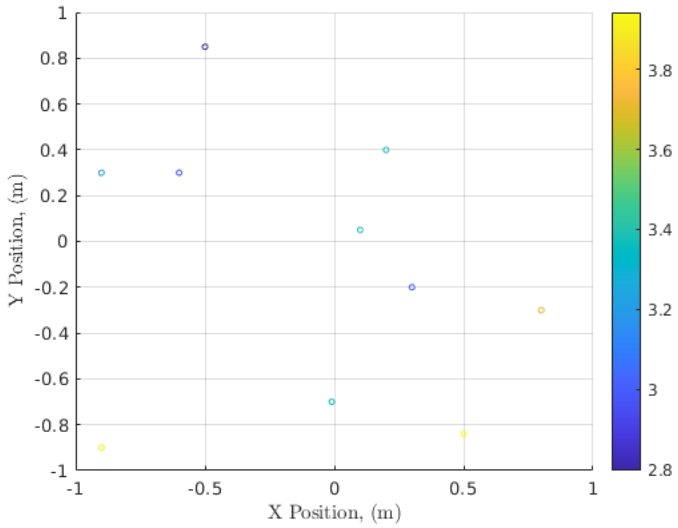
Contents

Drew Hanover Proj 2A

```
rng('default')
clc
close all
clear
```

Plot initial data

```
X = [0.1 -0.9 0.2 0.8 -0.6 0.3 0.5 -0.5 -0.01 -0.9];
Y = [0.05 0.3 0.4 -0.3 0.3 -0.2 -0.84 0.85 -0.7 -0.9];
Sensor_0 = [3.39382006 3.2073034 3.39965035 3.68810201 2.96941623...
    2.99495501 3.94274928 2.7968011 3.34929734...
    3.9129616];
figure
scatter(X,Y,10,Sensor_0)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')
```

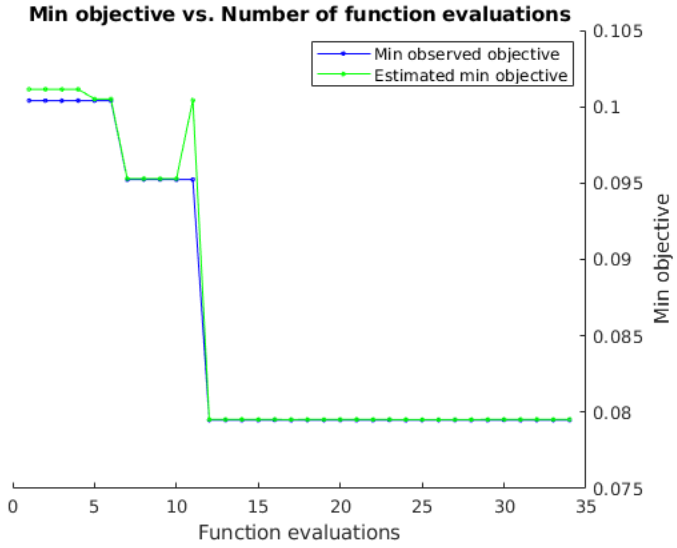


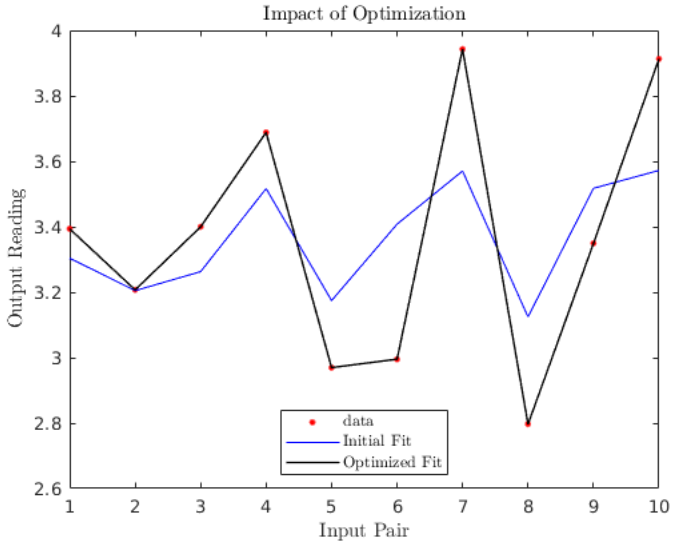
Train Model and Plot Hyperparam Optimization

```
tbl = readtable('init_data.txt','FileType','text','ReadVariableNames',true);
gprMdl1 = fitrgp(tbl,'Sensor','KernelFunction','squaredexponential');
gprMdl2 = fitrgp(tbl,'Sensor',...
      'FitMethod','sr','PredictMethod','fic','ActiveSetMethod','entropy',...
      'OptimizeHyperparameters','all','HyperparameterOptimizationOptions',...
      struct('MaxObjectiveEvaluations',30,'UseParallel',true));
ypred = resubPredict(gprMdl2);
train_Loss = resubLoss(gprMdl2)

ypred1 = resubPredict(gprMdl1);
ypred2 = resubPredict(gprMdl2);

figure
plot(tbl.Sensor,'r.','MarkerSize',10);
hold on
plot(ypred1,'b');
plot(ypred2,'k','LineWidth',1);
xlabel('Input Pair','Interpreter', 'latex');
```

```
ylabel('Output Reading','Interpreter', 'latex');
legend({'data','Initial Fit','Optimized Fit'},'Location','Best','Interpreter', 'latex');
title('Impact of Optimization','Interpreter', 'latex');
hold off
```

```
Copying objective function to workers...
Done copying objective function to workers.
|=====================================================================================================================================================|
| Iter | Active  | Eval   | Objective:  | Objective   | BestSoFar   | BestSoFar   |        Sigma | BasisFunction| KernelFuncti-|  KernelScale |  Stand
|      | workers | result | log(1+loss) | runtime     | (observed)  | (estim.)    |              |              | on           |              |
|=====================================================================================================================================================|
|    1 |       3 | Accept |      2.5229 |     0.24972 |      0.1004 |     0.10114 |       2.5911 |         none |  exponential |    0.0023619 |
|    2 |       3 | Best   |      0.1004 |     0.39947 |      0.1004 |     0.10114 |   0.00035538 |       linear | rationalquad |    0.0083092 |
|    3 |       3 | Accept |      2.5228 |     0.25448 |      0.1004 |     0.10114 |       3.2587 |         none |  exponential |     0.089288 |
|    4 |       3 | Accept |     0.12512 |     0.41535 |      0.1004 |     0.10114 |    0.0040627 |         none | rationalquad |       1.6959 |
|    5 |       5 | Accept |     0.38171 |     0.41854 |      0.1004 |     0.10051 |   0.00010505 |         none |  ardmatern52 |            - |
|    6 |       5 | Accept |     0.23081 |     0.46368 |      0.1004 |     0.10051 |     0.033024 | pureQuadrati | ardrationalq |            - |
|    7 |       2 | Best   |    0.095232 |     0.14778 |    0.095232 |    0.095293 |   0.00043514 |       linear | rationalquad |        1.295 |
|    8 |       2 | Accept |      0.1004 |     0.21183 |    0.095232 |    0.095293 |      0.20023 |       linear | ardsquaredex |            - |
|    9 |       2 | Accept |     0.15102 |     0.17522 |    0.095232 |    0.095293 |   0.00020362 |     constant |     matern32 |    0.0063745 |
|   10 |       2 | Accept |     0.19555 |     0.18941 |    0.095232 |    0.095293 |       1.2535 | pureQuadrati |  ardmatern32 |            - |
|   11 |       6 | Accept |      0.1004 |     0.12453 |    0.095232 |     0.10043 |       3.9178 |       linear | rationalquad |      0.21184 |
|   12 |       2 | Accept |      0.1004 |     0.16654 |    0.079482 |    0.079522 |       3.5496 |       linear | rationalquad |        0.904 |
|   13 |       2 | Accept |     0.19555 |     0.24212 |    0.079482 |    0.079522 |       0.0189 | pureQuadrati | rationalquad |     0.025334 |
|   14 |       2 | Best   |    0.079482 |     0.20179 |    0.079482 |    0.079522 |   0.00017234 |       linear |     matern32 |       1.5141 |
|   15 |       2 | Accept |      0.1004 |     0.20963 |    0.079482 |    0.079522 |    0.0002837 |       linear |     matern32 |     0.013906 |
|   16 |       2 | Accept |      0.1004 |     0.21447 |    0.079482 |    0.079522 |       2.6329 |       linear |  exponential |     0.017061 |
|   17 |       6 | Accept |     0.13031 |     0.13946 |    0.079482 |     0.07952 |   0.00018413 |         none | rationalquad |       1.1335 |
|   18 |       2 | Accept |     0.10896 |     0.11971 |    0.079482 |    0.079523 |   0.00086962 |       linear |     matern52 |       1.6916 |
|   19 |       2 | Accept |    0.099775 |     0.20593 |    0.079482 |    0.079523 |     0.020977 |       linear | ardsquaredex |            - |
|   20 |       2 | Accept |     0.15102 |     0.14572 |    0.079482 |    0.079523 |       1.1661 |     constant |     matern52 |     0.017037 |
|=====================================================================================================================================================|
| Iter | Active  | Eval   | Objective:  | Objective   | BestSoFar   | BestSoFar   |        Sigma | BasisFunction| KernelFuncti-|  KernelScale |  Stand
|      | workers | result | log(1+loss) | runtime     | (observed)  | (estim.)    |              |              | on           |              |
|=====================================================================================================================================================|
|   21 |       2 | Accept |     0.13684 |     0.19378 |    0.079482 |    0.079523 |    0.0022358 |         none | ardexponenti |            - |
|   22 |       2 | Accept |      2.5221 |     0.15258 |    0.079482 |    0.079523 |      0.13392 |         none | squaredexpon |     0.086261 |
|   23 |       6 | Accept |      0.1011 |     0.11687 |    0.079482 |    0.079521 |     0.079151 |       linear |     matern52 |      0.26901 |
|   24 |       2 | Accept |     0.55992 |     0.14677 |    0.079482 |    0.079517 |   0.00049947 |         none |     matern52 |       1.5928 |
|   25 |       2 | Accept |      0.1004 |     0.14366 |    0.079482 |    0.079517 |     0.015027 |       linear |     matern52 |     0.025967 |
|   26 |       2 | Accept |      0.1004 |     0.15587 |    0.079482 |    0.079517 |       3.1727 |       linear |     matern52 |     0.019415 |
|   27 |       2 | Accept |     0.13576 |     0.17538 |    0.079482 |    0.079517 |    0.0052646 |         none | rationalquad |     0.046469 |
|   28 |       2 | Accept |      0.1004 |     0.17536 |    0.079482 |    0.079517 |      0.41278 |       linear |  exponential |     0.022653 |
|   29 |       6 | Accept |      2.5229 |    0.097981 |    0.079482 |    0.079525 |        0.868 |         none |     matern52 |     0.030178 |
|   30 |       2 | Accept |     0.13933 |     0.15435 |    0.079482 |    0.079526 |    0.0074753 |     constant |     matern52 |      0.68561 |
|   31 |       2 | Accept |     0.19555 |     0.16367 |    0.079482 |    0.079526 |       0.1937 | pureQuadrati |  exponential |       1.2954 |
|   32 |       2 | Accept |      0.1004 |     0.18026 |    0.079482 |    0.079526 |       0.5407 |       linear | rationalquad |     0.015395 |
|   33 |       2 | Accept |     0.10067 |     0.21635 |    0.079482 |    0.079526 |    0.0017622 |       linear |  ardmatern32 |            - |
|   34 |       2 | Accept |      2.5229 |     0.15206 |    0.079482 |    0.079526 |   0.00022742 |         none |     matern52 |    0.0027235 |

__________________________________________________________
Optimization completed.
MaxObjectiveEvaluations of 30 reached.
Total function evaluations: 34
Total elapsed time: 11.2838 seconds
Total objective function evaluation time: 6.9203

Best observed feasible point:
      Sigma       BasisFunction    KernelFunction    KernelScale    Standardize
    __________    _____________    ______________    ___________    ___________

    0.00017234       linear           matern32         1.5141          true

Observed objective function value = 0.079482
Estimated objective function value = 0.079526
Function evaluation time = 0.20179

Best estimated feasible point (according to models):
      Sigma       BasisFunction    KernelFunction    KernelScale    Standardize
    __________    _____________    ______________    ___________    ___________

    0.00017234       linear           matern32         1.5141          true

Estimated objective function value = 0.079526
Estimated function evaluation time = 0.20179


train_Loss =

   3.4303e-08
```

