

Homogenous ensemble of undersampled majority class for highly imbalanced data binary classification

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Abstract

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Keywords: classification, classifier ensemble, undersampling, imbalanced data

1. Introduction

Most of existing classification models benefit from the assumption that there are no significant disparities between the classes of the considered problem. Nevertheless, in the real world, there are many situations in which the number of objects from one of the classes (called the *majority class*) significantly exceeds the number of objects of the remaining classes (*minority classes*), which often leads to decisions biased towards the *majority class*. However, when considering cases such as spam filtering, medical tests or fraud detection, we may come to the conclusion that the cost of making an incorrect decision against a minority class is much greater than in other cases. The above-mentioned problem in the literature is called the *imbalanced data classification* (Wang et al., 2017; Sun et al., 2009).

Following work focuses on the binary classification of the highly imbalanced problems, with an IR(*imbalanced ratio*) greater than 9, which is an important issue not only in the context of the construction of appropriate models, but even a proper quality measurement (Elazmeh et al., 2006). One of the important problems is also the fact that the number of patterns in the *minority class* may be so small that it will not allow to achieve the appropriate discriminatory power of the model, which may lead to its *overfitting* (Chen and Wasikowski, 2008). Most of these problems are the subject of extensive research (Bunkhumpornpat et al., 2009; Chawla et al., 2002).

One of the possible approaches to solve such problems are *inbuild mechanisms*, trying to adapt existing classification models to balance the accuracy between classes. Popular here is the learning approach without counter-examples, using *one-class classification* (Japkowicz

et al., 1995; Krawczyk et al., 2014), where the aim is to get to know the decision boundaries within minority classes. The solution may also be the *cost sensitive solutions*, assuming the asymmetric loss function (Lopez et al., 2012; He and Garcia, 2009).

Another approach, more connected with the following paper, is the group of *data pre-processing methods*, which focuses on reducing the number of majority class objects (*undersampling*) or generating patterns of minority class (*oversampling*) to balance a dataset. Overview of methods from this group is presented in Figure 1.

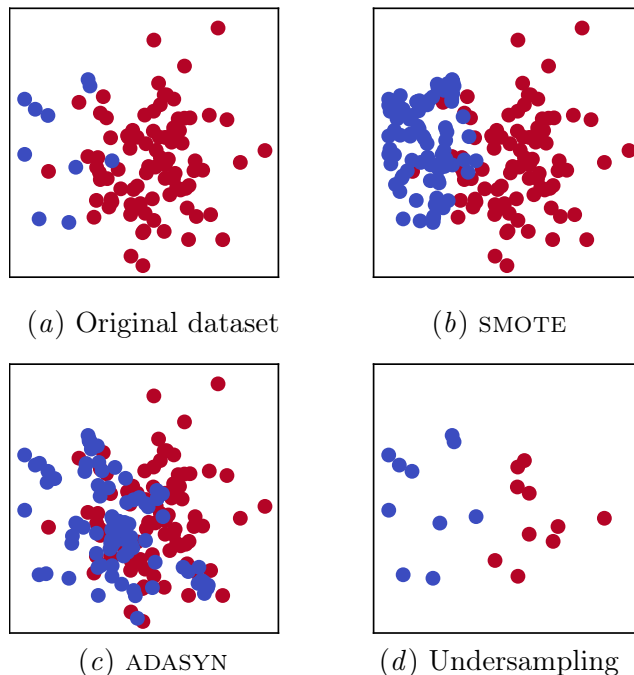


Figure 1: Examples of data preprocessing methods.

These algorithms are addressing the task of balancing the number of objects within the problem classes. In the case of basic *oversampling*, new objects are created as random copies of those already existing in the training set¹. Currently, the most common kind of *oversampling* is SMOTE (Chawla et al., 2011), shown in Figure 1(b), creating new, synthetic objects based on k averaged examples nearest to a random points from the space occupied by a minority class. An active version of SMOTE is the ADASYN algorithm (He et al., 2008), shown in Figure 1(c), which takes into account the difficulty of synthetic samples. This approach allows to solve the problem of repeating samples in the training set, but can also lead to *overfitting*, which is presented in Figure 2.

