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Sequential Quadratic Programming (SQP) for Solving Constrained Production Optimization — Case Study from Brugge Field

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Abstract

Normally only about 30% of the oil in a reservoir can be extracted, but using secondary production methods such as water or gas injection it is often possible to increase that percentage significantly and maintain the production rate of a reservoir over a longer period of time. In reservoirs under water or gas injection additional gains can be obtained through an efficient strategy management of front movement and reservoir sweep. The objective of reservoir production optimization is to maximize an outcome such as sweep efficiency or net present value (NPV) through the control of completion rates or pressures. Using optimization methods, it is possible to compute control settings that result in increased oil production and decreased water production compared to production from standard practices. In this paper, we focus on optimization using SQP with an ensemble-based approach to estimate the gradient for the optimization. Although uncertainty in reservoir properties is usually important for the computation of optimal controls, here we use a single realization of the reservoir to evaluate the efficiency of the optimization algorithm.

The most expensive aspect of gradient-based optimization is usually the computation of gradients. Most practical production optimization problems involve large-scale, highly complex reservoir models with thousands of constraints, which makes numerical calculation of the gradient time consuming. Here, we use an ensemble-based approach for finding gradients and use localization to improve estimation of the gradient from a small number of realizations. The BFGS method is used for maximizing the objective function, with the Hessian estimated from a sequence of estimates of the gradient. Improving the gradient approximation using localization results in improvement to the Hessian approximation. A second important aspect of the efficiency of the method is the identification of active constraints. In this paper, we use a method for eliminating non-negativity constraints to decrease computation time and an updating procedure to solve each iteration of SQP much faster than the base case. Both the speed of the algorithm and the final NPV were increased significantly.

We evaluate the method by applying it to optimization of control settings in the Brugge field. Brugge is a 3D synthetic model designed by TNO with 20 vertical producers and 10 vertical peripheral water injectors. All of the producers and injectors are smart wells whose downhole chokes must be adjusted to optimize NPV. The total number of completion flow rates to be controlled is 84 at each time step, with 40 time steps (every six months). There are 1200 inequality constraints on total well liquid rates and 3360 non-negativity constraints on completion liquid rates. There are also inequality constraints on the bottom-hole pressure for wells at each time period.

Introduction

At the time that a reservoir reaches its economic limit for production, there is usually a large fraction of the original oil-in-place remaining in the reservoir. Additional oil can sometimes be recovered economically through the application of tertiary recovery methods, or by drilling additional wells into regions that were not drained during primary or secondary recovery. In many cases, however, some of the remaining oil is left behind simply as a result of lack of knowledge of the reservoir and less-than-optimal control of production during the life of the field. While there are clear limits on the controllability of reservoirs (Ramakrishnan, 2007; Jansen et al., 2009), and the flow properties between wells are usually highly uncertain, there is strong evidence that incremental recoveries of several percent are easily obtainable from optimization of rate or pressure controls on wells and completions.

Because the reservoir is only directly observable at well locations, the reservoir flow properties and saturations between wells are usually highly uncertain. The production and injection history at individual wells can be often be used to improve the reservoir simulation models and to reduce uncertainty. Peters et al. (2010) summarized results from an SPE comparative test case on closed-loop reservoir optimization. Nine groups participated in the comparison by submitting optimized control settings for 84 completion intervals, based on estimates of reservoir properties from history-matching of data prior to the time of setting controls. While it is clear that the ensemble Kalman filter generally outperformed other methods of data assimilation, it was difficult to identify a best optimization method because of the difficulty of separating the effect of quality in history matching from quality of optimization when evaluating the overall results. Chen and Oliver (2010), Chen et al. (2010), and Lorentzen et al. (2006) provide more detailed descriptions of the application of closed-loop optimization by individual groups to the Brugge comparative case. It was clear from the Brugge test case that history matching and production optimization are both important aspects of real field problems optimization problems, but development of algorithms to treat the two aspects can largely be de-coupled.

In this paper, the focus is purely on the optimization of controls to increase NPV, assuming that the geology is known. There is, of course, a fairly extensive background on production optimization as a subject of its own. When the control variables are allowed to take continuous values, gradient-based methods are often the most efficient approaches to optimization, even when the objective function has multiple minima. Of the several methods available for estimating the gradient, the most efficient and accurate method for computing the local gradient is through the solution of a set of adjoint equations for the reservoir flow and simulation equations. Asheim (1987, 1988) used the adjoint approach to gradient computation for optimization of net-present-value of water flooding by optimization of well flow rates. Brouwer and Jansen (2004) applied steepest ascent with the gradient computed from the adjoint system to compute optimal control settings for a heterogeneous, horizontal, two-dimensional, two phase (oil-water) reservoir with horizontal wells. Two types of controls were compared: BHP constraints and rate constraints without an inflow equation. Quite different solutions were obtained for the two types of control, neither of which might be actually attainable in practice. Sarma et al. (2005) used a formulation of the adjoint approach that allowed the adjoint system to be constructed without access to the code of the reservoir simulator. Nævdal et al. (2006); Kraaijevanger et al. (2007); Lien et al. (2008) used adjoint methods and associated optimization algorithms for finding optimal control values in water flooding projects. Chen et al. (2010) used the augmented Lagrangian method for maximizing the NPV of the Brugge field. The total flow rate of each completion was treated as a control variable and the total flow rate of each well as a constraint. The minimum bottom hole pressure constraint was set by the flow simulator.

In some cases, the use of the gradient computed from an adjoint solution may not be the best solution. This could be because the objective function is not differentiable with respect to the control variables, or because the functional relationship is so nonlinear that an accurate local gradient is not particularly useful far from the minimum, or because of the difficulty of obtaining accurate gradients from commercial simulators. In cases such as those, it might be advantageous to use approximate gradients that are slower in convergence near the minima, but perhaps have better average behavior far from a minimum. Lorentzen et al. (2006) introduced an optimization method that is similar in many respects to the ensemble Kalman filter. In this approach (and the approach of Nwaozo (2006)), the Kalman gain estimated from an ensemble of control variables is used to iteratively update individual control variables to match a target value of NPV (the 'data'). Chen et al. (2009) used an ensemble of realizations of controls for estimation of the direction of steepest ascent for production optimization of controls for a water-flood with controls of individual intervals, and later proposed a method of improving the robustness of the estimation of the gradient from small ensembles of controls through the use of localization and averaging of effects (Chen and Oliver, 2009). Chaudhri et al. (2009) showed that optimization using the conjugate gradient direction could converge to the maximum faster than steepest ascent search direction, while the gradient is still approximated from an ensemble of controls. Ensemble-based gradient-approximation methods have been applied to optimization of NPV of the Brugge field (Chen and Oliver, 2010, 2009), for optimization of horizontal wells equipped with inflow control valves (ICV) in a sector model of 200,000 cells (Su and Oliver, 2010), and to to find optimal values of production rates, injection rates, and surfactant concentration for optimizing NPV of a surfactant flood (Odi et al., 2010).

In addition to the gradient-based methods, there are other methods that are less likely to be trapped in local minima and can be used with nondifferentiable objective functions, but typically converge slowly and become inefficient when the number of variables to be optimized is large. Harding et al. (1998) evaluated several approaches to optimization of production scheduling including the genetic algorithm (GA), simulated annealing, SQP, and several hybrid approaches. In their study, the GA achieved significantly better results than the other methods for

random starting points. They noted that the SQP method performed poorly on a problem that required adjusting the starting and ending times as the objective function was discontinuous with respect to those variables. Yeten et al. (2003) and Tavakkolian et al. (2004) used GA for solving optimization problems that might have been difficult with gradient-based methods. Wang et al. (2009) compared three methods of optimization: (1) steepest ascent method using finite differencing for gradient estimation, (2) simultaneous perturbation stochastic approximation (SPSA), and (3) the ensemble Kalman filter approach for optimization. The objective function was NPV and constraints were bounds on the BHP. In their problem, the best results were obtained using steepest ascent method.