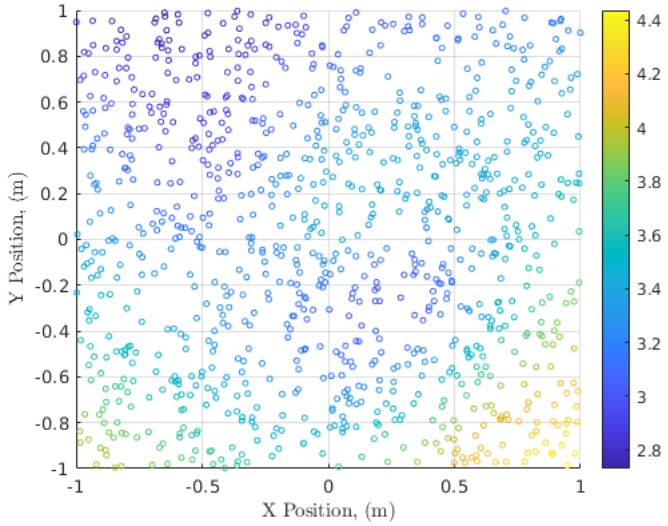


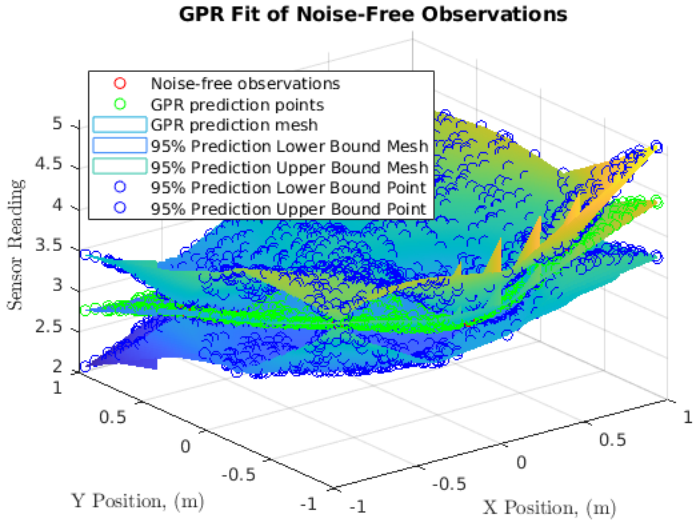


Test Model and Plot

```
X_test = 2*rand(1000,1)-1;
Y_test = 2*rand(1000,1)-1;
[sensor_pred, ~, intervals] = predict(gprMdl2,[X_test Y_test]);


[X_grid,Y_grid] = meshgrid(linspace(-1,1,1000),linspace(-1,1,1000)) ;
lower_interval_grid = griddata(X_test,Y_test,intervals(:,1),X_grid,Y_grid) ;
upper_interval_grid = griddata(X_test,Y_test,intervals(:,2),X_grid,Y_grid) ;
prediction_grid = griddata(X_test,Y_test,sensor_pred,X_grid,Y_grid) ;


figure
scatter(X_test,Y_test,10,sensor_pred)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')


figure
scatter3(tbl.X,tbl.Y,tbl.Sensor,'r') % Observed data points
hold on
scatter3(X_test, Y_test, sensor_pred,'g') % GPR predictions
mesh(X_grid,Y_grid,prediction_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,lower_interval_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,upper_interval_grid,'FaceAlpha','0.5')
scatter3(X_test, Y_test, intervals(:,1), 'b')
scatter3(X_test, Y_test, intervals(:,2), 'b')
hold off
title('GPR Fit of Noise-Free Observations')
legend({'Noise-free observations','GPR prediction points', 'GPR prediction mesh','95% Prediction Lower Bound Mesh','95% Prediction Upper Bound Mesh',
xlabel('X Position, (m)','Interpreter','latex')
```

```
ylabel('Y Position, (m)','Interpreter','latex')
zlabel('Sensor Reading','Interpreter','latex')
```





ReTrain Model with Request 1 and Plot Hyperparam Optimization

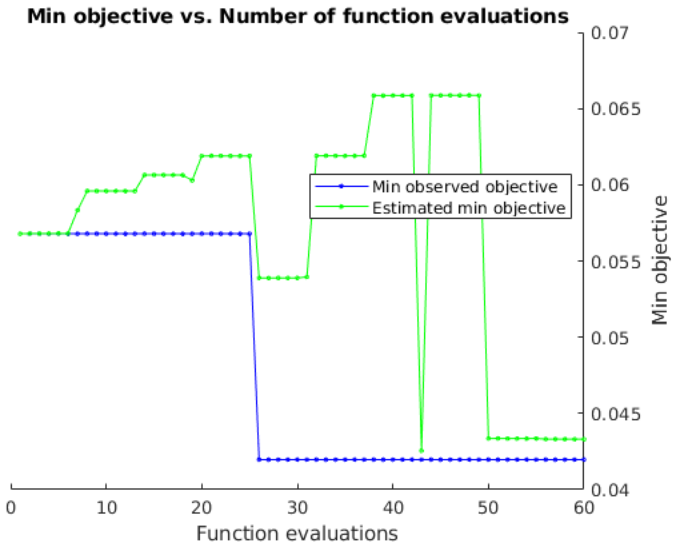
```
tbl = readtable('init_data_with_request_1.txt','FileType','text','ReadVariableNames',true);
gprMdl1 = fitrgp(tbl,'Sensor','KernelFunction','squaredexponential');
gprMdl2 = fitrgp(tbl,'Sensor',...
      'FitMethod','sr','PredictMethod','fic','ActiveSetMethod','entropy',...
      'OptimizeHyperparameters','all','HyperparameterOptimizationOptions',...
      struct('MaxObjectiveEvaluations',60,'UseParallel',true));
ypred = resubPredict(gprMdl2);
train_Loss = resubLoss(gprMdl2)

ypred1 = resubPredict(gprMdl1);
ypred2 = resubPredict(gprMdl2);

figure
plot(tbl.Sensor,'r.','MarkerSize',10);
hold on
plot(ypred1,'b');
plot(ypred2,'k','LineWidth',1);
xlabel('Input Pair','Interpreter', 'latex');
ylabel('Output Reading','Interpreter', 'latex');
legend({'data','Initial Fit','Optimized Fit'},'Location','Best','Interpreter', 'latex');
title('Impact of Optimization','Interpreter', 'latex');
hold off

figure
scatter(tbl.X,tbl.Y,10,tbl.Sensor)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')
```

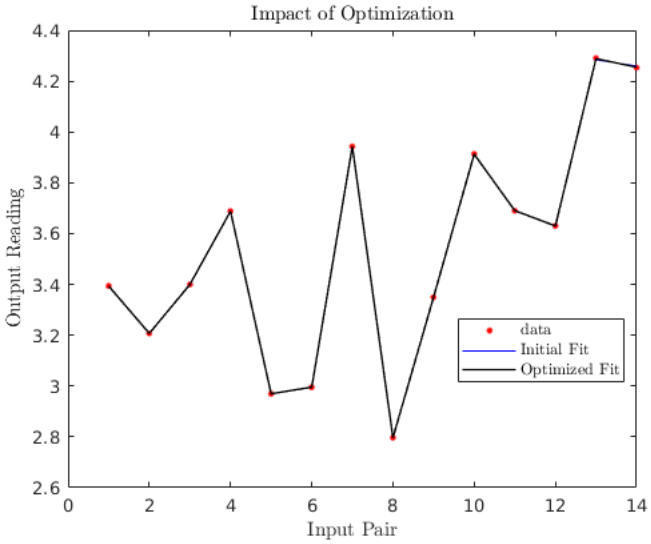
```
Copying objective function to workers...
Done copying objective function to workers.
```

