# 脑启发人工智能导论 Introduction to Brain-Inspired Artificial Intelligence

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# 基于速率的Hebb规则

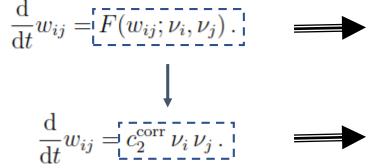
Rate-Based Hebb Rules



#### Rate-Based Hebbian Learning

➤ Hebbian学习规则公式:

▶ 两点特性:



- ✓ **局部性:** 突触效能的变化只能依赖 于局部变量(突触前后发放率 $v_i,v_j$ ) 和突触效能的实际值 $w_{ij}$ , 而不依赖 于其他神经元的活动。
- ✓ 协同性: 突触前和突触后神经元必 须同时活跃,才能发生突触权重变化。

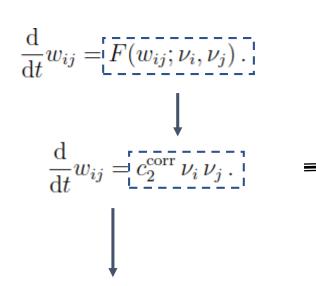
Cells that fire together, wire together

$$c_2^{corr} > 0$$
 Hebbian  $c_2^{corr} < 0$  Anti-Hebbian 同时激活时,突触削弱(反Hebb规则)



#### Rate-Based Hebbian Learning

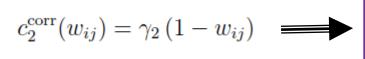
#### >突触权重约束:



如果仅依赖协同性,突触强度会无限增大,或始终处于饱和值!

**协同性**:突触前和突触后神经元必须同时活跃,才能发生突触权重变化。

#### 突触权重饱和:

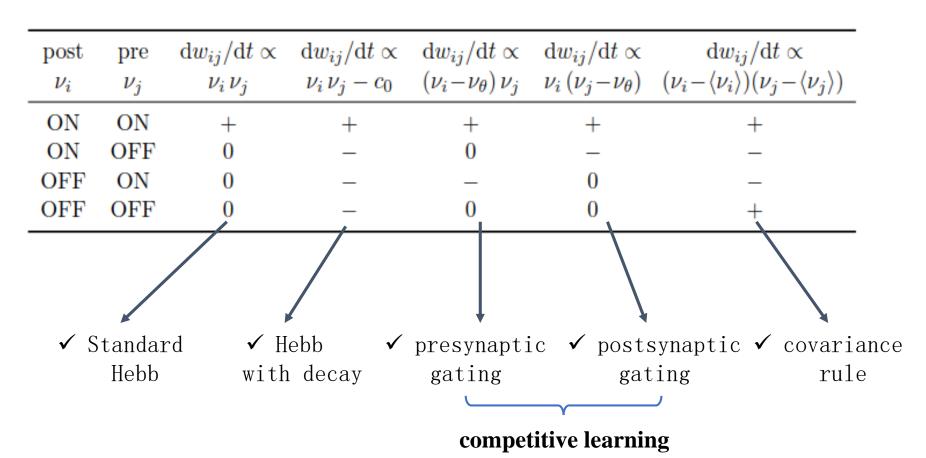


 $w_{ij}$ 不可独立于F, 否则持续施加相同的增强刺激,权重会无限增长; 需保证 $w_{ij}$ 达到最大值  $(w^{max}=1)$  时,  $c_2^{corr}$ 趋近于0.



### Hebb 学习规则变体

▶ 之前给出的Hebb学习规则公式只是一种可能,可以**指定突触重量的增长和衰减规则给出多种基于Hebb学习规则的变体**。



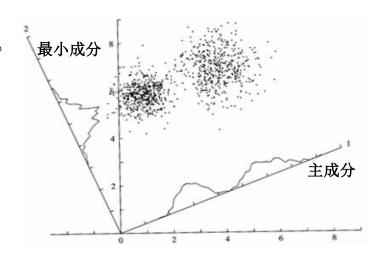


- ➤ 主成分分析 (principal component analysis, PCA): PCA是提取输入数据主成分的非神经网络方法
  - ✓ 找出一个方向分量w(主成分)能让高维数据样本中具有最大可变性(variability)。

样本点在这个方向分量上的 投影能最大程度分开

最大化样本点投影的方差







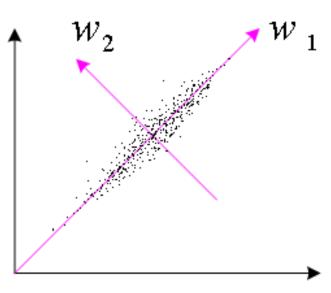
#### ▶ PCA计算过程:

- 1. 假定数据集:  $x = \{x_i, i = 1, ..., N\}$ , E[x]=0 (去中心化,0均值)
- 2. 设定一组正交方向分量:  $\mathbf{w} = \{\mathbf{w}_1, \mathbf{w}_2, ..., \mathbf{w}_d\}, w_i$ 是标准正交基( $\|\mathbf{w}_i\|_2 = 1$ ,  $w_i^T w_i = 0$ )

在欧几里德空间中,集合: $\{e_1=(1,0,0), e_2=(0,1,0), e_3=(0,0,1)\}$ 组成一个标准正交基。

3. 样本在方向分量上的投影:

$$y = w^T x$$



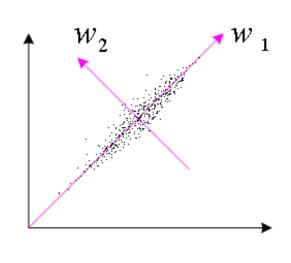


- ➤ PCA计算过程:
- 4.最大化样本点投影的方差, 计算协方差矩阵C:

maximizing 
$$J^{PCA}(w) = E[y^2]$$
  

$$= E[(w^T x)^2]$$

$$= w^T E[xx^T] w$$



C是协方差矩阵 (Correlation Matrix),特点:对称,非负定



- ➤ PCA计算过程:
  - 5.协方差矩阵C做特征值分解:

#### 6.根据拉格朗日乘子法求解:

特征值(Eigenvalues):  $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$ 

特征向量(Eigenvectors):  $e_1, e_2, \cdots, e_n$ 

$$w^{T}Cw = \lambda w^{T}w = \lambda$$

$$\lambda_{1} \geq w^{T}Cw \geq \lambda_{n}$$

$$w = e_{1}, \quad w^{T}Cw = \lambda_{1}$$

$$w = e_{n}, \quad w^{T}Cw = \lambda_{n}$$

样本点投影的方差就是协方差矩阵C的特征值,最大特征值  $\lambda_1$ 相关联的特征向量  $e_1$  (主成分)具有最大方差。

$$\max_{\|w\|=1} \{J^{PCA}(w)\} = \max_{\|w\|=1} \{w^T C w\} = e_1^T C e_1 = \lambda_1.$$



#### > Hebbian规则修改突触权重

$$h = \sum_{i} w_i v_i^{\text{pre}}$$

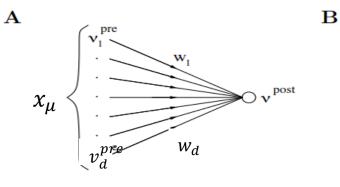
✓ 线性速率模型: 突触前发放率加权求和

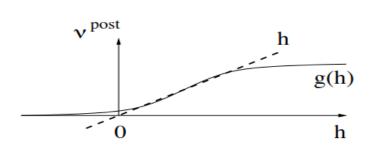
$$v_i^{post} = g(h)$$

✓ **非线性速率模型**:突触后神经元发放率 (firing rate)由总输入的非线性函数g给出

$$\Delta w_i = \gamma \, \nu_i^{\text{post}} \, \nu_i^{\text{pre}} =$$

✓ 学习率 (learning rate):  $0 < \gamma \ll 1$  与相关系数 $c_2^{corr}$ 有关。





A: 模型前馈结构,输入模式 $x_{\mu} = v_{i}^{pre}$  for 0 < j < d

B: 虚线表示**线性速率模型**的突触后发放率; 实线表示对线性速率模型加上非线性激活函数(sigmoidal)的突触后发放率。

线性速率模型类似于PCA的方向分量投影操作

能否通过hebb规则 学习到这种主成分

- ➤ Hebbian规则基于相关性学习,能够提取输入数据的第一主成分
- 1. 假定输入模式:  $x = \{x^{\mu}, \mu = 1, ..., N\}$ ,
- 2. 突触权重更新:  $w_i(\mu + 1) = w_i(\mu) + \gamma \sum_j w_j x_i^{\mu+1} x_i^{\mu+1}$
- 3. 考虑突触更新的长期行为, 权重期望的更新满足:

$$\langle w_{i}(\mu+1)\rangle = \langle w_{i}(\mu)\rangle + \gamma \sum_{j} \left[ \langle w_{j}(\mu)x_{j}^{\mu+1}x_{i}^{\mu+1} \rangle \right]$$
 输入模式独立,  

$$= \langle w_{i}(\mu)\rangle + \gamma \sum_{j} \langle w_{j}(\mu)\rangle \left\langle x_{j}^{\mu+1}x_{i}^{\mu+1} \right\rangle$$
 期望可拆分  

$$= \langle w_{i}(\mu)\rangle + \gamma \sum_{j} \overline{C}_{ij} \langle w_{j}(\mu)\rangle$$
  
