Why staggered grids?

Dr. Johnny Corbino

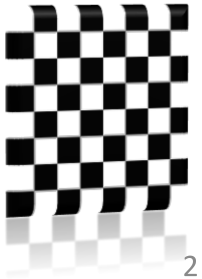
johnnycorbino@gmail.com

Justification

Mimetic finite-difference operators are constructed for staggered grids [*High-Order Mimetic Finite-Difference Operators Satisfying the Extended Gauss Divergence Theorem, J. Corbino and J. Castillo*].

This allows:

- More accurate solutions
- Expected physical behavior (Overcomes checkerboard pressure fields)



Checkerboard pressure fields

Suppose a two-dimensional incompressible flow without a body force, the governing equations are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{\partial p}{\partial x} \tag{2}$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) - \frac{\partial p}{\partial y} \tag{3}$$

Checkerboard pressure fields

Equations [1, 2, 3] are the continuity equation and the momentum equations in the x- and y-direction, respectively. The following figure represents a checkerboard pressure field,

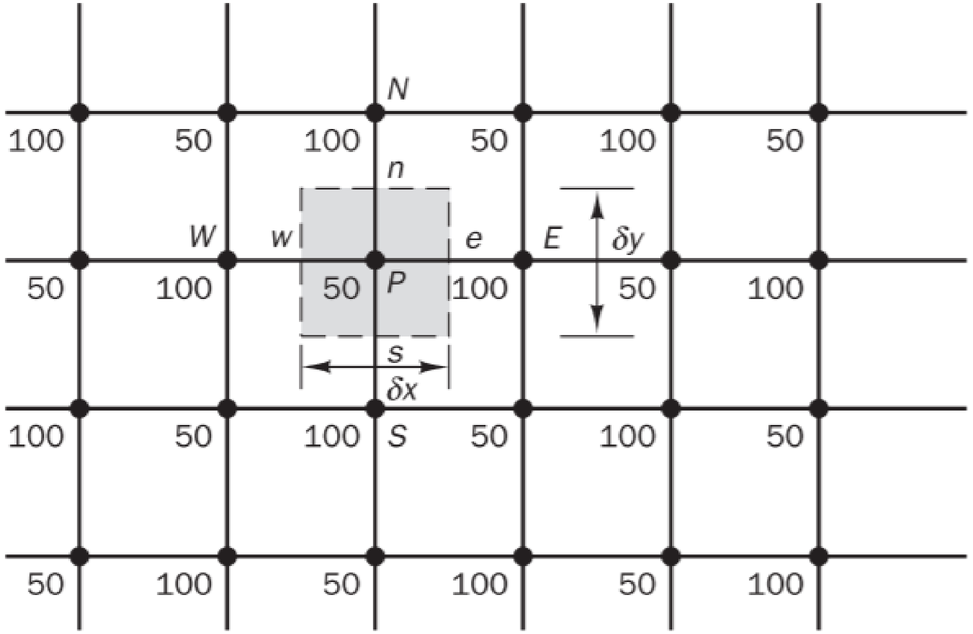


Fig. 1

Checkerboard pressure fields

The source term for the momentum equations in the x- and y-direction can be expressed as $S_u = -\frac{\partial p}{\partial x}$ and $S_v = -\frac{\partial p}{\partial y}$, respectively. If central difference is employed, the pressure gradient becomes,

$$\left(\frac{\partial p}{\partial x}\right)_P = \frac{p_e - p_w}{\Delta x} = \frac{\frac{p_E + p_P}{2} - \frac{p_P + p_W}{2}}{\Delta x} = \frac{p_E - p_W}{2\Delta x} \tag{4}$$

similarly,

$$\left(\frac{\partial p}{\partial y}\right)_P = \frac{p_N - p_S}{2\Delta y} \tag{5}$$

Checkerboard pressure fields

From equations [4, 5], we can see that **the pressure gradient at point *P* is related to the pressures of the neighbor grid points and not to its own pressure**. Therefore, if we have a pressure distribution as shown in figure [1], the discretization scheme represented by equations [4, 5] will obtain $\frac{\partial p}{\partial x} = 0$ and $\frac{\partial p}{\partial y} = 0$ throughout the computational domain. This means that **it will not recognize the difference between a checkerboard pressure field and an uniform pressure field**. This behavior is obviously non-physical.

Staggered grids

A remedy for the aforementioned problem is to use staggered grids. The idea is to store scalar variables at the ordinary nodal points, and vector variables at the cell faces in between the nodal points. The following figure [2] shows the new arrangement,

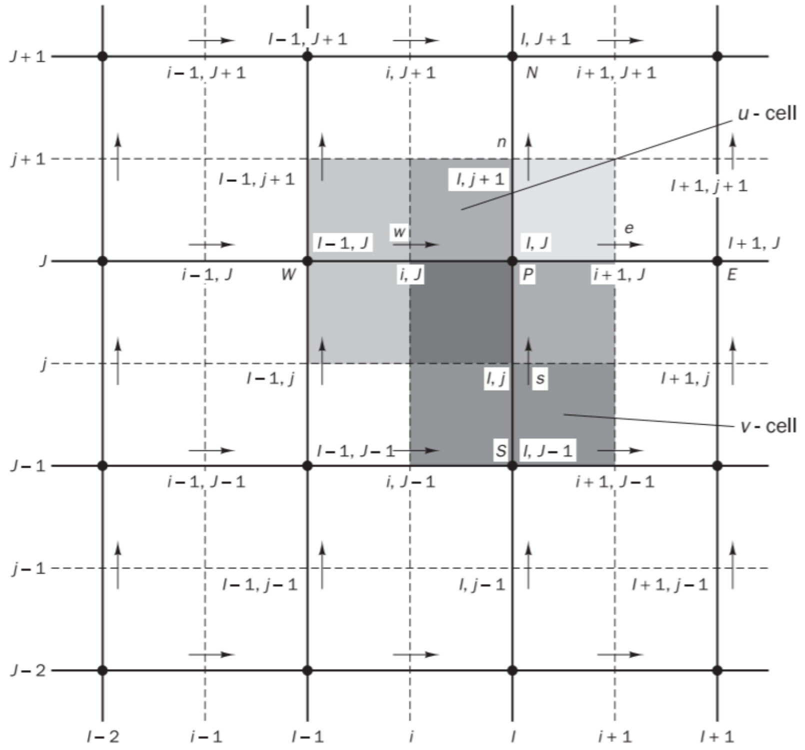


Fig. 2

Staggered grids

If we consider the checkerboard pressure field again, substitution of the appropriate nodal pressure values into equations [4, 5] now yields very significant non-zero pressure gradient terms. The staggering of the velocity avoids the unrealistic behavior for spatially oscillating pressure fields. In addition, this new arrangement does not require interpolation to calculate velocities at the cell faces (where they are needed for the scalar transport – convection-diffusion computations).