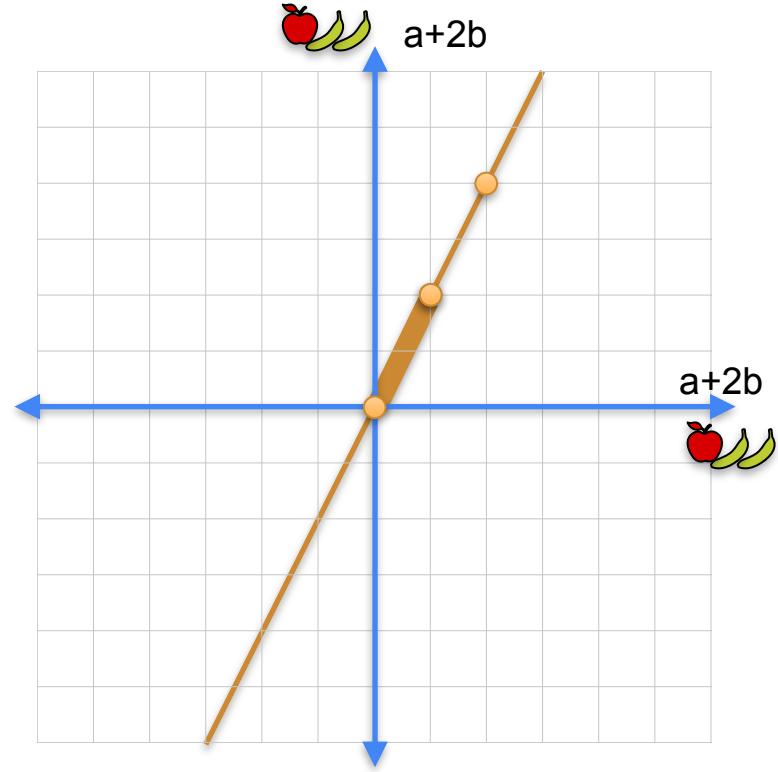
Row space



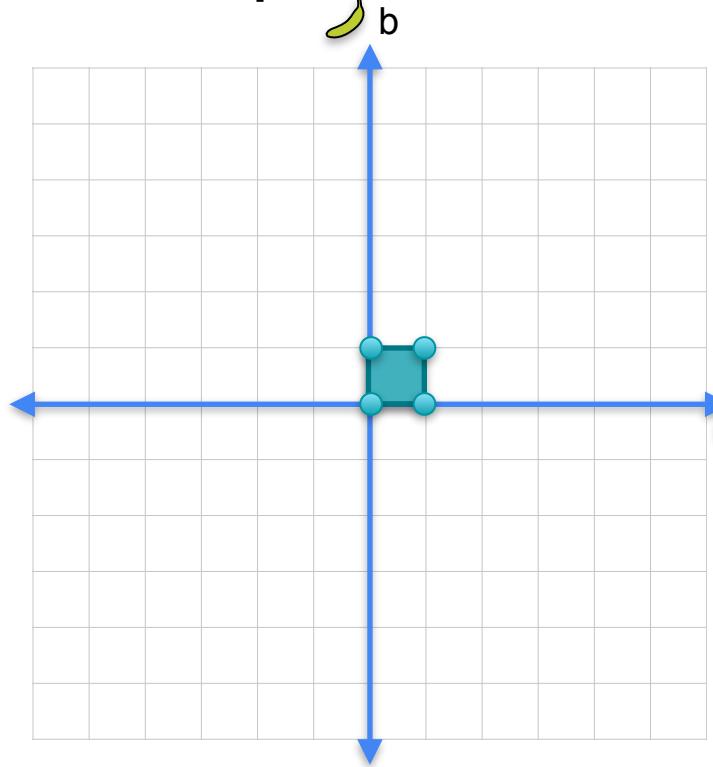
1	1
2	2

=

$$\begin{aligned}(0,0) &\rightarrow (0,0) \\ (1,0) &\rightarrow (1,2) \\ (0,1) &\rightarrow (1,2) \\ (1,1) &\rightarrow (2,4)\end{aligned}$$



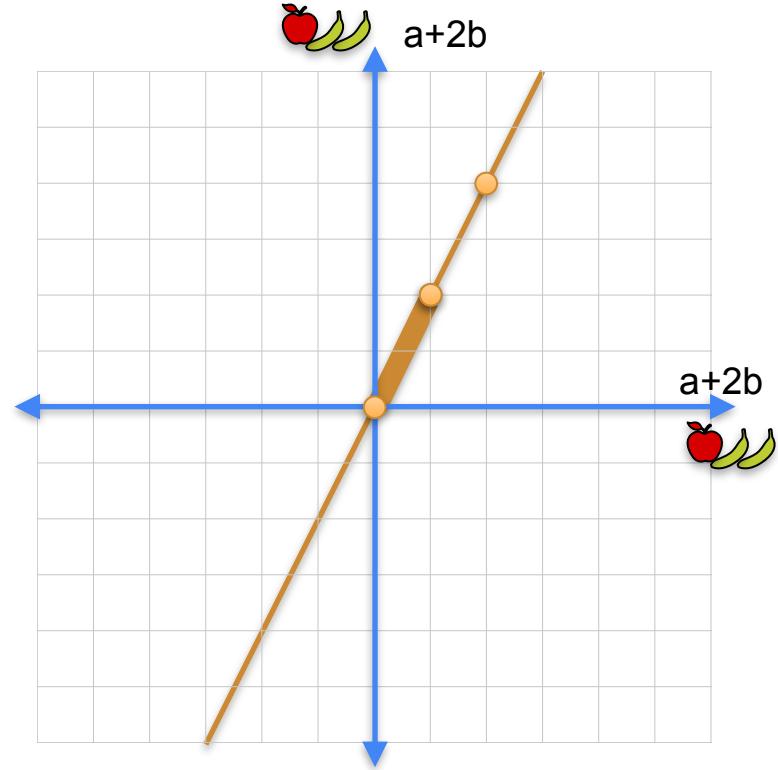
Row space



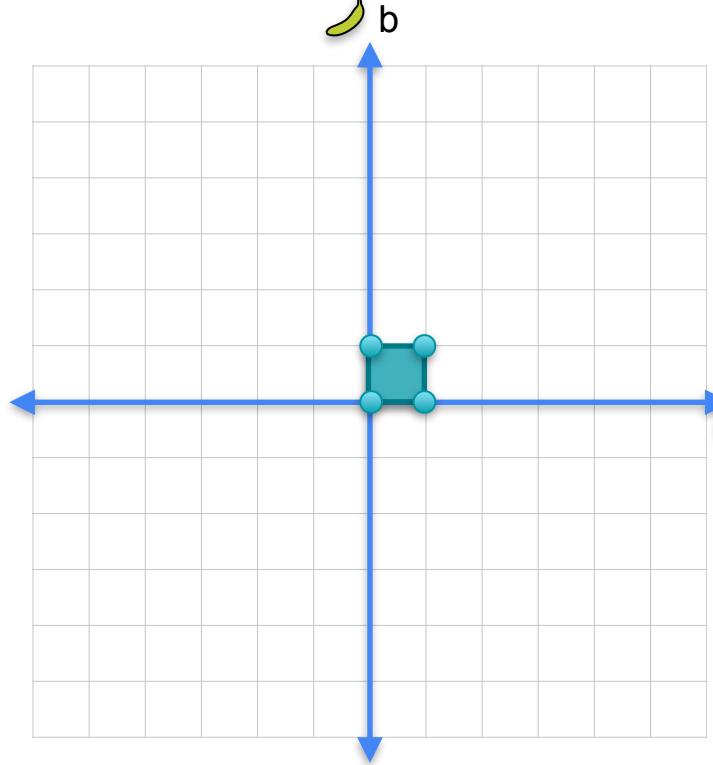
apple banana

$$\begin{matrix} 1 & 1 & 0 \\ 2 & 2 & 0 \end{matrix} =$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$

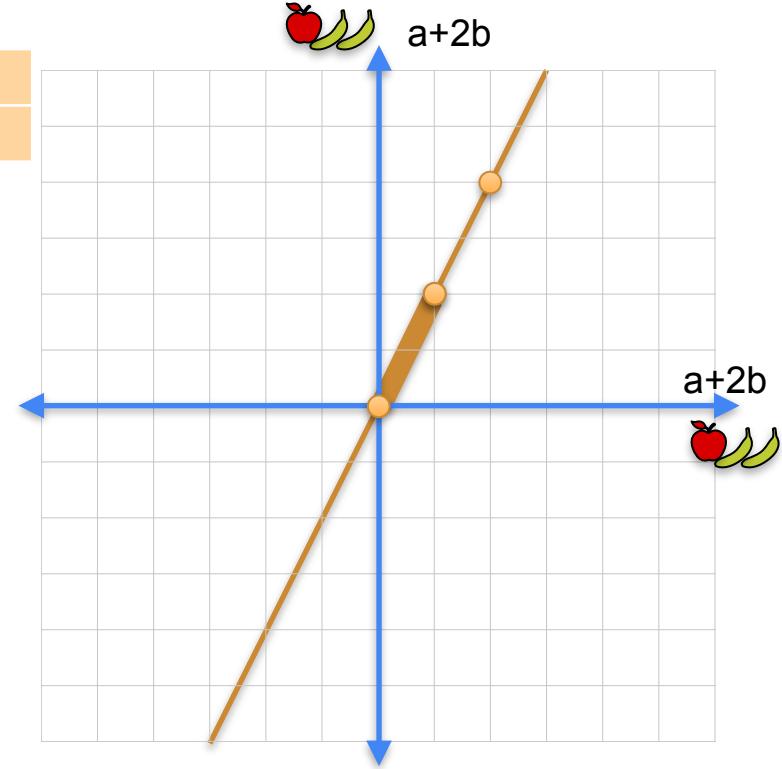


Row space

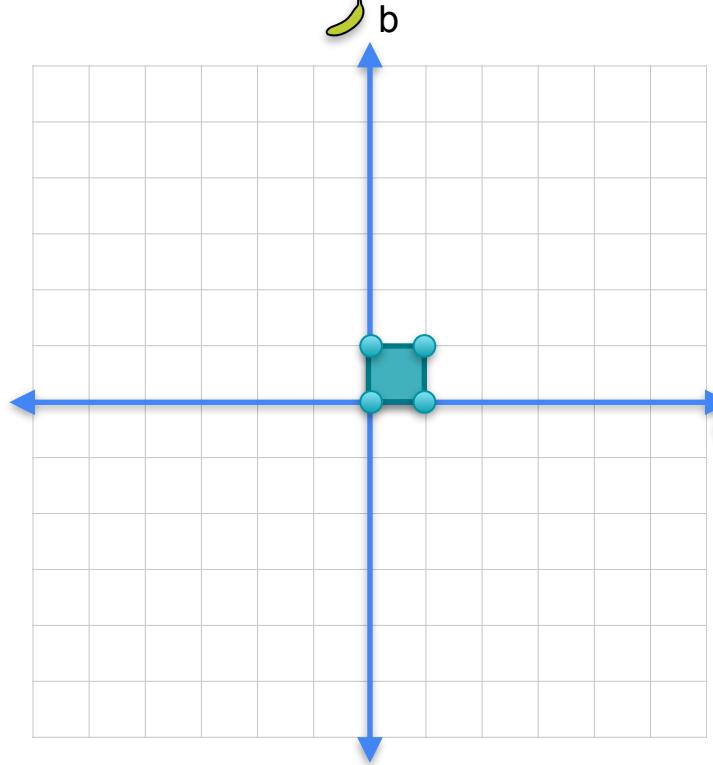


A 2x2 matrix equation. On the left, there is a 2x2 matrix with two red apple icons in the first column and two green banana icons in the second column. To its right is an equals sign. To the right of the equals sign is another 2x2 matrix with a green banana icon in the top-left cell and a red apple icon in the bottom-right cell. This represents the equation $\begin{pmatrix} \text{apple} & \text{banana} \\ 1 & 1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} \text{banana} \\ 0 \\ 0 \end{pmatrix}$.

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



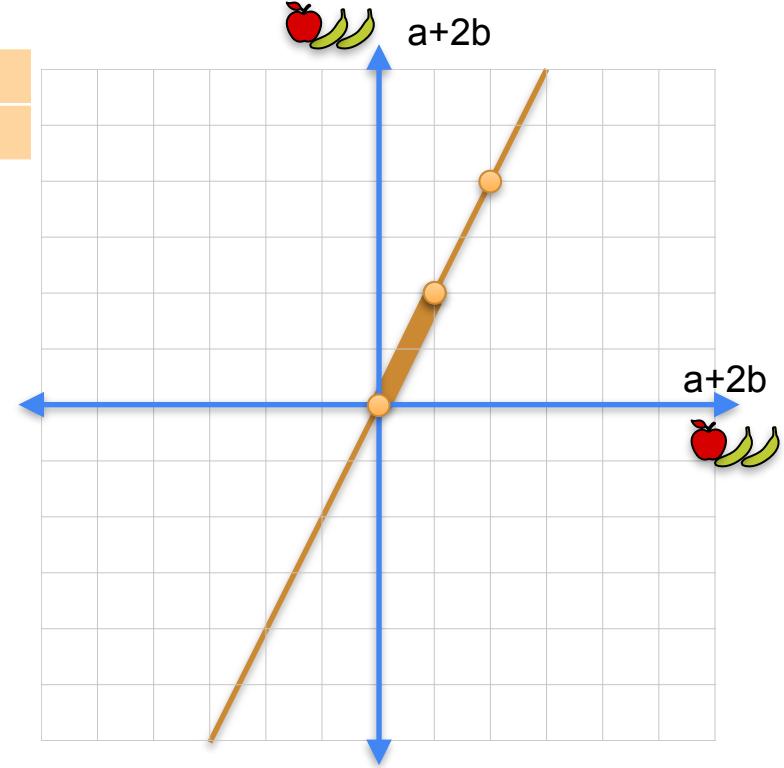
Row space



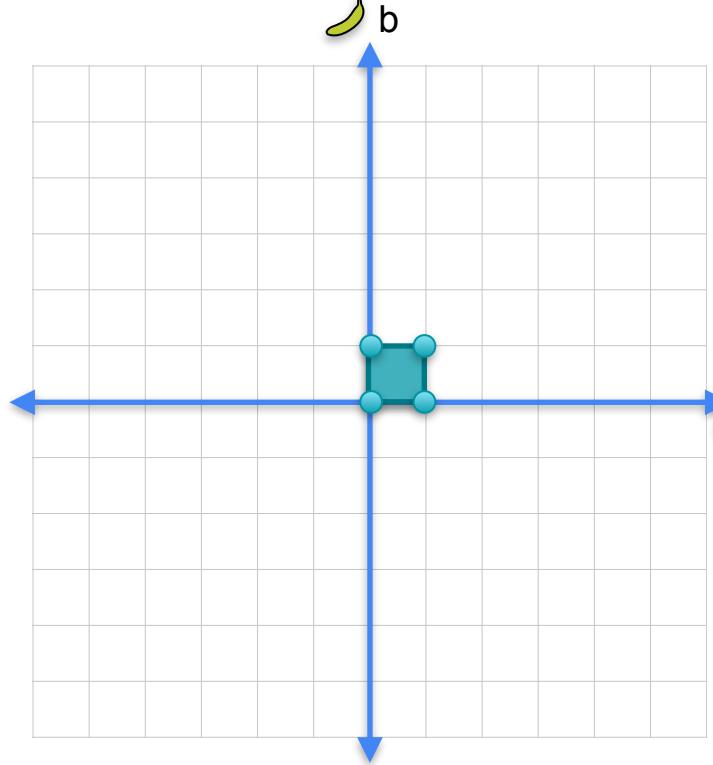
A 2x3 matrix equation. On the left, there are two fruit icons: a red apple above a blue 2x2 matrix and a green banana above a teal 2x1 column vector. To the right of the equals sign is another set of fruit icons: a red apple above an orange 2x1 column vector and a green banana above a blue 2x1 column vector.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} 1 \\ 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \\ \text{banana} & \text{apple} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



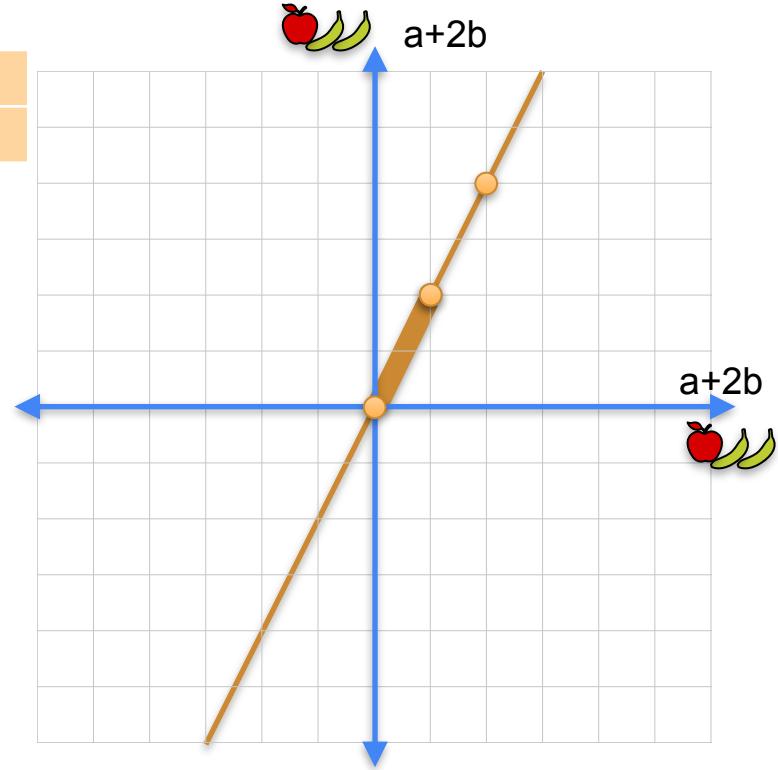
Row space



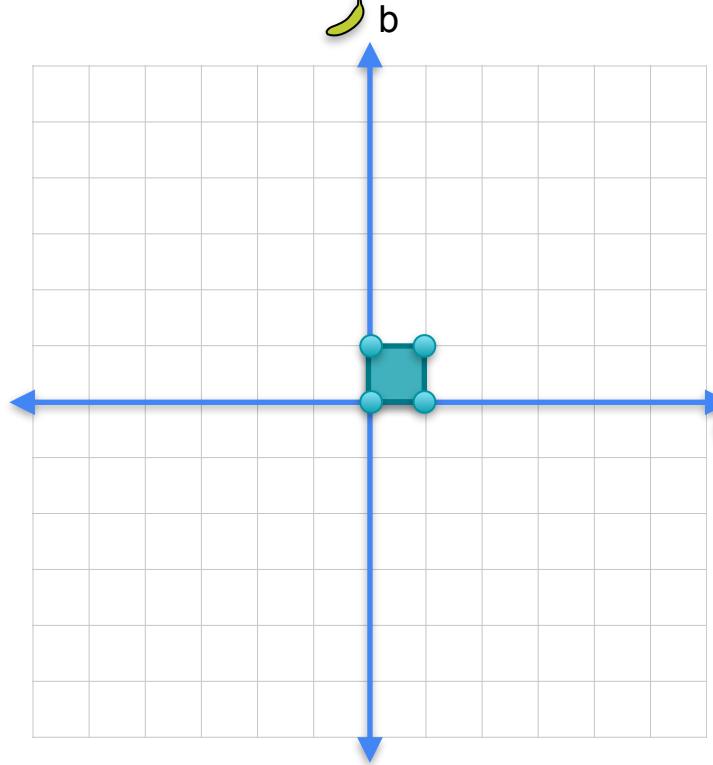
Apples Bananas

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 2 & 0 \end{matrix} = \begin{matrix} 3 \\ 1 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



