MA-374 LAB ASSIGNMENT -2 REPORT

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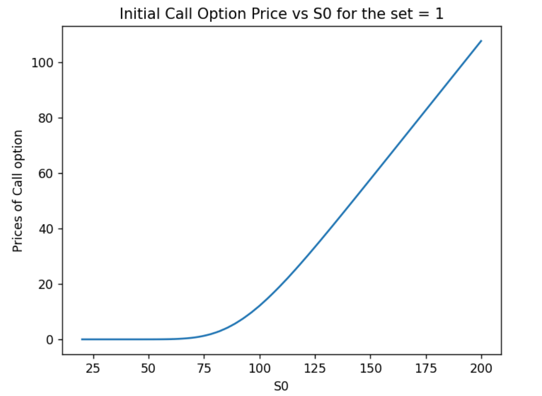
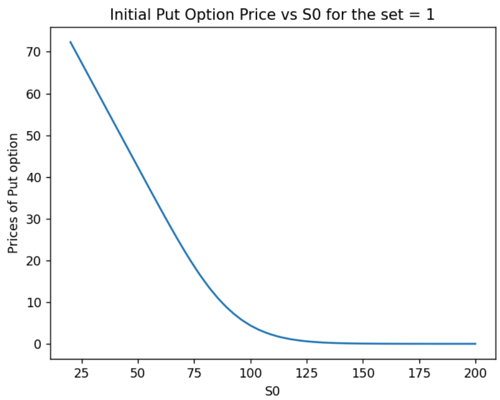
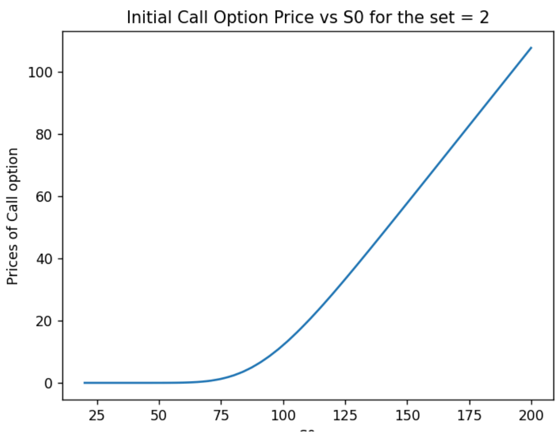
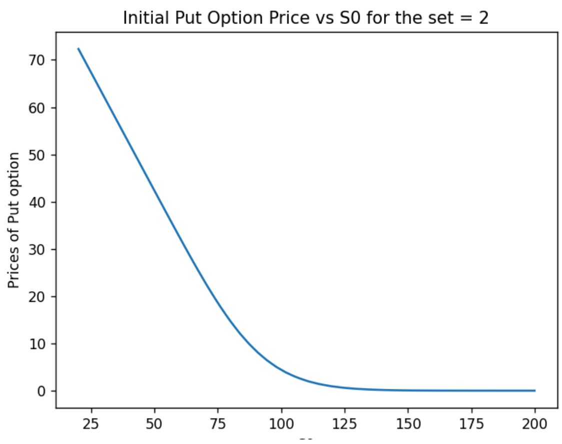
Q1)

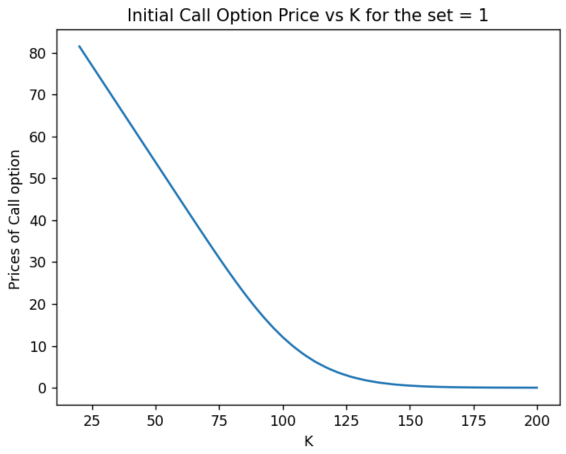
```
(base) h.ramineni@maths151:~/Desktop$ python 1.py
For Set = 1
Price of Call Option = 12.085380013710187
Price of Put Option = 4.397014652374166


For Set = 2
Price of Call Option = 12.12304707401251
Price of Put Option = 4.434681712676494
```

