7. To prove that for any $n \in N$

$$2+2^2+2^3+\cdots+2^n = 2^{n+1}-2$$

We prove this using induction.

For $n=1$

$2 = 2^{1+1}-2 = 2$. So the statement is valid for $n=1$.

Let's assume the statement is true for n, i.e.

$$2+2^2+\cdots+2^n = 2^{n+1}-2$$

Adding 2^{n+1} to both sides,

$$2+2^2+\cdots+2^n+2^{n+1} = 2^{n+1}-2+2^{n+1}$$

$$2+2^2+\cdots+2^n+2^{n+1} = 2*2^{n+1}-2$$

$$2+2^2+\cdots+2^n+2^{n+1} = 2^{n+2}-2 = 2^{(n+1)+1}-2$$

Therefore the statement is true for $n+1$ if it is true for n.

Hence proved.