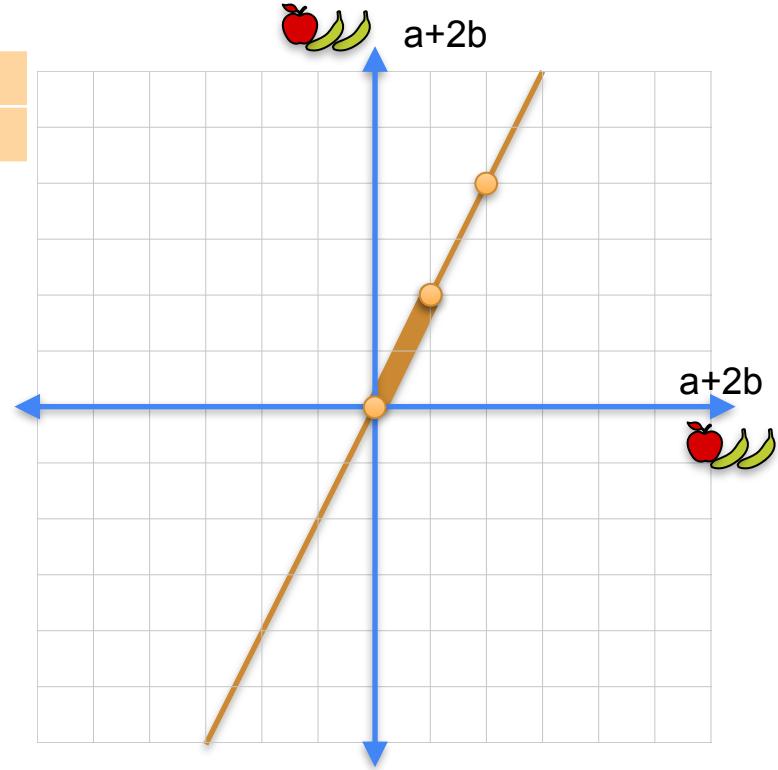
Row space



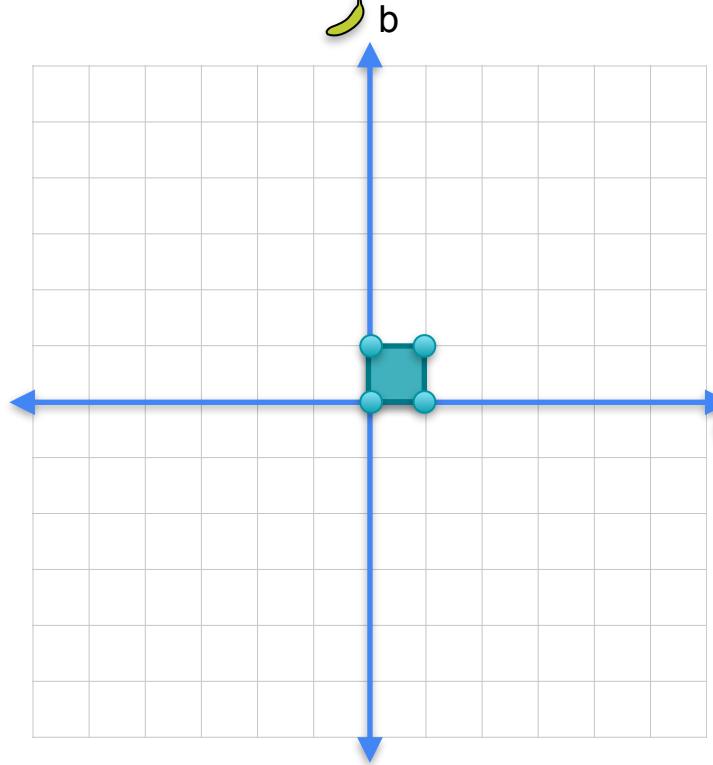
A 2x2 matrix equation. On the left, there is a red apple icon above the first column and a green banana icon above the second column. To the right of the equals sign is a green arrow pointing upwards, labeled $a+2b$. The matrix itself is a 2x2 grid with columns 1 and 2 colored blue, and rows 0 and 1 colored teal. The entries are 1, 1, 0, 2 in the first row and 2, 2, 1, 1 in the second row. To the right of the matrix is an orange arrow pointing upwards, labeled $a+2b$, which points to the same green arrow.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & \begin{matrix} 0 & 1 \end{matrix} \end{matrix} = \begin{matrix} \text{apple} \\ \begin{matrix} 3 & 1 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



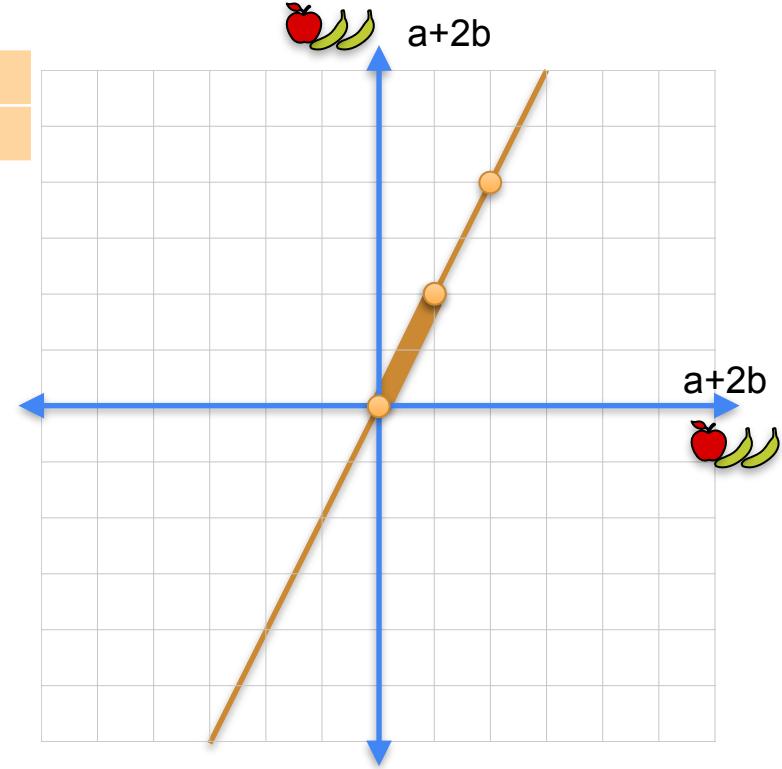
Row space



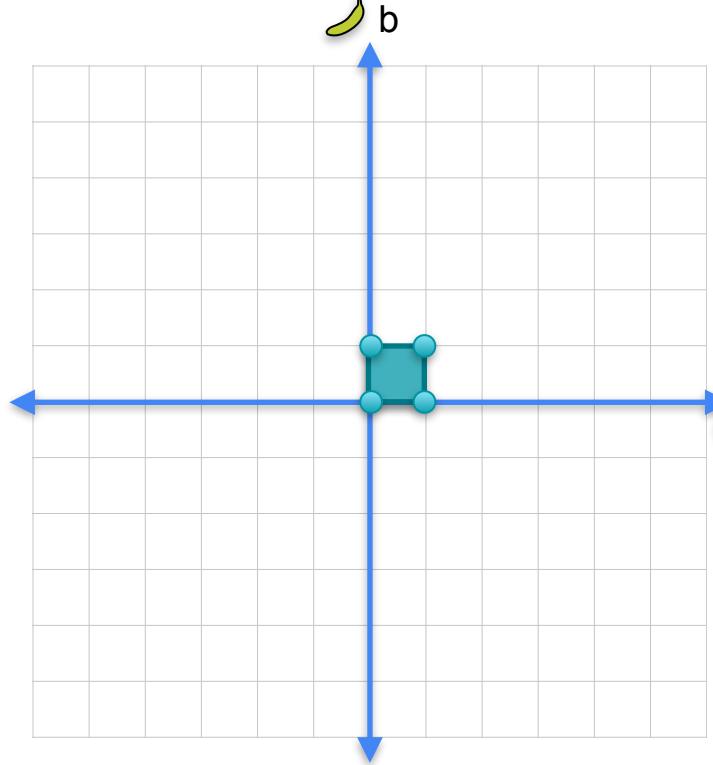
A 2x2 matrix equation. On the left, there is a 2x2 matrix with two red apple icons in the first column and two green banana icons in the second column. To its right is an equals sign. To the right of the equals sign is another 2x2 matrix with a single red apple icon in the top-left cell and a single green banana icon in the bottom-right cell. The entire equation is surrounded by a light gray border.

$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 1 \\ 2 & 2 \end{matrix} & = & \begin{matrix} 1 \\ 2 \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$

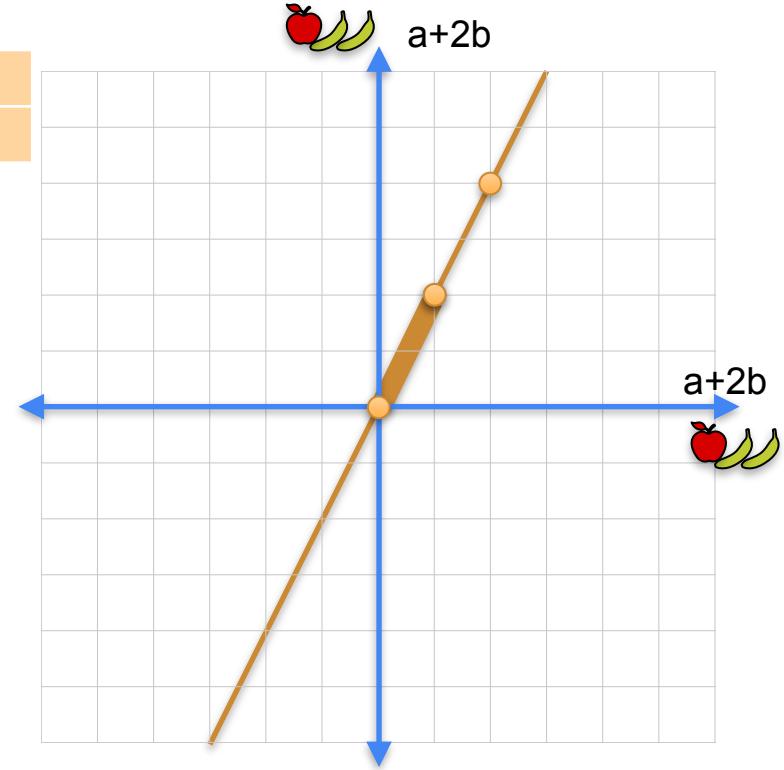


Row space

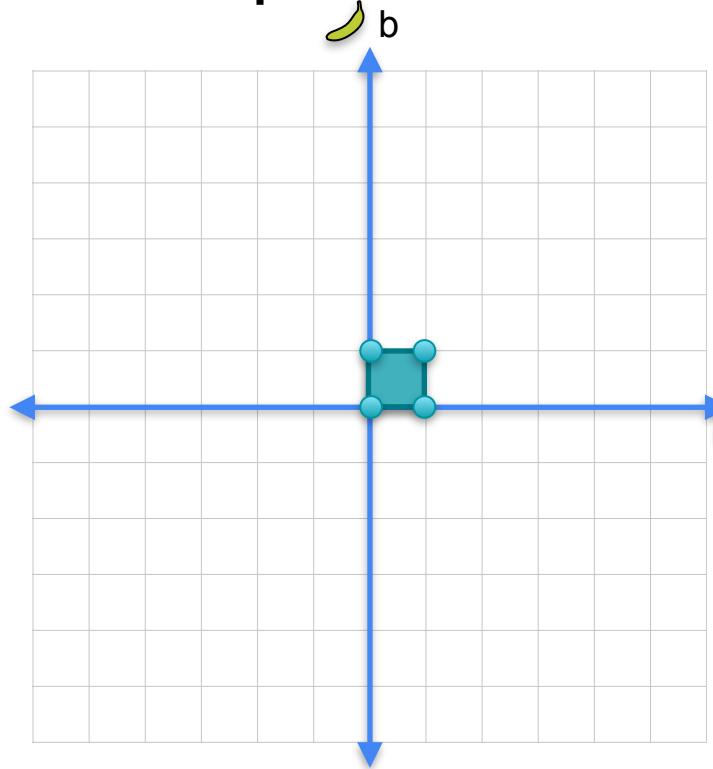


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 2 & 2 & 1 \\ \hline \end{array} & = \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$

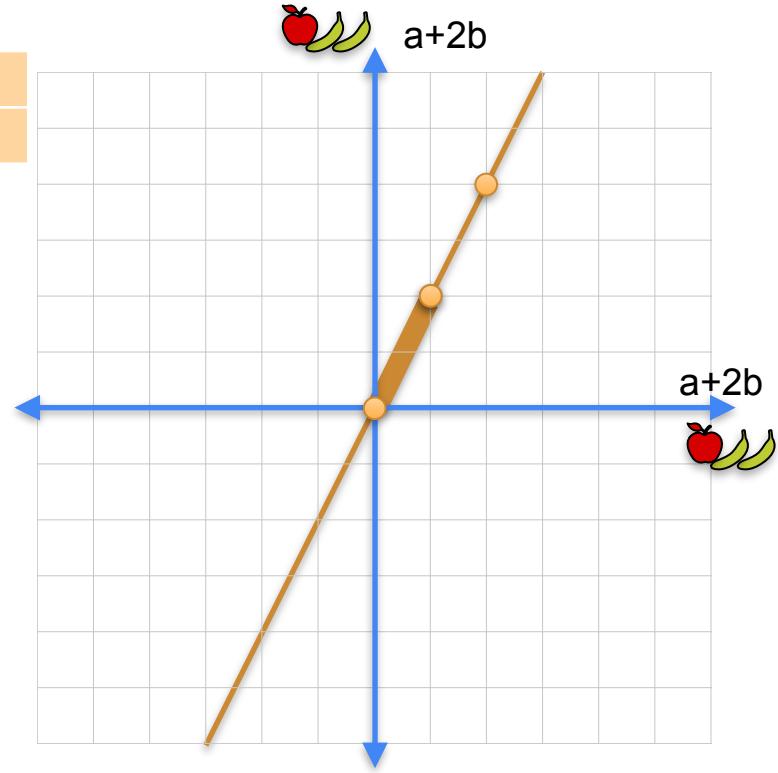


Row space

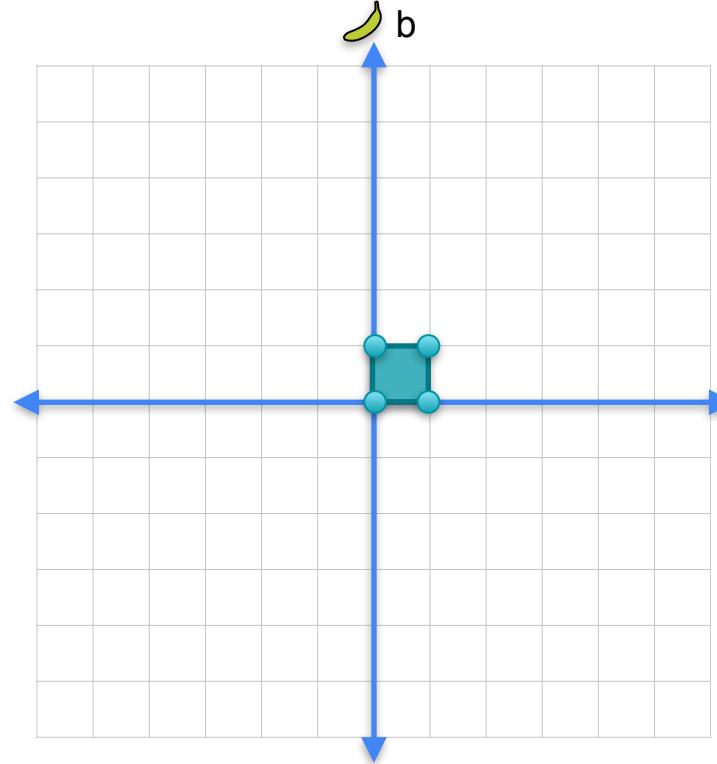


A 2x3 matrix equation. The left side shows a 2x3 matrix with columns: a red apple icon, a yellow banana icon, and a teal square. The right side shows the result of the multiplication: $= \begin{matrix} 4 \\ 3 \end{matrix}$. The matrix has rows: 1, 2 and columns: 1, 2, 1.

