WU #7 - Simulating CIs

Math 154 - Jo Hardin

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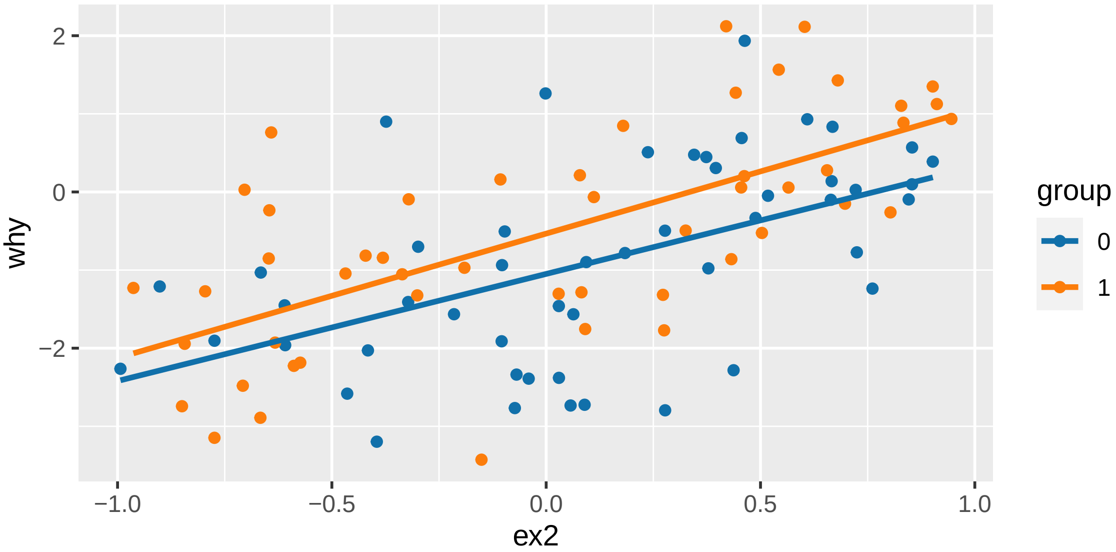
Name: ____

Let's say that you want to check the *mathematical formula* for creating CIs for the slope coefficient in a linear model. In particular, you want to know if the procedure you use actually does capture the population parameter 95% of the time. You are simulating from the following population model:

$$Y = -1 + 0.5X_1 + 1.5X_2 + \epsilon, \quad \epsilon \sim N(0,1)$$

Consider:

```
n <- 100
x1 <- rep(c(0,1), each=n/2)
x2 <- runif(n, min=-1, max=1)
```



```
## # A tibble: 3 x 7
##   term   estimate std.error statistic  p.value conf.low conf.high
##   <chr>     <dbl>     <dbl>     <dbl>    <dbl>    <dbl>     <dbl>
## 1 (Inte~   -1.06      0.151     -7.05 2.60e-10   -1.36     -0.765
## 2 x1        0.532     0.212      2.51 1.37e- 2    0.111     0.953
## 3 x2        1.50      0.196      7.64 1.59e-11    1.11      1.89
```

Write down the steps for the simulation that would assess whether the CI procedure is valid for the parameter on X_2. (No R code here, use words to explain / enumerate the process.)

Solution:

1. Note the data / population above and also the model you are working with. Your results will be based on those data and that model. Sometimes it will be straightforward to generalize, sometimes it won't.
2. Take a sample from the population you've set, and create a linear model.
3. Use the linear model created in 2. to find a CI for the β_2 slope parameter.
4. You know the truth! So check to see if β_2 is in the CI created in step 3.
5. Repeat steps 2 - 4 many times. Find the rate at which your CI captures the true parameter. If your interval in 3 is constructed as a 95% interval, you should be capturing β_2 about 95% of the time. (And furthermore, we'd expect β_2 to be lower than the interval 2.5% of the time and higher than the interval 2.5% of the time.)

R code: The gist of the idea for a single simulated CI. Notice that we *do* capture the true parameter β_2 in the confidence interval.

```
CI <- lm(y~x1+x2) %>% tidy(conf.int=TRUE) %>% data.frame()

CI
```

```
##          term estimate std.error statistic p.value conf.low
## 1 (Intercept)    -1.06      0.15      -7.1 2.6e-10    -1.36
## 2          x1     0.53      0.21       2.5 1.4e-02     0.11
## 3          x2     1.50      0.20       7.6 1.6e-11     1.11
##   conf.high
## 1     -0.77
## 2      0.95
## 3      1.89
```

```
between(beta2, CI[3,6], CI[3,7])
```

```
## [1] TRUE
```

A for-loop that repeats the process 1000 times.

```
beta2.in <- c()
beta0 <- -1; beta1 <- 0.5; beta2 <- 1.5
n_obs<- 100; reps=1000

set.seed(4747)
for(i in 1:reps){
  x1 <- rep(c(0,1), each=n_obs/2)
  x2 <- runif(n_obs, min=-1, max=1)
  y <- beta0 + beta1*x1 + beta2*x2 + rnorm(n_obs, mean=0, sd = 1)

  CI <- lm(y~x1+x2) %>% tidy(conf.int=TRUE) %>% data.frame()

  beta2.in <- c(beta2.in, between(beta2, CI[3,6], CI[3,7]))
}
sum(beta2.in)/reps
```

```
## [1] 0.94
```

A tidier way to run the code (the fewer for-loops, the faster your code will be). Note that the first thing to do is to create all 1000 datasets and put them into a tidy dataframe where one of the columns is a **column of dataframes**!

```
lin_data <-
  data.frame(row_id = seq(1, n_obs, 1)) %>%
  slice(rep(row_id, each = reps)) %>%
  mutate(
    sim_id = rep(1:reps, n_obs),
    x1 = rep(c(0,1), each = n()/2),
    x2 = runif(n(), min = -1, max = 1),
    y = beta0 + beta1*x1 + beta2*x2 + rnorm(n(), mean = 0, sd = 1)
  ) %>%
  arrange(sim_id, row_id) %>%
  group_by(sim_id) %>%
  nest()

lin_data
```

```
## # A tibble: 1,000 x 2
## # Groups:   sim_id [1,000]
##    sim_id data
##     <int> <list>
##  1      1 <tibble [100 x 4]>
##  2      2 <tibble [100 x 4]>
##  3      3 <tibble [100 x 4]>
##  4      4 <tibble [100 x 4]>
##  5      5 <tibble [100 x 4]>
##  6      6 <tibble [100 x 4]>
##  7      7 <tibble [100 x 4]>
##  8      8 <tibble [100 x 4]>
##  9      9 <tibble [100 x 4]>
## 10     10 <tibble [100 x 4]>
## # ... with 990 more rows
```

```
# the linear model function

beta_coef <- function(df){
  lm(y ~ x1 + x2, data = df) %>%
    tidy(conf.int = TRUE) %>%
    filter(term == "x2") %>%
    select(estimate, conf.low, conf.high)
}

lin_data %>%
  mutate(b2_vals = map(data, beta_coef)) %>%
  select(b2_vals) %>%
  unnest() %>%
  summarize(capture = between(beta2, conf.low, conf.high)) %>%
  summarize(capture_rate = sum(capture)/reps)
```

```
## # A tibble: 1 x 1
##   capture_rate
##          <dbl>
## 1        0.967
```

Note that our capture rate was 94.1%. Pretty good!