Developing a Boat: Understanding the Forces and Dynamics that goes into an Effective Design

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1 Proposed Design

1.1 Boat Shape

When designing a boat, deciding on an appropriate shape is an important consideration as it impacts the functions we assign to boats: floating level, the ability to right itself, and the speed at which the it cuts through the water. Our boat's shape is defined by a couple of parabolic curves. In the xy plane, the curve that defines the deck shape is:

$$\begin{aligned} x_1 &= |\frac{y}{c}|^2 - \frac{w}{2}, \\ x_2 &= -|\frac{y}{c}|^2 + \frac{w}{2}. \end{aligned} \tag{1}$$

where c is a constant that describes the curve's shape and w is the width of the boat. This curve can be seen in Figure 1. In the xz plane, the curve that defines the hull's cross-sectional shapes is:

$$z = (h - b)(\frac{x}{c})^2 + b \tag{2}$$

where h is the height of the boat, b is defined by the yz curve at points of z, and c is a constant that describes the curve's shape. In the yz plane, the curve that defines the boat's keel shape is:

$$z = (\frac{y}{c})^4 \tag{3}$$

where c is a constant that describes the curve's shape. This curve can be seen in Figure 2.

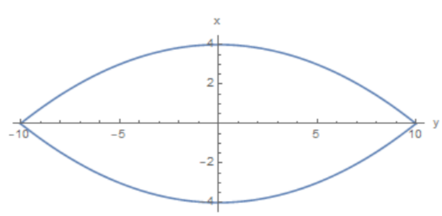

Figure 1: The deck of the boat in the xy plane.

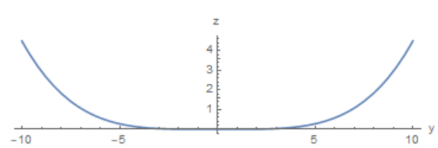

Figure 2: The keel of the boat in the yz plane.

Using these curves, the shape of the boat results in the rendering shown in Figure 3.

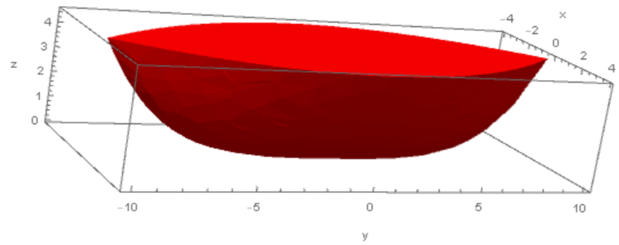

Figure 3: A 3D rendering of our boat's shape. Graphed in Mathematica using the functions described above in the xyz planes where the origin is at center of the keel of the boat. Each curve defines a region of the boat.

We decided upon this design for our boat as the parabolic shaped hull gives us a elongated, yet rounded bottom. This allows the boat to more easily float level in the water due to the lower set *center of mass (COM)* - the point in a body where the gravitational force may be taken to act. The sharp bow and stern also allow the boat to cut through water easier, resulting in a faster boat. Visually, the boat parallels actual hull designs, having a particular resemblance to that of a canoe. Our hope was that this similarity would reflect in the performance of our boat.

1.2 Boat Properties

After deciding on the general shape of our boat, we continuously adjusted the ballast mass and the position at which it is placed. Altering these parameters allow us to move the COM of the boat. This is an important ability as the changing the COM allows us to control how the boat responds to external torque and its ability to right itself. We call this ability the *righting moment* - the force that restores a boat back to it's resting position. The righting moment occurs when the *center of buoyancy (COB)* - the center of gravity for the volume of water which a boat displaces - is not vertically aligned with the COB. This causes a torque, or moment, on the boat which either restores the boat back to level position (as shown in Figure 2) or aids the boat in capsizing itself.

We ended up having a ballast that weighs 700 grams placed 0.5 inches above the origin. The boat weighs 186 grams by itself and 982 grams with the mast and ballast all together. We calculated this weight by designing the boat in SolidWorks (model shown in Figure 3) as per the the curves described above using function driven curves. Applying a material to the SolidWorks design with the same density as hardboard, the material we are using for the actual boat, the program calculated for us the boat's mass.