其中,  $C_{ij} = \frac{1}{N} \sum_{\mu=1}^{N} x_{i}^{\mu} x_{j}^{\mu} = \left\langle x_{i}^{\mu}x_{j}^{\mu} \right\rangle_{\mu}$  相关矩阵



采用矩阵表示

$$\langle w(\mu + 1) \rangle = (I + \gamma C) \langle \mathbf{w}(\mu) \rangle$$
  
=  $(I + \gamma C)^{\mu + 1} \langle \mathbf{w}(0) \rangle$ 

- 权重向量  $\mathbf{w}(\mu) = (w_i(\mu), ..., w_d(\mu))$
- 单位矩阵 /
- 4. 用C的特征向量 $e_k$ 表示权重向量:

$$\langle w(\mu) \rangle = \sum_{\mathbf{k}} \mathbf{a}_{\mathbf{k}}(\mu) \mathbf{e}_{\mathbf{k}}$$

$$\langle w(\mu) \rangle = \sum_{\mathbf{k}} (1 + \lambda_{\mathbf{k}})^{\mu} \mathbf{a}_{\mathbf{k}}(0) \mathbf{e}_{\mathbf{k}}$$

$$\langle w(\mu) \rangle \xrightarrow{\mathbf{n} \to \infty} (1 + \lambda_{\mathbf{1}}) \mathbf{a}_{\mathbf{1}}(0) \mathbf{e}_{\mathbf{1}}$$

权重向量指数增长,并由相关矩阵C最大特征值的特征向量(第一主成分)主导



#### >两者比较分析:

#### PCA方法:

- ✓ 需计算协方差矩阵C,并用**奇异值分解(SVD)**计算C的特征值和对应 的特征向量。该过程**计算复杂度较高**;
- ✓ 不适合在线提取主成分。

#### Hebb学习方法:

- ✔ 不计算协方差矩阵C ,可降低计算复杂度
- ✓可在线计算。



- ▶ Hebb学习问题(不稳定性):
  - **1. 输入归一化**: 只有在输入数据归一化(零均值,原点分布)时,才可得到输入数据的主成分
  - 2. 权值发散: 权重向量的方向向主成分的方向移动时, 其范数无限增长;

$$||w(k)|| \rightarrow +\infty$$
  
改进  
权重归一化



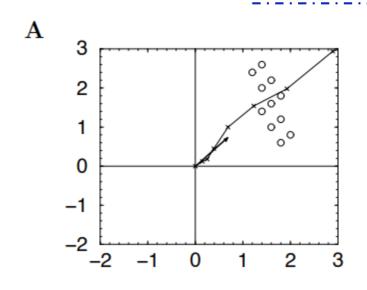
▶ 输入归一化影响:

#### 基于标准Hebb规则

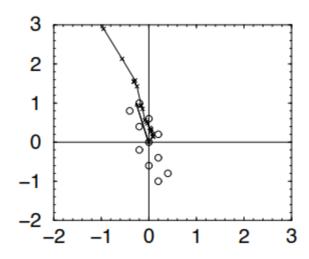
圆圈:输入模式分布;

十字连线: 权重更新过程

箭头: 相关矩阵主要特征向量



 $\mathbf{B}$ 



A: 未归一化输入模式,权重更新方向与相关矩阵的主要特征向量的方向一致;但与输入数据的第一个主成分 e1方向不同;

B: 归一化输入模式,相关矩阵的主要特征向量与输入数据的第一个主成分 e1 重合,权重更新可提取到数据集第一主成分e1。

### 权重归一化 Weight Normalization

- ▶ 减法归一化(Subtractive Normalization):
  - ✔ 通过减去每个突触的平均总权重变化,保持突触权重和恒定

$$\Delta w_{i} = \Delta \widetilde{w}_{i} - N^{-1} \sum_{j} \Delta \widetilde{w}_{j}$$

$$= \gamma \left( \sum_{j} w_{j} x_{j}^{\mu} x_{i}^{\mu} - N^{-1} \sum_{k} \sum_{j} w_{j} x_{j}^{\mu} x_{k}^{\mu} \right)$$

 $\sum_{i} \Delta w_{i} = 0 \rightarrow \sum_{i} w_{i} = const$ 

Δwi: 标准Hebb学习的权重变化

学习非局部:权重的变化取决于所有突触前神经元的活动

✓ 计算权重期望  $\langle w(\mu) \rangle$  , 得到矩阵表达:

$$\langle w(\mu) \rangle = [(1 + \gamma(C - \bar{C}))]^{\mu} \langle \mathbf{w}(0) \rangle$$

$$\overline{C_{ij}} = N^{-1} \sum_{\mathbf{k}} C_{kj}$$

权重更新方向与( $C - \overline{C}$ )矩阵特征向量的方向(对角向量正交方向)一致

问题:权重之和保持不变,但单个权重仍在增长。需要其他一些约束防止权重的某些分量超出所有边界,比如设置"硬"边界(hard bounds);



### 重归一化 Weight Normalization

- ▶ 乘法归一化(Multiplicative Normalization):
  - ✔ 所有权重都乘以一个公共因子(权重范数倒数),保持平方权重之和(权重 长度)不变。

$$w(\mu + 1) = \frac{w(\mu) + \Delta \widetilde{w}(\mu)}{\|w(\mu) + \Delta \widetilde{w}(\mu)\|}$$

$$\left\{ \sum_{i} w_{i}^{2} = const \right\}$$

$$\Delta \widetilde{\boldsymbol{w}}(\mu) = \gamma \boldsymbol{y}(\mu) \boldsymbol{x}(\mu)$$

基于乘法归一化可推导出0ja学习规则

$$w_i(\mu+1) = \frac{w_i(\mu) + \gamma \cdot \boldsymbol{y}(\mu) x_i(\mu)}{\sqrt{\sum_{j=1}^m \left[w_j(\mu) + \gamma \cdot \boldsymbol{y}(\mu) x_j(\mu)\right]^2}}$$

$$w_{i}(\mu + 1) = w_{i}(\mu + 1)|_{\gamma=0} + \left[\frac{dw_{i}(\mu + 1)}{d\gamma}\right]|_{\gamma=0} \gamma + o(\gamma^{2})$$
$$= w_{i}(\mu) + \gamma[\mathbf{y}(\mu)x_{i}(\mu) - \mathbf{y}^{2}(\mu)w(\mu)] + o(\gamma^{2})$$

0 ja规则更新方向与 权重正交,保证权 重向量长度不变

假设权重仅发生非常小的变化 $(\gamma \ll 1)$ ,可去掉高阶项 $o(\gamma^2)$ 

$$w_i(\mu + 1) = w_i(\mu) + \gamma \cdot [\mathbf{y}(\mu)x_i(\mu) - \mathbf{y}^2(\mu)w_i(\mu)] \mid$$

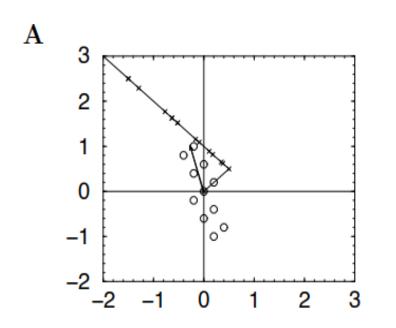
发放率形式  $\Delta w_i = \gamma \cdot \left[ v^{post} v_i^{pre} - w_i(\mu) (v^{post})^2 \right]$  **一)** 0 ja 规则

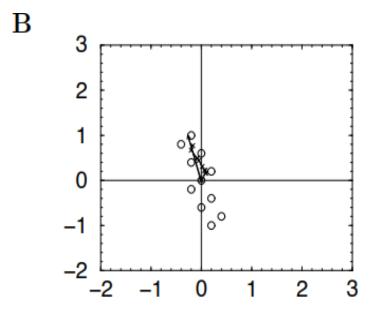




### 权重归一化 Weight Normalization

▶ 权值归一化影响:





A: 减法归一化: 与对角向量(1,1)的正交方向一致,如果没有其他约束,权重向量的长度将无限增长;

B: 乘法归一化(Oja): 权重向量快速收敛到数据集的第一个主成分(箭头)可以保证权重向量长度不变。



# 基于时间的Hebb规则

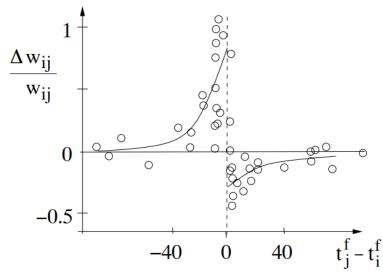
Temporal Hebb Rules



## 基于时间的Hebb学习

基于速率的Hebb假设核心:突触前和突触后神经元都必须活跃,以诱导突触的增强(Both pre- and postsynaptic neuron have to be active in order to induce a strengthening of the synapse.)这样<mark>忽略了权重变化的时间要求。两个神经元何时被视为一起活动?</mark>

#### > 突触前后脉冲的时间需求:



在两阶段学习窗口内(毫秒ms),**突触变化与**  $(t_j^f - t_i^f)$  时间差函数的关系:

- $\checkmark$  只有当突触前活动( $\mathbf{t}_{i}^{f}$ 时刻)和突触后活动 ( $\mathbf{t}_{i}^{f}$ 时刻)的彼此**非常接近**时,突触才会变 化 $\Delta \mathbf{w}_{ij}$ ;
- ✔ 两种变化形式:

LTP: 突触前脉冲先于突触后脉冲; LTD: 突触后脉冲先于突触前脉冲。



- ➤ Spike-Time Dependent Plasticity (STDP)

  STDP关注**突触前脉冲和突触后脉冲时间相关性**所引起的突触效能变化
- > 突触前脉冲触发更新

$$\frac{\mathrm{d}}{\mathrm{d}t}w_{ij}(t) = S_j(t) \left[ a_1^{\mathrm{pre}} + \int_0^\infty a_2^{\mathrm{pre,post}}(s) S_i(t-s) \, \mathrm{d}s \right]$$

$$S_j(t) = \sum_f \delta(t - t_j^{(f)})$$
 突触前脉冲序列  $S_i(t) = \sum_f \delta(t - t_i^{(f)})$  突触后脉冲序列

 $a_1^{\text{pre}}$  突触前non-Hebbian LTP 导致的突触效能变化幅度,为正值;

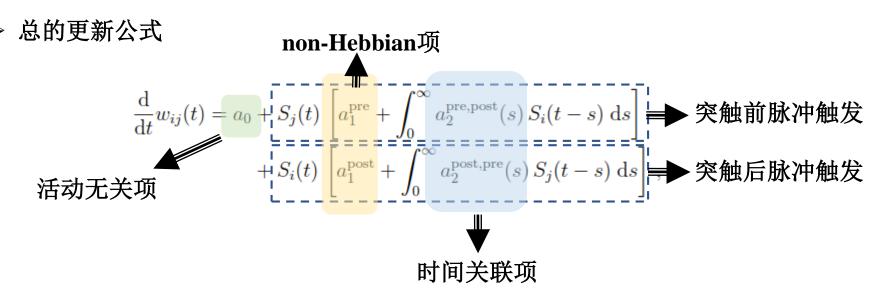
 $a_2^{\text{pre,post}}(s)$  积分核描述另一种时间相关性的突触效能变化幅度,表示在突触前的脉冲到达延时s后产生突触后动作电位时的权重变化

✔ 没有额外的突触后动作电位,突触前每一个脉冲都可以触发突触效能的变化。



#### > 突触后脉冲触发更新

突触效能的变化也可以由突触后脉冲触发,与突触前触发的变化类似,同样包括 $\mathbf{non-Hebbian}$ 项  $a_1^{\mathbf{post}}$  和**时间关联项**  $a_2^{\mathbf{post,pre}}$ 



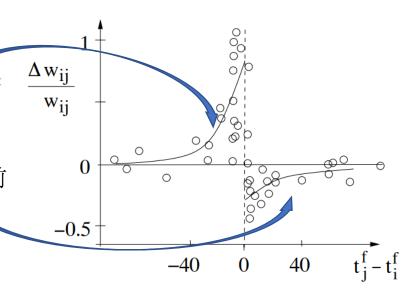
✓ 所有触发项(**活动无关项,non-Hebbian项,时间关联项**)同样**取决于突触 效能的实际值,**这种依赖性导致加强已经增强的突触或削弱被抑制的突触变得越来越困难。



#### > 学习窗口 (learning window)

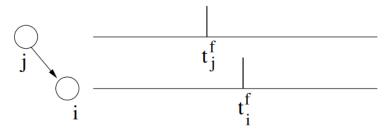
所有触发项都包含了学习窗口的概念:

- $\checkmark$   $a_2^{post, pre}$ : 突触前脉冲到达后延迟s才产生突触后动作电位时的权重变化量;
- $\checkmark$   $a_2^{pre,post}$ :突触后脉冲产生后延迟s突触前 动作电位才到达时的权重变化量;



#### 学习窗口定义如下:

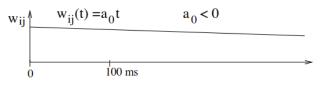
$$W(s) = \begin{cases} a_2^{\text{post,pre}}(-s) & \text{if } s < 0, \\ a_2^{\text{pre,post}}(s) & \text{if } s > 0, \end{cases}$$



$$s = t_j^{(f)} - t_i^{(f)}$$
 突触前脉冲到达和突触脉冲之间的延迟。

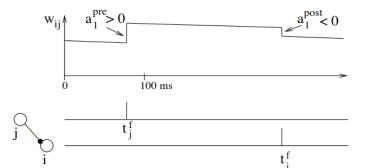


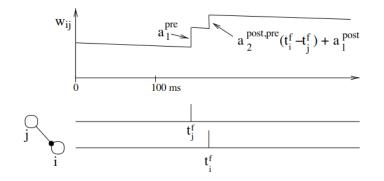
 $\mathbf{A}$ 





 $\mathbf{B}$ 





**A:**突触前后神经元无脉冲活动,0阶活动 无关项 $a_0 < 0$ 导致突触权重线性衰减;

**B:**突触脉冲到达或突触后脉冲发放,1阶 non-Hebbian项改变权重:

✓ 
$$a_1^{pre} > 0$$
 (突触脉冲  $t_i^f$  到达): 突触增强;

✓ 
$$a_1^{\text{post}}$$
 < 0(突触脉冲 $t_i^f$ 到达): 突触减弱;

C:假设时间关联项 $a_2^{post, pre}$ 满足:

$$a_2^{\text{post,pre}}(t_i^{(f)} - t_j^{(f)}) = W(t_j^{(f)} - t_i^{(f)}) > 0 \text{ for } t_j^{(f)} < t_i^{(f)}.$$

如果突触前脉冲到达 $t_j^f$ 后不久出现突触后脉冲 $t_i^f$ ,则权重变化在突触后脉冲时刻可以是为正值,即使 $a_1^{\text{post}} < 0$ 。



▶ 指数学习窗口 (Exponential learning window)

$$W(s) = \begin{cases} A_{+} \exp[s/\tau_{1}] & \text{for } s < 0, \\ A_{-} \exp[-s/\tau_{2}] & \text{for } s > 0, \end{cases}$$

#### 两种动力学过程结合

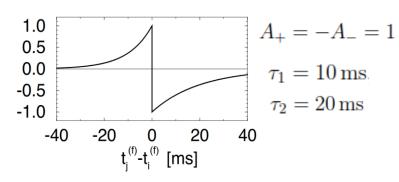
- ✓ 突触增强: 突触前脉冲稍早于突触后脉冲 (s < 0时W(s) > 0
- ✓ 突触减弱: 突触前脉冲稍晚于突触后脉冲 (W(s) < 0);
- ➤ 权重依赖 ( weight dependence ):

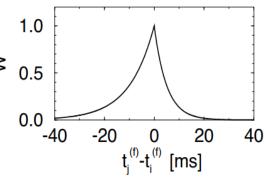
#### 突触权重需保持一定范围

- ✓ 突触增强项(正): 权重依赖 $\propto$  (1  $w_{ij}$ ) e.g.,  $A_{+}$ = (1 –  $w_{ij}$ )  $a_{+}$
- ✓ 突触减弱项(负): 权重依赖 × w<sub>ij</sub>e.g., A<sub>-</sub>= w<sub>ij</sub>a<sub>-</sub>

