

EXAMPLE PROBLEMS

Example 2.9 : Minimization of a three-variable function using a K-map

Y \ AB	00		01		11		10	
	C		C		C		C	
0	1		0		1		1	
1	1		0		0		1	

Thus, $Y = A\bar{C} + \bar{B}$

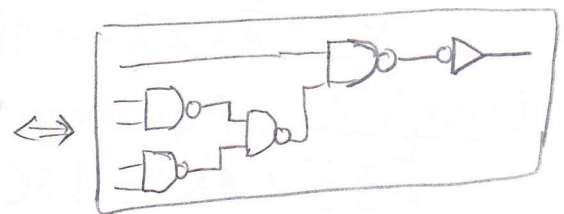
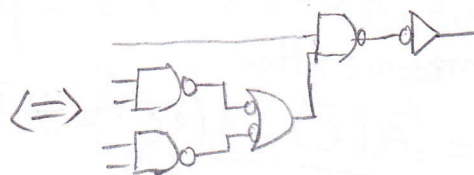
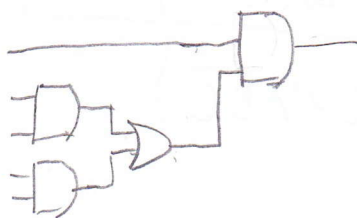
Variation:

Y \ AB	00		01		10		11	
	C		C		C		C	
0	1		0		0		0	
1	0		1		1		1	

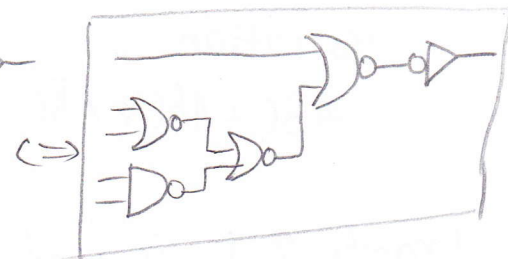
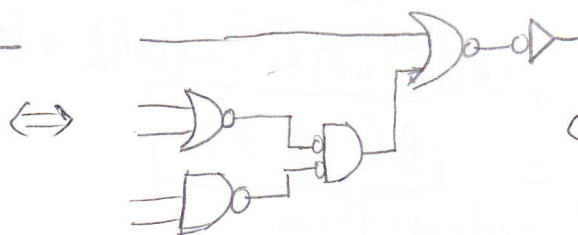
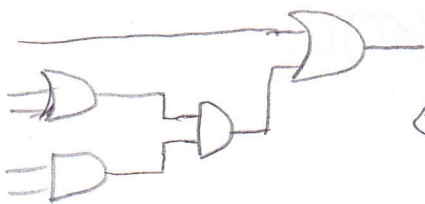
$\bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC$

$$\begin{aligned}
 \text{Thus, } Y &= \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + ABC \\
 &= \bar{A}\bar{B}\bar{C} + (\bar{A}B + A\bar{B} + AB)C \\
 &= \bar{A}\bar{B}\bar{C} + (\bar{A}B + A)C \\
 &= \bar{A}\bar{B}\bar{C} + (A+B)C
 \end{aligned}$$

Example 2.8 : Bubble pushing for CMOS logic $Y = \bar{A}\bar{B}\bar{C} + AC + BC$



variation:



Example 2.4: Derive the product-of-sums form

A	B	Y	\bar{Y}	minterm of \bar{Y}
0	0	0	1	$\bar{A}\bar{B}$
0	1	0	1	$\bar{A}B$
1	0	1	0	$A\bar{B}$
1	1	1	0	AB

\Rightarrow sum-of-products form of \bar{Y} :

$$\bar{Y} = \bar{A}\bar{B} + \bar{A}B$$

Taking the complement of both sides:

$$Y = \overline{\bar{A}\bar{B} + \bar{A}B}$$

Using De Morgan's theorem twice:

$$Y = \overline{\bar{A}\bar{B}} \cdot \overline{\bar{A}B} = \boxed{(A+B)(A+\bar{B})}$$

Variation:

A	B	Y	\bar{Y}	minterm of \bar{Y}
0	0	0	1	$\bar{A}\bar{B}$
0	1	0	1	$\bar{A}B$
1	0	0	1	$A\bar{B}$
1	1	1	0	AB

\Rightarrow sum-of-products form of \bar{Y} :

$$\bar{Y} = \bar{A}\bar{B} + \bar{A}B + A\bar{B}$$

Taking the complement of both side and applying DeMorgan's theorem:

$$Y = \overline{\bar{A}\bar{B} + \bar{A}B + A\bar{B}} = \overline{\bar{A}\bar{B}} \cdot \overline{\bar{A}B} \cdot \overline{A\bar{B}} = \boxed{(A+B)(A+\bar{B})(\bar{A}+B)}$$

Example 2.6: Equation minimization

$$\begin{aligned} \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C &= (\bar{A}(\bar{B}\bar{C}) + A(\bar{B}C)) + (\bar{A}B\bar{C} + A\bar{B}C) \\ &= \boxed{\bar{B}\bar{C} + A\bar{B}} \end{aligned}$$

Variation:

$$\begin{aligned} \bar{A}BC + ABC + A\bar{B}C &= (\bar{A}(BC) + A(BC)) + (\bar{A}B\bar{C} + A\bar{B}C) \\ &= \boxed{BC + A\bar{B}} \end{aligned}$$

Example 2.1: Sum-of-products form

A = There are ants, R = It rains, E = Ben enjoys the picnic

$$\Rightarrow E = \bar{A}\bar{R} = \Sigma(0) \Rightarrow \text{circuit: } \begin{array}{c} \text{A} \text{---} \text{AND} \text{---} \text{E} \\ \text{R} \text{---} \end{array}$$

Variation:

L = Ben feels lazy, R = It rains, S = Ben skips class

$$\Rightarrow S = LR = \Sigma(3) \Rightarrow \text{circuit: } \begin{array}{c} \text{L} \text{---} \text{AND} \text{---} \text{S} \\ \text{R} \text{---} \end{array}$$