Homogenous ensemble of undersampled majority class for highly imbalanced data binary classification

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Abstract

This is the abstract for this article.

Keywords: classification, classifier ensemble, undersampling, imbalanced data

1. Introduction

The main contributions of this work are:

- metoda konstrukcji komitetu na k-fold,
- propozycje reguł decyzyjnych,
- metoda pruningu dostosowującego regułę decyzyjną do zbioru testowego
- implementacja i ewaluacja eksperymentalna

2. Homogenous ensemble based on undersampling the majority class

2.1. Establishing ensemble

Complex oversampling methods, such as SMOTE or ADASYN, despite the large possibilities in most of the imbalanced problems, are not applicable to extreme situations where the minority class is represented by only a few samples, which makes it impossible to designate the nearest neighbors to create a new synthetic object. This could lead to the use of *undersampling* in such problems, but it is characterized, due to high randomness, by a strong instability in a situation of high IR (*imbalance ratio*), which does not allow for the development of a reliable solution.

A popular answer to the above-mentioned problem are the ensemble methods of *Bagging* or *Boosting*, characterized by random sampling with replacement of the training set, breaking a large problem, into a set of smaller problems. This work proposes a basic method, which

also breaks the imbalanced task, but with ensuring the use of all the patterns available in the data set, but without a risk of overlapping. Its description can be found in Algorithm 1.

Algorithm 1: Training classifier ensemble from multiple balanced training datasets separated from one imbalanced dataset of binary problem Given a dataset DS:

- 1. Divide DS into subsets of minority- MinC and majority-class MajC
- 2. Calculate imbalanced ratio IR as the proportion of the number of patterns in MinC and MajC
- 3. Establish k by rounding IR to nearest integer
- 4. Perform a shuffled k-fold division of MajC to produce a set of subsets $MajC_1, MajC_2, \dots, MajC_k$
- 5. For every i in range to k
 - 6. Join $MajC_i$ with MinC to prepare a training set TS_i ,
 - 7. Train classifier Ψ_i on TS_i and add it into ensemble

After dividing the dataset with imbalanced binary problem into separated minority (MinC) and majority class (MajC), we are calculating the IR $(imbalanced\ ratio)$ between given classes. Rounding IR to the nearest integer value k allows us to find the optimal division coefficient of the majority class samples in the context of maximizing the balance between the MinC and any $MajC_i$ subsets while ensuring that all MajC patterns are used in learning process with no overlapping between the individual $MajC_i$'s. Each of k classifiers Ψ_i is trained on union of MinC and $MajC_i$ sets.

Extending pool with oversampling As an extension of the method of classifier ensemble construction, it is also proposed to extend its pool by a model learned on an additional data set, which is a full set of data subjected to *oversampling*. It is worth testing if the knowledge gained from this method may be a valuable contribution to the ensemble decision. Due to impossibility to use SMOTE or ADASYN for oversampling the minority class with only few instances, only its basic variant will be used.

2.2. Fuser design

In addition to ensuring the diversity of the classifiers pool, which we achieve by a homogenous committee built on disjoint subsets of the majority class supplemented by minority patterns, the key aspect of the hybrid classification system is the appropriate design of its fuser – the element responsible for making decisions based on the answers of the base classifiers.

There are two groups of solutions here. The first are based on component *decisions* of the committee, most often employing the *majority voting* to produce a final decision. The decision rules proposed in this work are, however, part of the second group, where the *fuser* is carried out by *averaging* (or *accumulating*) the *support vectors* received from the members of a pool.

Note:

It should be remembered that in such methods, it is necessary to use a *probabilistic classi-fication model*, which also requires *quantitative* and not *qualitative data*.

Five fusers were proposed:

- 1. **REG** regular accumulation of support, without weighing the members of the committee.
- 2. **WEI** accumulation weighted after members of the committee.

The weight of the classifier in the pool is its quality achieved for the training set. We can not use here the measure of *accuracy*, which does not fit with the task of the imbalanced classification, so we decided on a *balanced accuracy* (Brodersen et al., 2010).

- 3. **NOR** akumulacja znormalizowanych wag członków,
- 4. con akumulacja ważona po wzorcach, przez kontrast,
- 5. nci iloczyn znormalizowanych wag i kontrastu

Wyliczanie wag. Accuracy się nie sprawdzi, więc BAC.

Jeśli klasyfikujemy nie jeden wzorzec, a wiele, wagi mogą być też dla pojedynczych próbek, dla podbicia, a więc pojawia się KONTRAST. Mamy takie ładne ilustracje z badań, dodajmy rysunek chociaż jeden poglądowo.

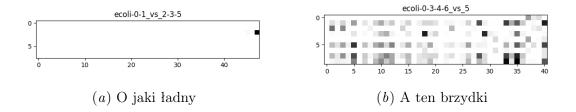


Figure 1: Rysunek.

Potencjał kontrastu dla danych strumieniowych.

Proponowane metody decyzyjne.

W konstrukcji reguły decyzyjnej opieramy się na wsparciu dla klasy pozytywnej.

