4. To prove that every odd natural number is of one of the forms $4n+1$ or $4n+3$ where $n \in \mathbb{Z}$

We see that the statement is true for the first two odd natural numbers, i.e.

$$4(0)+1 = 1$$
$$4(0)+3 = 3$$

For any possible natural number $p > 4$, we can express $p$ as

$$p = 4n + r \quad \text{where } n, r \in \mathbb{Z}$$
$$\text{and } n > 0,\ 0 \leq |r| \leq 3$$
(Division Theorem)

So $p$ can be of the form $4n$, $4n+1$, $4n+2$, $4n+3$.

Since $4n$ is clearly even $\forall n \in \mathbb{Z}$, $4n+1$ and $4n+3$ must be odd. Further, since every natural number can be expressed as $4n+r$, every odd number should be contained in the form $4n+1$ or $4n+3$. Hence proved.