The handling of constraints during the optimization can be a significant part of the optimization problem. In the adjoint approach, constraints that the state variables must satisfy to be solutions of the flow and transport problem are included in the Lagrangian. In this paper, we use sequential quadratic programming (SQP) for constrained nonlinear production optimization. This approach has been used previously for various aspects of production optimization (Davidson and Beckner, 2003; Díez et al., 2005; Sarma et al., 2006, 2008; Lorentzen et al., 2009). In this paper, we focus on two novel aspects: use of an ensemble-based gradient computation with localization and efficiency of the addition and removal of constraints. The method is evaluated on a channelized model of the Brugge field.

Methodology

The general problem to be solved is minimization of an objective or cost function, $f(x)$, subject to constraints $a_i(x) = 0$ for $i = 1, 2, \ldots, p$ and $c_j(x) > 0$ for $j = 1, 2, \ldots, q$. $f(x)$, $a_i(x)$, and $c_j(x)$ are assumed to be continuous functions and have continuous second partial derivatives. In addition, the feasible region of this problem is assumed to be nonempty.

The solution of this problem must satisfy the Karush-Kuhn-Tucker (KKT) conditions (see, e.g., Antoniou and Lu, 2007):

$$\begin{aligned} \nabla_x \mathcal{L}(x, \lambda, \mu) &= 0 \\ a_i(x) &= 0 \qquad \text{for } i = 1, 2, \ldots, p \\ c_j(x) &\geq 0 \qquad \text{for } j = 1, 2, \ldots, q \\ \mu &\geq 0 \\ \mu_j c_j(x) &= 0 \qquad \text{for } j = 1, 2, \ldots, q \end{aligned} \tag{1}$$

where $\mathcal{L}(x, \lambda, \mu)$ is the Lagrangian of the problem, defined as

$$\mathcal{L}(x, \lambda, \mu) = f(x) - \sum_{i=1}^{p} \lambda_i a_i(x) - \sum_{j=1}^{q} \mu_j c_j(x) \tag{2}$$

and λ and μ are vectors of Lagrange multipliers.

For solving the optimization problem iteratively, we approximate each KKT equation with the first terms from a Taylor series expansion and replace the KKT conditions (Eq. 1) with linearized approximations,

$$\mathbf{Y}_k \delta_x + \mathbf{g}_k - \mathbf{A}_{ek}^T \lambda_{k+1} - \mathbf{A}_{ik}^T \mu_{k+1} = 0 \tag{3a}$$

$$\mathbf{A}_{ek} \delta_x = -a_k \tag{3b}$$

$$\mathbf{A}_{ik} \delta_x \geq -c_k \tag{3c}$$

$$\mu_{k+1} \geq 0 \tag{3d}$$

$$(\mu_{k+1})_j (\mathbf{A}_{ik} \delta_x + c_k)_j = 0 \qquad \text{for } j = 1, 2, \cdots, q \tag{3e}$$

where

$$\mathbf{a}_k = \begin{bmatrix} a_1(x_k) & a_2(x_k) & \cdots & a_p(x_k) \end{bmatrix}$$

and

$$\mathbf{c}_k = \begin{bmatrix} c_1(x_k) & c_2(x_k) & \cdots & c_q(x_k) \end{bmatrix}$$

and $\mathbf{A}_{ek}$ and $\mathbf{A}_{ik}$ are the Jacobians of the equality and inequality constraints at x_k.

The above system of equations are the exact KKT conditions of the following QP problem:

$$\begin{aligned} \text{minimize:} \quad & \frac{1}{2}\delta^T \mathbf{Y}_k \delta + \delta^T \mathbf{g}_k \\ \text{subject to:} \quad & A_{ek}\delta = -\mathbf{a}_k \\ & A_{ik}\delta \geq -\mathbf{c}_k. \end{aligned} \tag{4}$$

Because the active set of constraints is not known at the beginning of the iteration, it is necessary to iteratively solve a subproblem,

$$d_k = \underset{d}{\operatorname{argmin}}\, f(\mathbf{d}) = \frac{1}{2}\mathbf{d}^T\mathbf{H}\mathbf{d} + \mathbf{d}^T\mathbf{g}_k \quad \text{subject to} \quad \mathbf{a}_i^T\mathbf{d} = 0 \quad \text{for } i \in \mathcal{J}_k \tag{5}$$

where $\mathbf{a}_i$ is the i vector of coefficients of all of equality and active inequality constraints.

Solving this QP problem provides an appropriate search direction, δ_k, for the nonlinear minimization problem. If the matrix $\mathbf{A}_{aik}$ is defined to be composed of those rows of $\mathbf{A}_{ik}$ that satisfy the equality $\mathbf{A}_{ik}\delta_x + \mathbf{c}_k = 0$, and $\hat{\mu}$ denotes the associated Lagrange multiplier we can rearrange Eq. 3a:

$$\mathbf{Y}_k\delta_x + \mathbf{g}_k - \begin{bmatrix}\mathbf{A}_{ek}^T & \mathbf{A}_{aik}^T\end{bmatrix}\begin{bmatrix}\lambda_{k+1} \\ \hat{\mu}_{k+1}\end{bmatrix} = 0.$$

The Lagrange multipliers $\lambda_{\mathbf{k+1}}$ and $\hat{\mu}_{k+1}$ can then be computed by

$$\begin{bmatrix}\lambda_{\mathbf{k+1}} \\ \hat{\mu}_{k+1}\end{bmatrix} = (\mathbf{A}_{ak}\mathbf{A}_{ak}^T)^{-1}\mathbf{A}_{ak}(\mathbf{Y}_k\delta_x + \mathbf{g}_k) \tag{6}$$

where

$$\mathbf{A}_{ak} = \begin{bmatrix}\mathbf{A}_{ek} \\ \mathbf{A}_{aik}\end{bmatrix}.$$

Also, from the complementary condition in Eq. 3e we see that the Lagrange multipliers of other inequality constraints, which are not active, are equal to zero. Because in each iteration of the nonlinear optimization problem we solve a quadratic problem to find the search direction, this method is called SQP or sequential quadratic programming. The most important part of each SQP iteration is solving the QP problem of Eq. 4.

The initial feasible solution is updated using the formula

$$x_{k+1} = x_k + \alpha\delta_k,$$

where the search direction δ_k is found by solving Eq. 4 and α_k is a scalar obtained by an inexact line search as proposed by Powell (1978), followed by a second line search to find the maximum value of α that does not violate inequality constraints. The value of α that is used is the smaller of the two values. After determining the search direction and the step length α, the updated control variables can be computed. Then, using the BFGS algorithm, the updated Hessian matrix is computed. With the updated constraint matrix c_k, Jacobian matrix A_k and gradient matrix g_k, a new SQP iteration can be started.

Summary of the SQP Algorithm The entire SQP algorithm can be summarized as follows (Antoniou and Lu, 2007).

1. Set $k = 0$. Select initialize values of the control variables, x_0, such that $c_i(x_0) \geq 0$ for $i = 1, 2, \ldots, q$ and initialize Lagrange multipliers $\mu_0 \geq 0$. Set the initial Hessian matrix to the identity matrix.

2. Compute the gradient of the objective function with respect to control variables, $\mathbf{g}_k$, the constraint matrices $\mathbf{a}_k$ and $\mathbf{c}_k$ and the gradient of equality and inequality constraints with respect to control variables, $\mathbf{A}_{ek}$ and $\mathbf{A}_{ik}$, using the current feasible solution.