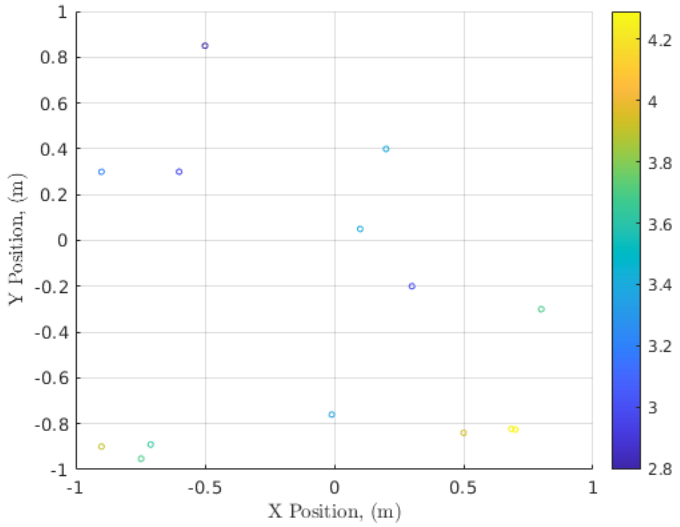
Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
1	6	Best	0.056788	0.20676	0.056788	0.056788	0.00035819	linear	exponential	1.5071	
2	2	Accept	0.19877	0.26442	0.056788	0.056807	2.2044	constant	rationalquad	0.0025993	
3	2	Accept	0.068484	0.34291	0.056788	0.056807	0.00035593	linear	ardrationalq	-	
4	2	Accept	0.19877	0.24142	0.056788	0.056807	0.69917	constant	matern52	0.04067	
5	2	Accept	0.083304	0.247	0.056788	0.056807	0.44586	linear	ardrationalq	-	
6	2	Accept	0.083304	0.25703	0.056788	0.056807	0.84122	linear	matern32	1.411	
7	6	Accept	0.058978	0.25473	0.056788	0.058336	0.00027577	none	ardexponenti	-	
8	2	Accept	0.083084	0.17621	0.056788	0.059589	0.25727	linear	exponential	0.096751	
9	2	Accept	2.618	0.19878	0.056788	0.059589	0.00025389	none	squaredexpon	0.0038683	
10	2	Accept	0.24118	0.14338	0.056788	0.059589	0.69108	none	matern52	0.68735	
11	2	Accept	0.12328	0.24993	0.056788	0.059589	0.0042242	pureQuadrati	ardsquaredex	-	
12	2	Accept	0.059252	0.21171	0.056788	0.059589	0.036649	linear	ardsquaredex	-	
13	6	Accept	0.059991	0.16273	0.056788	0.059589	0.00010018	linear	ardsquaredex	-	
14	2	Accept	0.073091	0.18475	0.056788	0.060638	0.17455	linear	exponential	1.3359	
15	2	Accept	0.12328	0.24505	0.056788	0.060638	0.00050583	pureQuadrati	ardsquaredex	-	
16	2	Accept	0.067307	0.23896	0.056788	0.060638	0.0002642	linear	ardexponenti	-	
17	2	Accept	1.8217	0.13775	0.056788	0.060638	0.00020291	none	exponential	0.17065	
18	2	Accept	0.19877	0.22859	0.056788	0.060638	1.09	constant	ardrationalq	-	
19	6	Accept	0.083304	0.1682	0.056788	0.060283	4.6303	linear	ardrationalq	-	
20	2	Accept	0.17187	0.16983	0.056788	0.061891	4.093	pureQuadrati	exponential	0.69525	
21	2	Accept	2.4667	0.17244	0.056788	0.061891	0.11919	none	matern52	0.014685	
22	2	Accept	0.17187	0.22124	0.056788	0.061891	1.7366	pureQuadrati	ardsquaredex	-	
23	2	Accept	0.063459	0.17264	0.056788	0.061891	0.0020713	linear	matern32	0.044369	
24	2	Accept	0.063531	0.23298	0.056788	0.061891	0.11187	linear	ardsquaredex	-	
25	6	Accept	0.17187	0.17223	0.056788	0.061892	4.4899	pureQuadrati	ardrationalq	-	
26	2	Accept	2.4953	0.15254	0.04197	0.053887	0.00020201	none	exponential	0.015401	
27	2	Accept	0.057865	0.18233	0.04197	0.053887	0.0075762	linear	matern52	0.30597	
28	2	Accept	0.12328	0.27756	0.04197	0.053887	0.0010498	pureQuadrati	ardrationalq	-	
29	2	Accept	0.0626	0.182	0.04197	0.053887	0.0070512	linear	matern52	0.034513	
30	2	Best	0.04197	0.2179	0.04197	0.053887	0.00010809	linear	matern32	1.4685	
31	6	Accept	0.14051	0.11041	0.04197	0.053951	0.00021139	none	exponential	1.111	
32	2	Accept	0.083304	0.14248	0.04197	0.061899	0.0022411	linear	matern32	0.0018437	
33	2	Accept	0.061052	0.21577	0.04197	0.061899	0.0030875	linear	rationalquad	0.81621	
34	2	Accept	0.060134	0.23197	0.04197	0.061899	0.00043567	linear	ardmatern52	-	
35	2	Accept	0.11675	0.23619	0.04197	0.061899	0.0019612	none	ardmatern52	-	
36	2	Accept	0.068484	0.28768	0.04197	0.061899	0.00012768	linear	ardrationalq	-	
37	6	Accept	0.17187	0.12951	0.04197	0.061901	4.1053	pureQuadrati	exponential	0.025722	
38	2	Accept	0.070015	0.16171	0.04197	0.065852	0.15677	linear	exponential	0.067285	
39	2	Accept	0.044449	0.31994	0.04197	0.065852	0.0047889	constant	ardrationalq	-	
40	2	Accept	0.073593	0.26071	0.04197	0.065852	0.20495	linear	ardsquaredex	-	
41	2	Accept	0.083304	0.17649	0.04197	0.065852	0.11133	linear	matern32	0.002508	
42	2	Accept	0.20263	0.23673	0.04197	0.065852	1.549	none	ardmatern52	-	
43	6	Accept	0.17187	0.15961	0.04197	0.04255	1.2494	pureQuadrati	exponential	0.00185	
44	2	Accept	0.083299	0.20113	0.04197	0.065863	0.11486	linear	exponential	0.0018049	
45	2	Accept	0.17187	0.21667	0.04197	0.065863	0.41361	pureQuadrati	matern32	0.42595	
46	2	Accept	0.24786	0.2716	0.04197	0.065863	3.0698	none	ardsquaredex	-	
47	2	Accept	0.069865	0.24208	0.04197	0.065863	0.16706	linear	rationalquad	0.26572	
48	2	Accept	0.40201	0.18435	0.04197	0.065863	0.00020709	none	exponential	0.9438	
49	6	Accept	0.083304	0.15449	0.04197	0.065863	0.32713	linear	exponential	0.0018538	
50	2	Accept	0.083304	0.22639	0.04197	0.043355	4.5751	linear	rationalquad	0.019419	
51	2	Accept	0.080158	0.28314	0.04197	0.043355	0.028929	linear	rationalquad	0.0037314	
52	2	Accept	0.17187	0.34954	0.04197	0.043355	0.30926	pureQuadrati	ardrationalq	-	
53	2	Accept	0.083304	0.31833	0.04197	0.043355	0.43439	linear	ardmatern32	-	
54	2	Accept	0.062332	0.21267	0.04197	0.043355	0.0034657	linear	matern32	0.088824	
55	6	Accept	0.083304	0.14572	0.04197	0.043357	0.0057919	linear	matern52	0.0018158	
56	2	Accept	0.083304	0.2653	0.04197	0.043313	0.0035212	linear	matern32	0.0018173	
57	2	Accept	0.083304	0.26537	0.04197	0.043313	3.4011	linear	rationalquad	1.6122	
58	2	Accept	0.088803	0.22244	0.04197	0.043313	0.11469	linear	squaredexpon	1.2568	
59	2	Accept	0.083302	0.27769	0.04197	0.043313	0.00029298	linear	matern52	0.0024508	
60	2	Accept	0.054733	0.20667	0.04197	0.043313	0.00045153	linear	matern52	0.3603	

```
Optimization completed.
MaxObjectiveEvaluations of 60 reached.
Total function evaluations: 60
Total elapsed time: 27.5135 seconds
Total objective function evaluation time: 13.0268

Best observed feasible point:
      Sigma       BasisFunction    KernelFunction    KernelScale    Standardize
    __________    _____________    ______________    ___________    ___________

    0.00010809       linear           matern32         1.4685          true

Observed objective function value = 0.04197
```

```
Estimated objective function value = 0.043313
Function evaluation time = 0.2179

Best estimated feasible point (according to models):
      Sigma       BasisFunction    KernelFunction    KernelScale    Standardize
    __________    _____________    ______________    ___________    ___________

    0.00010809       linear           matern32         1.4685          true

Estimated objective function value = 0.043313
Estimated function evaluation time = 0.21765


train_Loss =

   5.8965e-07
```

