3. To prove the statement "For any integer n, the number n^2+n+1 is odd".

We attempt to prove this using the method of induction.

For $n=1$, $n^2+n+1 = 3$ which is odd.

Let's assume the statement is true for n

i.e., n^2+n+1 is odd.

The equivalent expression for $(n+1)$ is

$$
\begin{aligned}
(n+1)^2 + (n+1) + 1 &= (n^2+2n+1) + (n+1) + 1 \\
&= n^2 + 3n + 3 \\
&= n^2 + 3n + 2 + 1 \\
&= (n+1)(n+2) + 1
\end{aligned}
$$

We see that $n+1$ & $n+2$ are any two consecutive integers. So at least one of them is even which in turn means that the product $(n+1)(n+2)$ is also even. This in turn means that $(n+1)(n+2)+1$ is always odd.

Hence the statement is true for $n+1$ as well and by induction is true for all n i.e ($\forall n \in \mathbb{Z}$).