3. Solve the QP problem in Eq. 4 for the direction of correction to the control variables, δ_x, and compute Lagrange multipliers λ_{k+1} and $\hat{\mu}_{k+1}$ using Eq. 6.

4. Compute the step length multiplier α_k using the method of Powell (1978).

5. Set $x_{k+1} = x_k + \alpha_k\delta_x$ and compute new values of $\hat{\mu}_{k+1}$.

6. If $\|\delta_x^*\| \leq \varepsilon$ or decrease in objective function is not significant, output solution $x^* = x_{k+1}$ and stop, otherwise continue to next step.
7. Update the estimate of the Hessian matrix $\mathbf{Y}_k$ using the BFGS algorithm (Powell, 1978).
8. Return to step 2.

Reducing the Cost of Optimization The cost of the SQP method increases rapidly with increasing number of inequality constraints. For solving large-scale optimization problems with many inequality constraints, it is important to implement techniques to efficiently solve Eq. 5 multiple times with small changes in the constraint equations. Also, because we use an ensemble-based method for gradient estimation, it is critical to reduce the effect of spurious correlations from small ensemble size. In this section we describe methods to update the QR decomposition of the equality and active inequality constraint equations for changing numbers of active constraints, to reduce the effect of spurious correlations on the estimation of the gradient, and to reduce the number of non-negativity constraints in the problem.

Updating the QR decomposition In each iteration of SQP it is necessary to solve a QP problem and in each iteration of QP problem it is necessary to solve an equality QP sub-problem with equality constraints of the general form $Ax = B$. One of the most time consuming parts of the SQP algorithm is related to solving this equality QP sub-problem which requires repeated decomposition of the matrix related to coefficients of active constraints (Nocedal and Wright, 2006). Because in each iteration of the QP problem we can at most add or delete one constraint from the active set (one row from matrix A or one column from matrix A^T), we use a method for updating Q and R matrices after adding or deleting one column of matrix A^T. The factorization of a square matrix of size $n \times n$ has computational complexity $O(n^3)$ but the computational complexity of this updating procedure is $O(n^2)$, which is important for large scale problems. The method is based on use of the Givens rotation matrix (Björck, 1996; Gill et al., 1981).

Computation of the gradient One of the greatest expenses associated with the optimization is the cost involved in computation of the gradient as it generally requires running the reservoir simulator. There are several options for computing the gradient, including the adjoint method (Brouwer and Jansen, 2004; Sarma et al., 2005), simultaneous perturbation stochastic approximation (SPSA) (Bangerth et al., 2006; Gao et al., 2007; Wang et al., 2009), and also approximating gradient from ensembles (Nwaozo, 2006; Chen et al., 2009). In this study, we approximate the gradient from an ensemble of control settings and predicted outcomes. Each ensemble member consists of a realization of control settings and an associated NPV obtained by by applying the realization of control settings to the reservoir model and running the simulator:

$$M = \begin{bmatrix} x_1 & x_2 & \cdots & x_{N_{xe}} \\ S(x_1) & S(x_2) & \cdots & S(x_{N_{xe}}) \end{bmatrix} \tag{7}$$

where the x_i are vectors of realizations of control variables and $S(x_i)$ is the corresponding NPV. We denote the covariance matrix for control variables by $C_{x,x}$ and the cross covariance between control variables and NPV by $C_{x,S(x)}$ and estimate them from the ensemble,

$$C_{x,x} = \frac{1}{N_{xe}-1} \sum_{j=1}^{N_{xe}} (x_j - \langle x \rangle)(x_j - \langle x \rangle), \tag{8}$$

$$C_{x,S(x)} = \frac{1}{N_{xe}-1} \sum_{j=1}^{N_{xe}} (x_j - \langle x \rangle)(S(x) - \langle S(x) \rangle), \tag{9}$$

where

$$\langle x \rangle = \frac{1}{N_{xe}} \sum_{j=1}^{N_{xe}} x_j, \tag{10}$$

and

$$\langle S(x) \rangle = \frac{1}{N_{xe}} \sum_{j=1}^{N_{xe}} S(x_j). \tag{11}$$

The gradient for SQP is can be approximated from the following formula (Nwaozo, 2006; Chen et al., 2009)

$$g = C_{x,x}^{-1} C_{x,S(x)}. \tag{12}$$

where the $C_{x,x}^{-1}$ denotes a generalized inverse which we compute using singular value decomposition. Note that $C_{x,x}^{-1}$ can be computed efficiently without forming the matrix $C_{x,x}$.

When we need to estimate gradient using this method, the number of realizations could be important. By increasing the number of realizations we use for estimating gradient, we can generally improve the final NPV and achieve faster convergence. Increasing the number of realizations, however, means increasing the number of times that the simulator must be called.

The following procedure was used to generate realizations.

1. Generate the covariance matrix for regularizing the sequence of controls:

$$C(t_{ij}) = \exp\left(\frac{-3(t_i - t_j)^2}{s^2}\right)$$

   In this formula $t_i - t_j$ is difference between time steps i and j, and s is the correlation length.

2. Compute the Cholesky decomposition of the covariance matrix. $LL^{\mathrm{T}} = C$.

3. Sample a random mean μ between minimum and maximum of control rates. In our case, the mean of the controls is drawn from a uniform distribution within range $[q_l, q_u]$. This range for the injectors is $[800, 3200]$ and for the producers is $[600, 2400]$.

4. Sample perturbation parameters independently from a standard normal distribution.

5. Sample correlated realizations r from the following formula:

$$r = \mu + \sigma LZ$$

   where σ is the standard deviation, used to adjust the magnitude of the perturbations.

6. Compare the generated realizations with constraints and truncate their values if necessary.

After generating realizations we need to check generated values with constraints on wells and completions for possible truncating of their values. Using this method 50 realizations were generated in the first iteration, and using these realization the estimate of the gradient matrix is computed. In Fig. 1 you can see plot of some of generated realizations for completions in injector or producer wells using this method.