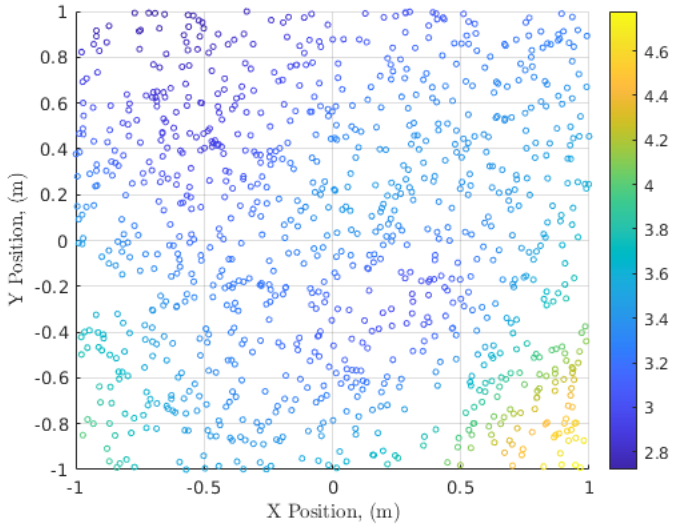


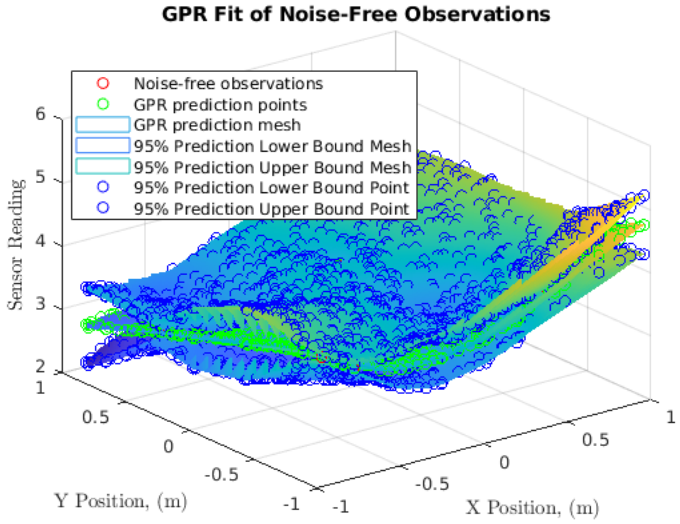




Test Model and Plot with data from Request 1

```
X_test = 2*rand(1000,1)-1;
Y_test = 2*rand(1000,1)-1;
[sensor_pred, ~, intervals] = predict(gprMdl2,[X_test Y_test]);


[X_grid,Y_grid] = meshgrid(linspace(-1,1,1000),linspace(-1,1,1000)) ;
lower_interval_grid = griddata(X_test,Y_test,intervals(:,1),X_grid,Y_grid) ;
upper_interval_grid = griddata(X_test,Y_test,intervals(:,2),X_grid,Y_grid) ;
prediction_grid = griddata(X_test,Y_test,sensor_pred,X_grid,Y_grid) ;



figure
scatter(X_test,Y_test,10,sensor_pred)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')


figure
scatter3(tbl.X,tbl.Y,tbl.Sensor,'r') % Observed data points
hold on
scatter3(X_test, Y_test, sensor_pred,'g') % GPR predictions
mesh(X_grid,Y_grid,prediction_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,lower_interval_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,upper_interval_grid,'FaceAlpha','0.5')
scatter3(X_test, Y_test, intervals(:,1), 'b')
scatter3(X_test, Y_test, intervals(:,2), 'b')
hold off
title('GPR Fit of Noise-Free Observations')
legend({'Noise-free observations','GPR prediction points', 'GPR prediction mesh','95% Prediction Lower Bound Mesh','95% Prediction Upper Bound Mesh',
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')
zlabel('Sensor Reading','Interpreter','latex')
```





ReTrain Model with Request 1 and 2, and Plot Hyperparam Optimization

```
tbl = readtable('init_data_with_request_1and2.txt','FileType','text','ReadVariableNames',true);
gprMdl1 = fitrgp(tbl,'Sensor','KernelFunction','squaredexponential');
gprMdl2 = fitrgp(tbl,'Sensor',...
      'FitMethod','sr','PredictMethod','fic','ActiveSetMethod','entropy',...
      'OptimizeHyperparameters','all','HyperparameterOptimizationOptions',...
      struct('MaxObjectiveEvaluations',180,'UseParallel',true));
ypred = resubPredict(gprMdl2);
train_Loss = resubLoss(gprMdl2)

ypred1 = resubPredict(gprMdl1);
ypred2 = resubPredict(gprMdl2);

figure
plot(tbl.Sensor,'r.','MarkerSize',10);
hold on
plot(ypred1,'b');
plot(ypred2,'k','LineWidth',1);
xlabel('Input Pair','Interpreter', 'latex');
ylabel('Output Reading','Interpreter', 'latex');
legend({'data','Initial Fit','Optimized Fit'},'Location','Best','Interpreter', 'latex');
title('Impact of Optimization','Interpreter', 'latex');
hold off

figure
scatter(tbl.X,tbl.Y,10,tbl.Sensor)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')
```

```
Copying objective function to workers...
Done copying objective function to workers.
```

Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
1	6	Best	0.21959	0.1891	0.21959	0.21959	0.00020627	constant	exponential	0.015186	
2	2	Accept	0.062507	0.30343	0.05836	0.090416	0.07435	pureQuadrati	ardsquaredex	-	
3	2	Accept	2.3118	0.21456	0.05836	0.090416	1.0433	none	squaredexpon	0.10123	
4	2	Best	0.05836	0.28965	0.05836	0.090416	0.34307	pureQuadrati	ardmatern32	-	
5	2	Accept	0.067707	0.31102	0.05836	0.090416	0.11851	none	ardexponenti	-	
6	2	Accept	0.10028	0.24587	0.05836	0.090416	0.0004829	linear	squaredexpon	0.010855	
7	6	Accept	0.2029	0.13297	0.05836	0.087039	0.00087812	constant	matern52	0.015665	
8	2	Accept	0.22407	0.14242	0.05836	0.058407	0.00062942	constant	exponential	0.012406	
9	2	Accept	2.5706	0.22703	0.05836	0.058407	0.022051	none	matern32	0.022052	
10	2	Best	0.05836	0.20687	0.05836	0.058407	0.34312	pureQuadrati	ardmatern52	-	
11	2	Accept	0.05836	0.1731	0.05836	0.058407	2.1935	pureQuadrati	squaredexpon	0.087134	
12	2	Accept	0.10041	0.22456	0.05836	0.058407	0.44463	linear	ardexponenti	-	
13	6	Accept	0.20225	0.10502	0.05836	0.058404	0.0023425	constant	matern52	0.016105	
14	2	Accept	0.07753	0.2575	0.05836	0.058409	0.00010252	linear	ardmatern32	-	
15	2	Accept	2.4706	0.14856	0.05836	0.058409	0.00046594	none	exponential	0.03954	
16	2	Accept	0.61383	0.21694	0.05836	0.058409	0.0001025	none	ardsquaredex	-	
17	2	Accept	0.063255	0.27308	0.05836	0.058409	0.15983	none	ardrationalq	-	
18	2	Accept	0.05836	0.17057	0.05836	0.058409	1.57	pureQuadrati	exponential	0.33151	
19	6	Accept	0.065459	0.18303	0.05836	0.058406	0.00010693	pureQuadrati	ardexponenti	-	
20	2	Accept	0.05836	0.16414	0.05836	0.058397	0.75826	pureQuadrati	exponential	0.3308	

Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
21	2	Accept	0.22255	0.15828	0.05836	0.058397	0.45045	constant	matern52	0.045217	
22	2	Accept	0.10225	0.22872	0.05836	0.058397	0.30733	linear	ardmatern32	-	
23	2	Accept	0.094926	0.14895	0.05836	0.058397	0.0038734	linear	exponential	0.14609	
24	2	Accept	0.080901	0.2043	0.05836	0.058397	0.014115	linear	ardsquaredex	-	
25	6	Accept	0.10041	0.17871	0.05836	0.058395	1.2705	linear	ardmatern52	-	
26	2	Accept	0.081834	0.24099	0.05836	0.058392	0.0081741	linear	ardsquaredex	-	
27	2	Accept	1.3916	0.15271	0.05836	0.058392	0.045012	none	squaredexpon	0.39988	
28	2	Accept	0.07753	0.25362	0.05836	0.058392	0.00010167	linear	ardmatern32	-	
29	2	Accept	0.10041	0.23582	0.05836	0.058392	0.88156	linear	ardrationalq	-	
30	2	Accept	0.10009	0.20191	0.05836	0.058392	0.00012407	linear	matern32	0.02257	
31	6	Accept	0.10041	0.20154	0.05836	0.058391	0.41682	linear	ardrationalq	-	
32	2	Accept	0.05836	0.16588	0.055101	0.058244	0.50585	pureQuadrati	exponential	0.015325	
33	2	Accept	2.676	0.19121	0.055101	0.058244	0.0052923	none	exponential	0.0022871	
34	2	Accept	0.10041	0.1553	0.055101	0.058244	0.020633	linear	matern52	0.0037821	
35	2	Best	0.055101	0.26474	0.055101	0.058244	0.014389	none	ardrationalq	-	
36	2	Accept	0.37698	0.22374	0.055101	0.058244	0.020838	none	exponential	1.2773	
37	6	Accept	0.058361	0.15678	0.055101	0.058226	0.0069415	pureQuadrati	exponential	0.015532	
38	2	Best	0.053161	0.20636	0.053161	0.053198	0.013053	constant	matern52	1.7555	
39	2	Accept	0.23525	0.16957	0.053161	0.053198	0.00017282	constant	exponential	0.0038319	
40	2	Accept	0.10041	0.22374	0.053161	0.053198	1.985	linear	ardmatern52	-	

Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
41	2	Accept	0.23673	0.2417	0.053161	0.053198	1.3175	constant	rationalquad	0.0074409	
42	2	Accept	0.23673	0.25696	0.053161	0.053198	1.0706	constant	ardmatern32	-	
43	6	Accept	0.10041	0.1471	0.053161	0.053198	0.0039536	linear	exponential	0.003351	
44	2	Accept	0.05836	0.24272	0.053161	0.053206	4.897	pureQuadrati	squaredexpon	0.0024671	
45	2	Accept	0.079338	0.2647	0.053161	0.053206	0.0007523	linear	ardmatern52	-	
46	2	Accept	0.1618	0.28794	0.053161	0.053206	0.56053	constant	ardrationalq	-	
47	2	Accept	2.6723	0.22825	0.053161	0.053206	0.015532	none	exponential	0.0052996	
48	2	Accept	0.079338	0.23306	0.053161	0.053206	0.00092592	linear	ardmatern52	-	
49	6	Accept	0.10041	0.14434	0.053161	0.053207	0.67961	linear	matern52	0.11923	
50	2	Accept	0.054753	0.24844	0.053161	0.053198	0.0040384	linear	matern32	1.7787	
51	2	Accept	0.077286	0.27951	0.053161	0.053198	0.012991	linear	ardmatern52	-	
52	2	Accept	0.11742	0.29026	0.053161	0.053198	0.445	none	ardsquaredex	-	
53	2	Accept	1.6045	0.2358	0.053161	0.053198	0.043463	none	exponential	0.33296	
54	2	Accept	0.10041	0.28848	0.053161	0.053198	4.2532	linear	ardmatern52	-	
55	6	Accept	0.05836	0.16153	0.053161	0.053198	0.15452	pureQuadrati	squaredexpon	1.8708	
56	2	Accept	0.055776	0.2973	0.053161	0.053198	0.0045248	linear	matern52	1.882	
57	2	Accept	0.23673	0.20281	0.053161	0.053198	3.7783	constant	matern32	0.29779	
58	2	Accept	0.082951	0.29243	0.053161	0.053198	0.0060502	linear	ardsquaredex	-	
59	2	Accept	0.1924	0.22332	0.053161	0.053198	0.0025921	constant	exponential	0.032333	
60	2	Accept	0.093722	0.1642	0.053161	0.053198	0.015213	constant	exponential	0.17709	

Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
61	6	Accept	0.069886	0.14677	0.053161	0.053198	0.0011104	linear	exponential	1.8808	
62	2	Accept	0.10041	0.21561	0.053161	0.053206	0.50621	linear	squaredexpon	1.5991	
63	2	Accept	0.054787	0.18387	0.053161	0.053206	0.017245	linear	matern32	1.2952	
64	2	Accept	2.6279	0.25178	0.053161	0.053206	0.00088377	none	exponential	0.0092555	
65	2	Accept	0.23673	0.2381	0.053161	0.053206	0.91246	constant	rationalquad	1.7029	
66	2	Accept	0.053605	0.3142	0.053161	0.053206	0.046937	none	ardrationalq	-	
67	6	Accept	0.1211	0.15219	0.053161	0.053206	0.45147	constant	exponential	1.8821	
68	2	Accept	2.0772	0.23059	0.051038	0.051557	0.024834	none	matern52	0.11714	
69	2	Accept	0.23673	0.27713	0.051038	0.051557	1.1529	constant	ardrationalq	-	
70	2	Best	0.051038	0.3542	0.051038	0.051557	0.00018075	none	ardrationalq	-	
71	2	Accept	0.077915	0.35518	0.051038	0.051557	0.019876	linear	ardmatern52	-	
72	2	Accept	0.091212	0.19578	0.051038	0.051557	0.0093339	linear	matern32	0.20474	
73	6	Accept	0.063221	0.13692	0.051038	0.051553	0.20398	constant	matern32	0.39519	

74	2	Accept	0.10041	0.16797	0.051038	0.051525	0.0062181	linear	matern32	0.0018867
75	2	Accept	2.5623	0.18973	0.051038	0.051525	0.33676	none	squaredexpon	0.013756
76	2	Accept	0.10041	0.22166	0.051038	0.051525	0.94553	linear	matern32	0.064821
77	2	Accept	0.099783	0.27363	0.051038	0.051525	0.16465	linear	ardsquaredex	-
78	2	Accept	0.10007	0.19807	0.051038	0.051525	0.00083924	linear	matern32	0.012465
79	6	Accept	0.2313	0.13377	0.051038	0.05152	0.0026713	constant	matern32	0.0060885
80	2	Accept	0.058712	0.18713	0.051038	0.051589	0.0033709	pureQuadrati	matern32	0.039621

Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFuncti-on	KernelScale	Stanc
81	2	Accept	0.61383	0.26694	0.051038	0.051589	0.003268	none	ardsquaredex	-
82	2	Accept	2.3134	0.18332	0.051038	0.051589	0.0083056	none	exponential	0.071868
83	2	Accept	0.099422	0.21052	0.051038	0.051589	0.00050467	linear	exponential	0.059227
84	2	Accept	1.5027	0.19393	0.051038	0.051589	0.80525	none	exponential	0.38119
85	6	Accept	0.05836	0.17829	0.051038	0.051584	5.0641	pureQuadrati	matern32	0.10501
86	2	Accept	0.05836	0.21161	0.051038	0.051396	0.89675	pureQuadrati	matern32	1.8695
87	2	Accept	0.084444	0.35624	0.051038	0.051396	0.18894	none	ardrationalq	-
88	2	Accept	0.078168	0.28176	0.051038	0.051396	0.025039	linear	ardmatern32	-
89	2	Accept	0.23673	0.23909	0.051038	0.051396	0.81569	constant	matern32	0.0092908
90	2	Accept	0.10041	0.28103	0.051038	0.051396	1.0702	linear	ardmatern52	-
91	6	Accept	0.05836	0.23317	0.051038	0.051395	0.0014577	pureQuadrati	matern32	0.001885
92	2	Accept	0.05955	0.39486	0.051038	0.051586	0.00010015	pureQuadrati	ardsquaredex	-
93	2	Accept	0.091005	0.32217	0.051038	0.051586	0.0042365	linear	ardexponenti	-
94	2	Accept	0.077143	0.22273	0.051038	0.051586	0.029976	constant	matern32	0.20552
95	2	Accept	0.23679	0.55952	0.051038	0.051586	0.97584	none	rationalquad	0.0029888
96	2	Accept	0.10041	0.24222	0.051038	0.051586	0.0019754	linear	matern32	0.0032456
97	6	Accept	0.058952	0.14579	0.051038	0.051586	0.0028771	pureQuadrati	matern52	0.032712
98	2	Accept	0.066002	0.43984	0.051038	0.051362	0.00010591	pureQuadrati	ardrationalq	-
99	2	Accept	0.08145	0.26437	0.051038	0.051362	0.023214	linear	ardsquaredex	-
100	2	Accept	0.33493	0.19782	0.051038	0.051362	0.0022248	none	matern52	0.76285

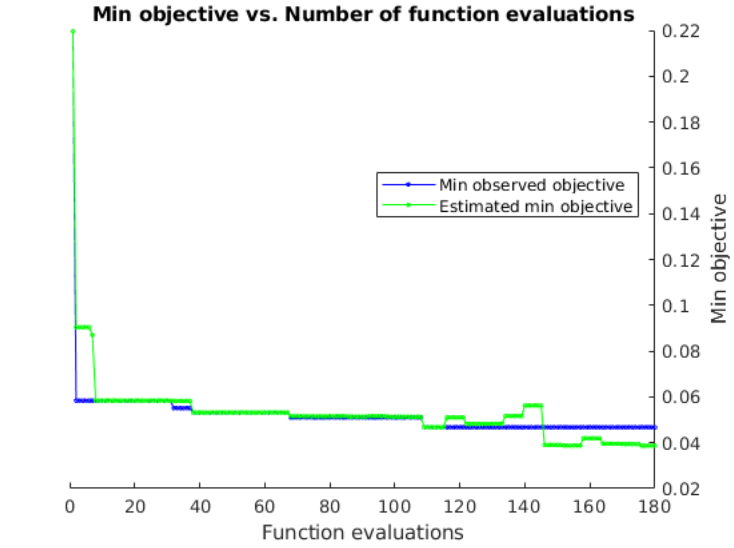
Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFuncti-on	KernelScale	Stanc
101	2	Accept	0.083709	0.37269	0.051038	0.051362	0.0010502	linear	ardrationalq	-
102	2	Accept	0.10041	0.34216	0.051038	0.051362	0.42525	linear	ardmatern52	-
103	6	Accept	0.063848	0.16616	0.051038	0.051336	0.00014375	pureQuadrati	matern52	1.8821
104	2	Accept	0.060234	0.27263	0.051038	0.051301	0.015214	pureQuadrati	matern52	0.099824
105	2	Accept	0.23673	0.25131	0.051038	0.051301	2.6105	constant	matern32	0.0045848
106	2	Accept	0.06041	0.18177	0.051038	0.051301	0.00014453	linear	matern52	0.76172
107	2	Accept	0.067549	0.3029	0.051038	0.051301	0.0012097	constant	ardsquaredex	-
108	2	Accept	0.10041	0.33279	0.051038	0.051301	1.2837	linear	ardexponenti	-
109	6	Best	0.046752	0.14473	0.046752	0.046778	0.044933	constant	matern52	0.81753
110	2	Accept	0.10041	0.17137	0.046752	0.046787	0.00027763	linear	matern52	0.0026518
111	2	Accept	0.10041	0.28775	0.046752	0.046787	1.8267	linear	ardmatern32	-
112	2	Accept	0.083709	0.35356	0.046752	0.046787	0.00024282	linear	ardsquaredex	-
113	2	Accept	0.10666	0.34837	0.046752	0.046787	0.26063	linear	ardsquaredex	-
114	2	Accept	0.098677	0.25153	0.046752	0.046787	0.088576	linear	rationalquad	0.024773
115	6	Accept	0.23673	0.13154	0.046752	0.046787	0.042621	constant	matern52	0.0035329
116	2	Accept	0.05836	0.29025	0.046752	0.051056	0.89424	pureQuadrati	rationalquad	0.055963
117	2	Accept	0.064073	0.43423	0.046752	0.051056	0.031719	constant	ardrationalq	-
118	2	Accept	0.083908	0.31573	0.046752	0.051056	0.06155	linear	ardmatern52	-
119	2	Accept	0.066227	0.43286	0.046752	0.051056	0.011427	constant	ardrationalq	-
120	2	Accept	0.06872	0.39677	0.046752	0.051056	0.0011492	constant	ardrationalq	-

