7370 - Probability & Statistics for Scientists and Engineers

PROJECT

“Normal Approximation to Binomial and Poisson’s Distribution”

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Binomial and its Normal Approximation

1.Theory Claim

For a Binomial distribution,

- **Mean(μ) = np**, where n is the no of values and P is the probability
- **Variance(σ^2) = npq**, where q = 1-p
- **Standard Deviation(σ) = √npq** or √Variance

Conditions for Normal approximation to Binomial (For reliable results):

- **np >= 5**
- **n(1-p) >= 5**

W.r.t Central Limit Theorem, if X is a binomial random variable with mean μ = np and variance σ^2 = npq, then the limiting form of the distribution of:

Z = X − np/ √npq

as n → ∞, is the standard normal distribution n(z; 0, 1).

Let us calculate the exact probability w.r.t Binomial first and then Normal approximation to compare observe the results according to the theory above,

Consider n=10 and p=0.5

P(Y=5) = P(Y<=5) - P(Y<=4)

= 0.6230 - 0.3770

= **0.2460 or 24.60% ---------------Binomial**

Y in the above example is defined as a sum of independent, identically distributed random variables. Therefore, as long as 'n' is sufficiently large, we can use the Central Limit Theorem as stated above to calculate probabilities for Y.

Now to approximate the same using Normal distribution,

μ = np =10(0.5) = 2

σ^2 = npq = √(10)(0.5)(0.5) = 2.5

We are using a continuous distribution to approximate a discrete distribution; therefore, we are required to make some corrections for this for better estimation. This is done by applying '**continuity correction**' which reduces the calculation to a normal probability calculation.

P(Y=5) = P(4.5 < Y < 5.5)

= P((4.5-5)/√2.5 < Y < (5.5-5)/√2.5)

= P(-0.32 < Z < 0.32)

= P(Z<0.32) - P(Z < -0.32)

= 0.6255 - 0.3745

= **0.2510 or 25.10% ---------------Normally Approximated**

The values obtained from the Binomial calculation and the Normally approximated calculation is 24.6% and 25.1%. **This is not at all a bad approximation considering we only used a relatively small n value of 10 to represent the concept.**

The same theory holds true for any value while doing Normal approximation to Binomial distribution, when the conditions are met, or the values are almost meet the condition on or the other way.

2.Qualitative Claim

Let us consider a population of 1000 random binomial values generated randomly with the function **rbinom(1000, N, P)** in R Studio; where the Probability and N values are as depicted below:

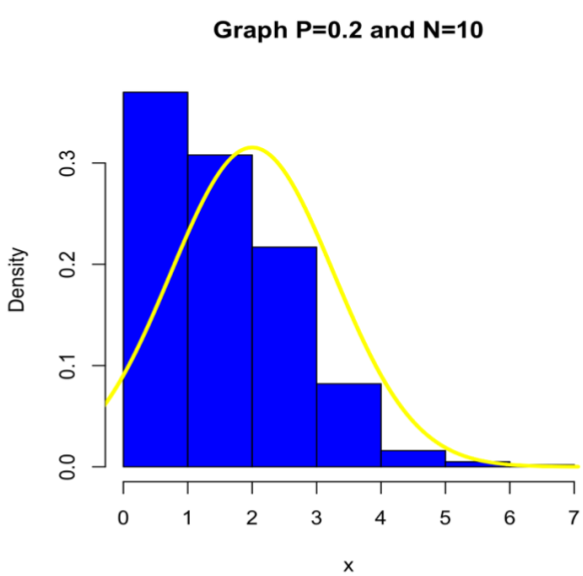
We keep the '**probability constant**' while '**changing the N value**', [N=10,25,50,75,90] to observe and study the characteristic behavior of the graph.

- Case 1: (p=0.2, n=10)

  Condition – np>=5 and nq>=5

  np = 0.2(10) = 2 [Fails condition 1]

  nq = 0.8(10) = 8 [Passes condition 2]

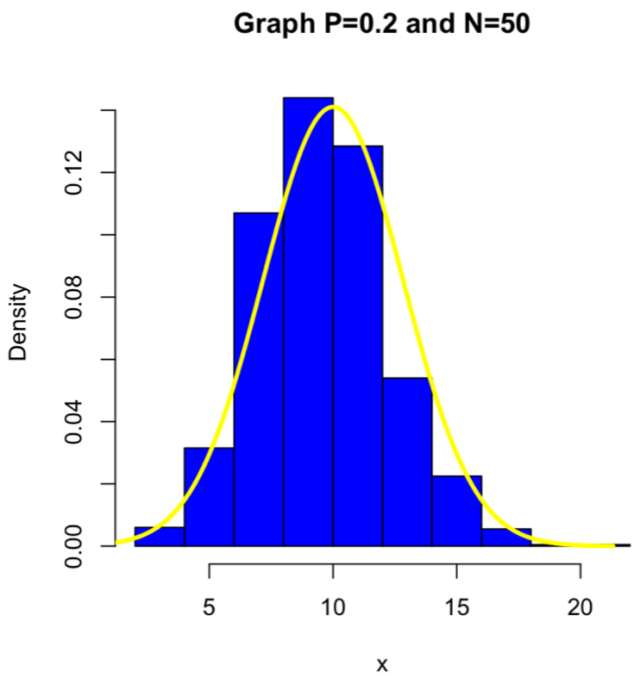


- Case 2: (p=0.2, n=50)