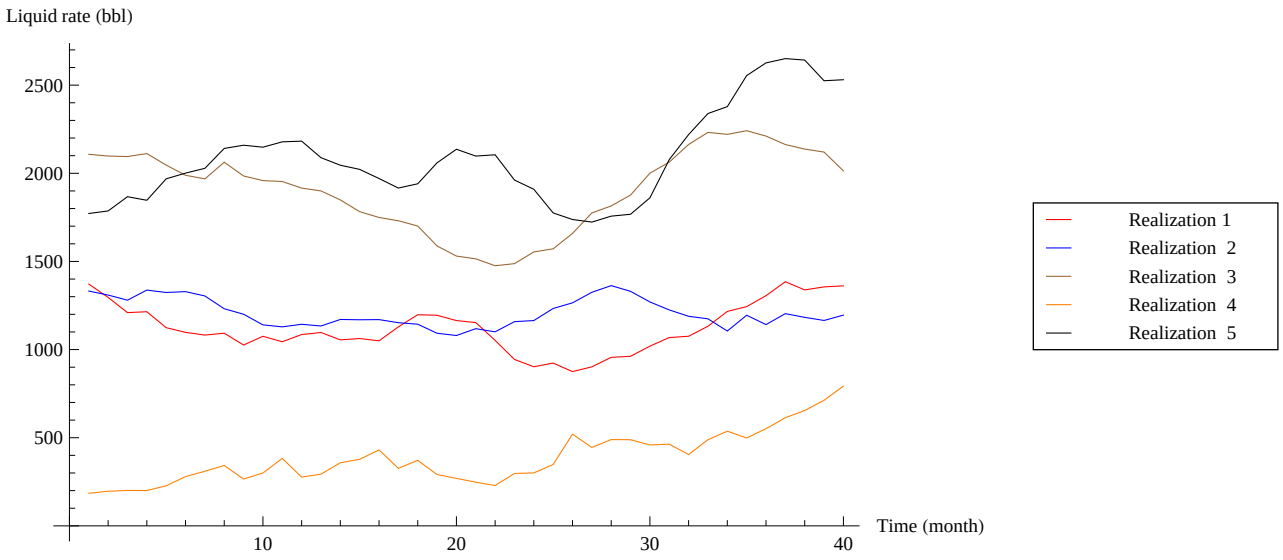


Figure 1: Plot of controls realizations for some of controls in producer wells

In subsequent iterations, it is not necessary to generate a random mean as the updated controls are taken to be the mean values. Perturbations are added to the mean, however, in order to generate new realizations. Previous investigations have shown that the results can be quite sensitive to the choice of correlation length and the magnitude of the perturbations (Su and Oliver, 2010). Also Chen and Oliver (2009) showed that the rate of increase of NPV with iterations was sensitive to the number of realizations.

Improving gradient estimation using covariance localization The efficiency of ensemble-based optimization methods depends strongly on the number of realizations of controls used for estimating the gradient. The approximation improves as ensemble size increases, and previous studies have shown that the ultimate NPV also depends on ensemble size (Chen et al., 2009). The cost of computation also increases with ensemble size, however, so the goal is to achieve reliable results from relatively small ensembles. In general, the problem with small ensemble size is that the estimate of the cross-covariance between control variables and field NPV is not accurate because the correlations are often weak and the estimates are contaminated by sampling error. Here, we follow the approach of Chen and Oliver (2009) who showed that regularization techniques could be used to improve the estimate of the cross-covariance. Briefly, the field NPV is written as the summation of well NPVs. We compute an estimate of the cross-covariance of control variables with the NPV of each well,

$$C_{x,S_w} = \begin{bmatrix} C_{x_1,S_1} & C_{x_1,S_2} & \cdots & C_{x_1,S_{N_w}} \\ C_{x_2,S_1} & C_{x_2,S_2} & \cdots & C_{x_2,S_{N_w}} \\ \vdots & \vdots & & \vdots \\ C_{x_{N_x},S_1} & C_{x_{N_x},S_2} & \cdots & C_{x_{N_x},S_{N_w}} \end{bmatrix} \tag{13}$$

where N_w is the number of wells. In general, the production (and consequently the NPV) of an individual well is most sensitive to the variables controlling completion intervals on that well, and only sensitive to controls on other wells to a lesser extent. In this approach, the sensitivity of distant wells are lumped to provide more robustness in estimation.

Using parallel computations The ensemble-based estimation of the gradient is naturally parallelizable, as each simulation run could potentially be performed on an independent processor. In general, ensemble-based methods for production optimization of petroleum reservoirs have used multiple processors to compute the ensemble of outcomes, but used a single processor to estimate the gradients or covariances. In the SQP method, the problems of variable elimination and matrix multiplications can become a significant part of the computation time so parallelizing this part of the method using MPI (Message Passing Interface) is a key aspect of making the method practical.

Reducing number of constraints by eliminating non-negative constraints The production optimization problem that we consider has a large number (3360) non-negativity constraints on the production rates for each completion interval for each time interval. Since the efficiency of the method depends strongly on the number of inequality constraints, the elimination of the 3360 non-negativity constraints has the potential of decreasing computation time substantially. To eliminate these constraints, we introduce a transformation of control variables so that instead of solving the original problem, the following problem is solved:

$$y_{\text{opt}} = \underset{y}{\operatorname{argmin}}\, S(y^2)$$

subject to the constraints

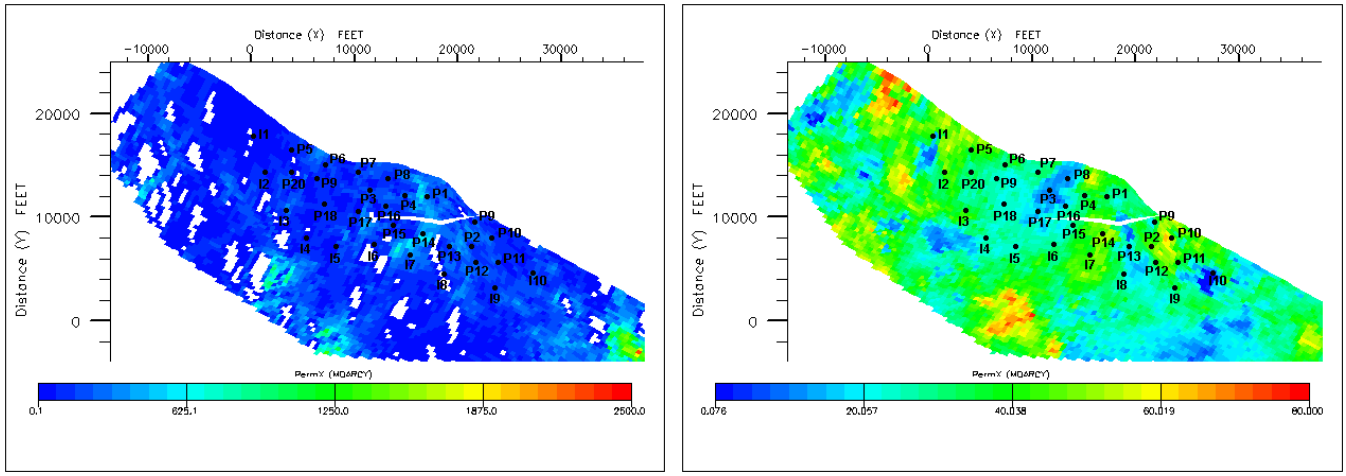
$$c_i(y^2) \geq 0$$

for $i = 1, 2, \ldots, p$ where $y = \sqrt{x}$. In this problem y can be negative, zero or positive and after finding optimal values of y we can find optimal values of original problem from $x = y^2$. The drawback to this approach is that it makes the optimization problem more nonlinear, but as we will show in the results section, the overall result is a reduction in computation time.

Application to Brugge Field

In this section, we apply SQP with ensemble-based gradient estimation to production optimization of the Brugge field. The Brugge field is a synthetic reservoir model designed by TNO as an SPE comparative study for closed-loop optimization (Peters et al., 2010). Although the organizers created 104 realizations conditioned on well logs, we simply selected one of the more complex models for the optimization study. Fig. 2 shows horizontal permeability in layers 1 and 5 of the realization that we used for production optimization. Layers 1 and 2 are fairly complex with channels that limit connectivity in directions perpendicular to the channel axis. The locations of the

10 injectors and 20 producers are also on the permeability grid. The wells have a total of 84 separate completions intervals, each of which can be controlled separately. All injectors have 3 completions, but the number of completion intervals in producers varies from 1 to 3. Although historical production data for 10 years were available, in this project we assumed that the geology and reservoir model is known so we could focus on the problem of production optimization for years 10 to 30. The total number of control steps is 40, as we only allow the controls to be adjusted at 6 months intervals. The total number of control variables is equal to 84×40 or 3360.



(a) Horizontal permeability in layer 1

(b) Horizontal permeability in layer 5

Figure 2: Changing horizontal permeability in layers 1 and 5 for active grids

The objective in this work is to maximize net present value (NPV), which we phrase as minimizing the negative of the NPV so that our objective function is

$$S(x) = \sum_{i=1}^{N_t} \frac{\nu_w Q_{w_i}(x) - \nu_o Q_{o_i}(x)}{(1+r_\tau)^{t_i/\tau}}.$$

where i is the time step index, N_t is the total number of control steps, r_τ is the discount rate in terms of time span τ, t_i is the cumulative time since the start of the production, ν_o and ν_w are the price of oil and the cost of water production over time step Δt_i, and x is vector of control variables.