Duża skala niezbalansowania to duża wielkość komitetu (ilustracja zależności na wykresie). Przyda się więc przycinanie (pruning).

2.3. Ensemble pruning

Wyjaśnienie podejścia do pruningu. Wyliczamy wzajemną zależność statystyczną (Wilcoxonem) pomiędzy wsparciami członków i grupujemy – omijając kwestię 1z2 2z3 ale nie 1z3 – je uśredniając wsparcia w obrębie grupy. Uśredniamy też wagi i tworzymy tak dwupoziomowy system fuzji (potrzebna ilustracja).

Pruning też jest w kontekście klasyfikacji wielu wzorców na raz. Wyjaśnienie kwestii wspomnianej wcześniej i uzasadnienie pominięcia jej analizy.

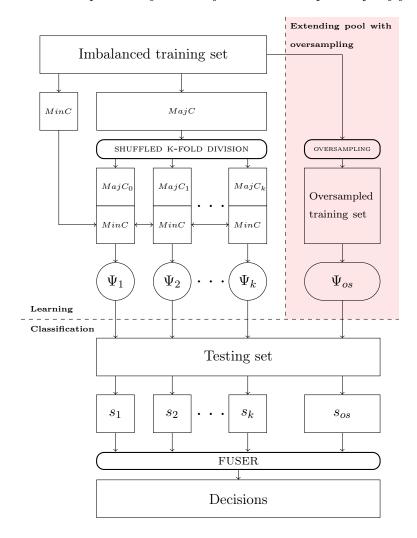


Figure 2: Scheme of using k-Fold division in ensemble construction

3. Experiment design

For the experimental evaluation of the proposed method, a collection of datasets made available with KEEL (Alcalá-Fdez et al., 2011) was used, focusing on a section containing highly unbalanced data, with IR greater than 9 (Fernández et al., 2009). From among the available datasets, 40 were selected presenting only binary problems with quantitative attributes. A review of selected datasets, including information on their number of features, the number of patterns in each class and the unbalance ratio is presented in Table 1.

As may be observed in the summary, the experiments are based on datasets with relatively small spatiality (up to 13 dimensions), with imbalance ratio from 9 to even 40. The

·	-		Sa	ample	s	
#	Dataset	Features	ALL	MAJ	MIN	IR
1	ecoli-0-1-3-7-vs-2-6	7	281	274	7	39.14
2	ecoli4	7	336	316	20	15.80
3	glass-0-1-6-vs-2	9	192	175	17	10.29
4	glass-0-1-6-vs-5	9	184	175	9	19.44
5	glass2	9	214	197	17	11.59
6	glass4	9	214	201	13	15.46
7	glass5	9	214	205	9	22.78
8	page-blocks-1-3-vs-4	10	472	444	28	15.86
9	shuttle-c0-vs-c4	9	1829	1706	123	13.87
10	shuttle-c2-vs-c4	9	129	123	6	20.50
11	vowel0	13	988	898	90	9.98
12	yeast-0-5-6-7-9-vs-4	8	528	477	51	9.35
13	yeast-1-2-8-9-vs-7	8	947	917	30	30.57
14	yeast-1-4-5-8-vs-7	8	693	663	30	22.10
15	yeast-1-vs-7	7	459	429	30	14.30
16	yeast-2-vs-4	8	514	463	51	9.08
17	yeast-2-vs-8	8	482	462	20	23.10
18	yeast4	8	1484	1433	51	28.10
19	yeast5	8	1484	1440	44	32.73
20	yeast6	8	1484	1449	35	41.40
21	ecoli-0-1-4-6-vs-5	6	280	260	20	13.00
22	ecoli-0-1-4-7-vs-2-3-5-6	7	336	307	29	10.59
23	ecoli-0-1-4-7-vs-5-6	6	332	307	25	12.28
24	ecoli-0-1-vs-2-3-5	7	244	220	24	9.17
25	ecoli- 0 - 1 - vs - 5	6	240	220	20	11.00
26	ecoli-0-2-3-4- vs -5	7	202	182	20	9.10
27	ecoli-0-2-6-7- vs -3-5	7	224	202	22	9.18
28	ecoli-0-3-4-6- vs -5	7	205	185	20	9.25
29	ecoli-0-3-4-7-vs-5-6	7	257	232	25	9.28
30	ecoli-0-3-4- vs -5	7	200	180	20	9.00
31	ecoli-0-4-6- vs -5	6	203	183	20	9.15
32	ecoli-0-6-7-vs-3-5	7	222	200	22	9.09
33	ecoli-0-6-7- vs -5	6	220	200	20	10.00
34	glass-0-1-4-6-vs-2	9	205	188	17	11.06
35	glass-0-1-5-vs-2	9	172	155	17	9.12
36	glass-0-4-vs-5	9	92	83	9	9.22
37	glass-0-6-vs-5	9	108	99	9	11.00
38	yeast-0-2-5-6-vs-3-7-8-9	8	1004	905	99	9.14
39	yeast - 0 - 2 - 5 - 7 - 9 - vs - 3 - 6 - 8	8	1004	905	99	9.14
40	yeast-0-3-5-9-vs-7-8	8	506	456	50	9.12

Table 1: Summary of imbalanced datasets chosen for evaluation

datasets provided by KEEL, to ensure easy comparison between results presented in various research, are already pre-divided into five parts, which forces the use of k-fold cross-validation with k = 5 in experiments (Alpaydin, 2009).