  Conditions – np>=5 and nq>=5

  np = 0.2(50) = 10 [Passes condition 1]

  nq = 0.8(50) = 40 [Passes condition 2]

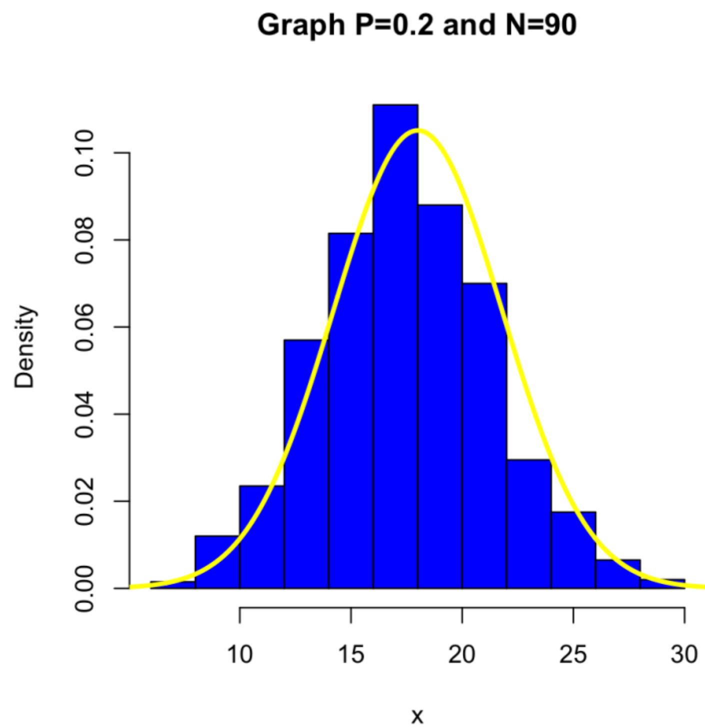


- Case 3: (p=0.2, n=90)

  Condition – np>=5 and nq>=5

  np = 0.2(90) = 18 [Passes condition 1]

  nq = 0.8(90) = 72 [Passes condition 2]



Inferences:

From the above graphs we can conclude **qualitatively that the normal fit and quality of normal approximation gets better with the value of N when p is kept constant**.

- Case 1: the N value is **very small**, it doesn't not pass the required conditions. It doesn't provide a normal graph.
- Case 2: the N value is **marginally acceptable**, it passes the required conditions by a small margin. It gives an almost normal graph at the threshold as the values are close by and we can rely on this graph with caution.
- Case 3: the N value is **large enough**, it comfortably passes the required conditions. This gives a perfectly normal graph which we can easily rely on.

This shows that the **quality of the normal approximated graph increases with N**, and it **also verifies the condition for normal approximation as it does not give a perfectly normal graph when the np and n(1-p) are less than a minimum of 5**.

3.Quantitative Claim

Case 1:

When we keep N constant (small) and vary P as shown below.

BINOMIAL	N=10					
	p=0.1;		p=0.25;		p=0.5;	
r	Binomial	Normal	Binomial	Normal	Binomial	Normal
0	0.34867844	0.29906466	0.05631351	0.07206258	0.000976563	0.00221278
1	0.73609893	0.70093534	0.24402523	0.23260337	0.010742188	0.01342648
2	0.92980917	0.94309272	0.5255928	0.5	0.0546875	0.05691871
3	0.9872048	0.99579887	0.77587509	0.76739663	0.171875	0.17138493
4	0.99836506	0.99988772	0.92187309	0.92793742	0.376953125	0.37591187
5	0.9998531	0.99999895	0.98027229	0.9857705	0.623046875	0.62408813
6	0.99999088	1	0.99649429	0.99825657	0.828125	0.82861507
7	0.99999963	1	0.9995842	0.99986964	0.9453125	0.94308129
8	0.99999999	1	0.99997044	0.99999411	0.989257813	0.98657352
9	1	1	0.99999905	0.99999984	0.999023438	0.99778722
10	1	1	1	1	1	0.99974797
11		1		1		0.99998031
12		1		1		0.99999895
12		1		1		0.99999895
14		1		1		1
15		1		1		1

<u>Inference Case 1:</u>

We can infer the relationship between small N values with respect to various P values from the table computed above.

- This indicates the **quality of the approximation is quite good for large values of n**.
- If **p is close to 0.5, even a moderate or small sample size will be sufficient for a reasonable approximation**. Like the above case where we get a decent approximation compared to others at p = 0.5.
- **The approximation is fairly crude for n = 10**, **at p = 0.1 and p = 0.25.**

However, **even for n = 10, there is a significant improvement in approximation at p = 0.50**.

Therefore, as proven above, we generally go for the approximation with p value **closest to 0.5 or at 0.5 to get the best results when N is fixed**.

In addition to it, assuming we need the best approximation from the above table,

Not only we would select p = 0.5 but also check the place at which the approximation gives reliable values w.r.t the actual binomial distribution.

This is calculated by checking the difference between the results of both distributions.