In this problem, the price of oil is set to 80 \$/bbl, the cost of both water injection and water production is 5 \$/bbl and discount rate is 10% per year (reference year is year ten). There are 2 types of constraints in the Brugge field, bottom hole pressure (BHP) and total well fluid rate. Maximum injection rate of each injector is 4000 bbl/day, while the maximum production rate of each producer is 3000 bbl/day. Minimum bottom-hole pressure for producers is 725 and maximum bottom-hole pressure for injectors is 2611 psi. The total number of upper bound constraints on control variables are 1200. There are 3360 lower constraints for control variables. In this problem, all constraints inequality constraints.

In order to assess the efficiency of our implementation of the SQP method, we compare the results with results based on a ensemble-based preconditioned steepest ascent method (Chen et al., 2009). The updating formula for steepest ascent method is

$$x_{l+1} = x_l + \frac{1}{\alpha_l} C_{x,x} C_{x,s}. \tag{14}$$

Covariance matrices are approximated from the ensemble of realizations. By substituting them into Eq. 14 we can update our solution. Multiplication by $C_{x,x}$ provides filtering (smoothing) for regularizing changes of control variables. For all examples, we used 50 realizations of control variables to estimate the gradient of the objective function with respect to control variables. The number of realizations and random numbers were the same for both the SQP and the steepest ascent methods.

Results and Discussion Since the goal of this work is to evaluate the SQP algorithm, we selected one realization from the initial ensemble of reservoir models as a test realization. In this work we simply compare SQP and steepest ascent for this single realization. The NPV level can not be compared with the results in Peters et al. (2010) and Chen and Oliver (2010), which are based on history matched models. The optimization starts at the end of year 10 when the reservoir has been under peripheral water flood for about 8 years. The total optimization time is 20 years and because each time step is 6 month, the total number of time steps is 40.

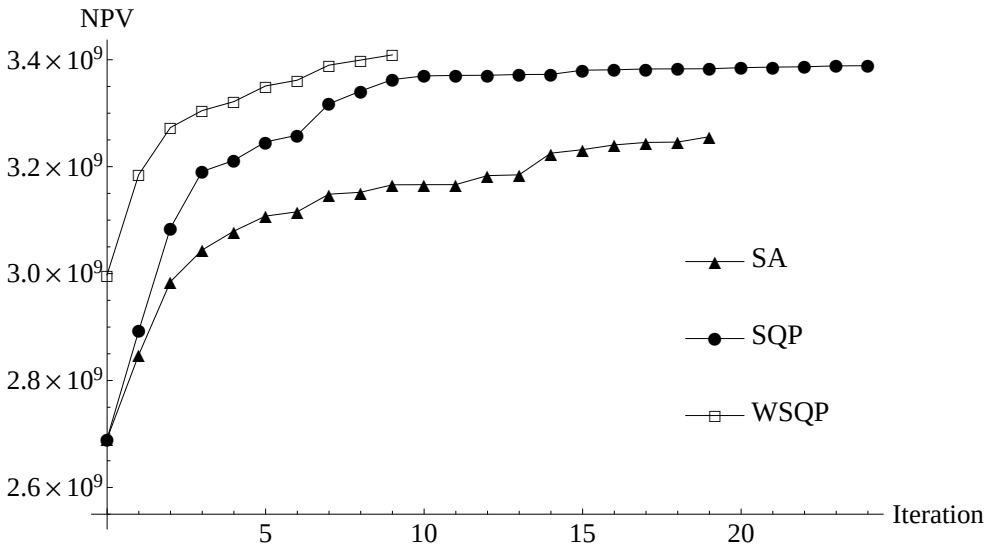


Figure 3: NPV vs. iteration for steepest ascent (SA) and the SQP methods before and after using water cut constraint

Fig. 3 shows the net present value (NPV) as a function of iteration for steepest ascent (SA) and the SQP methods. The same number of realizations and the same seed number were used for calculating the gradient in both methods. Although it is impossible to draw general conclusions from a single example, SQP was able to achieve a significantly higher value of NPV in this case. The rate of NPV increase in early iterations in SQP is also significantly higher than in the steepest ascent method. The same stopping criterion was used for both methods,

$$\frac{S_{\text{new}} - S_{\text{old}}}{S_{\text{old}}} \leq 10^{-5}.$$

We have observed that some wells appear to produce at very high water cut, even after optimization of rates. Although it is possible that the high water cut on one well improves the field NPV by improving the sweep, we observed in some cases that by combining an ad hoc reactive control with SQP, the field NPV can be improved. Therefore, in some examples, we simply set a water cut limit of 0.84 for all completions. Fig. 3 shows the results of the steepest ascent algorithm, and the SQP algorithm before and after using water cut constraints. It is clear that using water cut constraint resulted in a higher NPV with fewer iterations.

Because we use a stochastic method for computing gradient and this method is based on random sampling of control settings, the final result of optimization algorithm may vary from one ensemble to another. For comparing efficiency of optimization methods we ran the optimization code multiple times with different seed numbers. Fig. 4 shows the distribution of NPV results of SQP and steepest ascent methods for 40 different random seeds. In Fig. 4 the bounds of each box are 25% and 75% quantiles, the whiskers show the extremes, the line in the box is the median and the crosses show outliers. The first box plot (from left) shows results from the preconditioned steepest ascent method with no localization. The second and third boxes show results from the SQP method in which the numbers of iterations were limited to 20 and 25 iterations, respectively. The last box plot is related to results of SQP after using water cut constraints. For the same ensemble size, results of SQP are better than results from the steepest ascent method.

Figure 5 shows typical fluid injection and production rates for several wells before and after optimization. Rates at injectors were typically decreased in later years compared to rates in the base case (Fig. 5(a)), as production

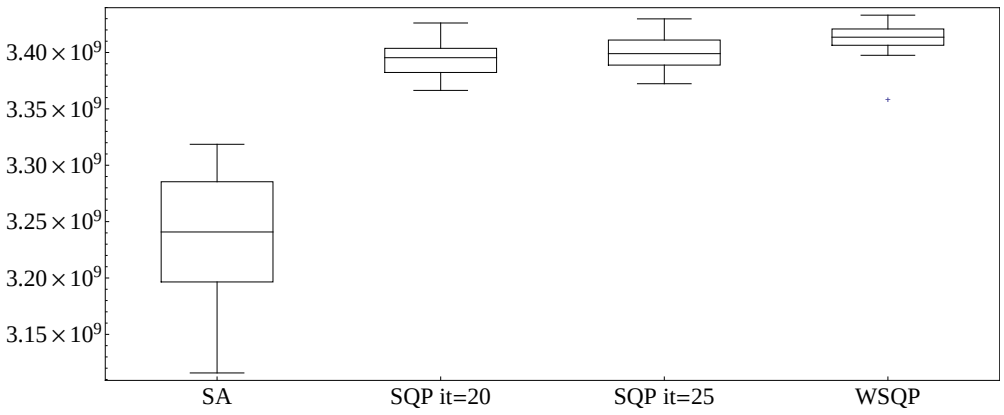
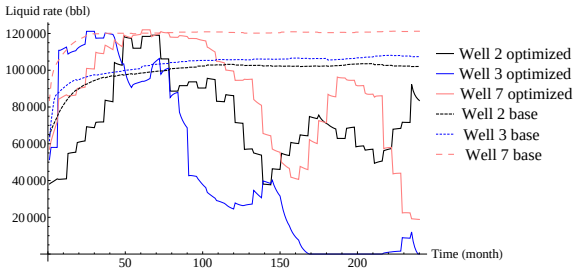
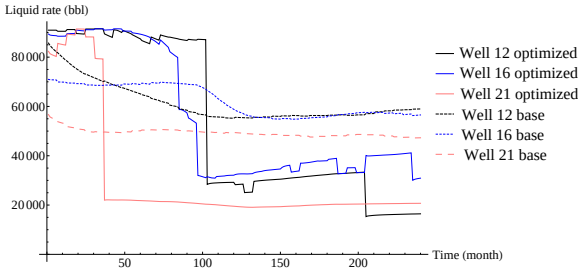


Figure 4: Box plot of running steepest ascent and SQP methods for 40 different seed numbers

from producing wells were restricted. Similarly, several of the injectors had much reduced liquid production rates at later times due to the shut-off of water producing intervals.