1. Since the characteristics of the new patterns will be identical to those already present in the dataset, we can consider Figure 1(a), an illustration of the original dataset, also as the presentation of pattern distribution after oversampling.

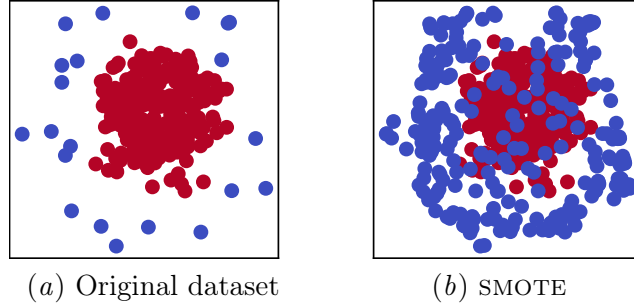


Figure 2: Example of wrong SMOTE oversampling.

In the case of *undersampling*, shown in Figure 1(d), in which we draw as many objects from the majority class as in the minority class, there is no risk of erroneous mixing of the distribution of classes.

The last group of methods to be mentioned are *hybrid approaches*, combining *over-* and *undersampling* algorithms with *ensemble classifiers* (Galar et al., 2012). The *Bagging* and *Boosting* variants, such as *AdaBoost.NC* (Wang et al., 2010) or *SMOTEBoost* (Chawla et al., 2003), have become particularly popular in this area.

The main contributions of this work are:

- a method of establishing a homogenous ensemble using a *k-fold undersampling* of majority class,
- proposition of five fusers to generate ensemble decision,
- a pruning method adjusting the decision rule to the *testing set*,
- implementation and experimental evaluation of proposed method.

2. Homogenous ensemble based on undersampling the majority class

2.1. Establishing ensemble

Complex oversampling methods, such as SMOTE or ADASYN, despite the large possibilities in most of the imbalanced problems, are not applicable to extreme situations where the minority class is represented by only a few samples, which makes it impossible to designate the nearest neighbors to create a new synthetic object. This could lead to the use of *undersampling* in such problems, but it is characterized, due to high randomness, by a strong instability in a situation of high IR (*imbalance ratio*), which does not allow for the development of a reliable solution.

A popular answer to the above-mentioned problem are the ensemble methods of *Bagging* or *Boosting*, characterized by random sampling with replacement of the training set, breaking a large problem, into a set of smaller problems. This work proposes a basic method, which also breaks the imbalanced task, but with ensuring the use of all the patterns available in the data set, but without a risk of overlapping. Its description can be found in Algorithm 1.

Algorithm 1: Training classifier ensemble from multiple balanced training datasets separated from one imbalanced dataset of binary problem
Given a dataset DS :

1. Divide DS into subsets of minority- $MinC$ and majority-class $MajC$
2. Calculate imbalanced ratio IR as the proportion of the number of patterns in $MinC$ and $MajC$
3. Establish k by rounding IR to nearest integer
4. Perform a *shuffled k-fold division* of $MajC$ to produce a set of subsets $MajC_1, MajC_2, \dots, MajC_k$
5. For every i in range to k
 6. Join $MajC_i$ with $MinC$ to prepare a training set TS_i ,
 7. Train classifier Ψ_i on TS_i and add it into ensemble

After dividing the dataset with imbalanced binary problem into separated minority ($MinC$) and majority class ($MajC$), we are calculating the IR (*imbalanced ratio*) between given classes. Rounding IR to the nearest integer value k allows us to find the optimal division coefficient of the majority class samples in the context of maximizing the balance between the $MinC$ and any $MajC_i$ subsets while ensuring that all $MajC$ patterns are used in learning process with no overlapping between the individual $MajC_i$'s. Each of k classifiers Ψ_i is trained on union of $MinC$ and $MajC_i$ sets.

Extending pool with oversampling As an extension of the method of classifier ensemble construction, it is also proposed to extend its pool by a model learned on an additional data set, which is a full set of data subjected to *oversampling*. It is worth testing if the knowledge gained from this method may be a valuable contribution to the ensemble decision. Due to impossibility to use SMOTE or ADASYN for oversampling the minority class with only few instances, only its basic variant will be used.

