Project 1_2

October 20, 2021

0.0.1 2: Warmup - life without a CLT

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

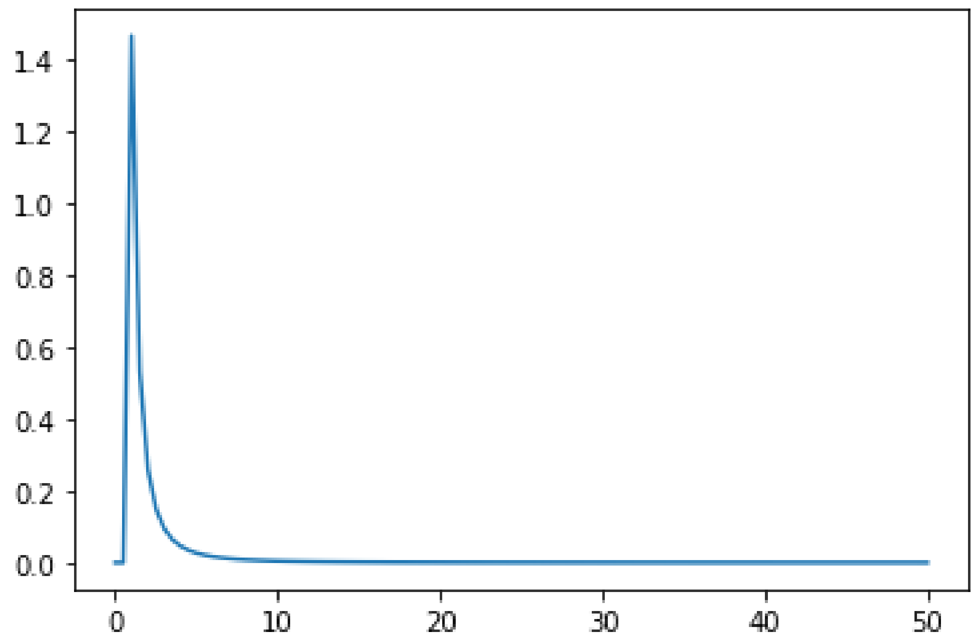
[2]:
```
def monte_carlo(alpha,num_samples, sample_generator, g_evaluator, cumsum=False):
    """Perform Monte Carlo sampling

    Inputs
    ------
    num_samples: integer, number of samples
    sample_generator: A function that generates samples with signature␣
↪sample_generator(nsamples)
    g_evaluator: a function that takes as inputs the samples and outputs the␣
↪evaluations.
                 The outputs can be any dimension, however the first dimension␣
↪should have size *num_samples*
    cumsum: Boolean, an option to return estimators of all sample sizes up to␣
↪num_samples

    Returns
    -------
    A Monte Carlo estimator of the mean, samples, and evaluations
    """
    samples = sample_generator(alpha,num_samples)
    evaluations = g_evaluator(samples)
    if cumsum is False:
        estimate =  np.sum(evaluations, axis=0) / float(num_samples)
    else:
        estimate = np.cumsum(evaluations, axis=0) / np.arange(1,num_samples+1,␣
↪dtype=np.float)

    return estimate, samples, evaluations
```

[ ]:
```

```

[3]:
```python
# Pareto PDF
def pareto_rv(alpha,xinput):
    p = [alpha/( x**(alpha+1) ) if x>=1 else 0 for x in xinput]
    return p
```

[4]:
```python
alpha = 3/2
x = np.linspace(0,50,100)
y = pareto_rv(alpha,x)
plt.plot(x,y)
```

[4]:
```
[<matplotlib.lines.Line2D at 0x7f4505b3a940>]
```



CDF of pareto is given by

$$CDF = \int_1^x \frac{\alpha}{x^{\alpha+1}} = 1 - \frac{1}{x^\alpha} \tag{1}$$

Now creating a pareto sampler with Inverse CDF sampler

[5]:
```python
np.random.seed(10)

def pareto_cdf(alpha,xinput):
    return 1 - 1/xinput**alpha

def pareto_sampler(alpha,num_samples):
    # Function inefficient since it generates sample_space_x_cdf every time it␣
 ↪is called
```

