

WU #11

Math 58B, Spring 2022

Tuesday, Feb 22, 2022

Your Name: _____

Names of people you worked with: _____

Instructions: Work on this problem in class with your group. Do your best. This piece of paper will be collected during class.

Task: You are trying to convince your college to offer more vegetarian options at mealtime, and you believe that 40% of students are vegetarians. The college says they disagree, and they won't change the offerings unless you can convince them that 35% or more of the student body is vegetarian.

Let's say it is a one sided test with level of significance of 0.1. Also, assume that the sample size will be big enough so that the central limit theorem holds. Start by suggesting a sample size of 50 people.

- What are the null and alternative hypotheses?
- What is the form of the Z-score which will assess whether or not you reject H_0 ?
- What is the Z^* value above which you will reject H_0 ? (If you draw the picture, I'll tell you the number.)
- What \hat{p} do you need to get to reject H_0 (with $n = 50$)?
- If, in fact, the true proportion of vegetarians is $p = 0.4$, what is the probability that you will reject? (Again, if you can draw the correct picture, I'll tell you the number.)
- What is your power?
- What would you do here to increase your power? Is your solution always feasible in other experiments?

Solution:

- $H_0 : p = 0.35, H_a : p > 0.35$
-

$$Z = \frac{\hat{p} - p}{\sqrt{p \cdot (1 - p)/n}} = \frac{\hat{p} - 0.35}{\sqrt{0.35 \cdot 0.65/50}}$$

c.

```
xqnorm(0.9, mean = 0, sd = 1)
```

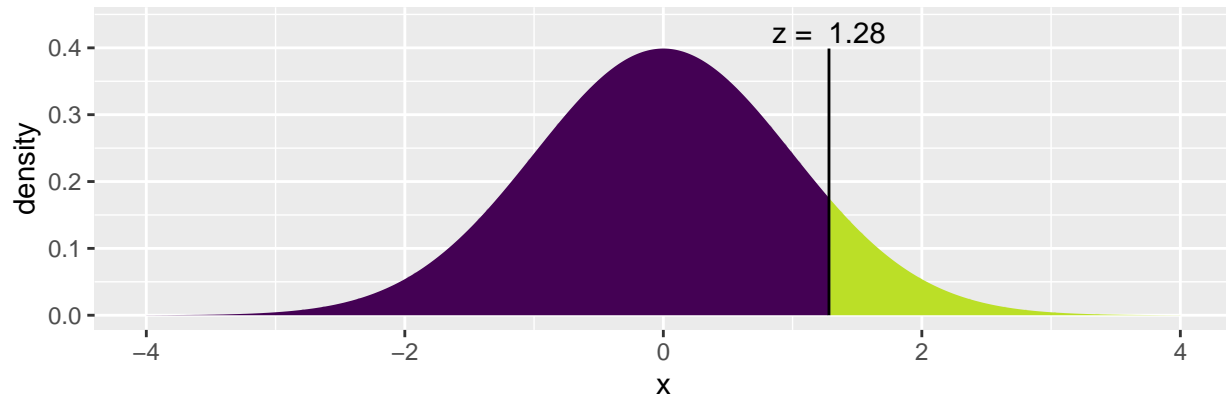
```
##
```

```
## If X ~ N(0, 1), then
```

```
## P(X <= 1.281552) = 0.9
```

```
## P(X > 1.281552) = 0.1
```

```
##
```



```
## [1] 1.281552
```

d.

$$\frac{\hat{p} - 0.35}{\sqrt{0.35 \cdot 0.65/50}} > 1.28 \rightarrow \hat{p} > 0.436$$

e.

```
1 - xpnorm(0.436, mean = 0.4, sd = sqrt(0.4*0.6/50))
```

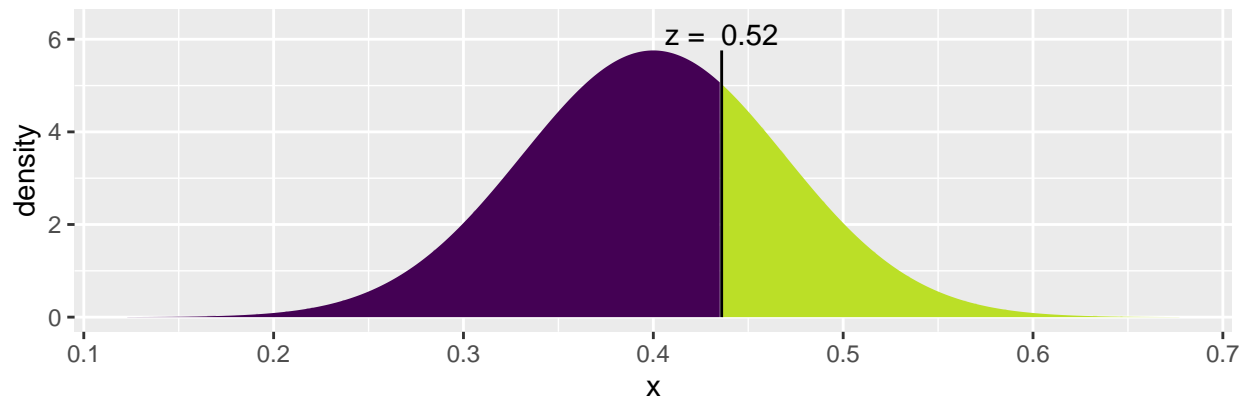
```
##
```

```
## If X ~ N(0.4, 0.06928), then
```

```
## P(X <= 0.436) = P(Z <= 0.5196) = 0.6983
```

```
## P(X > 0.436) = P(Z > 0.5196) = 0.3017
```

```
##
```



```
## [1] 0.3016659
```

- f. With 50 observations, there is only a 0.3 probability (power) that the random sample would reject H_0 **even if** 40% of students were truly vegetarians.
- g. To get to a higher power, more observations need to be sampled. That isn't always possible because often collecting data is expensive and time consuming.

Questions you should be able to answer:

- Why is p used in the denominator of the Z-score (instead of \hat{p}) ?
- Why is $p = 0.35$ instead of $p = 0.4$ in H_0 ?
- Why is the alternative direction $>$ instead of $<$?
- Why is the `xpnorm()` in part (e) centered at 0.4 with a SD of $\sqrt{0.4 \cdot 0.6/50}$?
- Is power always positively related to sample size? Why?