该学习窗口公式只描述LTP或者LTD中的一种, 不能同时描述两者 常数 $A_+>0$  and  $A_-<0$ 和 $au_{1,2}$ 

突触前脉冲到达和突触后发放时间差  $\mathbf{s} = t_i^f - t_i^f$  的两阶段学习窗口  $\mathbf{W}(\mathbf{s})$ :





$$\tau_1 = 20 \,\mathrm{ms}$$
$$\tau_2 = 10 \,\mathrm{ms}$$



#### ➤ 描述LTP和LTD的时间窗口

假设变量[a]和[b]对LTP有贡献

$$\frac{\mathrm{d}}{\mathrm{d}t}w_{ij}^{\mathrm{LTP}} = \gamma^{\mathrm{LTP}} \left[ a(t) \right] \left[ b(t) \right].$$

假设变量[c]和[d]对LTD有贡献  $\frac{\mathrm{d}}{\mathrm{d}t}w_{ij}^{\mathrm{LTD}} = -\gamma^{\mathrm{LTD}}\left[c(t)\right]\left[d(t)\right].$ 

总权重变化:

$$w_{ij}^{\text{corr}} = w_{ij}^{\text{LTP}} + w_{ij}^{\text{LTD}},$$

学习窗口: 
$$W(s) = \begin{cases} A_+ \exp[s/\tau_a] - A_- \exp[s/\tau_c] & \text{for } s < 0 \\ A_+ \exp[-s/\tau_b] - A_- \exp[-s/\tau_d] & \text{for } s > 0 \end{cases}$$

设置 
$$\begin{cases} A_{+} = \gamma^{\text{LTP}} d_a d_b \tau_a \tau_b / (\tau_a + \tau_b) \text{ and } A_{-} = \gamma^{\text{LTD}} d_c d_d \tau_c \tau_d / (\tau_c + \tau_d) \\ d_b = 1 / \tau_b \text{ and } d_c = 1 / \tau_c, \quad \tau_b \to 0 \text{ and } \tau_c \to 0 \end{cases}$$

突触前脉冲到达后触发突触后脉冲,读出[a]的电流值并诱导LTP

$$W(t_j^{(f)} - t_i^{(f)}) = \gamma^{\text{LTP}} d_a \exp\left(-\frac{t_i^{(f)} - t_j^{(f)}}{\tau_a}\right) \quad \text{for} \quad t_j^{(f)} < t_i^{(f)}.$$

突触前脉冲在突触后脉冲之后到达,读出[d]的电流值并诱导LTD

$$W(t_j^{(f)} - t_i^{(f)}) = -\gamma^{\text{LTD}} d_d \exp\left(-\frac{t_j^{(f)} - t_i^{(f)}}{\tau_d}\right) \quad \text{for} \quad t_j^{(f)} > t_i^{(f)}.$$



### 广义脉冲学习规则

- > 突触前后膜电位定义:
  - $\checkmark$  每个动作电位的持续时间较短,突触前膜电位可用一系列 δ 函数来近似  $u_j^{\text{pre}}(t) = \sum_f \delta(t-t_j^{(f)}) \ t_j^{(f)}$ 表示突触前终端(presynaptic terminal)的脉冲到达时间
  - $\checkmark$  采用脉冲响应模型(spike response model, SRMO)突触后膜电位  $u_i^{\text{post}}(t) = \eta(t \hat{t}_i) + h_i(t)$

$$\hat{t}_i$$
 最后一次突触放电时间  $h_i(t)$  局部突触后电位

η 突触位置的动作电位反向传播 (back propagating action potential) 的时间过程 (time course)



### 广义脉冲学习规则

- > Sharply peaked back propagating action potential:
  - ✔ 假设反向传播的动作电位是振幅大且持续时间短的峰值(Sharply peaked): 突触后神经元的膜电位由反向传播动作电位控制。

$$u_i^{\mathrm{post}}(t) = \sum_f \delta(t - t_i^{(f)})$$
   
 $h(t)$  项忽略, $\eta$ 用 $\delta$ 函数近似   
膜电位退化为一系列脉冲表示   
 $u_j^{\mathrm{pre}}(t) = \sum_f \delta(t - t_j^{(f)})$    
 $\frac{\mathrm{d}}{\mathrm{d}t}w_{ij}(t) = a_0 + S_j(t) \left[ a_1^{\mathrm{pre}} + \int_0^\infty a_2^{\mathrm{pre,post}}(s) \, S_i(t-s) \, \mathrm{d}s \right]$    
 $+ S_i(t) \left[ a_1^{\mathrm{post}} + \int_0^\infty a_2^{\mathrm{post,pre}}(s) \, S_j(t-s) \, \mathrm{d}s \right],$ 

$$\frac{\mathrm{d}w_{ij}}{\mathrm{dt}} = a_0 + \sum_{f} \alpha_1^{\mathrm{pre}}(t - t_j^{(f)}) + \sum_{f} \alpha_1^{\mathrm{post}}(t - t_i^{(f)}) + \sum_{f} \sum_{f} \alpha_2^{\mathrm{corr}}(t - t_i^{(f)}, t - t_j^{(f)}) + \dots,$$

- ✔ 权重变化连续
- $\checkmark t_j^f$ 时刻的突触前脉冲会导致权 重在 $t_j^f$ 后一些时间产生变化



### \*义脉冲学习规则

#### No back propagating action potential

✓ 突触位置的膜电位由缓慢变化的电位(potential)  $h_i(t)$ 控制。 假设 $h_i(t)$ 保持在常数值 $h^{post}$ 

$$u_{j}^{\text{pre}}(t) = \sum_{f} \delta(t - t_{j}^{(f)})$$

$$u_{i}^{\text{post}}(t) = h^{\text{post}}$$

$$\frac{d}{dt}w_{ij}(t) = a_{0} + S_{j}(t) \left[ a_{1}^{\text{pre}} + \int_{0}^{\infty} a_{2}^{\text{pre,post}}(s) S_{i}(t - s) \, ds \right]$$

$$+ S_{i}(t) \left[ a_{1}^{\text{post}} + \int_{0}^{\infty} a_{2}^{\text{post,pre}}(s) S_{j}(t - s) \, ds \right],$$

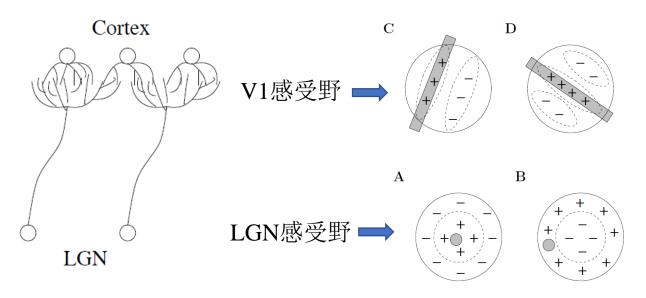
$$\frac{\mathrm{d}w_{ij}}{\mathrm{d}t} = a_0 + \sum_f a_1^{\mathrm{pre}} \delta(t - t_j^{(f)})$$
 ✓ 权重变化是由突触前脉冲触发的。 ✓ 权重更新方向和值取决于突触后膜电位。

$$+a_1^{\text{post}} h^{\text{post}} + a_2^{\text{corr}} h^{\text{post}} \sum \delta(t - t_j^{(f)}) + \dots$$

Fusi et al. (2000) [1] 基于脉冲学习理论的起点

#### Receptive Field Development

#### 从LGN到V1的感受野的形成过程

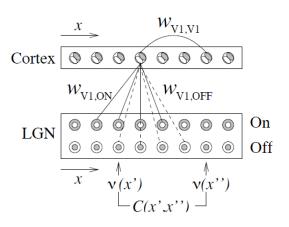


Receptive fields (schematic). A, B. Circularly symmetric receptive field as typical for neurons in the LGN. ON-center cells (A) are excited by light spots (gray) falling into the center of the receptive field. In OFF-center cells (B) the arrangement of excitatory and inhibitory regions is reversed. C, D. Two examples of receptive fields of simple cells in the primary visual cortex. The cells are best stimulated by a light bar oriented as indicated by the grey rectangle.

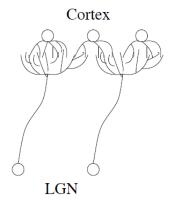


#### Receptive Field Development

 $\mathbf{A}$ 



В



A. Wiring diagram between LGN and cortex (schematic). B. Axons from LGN cells project only to a small region of cortex. Synaptic contacts are limited to a localized cluster of V1 neurons.