In the task of imbalanced data classification, due to its strong bias towards majority class, the accuracy measure is not a proper tool. For a reliable result, a measure of balanced accuracy is given as test results.

Both the implementation of the proposed method and the experimental environment have been constructed using the scikit-learn library (Pedregosa et al., 2011) in version $0.20.dev0^1$. Among the available classification models, the MLP ($Multilayer\ Perceptron$) and SVC ($Support\ Vector\ Machine$) were rejected. First one was not able to build a correct model due to the lack of convergence on the small datasets (minority class of data chosen for experiments is often represented by only two patterns in cross-validated folds) and second, whose probabilistic interpretation is measurable only with sufficiently large data sets, did not allow credible construction of a fuser. As base classifiers, the following algorithms were used:

- Gaussian Naive Bayes (GNB) (Chan et al., 1982),
- k-Nearest Neighbors (knn) with 5 neighbors and Minkowski metric,
- Decision Tree Classifier (DTC) with Gini criterion (Loh, 2011).

To provide a comparative result for the method presented in the following paper, each base classifier was also tested for the raw, imbalanced dataset and its under- and oversampled versions. Undersampling, due to high instability of results, was repeated five times on each fold. Used statistical analysis tool was a paired dependency between the classifier, which achieved the highest result and each of the others, calculated using the signed-rank *Wilcoxon* test (Wilcoxon, 1945).

Pełną implementację zaproponowanej metody i skrypt umożliwiający powtórzenie zaprezentowanych badań można odnaleźć w repozytorium abc.

The full implementation of the proposed method and the script allowing the repetition of the presented research may be found in the git repository available at url-removed-due-to-blind-review.

4. Experimental evaluation

Przedstawienie tabel.

Tabela zbiorcza zwycięstw w zależności od parametrów (z grupowaniem). Interpretacja wyników, czyli co zostało należycie uprawdopodobnione.

5. Conclusions

Co zostało zaproponowane.

Na co pozwala taka metoda.

Do jakich rezultatów doprowadziła.

Jakie sa plany na przyszłość (czyli co robisz w wakacje).

Acknowledgments

Acknowledgements go here.

^{1.} At the time of conducting research, only the development version of the package already has the implementation of balanced accuracy measure.