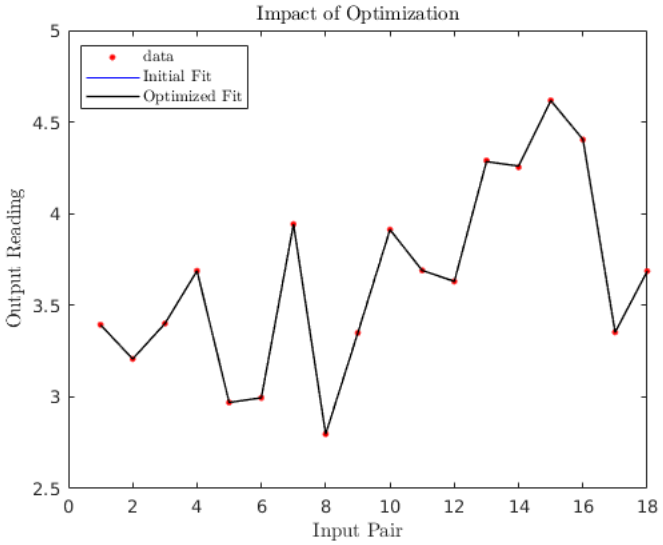
Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFuncti-on	KernelScale	Stanc
121	6	Accept	0.091787	0.20691	0.046752	0.051055	0.015053	linear	rationalquad	0.095331
122	2	Accept	0.05836	0.24709	0.046752	0.048204	1.1281	pureQuadrati	rationalquad	0.11883
123	2	Accept	0.060501	0.28781	0.046752	0.048204	0.036316	pureQuadrati	ardsquaredex	-
124	2	Accept	0.047818	0.28141	0.046752	0.048204	0.009954	constant	ardmatern32	-
125	2	Accept	0.095477	0.3024	0.046752	0.048204	0.00035334	linear	rationalquad	0.054767
126	2	Accept	0.082195	0.40119	0.046752	0.048204	0.032896	linear	ardrationalq	-
127	6	Accept	0.05836	0.2432	0.046752	0.048209	0.00042623	pureQuadrati	matern52	0.0018901
128	2	Accept	0.058804	0.32908	0.046752	0.048271	0.00011061	pureQuadrati	rationalquad	0.0019032
129	2	Accept	0.062363	0.31089	0.046752	0.048271	0.0079603	pureQuadrati	ardmatern52	-
130	2	Accept	2.6489	0.27064	0.046752	0.048271	0.0005202	none	exponential	0.0068079
131	2	Accept	0.10041	0.23585	0.046752	0.048271	1.3198	linear	matern52	0.018073
132	2	Accept	0.16028	0.27459	0.046752	0.048271	0.49005	none	rationalquad	0.24182
133	6	Accept	0.05836	0.16485	0.046752	0.048281	2.5056	pureQuadrati	squaredexpon	0.066294
134	2	Accept	0.066002	0.45711	0.046752	0.051661	0.00010153	pureQuadrati	ardrationalq	-
135	2	Accept	0.06872	0.41309	0.046752	0.051661	0.00021551	constant	ardrationalq	-
136	2	Accept	0.10022	0.16644	0.046752	0.051661	0.0081912	linear	matern32	0.012305
137	2	Accept	0.066002	0.44589	0.046752	0.051661	0.0011174	pureQuadrati	ardrationalq	-
138	2	Accept	0.065459	0.36898	0.046752	0.051661	0.0022143	pureQuadrati	ardexponenti	-
139	6	Accept	0.05836	0.19574	0.046752	0.051662	1.7371	pureQuadrati	squaredexpon	0.0018967
140	2	Accept	0.063349	0.38495	0.046752	0.056251	0.00010156	pureQuadrati	ardmatern32	-

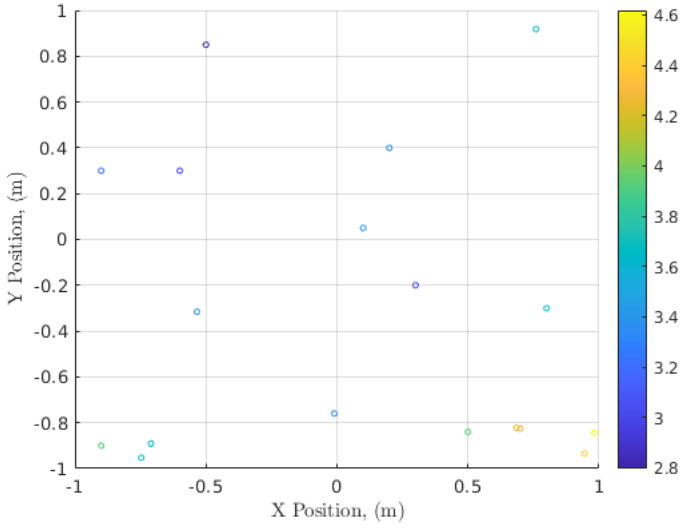
Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFuncti-on	KernelScale	Stanc
141	2	Accept	0.053861	0.3779	0.046752	0.056251	0.0002964	pureQuadrati	rationalquad	1.6792
142	2	Accept	0.051038	0.56382	0.046752	0.056251	0.0015774	none	ardrationalq	-
143	2	Accept	0.059755	0.40466	0.046752	0.056251	0.10254	none	ardrationalq	-
144	2	Accept	2.5533	0.19279	0.046752	0.056251	0.14231	none	squaredexpon	0.015096
145	6	Accept	0.062098	0.23564	0.046752	0.056251	0.00011035	pureQuadrati	ardmatern52	-
146	2	Accept	0.091005	0.34299	0.046752	0.03904	0.00010082	linear	ardexponenti	-
147	2	Accept	0.066099	0.4152	0.046752	0.03904	0.012236	constant	ardrationalq	-
148	2	Accept	0.098902	0.32922	0.046752	0.03904	0.15927	linear	ardsquaredex	-

Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Standardize
149	2	Accept	0.061431	0.25362	0.046752	0.03904	0.00098622	constant	matern32	0.44216	
150	2	Accept	0.080138	0.29887	0.046752	0.03904	0.038783	linear	ardmatern32	-	
151	6	Accept	0.064763	0.2063	0.046752	0.039046	0.00010374	none	ardmatern32	-	
152	2	Accept	0.064763	0.27985	0.046752	0.038774	0.00010091	none	ardmatern32	-	
153	2	Accept	0.05836	0.2813	0.046752	0.038774	0.20446	pureQuadrati	ardsquaredex	-	
154	2	Accept	2.6731	0.19505	0.046752	0.038774	3.2815	none	matern32	0.0058923	
155	2	Accept	0.05836	0.2452	0.046752	0.038774	0.66127	pureQuadrati	exponential	0.0124	
156	2	Accept	2.5217	0.2046	0.046752	0.038774	0.00034425	none	matern52	0.038915	
157	6	Accept	0.087523	0.29175	0.046752	0.03878	0.00010262	none	ardmatern52	-	
158	2	Accept	0.065138	0.4049	0.046752	0.041972	0.00010979	none	ardexponenti	-	
159	2	Accept	0.05836	0.28429	0.046752	0.041972	4.0269	pureQuadrati	ardsquaredex	-	
160	2	Accept	0.064763	0.28894	0.046752	0.041972	0.00062963	none	ardmatern32	-	
161	2	Accept	0.052338	0.37869	0.046752	0.041972	0.12359	constant	ardrationalq	-	
162	2	Accept	0.10041	0.2495	0.046752	0.041972	0.62577	linear	matern52	0.18512	
163	6	Accept	0.087523	0.33827	0.046752	0.041976	0.00010676	none	ardmatern52	-	
164	2	Accept	0.04763	0.30977	0.046752	0.039541	0.00010349	constant	ardmatern32	-	
165	2	Accept	2.0426	0.30029	0.046752	0.039541	0.00022798	none	matern52	0.12549	
166	2	Accept	0.051374	0.37858	0.046752	0.039541	0.00021441	constant	ardmatern52	-	
167	2	Accept	0.067267	0.54391	0.046752	0.039541	0.0052561	constant	ardrationalq	-	
168	2	Accept	0.10041	0.31453	0.046752	0.039541	3.1894	linear	exponential	0.0032596	
169	6	Accept	0.056529	0.18429	0.046752	0.039546	0.05771	constant	rationalquad	0.39925	
170	2	Accept	0.051374	0.29935	0.046752	0.039405	0.00010514	constant	ardmatern52	-	
171	2	Accept	0.10041	0.34986	0.046752	0.039405	2.1268	linear	ardmatern52	-	
172	2	Accept	0.061776	0.35399	0.046752	0.039405	0.00061903	pureQuadrati	rationalquad	0.15088	
173	2	Accept	0.05836	0.35636	0.046752	0.039405	2.2253	pureQuadrati	ardrationalq	-	
174	2	Accept	0.059114	0.26795	0.046752	0.039405	0.047843	pureQuadrati	matern32	0.041557	
175	6	Accept	0.14309	0.18269	0.046752	0.039404	0.088078	constant	rationalquad	0.0018853	
176	2	Accept	0.23673	0.28878	0.046752	0.038708	5.1277	constant	ardexponenti	-	
177	2	Accept	0.076312	0.31681	0.046752	0.038708	0.0056024	none	rationalquad	0.12686	
178	2	Accept	0.2379	0.37331	0.046752	0.038708	1.6217	none	ardmatern32	-	
179	2	Accept	0.087523	0.44061	0.046752	0.038708	0.0041404	none	ardmatern52	-	
180	2	Accept	0.05836	0.2602	0.046752	0.038708	5.0985	pureQuadrati	exponential	0.91779	

```
Optimization completed.
MaxObjectiveEvaluations of 180 reached.
Total function evaluations: 180
Total elapsed time: 97.5248 seconds
Total objective function evaluation time: 47.0765

Best observed feasible point:
     Sigma      BasisFunction    KernelFunction    KernelScale    Standardize
    ________    _____________    ______________    ___________    ___________

    0.044933      constant          matern52         0.81753         true

Observed objective function value = 0.046752
Estimated objective function value = 0.046949
Function evaluation time = 0.14473

Best estimated feasible point (according to models):
      Sigma      BasisFunction       KernelFunction       KernelScale    Standardize
    _________    _____________    ____________________    ___________    ___________

    0.0011492      constant       ardrationalquadratic        NaN           true

Estimated objective function value = 0.038708
Estimated function evaluation time = 0.39813


train_Loss =

   4.5724e-06
```



