1. Write a program to determine the initial price of an European call and an European put option in the binomial model with the following data:

$$S(0) = 100; K = 100; T = 1; M = 100; r = 8\%; \sigma = 20\%.$$

Use the following two sets of u and d for your program.

(A) Set 1 : $u = e^{\sigma\sqrt{\Delta t}}$; $d = e^{-\sigma\sqrt{\Delta t}}$.

(B) Set 2 : $u = e^{\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}$; $d = e^{-\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}$.

Here $\Delta t = \dfrac{T}{M}$, with M being the number of subintervals in the time interval $[0, T]$. Use the continuous compounding convention in your calculations (i.e., both in $\tilde{p}$ and in the pricing formula).

Now, plot the initial prices of both call and put options (for both the above sets of u and d) by varying one of the parameters at a time (as given below) while keeping the other parameters fixed (as given above):

(A) $S(0)$.

(B) K.

(C) r.

(D) σ.

(E) M (Do this for three values of K, namely, $K = 95, 100, 105$).

2. Write a program to determine the initial price of a *lookback* (European) option in the binomial model, using the basic binomial algorithm, with the following data:

$$S(0) = 100; T = 1; r = 8\%; \sigma = 20\%.$$

The payoff of the *lookback* option is given by:

$$V = \max_{0 \le i \le M} S(i) - S(M),$$

where $S(i) = S(i\Delta t)$ with $\Delta t = \dfrac{T}{M}$ (M being the number of subintervals of the time interval $[0, T]$). Use the continuous compounding convention in your calculations (i.e., both in $\tilde{p}$ and in the pricing formula). Use the following values of u and d for your program:

$$u = e^{\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}; \qquad d = e^{-\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}$$

(A) Obtain the initial price of the option for $M = 5, 10, 25, 50$.

(B) How do the values of options at time $t = 0$ compare for the above values of M that you have taken ?

(C) Tabulate the values of the options at all intermediate time points for $M = 5$.

3. Repeat Problem 2 using the (Markov based) computationally efficient binomial algorithm. Make a comparative analysis of the two algorithms, like computational time, the value of M it can handle, etc.

Submission Deadline: 18th January 2025, 11:59 PM