CS544: Graph Algorithms, Social Networks & NLP applications

March 22, 2011

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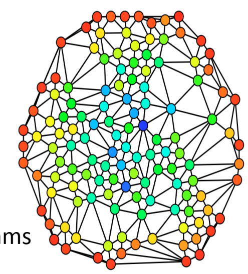
Graph Theory

- General introduction (terminology)
- Directed Graphs
- Undirected Graphs
- Refresh some algorithms



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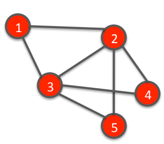
What is a Graph?

- A *graph* G=(***V***,***E***) is composed of:
  - ***V***: set of ***vertices***
  - ***E***: set of ***edges*** connecting the vertices
- An ***edge*** e=(u,v) is a pair of ***vertices***



V = {1,2,3,4,5}

E = {(1,2);(1,3);(2,3);(2,4);(2,5);(3,4);(3,5)}

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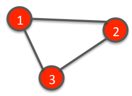
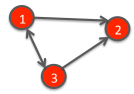
Undirected and Directed Graphs

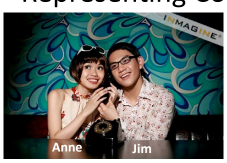
- An ***undirected graph*** is one in which the pair of vertices in an edge is unordered
  - $(v_0,v_1) = (v_1,v_0)$



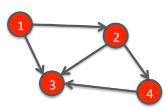
- A ***directed graph*** is one in which each edge is a directed pair of vertices
  - $(v_0,v_1) \;!\!= (v_1,v_0)$



Representing Conversations as Graph



Anne:	Jim, tell the Murrays they're invited
Jim:	Don, you and your dad should come for dinner!
Jim:	Mr. Murray, you should both come for dinner
Anne:	Don, did Jim tell you about the dinner? You must come.
Don:	Dad, we are invited for dinner tonight
Sam:	Anne, we're going, it's settled!

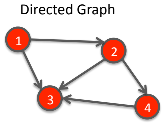


vertex (node)

edge (link)

Data Representation - Directed Graph

Directed Graph



vertex (node)

edge (link)

Edge list

Vertex	Vertex
1	2
1	3
2	3
2	4
3	4

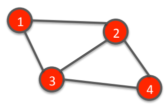
Adjacency matrix

Vertex	1	2	3	4
1	-	1	1	0
2	0	-	1	1
3	0	0	-	0
4	0	0	1	-

Directed graph captures who speaks to whom in the conversation

Data Representation - Undirected Graph

Undirected Graph



Edge list

Vertex	Vertex
1	2
1	3
2	3
2	4
3	4

Adjacency matrix (symmetric)

Vertex	1	2	3	4
1	-	1	1	0
2	1	-	1	1
3	1	1	-	1
4	0	1	1	-

Undirected graph captures who knows who in the conversation

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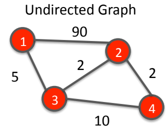
Adding weights to edges

Undirected Graph



Edge list

Vertex	Vertex	Weight
1	2	90
1	3	5
2	3	2
2	4	2
3	4	10

Adjacency matrix has the weights instead 0 and 1

Vertex	1	2	3	4
1	-	90	5	0
2	90	-	2	2
3	5	2	-	10
4	0	2	10	-

Weights can represent the frequency of interaction

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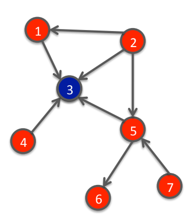
Degree Centrality

- The activity of a node can be captured through degrees
- The degree of a node corresponds to the number of direct connections it has
- <u>Degree measures:</u>

  − $inDegree(u) = \sum_{\forall (u,v) \in E} (v,u)$

  is the sum of all incoming edges to *u*



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Degree Centrality

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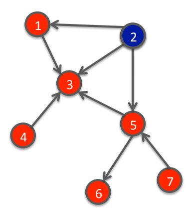

