Math 154 – Fall 2021
Jo Hardin
warm-up #10 (bootstrap SE)

Your Name: ________________________________

Names of people you worked with: ________________________________

Task: Consider the distribution of: $\frac{\overline{X}-\mu}{s/\sqrt{n}}$ (which, incidentally, we know is distributed according to t_{n-1} if $X_i \stackrel{iid}{\sim} N(\mu, \sigma^2)$.)

Let

$$\begin{aligned}\hat{\theta}^*_b &= \text{estimate of } \theta \text{ from the } b^{th} \text{ resample}\\ \hat{SE}^*_B &= \left[\frac{1}{B-1}\sum_{b=1}^{B}(\hat{\theta}^*_b - \hat{\theta}^*)^2\right]^{1/2}\end{aligned}$$

1. If you sample B times from a population, how many copies of $\overline{X}$ will there be? How many copies of $s/\sqrt{n}$ will there be?
2. If you re-sample B times from a dataset, how many copies of $\hat{\theta}^*_b$ will there be? How many copies of $\hat{SE}^*_B$?
3. To address the problem, suggest a way of estimating the SE of $\hat{\theta}$ separately for each b.

Solution:

- When sampling from a population, there will be B copies each of $\overline{X}$ and $s/\sqrt{n}$.
- When re-sampling from a dataset, there will be B copies of $\hat{\theta}^*_b$ and 1 copy of $\hat{SE}^*_B$.

To find $\hat{SE}^*(b)$, we must bootstrap twice. The algorithm is as follows:

1. Generate B_1 bootstrap samples (resamples from the original data), and for each sample $\underline{X}^{*b}$ compute the bootstrap estimate $\hat{\theta}^*_b$.
2. Take B_2 bootstrap samples (resamples from the bootstrapped data) from $\underline{X}^{*b}$, and estimate the standard error, $\hat{SE}^*(b)$.
3. The resulting distribution will be based on B_1 values for $T^*(b) = \frac{\hat{\theta}^*_b - \hat{\theta}}{\hat{SE}^*(b)}$.