2.2. Fuser design

In addition to ensuring the diversity of the classifiers pool, which we achieve by a homogenous committee built on disjoint subsets of the majority class supplemented by minority patterns, the key aspect of the hybrid classification system is the appropriate design of its *fuser* – the element responsible for making decisions based on the answers of the base classifiers.

There are two groups of solutions here. The first are based on component *decisions* of the committee, most often employing the *majority voting* to produce a final decision. The decision rules proposed in this work are, however, part of the second group, where the *fuser* is carried out by *averaging* (or *accumulating*) the *support vectors* received from the members of a pool.

NOTE:

It should be remembered that in such methods, it is necessary to use a *probabilistic classification model*, which also requires *quantitative* and not *qualitative data*.

Five fusers were proposed:

1. **REG** — regular accumulation of support.

A basic method without weighing the members of a committee.

2. **WEI** — accumulation weighted after members of a committee.

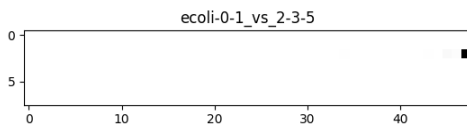
The weight of the classifier in the pool is its quality achieved for the training set. We can not use here the measure of *accuracy*, which does not fit with the task of the imbalanced classification, so we decided on a *balanced accuracy* (Brodersen et al., 2010).

3. **NOR** — same as **WEI**, but with normalization of weights,

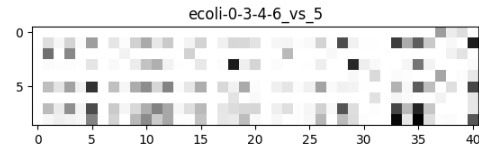
To reward classifiers with a higher *discriminative power*, weights are subjected to normalization by a *MinMaxScaler*.

4. **CON** — accumulation weighted by tested patterns.

In order to reward classifiers with greater *"certainty"* for given object, the decision for each pattern is weighted by the absolute difference between class support, for the needs of research called the *contrast*. Individual classifiers in the pool do not have to be better or worse for each of the tested patterns. This is illustrated in Figure 3, where we can see two cases of ensembles. There are tested patterns on the X axis and classifiers in the pool on the Y axis. A white square means the *contrast* of 1, and therefore a *sure* decision, and the black square the *contrast* of 0, which describes the pattern that is exactly on the decision boundary.



(a) Example of a *"sure"* ensemble



(b) Example of *"unsure"* ensemble

Figure 3: Illustration of the *contrast* in committees built on two different datasets.

5. **NCI** — accumulation by a product of normalized weights **NOR** and a *contrast* **CON**.

The proposed method of constructing the committee makes its size directly dependent on the IR, which, given the highly unbalanced data (for example with IR greater than 40), leads to the construction of an extensive hybrid model. Therefore, the method of pruning it to a smaller size was also considered.

2.3. Ensemble pruning

In typical methods of *ensemble pruning*, it follows the phase of training the committee, for example, by eliminating the classifiers that achieve the lowest quality on the *training* or separated *validation set*. This paper proposes a method of *response pruning* based on the assumption that during the testing phase we analyze not just a single test pattern, but the entire *testing set*.