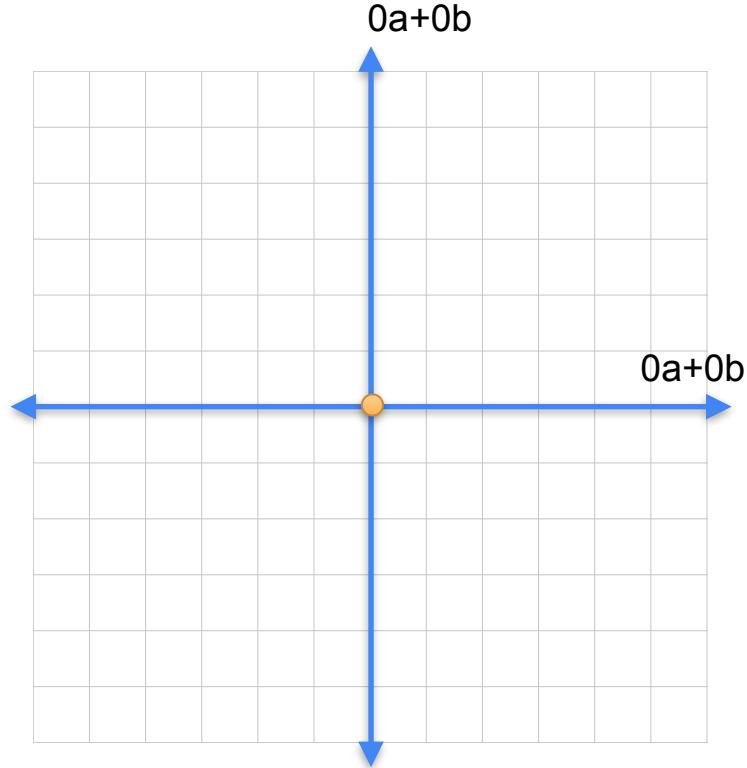
$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (1,2)$
 $(0,1) \rightarrow (1,2)$
 $(1,1) \rightarrow (2,4)$



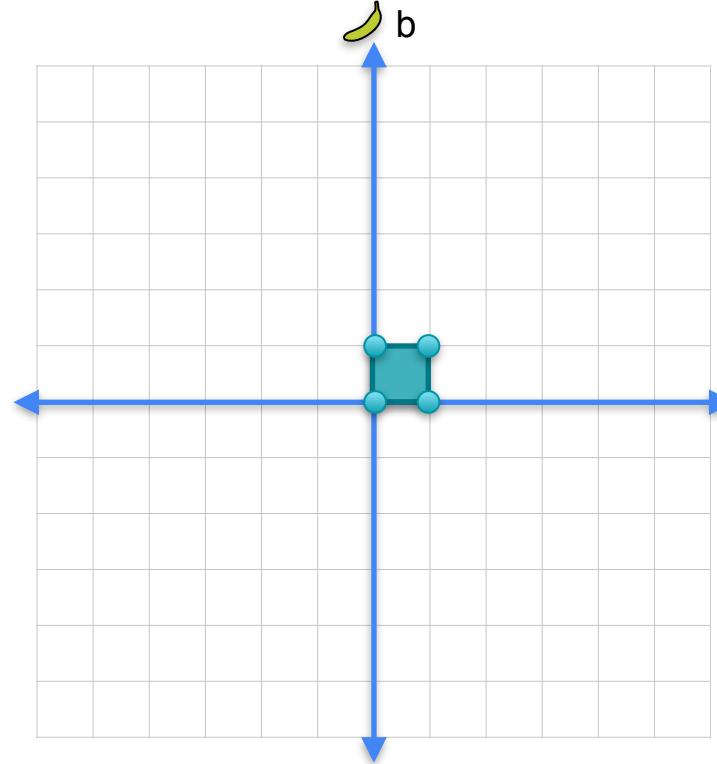
Row space



$$\begin{array}{c} \text{apple} \quad \text{banana} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \end{array} = \begin{array}{l} (0,0) \rightarrow (0,0) \\ (1,0) \rightarrow (0,0) \\ (0,1) \rightarrow (0,0) \\ (1,1) \rightarrow (0,0) \end{array}$$



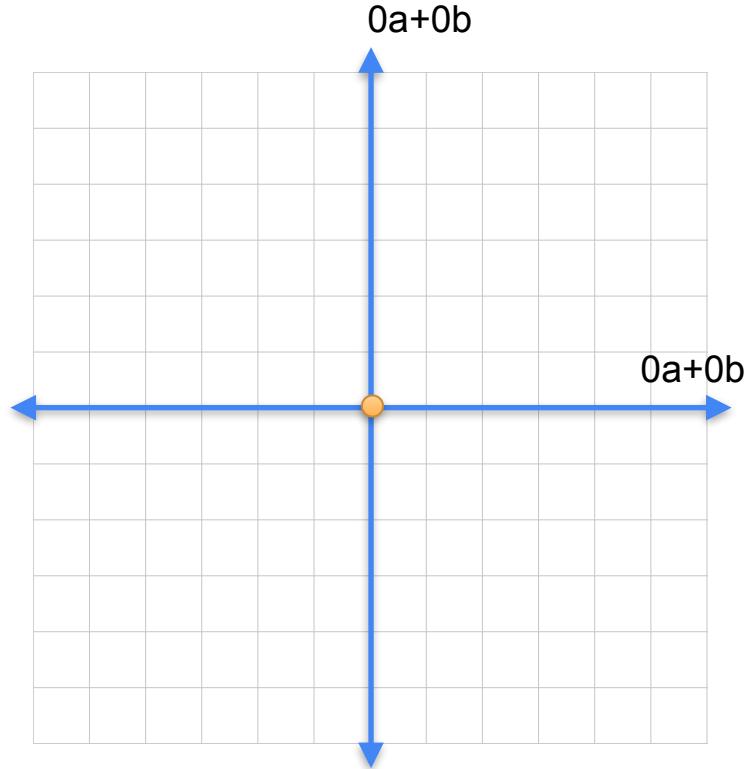
Row space



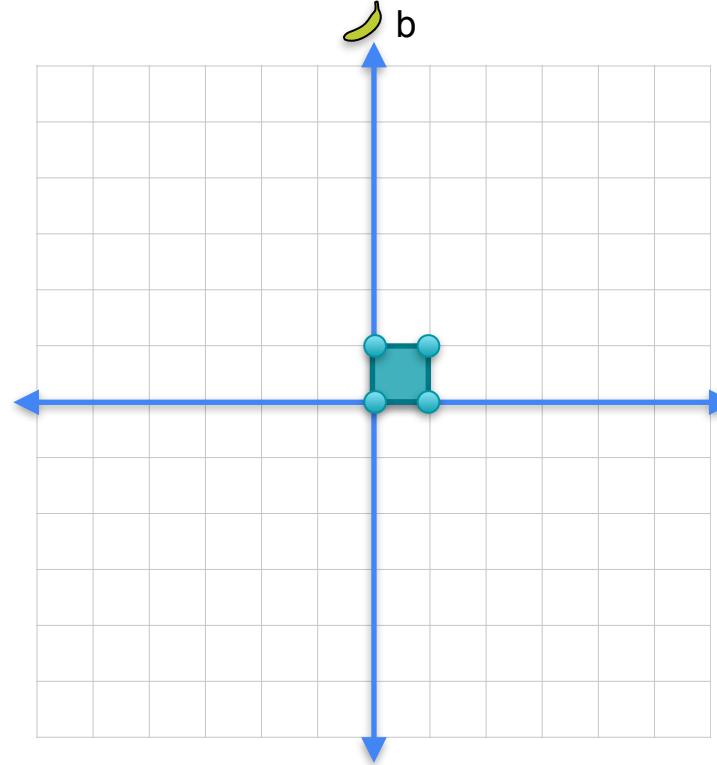
=

0	0	a
0	0	b

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (0,0)$
 $(0,1) \rightarrow (0,0)$
 $(1,1) \rightarrow (0,0)$



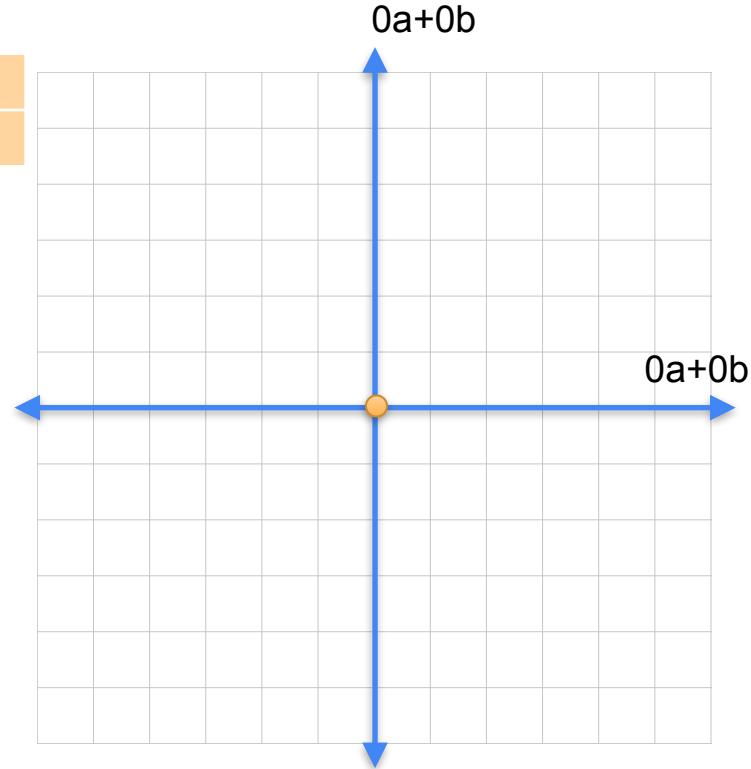
Row space



A 2x2 matrix with fruit icons. The first column contains a red apple icon and a blue background cell. The second column contains a yellow banana icon and a green background cell. To the right of the matrix is a multiplication sign (=).

$$\begin{matrix} \text{apple} & \text{banana} \\ 0 & 0 \\ 0 & 0 \end{matrix} = \begin{matrix} a \\ b \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$(0,0) \rightarrow (0,0)$
 $(1,0) \rightarrow (0,0)$
 $(0,1) \rightarrow (0,0)$
 $(1,1) \rightarrow (0,0)$

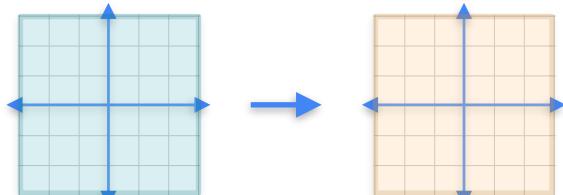


Row space

Non-singular

3	1
1	2

Rank = 2

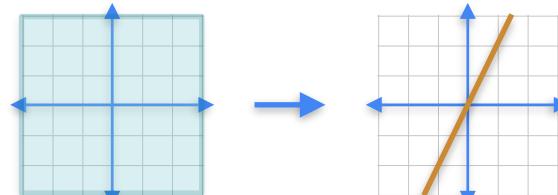


Dimension = 2

Singular

1	1
2	2

Rank = 1

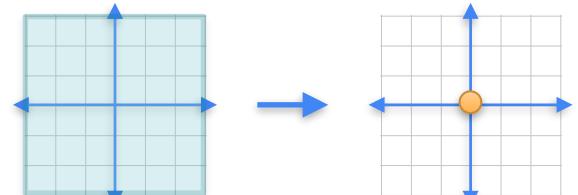


Dimension = 1

Singular

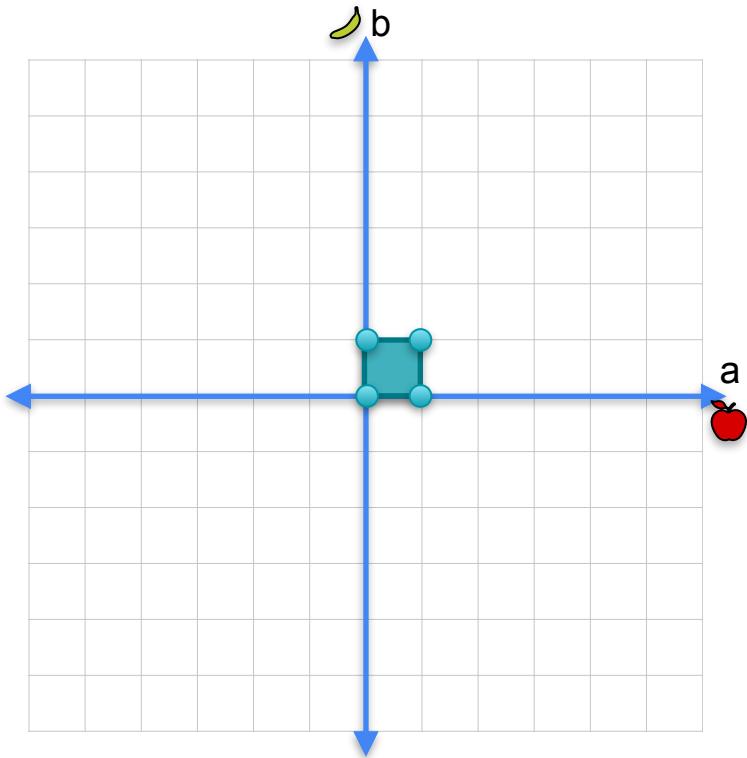
0	0
0	0

Rank = 0



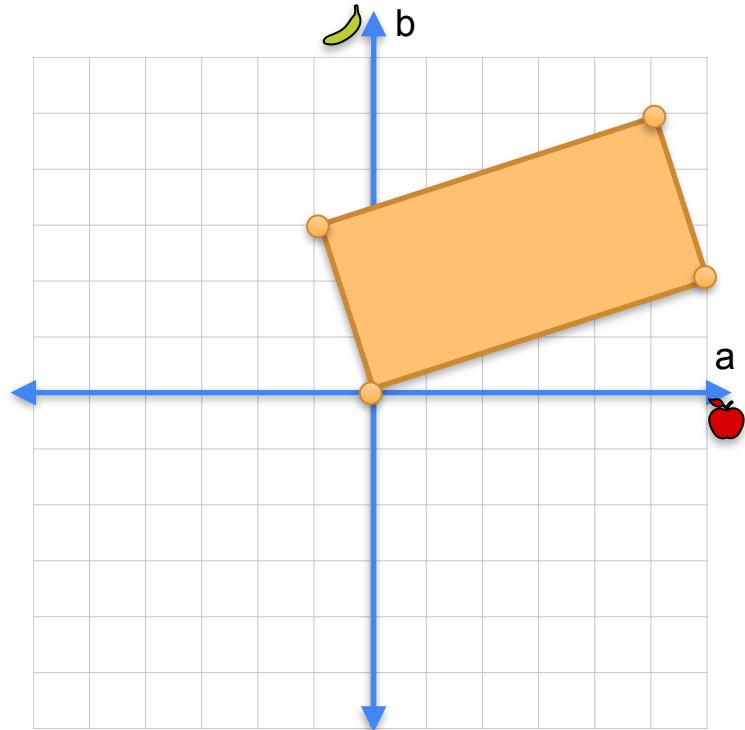
Dimension = 0

Orthogonal matrix

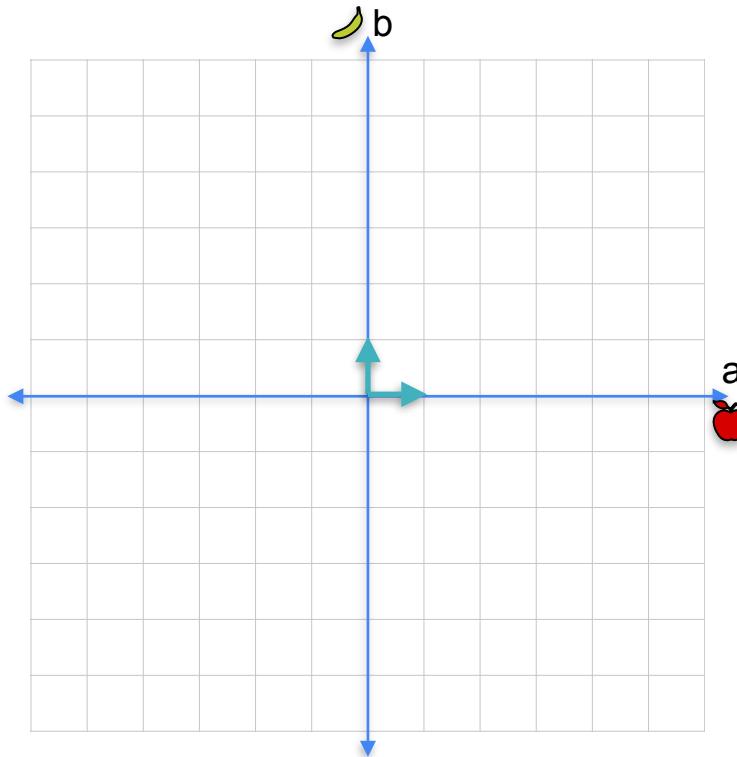


6	-1
2	3

- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (6,2)$
- $(0,1) \rightarrow (-1,3)$
- $(1,1) \rightarrow (5,5)$

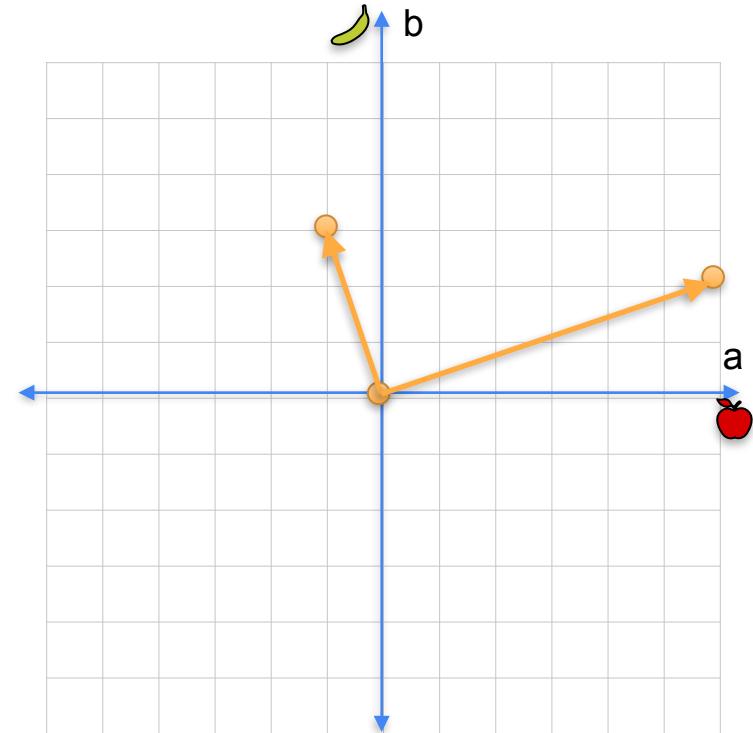


Orthogonal matrix



apple	banana
6	-1
2	3

$$\begin{aligned}(1,0) &\rightarrow (6,2) \\ (0,1) &\rightarrow (-1,3)\end{aligned}$$



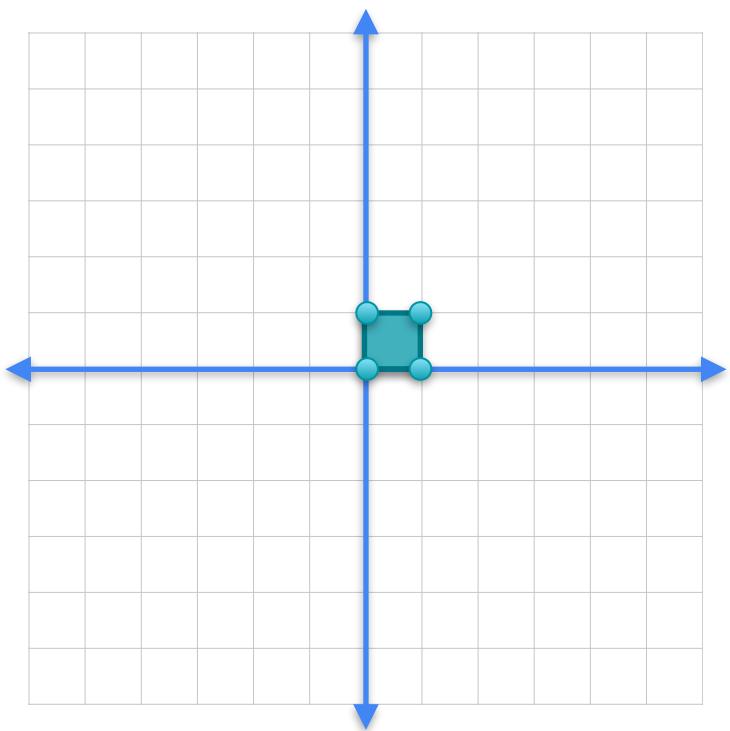
Orthogonal matrices have orthogonal columns

$$\begin{array}{|c|c|} \hline 6 & -1 \\ \hline 2 & 3 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & -1 \\ \hline 2 & 3 \\ \hline \end{array} = 0$$