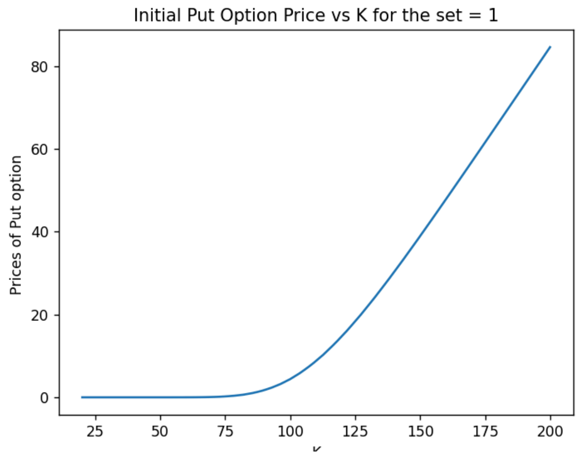


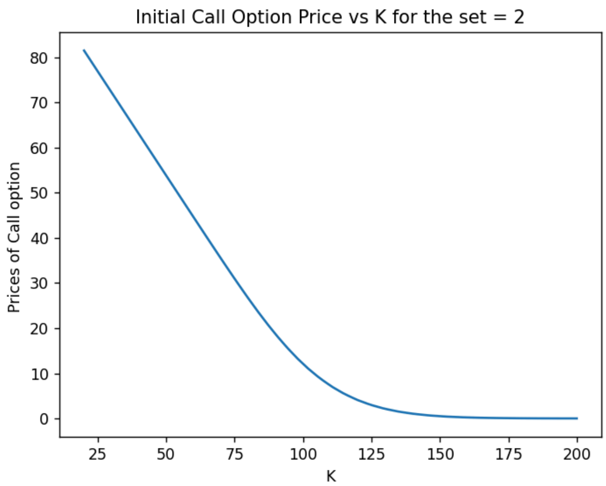


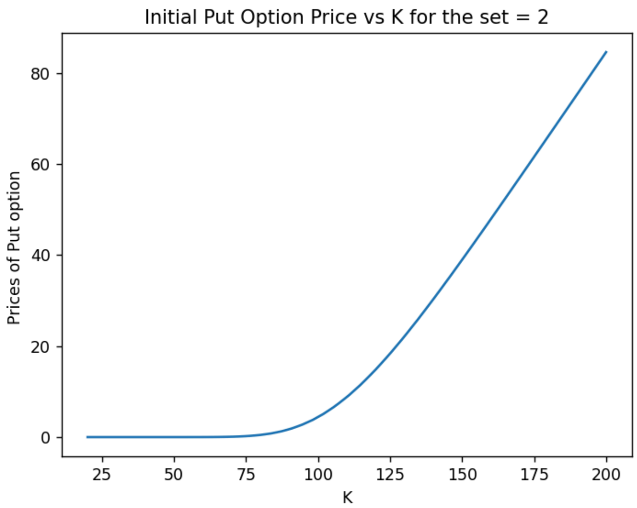


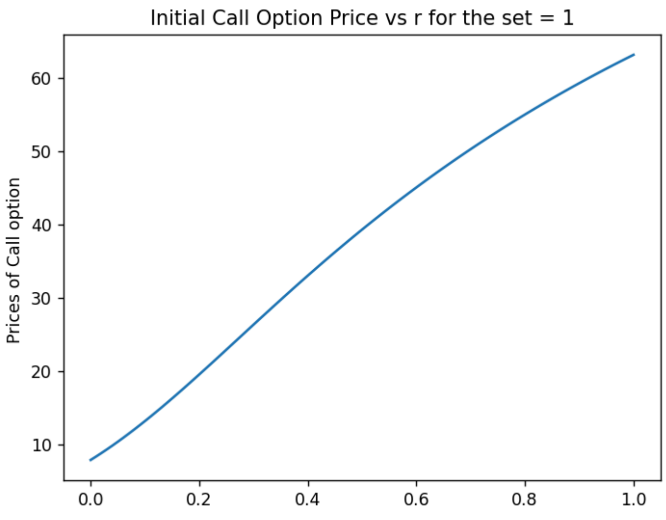


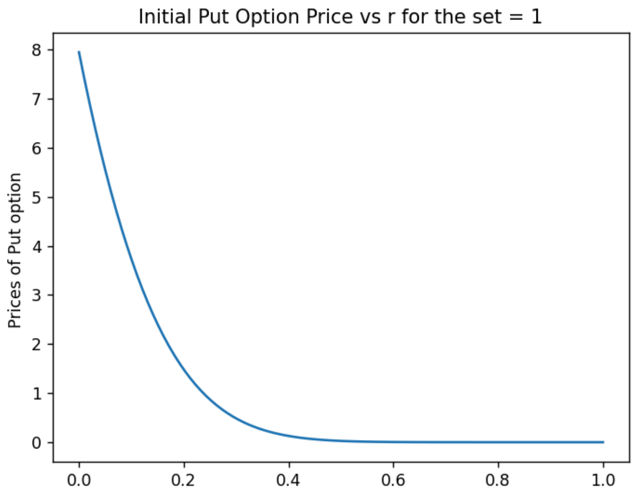

Initial Call Option Price vs K for the set = 1
Prices of Call option
K

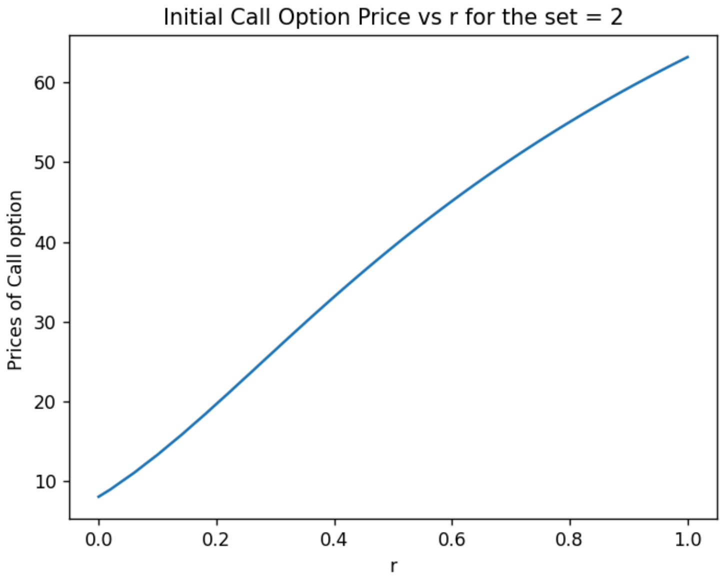

Initial Put Option Price vs K for the set = 1
Prices of Put option

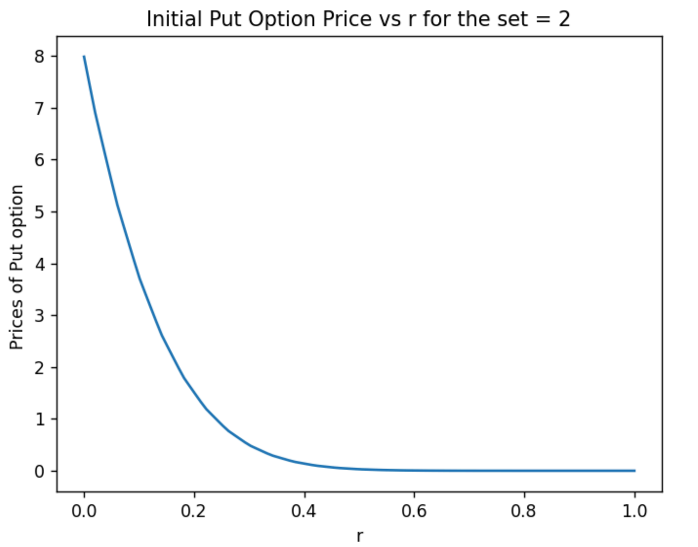

Initial Call Option Price vs K for the set = 2