With the calculated total mass, we can find the COM of the boat itself. Using

$$\begin{gathered} \frac{\rho}{m}\iiint_R r\,dV, \\ COM_r = \frac{\sum_i m_i r_i}{\sum_i m_i} \end{gathered} \tag{4}$$

where ρ is the density of the boat, m is the mass of the boat and r is the directional component to solve for, the COM for each component can be determined. Then to account for the masses of the mast and ballast (treated as point masses), we can use the second equation in formula 4 (weighted average) to find the true COM. We found that the COM of our boat is at (0,0,1.99847) which translates to roughly 2 inches above the center of the keel.

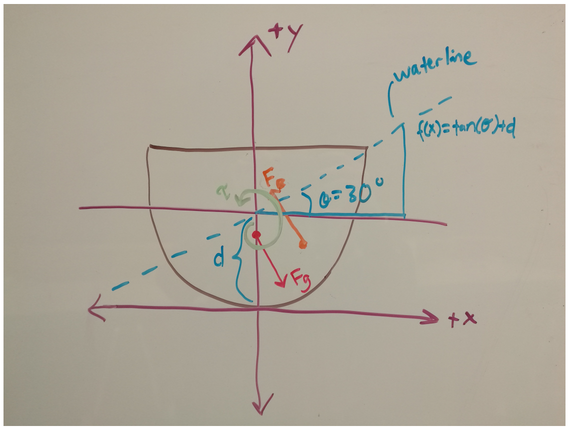

Figure 4: Free body diagram detailing the righting moment on a boat. This torque is caused by the gravitational force and buoyant force not being vertically inline with each other.

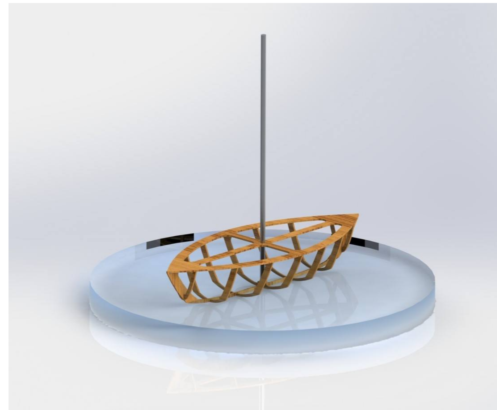

Figure 5: CAD model of our boat.

2 Design Justification

Our design specification was to build a boat that would float level and have an AVS curve (*angle of vanishing stability* - the angle from the vertical at which a boat will no longer stay upright but will capsize) between 120° and 140°.

We know a boat will float if considering the boat system at static equilibrium, a *waterline* - the distance between the bottom of the boat and the top of the water - exists at a point within the region of the boat. If this condition is satisfied then it means that the mass of water displaced is equal to the mass of the whole boat. Therefore, the gravitational force and buoyant force are then equal and in opposite directions, canceling each other out. We can tell a boat floats level when its COM and COB are vertically aligned. To determine if our boat satisfies this parameter, we first must solve for the unknowns.

2.1 The Water

Assuming the boat is floating, then the net forces acting upon the boat is equal to zero and the system is in static equilibrium. We can then find the volume of water displaced by using the equation for the buoyant force and rearraging to solve for volume.

$$\begin{aligned} F_b &= \rho g V_f, \\ V_f &= \frac{F_b}{\rho g}. \end{aligned} \tag{5}$$

where F_b is the buoyant force, ρ is the fluid density, g is the gravitational acceleration, and V_f is the volume of the fluid displaced.

Using the volume of fluid displaced, we can calculate where the waterline would be as it occurs at the height that causes the displaced water to equal the weight of the whole boat as stated by Archimedes' principle. This means that the volume of the displaced fluid must be equal to the volume of the submerged region of the boat. Since the position of the waterline defines the volume of the submerged region, we can set up an equation for the volume of the submerged region involving the waterline. Substituting in the volume of fluid displaced, we can then solve for our unknown (the waterline or draft). This equation is set up as follows:

$$V_f = \int_0^{Length} \int_{-w}^{w} \int_{HullShape}^{tan(\theta)x+d} dy\, dx\, dz \tag{6}$$

where V_f is the volume of water displaced, w is the x coordinate at which the *HullShape* intersects with the waterline, θ is the heel angle, and d is the water line. To find the water line, we use the above equation and solve for d, substituting V_d with the volume of water solved prior.