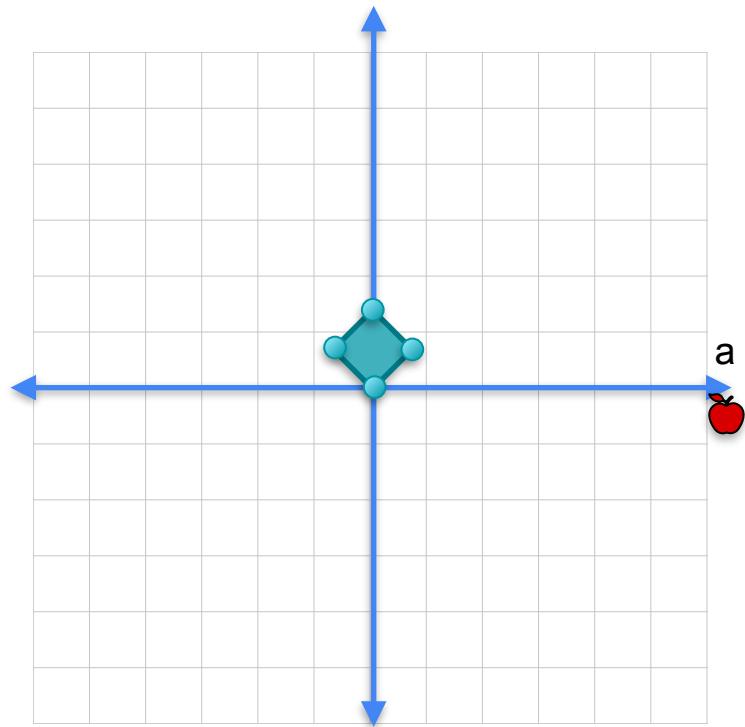
$$\begin{array}{|c|c|c|c|} \hline 6 & -1 & 2 & 3 \\ \hline \end{array} = 0$$

Orthogonal matrix

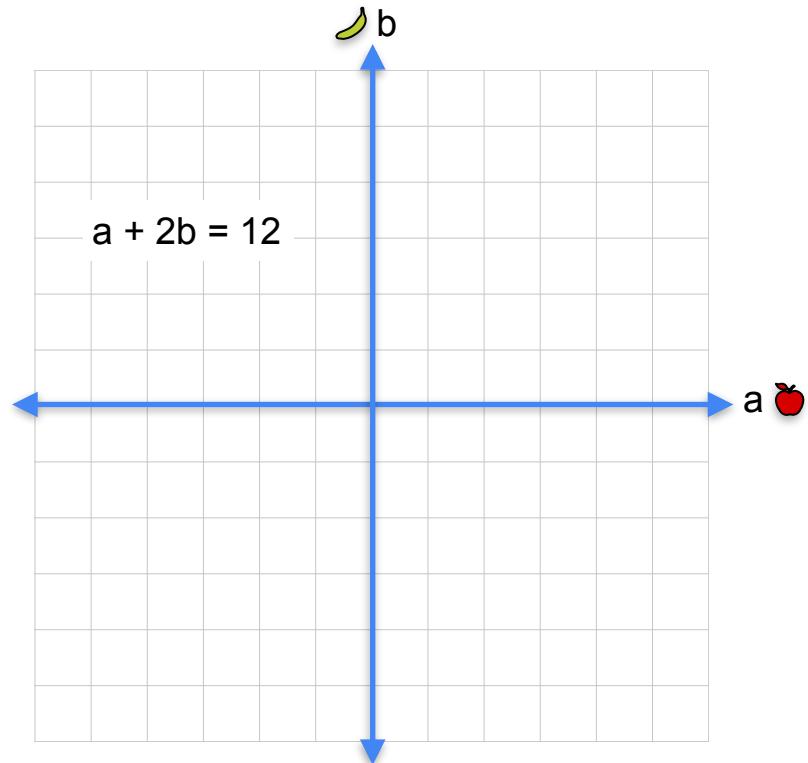
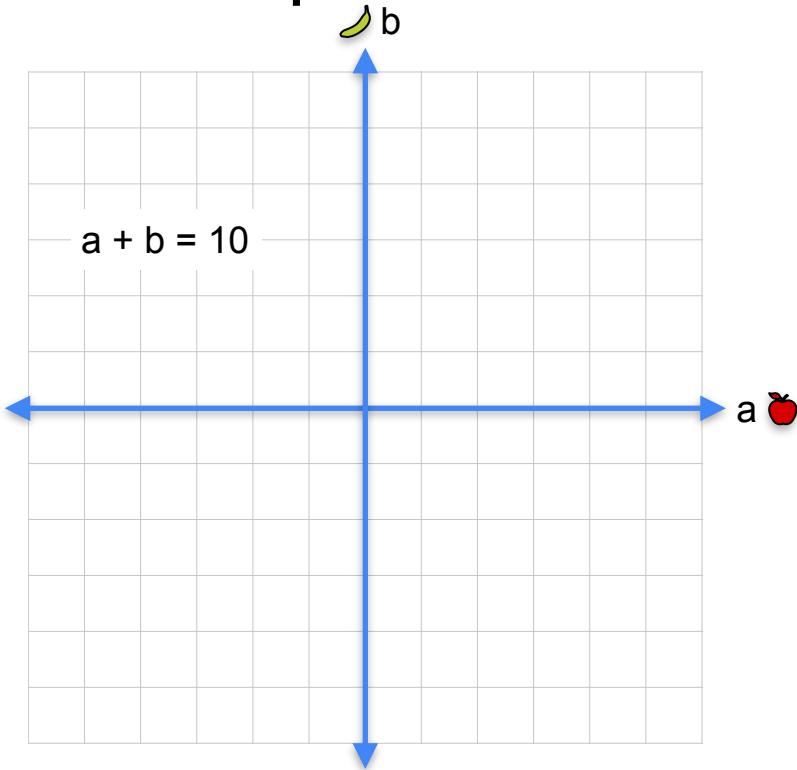


0.7071	0.7071
-0.7071	0.7071

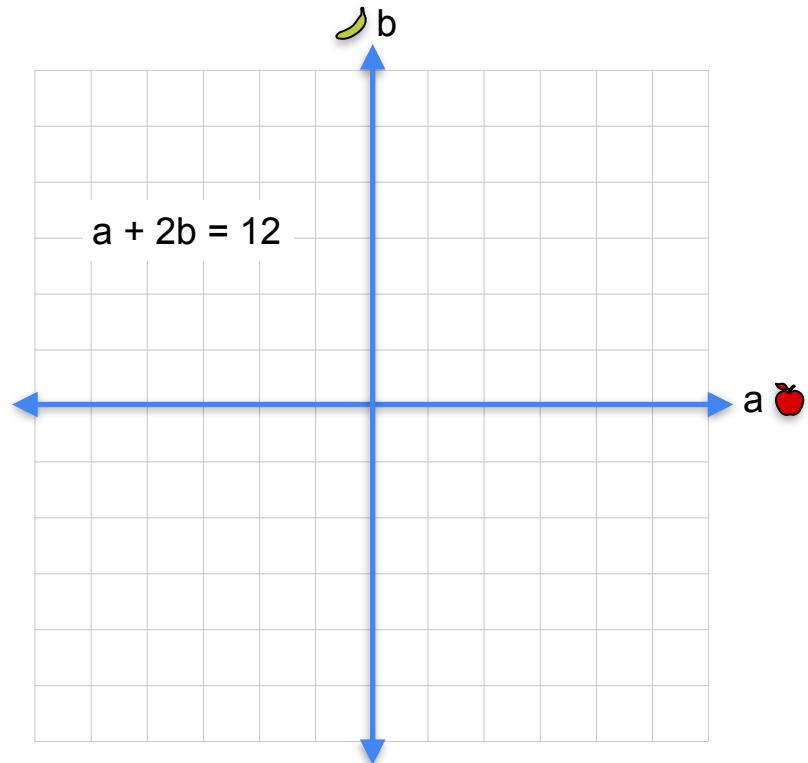
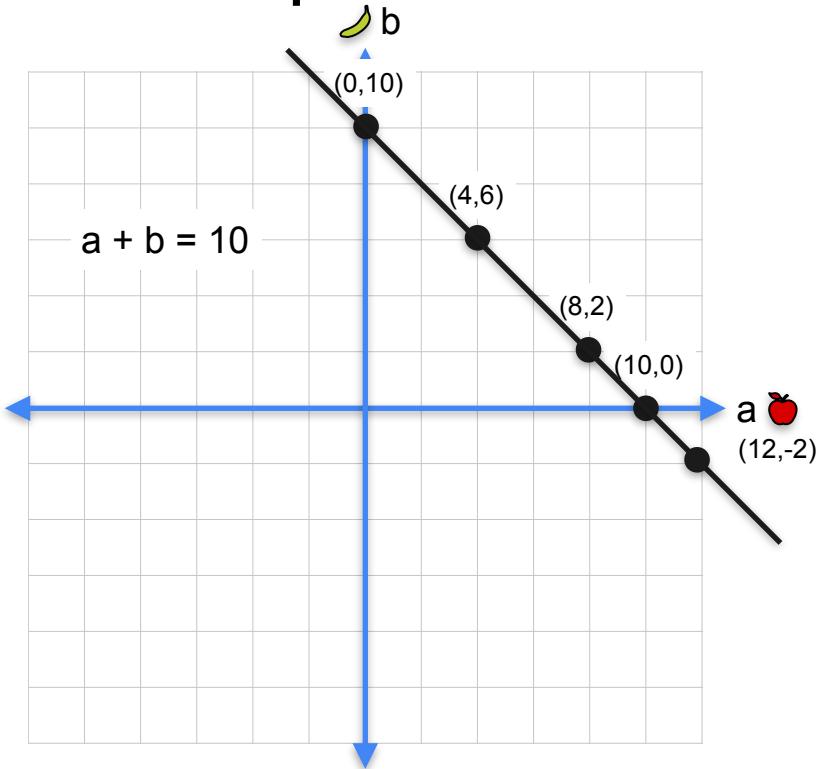
- (0,0) → (0,0)
- (1,0) → (0.7071, 0.7071)
- (0,1) → (-0.7071, 0.7071)
- (1,1) → (0, 1.4142)



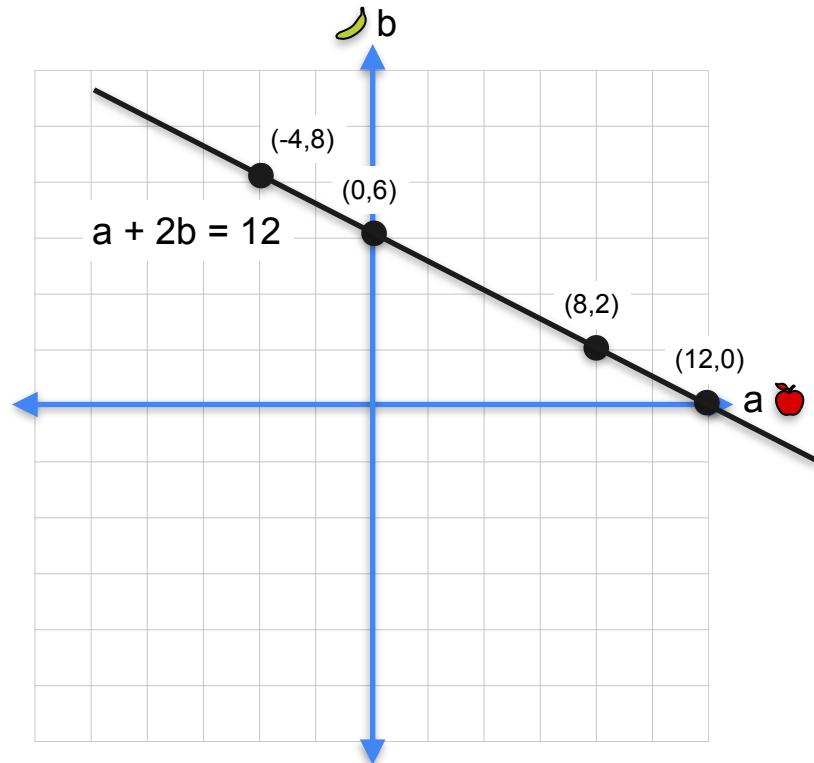
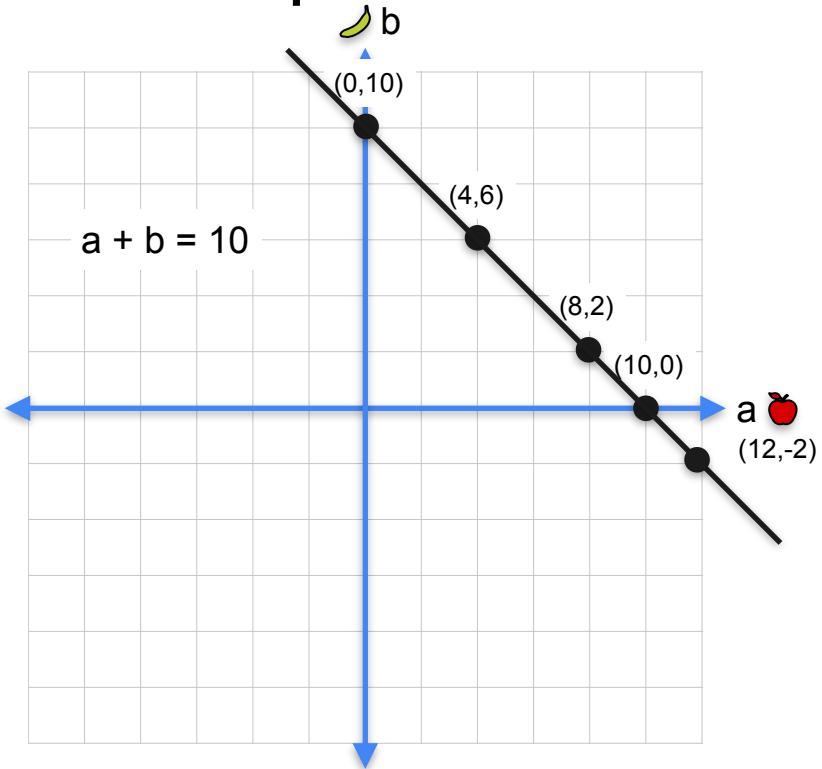
Linear equation in 2 variables -> Line



Linear equation in 2 variables -> Line

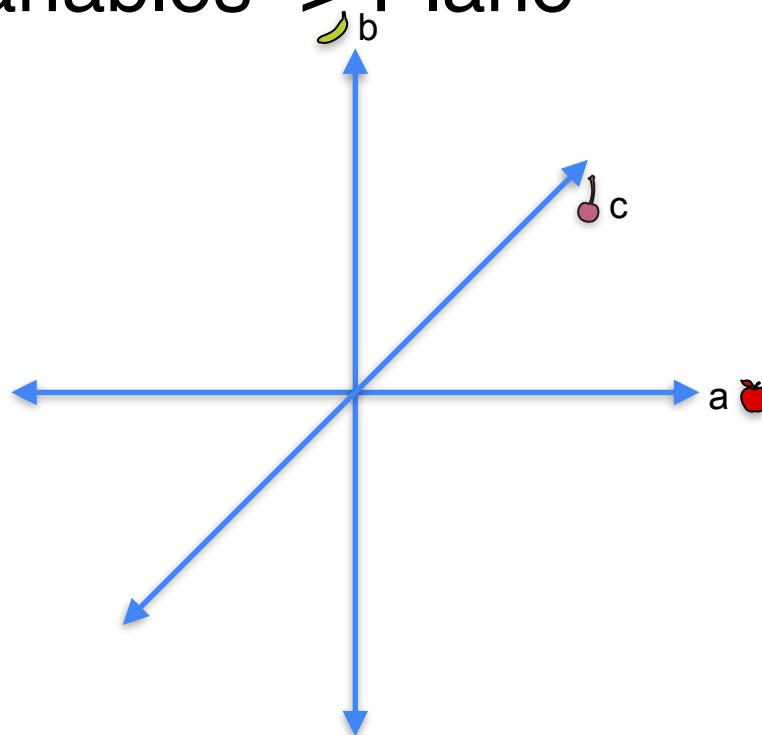


Linear equation in 2 variables -> Line



Linear equation in 3 variables -> Plane

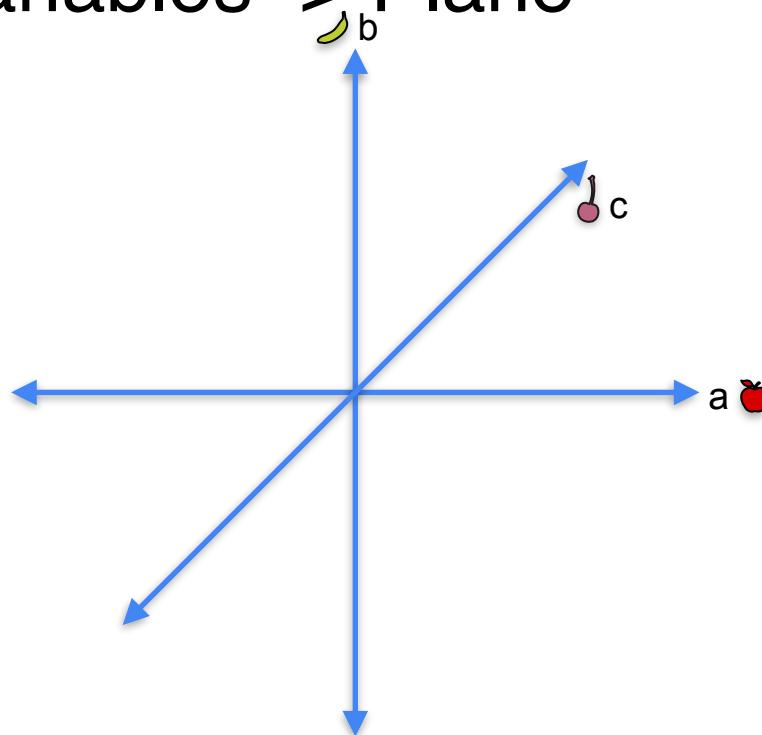
$$a + b + c = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