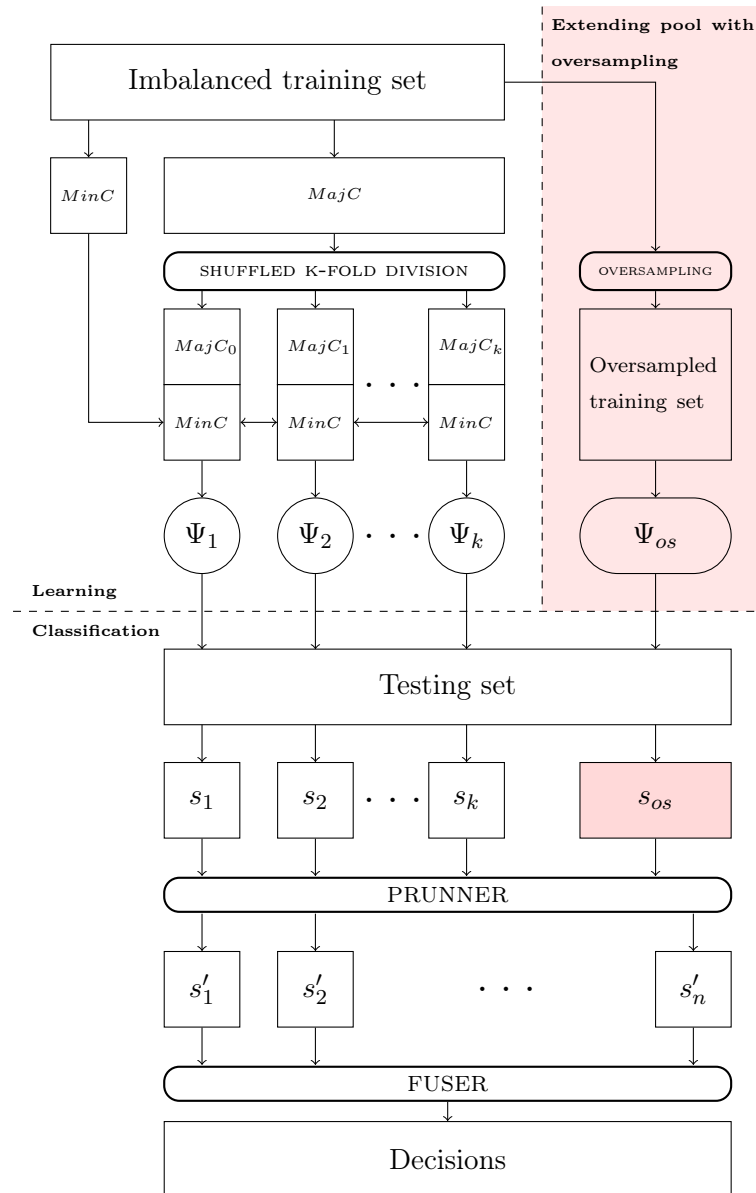


Figure 4: Scheme of using k-Fold division in ensemble construction.

Ensemble, receiving a *testing set*, generates *support vectors* (s_i) for each classified object, so, with a binary problem, we can treat received support for one of the problem classes as values from the *random variables* to analyze their mutual statistical dependence.

In the proposed method, using the signed-rank test, we are *clustering* the pool of k (or $k + 1$ on the *oversampling* variation of a method) classifiers to n groups (where $n \leq k$), to average the support and weight classes within groups to create a new set of supports from s'_1 to s'_n , passed later on to *fuser*.

The scheme of the full decision model of the proposed method is shown in Figure 4.

NOTE:

In the considered case of pruning, we ignore the possible situation in which the answer Ψ_1 is dependent on Ψ_2 , the answer Ψ_2 is dependent on Ψ_3 , but Ψ_1 is not dependent on Ψ_3 . This is an interesting issue that will be addressed in future research, but to simplify the proposal, a simplified approach has been used.

3. Experiment design

For the experimental evaluation of the proposed method, a collection of datasets made available with KEEL (Alcalá-Fdez et al., 2011) was used, focusing on a section containing highly unbalanced data, with IR greater than 9 (Fernández et al., 2009). From among the available datasets, 40 were selected presenting only binary problems with quantitative attributes. A review of selected datasets, including information on their number of features, the number of patterns in each class and the unbalance ratio is presented in Table 1.