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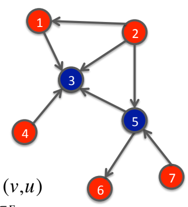
Degree Centrality

- The activity of a node can be captured through degrees
- The degree of a node corresponds to the number of direct connections it has
- <u>Degree measures:</u>
  - $totalDegree(u) = \sum_{\forall (u,v) \in E} (u,v) + \sum_{\forall (u,v) \in E} (v,u)$

  is the sum of all outgoing and incoming edges to *u*



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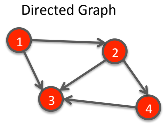
Calculate Centrality with Adjacency Matrix

Directed Graph



Adjacency matrix

Vertex	1	2	3	4
1	-	1	1	0
2	0	-	1	1
3	0	0	-	0
4	0	0	1	-

- The row sum is the *outDegree* of a node
- The column sum is the *inDegree* of a node

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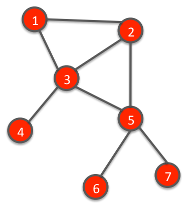
Degree Centrality

- How much are the *inDegree*, *outDegree* and *totalDegree* of a node in the undirected graph?

Answer: They are identical.



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Why using Centrality ?

- Centrality measure captures the connectedness of a node, hence we can measure influence and/or popularity
- Useful in assessing which nodes are central with respect to spreading information and influencing others in their immediate neighborhood
- Analyze your own networks

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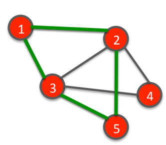
Paths and short path

- A ***path*** between two nodes is any sequence of vertices $v_1, v_2, \ldots v_k$ that connect two nodes
- The ***shortest path*** between two nodes is the path that connects the two nodes with the shortest number of edges



- How much is the shortest path between nodes 1 and 5?

Length 2: {1,3,5} and {1,2,5}

- What are the longer paths between the two nodes?

{1,2,3,5}, {1,2,3,5}, {1,2,4,3,5}, {1,3,2,4,5}

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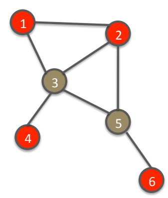
Betweenness Centrality

- The ***betweenness*** of node v is $BE(v) = \sum_{\substack{s \neq v \neq t \in V \\ s \neq t}} \frac{\sigma_{st}(v)}{\sigma_{st}}$ the number of shortest paths that pass through a node divided by all shortest paths in the network
- Reflects which nodes are more likely to be in communication paths between other nodes
- What happens if nodes 3 and 5 are removed from the network?



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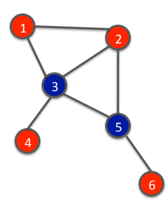
Closeness Centrality

- The ***closeness*** of a node v_i is
$$CL(v_i) = \frac{n-1}{\sum_{j=1}^{n} d(v_i, v_j)}$$
where *n* is the total number of nodes in the graph, $d(v_i,v_j)$ is the shortest path of v_i to all other nodes in the network
(how many hops on average are necessary to reach every other node)
- Measures the ***reach***, i.e. how long will it take to reach other nodes from a given starting node
- Useful when information dissemination is main concern



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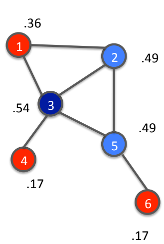
Eigenvector Centrality

- The ***eigenvector centrality*** is the sum of the eigenvector centralities of all nodes directly connected to it
- A node with high eigenvector centrality is connected to other nodes with high eigenvector centralities
- Useful to determine who is connected to the most connected nodes



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Interpretations in Social Networks

- **Degree** How many people can this person reach directly?
- **Betweenness** How likely is this person to be the most direct route between two people in the network?
- **Closeness** How fast can this person reach everyone in the network?
- **Eigenvector** How well is this person connected to other well-connected people?