In this case, we get reliable values at p=0.5 from r>=14

[Reason : No difference between the Binomial and Normal approximated results at that point and from there on]

<u>Case 2:</u>

When we keep <u>P constant and vary N</u> as shown below.

i) Variation 1, P = 0.06 [Low P value]

BINOMIAL	P=0.06									
	N=5		N=10		N=25		N=80		N=95	
r	Binomial	Normal	Binomial	Normal	Binomial	Normal	Binomial	Normal	Binomial	Normal
0	0.73390402	0.64678233	0.53861511	0.447028074	0.21291014	0.199844672	0.00708318	0.021465184	0.002799923	0.01233563
1	0.96812871	0.98808587	0.882412	0.884650796	0.55266036	0.5	0.043252611	0.060140029	0.01977818	0.03480116
2	0.99802973	0.99998287	0.98116216	0.994301699	0.81289457	0.800155328	0.13444575	0.13944604	0.07071295	0.08341394
3	0.99993831	1	0.9979707	0.999943776	0.94024322	0.953943205	0.285787554	0.270260967	0.171498771	0.17094255
4	0.99999922	1	0.99984825	0.999999897	0.98495073	0.994240199	0.471744772	0.44384164	0.319460934	0.30208064
5	1	1	0.99999206	1	0.99693615	0.999622389	0.652162839	0.629130502	0.491348894	0.46557252
6		1	0.99999971	1	0.99948624	0.999987281	0.796113424	0.788242784	0.655922473	0.63518477
7		1	0.99999999	1	0.99992804	0.999999783	0.893247253	0.898158192	0.789482185	0.78160894
8		1	1	1	0.9999915	0.999999998	0.949822542	0.959238273	0.883258154	0.88679531
9		1	1	1	0.99999915	1	0.978712051	0.986540896	0.941119922	0.94967198
10		1	1	1	0.99999993	1	0.991804531	0.996357087	0.972882339	0.98094654
11		1		1	0.99999999	1	0.997122559	0.999195519	0.988548521	0.99388996
12		1		1	1	1	0.999074389	0.999855555	0.995548305	0.99834696
12		1		1	1	1	0.999074389	0.999855555	0.995548305	0.99834696
14		1		1	1	1	0.999925131	0.999997522	0.999467396	0.99992817
15		1		1	1	1	0.99998104	0.999999764	0.99983499	0.99998851
16		1		1	1	1	0.999995537	0.999999982	0.999952308	0.99999846
17		1		1	1	1	0.999999021	0.999999999	0.999987106	0.99999983
18		1		1	1	1	0.999999799	1	0.999996732	0.99999998
19		1		1	1	1	0.999999962	1	0.999999221	1
20		1		1	1	1	0.999999993	1	0.999999825	1
21		1		1	1	1	0.999999999	1	0.999999963	1
22		1		1	1	1	1	1	0.999999993	1
23		1		1	1	1	1	1	0.999999999	1
24		1		1	1	1	1	1	1	1
25		1		1	1	1	1	1	1	1
	np=0.3		np=0.6		np=1.5		np=4.8		np=5.7	
	√npq=0.5310		√npq=0.7509		√npq=1.1874		√npq=2.1241		√npq=2.3147	

ii) Variation 2, P = 0.4 [Close to 0.5, which is considered a good P value]

BINOMIAL	P=0.4									
	N=5		N=10		N=25		N=80		N=95	
r	Binomial	Normal	Binomial	Normal	Binomial	Normal	Binomial	Normal	Binomial	Normal
0	0.07776	0.08544295	0.006046618	0.01192999	2.84303E-06	5.25511E-05	1.7869E-18	3.2636E-13	8.40173E-22	2.02183E-15
1	0.33696	0.32403168	0.046357402	0.05328104	5.02268E-05	0.000259999	9.7088E-17	1.6921E-12	5.40511E-20	1.05174E-14
2	0.68256	0.67596832	0.167289754	0.16644624	0.000429297	0.001099411	2.6067E-15	8.3359E-12	1.72133E-18	5.23985E-14
3	0.91296	0.91455705	0.382280602	0.37343545	0.002366769	0.003980598	4.6106E-14	3.9022E-11	3.61784E-17	2.50031E-13
4	0.98976	0.98876334	0.633103258	0.62656455	0.009470831	0.012369698	6.0435E-13	1.7358E-10	5.6452E-16	1.14275E-12
5	1	0.99930128	0.833761382	0.83355376	0.029362205	0.033091323	6.2613E-12	7.3382E-10	6.97506E-15	5.00268E-12
6		0.99998005	0.945238118	0.94671896	0.073565258	0.076513417	5.3402E-11	2.9484E-09	7.10805E-14	2.09785E-11
7		0.99999974	0.987705446	0.98807001	0.153551735	0.153708221	3.8563E-10	1.126E-08	6.1445E-13	8.42722E-11
8		1	0.998322278	0.99816322	0.27353145	0.270138267	2.4067E-09	4.0877E-08	4.59916E-12	3.24306E-10
9		1	0.999895142	0.99980771	0.424617018	0.419125321	1.3186E-08	1.4108E-07	3.02784E-11	1.19567E-09
10		1	1	0.99998642	0.585774956	0.580874679	6.4207E-08	4.6294E-07	1.77506E-10	4.2236E-09
11		1		0.99999936	0.732282173	0.729861733	2.8066E-07	1.4446E-06	9.35952E-10	1.42954E-08
12		1		0.99999998	0.846232231	0.846291779	1.1104E-06	4.287E-06	4.47537E-09	4.63642E-08
12		1		0.99999998	0.846232231	0.846291779	1.1104E-06	4.287E-06	4.47537E-09	4.63642E-08
14		1		1	0.965608482	0.966908677	1.3235E-05	3.25E-05	7.83666E-08	4.29266E-07
15		1		1	0.986830927	0.987630302	4.0314E-05	8.3054E-05	2.9014E-07	1.22564E-06
16		1		1	0.995673612	0.996019402	0.00011365	0.00020201	9.96053E-07	3.35459E-06
17		1		1	0.998794559	0.998900589	0.00029772	0.00046778	3.183E-06	8.80249E-06
18		1		1	0.999719285	0.999740001	0.0007272	0.00103155	9.50083E-06	2.21473E-05
19		1		1	0.99994641	0.999947449	0.00166152	0.00216697	2.65701E-05	5.3438E-05
20		1		1	0.999991835	0.999990935	0.0035613	0.00433815	6.98122E-05	0.00012367
21		1		1	0.999999046	0.999998667	0.00717993	0.00828008	0.00017277	0.000274566
22		1		1	0.99999992	0.999999833	0.01364959	0.01507522	0.000403644	0.000584904
23		1		1	0.999999996	0.999999982	0.02452613	0.02619675	0.00089216	0.001195866
24		1		1	1	0.999999998	0.04174732	0.04347926	0.001869192	0.002347231
25		1		1	1	1	0.0674643	0.06897844	0.00371904	0.004424208
	np=2		np=4		np=10		np=32		np=38	
	√npq=1.0954		√npq=1.5491		√npq=2.4494		√npq=4.3817		√npq=4.7749	