With the water line solved for, we can now determine the COB. Using the same equation we used for COM, we can solve for the COB as it is at the COM of the region of the boat that is submerged underwater. Note that regions of integration must be adjusted so that the integration happens over the volume of the boat submerged underwater.

Graphing the COM and COB at different heel angles to see how the boat would react offers us the diagram shown in Figure 4.

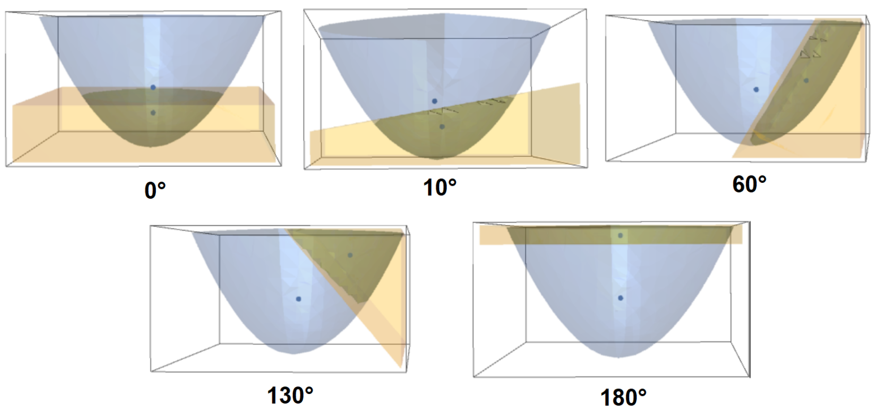

Figure 6: A diagram showing the waterline, COM and COB on our boat at different heel angles. The orange tinted region is the water.

2.2 AVS Curve

Knowing the COM of the boat and the COB at a given theta, we can now calculate the righting moment of the boat at any given heel angle. This is calculated using

$$\tau = r \times F_b = (COB - COM) \times (0, mg\hat{\mathbf{j}}, 0) \tag{7}$$

where τ is the righting moment, r is the position vector at the point of rotation with respect to the COM, and F_b is the buoyant force. Plotting the righting moment vs. the heel angle gives us the Stability or AVS Curve seen in Figure 4.

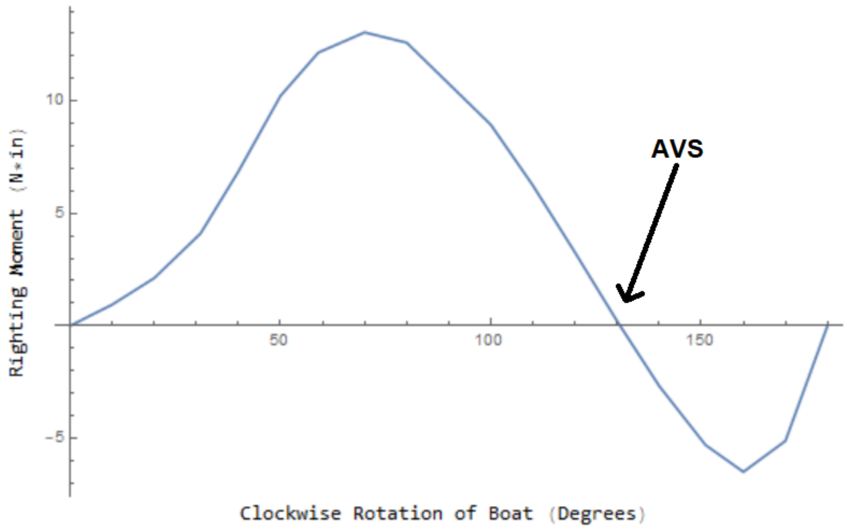

Figure 7: A graph showing the righting moment vs. the heel rotation. The curve tells us about the angle of vanishing stability.

The curve tells us that as our boat turns from 0° to 180°, it has a righting moment that gradually increases until it reaches a maximum force at roughly 73°. The boat's righting moment will then decrease until the boat turns 130° where it's AVS is located. From there on, the torque will aid the capsizing motion until 180° where it will finally float level once again.

3 Expected Performance

We predict our boat to float flat as at 0° the boat should have a righting moment of 0 N*in, depicted in the AVS Curve. This means that it should experience no torque at that angle. The AVS should also occur at 130°. We estimate that our boat will fare relatively well in the speed category as it's not particularly heavy and has a sharp bow and stern to cut through water reducing drag.