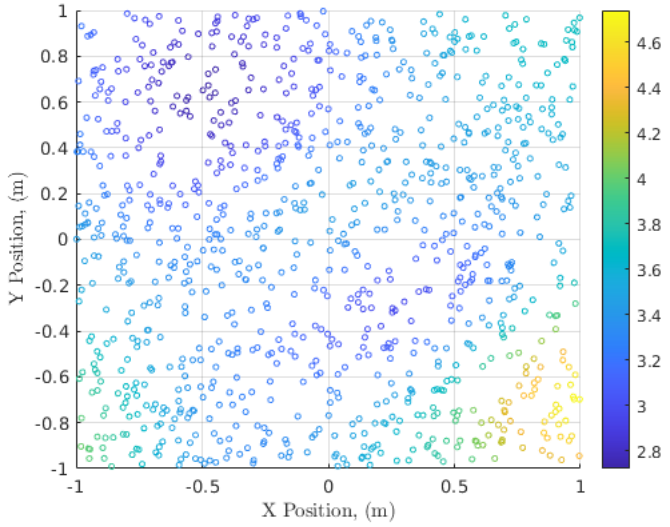


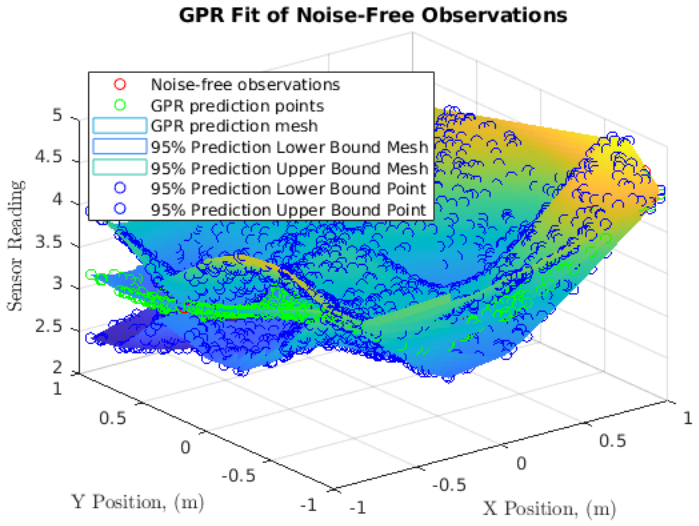


Test Model and Plot with data from Request 1 and 2

```
X_test = 2*rand(1000,1)-1;
Y_test = 2*rand(1000,1)-1;
[sensor_pred, ~, intervals] = predict(gprMdl2,[X_test Y_test]);


[X_grid,Y_grid] = meshgrid(linspace(-1,1,1000),linspace(-1,1,1000)) ;
```

```
lower_interval_grid = griddata(X_test,Y_test,intervals(:,1),X_grid,Y_grid) ;
upper_interval_grid = griddata(X_test,Y_test,intervals(:,2),X_grid,Y_grid) ;
prediction_grid = griddata(X_test,Y_test,sensor_pred,X_grid,Y_grid) ;



figure
scatter(X_test,Y_test,10,sensor_pred)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')


figure
scatter3(tbl.X,tbl.Y,tbl.Sensor,'r') % Observed data points
hold on
scatter3(X_test, Y_test, sensor_pred,'g') % GPR predictions
mesh(X_grid,Y_grid,prediction_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,lower_interval_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,upper_interval_grid,'FaceAlpha','0.5')
scatter3(X_test, Y_test, intervals(:,1), 'b')
scatter3(X_test, Y_test, intervals(:,2), 'b')
hold off
title('GPR Fit of Noise-Free Observations')
legend({'Noise-free observations','GPR prediction points', 'GPR prediction mesh','95% Prediction Lower Bound Mesh','95% Prediction Upper Bound Mesh',
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')
zlabel('Sensor Reading','Interpreter','latex')
```





ReTrain Model with Request 1 and Plot Hyperparam Optimization

```
tbl = readtable('init_data_with_request_1and2and3.txt','FileType','text','ReadVariableNames',true);
gprMdl1 = fitrgp(tbl,'Sensor','KernelFunction','squaredexponential');
```

```
gprMdl2 = fitrgp(tbl,'Sensor',...
      'FitMethod','sr','PredictMethod','fic','ActiveSetMethod','entropy',...
      'OptimizeHyperparameters','all','HyperparameterOptimizationOptions',...
      struct('MaxObjectiveEvaluations',60,'UseParallel',true));
ypred = resubPredict(gprMdl2);
train_Loss = resubLoss(gprMdl2)

ypred1 = resubPredict(gprMdl1);
ypred2 = resubPredict(gprMdl2);

figure
plot(tbl.Sensor,'r.','MarkerSize',10);
hold on
plot(ypred1,'b');
plot(ypred2,'k','LineWidth',1);
xlabel('Input Pair','Interpreter', 'latex');
ylabel('Output Reading','Interpreter', 'latex');
legend({'data','Initial Fit','Optimized Fit'},'Location','Best','Interpreter', 'latex');
title('Impact of Optimization','Interpreter', 'latex');
hold off

figure
scatter(tbl.X,tbl.Y,10,tbl.Sensor)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')
```

```
Copying objective function to workers...
Done copying objective function to workers.
```

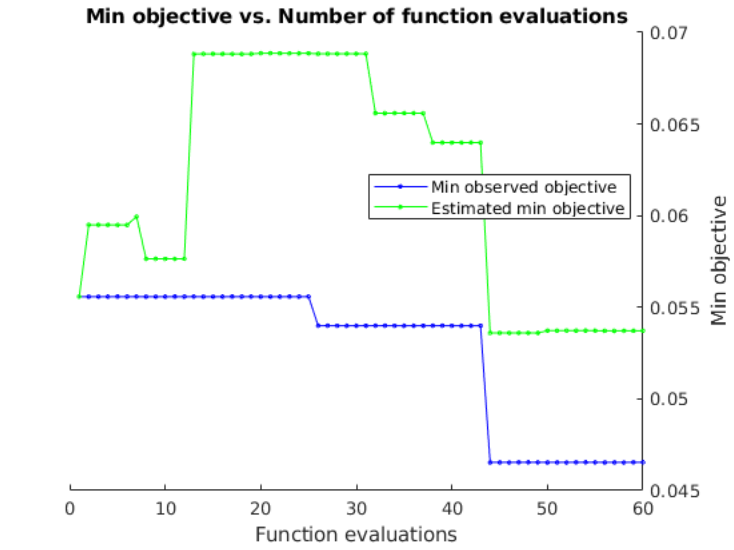
Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
1	6	Best	0.055582	0.24568	0.055582	0.055582	0.0001078	linear	matern32	0.57602	
2	2	Accept	0.067064	0.34248	0.055582	0.059495	0.27744	none	ardexponenti	-	
3	2	Accept	0.090437	0.35189	0.055582	0.059495	0.023839	pureQuadrati	ardexponenti	-	
4	2	Accept	0.096656	0.31563	0.055582	0.059495	0.3965	linear	ardsquaredex	-	
5	2	Accept	0.078318	0.35283	0.055582	0.059495	0.026292	pureQuadrati	ardmatern32	-	
6	2	Accept	0.41996	0.46302	0.055582	0.059495	0.0010428	none	ardsquaredex	-	
7	6	Accept	0.21487	0.17454	0.055582	0.059945	0.00011255	constant	matern32	0.0062545	
8	2	Accept	0.059229	0.21124	0.055582	0.057642	0.019669	linear	matern32	0.89288	
9	2	Accept	0.10624	0.22744	0.055582	0.057642	0.16629	pureQuadrati	ardsquaredex	-	
10	2	Accept	2.5811	0.17073	0.055582	0.057642	0.012576	none	squaredexpon	0.018277	
11	2	Accept	0.36337	0.26836	0.055582	0.057642	0.012362	none	ardsquaredex	-	
12	2	Accept	0.1123	0.21456	0.055582	0.057642	0.47308	constant	ardmatern52	-	
13	6	Accept	0.091192	0.15449	0.055582	0.068818	0.00014389	linear	matern32	0.026496	
14	2	Accept	0.069172	0.28113	0.055582	0.068826	0.00010141	pureQuadrati	ardsquaredex	-	
15	2	Accept	0.069083	0.36515	0.055582	0.068826	0.006751	linear	ardrationalq	-	
16	2	Accept	0.07486	0.38175	0.055582	0.068826	0.015808	constant	ardrationalq	-	
17	2	Accept	1.7744	0.17583	0.055582	0.068826	0.00020332	none	matern52	0.27914	
18	2	Accept	2.6379	0.25741	0.055582	0.068826	0.013809	none	exponential	0.017708	
19	6	Accept	0.089878	0.18367	0.055582	0.068824	0.00010515	pureQuadrati	ardexponenti	-	
20	2	Accept	0.096656	0.21836	0.055582	0.06886	4.6199	linear	ardsquaredex	-	

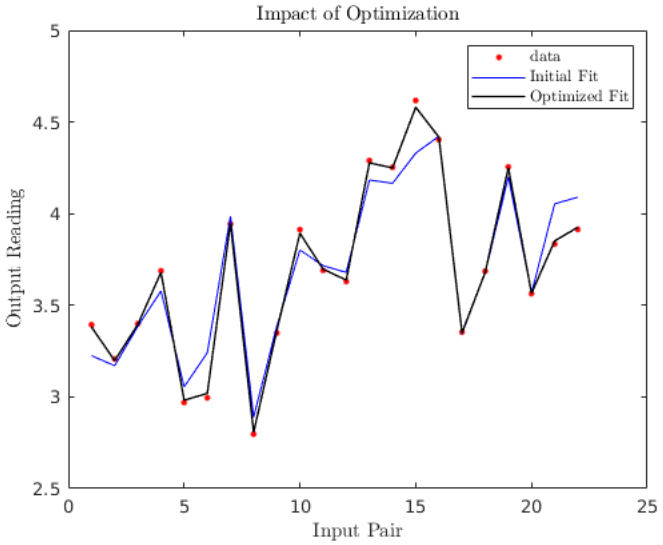
Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
21	2	Accept	0.096656	0.2695	0.055582	0.06886	0.46982	linear	ardrationalq	-	
22	2	Accept	0.083188	0.33189	0.055582	0.06886	0.001278	linear	rationalquad	0.056268	
23	2	Accept	2.6964	0.35494	0.055582	0.06886	0.0063949	none	exponential	0.0036229	
24	2	Accept	0.067961	0.33773	0.055582	0.06886	0.0011512	linear	ardmatern52	-	
25	6	Accept	0.070602	0.17214	0.055582	0.068859	0.00010226	linear	ardexponenti	-	
26	2	Accept	0.09374	0.15537	0.053993	0.068835	0.0064382	linear	exponential	0.016601	
27	2	Accept	0.081019	0.23447	0.053993	0.068835	0.31758	constant	ardmatern52	-	
28	2	Best	0.053993	0.15263	0.053993	0.068835	0.12266	linear	squaredexpon	1.4211	
29	2	Accept	0.093015	0.2492	0.053993	0.068835	0.24639	linear	ardmatern52	-	
30	2	Accept	0.36543	0.21624	0.053993	0.068835	0.01173	none	ardsquaredex	-	
31	6	Accept	0.0676	0.25544	0.053993	0.068834	0.00010219	linear	ardmatern32	-	
32	2	Accept	0.1034	0.22899	0.053993	0.065594	0.021622	pureQuadrati	exponential	0.013444	
33	2	Accept	0.096656	0.20713	0.053993	0.065594	1.0737	linear	squaredexpon	0.0071485	
34	2	Accept	0.10928	0.22516	0.053993	0.065594	3.4723	pureQuadrati	exponential	0.011564	
35	2	Accept	0.069131	0.36675	0.053993	0.065594	0.00048407	linear	ardrationalq	-	
36	2	Accept	0.096656	0.23764	0.053993	0.065594	1.09	linear	ardmatern52	-	
37	6	Accept	0.067961	0.21322	0.053993	0.065594	0.00010246	linear	ardmatern52	-	
38	2	Accept	0.093459	0.17737	0.053993	0.063985	0.00023269	pureQuadrati	matern32	1.8732	
39	2	Accept	0.10928	0.2295	0.053993	0.063985	0.23802	pureQuadrati	rationalquad	0.051974	
40	2	Accept	0.068058	0.17275	0.053993	0.063985	0.00087149	linear	matern32	0.18264	