Prices of Call option
K

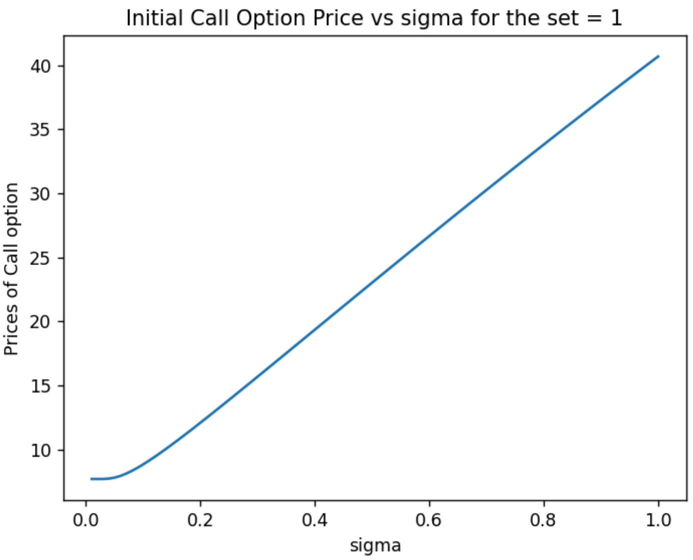

Initial Put Option Price vs K for the set = 2
Prices of Put option
K


Initial Call Option Price vs r for the set = 1
Prices of Call option

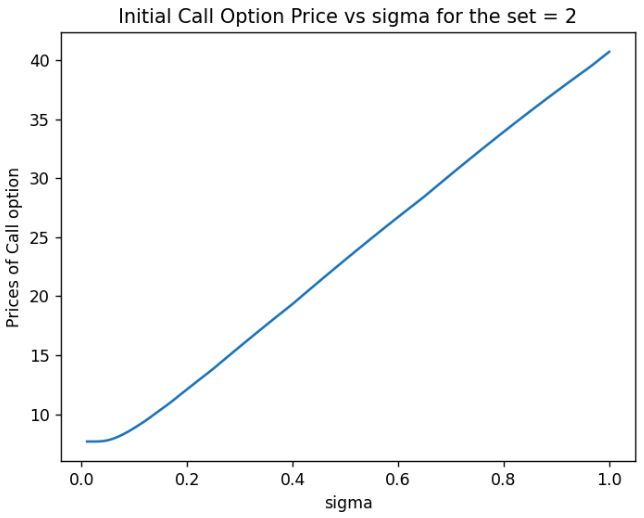

Initial Put Option Price vs r for the set = 1
Prices of Put option


Initial Call Option Price vs r for the set = 2
Prices of Call option
r

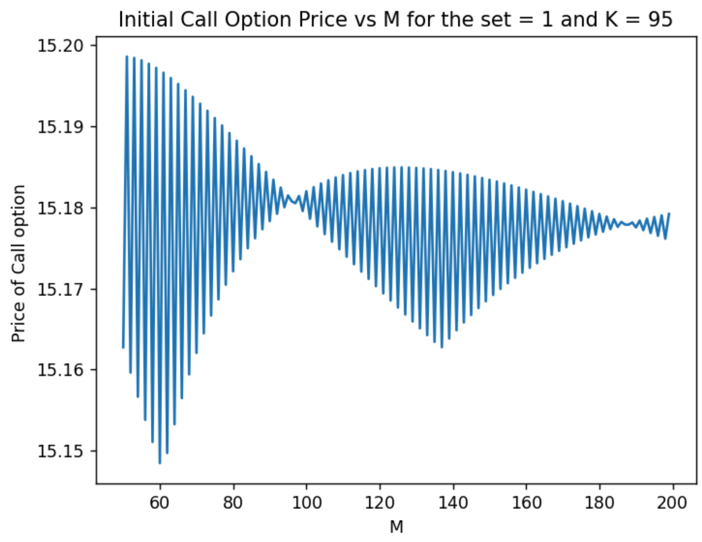

Initial Put Option Price vs r for the set = 2
Prices of Put option
r

