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CS221-Assignment 3

1.

(a)

- 0 → 110
- 1
- 2
- 3
- 4 → 70
- 5 → 93 → 60
- 6 → 50
- 7 → 150 → 40
- 8 → 96 → 140 → 30
- 9 → 97 → 20 → 130
- 10 → 10 → 120

(b)

	0	93	•(93 #3 probe)
	1	70	•(93 #1 probe)
	2	140	
	3		
	4	50	•(96 #1 probe)
	5	120	•(97 #1 probe) •(96 #4 probe)
	6		
	7	30	
	8		
	9		
	10	10	
	11		
•(93 #2 probe)	12	150	•(97 #2 probe) •(96 #2 probe)
	13	96	•(96 #5 probe)
	14	60	
	15	130	
	16		
	17	40	
	18	110	
	19	97	•(97 #3 probe)
	20	20	•(96 #3 probe)
	21		
	22		

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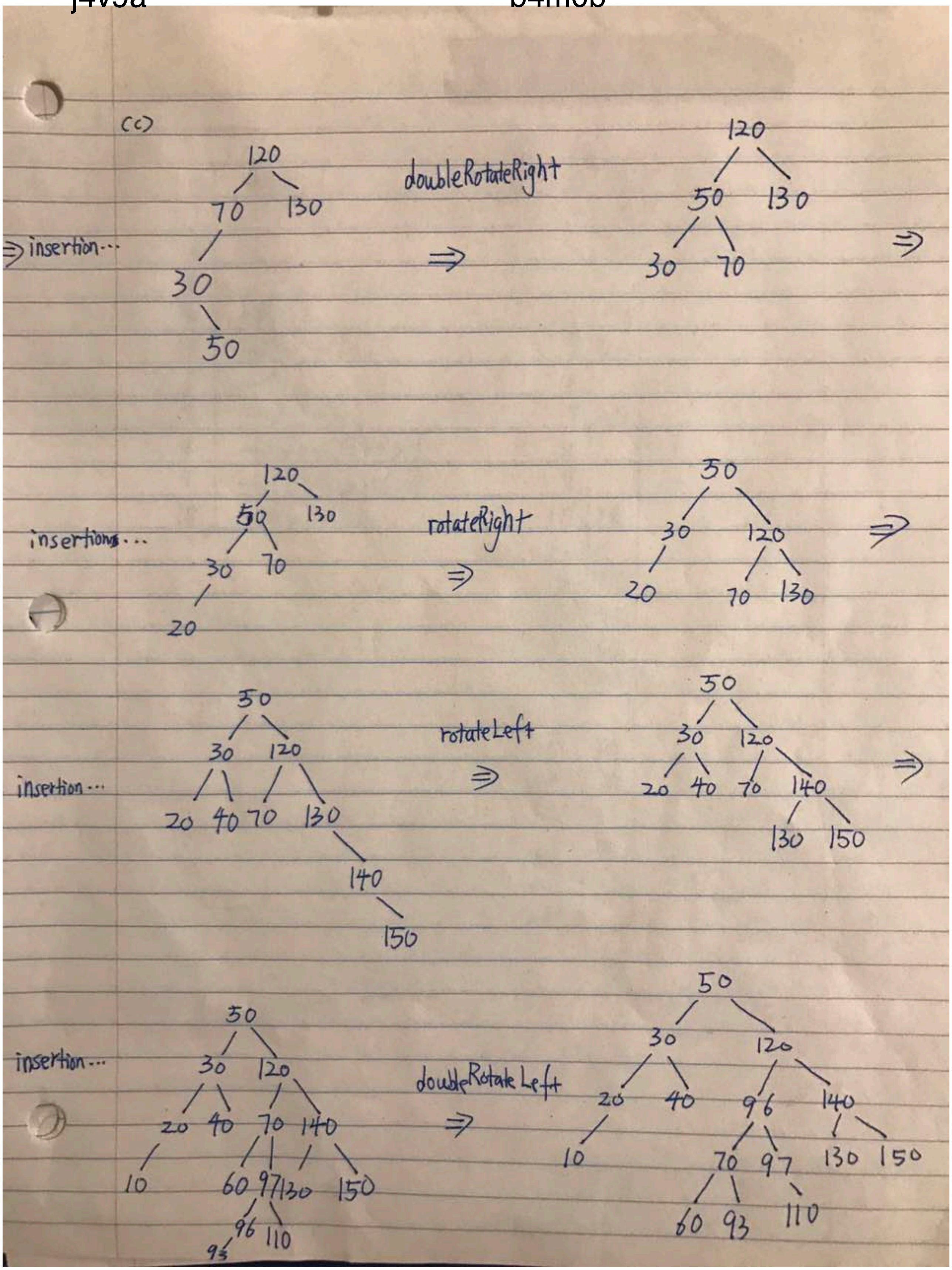