Γ	Data	set																						9-9																8-8	8-9-	
			ecoli-0-1-3-7-vs-2-6	ecoli4	glass-0-1-6-vs-2	glass-0-1-6-vs-5	glass2	glass4	glass5	page-blocks-1-3-vs-4	shuttle-c0-vs-c4	shuttle-c2-vs-c4	vowel0	yeast-0-5-6-7-9-vs-4	yeast-1-2-8-9-vs-7	yeast-1-4-5-8-vs-7	yeast-1-vs-7	yeast-2-vs-4	yeast-2-vs-8	yeast4	yeast5	yeast6	ecoli-0-1-4-6-vs-5	ecoli-0-1-4-7-vs-2-3-5-6	ecoli-0-1-4-7-vs-5-6	ecoli-0-1-vs-2-3-5	ecoli-0-1-vs-5	ecoli-0-2-3-4-vs-5	ecoli-0-2-6-7-vs-3-5	ecoli-0-3-4-6-vs-5	ecoli-0-3-4-7-vs-5-6	ecoli-0-3-4-vs-5	ecoli-0-4-6-vs-5	ecoli-0-6-7-vs-3-5	ecoli-0-6-7-vs-5	glass-0-1-4-6-vs-2	glass-0-1-5-vs-2	glass-0-4-vs-5	glass-0-6-vs-5	yeast-0-2-5-6-vs-3-7-8-9	yeast-0-2-5-7-9-vs-3-6-8	yeast-0-3-5-9-vs-7-8
	Fu	11	.825	.878	.580	.941	.591	.587	.938	.763	.991	966.	.917	.504	.544	.547	.604	.561	.657	.551	.831	.650	877	.630	.735	.638	.782	.754	.563	.784	.775	.817	.854	.508	.780	.577	.519	.994	.945	0.29	.577	.557
	US	S	.838	787.	.589	975	.620	.745	.945	908.	.993	.946	.905	.601	.598	.566	989.	.739	.762	099.	910	.795	629.	.634	899.	.578	.658	.657	.595	.716	.665	.657	.725	.571	.682	.590	.555	.984	686.	.605	.785	.633
	os	S	908.	.859	.569	.941	.617	.731	.938	.791	066.	886.	906.	.498	.540	.541	.586	.529	.616	.526	.780	.628	.885	299.	.863	.639	797	.638	.592	.725	.734	.730	.890	.544	.847	.597	.508	.984	.945	.782	.524	.539
	ro	NC	.837	.922	.585	686.	.641	.771	.938	.828	166.	966:	606:	.791	639	.568	.719	.861	.773	.811	955	878.	.913	629	.831	.658	.816	895	.628	895	.791	.883	.901	269.	.830	.620	.584	.994	995	.783	900	.605
	mbers	NOR	.837	.923	585	686	.641	.771	.938	.830	.991	966	606	.785	.625	.564	.725	698.	.773	.820	.957	788.	910	.657	.826	.658	.816	.895	.623	.901	.789	.883	.903	.618	.853	.620	.584	.994	.995	.781	268.	.605
	d me	CON	.845	895	277	686	644	774	938	.831	.994	966	606	.710	.556	.562	674	827	.773	692	935	.845	862	.630		.618		.758	.613		.673	.783	928	.508	.755	.620	.558	.994	. 395	. 276		009.
d set	Reduced members	WEI	.845	968	. 277	686	.641	774	938	.831	.994	966	606	724	564	.555	.695	.833	773	. 292	930	. 098	837	.630		.618	.691	. 758	.613	. 870	.673	.783	878	208	.755	622	.551	.994	. 366	. 276		.612
mple	\mathbf{R} e	REG	.845	.746	. 577	. 686	641	.812	. 938	831	994	966	910	. 289	556	588	700	805	773	710	927	.818	. 785	. 089	.617	558		.731	.588	-	653	. 758	. 878	548	. 889	622	533	. 994	. 395	. 576		.611
oversampled set		NC		904	591	941	. 919	. 677	938	845	991	. 888	903	715	566	590	700	800	773	746	934	795	906	. 663	.855	. 638	.830	803	628	851	747	758	901	575	825	595	533	994	995	. 892	_	633
With o	ers	NOR	'	913	560	941	. 919	. 622	. 938	. 846	. 166.	. 886	904	. 726	574	.555	705	. 662	. 773	. 781	. 934	.843	906	. 662	.852	. 638	.834	. 828	625	. 857	785	. 772	901	. 575	828	595	530	. 994	. 395	. 777		. 619
	members	CON	. 688	868	552	941 .	619	.774	. 938	803	995	. 626	910	. 229	553	554	674	781	. 967.	. 637	919	. 892.	. 098	. 630	. 755	. 889	. 789	. 908.	.588	-	. 726	.811	901	. 557	838	558	.527	. 994	. 395	. 692	1	.589
