

5. To prove that for any integer n , at least one of $n, n+2, n+4$ is divisible by 3.

For any integer n , three consecutive integers can be written as $n, n+1, n+2$. Out of three consecutive integers at least one is always divisible by 3 because of the division theorem, i.e. $n = 3q + r, 0 \leq |r| \leq 2$

If either of n or $n+2$ is divisible by 3, the statement is proved.

Let's assume $n+1$ is divisible by 3 instead i.e. $n+1 = 3q, q \in \mathbb{Z}$

Adding 3 on both sides, $n+1+3 = 3q+3$
 $n+4 = 3(q+1)$

So, if $n+1$ is divisible by 3, $n+4$ is also divisible by 3.

This proves that at least one of $n, n+1$ or $n+4$ is divisible by 3.