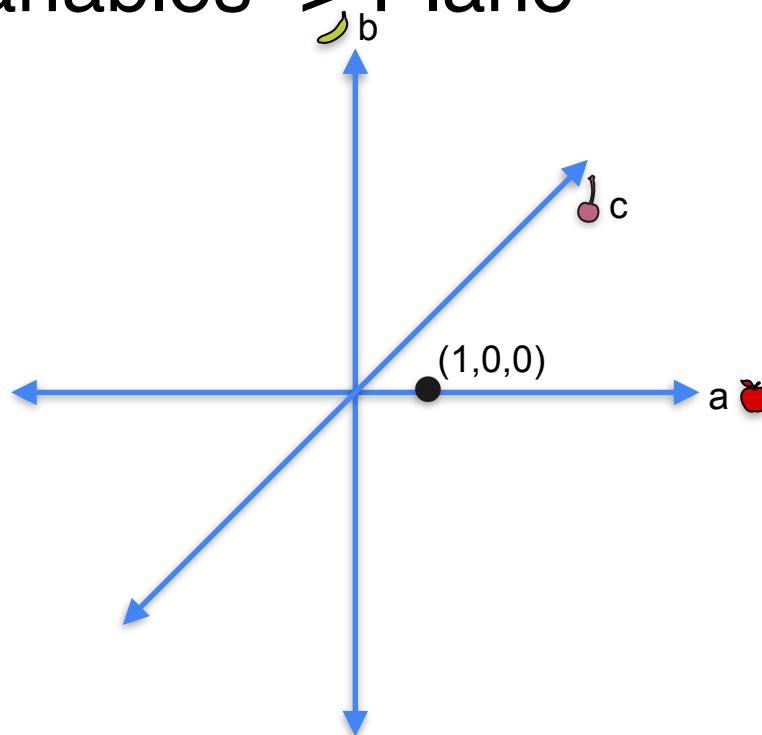
$$1 + 0 + 0 = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

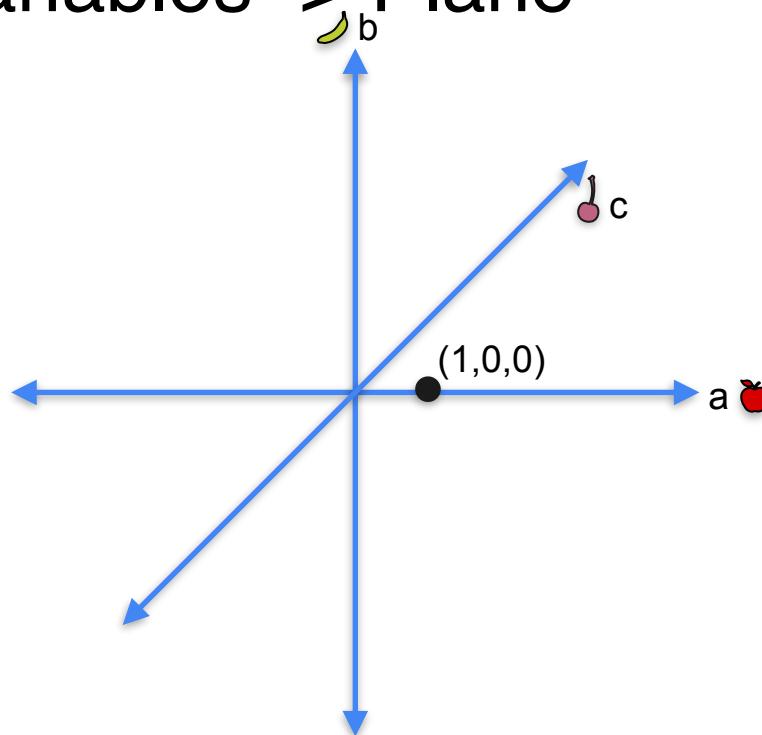


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

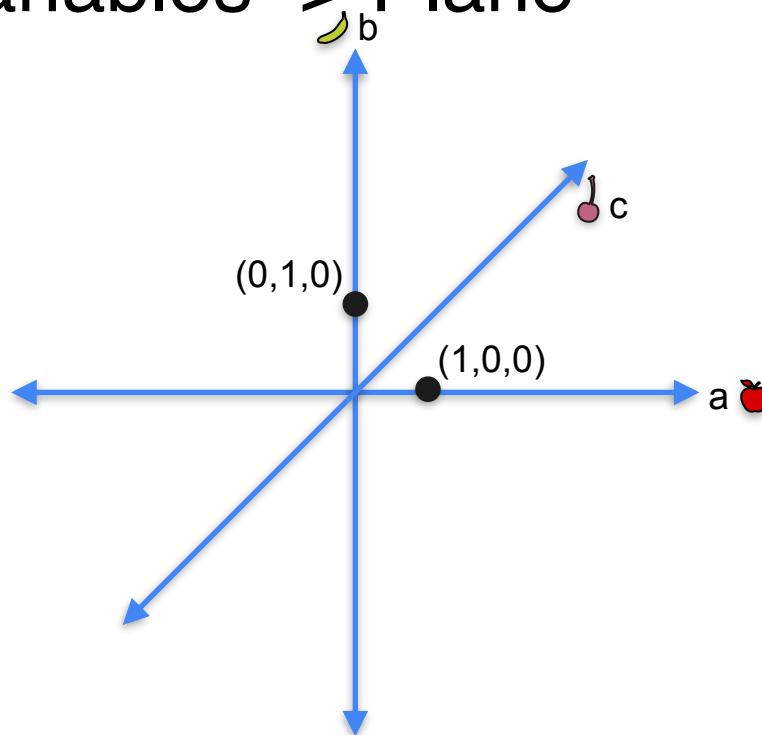


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$



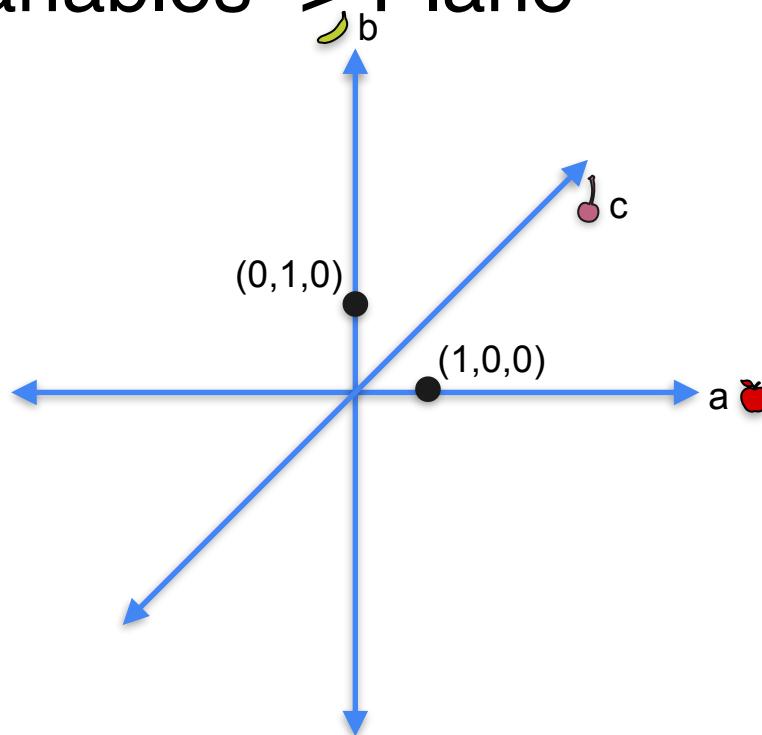
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



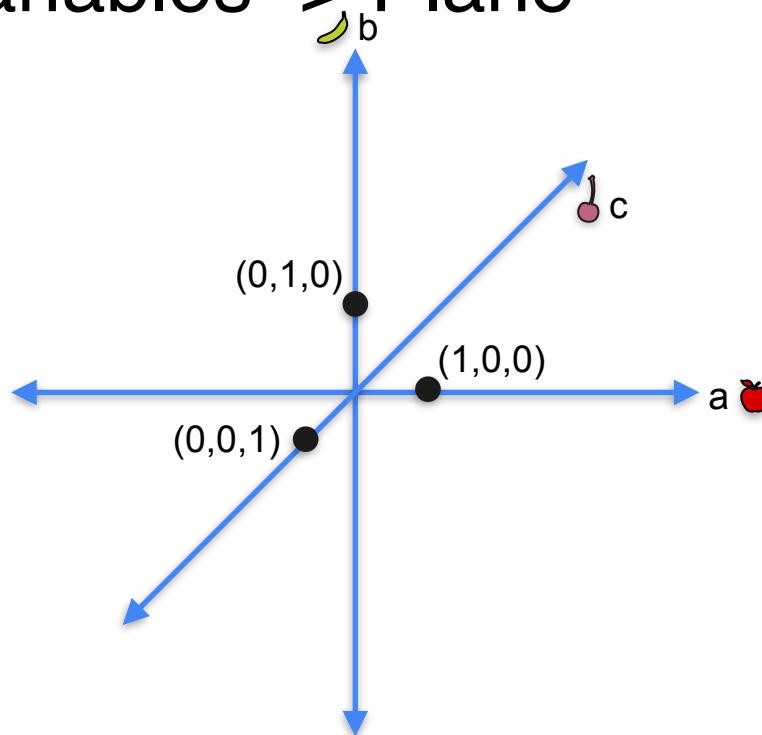
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



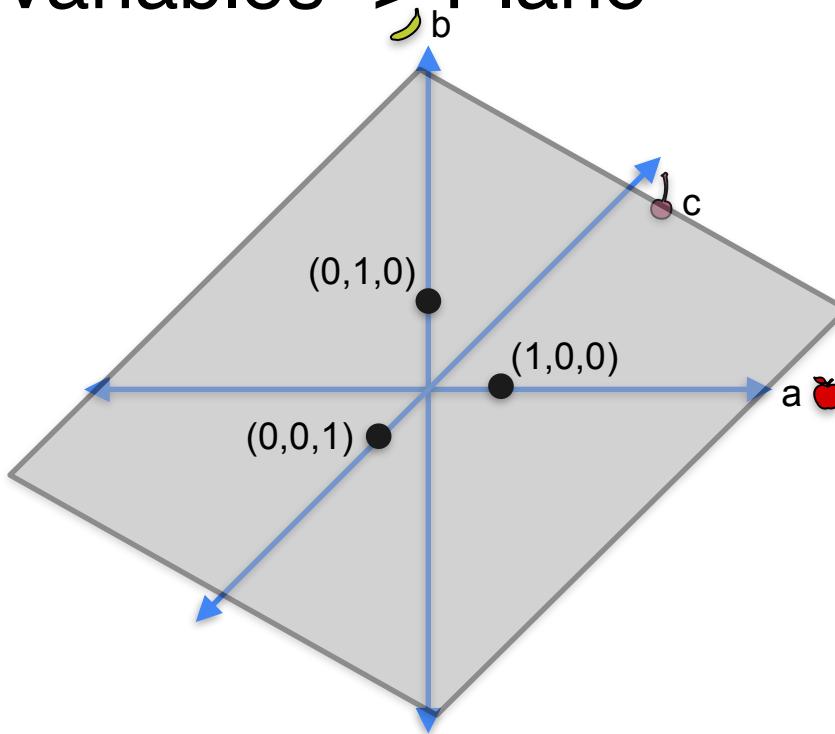
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

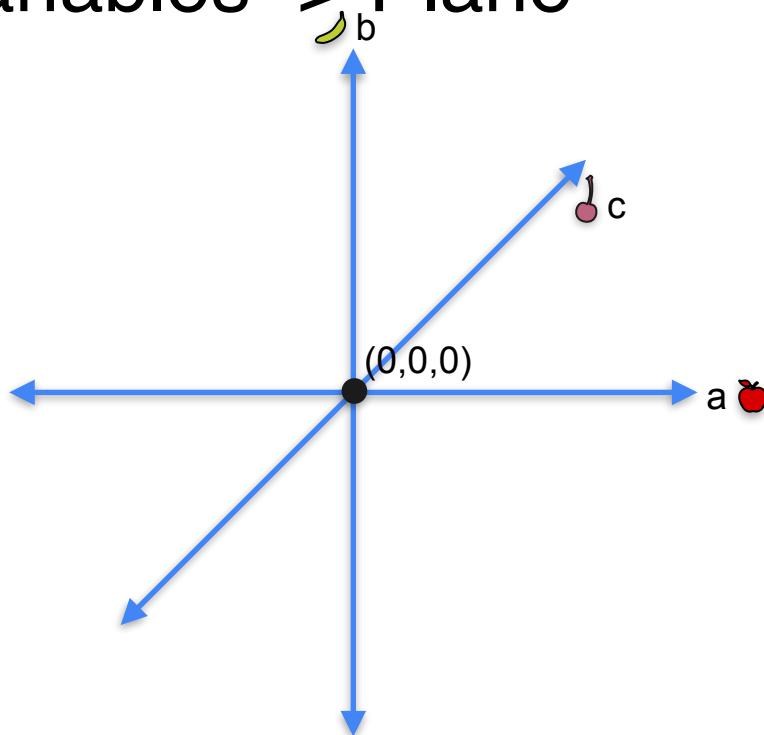
$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



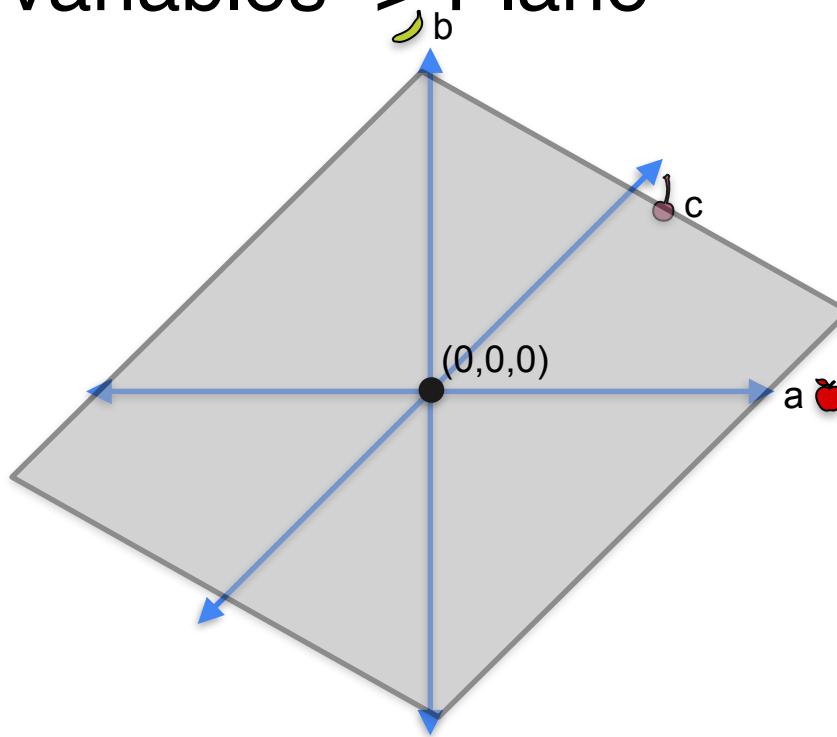
Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = \mathbf{0}$$



Linear equation in 3 variables -> Plane

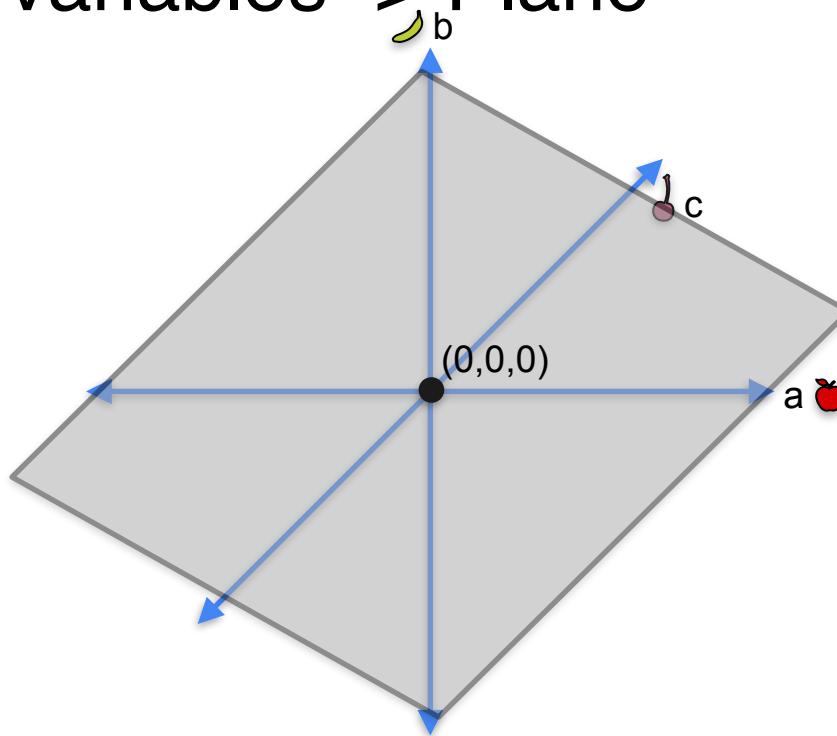
$$3a - 5b + 2c = \mathbf{0}$$



Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = \mathbf{0}$$

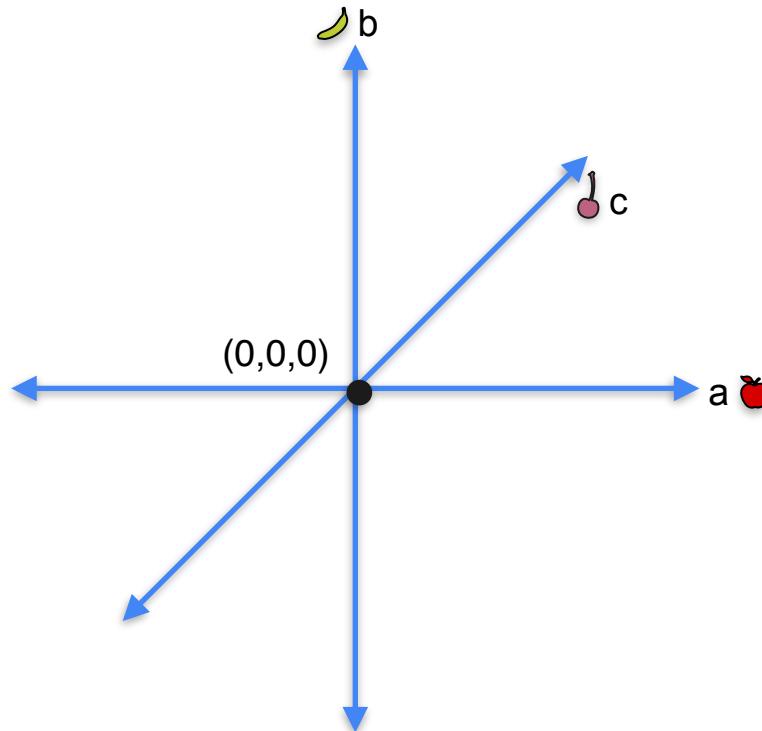
$$3(0) + 5(0) + 2(0) = \mathbf{0}$$



System 1

System 1

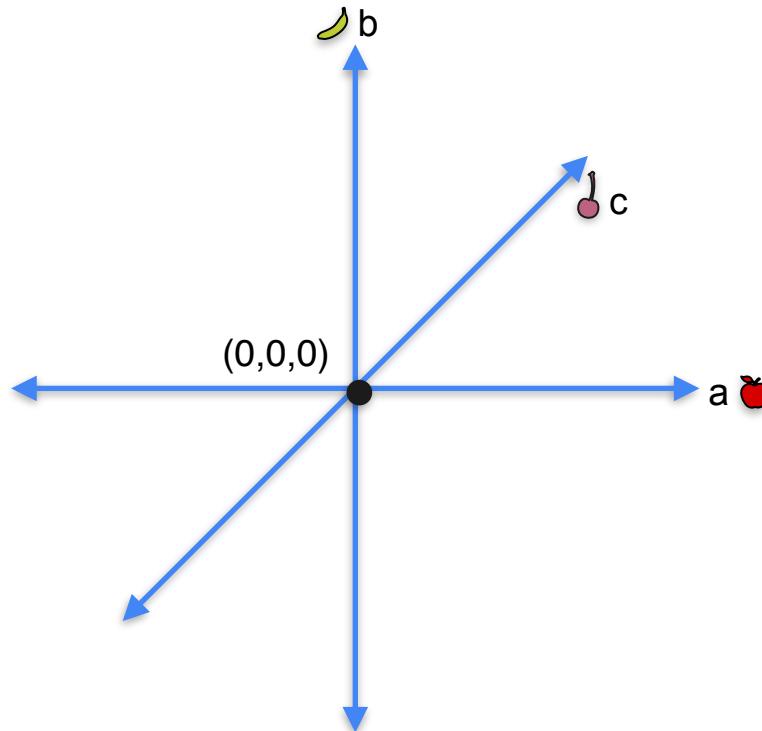
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