iii) Variation 3, P = 0.9 [High P value]

BINOMIAL	P=0.9									
	N=5		N=10		N=25		N=80		N=95	
r	Binomial	Normal	Binomial	Normal	Binomial	Normal	Binomial	Normal	Binomial	Normal
0	1E-05	1.23802E-09	1E-10	1.61497E-19	1E-25	5.26985E-49	1E-80	9.6204E-157	1E-95	4.3289E-186
1	0.00046	3.86965E-06	9.1E-09	1.32482E-15	2.26E-23	7.79354E-45	7.21E-78	1.8714E-152	8.56E-93	8.5856E-182
2	0.00856	0.001434132	3.736E-07	3.63606E-12	2.4526E-21	7.40641E-41	2.56681E-75	3.169E-148	3.62521E-90	1.515E-177
3	0.08146	0.068012612	9.1216E-06	3.35516E-09	1.70123E-19	4.5239E-37	6.01513E-73	4.6714E-144	1.01267E-87	2.3787E-173
4	0.40951	0.5	0.000146903	1.04856E-06	8.46979E-18	1.77648E-33	1.04369E-70	5.9942E-140	2.09885E-85	3.3231E-169
5	1	0.931987388	0.001634937	0.000112282	3.22197E-16	4.48619E-30	1.42998E-68	6.6956E-136	3.44232E-83	4.1304E-165
6		0.998565868	0.012795198	0.004201132	9.73402E-15	7.2881E-27	1.61128E-66	6.5107E-132	4.65322E-81	4.568E-161
7		0.99999613	0.070190826	0.056907281	2.39651E-13	7.61985E-24	1.53553E-64	5.5111E-128	5.33175E-79	4.495E-157
8		0.999999999	0.263901071	0.299064657	4.89548E-12	5.12963E-21	1.26317E-62	4.061E-124	5.28569E-77	3.9356E-153
9		1	0.65132156	0.700935343	8.40445E-11	2.22478E-18	9.11061E-61	2.605E-120	4.60502E-75	3.0659E-149
10		1	1	0.943092719	1.22379E-09	6.22096E-16	5.83207E-59	1.4547E-116	3.56942E-73	2.1252E-145
11		1		0.995798868	1.52116E-08	1.12249E-13	3.34633E-57	7.0717E-113	2.48604E-71	1.3107E-141
12		1		0.999887718	1.62083E-07	1.30839E-11	1.73501E-55	2.9928E-109	1.56858E-69	7.193E-138
12		1		0.999887718	1.62083E-07	1.30839E-11	1.73501E-55	2.9928E-109	1.56858E-69	7.193E-138
14		1		0.999999997	1.1681E-05	4.8213E-08	3.53201E-52	3.5365E-102	4.76626E-66	1.526E-130
15		1		1	7.89819E-05	1.53063E-06	1.40159E-50	9.875E-99	2.32019E-64	5.8995E-127
16		1		1	0.000457549	3.16712E-05	5.13557E-49	2.40054E-95	1.04584E-62	2.0294E-123
17		1		1	0.002261312	0.00042906	1.74392E-47	5.08039E-92	4.38162E-61	6.2117E-120
18		1		1	0.009476361	0.003830381	5.50597E-46	9.36067E-89	1.71186E-59	1.6918E-116
19		1		1	0.033399945	0.022750132	1.62086E-44	1.50156E-85	6.25516E-58	4.1001E-113
20		1		1	0.097993621	0.09121122	4.4602E-43	2.09708E-82	2.14327E-56	8.8417E-110
21		1		1	0.236408642	0.252492538	1.14983E-41	2.5499E-79	6.90235E-55	1.6966E-106
22		1		1	0.46290595	0.5	2.78261E-40	2.69946E-76	2.09367E-53	2.8969E-103
23		1		1	0.728794094	0.747507462	6.33261E-39	2.48819E-73	5.99281E-52	4.4016E-100
24		1		1	0.928210201	0.90878878	1.35744E-37	1.99686E-70	1.62146E-50	5.95106E-97
25		1		1	1	0.977249868	2.74468E-36	1.39534E-67	4.15342E-49	7.15975E-94
	np=4.5		np=9		np=22.5		np=72		np=85.5	
	√npq=0.6708		√npq=0.9486		√npq=1.5		√npq=2.6832		√npq=2.9240	