	All m	WEI	.841	868	552	941	616	781	. 886	. 208	. 366.	. 626	910	. 700	552	.558	. 189	. 794	. 1771	. 655	921	. 2773	.812	. 089	. 229	. 819		. 758	.588	-	735	. 794	901	.557	838	592	.527	. 994	. 366	. 657	1	. 209.
		REG		. 874	552	941	. 619	. 781	. 938	. 208	. 395	984	910	. 702	563	.551	. 671	. 773	. 796	. 644	. 716.	. 092.	.810	. 630		. 618	.714	. 927.	.588		. 269	. 697.	901	.557	.845	.592	.527	. 994	. 395	. 634		. 601
		NC		. 898	.580	. 989	.641	. 166	. 938	828	.991	. 966.	. 606	.774	642	. 575	. 722	862	.773	.813	_	. 878	.913	.654	. 829	. 658	.741	.853	. 630	_	. 667.	_	.878	. 597	.830	. 620	.582	994	995	. 782		. 605
	members		8. 788.	8. 926	583 .5	3. 686	641 .6	7. 992.	938 .9	828 .8	991 .9	3. 966.	606	7. 987.	627 .6	550 .5	.726 .7	8. 078.	7. 877	-	3. 736.	8. 788.	3. 906	.652 .6		. 658	.741 .7	853 .8	625 .6	8. 106.	. 781	883 .8	8. 808	.640 .5	853 .8	620 .6	.604	994 . 9	3. 366	782 .7		.605 .6
			45		5777	3. 686.	.641 .6	7. 408.	3. 886.	828 .8	. 994	3. 966.	3. 606.	7. 727.	.562 .6	554 .5	7. 689.	8. 728.	7. 877.	-			3. 018.			. 578		8. 887.	.588			8. 887.	∞.		8. 207.	·	.542	.994	3. 366.	. 92		9. 909.
d set	$\mathbf{Reduced}$	WEI	845 .8	.8.008.	5.777	3. 686.	.641 .6	8. 708.	3. 886.	828 .8	994	3. 966.	3. 606	7. 787.	557 .5	5. 566	. 710	822 .8	7. 877.				8. 018.	9. 089.		5.873		. 733	588 .5		. 673		.876 .87	548 .5	.755 .7	.622 .6	.536 .5	994	3. 366.	.576 .5′		.605
mple	Rec	REG		.748 .8	5777	3. 686.	.641 .6	8. 708.	3. 886.	828 .8	. 994	3. 966.	3. 606.	7. 089.	.562 .5	567	7. 807.	8. 028.	7. 877.			8. 829.	8. 099.	9. 089.		. 558		. 658	.588		. 633	. 7111	853 .8	.548 .5	. 685	.622 .6	.542	.994	3. 266.	. 576		9. 909.
Without oversampled		NC	_	.857	.610	.941	9. 919.	802 .8	3. 886	8. 845	. 994	3. 626.	303	.731 .6	570	.563	7.03	8. 008.	7. 477		_	_	.804	9. 089.	_	. 578		. 753 .6				. 792 .7	8. 803		812 .6	.581	.534	. 994		. 576		.621
ont c	'n		•	8. 678.	580 .6	939	9. 919.	802 .8	938 .9	846 .8	995 .9	9. 626.	902 .9	.728 .7	576 .5	557 .5	7. 207.	8. 667.	7. 877.				804 .8	9. 089.		575 .5		7. 877.	635 .6	-	. 765 . 7	.833	901 .9		805 .8	615 .5	.527 .5	994 .9		576 .5		.620 .6
With	members	CON		8. 658	610 .5	6. 686.	9. 619.	8. 008.	938 .9	817 .8	994 .9	9. 276.	6. 606.	7. 689.	567 .5	550 .5	7. 669.	7. 803.7	7. 877.			8. 787.	8. 887.	9. 089.		5.873		7. 807.			.633 .7	. 756 .8	6. 978.		8. 867.	-	582 .5	994 .9	995 .9	5. 975.		9. 909.
	All me	WEI		834 .8	580 .6	6. 686	9. 619	8. 797	938 .9	817 .8	994 .9	975 .9	6. 606	Ċ	570 .5	553 .5	9. 869.	801 .8	7. 877.				7. 187.	9. 089.		578 .5					633 .6		8. 978.	598 .5	7. 787.	589 .5	582 .5	. 994	995 .9	576 .5	1	.632 .6
	∢	REG		8. 608.	580 .5	6. 686	9. 619	7. 008	·	817 .8		97.1	6. 606.	-	563 .5	552 .5		804 .8	7. 877.	-	ľ	-		9. 089.	·	578 .5			.563 .5	-	633 .6		8. 978.		7. 367.	558 .5	567 .5	.994	. 995	576 .5	1	.597 .6
		REG	∞	∞.	īÿ.	õ	9	∞.	<u>6</u>	<u>∞</u>	<u>ő</u> ;	.9.	<u> </u>	99.	ī.	ις	<u>.</u>	<u>∞</u> .	7	.6	9	~	~	.6.	9.	χċ		9	ī.	χö	9.	7.	<u>,</u>	χċ	7.	ΐ	ī.	ğ.	Ğ.	χċ	∞ <u>.</u>	ij