IR	Samples			Features	DS
	ALL	MAJ	MIN		
39.14	281	274	7	7	<i>ecoli-0-1-3-7-vs-2-6</i>
15.80	336	316	20	7	<i>ecoli4</i>
10.29	192	175	17	9	<i>glass-0-1-6-vs-2</i>
19.44	184	175	9	9	<i>glass-0-1-6-vs-5</i>
11.59	214	197	17	9	<i>glass2</i>
15.46	214	201	13	9	<i>glass4</i>
22.78	214	205	9	9	<i>glass5</i>
15.86	472	444	28	10	<i>page-blocks-1-3-vs-4</i>
13.87	1829	1706	123	9	<i>shuttle-c0-vs-c4</i>
20.50	129	123	6	9	<i>shuttle-c2-vs-c4</i>
9.98	988	898	90	13	<i>vowel0</i>
9.35	528	477	51	8	<i>yeast-0-5-6-7-9-vs-4</i>
30.57	947	917	30	8	<i>yeast-1-2-8-9-vs-7</i>
22.10	693	663	30	8	<i>yeast-1-4-5-8-vs-7</i>
14.30	459	429	30	7	<i>yeast-1-vs-7</i>
9.08	514	463	51	8	<i>yeast-2-vs-4</i>
23.10	482	462	20	8	<i>yeast-2-vs-8</i>
28.10	1484	1433	51	8	<i>yeast4</i>
32.73	1484	1440	44	8	<i>yeast5</i>
41.40	1484	1449	35	8	<i>yeast6</i>
13.00	280	260	20	6	<i>ecoli-0-1-4-6-vs-5</i>
10.59	336	307	29	7	<i>ecoli-0-1-4-7-vs-2-3-5-6</i>
12.28	332	307	25	6	<i>ecoli-0-1-4-7-vs-5-6</i>
9.17	244	220	24	7	<i>ecoli-0-1-vs-2-3-5</i>
11.00	240	220	20	6	<i>ecoli-0-1-vs-5</i>
9.10	202	182	20	7	<i>ecoli-0-2-3-4-vs-5</i>
9.18	224	202	22	7	<i>ecoli-0-2-6-7-vs-3-5</i>
9.25	205	185	20	7	<i>ecoli-0-3-4-6-vs-5</i>
9.28	257	232	25	7	<i>ecoli-0-3-4-7-vs-5-6</i>
9.00	200	180	20	7	<i>ecoli-0-3-4-vs-5</i>
9.15	203	183	20	6	<i>ecoli-0-4-6-vs-5</i>
9.09	222	200	22	7	<i>ecoli-0-6-7-vs-3-5</i>
10.00	220	200	20	6	<i>ecoli-0-6-7-vs-5</i>
11.06	205	188	17	9	<i>glass-0-1-4-6-vs-2</i>
9.12	172	155	17	9	<i>glass-0-1-5-vs-2</i>
9.22	92	83	9	9	<i>glass-0-4-vs-5</i>
11.00	108	99	9	9	<i>glass-0-6-vs-5</i>
9.14	1004	905	99	8	<i>yeast-0-2-5-6-vs-3-7-8-9</i>
9.14	1004	905	99	8	<i>yeast-0-2-5-7-9-vs-3-6-8</i>
9.12	506	456	50	8	<i>yeast-0-3-5-9-vs-7-8</i>

Table 1: Summary of imbalanced datasets chosen for evaluation

As may be observed in the summary, the experiments are based on datasets with relatively small spatiality (up to 13 dimensions), with imbalance ratio from 9 to even 40. The datasets provided by KEEL, to ensure easy comparison between results presented in various research, are already pre-divided into five parts, which forces the use of *k-fold cross-validation* with $k = 5$ in experiments (Alpaydin, 2009).

In the task of imbalanced data classification, due to its strong bias towards majority class, the *accuracy* measure is not a proper tool. For a reliable result, a measure of *balanced accuracy* is given as test results.

Both the implementation of the proposed method and the experimental environment have been constructed using the *scikit-learn* library (Pedregosa et al., 2011) in version *0.20.dev0*². Among the available classification models, the MLP (*Multilayer Perceptron*) and SVC (*Support Vector Machine*) were rejected. First one was not able to build a correct model due to the lack of convergence on the small datasets (minority class of data chosen for experiments is often represented by only two patterns in cross-validated folds) and second, whose probabilistic interpretation is measurable only with sufficiently large data sets, did not allow credible construction of a fuser. As base classifiers, the following algorithms were used:

- *Gaussian Naive Bayes* (GNB) (Chan et al., 1982),
- *k-Nearest Neighbors* (kNN) — with 5 neighbors and *Minkowski* metric,
- *Decision Tree Classifier* (DTC) — with *Gini* criterion (Loh, 2011).