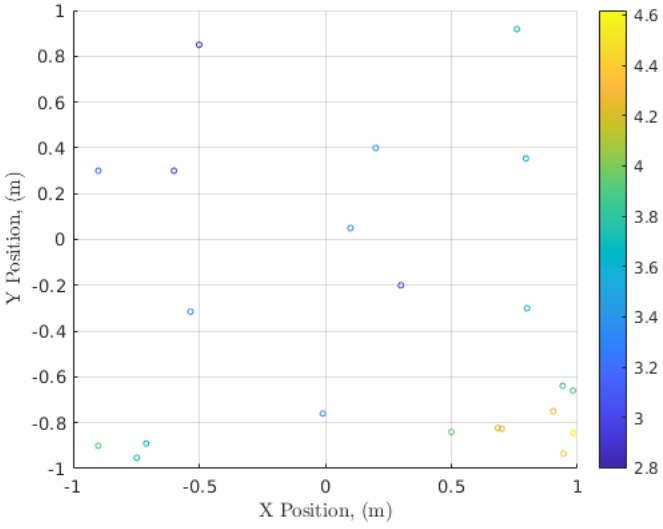
Iter	Active workers	Eval result	Objective: log(1+loss)	Objective runtime	BestSoFar (observed)	BestSoFar (estim.)	Sigma	BasisFunction	KernelFunction	KernelScale	Stand
41	2	Accept	0.095273	0.22423	0.053993	0.063985	0.00058065	linear	rationalquad	0.0024584	
42	2	Accept	0.10804	0.23292	0.053993	0.063985	0.025447	none	ardmatern52	-	
43	6	Accept	0.11054	0.19567	0.053993	0.063985	0.00088369	none	ardmatern52	-	
44	2	Accept	0.40234	0.1803	0.046537	0.053601	0.0004909	linear	matern52	1.8251	
45	2	Accept	0.11054	0.29358	0.046537	0.053601	0.00070502	none	ardmatern52	-	
46	2	Accept	0.096656	0.23239	0.046537	0.053601	4.1282	linear	exponential	0.2861	
47	2	Best	0.046537	0.2802	0.046537	0.053601	0.0021711	none	ardexponenti	-	

48	2	Accept	0.046985	0.31459	0.046537	0.053601	0.05653	none	ardexponenti	-
49	6	Accept	0.066559	0.20078	0.046537	0.053601	0.00011378	none	ardmatern32	-
50	2	Accept	0.070602	0.31783	0.046537	0.053726	0.0001015	linear	ardexponenti	-
51	2	Accept	0.0676	0.34403	0.046537	0.053726	0.00011455	linear	ardmatern32	-
52	2	Accept	0.082224	0.30859	0.046537	0.053726	0.0031382	linear	rationalquad	0.06305
53	2	Accept	0.50639	0.38107	0.046537	0.053726	0.036641	none	ardsquaredex	-
54	2	Accept	0.10928	0.31934	0.046537	0.053726	1.0403	pureQuadrati	rationalquad	0.65956
55	6	Accept	0.070051	0.25215	0.046537	0.053726	0.00024177	constant	ardmatern32	-
56	2	Accept	0.30274	0.38507	0.046537	0.053717	4.6108	none	ardmatern32	-
57	2	Accept	0.082015	0.25132	0.046537	0.053717	0.018431	pureQuadrati	matern52	0.14167
58	2	Accept	2.6816	0.27652	0.046537	0.053717	3.6178	none	squaredexpon	0.043604
59	2	Accept	0.057241	0.51364	0.046537	0.053717	0.0015289	none	ardrationalq	-
60	2	Accept	0.13355	0.24669	0.046537	0.053717	0.33455	constant	exponential	0.13933

```
__________________________________________________________
Optimization completed.
MaxObjectiveEvaluations of 60 reached.
Total function evaluations: 60
Total elapsed time: 20.6851 seconds
Total objective function evaluation time: 15.8951

Best observed feasible point:
      Sigma      BasisFunction    KernelFunction    KernelScale    Standardize
    _________    _____________    ______________    ___________    ___________

    0.0021711        none         ardexponential        NaN           true

Observed objective function value = 0.046537
Estimated objective function value = 0.053717
Function evaluation time = 0.2802

Best estimated feasible point (according to models):
     Sigma     BasisFunction    KernelFunction    KernelScale    Standardize
    _______    _____________    ______________    ___________    ___________

    0.05653        none         ardexponential        NaN           true

Estimated objective function value = 0.053717
Estimated function evaluation time = 0.28704


train_Loss =

   1.8690e-04
```



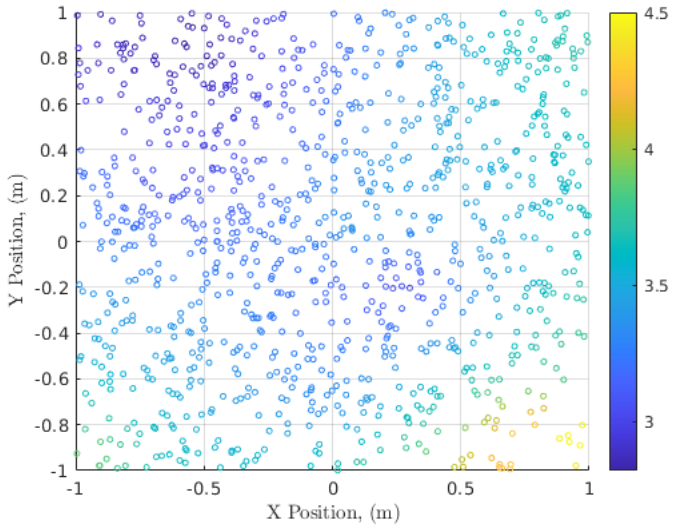


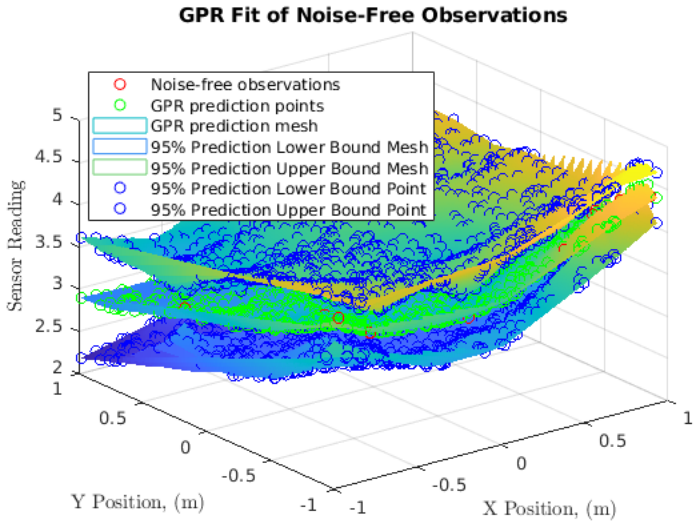


Test Model and Plot with data from Request 1,2 and 3

```
X_test = 2*rand(1000,1)-1;
Y_test = 2*rand(1000,1)-1;
[sensor_pred, ~, intervals] = predict(gprMdl2,[X_test Y_test]);


[X_grid,Y_grid] = meshgrid(linspace(-1,1,1000),linspace(-1,1,1000)) ;
```

```
lower_interval_grid = griddata(X_test,Y_test,intervals(:,1),X_grid,Y_grid) ;
upper_interval_grid = griddata(X_test,Y_test,intervals(:,2),X_grid,Y_grid) ;
prediction_grid = griddata(X_test,Y_test,sensor_pred,X_grid,Y_grid) ;



figure
scatter(X_test,Y_test,10,sensor_pred)
colormap(gca,'default')
colorbar
grid on
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')


figure
scatter3(tbl.X,tbl.Y,tbl.Sensor,'r') % Observed data points
hold on
scatter3(X_test, Y_test, sensor_pred,'g') % GPR predictions
mesh(X_grid,Y_grid,prediction_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,lower_interval_grid,'FaceAlpha','0.5')
mesh(X_grid,Y_grid,upper_interval_grid,'FaceAlpha','0.5')
scatter3(X_test, Y_test, intervals(:,1), 'b')
scatter3(X_test, Y_test, intervals(:,2), 'b')
hold off
title('GPR Fit of Noise-Free Observations')
legend({'Noise-free observations','GPR prediction points', 'GPR prediction mesh','95% Prediction Lower Bound Mesh','95% Prediction Upper Bound Mesh',
xlabel('X Position, (m)','Interpreter','latex')
ylabel('Y Position, (m)','Interpreter','latex')
zlabel('Sensor Reading','Interpreter','latex')
```





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