(a) Liquid rate of 3 injector wells



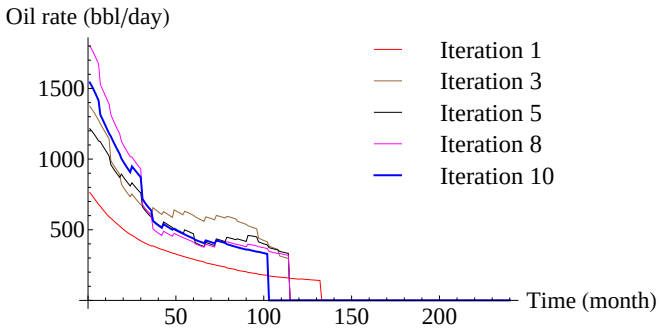
(b) Liquid rate of 3 producing wells

Figure 5: Changing liquid rate of several wells before and after optimization

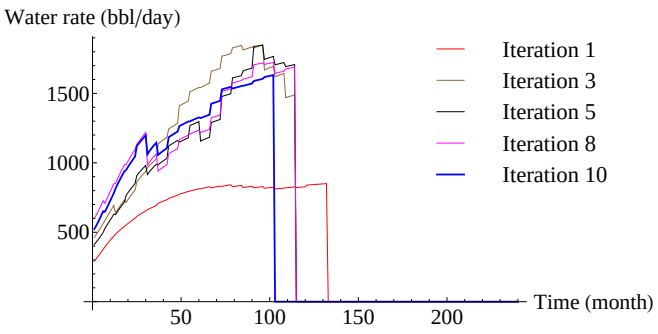
Fig. 6, shows the change in oil and water rates in two producer completions with iteration. Both wells have substantial water production from the beginning of the optimization period. Within a few iterations, the optimization algorithm has converged to final completion shut-in times, but the algorithm seems to have a more difficult time converging to rates for individual time intervals. On the other hand, it is quite probable that the optimal estimate of optimal control settings is insensitive to high frequency variations in the rates (Lien et al., 2008). The convergence of control variables seems well behaved, and the optimization algorithm seems to decrease water injection as much as possible and in some cases shut off the completion in a producing well completely when the water cut has become too high.

Fig. 7 shows oil saturation maps for layers 1 and 5 at the start of production optimization (Year 10) and at the end of production (Year 30). Note that at the start of optimization, many of the producers were already at fairly low oil saturation in both layers, although at that time much of the crest of the structure had not yet been swept. At the end of optimization, the remaining oil is primarily located updip of all producers and partially isolated by a sealing fault, where it is unlikely to be recoverable under any production strategy that does not involve either well conversion or new drilling.

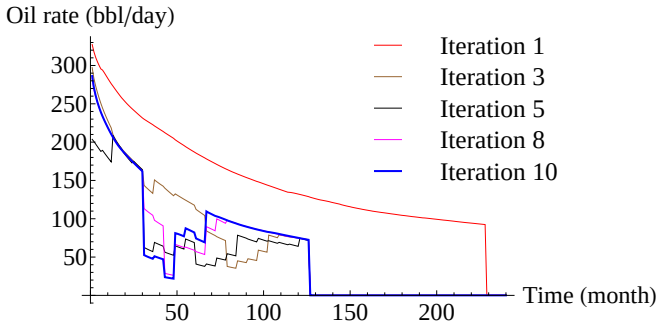
Timing of runs using QR updating and MPI In this section, we investigate the effect of updating of the QR factorization, compared to QR factorization, and the use of multiple processors for matrix operations and for reservoir simulation. Recall that the computation time of QR factorization and matrix multiplication in each iteration of the QP sub-problem depends on the number of active constraints. The number of active constraints will generally increase during the iterations and the starting number of active constraints has a great impact on computation time in that iteration. For this reason we evaluate CPU time (in minutes) for three different



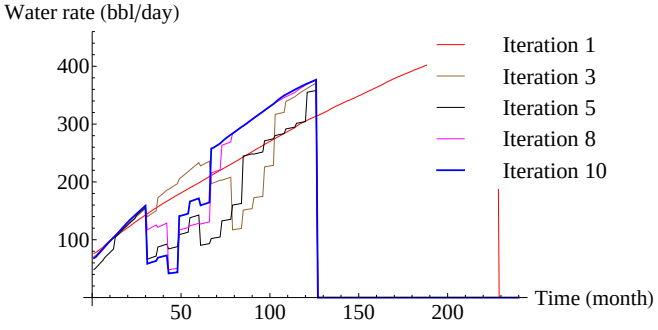
(a) Oil rate of bottom completion in producer#2



(b) Water rate of bottom completion in producer#2



(c) Oil rate of middle completion in producer#13



(d) Water rate of middle completion in producer#13

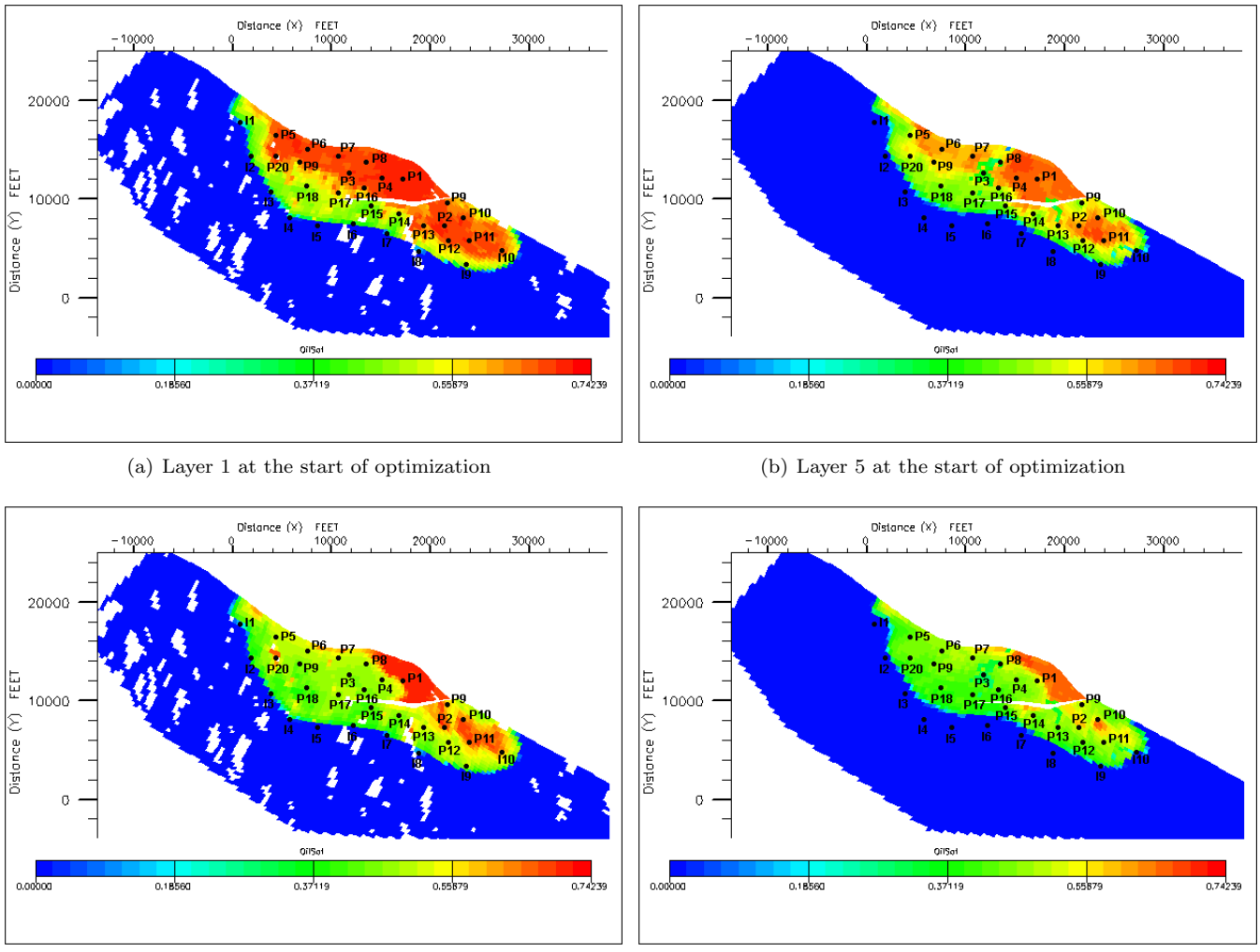
Figure 6: Improving production from different completions with changing iterations