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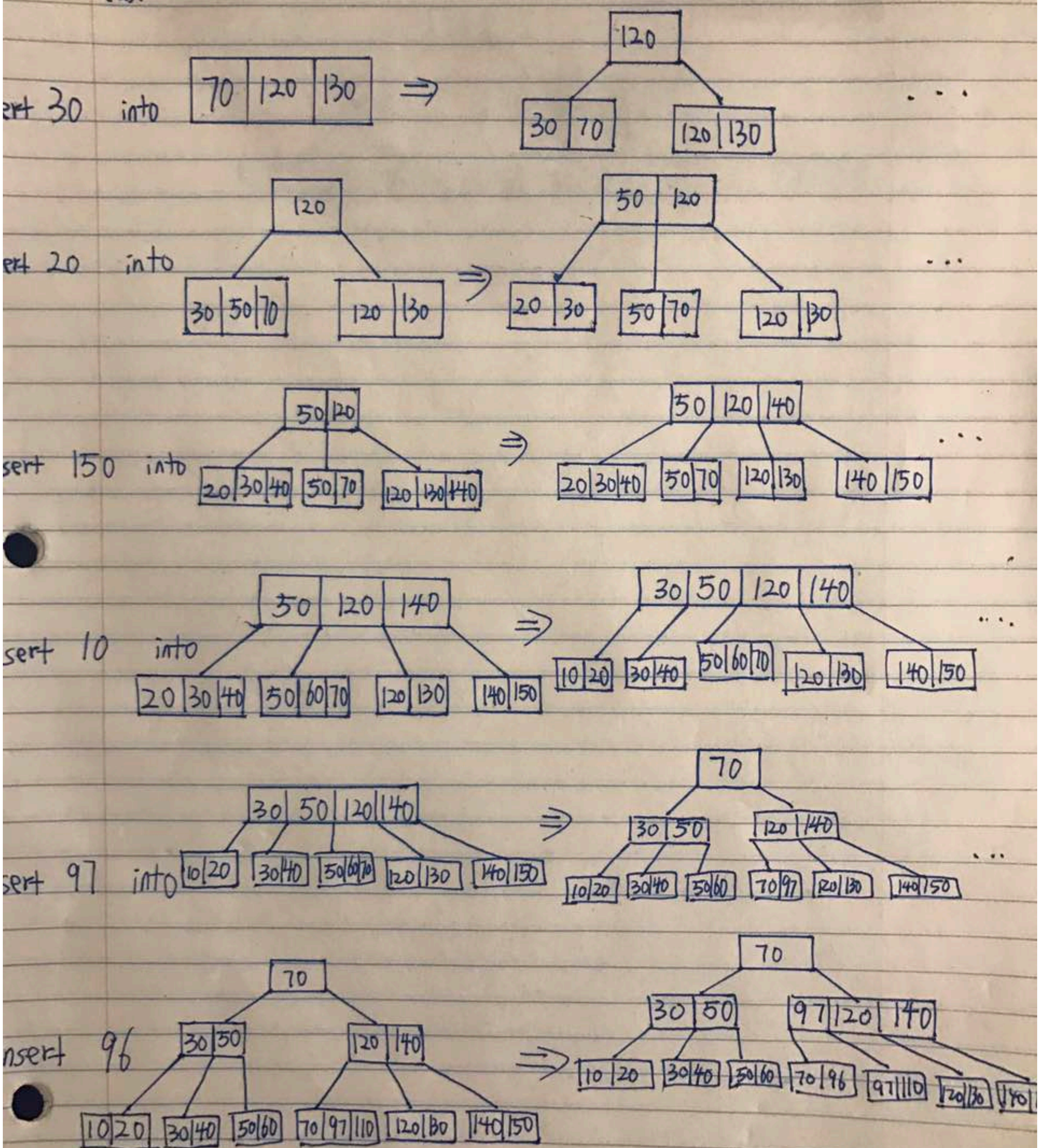
(d).