<u>Inference Case 2:</u>

In all the above variations of case 2, we can notice the following trends:

- The **higher the value of N the higher is the quality of the approximation**.
- Even when **p is lower a very large N value gives a good or close to good result** as depicted in the first variation.
- **When p is higher, and the N is lower it still gives us mediocre results**.
- We can clearly see the **trend in which the mean and standard deviation changes with respect to every p and N value**.
  - In variation 1, the mean is lower and so is the standard deviation when p value is less like 0.06.
  - In variation 2, mean and standard deviation both are better and close to the acceptable range when p is close to 0.5. This proves the reason for the existence of the conditions and the quantitative proof behind it.
  - In variation 3, the mean is higher but standard deviation is not as high when p is high like 0.9.
- Finally, **no matter whether the value of p is ideal or not (i.e., close to 0.5), a comparatively very large n value will always provide us with good results**.
- **An ideal p value enhances the result provided by an approximation with a very large N value.**

When p is fixed at p = 0.1, note the improvement of the approximation as we go from n = 5 to n = 95.

Like in the above case, the Normal approximation gives the most reliable results when the difference between Binomial and Normally approximated result is 0.

For instance, when we have to pick a value from variation 3, p=0.9 and N=

Usually in the other cases where p = 0.4 or p = 0.9, the values of np and n(1-p) are at least close to or more than 5 due to which we get acceptable and good quality values respectively.

It turns out that the normal distribution with $\mu = np$ and $\sigma^2 = np(1 - p)$ **not only provides a very accurate approximation to the binomial distribution when n is large and p is not extremely close to 0 or 1** but **also provides a fairly good approximation even when n is small, and p is reasonably close to 1/2**.

--End of Binomial--

Poisson and its Normal Approximation

1.Theory Claim

For a Poisson's distribution,

- **Mean (μ) = λ t**, where time t=1

  = λ

- **Variance(σ^2) = λ**
- **Standard Deviation(σ) = √λ**

In Normal approximation to Poisson note that the mean and variance are equal.

μ = λ

Condition for Normal approximation to Poisson (For best results):

- **λ > 20 or μ > 20**

Let us calculate the exact probability w.r.t Poisson's first and then Normal approximation and compare the results,

Consider λ = 6.5,

P(Y>=9) = 1 – P(Y<=8)

= 1 – 0.792

= **0.208 or 20.8% ------------------Poisson's**

Now to approximate the same using normal distribution,

P(Y>=9) = P(Y>8.5)

We are using a continuous distribution to approximate a discrete distribution; therefore, we are required to make some corrections for this for better estimation just like in the binomial approximation previously. Therefore, we have applied 'continuity correction' which reduces the calculation to a normal probability calculation further on.

P(Y > 8.5) = P(Z > (8.5-6.5)/√6.5), where Z=(X-μ)/σ = (X-λ)/√λ

= P(Z > 0.78)

= **0.218 or 21.8% ------------------Normal Approximation**

In summary, the probability estimation w.r.t Poisson's where P(Y>=9) is exactly 0.208 and w.r.t Normal approximation after continuity correction at P(Y>8.5) is approximately 0.218.

This is a good approximation considering the λ value used above is only 6.5.

This also shows how much better the quality of approximation can get when the lambda value is close to 20 or more than that.

2.Qualitative Claim

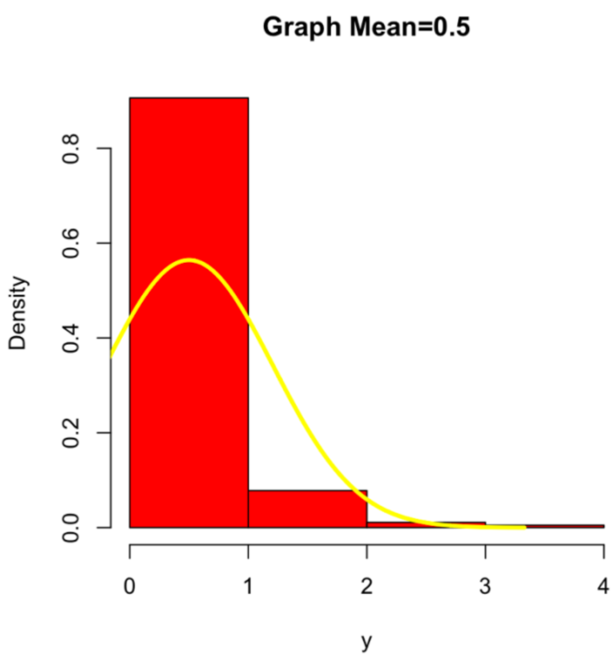
Let us consider a population sample size of 1000 random poisson values generated randomly with the function **rpois(1000, μ)** in R Studio; where the Mean values are as depicted below:

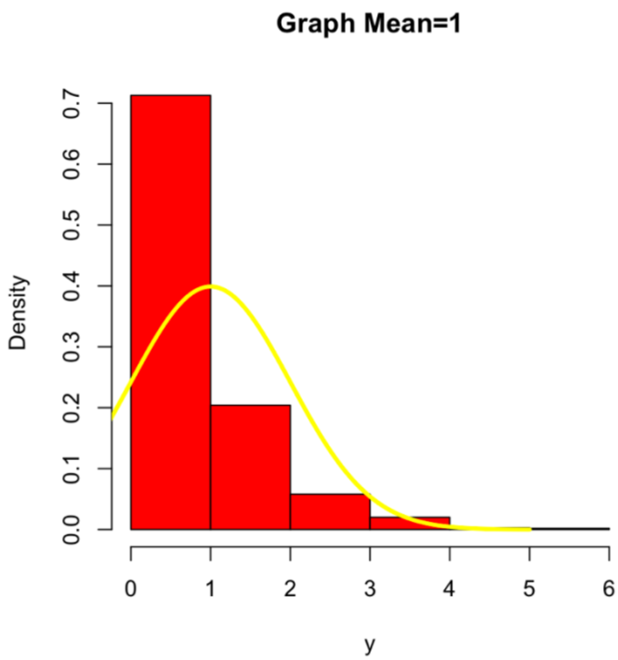
We keep **'change the μ value'**, [μ=0.5,1,3,5,20,50,80] to observe and study the characteristic behavior of the graph.

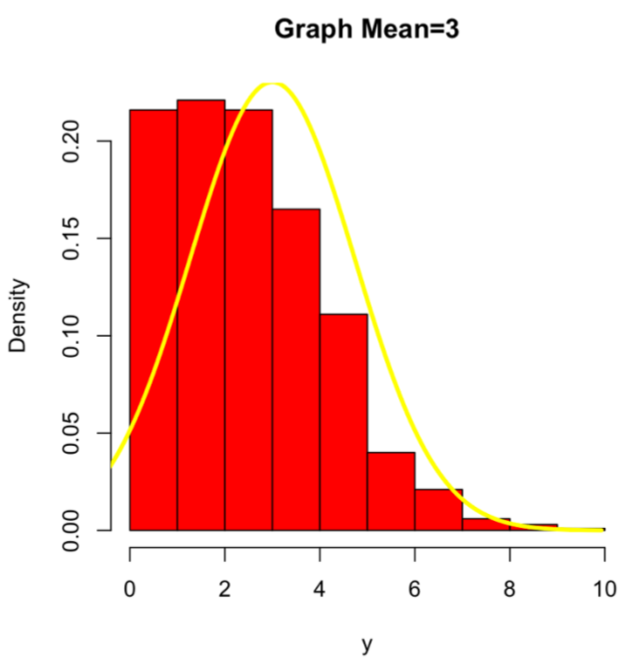
- **Case 1:** (Mean or $\lambda = 0.5, 1, 3, 5$)

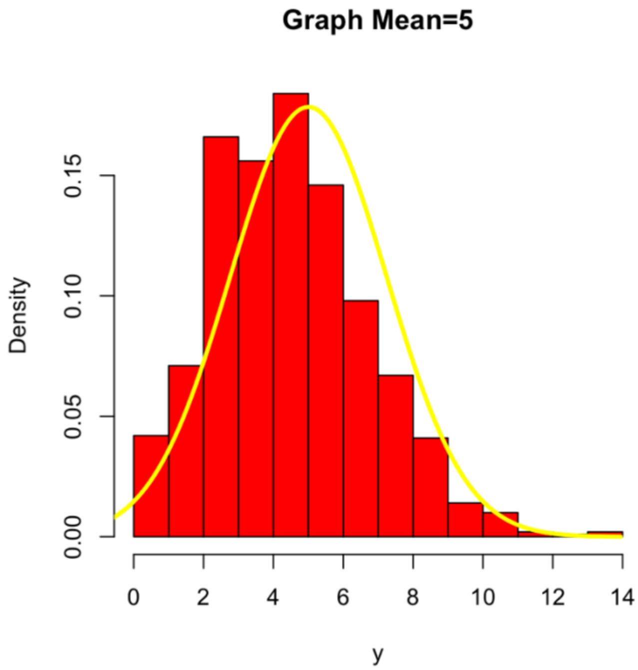
  Condition – $\lambda > 20$

  $\lambda = 0.5, 1, 3, 5$ [Fails the condition]




