

INTERVIEW QUESTION

Question 5.1: What is the largest possible result of multiplying two unsigned N -bit numbers?

$$\begin{array}{r} \underbrace{0 \ 1 \ 1 \ 1 \ \dots \ 1 \ 1 \ 1}_N \text{ bits}_2 = (2^{N-2})_{10} + (2^{N-3})_{10} + \dots + (2^2)_{10} + (2^1)_{10} + (2^0)_{10} \\ \times \underbrace{0 \ 1 \ 1 \ 1 \ \dots \ 1 \ 1 \ 1}_N \text{ bits}_2 = (2^{N-2})_{10} + (2^{N-3})_{10} + \dots + (2^2)_{10} + (2^1)_{10} + (2^0)_{10} \end{array}$$

$$\Rightarrow M_{\text{result}} = (2^{N-2} + 2^{N-3} + \dots + 2^2 + 2^1 + 2^0)(2^{N-2} + 2^{N-3} + \dots + 2^2 + 2^1 + 2^0)$$

$$M = (2^{N-1} - 1)(2^{N-1} - 1)$$

$$M = (2^{N-1} - 1)^2$$

$$M = 2^{2(N-1)} - 2 \cdot 2^{N-1} \cdot 1 + 1^2$$

$$M = 2^{2N-2} - 2^N + 1$$