iterations for 2 different cases: SQP using only QR factorization and SQP using QR factorization and MPI. In these experiments, the code is set to run on two nodes of a supercomputer. Each node consists of eight processors so a maximum of 16 processors are used. Only 5 processors were used for gradient calculations, however, and all of 16 processors were used for matrix multiplications.

In Table. 1 you can see these results for the first iteration. In this iteration at the start of iteration there is not any active constraints.

As you can see in Table 1, because at the start of iterations there are no active constraints, the effect of QR updating is not significant (time reduces from 102 to 94), but the effect of MPI on reducing running time is significant because of the reduction of cost in the running of the simulation model and for the cost of matrix multiplications. Running time difference between using 8 or 16 processors is not significant because total number of iterations for solving QP sub-problem is 245 iterations and this SQP iteration terminate pretty fast. In Table 1 (center column), we see that the effect of QR updating and MPI usage are both significant when there are a substantial number of constraints as at the start of the second iteration. As a result of using both methods, time was decreased by a factor of almost 4.5 for this iteration when there were 243 active constraints. Finally, in the last column of Table 1 we see results for the sixth iteration, which has 708 active constraints. As you can see in Table 1, because at the start of iteration there are 708 active constraints, effect of both of QR update and MPI is significant and the total time is reduced from more than 10 hours when no QR updating is done and the code is run on a single processor, to slightly more than one hour when 16 processors are used.

Using covariance localization for improving gradient estimation Similar to the previous comparison of results of SQP and steepest ascent methods, an investigation of the effect of localization should be done with multiple independent ensembles. In this case, we repeated each SQP experiment 20 times to reduce the variability in the estimate of the effect. Fig. 8 compares NPV results from SQP optimization using localization with ensemble size of 20 to results of SQP optimization without any regularization of the gradient using ensemble size of 20 and



(a) Layer 1 at the start of optimization

(b) Layer 5 at the start of optimization

(c) Layer 1 at the end of optimization

(d) Layer 5 at the end of optimization

Figure 7: Oil saturation in layers 1 and 5 at the beginning (Year 10) and the end (Year 30) of the production optimization period.

Table 1: Time in minutes for one outer iteration with various numbers of active constraints and various methods for reducing time. Note that only three processors are used for reservoir simulation in all cases.

Method	Processors	Active constraints		
		0	243	708
SQP	1	102	293	608
SQP+QR update	1	94	182	224
SQP+QR+MPI	8	36	72	96
SQP+QR+MPI	16	31	64	86

40. In this case, the use of localization allowed us to obtain results from a small ensemble that were equivalent in quality to an ensemble twice as large but for which localization was not used. This result is consistent with the findings of (Chen and Oliver, 2009) for optimization using steepest ascent.

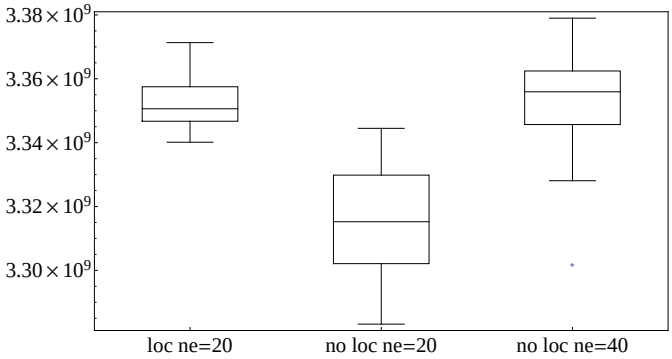


Figure 8: Box plot for running SQP with localization using ensemble size of 20 and SQP without localization using ensemble size of 20 and 40

Reducing number of constraints by eliminating non-negative constraints In this problem, there are 3360 non-negativity constraints. By eliminating the non-negative constraints, the computation time was reduced from approximately 100 minutes per iteration using 8 processors to approximately 50 minutes per iteration.

Using QR factorization and MPI for matrix multiplications In most ensemble-based optimization methods, the cost of running the reservoir simulator to estimate the gradient (and covariance) is the most time-consuming part of the computation. In this case, however, because the Brugge model is only two-phase, and relatively small (44,500 cells) the time for computing the gradient using 50 realizations is about 18 minutes on 5 processors. As a result, in this problem estimating the gradient is not the most time consuming part of solving the constrained optimization problem when there are a large number of constraints on control variables.

Summary and Conclusions

Ensemble-based methods are attractive for use in production optimization because they allow the use of the reservoir simulator as a 'black box' so that any type of reservoir simulator can be used. It can be particularly useful with closed-loop optimization as it provides a method for robust optimization with uncertainty in the reservoir properties.Although ensemble-based methods have previously been used with production optimization to compute the gradients, the handling of constraints has not been done rigorously. Here we implemented an SQP method that included the gradient of the constraints in the Lagrangian and applied the method to the problem of production optimization of the Brugge field. This problem requires determination of optimal liquid rates on 84 completion intervals over a 20 year production period. Interval rates are subject to total well liquid rates, non-negativity constraints for producers, and minimum pressure constraints.

In the Brugge example, SQP attained a higher NPV than the steepest ascent method when the same number of ensemble members were used to estimate the gradients in both methods. Although reduction in water production was not the objective of the minimization, the SQP method did result in less water production than steepest ascent method, presumably because of less regularization of the rates. We also found that applying a water cut constraint to the completions prevented the final solutions from including completions that produced at relatively high water cuts. This modification resulted in an increase in NPV and faster convergence to the final NPV using smaller number of iterations. The SQP method can be quite expensive when there are many constraints in the problem. We identified expensive aspects of the method and developed modifications to increase the efficiency. In particular, updating the QR factorization instead of recomputing the QR factorization from scratch decreased running time of each QP iteration significantly. Also, using MPI for matrix multiplications and gradient calculation helped to decrease the running time. Without using these methods, SQP was very difficult to apply to large optimization problems.