Graph Mean=1
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
1
2
3
4
5
6
y

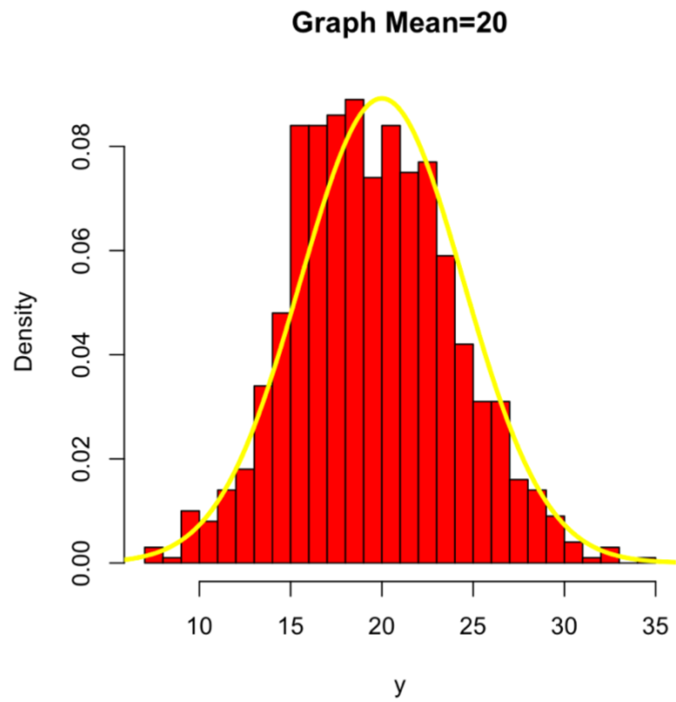

Graph Mean=3
Density
0.00
0.05
0.10
0.15
0.20
0
2
4
6
8
10
y



- **<u>Case 2:</u>** (Mean or λ = 20)

  Condition – λ > 20

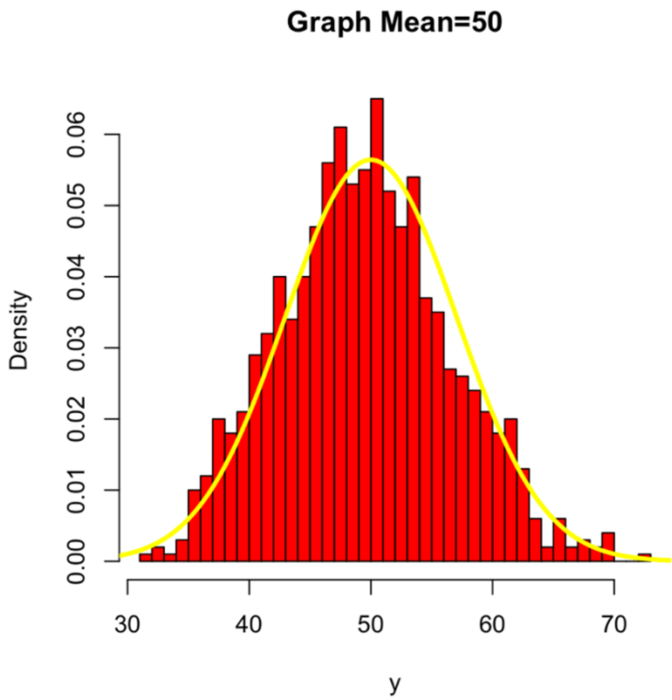
  λ = 20 [Narrowly passes the condition]

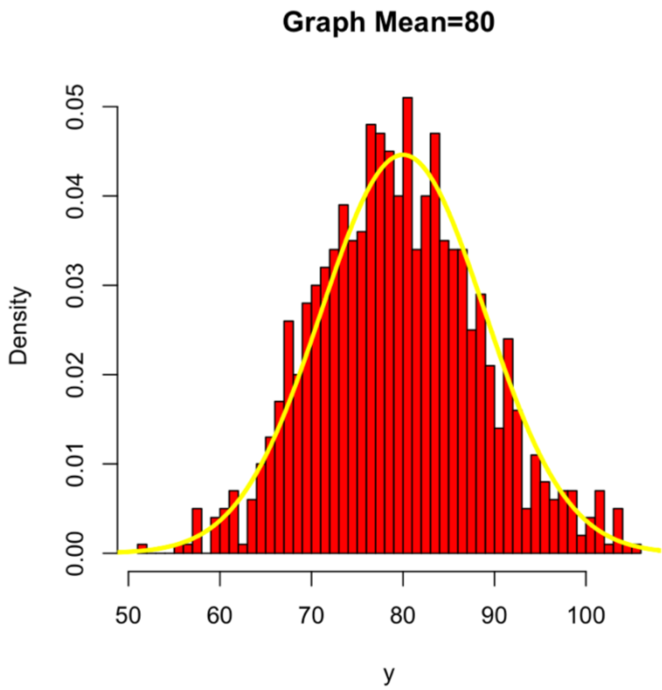


- **Case 3:** (Mean or λ = 50, 80)

  Condition – λ > 20

  λ = 50, 80 [Comfortably passes the condition]





<u>Inference:</u>

From the above graphs, we can conclude **qualitatively that the normal fit and quality of normal approximation gets better along with the mean or λ.**

- It is evident that the graphs fit inside the **Normal distribution bell curve better and better as the mean or lambda increases at a steady rate.**
- Case1: t**he graphs do not fit the normal curve, but the trend and quality of the fit increases with the increase in λ though it fails the condition.**
- Case 2: **the graph almost completely fits the normal curve as it approaches the threshold limit of λ >20 for a dependable quality of approximation**
- Case 3: **the graphs fit the normal curve comfortably as the value of λ increases more than 20, we can see the improvement in quality as it reaches λ=50 and finally 80 in the end.**

The above observation from the generated graph **not only shows us how the quality of the Normal approximation to Poisson's improves with N but also verifies the condition of λ > 20 by clearly stating the difference in output and quality** via the cases mentioned above.

3.Quantitative Claim

We have constructed a table with 25 rows to identify and compare the results of Poisson’s and its respective Normal approximation.

Five different lambda values are used to account for values from lower middle and higher spectrum of numbers.

