LAB-1 REPORT

Problem 1: Linear Congruential Generators

The first experiment involved generating sequences of numbers using **Linear Congruence Generators** with different parameters and initial conditions. The LCGs were defined by the recurrence relation **xi = (a * xi-1 + b) % m**, where **a**, **b**, and **m** are constants, and **xi** is the **ith** term of the sequence.

The results of the experiment are presented in the following tables:

Table 1: Sequences generated by LCG with a=6, b=0, m=11 varying x0 from 1 to 10.

x0	Sequence	Distinct Values Before Repetition
0	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	1
1	[1, 6, 3, 7, 9, 4, 2, 8, 5, 10]	10
2	[2, 12, 6, 14, 18, 10, 4, 16, 8, 20]	10
3	[3, 18, 9, 21, 27, 15, 6, 24, 12, 30]	10
4	[4, 24, 12, 28, 36, 20, 8, 32, 16, 40]	10
5	[5, 30, 15, 35, 45, 25, 10, 40, 20, 50]	10
6	[6, 36, 18, 42, 54, 30, 12, 48, 24, 60]	10
7	[7, 42, 21, 49, 63, 35, 14, 56, 28, 70]	10
8	[8, 48, 24, 56, 72, 40, 16, 64, 32, 80]	10
9	[9, 54, 27, 63, 81, 45, 18, 72, 36, 90]	10
10	[10, 60, 30, 70, 90, 50, 20, 80, 40, 100]	10

The sequences generated by the LCGs with a=6 has a period of 10, regardless of the initial condition **x0**.

Table 2: Sequences generated by LCG with a=3, b=0, m=11

x0	Sequence	Distinct Values Before Repetition
0	[0, 0, 0, 0, 0, 0, 0, 0, 0, 0]	1
1	[1, 3, 9, 5, 4, 1, 3, 9, 5, 4]	5
2	[2, 6, 7, 10, 8, 2, 6, 7, 10, 8]	5
3	[3, 9, 5, 4, 1, 3, 9, 5, 4, 1]	5
4	[4, 1, 3, 9, 5, 4, 1, 3, 9, 5]	5
5	[5, 4, 1, 3, 9, 5, 4, 1, 3, 9]	5
6	[6, 7, 10, 8, 2, 6, 7, 10, 8, 2]	5
7	[7, 10, 8, 2, 6, 7, 10, 8, 2, 6]	5
8	[8, 2, 6, 7, 10, 8, 2, 6, 7, 10]	5
9	[9, 5, 4, 1, 3, 9, 5, 4, 1, 3]	5
10	[10, 8, 2, 6, 7, 10, 8, 2, 6, 7, 10]	5

The sequences generated by the LCGs with a=3 has a period of 5, regardless of the initial condition **x0**.

A linear congruence generator that produces all (m-1) distinct values before repeating is said

to have a full period. We can observe that a=6 gives us a **full period**. So, a=6 is a better choice than a=3.

Problem 2:

The choices of **x0** are 12345, 67890, 11111, 22222, 33333.

For each **x0** value, a sequence of 10,000 numbers using the linear congruence generator is generated with **m = 244944** and values of ‘a’ being 1597 and 51749. The frequency of each range **[0, 0.05), [0.05, 0.10), ..., [0.95, 1)** is calculated and plotted on a bar graph to visualize the frequencies.

Below are the frequencies of each region given values of a and x0.

a = 1597

- **x0 = 12345:**

  [495 503 493 504 502 490 511 493 503 506 493 500 491 500 513 495 502 497 509 500]

- **x0 = 67890:**

  [497 496 514 495 497 513 495 496 495 515 494 493 514 492 491 517 492 488 493 513]

- **x0 = 11111:**

  [512 499 501 497 506 496 496 497 506 497 493 509 501 503 495 491 506 504 487 504]

- **x0 = 22222:**

  [502 503 500 508 504 498 503 497 502 499 489 498 502 500 498 503 499 494 500 501]

- **x0 = 33333**:

  [502 496 510 502 493 502 493 505 502 491 512 494 501 505 494 500 493 500 508 497]

a = 51749

- **x0 = 12345:**

  [495 494 526 494 494 526 492 494 494 493 494 491 527 496 493 524 495 493 494 491]

- **x0 = 67890:**

  [371 553 558 493 433 554 492 494 494 555 431 492 555 557 372 556 554 371 558 557]

- **x0 = 11111:**

  [504 502 501 516 490 492 495 492 509 501 500 510 493 515 493 492 491 500 501 503]