In addition, we attempt to use as small an ensemble as possible to estimate the gradient of the objective function with respect to control variables. Using covariance localization (Chen and Oliver, 2009), improved estimation of the gradient and the BFGS approximation to the Hessian. This would be more important when the reservoir model

is large so that simulation time dominates the computation. Finally, we showed that by eliminating non-negative constraints, computation time could be reduced significantly, even though it made the problem more nonlinear.

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References

Antoniou, A. and W. Lu, *Practical Optimization: Algorithms and Engineering Applications*, Springer-Verlag New York, Inc, 2007.

Asheim, H., Optimal control of water drive (SPE-15978), 1987, submitted to SPE for consideration for publication.

Asheim, H., Maximization of water sweep efficiency by controlling production and injection rates (SPE-18365), in *SPE European Petroleum Conference, London, UK October 16–19, 1988*, 1988.

Bangerth, W., H. Klie, M. F. Wheeler, P. L. Stoffa, and M. K. Sen, On optimization algorithms for the reservoir oil well placement problem, *Computational Geosciences*, **10**(3), 303–319, 2006.

Björck, Å., *Numerical Methods for Least Squares Problems*, SIAM, 1996.

Brouwer, D. R. and J. D. Jansen, Dynamic optimization of waterflooding with smart wells using optimal control theory, *SPE J.*, **9**(4), 391–402, 2004.

Chaudhri, M. M., H. A. Phale, N. Liu, and D. S. Oliver, An improved approach for ensemble-based production optimization, SPE-121305, *Proceedings of the SPE Western Regional Meeting, 24–26 March 2009, San Jose, California*, 2009.

Chen, C., Y. Wang, G. Li, and A. C. Reynolds, Closed-loop reservoir management on the Brugge test case, *Computat Geosci*, 2010.

Chen, Y. and D. S. Oliver, Localization of ensemble-based control setting updates for production optimization (SPE 125042), in *SPE Annual Technical Conference, 4–7 October 2009, New Orleans, Louisiana, U.S.A.*, 2009.

Chen, Y. and D. S. Oliver, Ensemble-based closed-loop optimization applied to Brugge Field, *SPE Reservoir Evaluation & Engineering*, **13**(1), 56–71, 2010.

Chen, Y., D. S. Oliver, and D. Zhang, Efficient ensemble-based closed-loop production optimization, *SPE Journal*, **14**(4), 634–645, 2009.

Davidson, J. E. and B. L. Beckner, Integrated optimization for rate allocation in reservoir simulation, *SPE Reservoir Evaluation & Engineering*, **6**(6), 426–432, 2003.

Díez, M. D., K. Brusdal, G. Evensen, T. Barkve, and A. Mjaavatten, Opportunities and challenges of using sequential quadratic programming (SQP) for optimization of petroleum production networks, *Computer Aided Chemical Engineering*, **20**, 169–174, 2005.

Gao, G., G. Li, and A. C. Reynolds, A stochastic optimization algorithm for automatic history matching, *SPE Journal*, **12**(2), 196–208, 2007.

Gill, P. E., W. Murray, and M. H. Wright, *Practical Optimization*, Academic Press, San Diego, CA, 1981.

Harding, T. J., N. J. Radcliffe, and P. R. King, Hydrocarbon production scheduling with genetic algorithms, *SPE Journal*, **3**(2), 99–107, 1998.

Jansen, J. D., J. F. M. Doren, M. Heidary-Fyrozjaee, and Y. C. Yortsos, Front controllability in two-phase porous media flow, in *Model-Based Control*, (edited by P. M. Hof, C. Scherer, and P. S. Heuberger), pp. 203–219, Springer US, 2009.

Kraaijevanger, J. F. B. M., P. J. P. Egberts, J. Valstar, and H. W. Buurman, Optimal waterflood design using the adjoint method, SPE-105764, in *SPE Reservoir Simulation Symposium, 26–28 February 2007, Houston, Texas, U.S.A.*, 2007.

Lien, M., D. R. Brouwer, T. Mannseth, and J. D. Jansen, Multiscale regularization of flooding optimization for smart field management, *SPE J.*, **13**(2), 195–204, 2008.

Lorentzen, R. J., A. M. Berg, G. Nævdal, and E. H. Vefring, A new approach for dynamic optimization of waterflooding problems, SPE-99690, in *SPE Intelligent Energy Conference and Exhibition, 11–13 April 2006, Amsterdam, The Netherlands*, 2006.

Lorentzen, R. J., A. Shafieirad, and G. Nævdal, Closed loop reservoir management using the ensemble Kalman filter and sequential quadratic programming, SPE-119101, in *SPE Reservoir Simulation Symposium, 2–4 February 2009, The Woodlands, Texas*, 2009.

Nævdal, G., D. R. Brouwer, and J.-D. Jansen, Waterflooding using closed-loop control, *Computational Geosciences*, **10**(1), 37–60, 2006.

Nocedal, J. and S. J. Wright, *Numerical Optimization*, Series in Operations Research and Financial Engineering, second edn., Springer, New York, 2006.

Nwaozo, J., *Dynamic Optimization of a Water Flood Reservoir*, Masters thesis, University of Oklahoma, 2006.

Odi, U. O., R. H. Lane, and M. A. Barrufet, Ensemble based optimization of EOR processes, SPE-132626, in *SPE Western Regional Meeting, 27–29 May 2010, Anaheim, California*, 2010.

Peters, L., R. Arts, G. Brouwer, C. Geel, S. Cullick, R. Lorentzen, Y. Chen, K. Dunlop, F. Vossepoel, R. Xu, P. Sarma, A. Alhutali, and A. Reynolds, Results of the Brugge benchmark study for flooding optimization and history matching, *SPE Reserv. Evalu. Eng.*, **13**(3), 391–405, 2010.

Powell, M. J. D., Algorithms for non-linear constraints that use Lagrangian functions, *Mathematical Programming*, **14**(2), 224–248, 1978.

Ramakrishnan, T., On reservoir fluid-flow control with smart completions, *SPE Production & Operations*, **22**(1), 4–12, 2007.

Sarma, P., K. Aziz, and L. J. Durlofsky, Implementation of adjoint solution for optimal control of smart wells, SPE-92864, in *SPE Reservoir Simulation Symposium, 31 January–2 Feburary 2005, The Woodlands, Texas*, 2005.

Sarma, P., W. H. Chen, L. J. Durlofsky, and K. Aziz, Production optimization with adjoint models under nonlinear control-state path inequality constraints, *SPE Reservoir Evaluation & Engineering*, **11**(2), 326–339, 2008.

Sarma, P., L. J. Durlofsky, K. Aziz, and W. H. Chen, Efficient real-time reservoir management using adjoint-based optimal control and model updating, *Computational Geosciences*, **10**(1), 3–36, 2006.

Su, H. and D. S. Oliver, Smart well production optimization using an ensemble-based method (SPE 126072), *SPE Reservoir Evaluation & Engineering*, **in press; posted 23 August 2010**, 2010.

Tavakkolian, M., F. Jalali F., and M. A. Emadi, Production optimization using genetic algorithm approach, SPE-88901, in *Nigeria Annual International Conference and Exhibition, 2-4 August 2004, Abuja, Nigeria*, 2004.

Wang, C., G. Li, and A. C. Reynolds, Production optimization in closed-loop reservoir management, SPE-109805, *SPE Journal*, **14**(3), 506–523, 2009.

Yeten, B., L. J. Durlofsky, and K. Aziz, Optimization of nonconventional well type, location, and trajectory, *SPE Journal*, **8**(3), 200–210, 2003.