POISSON'S										
	λ=0.4		λ=5		λ=20		λ=60		λ=90	
r	Poisson	Normal	Poisson	Normal	Poisson	Normal	Poisson	Normal	Poisson	Normal
0	0.67032005	0.56282194	0.00673795	0.012671433	2.06115E-09	3.87146E-06	8.75651E-27	4.74038E-15	8.19401E-40	1.19042E-21
1	0.93844806	0.95901826	0.04042768	0.036814755	4.32842E-08	1.07573E-05	5.34147E-25	1.29887E-14	7.45655E-38	3.25338E-21
2	0.99207367	0.99955101	0.12465202	0.089849632	4.55515E-07	2.84931E-05	1.62959E-23	3.50102E-14	3.39314E-36	8.79417E-21
3	0.99922375	0.99999953	0.26502592	0.185539413	3.20372E-06	7.19561E-05	3.3153E-22	9.28329E-14	1.0295E-34	2.35116E-20
4	0.99993876	1	0.44049329	0.327355515	1.69447E-05	0.000173291	5.06005E-21	2.42154E-13	2.34299E-33	6.21723E-20
5	0.99999596	1	0.61596065	0.5	7.19088E-05	0.000398076	6.18022E-20	6.21394E-13	4.26637E-32	1.62608E-19
6	0.99999977	1	0.76218346	0.672644485	0.000255122	0.000872485	6.29224E-19	1.56868E-12	6.47474E-31	4.20645E-19
7	0.99999999	1	0.86662833	0.814460587	0.00077859	0.001825081	5.49284E-18	3.89578E-12	8.42361E-30	1.07627E-18
8	1	1	0.93190637	0.910150368	0.002087259	0.003644944	4.197E-17	9.5182E-12	9.59051E-29	2.72371E-18
9	1	1	0.96817194	0.963185245	0.004995412	0.006952765	2.85151E-16	2.28781E-11	9.7072E-28	6.81766E-18
10	1	1	0.98630473	0.987328567	0.010811719	0.012673071	1.74424E-15	5.40995E-11	8.84405E-27	1.68789E-17
11	1	1	0.99454691	0.99635571	0.021386822	0.02208482	9.70288E-15	1.25858E-10	7.32622E-26	4.13325E-17
12	1	1	0.99798115	0.999127723	0.039011993	0.036817977	4.94961E-14	2.88064E-10	5.56399E-25	1.0011E-16
13	1	1	0.99930201	0.999826762	0.066127641	0.058760959	2.33157E-13	6.48669E-10	3.90119E-24	2.3983E-16
14	1	1	0.99977375	0.999971518	0.104864281	0.089854498	1.02028E-12	1.43711E-09	2.54034E-23	5.6829E-16
15	1	1	0.99993099	0.99999613	0.156513135	0.131774319	4.16875E-12	3.13252E-09	1.54417E-22	1.33193E-15
16	1	1	0.99998013	0.999999566	0.221074202	0.185544762	1.59755E-11	6.71804E-09	8.80117E-22	3.08771E-15
17	1	1	0.99999458	0.99999996	0.297028398	0.251165759	5.76465E-11	1.41757E-08	4.72206E-21	7.08011E-15
18	1	1	0.9999986	0.999999997	0.381421949	0.327359125	1.9655E-10	2.94311E-08	2.39318E-20	1.6058E-14
19	1	1	0.99999965	1	0.470257267	0.411530937	6.35192E-10	6.01226E-08	1.14925E-19	3.60242E-14
20	1	1	0.99999992	1	0.559092584	0.5	1.95112E-09	1.20849E-07	5.24395E-19	7.99372E-14
21	1	1	0.99999998	1	0.643697648	0.588469063	5.71091E-09	2.39022E-07	2.27927E-18	1.75452E-13
22	1	1	1	1	0.720611343	0.672640875	1.59649E-08	4.65186E-07	9.45829E-18	3.80912E-13
23	1	1	1	1	0.787492817	0.748834241	4.27144E-08	8.90885E-07	3.75501E-17	8.17992E-13
24	1	1	1	1	0.843227378	0.814455238	1.09588E-07	1.67893E-06	1.42894E-16	1.73754E-12
25	1	1	1	1	0.887815027	0.868225681	2.70085E-07	3.11368E-06	5.22134E-16	3.65078E-12
	σ=0.6324		σ=2.2360		σ=4.4721		σ=7.7459		σ=9.4868	

<u>Inference:</u>

From the following table it is evident that **the quality of Normal approximation to Poisson’s improves as the lambda or mean increases. We can see this increase in quality from 0.4 to 5 very clearly and**

it follows the same trend throughout the spectrum, giving promising results especially when lambda is more than 20.

With respect to the above table and the proven tends, **we can identify the threshold for the computed λ values to get a good quality result. We do this by comparing the difference between Normal and Poisson's result for the r values above**.

The lower the difference, the better the confidence. [Let us consider a very good quality approximation where the difference is 0 for the following claims]

- **For λ=0.4, we can rely on Normal approximation when r>8 as the difference is minimum**
- **For λ=5, we can rely on Normal approximation when r>22 and so on for λ=20, 60 and 90**

Note: Though the difference at r=8 and r=22 is zero, here we do not consider it as marginal values might contain any unseen error at the transition point and the difference might be very less (unseen) where λ is very large. This interval that we choose depends on the accuracy we need for our application [like the difference of 0 considered, provided we need the best quality in this case for ease of explanation of the selection criteria].

---------------------------------------End of Poisson---------------------------------------

End of Project