To provide a comparative result for the method presented in the following paper, each base classifier was also tested for the raw, imbalanced dataset and its under- and oversampled versions. Undersampling, due to high instability of results, was repeated five times on each fold. Used statistical analysis tool was a paired dependency between the classifier, which achieved the highest result and each of the others, calculated using the signed-rank *Wilcoxon* test (Wilcoxon, 1945).

The full implementation of the proposed method and the script allowing the repetition of the presented research may be found in the git repository available at [url-removed-due-to-blind-review](#).

4. Experimental evaluation

Przedstawienie tabel.

Tabela zbiorcza zwycięstw w zależności od parametrów (z grupowaniem).
Interpretacja wyników, czyli co zostało należyście uprawdopodobnione.

5. Conclusions

Co zostało zaproponowane.

Na co pozwala taka metoda.

Do jakich rezultatów doprowadziła.

Jakie są plany na przyszłość (czyli co robisz w wakacje).

2. At the time of conducting research, only the development version of the package already has the implementation of *balanced accuracy* measure.

Without oversampled set										With oversampled set										Dataset			
All members					Reduced members					All members					Reduced members					OS	US	Full	
REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC				
.845	.843	.843	.837	.837	.845	.845	.845	.837	.837	.841	.841	.839	.828	.825	.845	.845	.845	.837	.837	.806	.838	.825	ecoli-0-1-3-7-vs-2-6
.809	.834	.859	.879	.857	.748	.800	.820	.926	.898	.874	.898	.868	.913	.904	.746	.896	.895	.923	.922	.859	.787	.878	ecoli4
.580	.580	.610	.580	.610	.577	.577	.577	.583	.580	.552	.552	.552	.560	.591	.577	.577	.577	.585	.585	.569	.589	.580	glass-0-1-6-vs-2
.989	.989	.989	.939	.941	.989	.989	.989	.989	.989	.941	.941	.941	.941	.941	.989	.989	.989	.989	.989	.941	.975	.941	glass-0-1-6-vs-5
.619	.619	.619	.616	.616	.641	.641	.641	.641	.641	.619	.616	.619	.616	.616	.641	.641	.644	.641	.641	.617	.620	.591	glass2
.800	.797	.800	.802	.802	.807	.807	.804	.766	.766	.781	.781	.774	.779	.779	.812	.774	.774	.771	.771	.731	.745	.587	glass4
.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.938	.945	.938	glass5
.817	.817	.817	.846	.845	.828	.828	.828	.828	.828	.807	.807	.803	.846	.845	.831	.831	.831	.830	.828	.791	.806	.763	page-blocks-1-3-vs-4
.994	.994	.994	.995	.994	.994	.994	.994	.991	.991	.995	.995	.995	.991	.991	.994	.994	.994	.991	.991	.990	.993	.991	shuttle-c0-vs-c4
.971	.975	.975	.979	.979	.996	.996	.996	.996	.996	.984	.979	.979	.988	.988	.996	.996	.996	.996	.996	.988	.946	.996	shuttle-c2-vs-c4
.909	.909	.909	.902	.903	.909	.909	.909	.909	.909	.910	.910	.910	.904	.903	.910	.909	.909	.909	.909	.906	.905	.917	vowel0
