MA 322 - Scientific Computing

Lab - 3 (Date - 21/01/2025)

1. Write a program to solve the following system of nonlinear equations using Newton's method:

$$y\cos(xy) + 1 = 0$$
$$\sin(xy) + x - y = 0$$

Your program should:

- Accept the initial guesses $x_0 = 1$ and $y_0 = 2$ as the starting point.
- Iteratively calculate the next approximations for x and y using Newton's method (use the **jacobian matrix** for updating the solutions).
- Stop when the updates for x and y are smaller than the tolerance 10^{-6}.
- Display the solution (x, y) and the number of iterations.

2. Write a program to find the root of the following nonlinear equation using the **fixed-point iteration** method:

$$e^{-x}\cos(x) - x^2 = 0$$

Your program should:

1. Accept an initial guess x_0 from the user.
2. Iteratively calculate the next approximations for x.
3. Stop the iteration when the absolute difference between consecutive approximations is less than 10^{-6}.
4. Display:
    - The approximate root of the equation.
    - The number of iterations taken.

3. Solve the following polynomial equation to find all real roots using the **Müller method**:

$$f(x) = x^4 - 6x^3 + 11x^2 - 6x + 1 = 0$$

Instructions:

- **Initial Guesses:** Take different sets of initial guesses for all the real roots.
- **Stopping Criteria:** The method should stop when the absolute difference between successive approximations is smaller than a tolerance of 10^{-6}. Specifically, stop iterating when:

$$|x_{n+1} - x_n| < 10^{-6}$$

Hint: The Müller method is an iterative numerical technique to find the roots of nonlinear equations. It approximates the function by a quadratic polynomial passing through three points and refines the root estimate based on where the quadratic intersects the x-axis.