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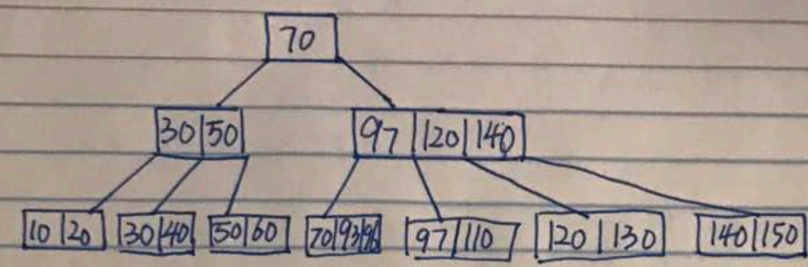
Final:



2.

Proof:

We ~~spitt~~ split the proof into ~~two~~ three cases:

Case 1: No computer is connected to 0 number of other computers

Thus, in case 1, every computer is at least connected to ~~one~~ 1 or more other computers.

Now, we have 6 computers, but with only 5 possilbe number of connections, namely 1, 2, 3, 4, 5.

Therefore, according to the Pigeonhole Principle, at least two computers are directly connected to the same number of other computers.

Case 2: There is only 1 computer does not connect to any other computers

Thus, in case 2, except the one connects to nothing, the remaining 5 computers are connected to some ~~other~~ number of other computers, with only 4 possible number of connections, namely, 1, 2, 3, 4.

Therefore, according to the Pigeonhole Principle, at least two computers are directly connected to the same number of other computers.

Case 3: There are at least two computers connect to 0 number of computers

Thus, there are at two computers connect to the same number of other computers (two or more 0's).

Q.E.D

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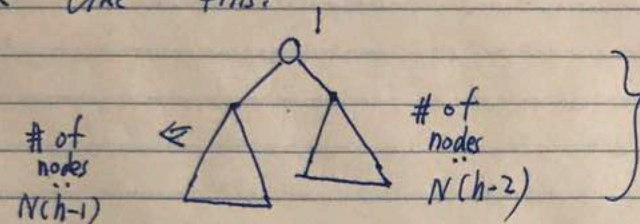
3.

Proof:

Claim: $N(h) = N(h-1) + N(h-2) + 1$

Since h is the height of an AVL tree, ~~so~~ and to keep ~~the~~ the tree to ~~A~~ have smallest number of nodes, the AVL tree should always be $|1|$, instead of 0. Thus, the root's left child's height minus the root's right child's height $= |1|$. Thus, it should look like this:



$N(h) = 1 + N(h-1) + N(h-2)$

$$\Rightarrow N(h) = \begin{cases} 0 & \text{if } h = -1 \\ 1 & \text{if } h = 0 \\ N(h-1) + N(h-2) + 1 & \text{otherwise} \end{cases}$$

Induction:

Base case: $(h=0)$

$$\begin{aligned} N(0) &= F(0+3) - 1 \\ &= F(3) - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

Inductive hypothesis: $(h < k)$ $\quad N(h) = F(h+3) - 1$

by $N(h) = N(h-1) + N(h-2) + 1$ $\Rightarrow$

Inductive step:

$$\begin{aligned} N(k) &= N(k-1) + N(k-2) + 1 \\ &= F(k-1+3) - 1 + F(k-2+3) - 1 + 1 \\ &= F(k+2) + F(k+1) - 1 \end{aligned}$$

by definition of $F(n)$ $= F(k+3) - 1$

$\Rightarrow N(h) = N(h-1) + N(h-2) + 1 = F(h+3) - 1$ Q.E.D