WU #18 - Support Vector Machines 3

Math 154 - Jo Hardin

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Name: ____

Consider two 2-dimensional data points: $\mathbf{A} = (1, 2)$, $\mathbf{B} = (2, 4)$. Use the following function to map the two data points into the indicated six-dimensional space:

$$\phi(x) = (x_1^2, x_2^2, \sqrt{2}x_1x_2, \sqrt{2}x_1, \sqrt{2}x_2, 1)$$

a. $\phi(\mathbf{A}) =$

b. $\phi(\mathbf{B}) =$

c. $\phi(\mathbf{A}) \cdot \phi(\mathbf{B}) = \phi(\mathbf{A})^t \phi(\mathbf{B}) =$

The kernel function corresponding to the projection above is,

$$K(\mathbf{x}, \mathbf{y}) = (\mathbf{x}^t\mathbf{y} + 1)^2$$

(A degree-two polynomial kernel. More generally, we could choose a polynomial kernel of arbitrary degree: $K(\mathbf{x}, \mathbf{y}) = (\mathbf{x}^t\mathbf{y} + const)^d$.)

d. $K(\mathbf{A}, \mathbf{B}) =$

Solution:

a. $\phi(\mathbf{A}) = (1, 4, 2\sqrt{2}, \sqrt{2}, 2\sqrt{2}, 1)$

b. $\phi(\mathbf{B}) = (4, 16, 8\sqrt{2}, 2\sqrt{2}, 4\sqrt{2}, 1)$

c. $\phi(\mathbf{A}) \cdot \phi(\mathbf{B}) = \phi(\mathbf{A})^t \phi(\mathbf{B}) = (4 + 64 + 16 \cdot 2 + 2 \cdot 2 + 8 \cdot 2 + 1) = 121$

$$K(\mathbf{x}, \mathbf{y}) = (\mathbf{x}^t\mathbf{y} + 1)^2$$

d. $K(\mathbf{A}, \mathbf{B}) = ((1 \cdot 2 + 2 \cdot 4) + 1)^2 = 11^2 = 121$