Table 2: Balanced accuracy scores obtained using GNB as a base classifier

Г	ata	set																						9-9																8-8	8-9-	
			ecoli-0-1-3-7-vs-2-6	ecoli4	glass-0-1-6-vs-2	glass-0-1-6-vs-5	glass2	glass4	glass5	page-blocks-1-3-vs-4	shuttle-c0-vs-c4	shuttle-c2-vs-c4	vowel0	yeast-0-5-6-7-9-vs-4	yeast-1-2-8-9-vs-7	yeast-1-4-5-8-vs-7	yeast-1-vs-7	yeast-2-vs-4	yeast-2-vs-8	yeast4	yeast5	yeast6	ecoli-0-1-4-6-vs-5	ecoli-0-1-4-7-vs-2-3-5-6	ecoli-0-1-4-7-vs-5-6	ecoli-0-1-vs-2-3-5	ecoli-0-1-vs-5	ecoli-0-2-3-4-vs-5	ecoli-0-2-6-7-vs-3-5	ecoli-0-3-4-6-vs-5	ecoli-0-3-4-7-vs-5-6	ecoli-0-3-4-vs-5	ecoli-0-4-6-vs-5	ecoli-0-6-7-vs-3-5	ecoli-0-6-7-vs-5	glass-0-1-4-6-vs-2	glass-0-1-5-vs-2	glass-0-4-vs-5	glass-0-6-vs-5	yeast-0-2-5-6-vs-3-7-8-9	yeast-0-2-5-7-9-vs-3-6-8	yeast-0-3-5-9-vs-7-8
	Ful	1	.850	.848	.555	.739	.485	.781	.695	808	966.	009.	977	299.	.499	.499	.517	.819	.774	.574	.850	.739	868.	.847	.838	.830	900	.894	787.	.875	.876	.875	.900	.835	.847	.512	.527	.850	.745	.762	.902	.639
	US	3	.835	.928	999.	.852	.678	.865	.811	.872	966.	.845	.939	.792	.652	.590	.682	806.	.734	.835	.952	878	988.	.882	.883	.895	.902	.904	.814	.881	887	888	888.	.844	.850	.681	.651	.917	.816	.760	.902	.702
	os	3	.835	606.	.656	.933	.715	.925	.830	.917	966.	000.	666.	.795	.627	.615	.705	.885	.803	.749	.964	.840	.917	.856	836	.887	.916	606.	.890	.911	.894	.911	.914	.893	.863	.732	.656	.988	.985	.784	.904	.718
	70	NC	856	.943	.735	.841	.746	868.	.833	.920	966:	966:	986	829	693	.643	.710	606:	.758	.852	096	895	892	.902	888	.863	.911	006:	.855	895	888.	.903	.895	.858	788.	.727	.758	.951	.933	.798	895	.734
	mber	NOR	.853	.940	.758	.833	.723	888	.821	.917	966	.927	086	.828	.692	.636	904	.921	.755	.841	096	.893	.892	.904	.887	.883	.911	006	.860	.895	.885	.903	.892	.863	.890	.734	.745	.951	.913	.795	968.	.723
	Reduced members	CON	.851	.945	.737	.836	.731	.861	.826	.881	966	906	.954	.819	.684	.623	.734	.921	.752	.843	.958	888	892	688.	.871	.872	.911	006	848	.890	268.	906	.890	.870	.872	.747	.712	.951	878	800	.912	.750
d set	duce	WEI	.853	.945	705	833	732	.861	813	875	966	887	948	800	099.	209	.755	920	738	844	958	887	892	988	.893	874	606	006	843	893	911	906	890	870	865	.749	695	.951	878	862	.913	.767
mple	$\mathbf{R}_{\mathbf{e}}$	REG	854	945	705	833	726	861	816	.874	966	887	948	. 662		594	.745	921	738	844	958	. 887	.892	988	.893	874	606	. 006	843	.893	913	906	890	865	865	741	929	.951	878	. 793		.770
versa		NC	834	606	728	. 628	756	913	873	924	966	000	966	820		609	200	906	803	774	296	865	906	988	894	885	914	006	833	901	890	903	895	863	885	716	740	951	953	. 867	- 1	730
With oversampled set	ers	NOR	835	. 606	750	. 628	. 756	905	. 801	. 606	. 966	. 000	. 266	. 830	•	612	700	906	. 803	. 771	962	.840	. 904	. 698.	. 892	. 885	911	. 006.	. 088	. 106	•	•	. 868	. 865	863	718	. 797	951	953	791	Ŀ.	. 731