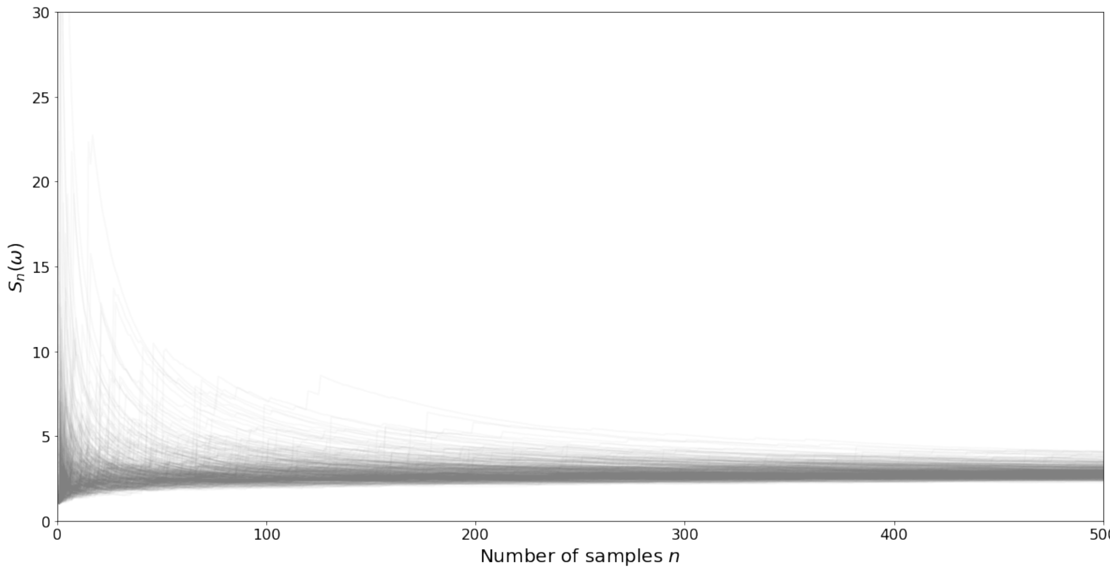
```
    # Following naming convention from lecture notes
    u = np.random.uniform(size = num_samples)
    sample_space_x = np.linspace(1,500,100000 )
    sample_space_x_cdf = pareto_cdf(alpha,sample_space_x)
    x_output = []
    for i in range(num_samples):
        index = np.argmax( sample_space_x_cdf > u[i] )
        x_output.append( sample_space_x[index] )
    return x_output
```

[6]:
```
# Generating Paths
alpha = 3/2
g = lambda x: x # identity function
num_trials = 500
num_samples = 10000
estimator_vals = np.zeros((num_trials, num_samples))
for trial in range(num_trials):
    estimator_vals[trial, :], _, _ = monte_carlo(alpha,num_samples,␣
 ↪pareto_sampler, g, cumsum=True)
```

0.0.2 Plotting

[9]:
```
print("estimates",np.shape(estimator_vals))
plt.figure(figsize=(20,10))
plt.plot(np.arange(1, num_samples+1), estimator_vals.T, color='grey', alpha=0.
 ↪05)
plt.ylabel(r'$S_n(\omega)$', fontsize=20)
plt.xlabel(r'Number of samples $n$', fontsize=20)
plt.xticks(fontsize=16)
plt.yticks(fontsize=16)
plt.xlim([0, 500])
plt.ylim([0, 30])
plt.show()
```

```
estimates (500, 10000)
```



0.0.3 Getting mean from all trials. actual mean = 3.0

[8]:
```
# print(estimator_vals[:,-1])
```

[7]:
```
print("Mean is ",np.mean(estimator_vals[:,-1]))
```

```
Mean is  2.8735793153731515
```