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General Interpretations …

- **Degree** How many people has this person collaborated with?
- **Betweenness** Who is the spy through whom most of the confidential information is likely to flow?
- **Closeness** How fast will a disease spread from a person to the rest of the network?
- **Eigenvector** Who is the author that is most cited by other well-cited authors?

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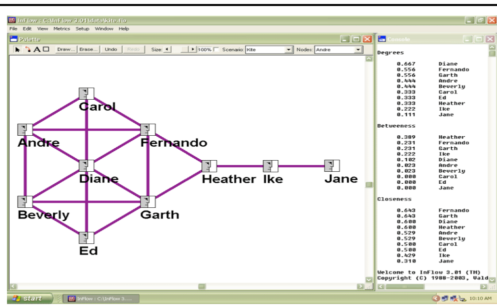

- Which node has the highest degree?
- Which node is the most central to the network?

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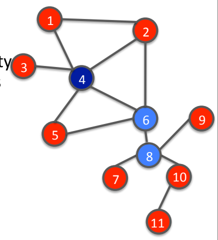
Key Player Problem

- The key player is calculated as

$$KPP(v) = \frac{\sum_{u \in V} \frac{1}{d(u,v)}}{|V| - 1}$$

high values indicate strong connectivity and proximity to the rest of the nodes

- Observations:
  - node 4 is the most central node
  - nodes 6 and 8 reach more nodes
  - if nodes 6 and 8 are removed, the network will become disrupted.
  - nodes 6 and 8 together are more 'key' to this network than node 4



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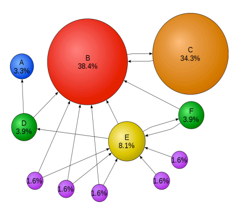
Page Rank *(Brin & Page 1998)*

- The page rank of a node v is

$$PR(v) = \frac{(1-\alpha)}{|V|} + \alpha \sum_{u,v \in E} \frac{PR(u)}{\mathrm{outD(u)}}$$

- Imagine a web surfer doing a simple random walk on the entire web for an infinite number of steps.
- Occasionally, the surfer will get bored and instead of following a link pointing outward from the current page will jump to another random page.



- At some point, the percentage of time spent at each page will converge to a fixed value.

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NLP APPLICATIONS, SEMANTIC CLASS LEARNING

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FLASHBACK

How are Max Planck, Angela Merkel and Dalai Lama related?

All have doctoral degrees from German universities

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FLASHBACK

Semantic Class Learning: Objectives

- Given a class and an instance, learn automatically with minimum supervision new instances of that class
- Examples:
  - *class_name*: Nobel prize winners
  - *instances*: Albert Einstein, Max Plank …
  - *class_name*: US states
  - *instances*: Georgia, Alabama, California …

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FLASHBACK

Bootstrapping

- Start with a pattern, *class_name* and <*seed*>
- Feed the newly learned terms on <*seed*> position
- Conduct a breadth-first search

states such as *Texas* *Alabama* *Mississippi* *Arkansas* and *

Florida
Texas
California
Mississippi
…
Arkansas
Tennessee
Nevada

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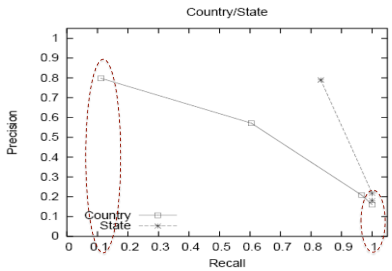
FLASHBACK

Performance of Bootstrapping



Problem: search needs guidance
Solution: rank the learned instances

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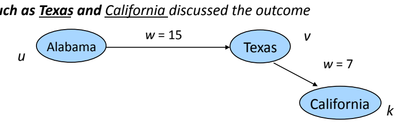
Hyponym Pattern Linkage Grap