🔰	members	CON	832	. 972	. 092	853	. 746	. 888	. 628	. 926	. 966	. 000	963	.832	-	614 .	.712	913	. 877.	. 839	. 096	. 892		. 886	. 887	. 298	911	. 006.	. 098	. 895	•		. 068	855	. 088	701	.748	.951	. 826	•		. 739
	All m	WEI	. 830	. 296	751	. 839	.757	898	.843	. 206.	. 966	. 000	.953	.824		.628	.743	. 921	. 782	. 829	. 956	. 006.	. 894	. 698.	. 928.	.865	. 911	. 006.	.858	. 893			. 890	. 875	. 878	717	. 728	. 951	. 898	.802	-	.742
		REG	. 830	. 964	717	. 836	. 737	. 898	.838	. 106.	. 966.	. 966.	. 952	.810	.702	.622	. 737	. 921	. 782	.838	. 956	. 899	. 894	. 698.	.874	. 872	. 911	. 006.	.853	. 893	-		. 890	. 875	. 878	. 731	. 705	.951	.883	-		. 759 .
		NC	845	. 945	. 669	. 088	.724	863	.816	. 268	966	850	945	. 787		584	.742	917	.734	.843	.958	. 886	. 890	. 968	.891	.872	. 111	. 903	811	. 890	_	_	. 068	. 298	872	. 717	650	. 944	.878	. 790		754
	members	NOR	8. 844	. 126.	746 .6	3. 088.	724 .7	863 .8	813 .8	3. 206.	3. 966.	883 . 8	945 .2	7.85		571	7.33 .7	9. 716.	7. 727	. 841	957	3. 788.	892 .8	3. 968.	. 913	892 . 8	9. 116	3. 006.	8. 918	3. 068.	-	-	8. 068	872 .8	8. 028	714 .7	653 .6	3. 686.	878		•	. 749
		CON		.940		3. 088.		. 861	813 .8	3. 998.	3. 966.	8. 058.		-		.602	. 736	-	. 728	.841 .8		885 .8		891 . 8	.881	·	3. 706.	903	833 .8	3. 678.	- 1	. 606.	·	8555	858.	•	. 650	.944		7. 067.	308	•
d set	$\mathbf{Reduced}$	WEI	8. 345	940	735	3. 778	.724	861 .8	.813	871 .8	. 966.	883		. 622.	_	. 173.	. 729	.914	. 721	.841 .8	. 226	886 .8	892 .8	. 895			. 305	3. 006.	. 836	8. 788.			.884	827 .8	3. 098.	. 712	. 674	. 686	898			748
	\mathbf{R}	REG		·	·	. 830	. 701	. 861	.813	·	966	·		. 622.	·	. 571	. 730	-	. 721		. 957	886 .	. 892				. 905	. 006.	.833	3. 788.				8. 738.	8. 098.	. 717.	. 674	. 939	898.			. 748
Without oversampled	_	NC	-	_	-		_	.875	830	_	966	8. 658.	_	_	_	. 592	. 269	806			. 954	.885	_				_	9. 268.	_	8. 068.	_	_		875 .	863	. 725	653	.932	898	_	_	753
out 6	s	NOR	8. 748	949	7.15	883 .8		878	3. 088.	3. 778.		·	·	7. 987.		. 593	. 705		7. 617.	3. 688.	. 355	3. 988.						8. 368.	841 .8	3. 068.			·	872 .8	898	.712	. 699	. 932	898	Ī	•	746
Witl	members	CON	836 .8	945 .9	7.18	3. 088.	.724	8. 198.	8. 826 .8	8. 874 .8	3. 966.	8. 658.	·	. 785		. 597	. 695	.914	.740		.954 .9	884 .8	3. 068.		888.	. 892		3. 006.	8. 988.					3. 098.	8. 028.	7. 907.	.653 .6	3. 686.	8. 873	•	•	. 755
	All me	WEI		. 941	ľ	8. 088		8. 198.						. 781		574 .5	9. 669.				. 955	883 .8	8. 068.			-	3. 206.	3. 006.	8. 988.	8. 788.				8. 098.	853 .8	7.15 .7	9. 029	. 932				. 757
	¥	REG	8. 988.		7. 817	8. 088	Ľ	861 .8			3. 966.	Ť		-		576 .5		-	ľ					888.	·			3. 006.	8. 988.	-				8. 098.	ľ	7.15 .7	.684 .6	932	Ċ	•		. 757
			<u>«</u>	ڊ -	7.	∞	9.	∞		∞.	ο.	ω	 	2.	9.	ı.	9.		7.					∞.	 ∞	∞.				∞.	 oj	o.		∞.		7.	9.			7.	 o:	.7