Intracortical projections are supposed be constant and dominated by short-range excitation

$$w_{\mathrm{V1,V1}}(\boldsymbol{x}_1,\boldsymbol{x}_2) \propto \exp\left(-\frac{\|\boldsymbol{x}_1-\boldsymbol{x}_2\|}{\sigma_{\mathrm{V1,V1}}^2}\right).$$

The firing rate  $v_{V1}(x)$  of a cortical neuron at position x

$$\begin{split} \nu_{\text{V1}}(\boldsymbol{x}) &= \sum_{\boldsymbol{x}'} w_{\text{V1,ON}}(\boldsymbol{x}, \boldsymbol{x}') \, \nu_{\text{ON}}(\boldsymbol{x}') + \sum_{\boldsymbol{x}'} w_{\text{V1,OFF}}(\boldsymbol{x}, \boldsymbol{x}') \, \nu_{\text{OFF}}(\boldsymbol{x}') \\ &+ \sum_{\substack{\boldsymbol{x}' \\ (\boldsymbol{x}' \neq \boldsymbol{x})}} w_{\text{V1,V1}}(\boldsymbol{x}, \boldsymbol{x}') \, \nu_{\text{V1}}(\boldsymbol{x}') \,, \end{split}$$

vON/OFF(x') is the firing rate of a neuron in the ON/OFF layer of the LGN.



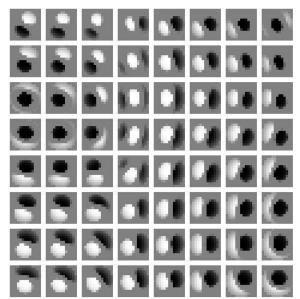
#### Receptive Field Development

Define an arborization function A(x, x') that gives the a priori probability that a connection between a LGN cell at location x and a cortical cell at x' is formed. The arborization is a rapidly decaying function of the distance

$$A(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{x}'\|^2}{\sigma_{\text{V1,LGN}}^2}\right).$$

The dynamics of the weight distribution is given by a modified form of Hebb's learning rule

$$\frac{\mathrm{d}}{\mathrm{d}t} w_{\mathrm{V1,ON/OFF}}(\boldsymbol{x}, \boldsymbol{x}') = \gamma A(\boldsymbol{x}, \boldsymbol{x}') \nu_{\mathrm{V1}}(\boldsymbol{x}) \nu_{\mathrm{ON/OFF}}(\boldsymbol{x}').$$



Receptive fields of 64 cortical neurons. Each small square shows the distribution of weights.



#### Visual Cortex Map

(a) Cortical sheet before learning.

(b) Cortical sheet before learning.



# 突触可塑性的多样性



## 突触可塑性(synaptic plasticity)的种类

#### ▶定义

- 1) 在神经科学中,突触可塑性是指神经细胞间的连接强度可调节,突触的形态和功能可发生较为持久的改变的特性或现象。突触会随着自身活动的加强与减弱相应得到加强与减弱。突触可塑性是学习和记忆的基本神经机制,已成为神经科学研究的热点领域。
- 2) 在人工神经网络中,突触可塑性是指利用神经科学中突触可塑性有关理论结合数学模型来构造神经元之间的连接。
- ▶不同的突触可塑性持续的时间具有差异性,按照持续的时间分类:

突触短时增强/抑制,即STP/STD → 神经适应 (Adaptation) (Short-term potential/depression)
 突触长时增强/抑制,即LTP/LTD → 学习、记忆 (Long-term potential/depression)



## 突触可塑性(synaptic plasticity)

1、短期突触可塑性(short-term synaptic plasticity, STP)

定义:短期突触可塑性指突触传递效率增强或减弱的变化现象,并且这种变化只持续数十毫秒到几分,其中效率增强与减弱分别叫做短时程增强(short-term enhancement)与短时程抑制(short-term depressed);而持续数百毫秒的增强又被称之为短时程易化(short-term facilitation)。

2、长期抑制可塑性(Long-term depression, LTD)

定义:长期抑制可塑性被认为是后突触接受体密度的改变导致的,但是前突触释放物的改变也可能有一定影响。

3、长期增强可塑性(Long-term potentiation, LTP)

定义:长期增强可塑性是由于同步刺激两个神经元而发生在两个神经元信号传输中的一种持久的增强现象。

● 输入特定性

一个突触的LTP一经诱导,不会扩散到其他突触,因而LTP具有输入特定性。

● 关联性

一条通路弱刺激尚不足以诱导LTP时,另一通路强刺激会同时诱导两条通路的LTP。

● 协同性

LTP可由强烈的强直刺激激发突触的单一通路,或通过许多较弱的刺激协作引发。

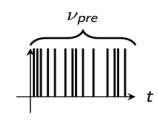
● 持久性

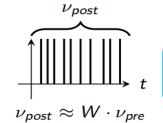
LTP作用时间可以持续 几分钟乃至几个月。 与其他突触可塑性的 根本区别。



▶分类2: 基于脉冲速率或时间,可以分为:

速率依赖

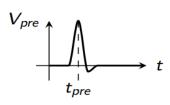


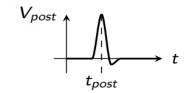


$$rac{dW}{dt} = f\left(
u_{ extsf{pre}}, 
u_{ extsf{post}}, W
ight)$$

例如Hebbian learning、Anti-Hebbian、Non-Hebbian learning

时间依赖



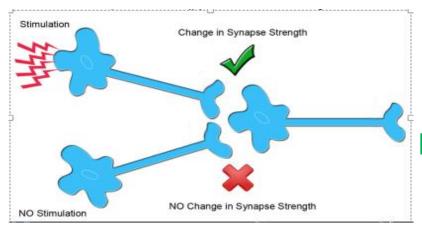


$$\int \frac{dW}{dt} = f(t_{pre}, t_{post}, W)$$

例如Spike-Timing Dependent Plasticity (STDP)



- ▶分类3:同源突触可塑性和异源突触可塑性
  - Homosynaptic plasticity **同源突触可塑性** 突触前细胞信号的传播是局部的,特异性刺激突触后细胞,突触可塑性依赖于突触前和突触后活动。



Hebbian 和Anti-Hebbian学习规则都属于只依赖于突触前后的发放速率的规则,即都属于同源突触可塑性的一种。

$$\tau_w \frac{dw_i(t)}{dt} = ax_i(t)y(t)$$

Hebbian: 即突触前神经元向突触后神经元的持续重复的刺激可以导致突触传递效能的增加(a > 0)。

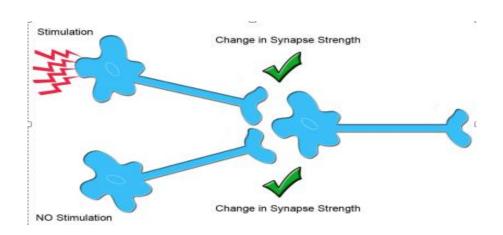
Anti-Hebbian:即突触前神经元向突触后神经元的持续重复的刺激可以导致突触传递效能的减小(a < 0)。



□ 几种基于Non-Hebbian的可塑性:

异源突触可塑性 Heterosynaptic plasticity

一个特定神经元的活动导致其他未激活神经元的突触 连接强度发生非特异性的变化,突触可塑性依赖于突 触后神经元,而不依赖于突触前神经元。





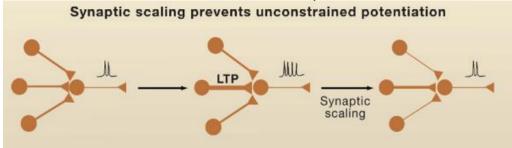
- □分类4: Hebb 和 Non-Hebbian突触可塑性:
- □几种Non-Hebbian可塑性:
  - 1、异源突触可塑性 Heterosynaptic plasticity
  - 2、突触缩放 synaptic scaling

根据神经元网络中的整体活动水平触发,对所有突触进行全局调整,以防止某些突触在修改过程中被减少到零或过度增加。

突触修改: Hebbian可塑项-非Hebbian项,即

$$\Delta w \propto r_{post}r_{post} - (r_{post})^2 w$$





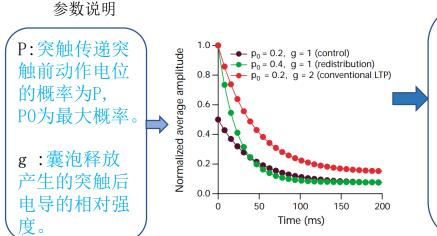
当一个输入的LTP增加突触后放 电时,突触缩放将降低所有突触 输入的强度,直到放电率恢复到 控制水平。



3、突触再分配(synaptic redistribution)