- HPLG=(V,E) where vertex $v \in V$ is an instance, and $e \in E$ is an edge between two instances

*certain **states, such as Alabama and** Texas, should forbid prayers that are led*
***states such as Texas and** California discussed the outcome*



- the weight *w* of an edge is the frequency with which *u* generated *v*

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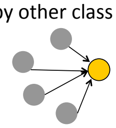
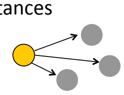
Properties of Graph Measures

- Observing two characteristics:
  – *Popularity*, the ability of an instance to be discovered by other class instances



  – *Productivity*, the ability of an instance to discover other class instances



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Ranking Functions

- Employed graph-based measures
  - inDegree
  - outDegree
  - totalDegree
  - Betweenness
  - Key player
- Use them to rank the learned instance elements

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Learning US states

number of learned instances →

	States						
	#N	inDegree	outDegree	totalDegree	Betweenness	KeyPlayer	Bootstrap.
	25	1.0	**1.0**	**1.0**	.88	1.0	.45
	50	.98	**1.0**	**1.0**	.86	.98	.10
	64	.77	.78	.78	.77	.78	--

- HPLGs perform better than bootstrapping
- outDegree and totalDegree discover all state instances
- if there are only 50 US states, why does the algorithm keep on learning

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The Troublesome Fourteen

- Instances after the learned 50 US states:

  Russia, Ukraine, Uzbekistan, Azerbaijan, Moldava, Tajikistan, Armenia, Chicago, Boston, Atlanta, Detroit, Philadelphia, Tampa, Moldavia

“authoritarian **former Soviet states** such as **Georgia** and **Ukraine**”

“Findlay has 20 restaurants in states such as **Florida** and **Chicago**”

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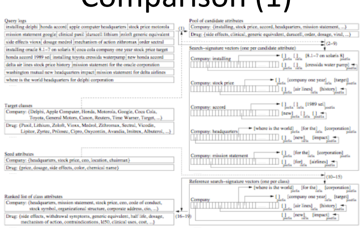
Learning Country Names

Countries		
#N	KeyPlayer	outDegree
10	.90	**1.0**
25	.88	**1.0**
50	.80	**1.0**
75	.69	.93
100	.68	.84
116	.65	.80

Error Analysis

- Type 1: incorrect proper name extraction
  "states such as Georgia and English speaking countries"
- Type 2: instances that formerly belonged to the semantic class
  "Serbia-Montenegro", *"Czechoslovakia"*
- Type 3: spelling variants
  "Kyrgystan" vs *"Kyrgyzhstan"*
- Type 4: sentences with wrong factual assertions
  "industry in countries such as France and North America"
- Type 5: broken expressions
  "issue has been tough for states such as Texas and New"

Comparison (1)



- Contextual vectors from query logs (Pasca,07)*

Learned Country Names	Pasca 07 (precision)	outDegree (precision)
100	95%	100%
150	82%	100%

***Organizing and Searching the World Wide Web of Facts - Step Two: Harnessing the Wisdom of the Crowds (Pasca,07)**

Comparison (2)

- KnowItAll system, details in Lecture #6
  - uses singly-anchored patterns
    "country such as *"
  - ranks with mutual information

Learned Country Names	KnowItAll 1	KnowItAll 2	outDegree
Precision	79%	97%	100%
Recall	89%	58%	77%

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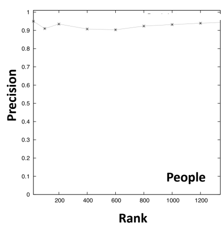
Evaluation against WordNet



	# harvested	PrWN	PrHUM	NotInWN
People Names	1344	.23	.95	986

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Lessons Learned so far …

- Graph algorithms can be employed to guide semantic class harvesting systems
- outDegree outperforms complex graph ranking algorithms
- Achieves higher recall and accuracy compared to existing knowledge harvesting algorithms
- Learns information missing from WordNet

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