- **x0 = 22222:**

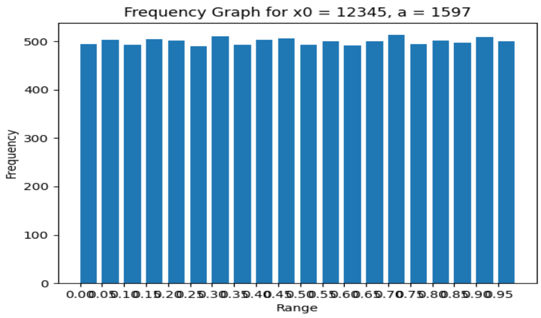
  [492 512 494 518 495 499 516 515 492 491 491 493 493 493 497 495 496 514 514 490]

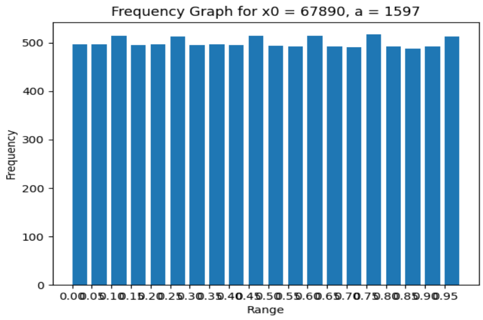
- **x0 = 33333:**

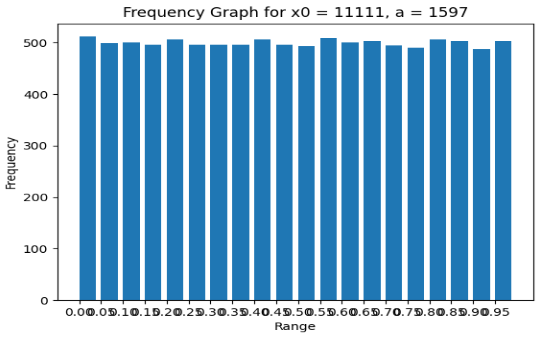
  [495 554 463 463 557 494 463 526 494 494 495 555 462 462 557 494 462 525 492 493]

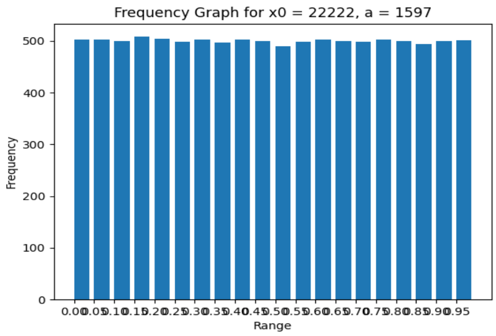
Observations:

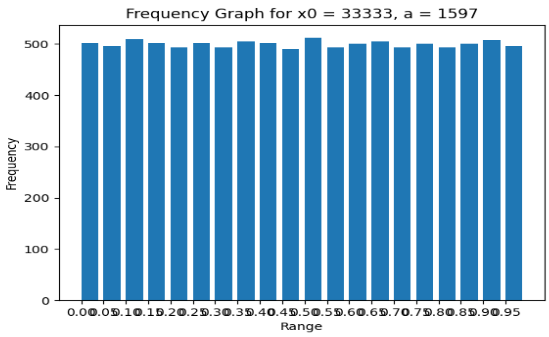
- For **a = 1597**, the frequencies are relatively uniform across all ranges, indicating a good distribution of random numbers.
- For **a = 51749**, the frequencies are more uneven, with some ranges having significantly higher frequencies than others. This may indicate a less uniform distribution of random numbers.
- The choice of **x0** does not seem to have a significant impact on the frequency distribution, although there are some minor variations.