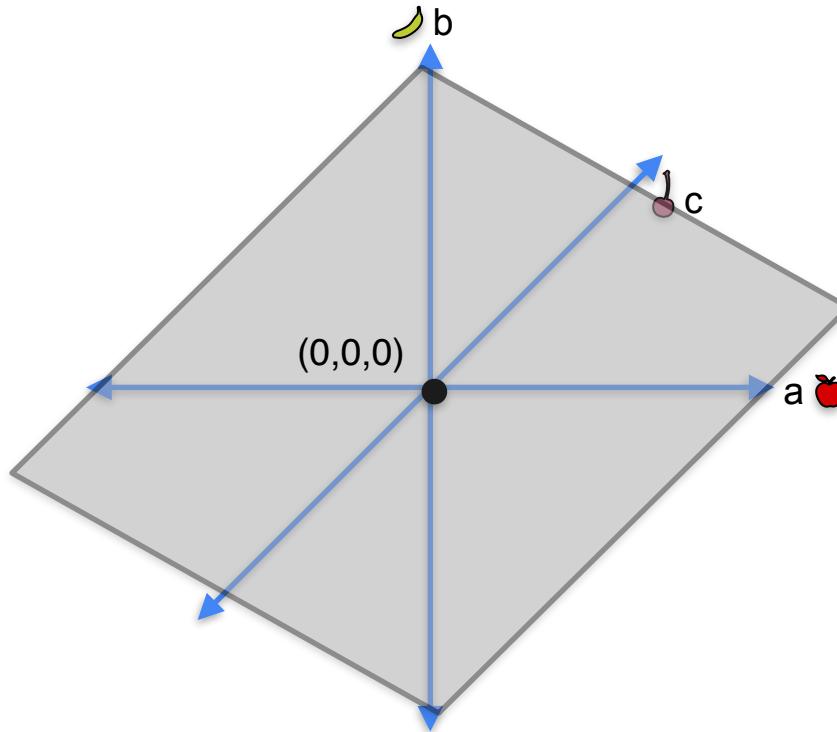
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

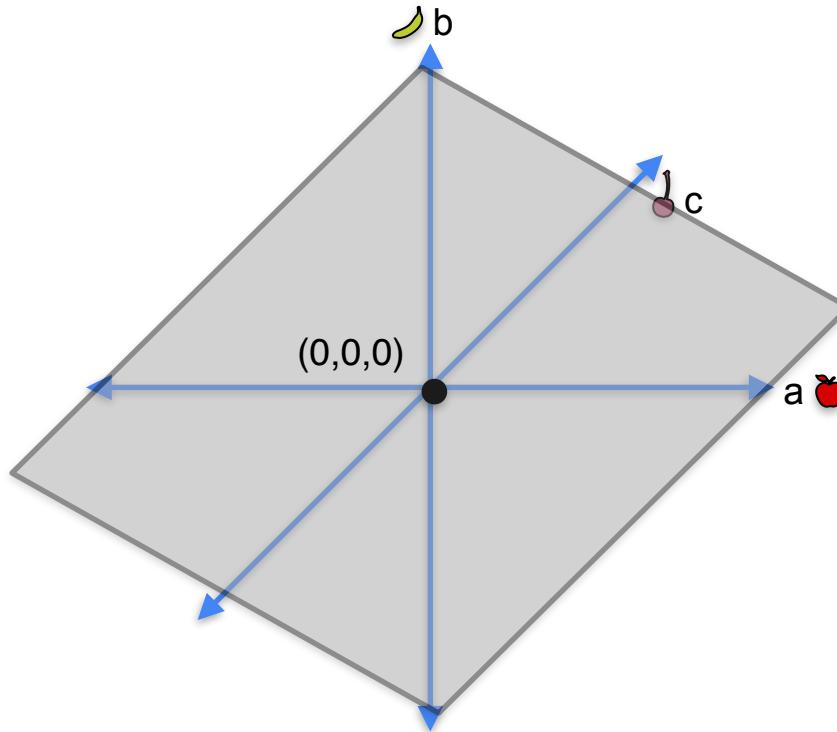
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

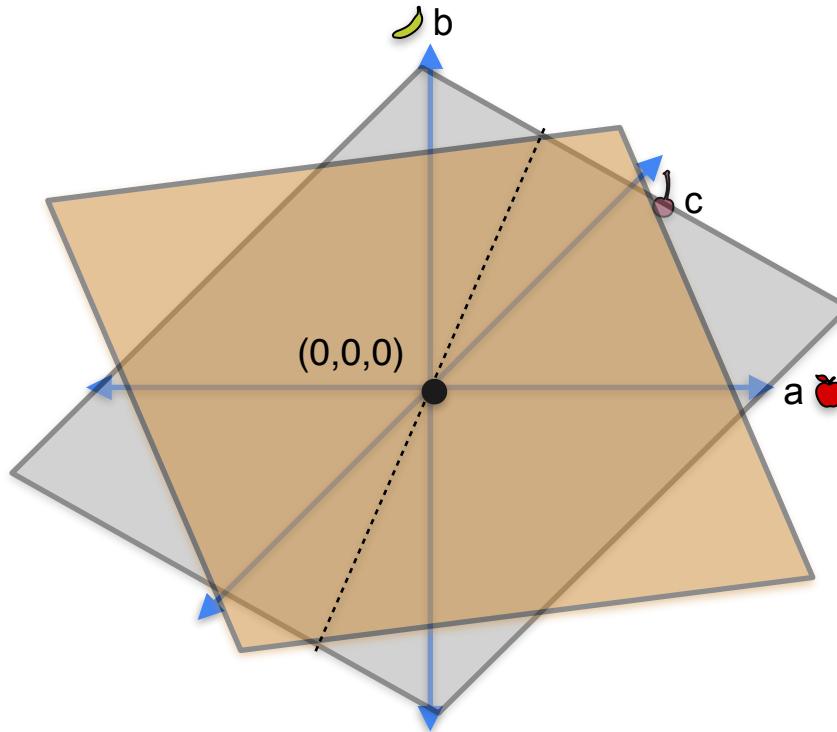
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

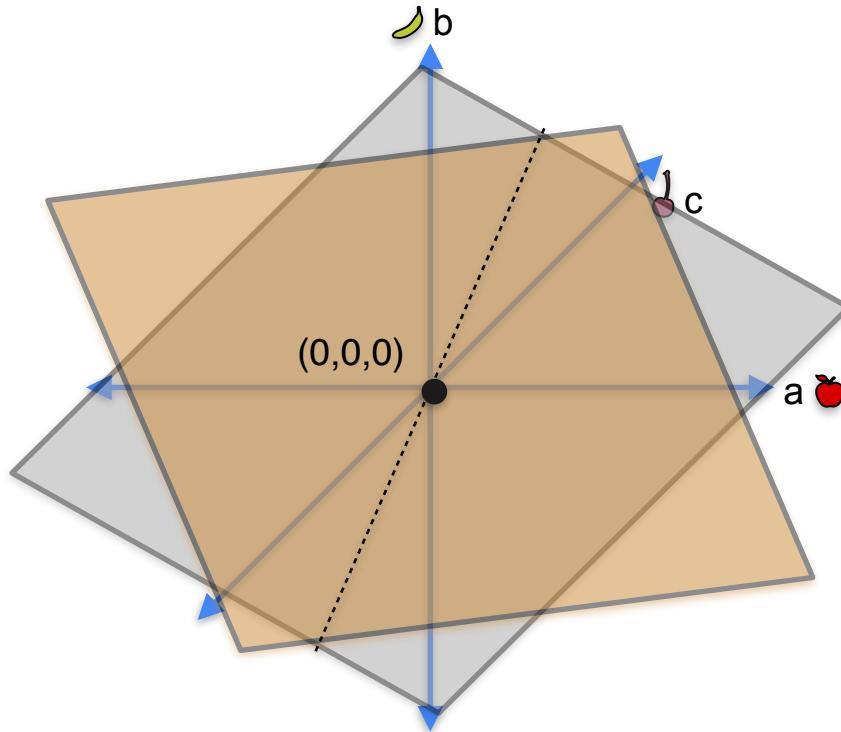
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

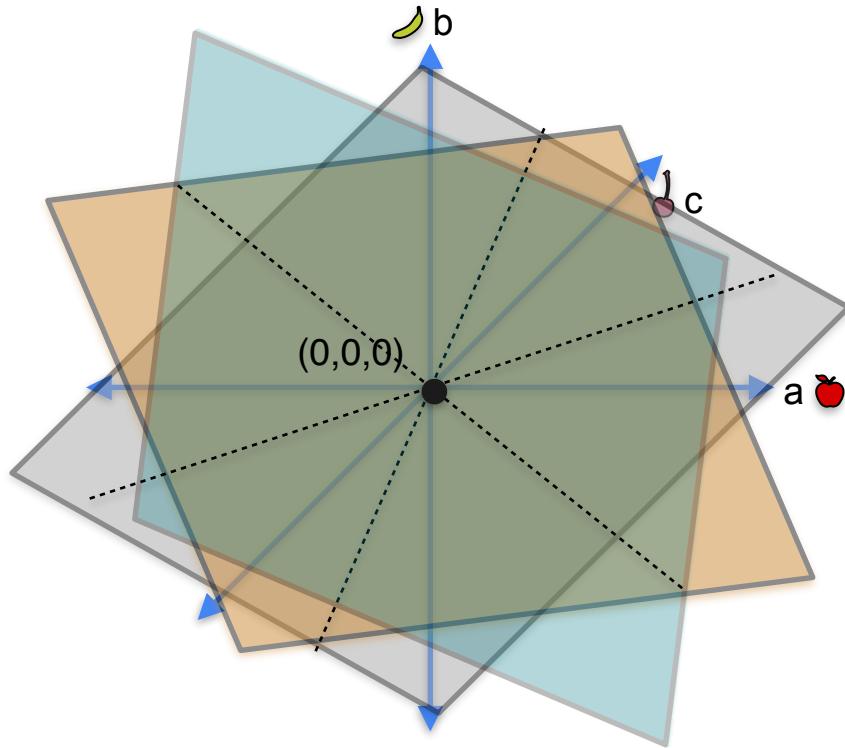
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

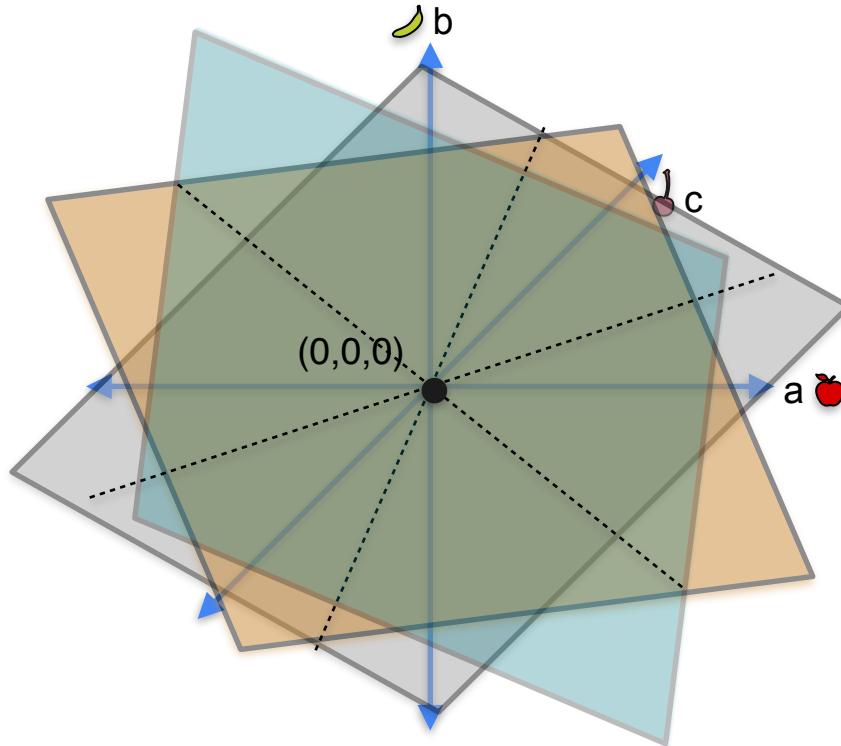
System 1

- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



Solution space

- $a = 0$
- $b = 0$
- $c = 0$



System 1

System 1

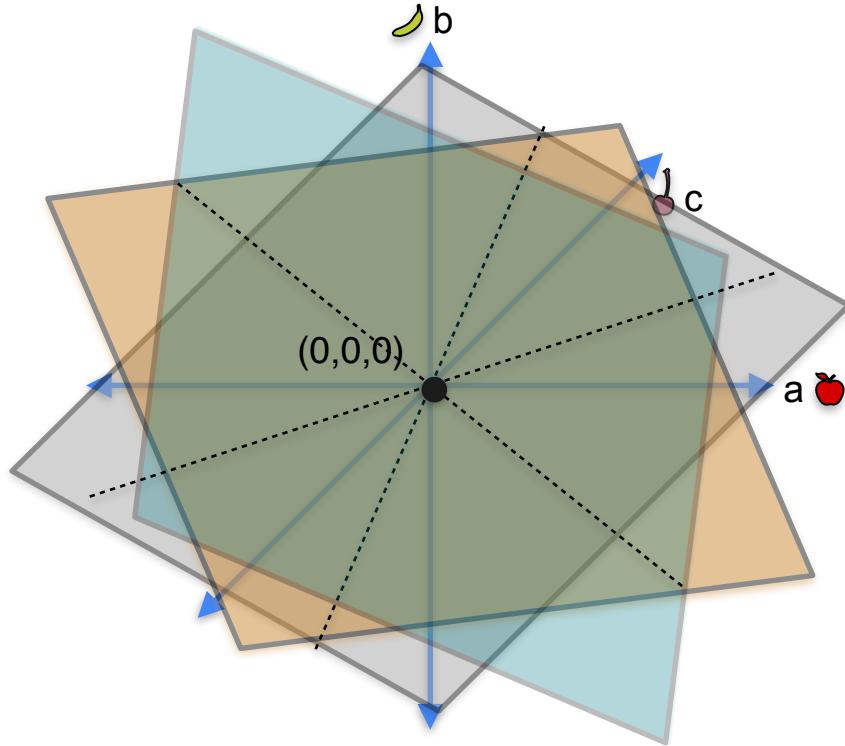
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



Solution space

- $a = 0$
- $b = 0$
- $c = 0$

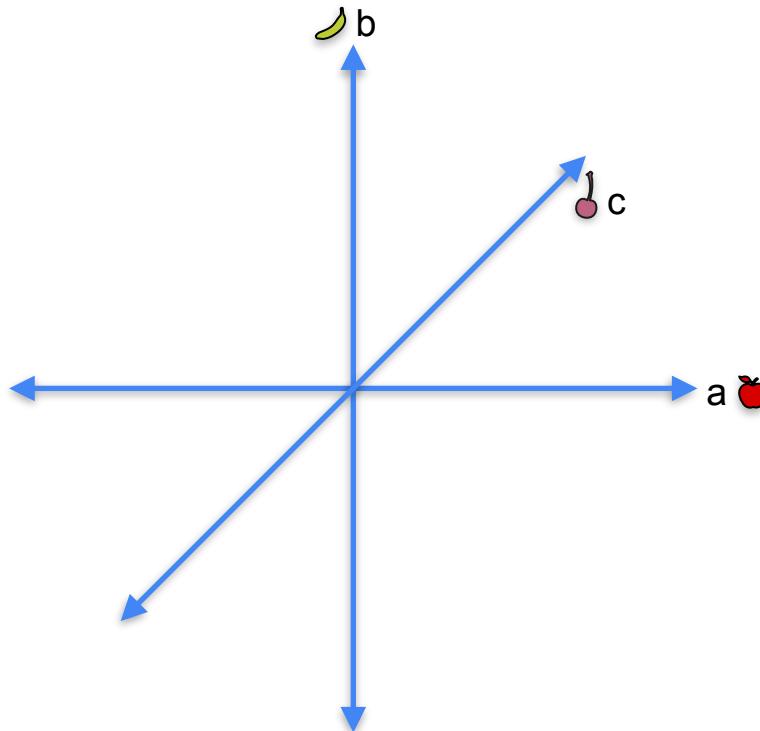
The point
 $(0,0,0)$



System 2

System 2

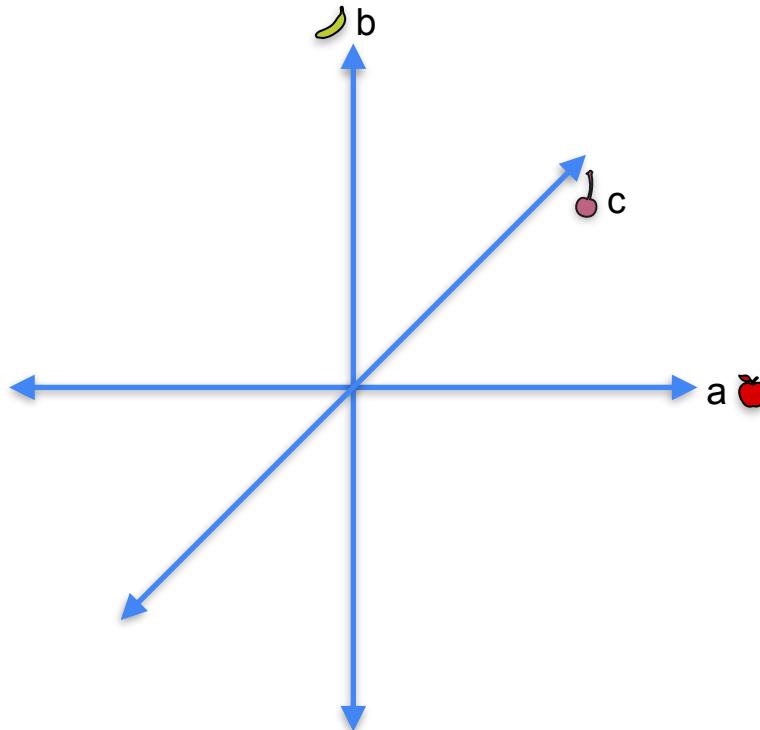
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