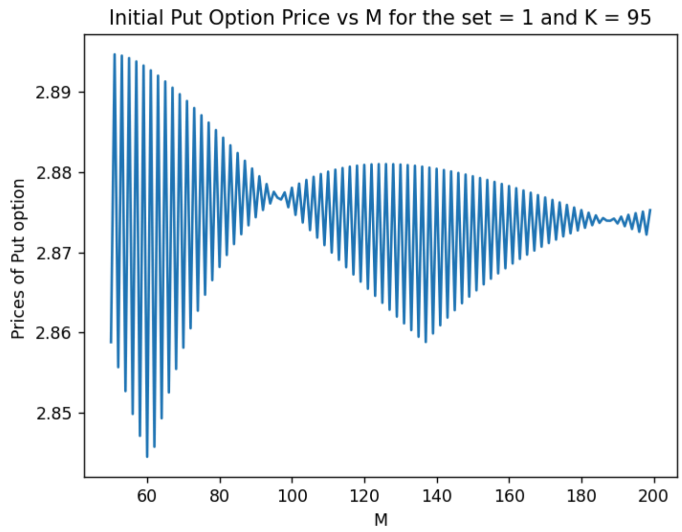

Initial Call Option Price vs sigma for the set = 1
Prices of Call option
sigma

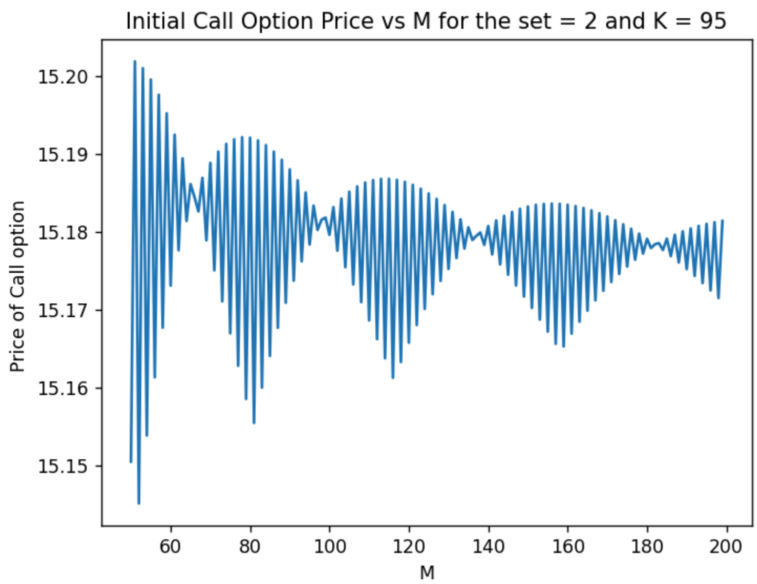

Initial Put Option Price vs sigma for the set = 1
Prices of Put option
sigma

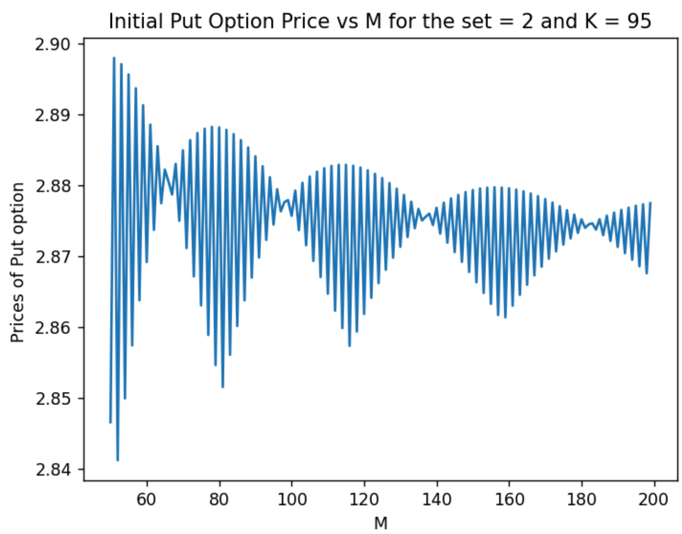

Initial Call Option Price vs sigma for the set = 2
Prices of Call option
sigma


Initial Put Option Price vs sigma for the set = 2
Prices of Put option
sigma

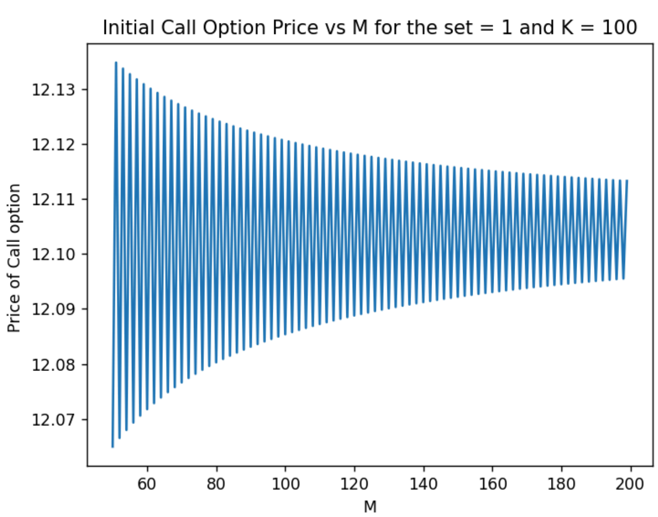