Table 3: Balanced accuracy scores obtained using knn as a base classifier

]	Dat	tas	et																						9-9																8-8	8-9.	
				ecoli-0-1-3-7-vs-2-6	ecoli4	glass-0-1-6-vs-2	glass-0-1-6-vs-5	glass2	glass4	glass5	page-blocks-1-3-vs-4	shuttle-c0-vs-c4	shuttle-c2-vs-c4	vowel0	yeast-0-5-6-7-9-vs-4	yeast-1-2-8-9-vs-7	yeast-1-4-5-8-vs-7	yeast-1-vs-7	yeast-2-vs-4	yeast-2-vs-8	yeast4	yeast5	yeast6	ecoli-0-1-4-6-vs-5	ecoli-0-1-4-7-vs-2-3-5-6	ecoli-0-1-4-7-vs-5-6	ecoli-0-1-vs-2-3-5	ecoli-0-1-vs-5	ecoli-0-2-3-4-vs-5	ecoli-0-2-6-7-vs-3-5	ecoli-0-3-4-6-vs-5	ecoli-0-3-4-7-vs-5-6	ecoli-0-3-4-vs-5	ecoli-0-4-6-vs-5	ecoli-0-6-7-vs-3-5	ecoli-0-6-7-vs-5	glass-0-1-4-6-vs-2	glass-0-1-5-vs-2	glass-0-4-vs-5	glass-0-6-vs-5	yeast-0-2-5-6-vs-3-7-8-9	yeast-0-2-5-7-9-vs-3-6-8	yeast-0-3-5-9-vs-7-8
	F	ʻull		.841	998.	.546	.936	.573	.804	868.	966.	000.	.950	.936	.659	.630	.537	.683	.843	069.	.643	.845	.730	.781	.820	787.	.760	.857	.781	.790	.786	.840	.831	.836	.850	.795	.610	.578	.994	.995	.733	.854	.688
	τ	IJS		.708	.848	.630	988.	.682	.835	867	.958	000.	959	.940	.750	.624	.581	.661	900	.715	.792	.936	.818	.823	.804	.803	.802	.841	.843	.791	.834	.839	.862	.838	.786	.819	.675	.634	.942	878	.732	898.	.635
	(os		.624	.817	.581	.859	.616	.819	.933	.994	000.	.990	.921	.674	.621	.533	.603	.822	269.	.626	.845	.750	.794	.827	.844	.764	.805	.832	.811	.812	.836	869	.813	.864	.827	929.	.572	.994	.955	.701	298.	.599
		vo.	NC	.842	.834	.700	.940	.801	098.	946	.992	000.	000.	.961	.759	.750	.587	.789	.931	802	.845	296.	098.	.860	998.	298.	.831	.855	897	.835	.901	.851	947	.859	.853	.838	.715	.535	.982	066.	.780	768.	.651
	,	mber	NOR	.842	.834	.700	.940	.801	098.	.949	.992	000.	000.	.961	.759	.750	.587	.789	.931	.802	.845	296.	.860	.860	998.	298.	.831	.855	268.	.835	.901	.851	.947	.859	.853	.838	.715	.535	.982	.990	.780	268.	.651
		d me	CON	.823	928.	.738	.934	.800	.903	939	.991	000.	000.	.957	777.	.748	.649	.782	.958	.789	.850	.964	.851	887	.848	928.	.820	.850	.917	.830	.901	.864	.936	.904	.835	.880	.739	.751	.982	.975	.795	006.	.715
d set	,	Reduced members	WEI	.823	928.	.738	.934	800	.903	.939	.991	000	000.	.957	777.	.748	.649	.782	.958	.789	.850	.964	.851	887	.848	928.	.820	.850	.917	.830	.901	.864	.936	.904	.835	.880	.739	.751	.982	975	.795	006.	.715
mple	. ,	<u> </u>	REG	.823	.876	.741	.934	.767	.905	939	.991	000	000	.957	822.	.749	.649	.782	096	.790	.850	.964	.852	.892	.848	928.	.820	.852	.917	.833	906.	898.	.911	906.	.840	.880	.739	.754	.982	975	.801	.903	.720
verse	-		NC	.725	688.	.584	.943	.644	898.	.973	.993	000	.950	.951	.705	.647	.562	.598	888.	.716	.642	.932	.782	.871	.852	298.	.838	.830	895	.838	.893	.859	.944	.831	298.	.812	.681	.557	.982	.985	.723	878.	.628
With oversampled set		ers	NOR	.715	.865	.603	.943	.634	898.	.971	.993	000	.950	.951	.704	.659	.570	.622	.895	.737	.651	.931	.804	.871	698.	298.	.838	.855	268.	.838	.893	.859	.944	.829	.865	.838	.711	.557	.982	.985	.742	928.	.627
>	•	members	CON	.784	.878	.722	.937	.684	.895	.946	066.	000	.950	.958	.751	.738	.645	.709	.959	.820	.830	.956	.856	.885	.849	.881	.840	.850	.917	.833	.893	998.	.936	906.	.840	.863	962.	.704	.982	696.	.805	968.	.724
		All n	WEI	.784	878	.716	.937	.684	868.	939	686.	000	.950	.959	.758	.742	.650	.741	.958	.813	.818	296.	.851	.885	.849	.881	.840	.850	.917	.833	.893	998.	.936	906:	.832	.863	.813	.704	.982	.964	803	968.	.724
			REG	.783	928.	.716	.934	.749	868.	.939	686	000	.950	.959	.760	.733	.656	.737	.958	808	.815	996	.850	.885	.848	.879	.840	.850	.917	.833	.893	998.	.936	906	.832	.863	.847	.704	.982	.964	.800	968.	.728
			NC	838	873	089	937	.784	206	891	992	000	000	926	787	.732	586	801	954	.782	.844	965	852	887	830	.876	865	820	911	.820	887	853	911	912	830	878	746	.819	.982	_		895	724
		members	NOR		873	. 089	. 786	. 784	. 206	891	. 266	. 000	. 000	956	. 787	732	. 586	. 801	.954	782	.844	965	.852	. 887	.830	. 928	. 865	. 850	. 111	. 820	-	853	. 111	912 .	. 830	878	746	819 .	. 282				.724
ىد ا			CON	. 23	•	. 727	.934		. 897		.991	. 000.	. 000	. 956		. 736	.624	.794	.948	. 787.	.846	.964	.847	.910	.845	.871	.878	.848	-	.823		.875	-	. 9	.818	ω.	.827	. 800	. 982			. 898.	
ed set		$\mathbf{Reduced}$	WEI			727	934	. 780	. 268		. 166	. 000		956	. 787	. 736	624	. 794	. 948	. 787		.964	.847	. 910		.871	. 878.		.911	.823				906:		.875	.827	. 800	. 982				.729
lame	,	Re	REG	.823	. 928.	. 732	.934	. 785	. 006.		. 991	.000	.000	. 957	. 787.	. 736	.633	. 783	.948	. 787		.964	.847	. 910		.878	. 878.		.911	.833				. 906.	.818	.882	.835	.734	.982		-		. 729
Without oversampled	-		NC	816			934	.757	.893		. 286	000		926	. 987.	_	.602	781	.955	774		964	.838	.873		_	.873	_	606:	.820		.853		.912		875		.840	.982	-		_	.716
hout		ırs	NOR		. 998	657	. 937	.803	. 868		. 286	. 000		. 956	. 783	.722	.621	. 784	. 955	. 778		. 965	.840	.873			.873		. 606.	. 820		.853		.912		. 875	.805	.836	. 282			1	. 709
Wit	•	members	CON		. 998.	. 731	. 626	. 739	.885		. 186	. 000		. 955	. 788		.643	. 764	.951	. 794		. 1961	. 839	. 868.			.871		. 111	.823		.875	.928	.912	.818	. 298.	. 830	.803	. 982				.737
		All m	WEI				. 626	742	. 885		. 286	. 000		955	. 788		. 639		. 950	. 795		. 962	.838	. 006.			. 871		. 911	. 823		. 875		. 912		. 298	.833	. 803	. 282				. 736
			REG	•	. 998.	.740	. 929.	. 739	.885	ľ	.982	. 000.	•	.955		. 731	. 583.	ľ	. 950	. 798	-	.962	.838	. 006.			. 873		.914		·	. 875	. 928	.912	·	. 298.	.835	. 806	.982		•		.736

Table 4: Balanced accuracy scores obtained using DTC as a base classifier

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