定义:在某些皮质突触上,LTP可以改变突触的短期可塑性,这种效应称为突触再分配。

机理:如果突触在足够长的时间内处于不活动状态,则p接近其最大值p0(增加突触传递的振幅)。当突触活跃时,p以与传输速率成比例的速率减少(削弱突触传递振幅),反之增加。



#### 实验说明

棕色:实验组,在突触长期不活跃时,LTP调节p 值为最大,从而达到第一个振幅最大的脉冲(t=0),随后逐渐减低p值,则突触振幅变小,直到最后稳定在0.1左右。

绿色: PO增大为原来的两倍,则LTP调节p值第一个脉冲振幅是原来的两倍,但是在突触再分配机制下,最后达到和实验组一样的振幅水平。

红色: g增大为原来的两倍,则第一个脉冲和最后 平均的突触振幅都增大。

结论:突触再分配可以显著增强序列中第一个脉冲的突触传递幅度,而对最终稳态幅度没有影响。

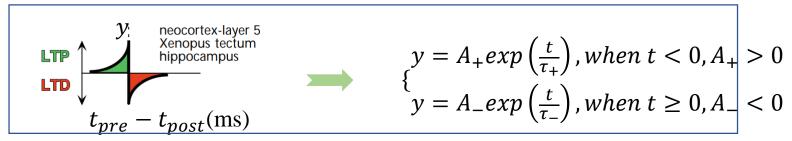
其他Non-Hebbian可塑性例如递质诱发塑性(Transmitter-induced plasticity)、 **结构可塑性、稳态可塑性**等。

- 4. 其他Non-Hebbian可塑性:
- 递质诱发塑性(Transmitter-induced plasticity)、
- 结构可塑性、
- 稳态可塑性…



- ▶基于时间的可塑性
  - 脉冲时间依赖可塑性 (spike-timing-dependent plasticity, STDP)

对突触前和突触后神经元动作电位的相对时间非常敏感,因此依赖于网络活动的细节(局部)。突触前活动先于突触后放电可以诱导LTP,而颠倒这种时间顺序会导致LTD。



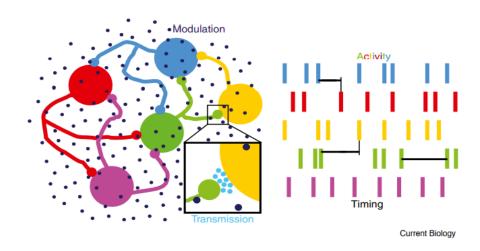


### 多种突触可塑性如何协同?



### 多突触可塑性协同模型

- 越来越多的实验和计算工作表明,为确保生物在不断变化的环境中生存,大脑依赖于多重可塑性和稳态机制(homeostatic plasticity),这些机制同时作用于突触和细胞的内在参数。
- 可能参与触发和调节细胞和突触可塑性的神经元网络的有电活动的整体水平、神经元内和神经元之间的精确时间、神经调节物质以及神经递质。



细胞和突触可塑性的潜在触发器和调节器



### 相关概念介绍

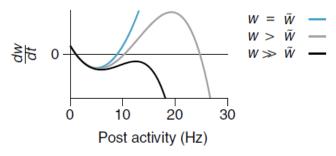
- **同突触可塑性** (homosynaptic plasticity): 共同依赖于突触前和突触后活动的突触可塑性表现
- **异突触可塑性** (heterosynaptic plasticity): 对突触可塑性的贡献仅取决于突触后神经元的状态,而不取决于突触前神经元的状态
- **递质诱发可塑性(transmitter-induced plasticity)**: 突触的变化只取决于递 质的释放,而不取决于突触后神经元的状态
- 内稳态 (homeostatic): 作用10分钟以上时间尺度的缓慢补偿过程

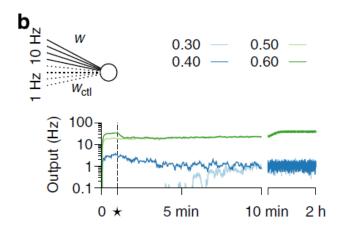
	Hebbian	Anti-Hebbian	Non-Hebbian
同突触可塑性	<b>√</b>	$\checkmark$	
异突触可塑性			$\checkmark$
递质诱发可塑性			√
内稳态可塑性			$\checkmark$



兴奋性突触的局部同突触、异突触和递质诱发可塑性协同作用下可实现记忆和回忆功能的脉冲神经网络模型

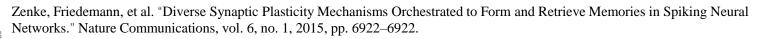




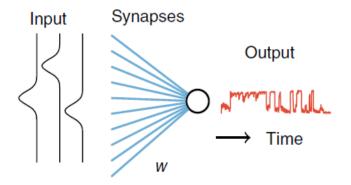


### Orchestrated forms of plasticity in a feed-forward network give rise to bistability and synaptic competition.

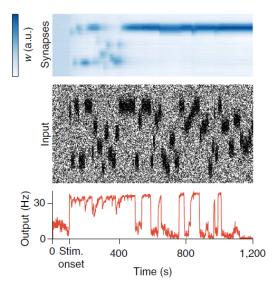
- (a) Postsynaptic rate dependence of orchestrated forms of plasticity. With a synaptic weight larger than the reference value ~w, synaptic depression is possible at high postsynaptic rates.
- (b) The mean firing rate over time of a single postsynaptic integrate-and-fire neuron with plastic synapses for different initial conditions. The postsynaptic neuron receives simultaneous Poisson input via an active pathway (80 synapses at 10 Hz) and a control pathway (80 synapses at 1 Hz, initial weight w\_ctl=0.1). Different colours signify different initial weights w in the active pathway.







Schematic of the simulation of a single integrate-and-fire neuron receiving spatiotemporally correlated input.

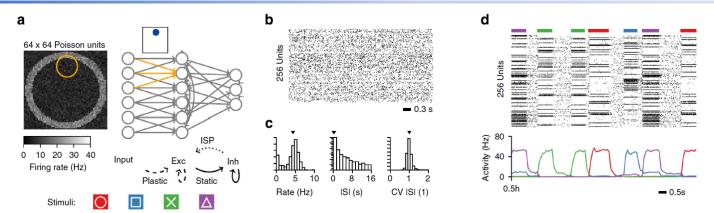


Evolution of the synaptic weights indicates receptive field development

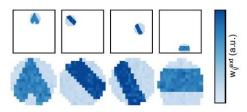
Spike raster of the input spike trains.

The mean firing rate of the postsynaptic neuron over time. After stimulus onset (at t=100 s) a Gaussian activity profile centred at a random position modulates the input spike trains. The centre of the Gaussian is shifted at random intervals (mean=20 s).

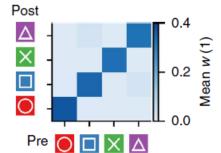




(a) Schematic representation of the network model (top right). Top left: input layer composed of 64x64 Poisson neurons. Each network neuron receives input from a circular subset (orange circle) of input units centred at a random position. Inset: initial receptive field of a network neuron. Connection types are indicated below. Solid lines: no long-term plasticity. Dashed lines: orchestrated Hebbian and non-Hebbian plasticity. Dotted line: inhibitory plasticity (ISP). Four overlapping geometric shapes (bottom) are presented at random times via the input units. (b) Spike raster of the initial network activity. (c) Network statistics. (d) Network activity after 30 min. Top: spike raster. Bottom: temporal evolution of the population firing rate of assembly neurons coding for the respective stimuli.



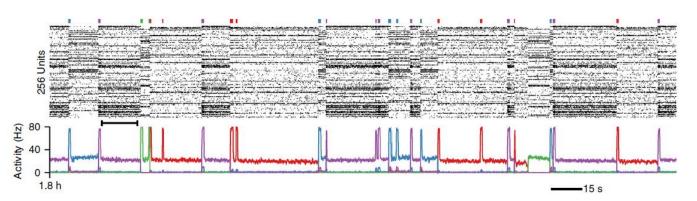
Receptive fields of four randomly chosen network neurons after learning.

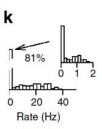


The mean weight strength of the recurrent weights between neurons ordered according to stimulus preference (t=1 h).









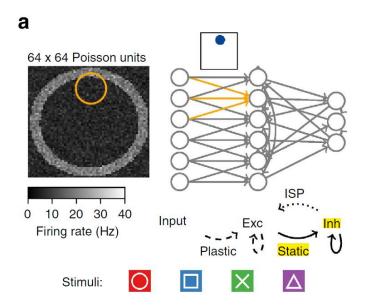


Network activity and the population firing rate of assembly neurons coding for the respective stimuli after 1.8 h of simulated time. Note the delay activity during the interstimulation interval.



