27-11-2020

Unit-9

Random variables

- Expressing outcomes of random processes as numbers.

$$X = \begin{cases} 1 & \text{if heads} \\ 0 & \text{if tails.} \end{cases}$$

- Random variables → Discrete, → Continuous

⇒ Probability distribution for discrete R.V.

- X = # H after 3 flips of a fair coin

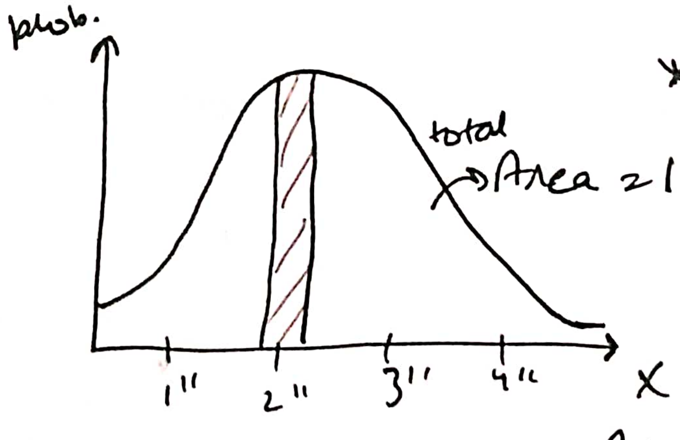
⇒ mean (expected value) of a discrete random variable.

X	P(x)
—	—
—	—

$$\mu(x) = \sum_{i=1}^{N} x_i \, P(x_i)$$

$$Var(X) = \sum_{i=1}^{N} (\mu - x_i)^2 \, P(x_i)$$

⇒ Probability distribution for continuous. R.V.



X → Amount of rain in Karnal

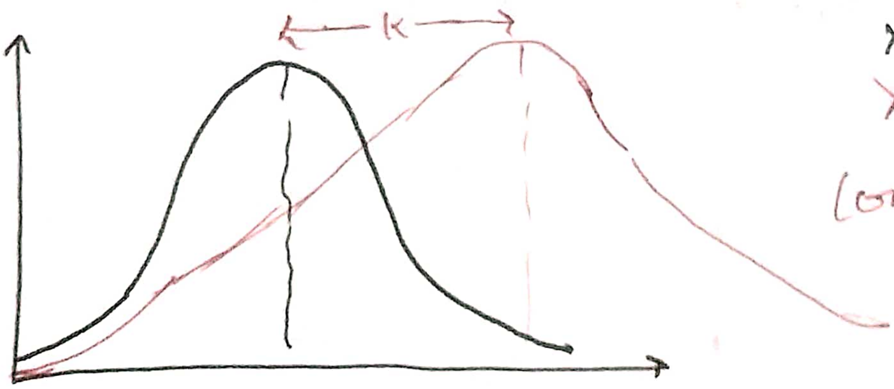
$P(X = 2) = 0$ (~~under~~ Area under a line)

$P(1.9 < X < 2.1)$ = Area under the shaded region

⇒ Transforming random variable

• adding a constant.

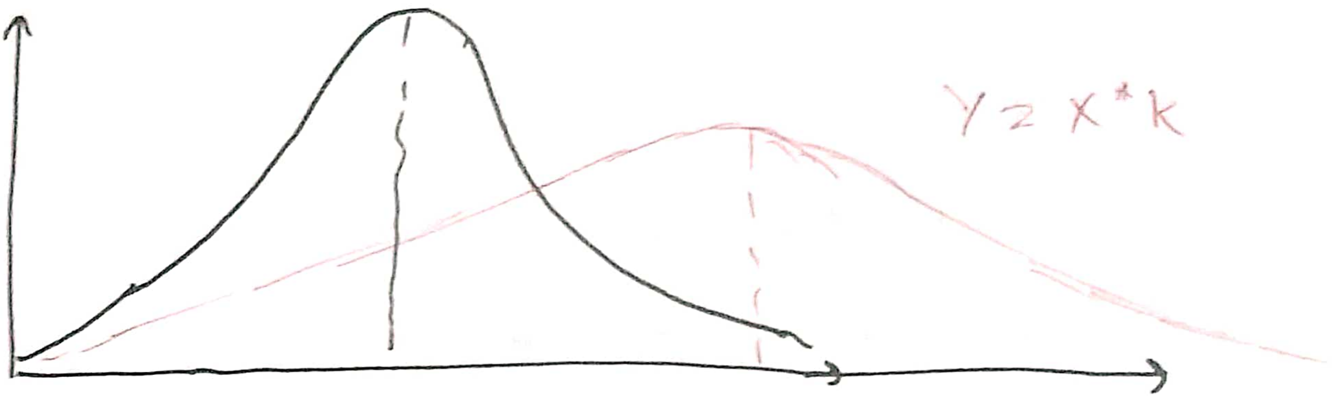
$Y = X + K$



R.V.	mean	std. dev
X	μ_x	σ_x
Y	$\mu_x + K$	σ_x

(only mean changes)