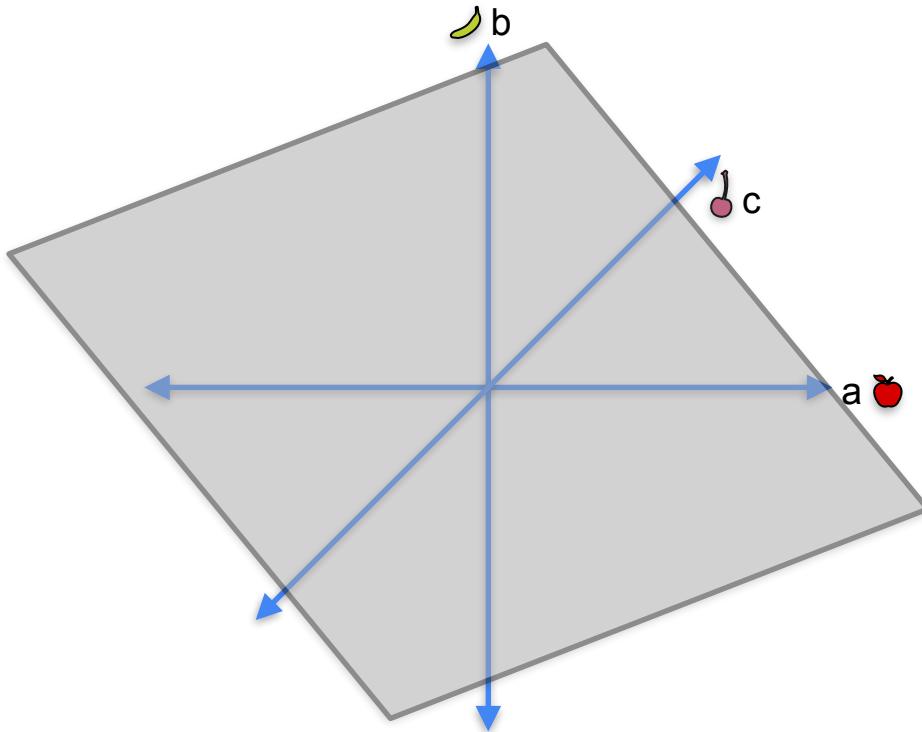
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

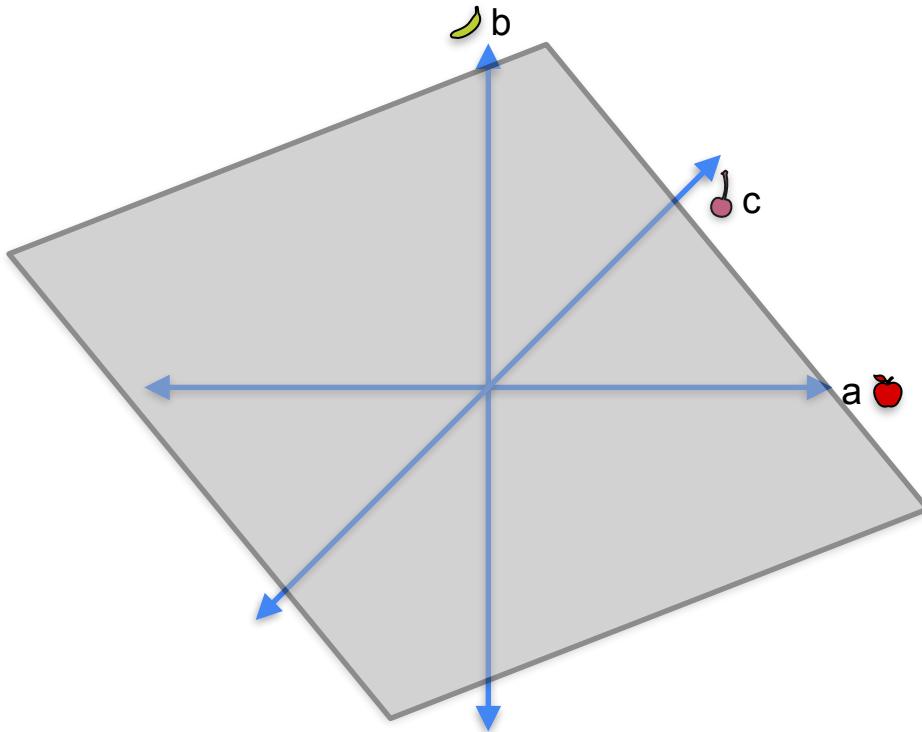
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

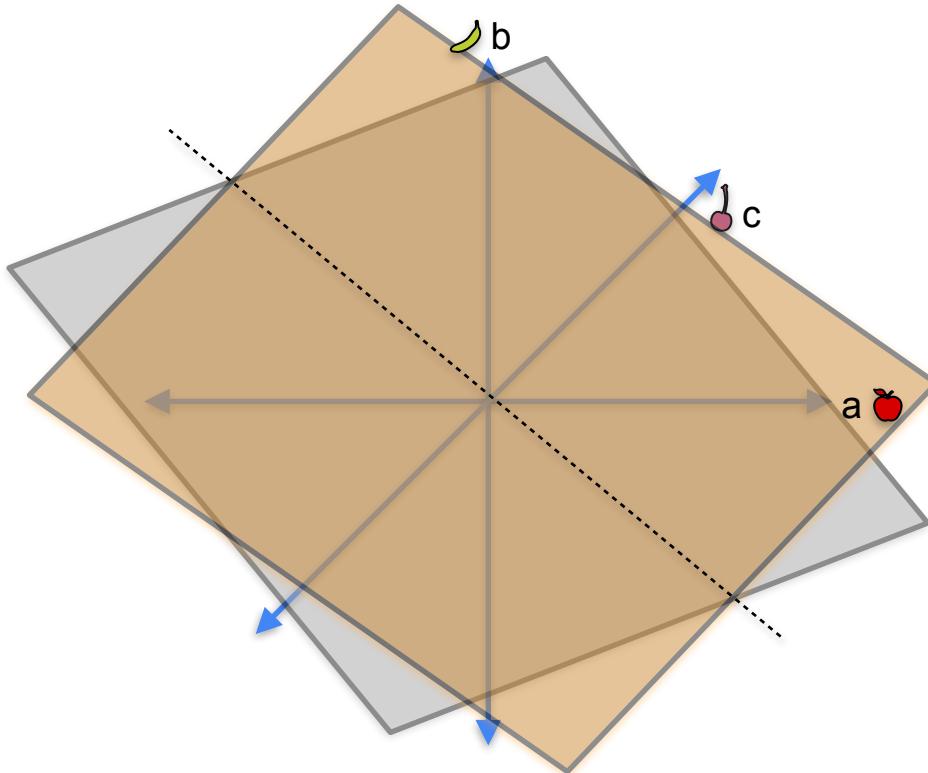
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

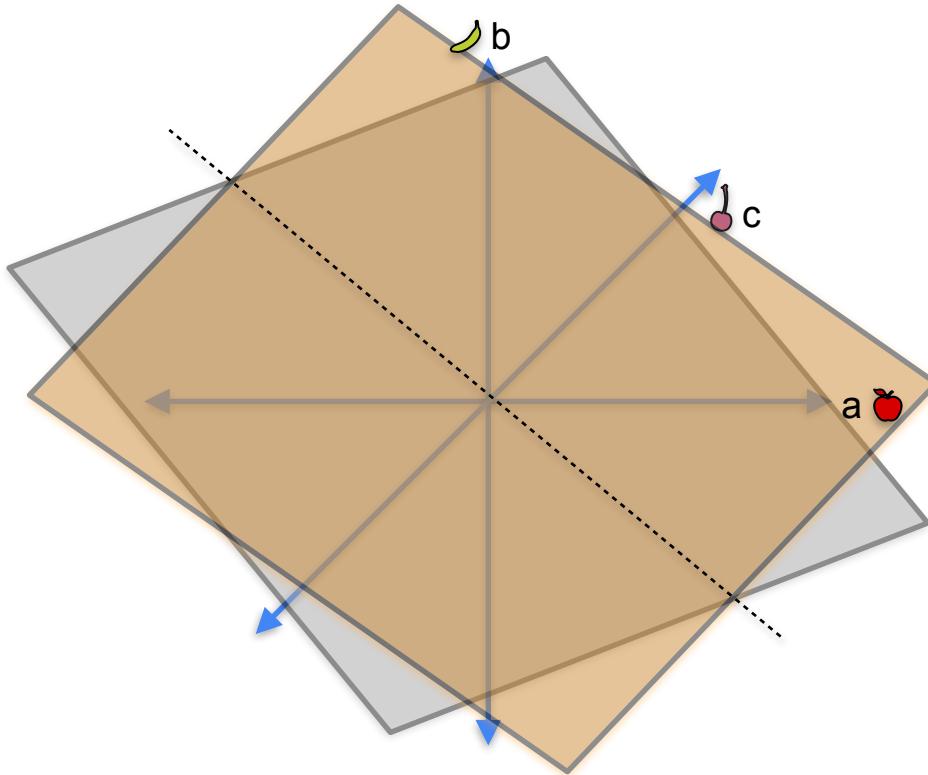
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

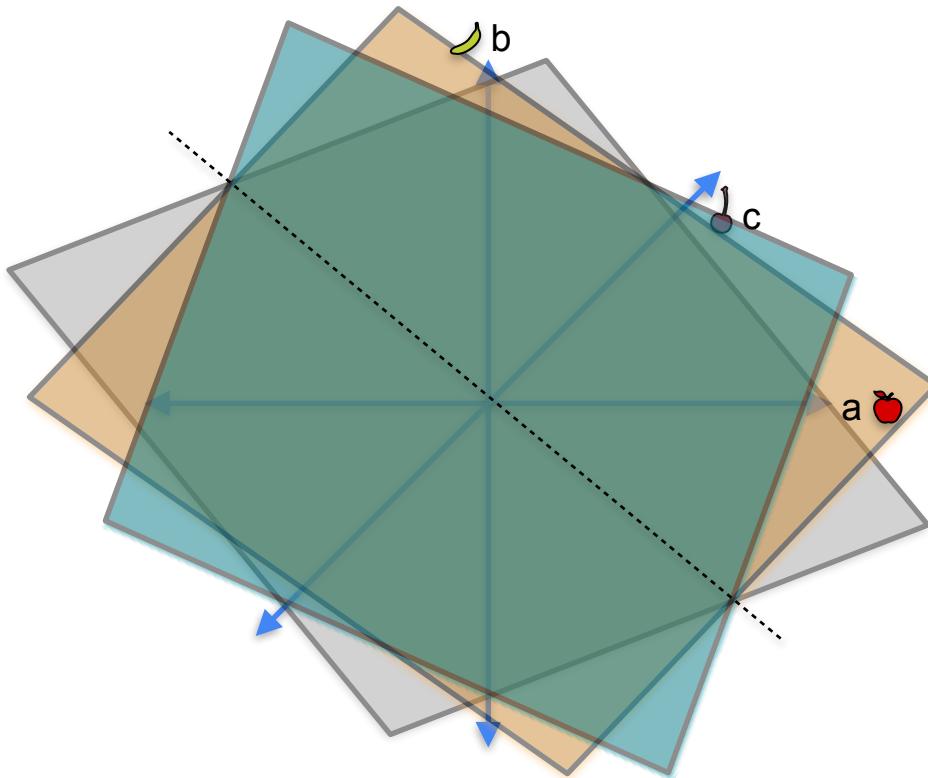
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

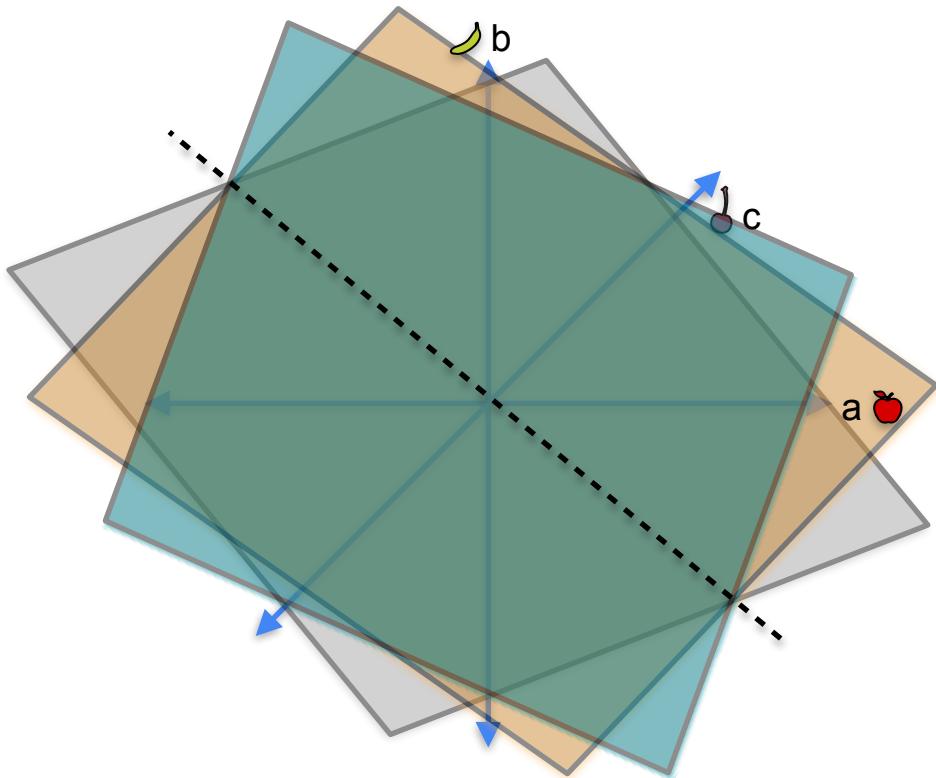
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

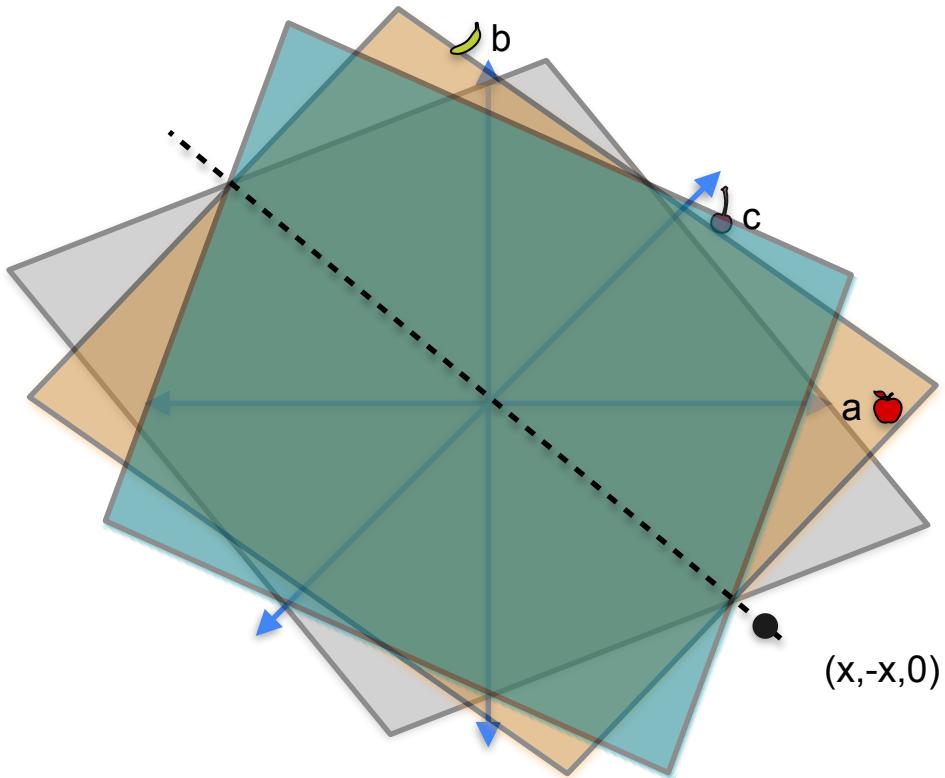
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

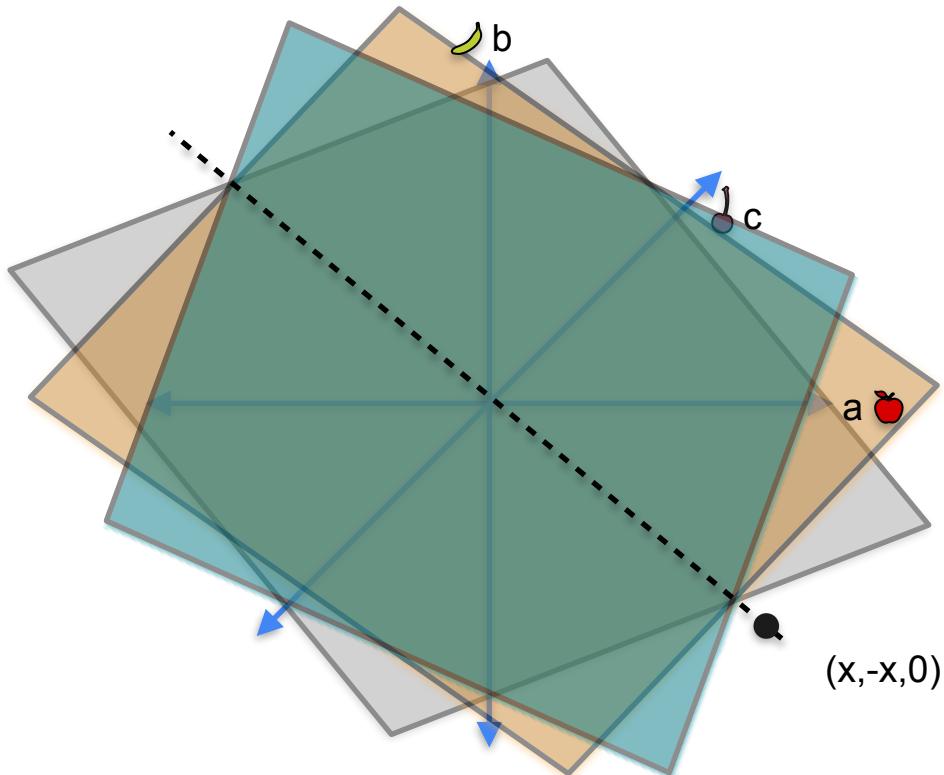
System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