Diverse synaptic plasticity mechanisms orchestrated to form and retrieve memories in spiking neural networks

- ➤ Input layer composed of 64 \* 64 Poisson neurons.
- Each network neuron receives input from a circular subset (orange circle) of input units centred at a random position.
- Four overlapping geometric shapes (bottom) are presented at random times via the input units.



- ✓ Short term plasticity acts on EE, EI, StimE
- ✓ Triplet STDP rule with pre and post offset terms and slower consolidation dynamics acts on EE and StimE
- ✓ Inhibitory Spike Timing Dependent Plasticity (iSTDP) acts on IE

Solid lines: no long-term plasticity

Dashed lines: orchestrated Hebbian and non-

Hebbian plasticity

Dotted line: inhibitory plasticity

$$\frac{d}{dt}w_{ij} = S_i(t) \left( \underbrace{Az_j^+ z_i^{\mathrm{slow}}(t-\epsilon) - \beta \left( z_i^-(t-\epsilon) \right)^3 \left( w_{ij} - \tilde{w}_{ij} \right)}_{-S_j(t)} \right) \mathbf{LTD}$$

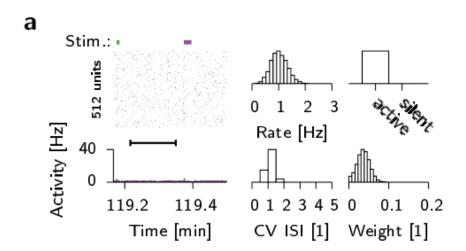
$$\frac{d}{dt}w_{ij}(t) = \eta \, G(t) \left[ \left( z_i(t) + 1 \right) S_j(t) + z_j(t) \, S_i(t) \right]$$

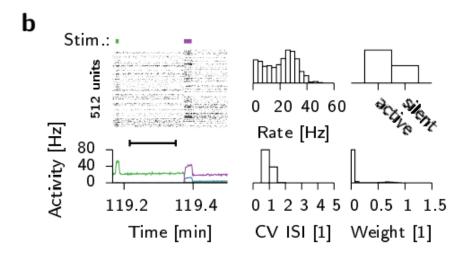
$$\mathbf{Transmitter-induced}$$

Zenke, Friedemann, E. J. Agnes, and W. Gerstner. "Diverse synaptic plasticity mechanisms orchestrated to form and retrieve memories in spiking neural networks." Nature Communications 6:6922, 2015.

Blocked homeostatic metaplasticity of LTD in a network with weak random initial input. The network fails to develop selectivity.

Blocking **transmitter-induced** plasticity causes many neurons to **fall and remain silent**.



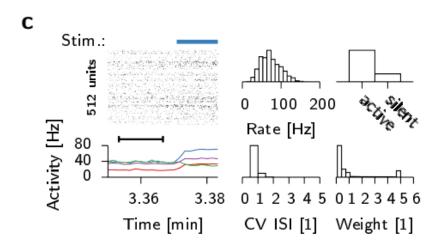


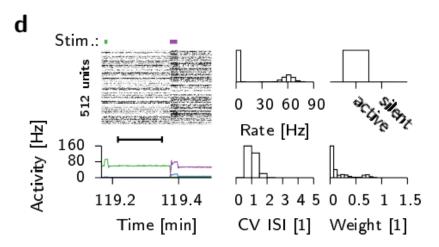


Zenke, Friedemann, E. J. Agnes, and W. Gerstner. "Diverse synaptic plasticity mechanisms orchestrated to form and retrieve memories in spiking neural networks." Nature Communications 6:6922, 2015.

Blocked heterosynaptic plasticity causes rapidly increasing firing rates without learning or emergence of delay activity (simulation stopped after 300 s).

Blocking **inhibitory plasticity** allows the network to develop delay activity, but at **highly elevated** firing rates.





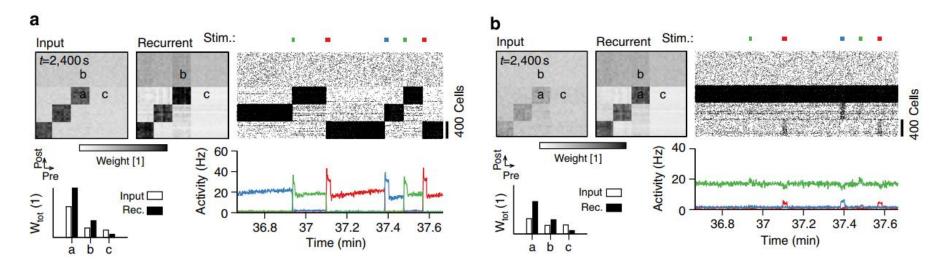


Zenke, Friedemann, E. J. Agnes, and W. Gerstner. "Diverse synaptic plasticity mechanisms orchestrated to form and retrieve memories in spiking neural networks." Nature Communications 6:6922, 2015.

#### Blocking consolidation causes cell assemblies to decouple from external input.

network with normal consolidation dynamics

network without consolidation



Top left: 2D histogram of a 2,000-unit section of the excitatory weight matrix that connects input neurons with the excitatory neurons in the network. Characteristic values have been plotted in the bar plot at the bottom (highlighted letters).

Top middle: as before but showing recurrent weights between the selected 2,000 units in the network. Top right: spike raster of the same 2,000 units. Bottom right: population activity of the 3 \* 400 unit blocks shown in the spike raster.



Zenke, Friedemann, E. J. Agnes, and W. Gerstner. "Diverse synaptic plasticity mechanisms orchestrated to form and retrieve memories in spiking neural networks." Nature Communications 6:6922, 2015.

**吳生**▶神经元模型-有放电频率适应(spike frequency adaptation, in the spike S<sub>j</sub> in the spike

$$\begin{array}{c}
\text{pre } spike S_j \\
\text{post } \\
w_{ij} \\
\text{i}
\end{array}$$

$$\tau^m \frac{dU_i}{dt} = (U^{rest} - U_i) + g_i^{exc}(t) \left( U^{exc} - U_i \right) + (g_i^{gaba}(t) + g_i^a(t)) (U^{inh} - U_i)$$

其中,
$$g_i^{gaba}(t)$$
是抑制性突触输入, $\frac{dg_i^{gaba}}{dt} = -\frac{g_i^{gaba}}{\tau^{gaba}} + \sum_{j \in inh} w_{ij} S_j(t)$ 

 $g_i^a(t)$ 表示对脉冲触发适应的贡献,  $\frac{dg_i^a}{dt} = -\frac{g_i^a}{\tau^a} + \Delta^a S_i(t)$ , 其中S表示脉冲,  $\Delta^a$ 是适应强度。

$$g_i^{exc}(t)$$
表示兴奋性的电流输入:  $g_i^{exc}(t) = \alpha g_i^{ampa}(t) + (1-\alpha) g_i^{nmda}(t)$ 

快速变化的类 似于AMPA组 件

缓慢上升和衰变 的类似于NMDA 的组件

其中,
$$\frac{dg_i^{ampa}}{dt} = -\frac{g_i^{ampa}}{\tau^{ampa}} + \sum_{j \in exc} w_{ij} u_j(t) x_j(t) S_j(t)$$

$$\tau^{nmda} \frac{dg_i^{nmda}}{dt} = -g_i^{nmda} + g_i^{ampa}$$
短期可塑性

过程描述:一旦神经元的膜电压 $U_i$ 超过了阈值 $\theta_i$ ,触发一个脉冲,膜电压 $U_i$ 重置为 $U_i^{rest}$ ,同时阈值瞬间增加到 $\theta^{spike}$ ,进入不应期,在不应期期间没有 输入的时候,阈值会逐渐恢复到 $U_i^{rest}$ 。



此模型结合了不同形式的可塑性机制。兴奋性突触具有STP、STDP、异突触可塑性和递质诱导可塑性。兴奋性神经元上的抑制性突触同样具有可塑性,并遵循由分泌因子全局调节的STDP规则。

ightharpoonup 短期可塑性STP的状态变量  $x_j(t), u_j(t)$ 计算方式为:

$$\frac{d}{dt}x_j(t) = \frac{1 - x_j(t)}{\tau^{\mathrm{d}}} - u_j(t)x_j(t)S_j(t)$$

$$\frac{d}{dt}u_j(t) = \frac{U - u_j(t)}{\tau^{\mathrm{f}}} + U(1 - u_j(t))S_j(t)$$

