

Inference on proportions & means of one or two populations

Parameter	Statistic	Standard Error or Deviation	Distribution	Assumptions
$\pi$	$\hat{p}$	$\sqrt{\pi_0(1-\pi_0)/n}$ (HT) $\sqrt{\hat{p}(1-\hat{p})/n}$ (CI)	$Z$ $Z$	successes & failures $\geq 10$
$\pi_1 - \pi_2$	$\hat{p}_1 - \hat{p}_2$ (2 independent samples)	$\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}$ (HT) $\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ (CI)	$Z$ $Z$	successes & failures $\geq 5$ in each group
$\ln(RR) = \ln(\pi_1/\pi_2)$ take inverse $\ln(e)$ for CI	$\ln(\hat{p}_1/\hat{p}_2)$	$\sqrt{\frac{1}{a} - \frac{1}{a+c} + \frac{1}{b} - \frac{1}{b+d}}$	$Z$	large samples
$\ln(OR) = \ln(\tau)$ take inverse $\ln(e)$ for CI	$\ln(\hat{\tau}) = \ln(\hat{O}_1/\hat{O}_2)$	$\sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$	$Z$	large samples
$\mu$	$\bar{x}$	$\sigma/\sqrt{n}$ ( $\sigma$ known) $s/\sqrt{n}$ ( $\sigma$ unknown)	$Z$ $t, df = n - 1$	$n \geq 30$ or normal
$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$ (2 independent samples)	$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ ( $\sigma_1 \neq \sigma_2$ ) $\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}$ ( $\sigma_1 = \sigma_2$ )	$t, df \approx \min\{n_1, n_2\} - 1$ $t, df = n_1 + n_2 - 2$	$n_1, n_2 \geq 30$ or normal
$\mu_d$	$\bar{x}_d$ (matched pairs)	$s_d/\sqrt{n}$ ( $n = \# \text{pairs}$ )	$t, df = n - 1$	$n \geq 30$ or normal
$X_{n+1}$	$\bar{x}$	$\sqrt{s^2 + \frac{s^2}{n}}$	$t, df = n - 1$	normal

To find the  $p$ -value of a HT, look up the score  $\frac{\text{statistic} - \text{hypothesized value}}{\text{SE (or SD)}}$  on the specified distribution in the direction of  $H_a$ .

A  $(100 - \alpha)\%$  CI is of the form  $\text{statistic} \pm \text{multiplier} \times \text{SE (or SD)}$ . The multiplier is the  $(100 - \alpha/2)\%$  point from the specified distribution.

Pooled proportion (under the assumption  $\pi_1 = \pi_2$ ):  $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$ .

Pooled standard error (under the assumption  $\sigma_1 = \sigma_2$ ):  $s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

**Inference for two variables (explanatory variable not necessarily binary)**

**Two categorical variables ( $\chi^2$  test).**  $\chi^2$ -statistic =  $\sum_{i=1}^r \sum_{j=1}^c \frac{(\text{observed}_{ij} - \text{expected}_{ij})^2}{\text{expected}_{ij}}$ , where  $\text{expected}_{ij} = \frac{(\text{total of row } i) \times (\text{total of column } j)}{\text{grand total}}$ .

Under the null hypothesis assumption, the statistic follows a  $\chi^2$  distribution with  $(r-1)(c-1)$  degrees of freedom.

**Categorical explanatory, quantitative response variables (ANOVA).**  $F$ -statistic =  $\frac{\text{MST}}{\text{MSE}}$ , where  $\text{MST} = \frac{n_1(\bar{x}_1 - \bar{x})^2 + \dots + n_I(\bar{x}_I - \bar{x})^2}{I-1}$

and  $\text{MSE} = \frac{(n_1-1)s_1^2 + \dots + (n_I-1)s_I^2}{N-I}$ . Under the null hypothesis assumption, the statistic follows an  $F$  distribution with  $(I-1, N-I)$  degrees of freedom.

**Two quantitative variables (Simple Linear Regression).**

$$r = \frac{1}{n-1} \sum_{i=1}^n \frac{(x_i - \bar{x})(y_i - \bar{y})}{s_x s_y}, \quad b_1 = r \frac{s_y}{s_x}, \quad b_0 = \bar{y} - r\bar{x}, \quad \hat{y} = b_0 + b_1 x, \quad SE(b_1) = \frac{s}{s_x} \frac{1}{\sqrt{n-2}}.$$

Under the null hypothesis  $H_0 : \beta_1 = 0$ , the standardized slope statistic  $\frac{b_1 - 0}{SE(b_1)}$  follows a  $t$  distribution with  $n-2$  degrees of freedom.