

8. To prove that if the sequence $\{a_n\}_{n=1}^{\infty} \rightarrow L$ as $n \rightarrow \infty$, then for $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty} \rightarrow ML$.

Using the formal definition of limit of a sequence, $a_n \rightarrow L$ as $n \rightarrow \infty$ iff $(\forall \epsilon > 0) (\exists n \in \mathbb{N}) (\forall m \geq n) [|a_m - L| < \epsilon]$

Consider the expression $|a_m - L| < \epsilon$

Multiplying both sides by M does not change the inequality as $M > 0$

$$\text{i.e., } M|a_m - L| < M\epsilon$$

$$\Rightarrow |Ma_m - ML| < M\epsilon$$

We can designate $M\epsilon = \epsilon'$. Since $M > 0$, $M\epsilon > 0$ and only condition $(\forall \epsilon > 0)$ also includes $(\forall M\epsilon > 0)$ i.e. $(\forall \epsilon' > 0)$

$\therefore (\forall \epsilon' > 0) (\exists n \in \mathbb{N}) (\forall m \geq n) [|Ma_m - ML| < \epsilon']$ is valid which is the definition of $\{Ma_n\}_{n=1}^{\infty} \rightarrow ML$ as $n \rightarrow \infty$. Hence proved.