• Scaling Random variable



R.V.	μ	σ
X	μ_x	σ_x
Y	$\mu_x * k$	$\sigma_x * k$

(mean & std both are changed)

⇒ Combining Random Variables

Given 2 R.V., X & Y with mean ~~Ex~~ $E(X) = \mu_x$ & $E(Y) = \mu_y$ and std $var(X) = \sigma_x^2$ & $var(Y) = \sigma_y^2$

- $E(X + Y) = E(X) + E(Y)$ ↙ mean
- $E(X - Y) = E(X) - E(Y)$

⇒ Variance

- $Var(X + Y) = Var(X) + Var(Y)$
- $Var(X - Y) = Var(X) + Var(Y)$

} ~~Always~~ Same

Note →
For above equations to hold, A and B are independent.

Remember →
The ~~eq~~. equation is for variance & not ~~sd~~ std deviation. make sure to convert if std dev is given.

⇒ Binomial Random Variables

⇒ A R.V. is a binomial R.V. if :-

- Made up of independent trials.
- Each trial can be classified as either success or failure
- fixed number of trials
- Probability of success on each trial is constant.

eg X = # H after 10 flips

Y = # kings after taking 2 cards without replacement.

X is Binomial R.V. , Y is not.

Note :-

If given to check whether a R.V. is binomial or not. Just check whether it satisfies the above given cond or not.

⇒ Binomial distribution

- It describes the distribution of binary data from a finite sample.
- Thus it gives the probability of getting no events out of n trials.

⇒ How is it different from Normal distribution?

Normal dist distribution continous data whereas binomial distribution describes discreted data.

• Binomial → finite number of events

Normal → infinite number of events.

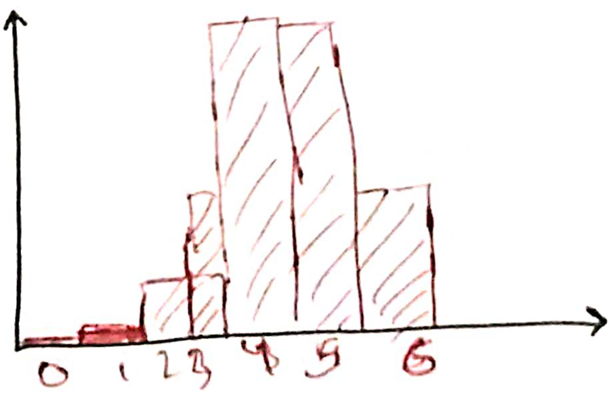
⇒ P(r scores in n attempts)

P(r scores (success) in n attempts) = $^nC_r\, p^r (1-p)^{n-r}$

where, p = prob. of success.

⇒ X = # successful free throws in 6 shots

p(success) = 0.7 = 70%.



⇒ mean and variance of Bernouli dist.

- In binomial distribution we were talking of n succes in n trials.
- In bernouli distribution there is only one trial (or Bernouli trial).

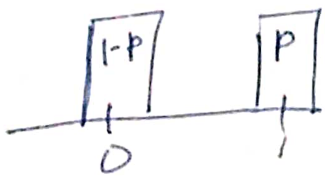
- Binomial R.V. with $n = 1$ is ~~binomial~~ bernouli R.V.

- Let X be a bernouli R.V.

  $X = 1$ if succes, $X = 0$ for failure

  $P(X=1)$ is p.

- Then dist is (X can be 0 or 1)



⇒ Expected value & variance

$$\mu_X = E(X) = (1-p)\cdot 0 + p \cdot 1 = p$$

$$\sigma^2 = 1-p\,(0-p)^2 + p(1-p)^2 = \boxed{p(1-p)}$$

$$= \boxed{pq}$$

⇒ Mean and variance of a binomial R.V.

X is a binomial R.V.

X = # success after n trials,
P(success) = p

(X can be 0, 1, 2 ... n)

⇒ Expected value and Variance

$E(X) = np$

$Var(X) = n \cdot p(1-p)$

⇒ Geometric R.V.

A R.V. of the form "How many attempts until success?"

• Number of trials is not fixed.

⇒ X be a Geometric R.V. ~~such~~
X = # independent trials to get success, where P(success) for each trial is p.

$E(X) = \frac{1}{p}$

↑

expected value of X

⇒ **Law of Large Numbers**

If from a population we take n samples & found their mean.

Then the sample mean will approach population mean if $n \to \infty$.

⇒ **Poisson distribution**

According to binomial dist,

$$P(X=k) = {}^{n}C_{k}(p)^{k}(1-p)^{n-k}$$

when $n \to \infty$, it becomes Poisson dist.

· Eg

X = # cars passing through in an hour

$$E(X) = \lambda$$

$$P(X=k) = \lim_{n\to\infty} {}^{n}C_{k}\left(\frac{\lambda}{n}\right)^{k}\left(1-\frac{\lambda}{n}\right)^{n-k}$$

$$\boxed{P(X=k) = \frac{\lambda^{k} e^{-\lambda}}{k!}}$$