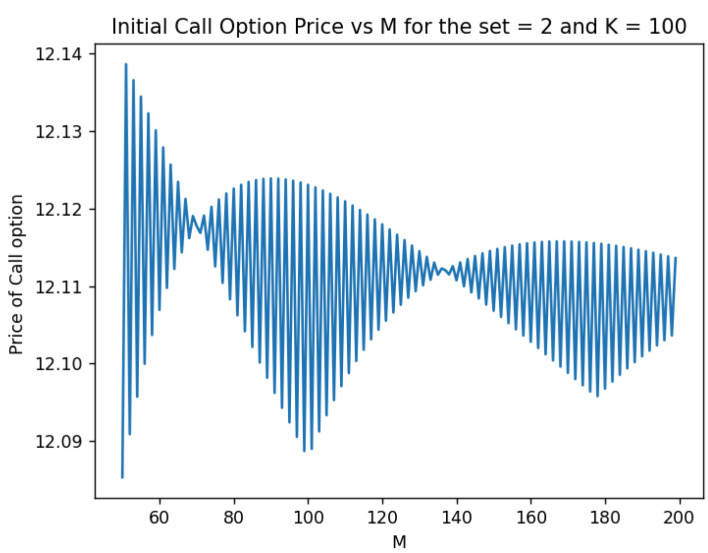






similarly for other values of K



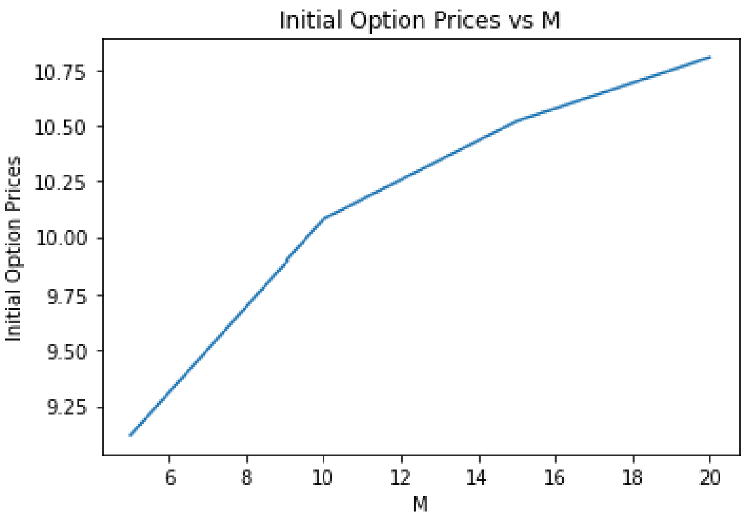


2) The initial option prices for the Loopback Option are:

SI No	M	Initial Option Price	Execution Time (in sec)
1.	5	9.119298985864683	0.000476837158203125
2.	10	10.080582906831	0.003813982009887695
3.	25	11.00349533564633	217.85930705070496
4.	26	11.036041506227305	450.0798707008362
5.	27	11.067921312247574	928.2180182933807
6.	50	Not feasible	Not feasible

For M = 50, the algorithm is unable to calculate the option price since it has exponential time complexity. For M = 26 and 27, we can see that as we increase M by single unit, the execution time almost doubles. So, computation for M = 50 is not feasible with this algorithm for finding out the loopback option.

(a) The plots comparing the above values are as follows:



As the value of M increases, the initial option price increases, and it seems that the prices tend to converge as M is increased further.