0.0.4 Evaluate convergence properties

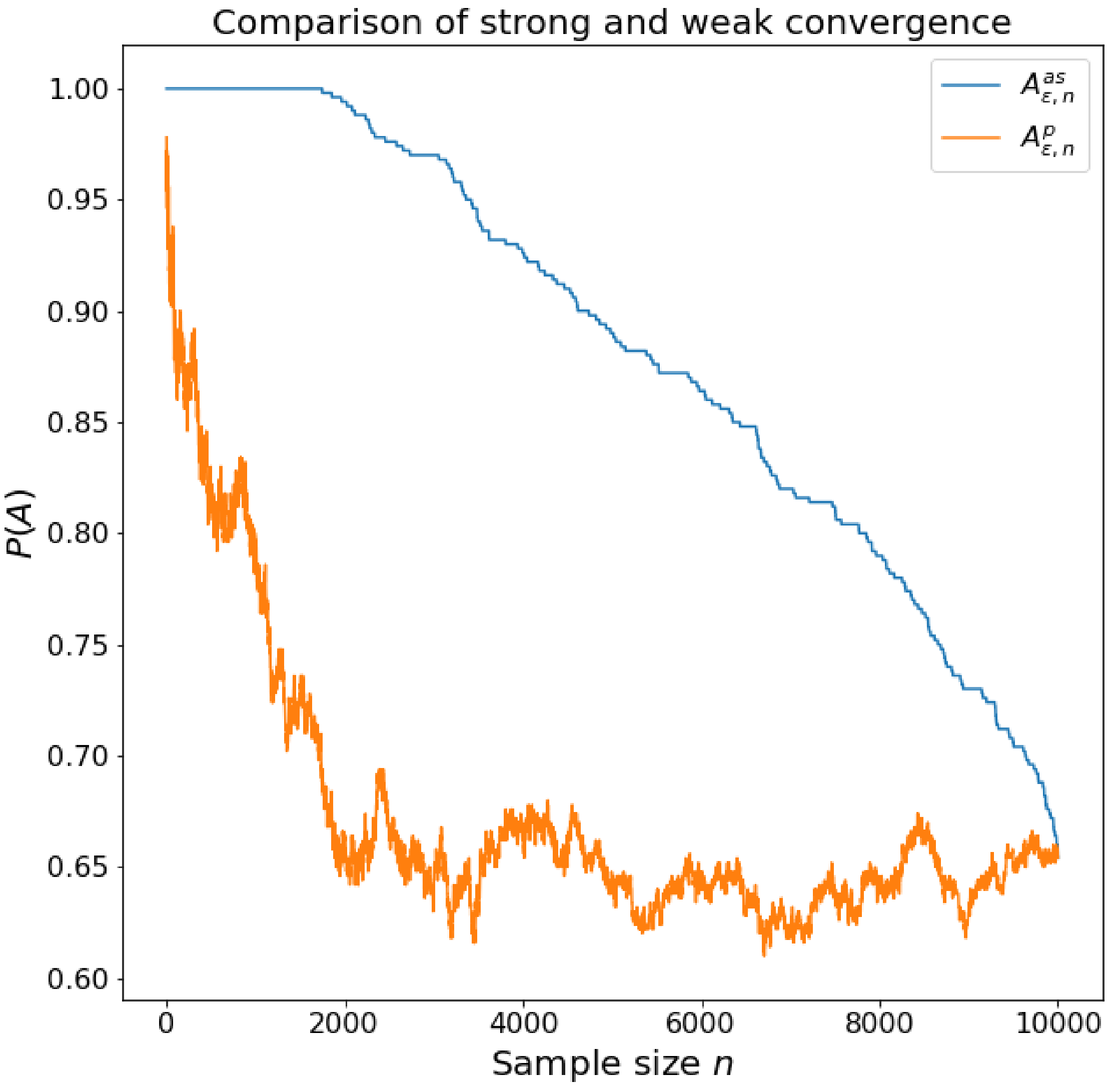
Copying functions from lecture notebooks on weak_vs_strong_convergence

[108]:
```
def estimate_probability_prob(sample_path_errs, epsilon, n):
    """ Estimate the probability of the event related to convergence in␣
 ↪probability

    sample_path_errs: (Npaths, Nsamples_per_path) array of errors of each path
    epsilon: float, target error region
    n: positive integer
    """

    Npaths, Nsamples_per_path = sample_path_errs.shape
    estimate = np.sum(sample_path_errs[:, n-1] > epsilon) / float(Npaths) #n-1␣
 ↪because indexing by zero
    return estimate

def estimate_probability_as(sample_path_errs, epsilon, n):
```

```
    """ Estimate the probability of the event related to convergence almost␣
↪surely

    sample_path_errs: (Npaths, Nsamples_per_path) array of errors of each path
    epsilon: float, target error region
    n: positive integer

    Note
    ----
    This function is a bit inefficient, it would be better if n could be a list␣
↪of variables so that we can reuse
    the calculations
    """

    Npaths, Nsamples_per_path = sample_path_errs.shape
    # Note the difference from in probability ---- we are looking into the␣
↪future
    # We are looking if any value in the path satisfies the error condition
    paths_satisfy_condition = np.any(sample_path_errs[:, n-1:] > epsilon,␣
↪axis=1)
    estimate = np.sum(paths_satisfy_condition) / float(Npaths)
    return estimate
```

[114]:
```
epsilon = 0.1
n = np.arange(1, num_samples+1)
prob_as = np.zeros((n.shape[0])) # probability almost surely
prob_p = np.zeros((n.shape[0]))
for ii, nn in enumerate(n):
    # we take absolute values because the true answer is 0
    prob_as[ii] = estimate_probability_as(np.abs(estimator_vals-3.0), epsilon,␣
↪nn)
    prob_p[ii] = estimate_probability_prob(np.abs(estimator_vals-3.0), epsilon,␣
↪nn)
```

[115]:
```
plt.figure(figsize=(10,10))
plt.plot(n, prob_as, label=r'$A_{\epsilon, n}^{as}$')
plt.plot(n, prob_p, label=r'$A_{\epsilon, n}^{p}$')
plt.title('Comparison of strong and weak convergence', fontsize=20)
plt.ylabel(r'$P(A)$', fontsize=20)
plt.xlabel(r'Sample size $n$', fontsize=20)
plt.xticks(fontsize=16)
plt.yticks(fontsize=16)
plt.legend(fontsize=16)
plt.show()
```



Comparison of strong and weak convergence
$A^{as}_{\varepsilon,n}$
$A^{p}_{\varepsilon,n}$
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
$P(A)$
0
2000
4000
6000
8000
10000
Sample size n


```
[ ]:
```