其中,常数参数 $\tau^d=200ms$ ,  $\tau^f=600ms$ , U=0.2。

>兴奋性突触的长期可塑性:

兴奋性突触连接之间通常有三种不同形式的突触可塑性机制,即三联体STDP,递质诱导和异突触可塑性。

$$\frac{d}{dt}w_{ij}(t) = Az_j^+(t)z_i^{\text{slow}}(t-\epsilon)S_i(t)$$
 三联体STDP —  $B_i(t)z_i^-(t)S_j(t)$  — 双重LTD

其中,A, $\beta$ , $\delta$ , $B_i$ =A是常数参数,其他参数见下页。



>兴奋性突触的长期可塑性:

$$\frac{d}{dt}w_{ij}(t) = Az_j^+(t)z_i^{\text{slow}}(t-\epsilon)S_i(t)$$

$$-B_i(t)z_i^-(t)S_j(t)$$

$$-\beta(w_{ij}-\tilde{w}_{ij}(t))(z_i^-(t-\epsilon))^3S_i(t)$$

$$+\delta S_j(t).$$

•  $z_{j/i}^{x}(t)$ 定义了突触轨迹,j表示突触前的轨迹,i表示突触后的轨迹,其计算方式以突触后为例:

$$\frac{dz_i^x}{dt} = -\frac{z^x}{\tau^x} + S_i(t)$$
 时间 常数

• 整合权重:

参数

描

述

其中, $w^P = 0.5$ ,  $\tau^{cons} = 20min$ , P是控制膜电压的参数。



▶ 长期抑制的稳态调节:

$$B_i(t) = \begin{cases} AC_i(t) & \text{for } C_i(t) \leq 1\\ A & \text{otherwise} \end{cases}$$

$$\frac{d}{dt}C_i(t) = -\frac{C_i(t)}{\tau^{\text{hom}}} + \left(z_i^{\text{ht}}(t)\right)^2$$

其中, $\tau^{hom} = 20min, z_i^{ht}(t)$ 是突触轨迹,满足

$$\frac{dz_i^{ht}}{dt} = -\frac{z^{ht}}{\tau^{ht}} + S_i(t)$$

其中, $\tau^{ht} = 100ms$ 。

▶兴奋性神经元抑制性突触的长期可塑性:

在此模型中,使用了一种抑制性突触可塑性,这种可塑性由一种全局分泌因子调节。即,当全局网络活动过高时,抑制性突触往往增强,该突触满足如下STDP规则:

# 常数 突触后i/前j活动轨迹 $\frac{d}{dt}w_{ij}(t) = \eta G(t) \left[ (z_i(t) + 1)S_j(t) + z_j(t)S_i(t) \right]$

突触前j/后i脉冲序列

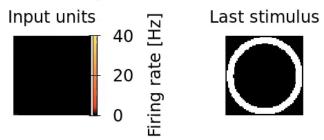
其中, $G(t) = H(t) - \gamma$ 线性依赖于全局分泌因子H(t):

$$\frac{d}{dt}H(t) = -\frac{H(t)}{\tau^{H}} + \sum_{i \in exc} S_{i}(t)$$

其中,时间常数 $\tau^H = 10s$ 。



记忆实验: 四种输入模式随机重复刺激,不同刺激下组装神经元群体发放率的时间变化(视频演示)



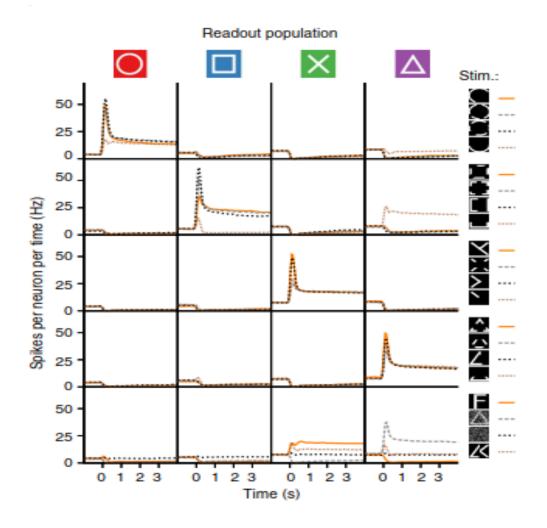
Stimulus:



**结论:学习后**,神经元响应特定的输入模式,并在短暂刺激其中一种模式后保持延迟活动

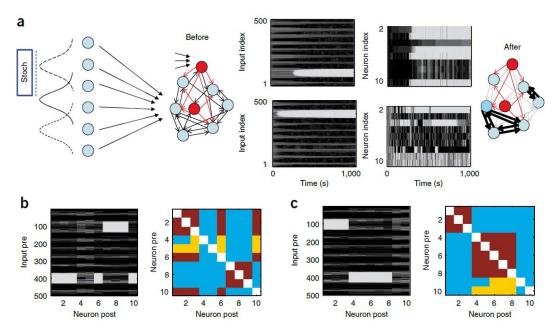


回忆实验:残缺输入/已知模式的模糊组合细胞组件被激活--记忆从部分线索中唤起





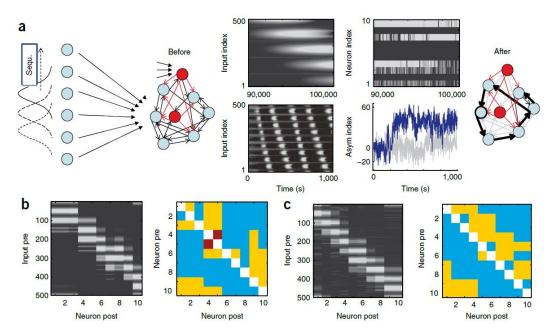
### 模型二



Plasticity during rate coding. (a) A network of ten excitatory (light blue) and three inhibitory neurons (red) received feedforward inputs from 500 Poisson spike trains with a Gaussian profile of firing rates. The center of the Gaussian was shifted randomly every 100 ms (schematic network before (left) and after the plasticity experiment (right)). The temporal evolution of the weights are shown (top, small amplitudes of plasticity; bottom, normal amplitudes of plasticity; left, feedforward connections onto neuron 1; right, recurrent connections onto neuron 1). (b) Mean feedforward weights (left) and recurrent excitatory weights (right) averaged over 100 s. The feedforward weights (left) indicate that the neurons developed localized receptive fields (light gray). The recurrent weights (right) were classified as weak (less than 2/3 of the maximal weight, light blue), strong unidirectional (more than 2/3 of the maximal weight, yellow) or strong reciprocal (brown) connections. The diagonal is white, as self-connections do not exist in the model.



### 模型二



**Temporal-coding procedure.** Input patterns were moved successively every 20 ms, corresponding to a step-wise motion of the Gaussian stimulus profile across the input neurons. (a) The schematic figure shows the network before and after the plasticity experiment. Shown are the temporal evolution of the weights (top panels: amplitude of synaptic plasticity for feedforward connections reduced by a factor of 100; left, feedforward weights onto neuron 6; right, lateral connections onto neuron 6; bottom panels: normal amplitude of plasticity; left, feedforward connections onto neuron 1; right, temporal evolution of asymmetry index of connection pattern. Positive values indicate the weights from neurons n to n + k are stronger than those from n to n - k for  $1 \le k \le 3$ ). (b) Receptive fields are localized (left). The recurrent network developed a ring-like structure with strong unidirectional connections from neuron 8 (vertical axis) to neurons 9 and 10 (horizontal axis), etc.



### 模型二

#### 基于电压的STDP稳态模型用于解释大脑中不同区域的不同编码原则

此模型包含LTP可塑性和LTD可塑性的模型。

▶神经元模型-自适应整合发放模型 adaptive exponential integrate-and-fire (AdEx) 电压在指数函数中(与其他

$$C\frac{d}{dt}u = -g_L(u-E_L) + g_L\Delta_T e^{\frac{u-V_T}{\Delta_T}} - w_{ad} + z + I$$
电压在指数函数中(与其他模型不同之处),可以更好地拟合数据

• 其中, u膜电压,C是膜电容, $g_L$ 是泄露电导, $E_L$ 是静息电位,I是刺激电流,指数项描述了Na电流激活影响, $\Delta T = 2mV$ 是斜率因子, $V_T$ 是阈值电压

• 变量
$$w_{ad}$$
表示为: 
$$\tau_{w_{ad}} \frac{d}{dt} w_{ad} = a(u - E_L) - w_{ad}$$
 参数

• 当膜电压 $\mathbf{u}$ 大于阈值电压 $V_T$ , $\mathbf{u}$ 则重置为 $V_{\mathrm{rest}}$ ,  $w_{ad}$ 则增加b。