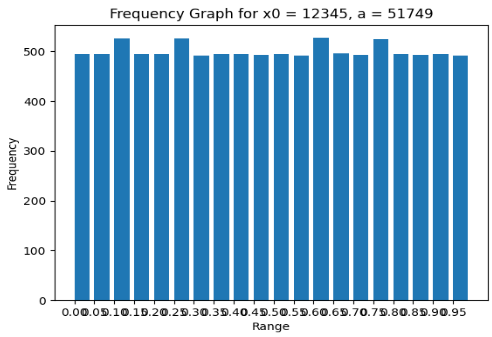

Frequency Graph for x0 = 12345, a = 1597
Frequency
Range

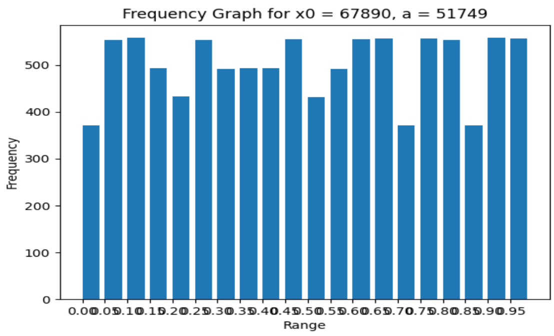

Frequency Graph for x0 = 67890, a = 1597
Frequency
Range

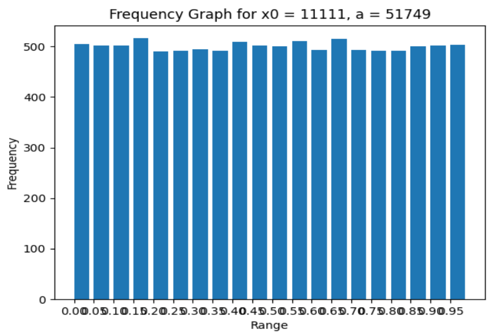

Frequency Graph for x0 = 11111, a = 1597
Frequency
Range

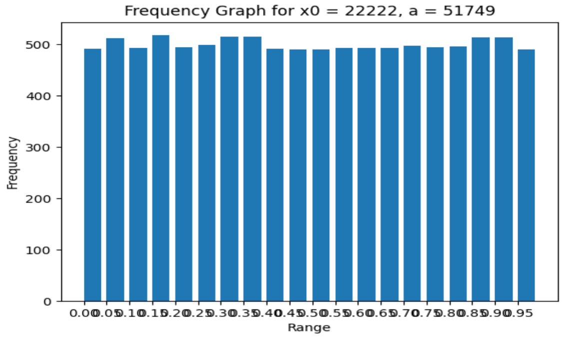

Frequency Graph for x0 = 22222, a = 1597
Frequency
Range

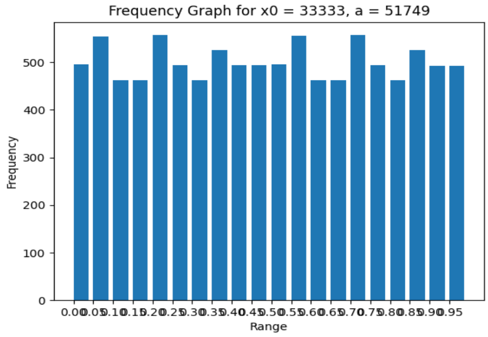

Frequency Graph for x0 = 33333, a = 1597
Frequency
Range


Frequency Graph for x0 = 12345, a = 51749
Frequency
Range

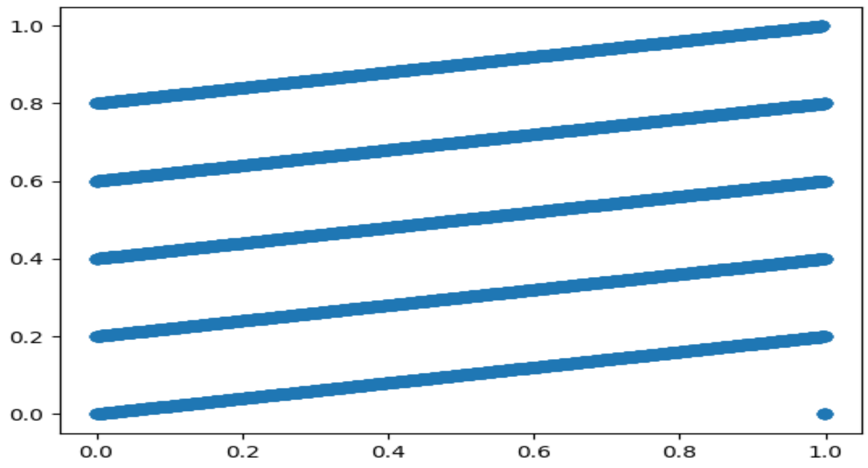

Frequency Graph for x0 = 67890, a = 51749
Frequency
Range


Frequency Graph for x0 = 11111, a = 51749
Frequency
Range


Frequency Graph for x0 = 22222, a = 51749
Frequency
Range


Frequency Graph for x0 = 33333, a = 51749
Frequency
Range

Problem 3:

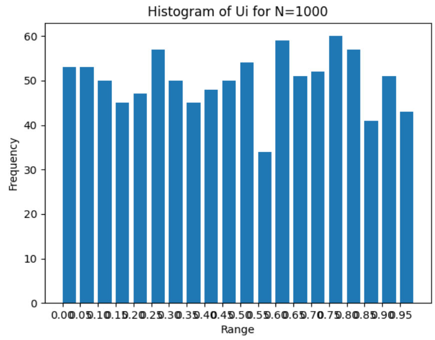


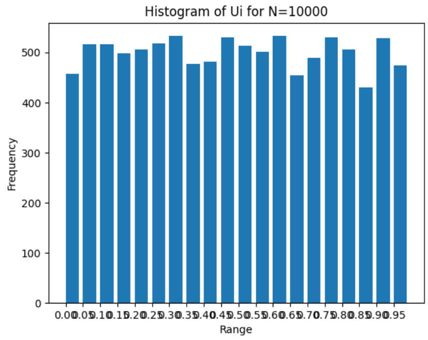
I chose x0 = 12345 for the LCG algorithm. This value was chosen arbitrarily.

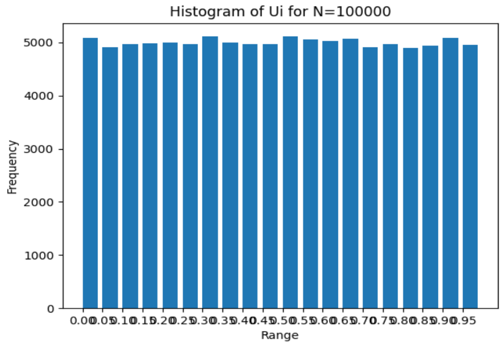
The plot shows a characteristic pattern of the LCG algorithm, with points scattered throughout the unit square. The plot does not show any obvious periodic behavior, indicating that the chosen x0 = 12345 is suitable for generating a sequence of random numbers.

Problem 4:

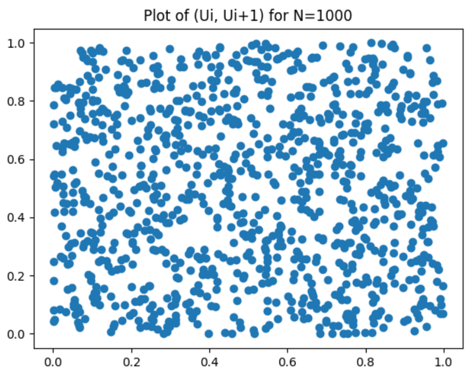
a) a = 1597
b = 1
m = 244944
x0 = 123
The first 17 numbers generated by the Linear Congruence Generator are:
0.8019465673786661, 0.7086721862956431, 0.7494855967078189, 0.9285020249526422, 0.8177379319354628, 0.9274813834999021, 0.18777353190933438, 0.8743345417728133, 0.3122672937487752, 0.6908721993598537, 0.32290646025213926, 0.6816211052322163, 0.5489091384153113, 0.6078981318178849, 0.8133205957280031, 0.8729954601868182, 0.54601868182, 0.1737540009144947

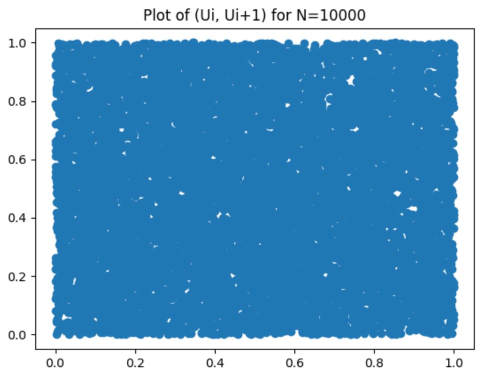


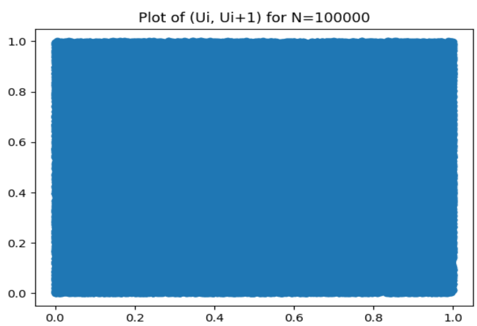




The plot shows a uniform distribution of points, which indicates that the random numbers generated are independent and identically distributed. we can observe that as N increases the histograms becomes more and more uniform.







As you increase the number of samples (N), the points on the scatter plot will become denser. Since each point is independent, they will be distributed evenly across the unit square, approaching a uniform distribution.