Solution space

- $c = 0$
- $b = -a$



System 2

System 2

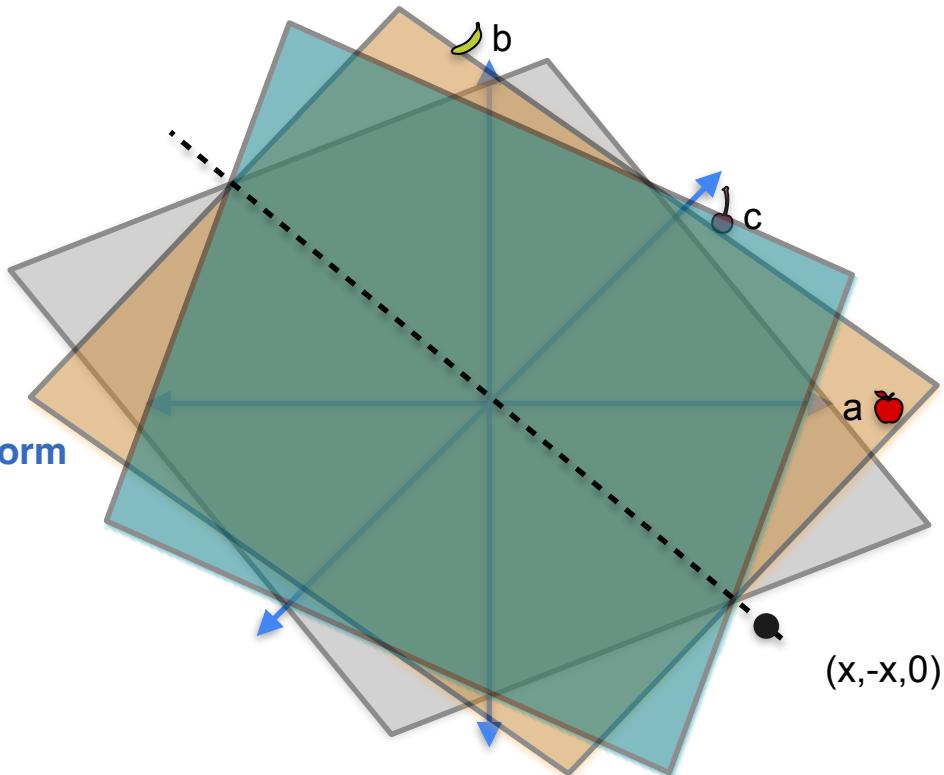
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



Solution space

- $c = 0$
- $b = -a$

All points of the form
 $(x, -x, 0)$



System 2

System 2

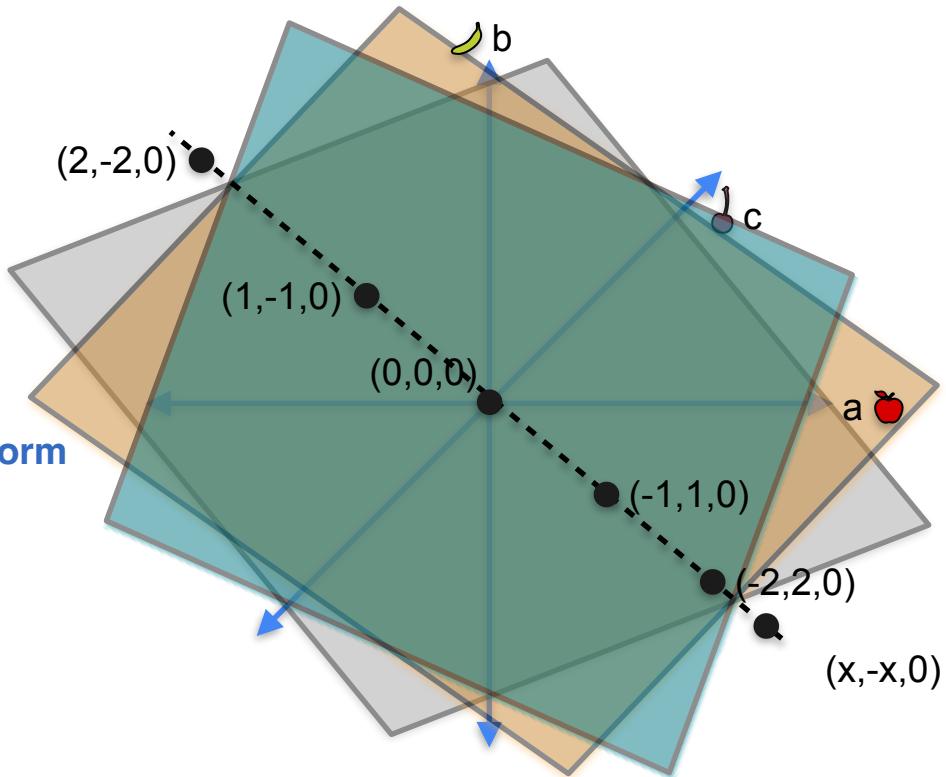
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



Solution space

- $c = 0$
- $b = -a$

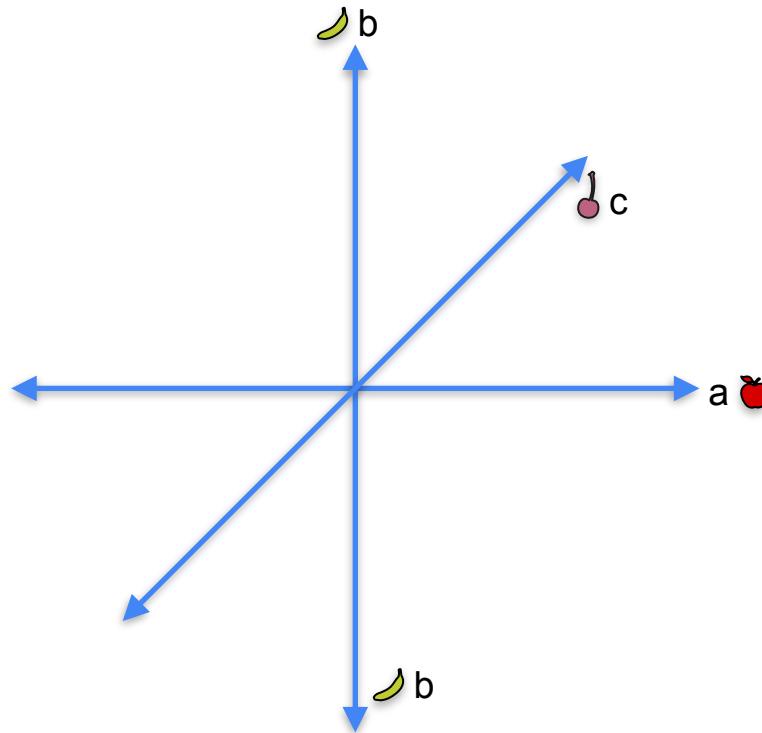
All points of the form
 $(x, -x, 0)$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

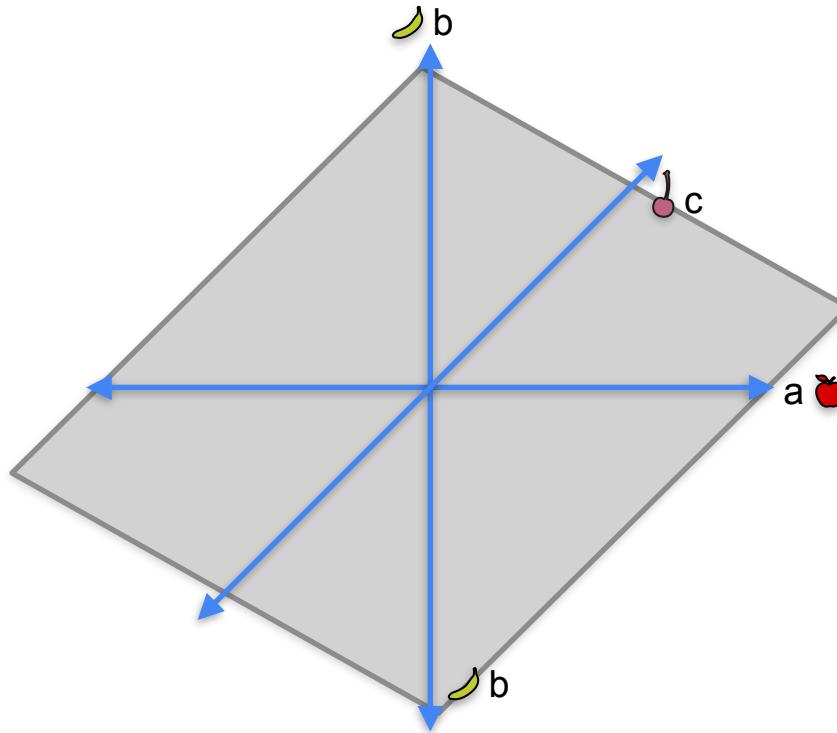
System 3

- $a + b + c = \mathbf{0}$



- $2a + 2b + 2c = \mathbf{0}$

- $3a + 3b + 3c = \mathbf{0}$



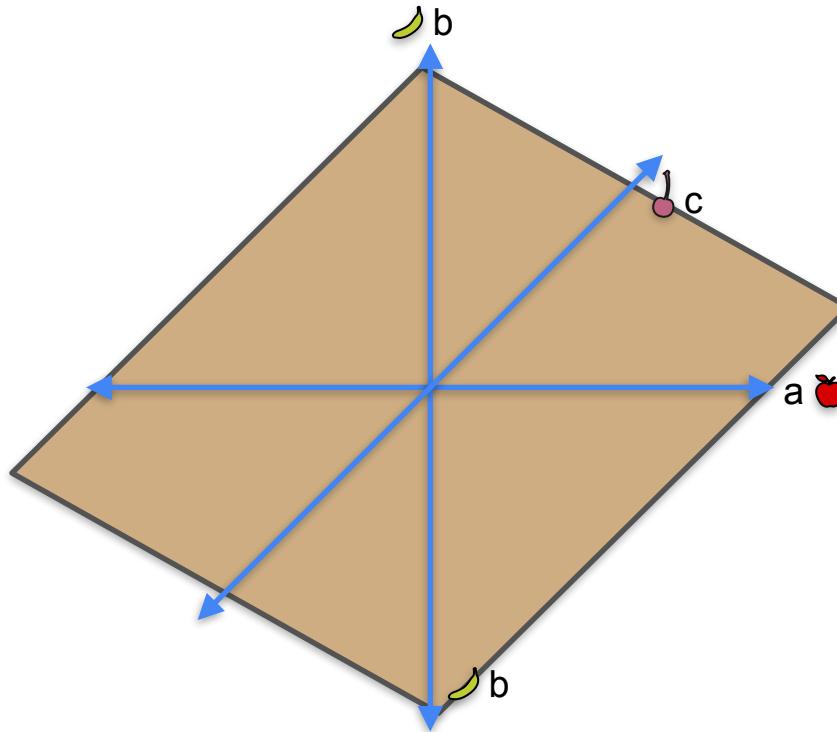
System 3

System 3

- $a + b + c = \mathbf{0}$

- $2a + 2b + 2c = \mathbf{0}$ 

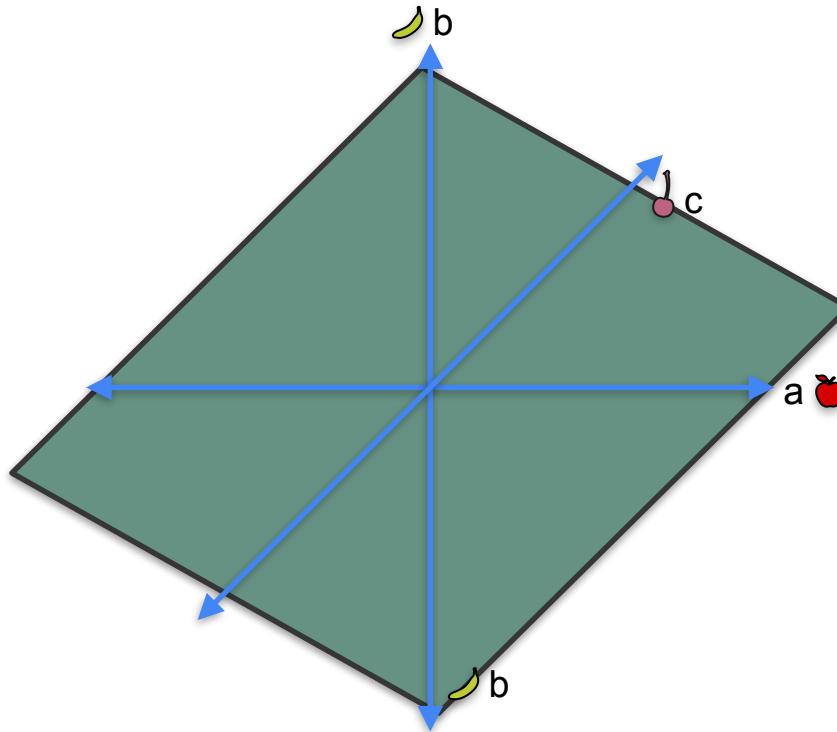
- $3a + 3b + 3c = \mathbf{0}$



System 3

System 3

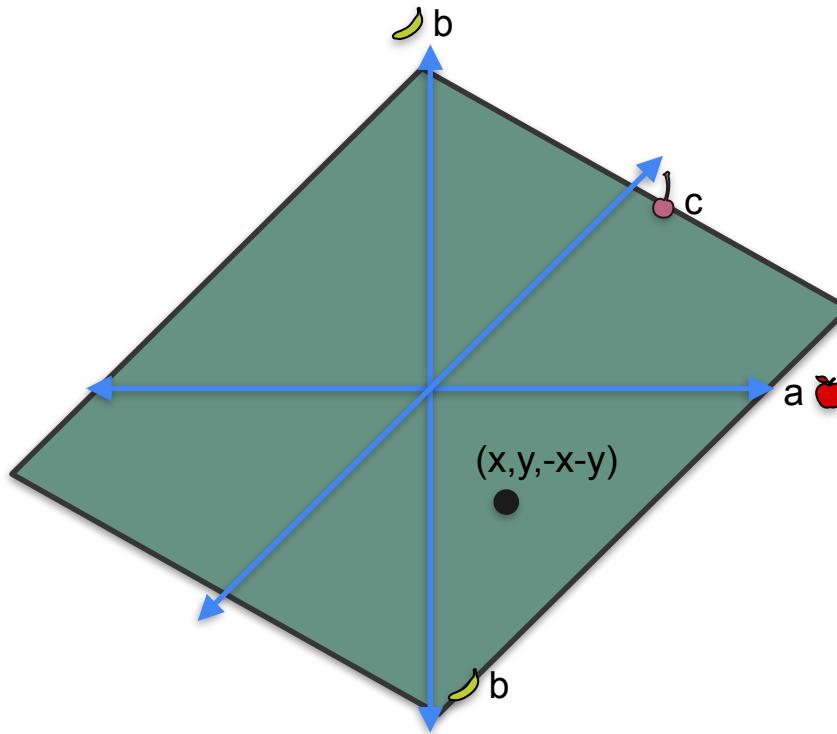
- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

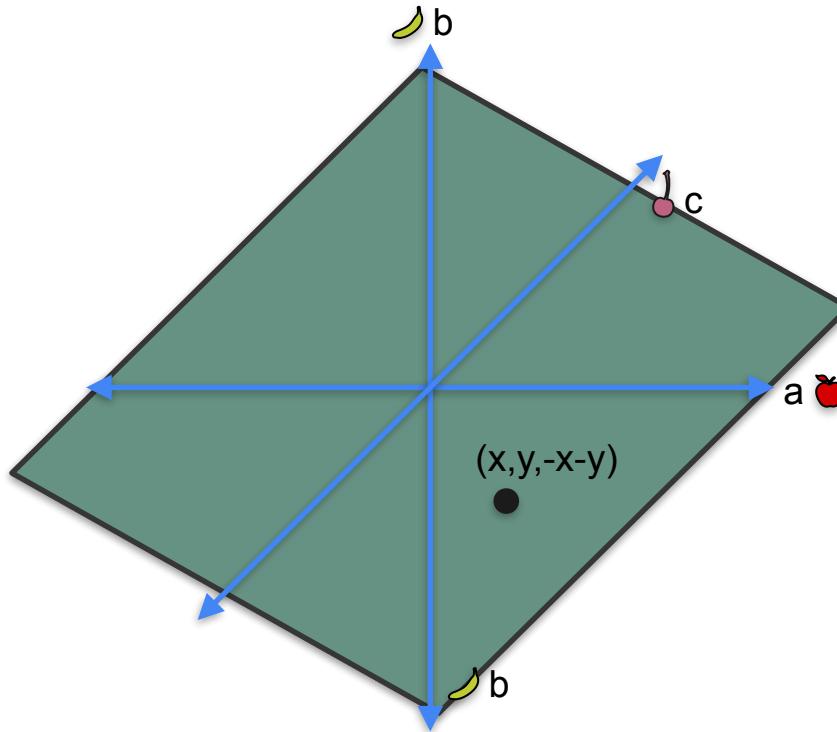
System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



Solution space

$$\bullet a + b + c = 0$$



System 3

System 3

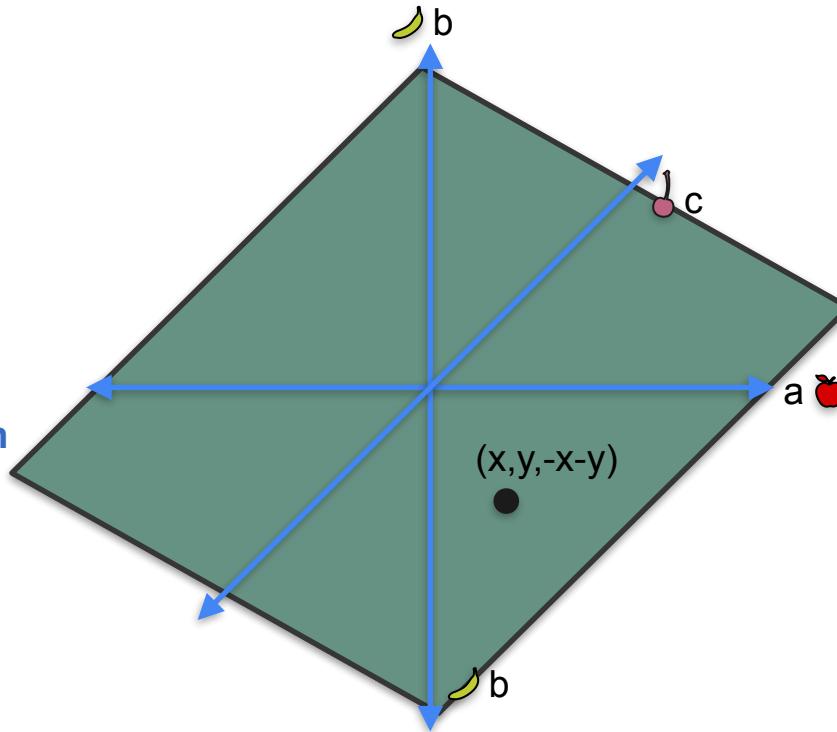
- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Solution space

$$\bullet a + b + c = 0$$

All points of the form

$$(x, y, -x-y)$$



System 3

System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

Solution space

$$\cdot a + b + c = 0$$

All points of the form

$$(x, y, -x - y)$$