(b) The values of the options at all intermediate time points for M = 5:
(<u>Note:</u> 't' shows time intervals wrt M)

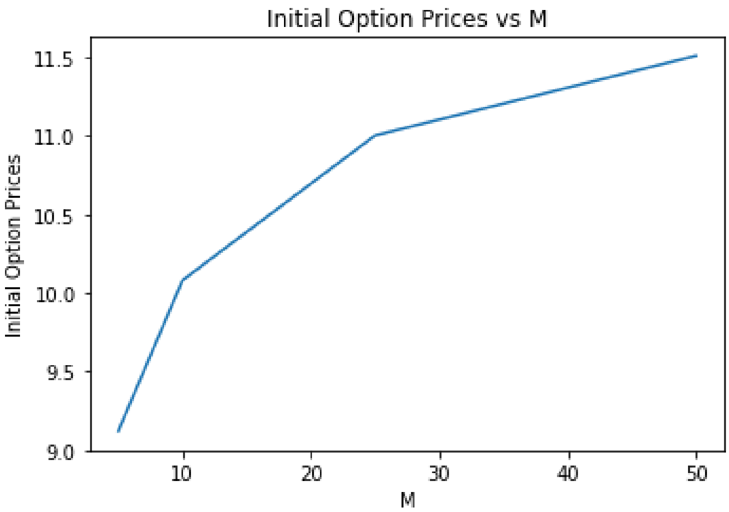
	t = 0	t = 1	t = 2	t = 3	t = 4	t = 5
1.	9.12	9.03	8.55	7.42	5.50	0.00
2.	x	9.50	9.78	9.96	9.57	11.18
3.	x	x	7.15	6.20	4.60	0.00
4.	x	x	12.17	13.71	15.63	19.45
5.	x	x	x	6.20	4.60	0.00
6.	x	x	x	8.32	8.00	9.35
7.	x	x	x	7.15	6.68	6.37
8.	x	x	x	17.58	21.19	25.39
9.	x	x	x	x	4.60	0.00
10.	x	x	x	x	8.00	9.35
11.	x	x	x	x	3.85	0.00
12.	x	x	x	x	13.07	16.27
13.	x	x	x	x	3.85	0.00
14.	x	x	x	x	10.68	13.58
15.	x	x	x	x	10.68	13.58
16.	x	x	x	x	25.05	29.48
17.	x	x	x	x	x	0.00
18.	x	x	x	x	x	9.35
19.	x	x	x	x	x	0.00
20.	x	x	x	x	x	16.27
21.	x	x	x	x	x	0.00
22.	x	x	x	x	x	7.82
23.	x	x	x	x	x	5.33
24.	x	x	x	x	x	21.23
25.	x	x	x	x	x	0.00
26.	x	x	x	x	x	7.82
27.	x	x	x	x	x	2.90
28.	x	x	x	x	x	18.81
29.	x	x	x	x	x	2.90
30.	x	x	x	x	x	18.81
31.	x	x	x	x	x	18.81
32.	x	x	x	x	x	32.11

(a) The initial option prices for the Loopback Option are:

SI No	M	Initial Option Price	Execution Time (in sec)
1.	5	9.119298985864683	0.00028252601623535156
2.	10	10.080582906831	0.0011289119720458984
3.	25	11.00349533564633	0.06793522834777832

4.	50	11.510862222177286	4.077192544937134

(b) The plots comparing the above values are as follows:



As the value of M increases, the initial option price increases, and it seems that the prices tend to converge as M is increased further.

(c) The values of the options at all intermediate time points for M = 5 (each state is characterised by a tuple denoting - (i) stock price at that instant, and (ii) maximum stock price among all the stock prices in the current path):

3)
t = 0 :

	Intermediate state	Option Price
1.	(100, 100)	9.119298985864683

t = 1 :

	Intermediate state	Option Price
1.	(110.676651999383, 110.676651999383)	9.027951165547751
2.	(92.54800352077254, 100)	9.504839866450853

t = 2:

	Intermediate state	Option Price
1.	(122.49321297792528, 122.49321297792528)	8.548076183576441
2.	(102.42903178906215, 110.676651999383)	9.799118753547026
3.	(102.42903178906214, 102.42903178906214)	7.147915756774744
4.	(85.65132955680926, 100)	12.168664659721792

t = 3:

	Intermediate state	Option Price
1.	(135.57138705044142, 135.57138705044142)	7.416771005131011
2.	(113.3650230595177, 122.49321297792528)	9.955271272957816
3.	(113.3650230595177, 113.3650230595177)	6.201916453882752
4.	(94.79602394643446, 110.676651999383)	13.712862965988533
5.	(113.36502305951768, 113.36502305951768)	6.201916453882752
6.	(94.79602394643445, 102.42903178906214)	8.32461466963314
7.	(94.79602394643445, 100)	7.14841820819012
8.	(79.26859549382432, 100)	17.582062714095418

t = 4:

	Intermediate state	Option Price
1.	(150.04587225655362, 150.04587225655362)	5.501638813873981
2.	(125.46861206060268, 135.57138705044142)	9.571391531700229
3.	(125.46861206060268, 125.46861206060268)	4.600479677676438
4.	(104.91706553244704, 122.49321297792528)	15.631851880479827
5.	(104.91706553244704, 113.3650230595177)	8.003613780975444
6.	(104.91706553244704, 110.676651999383)	6.6808429992566465
7.	(87.73182757949854, 110.676651999383)	21.18808934534565
8.	(125.46861206060267, 125.46861206060267)	4.600479677676438

9.	(104.91706553244703, 113.36502305951768)	8.003613780975444
10.	(104.91706553244701, 104.91706553244701)	3.8469288844156075
11.	(87.73182757949853, 102.42903178906214)	13.071380970928788
12.	(87.73182757949853, 100)	10.68090442602997
13.	(73.36150254849147, 100)	25.051229457037028

t = 5:

	Intermediate state	Option Price
1.	(166.06574787682462, 166.06574787682462)	0.0
2.	(138.86445913876912, 150.04587225655362)	11.181413117784501
3.	(138.8644591387691, 138.8644591387691)	0.0
4.	(116.118695507311, 135.57138705044142)	19.452691543130413
5.	(116.118695507311, 125.46861206060268)	9.349916553291678
6.	(116.11869550731102, 122.49321297792528)	6.374517470614265
7.	(97.09864950286031, 122.49321297792528)	25.39456347506497
8.	(116.11869550731102, 116.11869550731102)	0.0
9.	(97.09864950286031, 113.3650230595177)	16.266373556657385
10.	(97.09864950286031, 110.676651999383)	13.578002496522686
11.	(81.1940548771124, 110.676651999383)	29.48259712227059
12.	(116.11869550731099, 125.46861206060267)	9.349916553291678
13.	(116.11869550731099, 116.11869550731099)	0.0
14.	(97.0986495028603, 113.36502305951768)	16.266373556657385
15.	(116.11869550731097, 116.11869550731097)	0.0
16.	(97.0986495028603, 104.91706553244701)	7.8184160295867144
17.	(97.0986495028603, 102.42903178906214)	5.330382286201839
18.	(81.19405487711239, 102.42903178906214)	21.234976911949744
19.	(97.0986495028603, 100)	2.9013504971397026
20.	(81.19405487711239, 100)	18.805945122887607
21.	(67.89460596146952, 100)	32.10539403853048

Comparative Analysis:

1. The table compares the execution time for the different algorithms:

M	Unoptimised algorithm	Markov based algorithm
5	0.000476837158203125	0.00028252601623535156
10	0.003813982009887695	0.0011289119720458984
25	217.85930705070496	0.06793522834777832
50	Not feasible	4.077192544937134

2. The highest value of M the algorithms can handle:
    a. Unoptimised Algorithm - around 25 to 30. The execution time almost doubles up as M is increased by 1 unit. At M = 50, the google colab executing this code crashes due to insufficient RAM.
    b. Markov based Algorithm - around 85. At M = 90, the google colab executing this code crashes due to insufficient RAM.

3. The unoptimised algorithm has exponential space complexity, while the Markov based algorithm does not have exponential space complexity, since it utilizes the concept of memoization based on the ideas of dynamic programming.

4. Unoptimised algorithm is not a practical solution since the time complexity can blow up for small values of M (like 50), while the Markov based algorithm is able to cope up with this.