.690	.706	.689	.728	.731	.680	.737	.727	.786	.774	.702	.700	.677	.726	.715	.687	.724	.710	.785	.791	.498	.601	.504	yeast-0-5-6-7-9-vs-4
.563	.570	.567	.576	.570	.562	.557	.562	.627	.642	.563	.552	.553	.574	.566	.556	.564	.556	.625	.639	.540	.598	.544	yeast-1-2-8-9-vs-7
.552	.553	.550	.557	.563	.567	.566	.554	.550	.575	.551	.558	.554	.555	.590	.588	.555	.562	.564	.568	.541	.566	.547	yeast-1-4-5-8-vs-7
.698	.698	.699	.705	.703	.703	.710	.689	.726	.722	.671	.681	.674	.705	.700	.700	.695	.674	.725	.719	.586	.686	.604	yeast-1-vs-7
.804	.801	.803	.799	.800	.820	.822	.827	.870	.862	.773	.794	.781	.799	.800	.805	.833	.827	.869	.861	.529	.739	.561	yeast-2-vs-4
.773	.773	.773	.773	.774	.773	.773	.773	.773	.773	.796	.771	.796	.773	.773	.773	.773	.773	.773	.773	.616	.762	.657	yeast-2-vs-8
.675	.674	.674	.769	.728	.708	.766	.766	.822	.813	.644	.655	.637	.781	.746	.710	.765	.769	.820	.811	.526	.660	.551	yeast4
.934	.934	.934	.934	.934	.929	.934	.938	.957	.955	.917	.921	.919	.934	.934	.927	.930	.935	.957	.955	.780	.910	.831	yeast5
.785	.787	.787	.847	.796	.829	.863	.848	.887	.878	.760	.773	.768	.843	.795	.818	.860	.845	.887	.878	.628	.795	.650	yeast6
.785	.781	.783	.804	.804	.660	.810	.810	.906	.913	.810	.812	.860	.906	.906	.785	.837	.862	.910	.913	.885	.679	.877	ecoli-0-1-4-6-vs-5
.630	.630	.630	.630	.630	.630	.630	.630	.652	.654	.630	.630	.630	.662	.663	.630	.630	.630	.657	.659	.667	.634	.630	ecoli-0-1-4-7-vs-2-3-5-6
.617	.617	.617	.710	.693	.617	.617	.617	.823	.829	.637	.677	.755	.852	.855	.617	.617	.617	.826	.831	.863	.668	.735	ecoli-0-1-4-7-vs-5-6
.578	.578	.578	.575	.578	.558	.578	.578	.658	.658	.618	.618	.638	.638	.638	.558	.618	.618	.658	.658	.639	.578	.638	ecoli-0-1-vs-2-3-5
.639	.689	.689	.736	.732	.616	.666	.664	.741	.741	.714	.739	.789	.834	.830	.641	.691	.689	.816	.816	.797	.658	.782	ecoli-0-1-vs-5
.683	.733	.708	.778	.753	.658	.733	.733	.853	.853	.756	.758	.806	.828	.803	.731	.758	.758	.895	.895	.638	.657	.754	ecoli-0-2-3-4-vs-5
.563	.563	.588	.635	.613	.588	.588	.588	.625	.630	.588	.588	.588	.625	.628	.588	.613	.613	.623	.628	.592	.595	.563	ecoli-0-2-6-7-vs-3-5
.876	.876	.876	.876	.876	.818	.870	.873	.901	.898	.859	.865	.849	.857	.851	.843	.870	.870	.901	.895	.725	.716	.784	ecoli-0-3-4-6-vs-5
.633	.633	.633	.765	.729	.633	.673	.653	.781	.799	.697	.735	.726	.785	.747	.653	.673	.673	.789	.791	.734	.665	.775	ecoli-0-3-4-7-vs-5-6
.756	.756	.756	.833	.792	.711	.786	.783	.883	.883	.769	.794	.811	.772	.758	.758	.783	.783	.883	.883	.730	.657	.817	ecoli-0-3-4-vs-5
.876	.876	.876	.901	.903	.853	.876	.878	.903	.878	.901	.901	.901	.901	.901	.878	.878	.876	.903	.901	.890	.725	.854	ecoli-0-4-6-vs-5
.598	.598	.598	.573	.555	.548	.548	.548	.640	.597	.557	.557	.557	.575	.575	.548	.508	.508	.618	.597	.544	.571	.508	ecoli-0-6-7-vs-3-5
.795	.787	.793	.805	.812	.685	.755	.705	.853	.830	.845	.838	.838	.828	.825	.688	.755	.755	.853	.830	.847	.682	.780	ecoli-0-6-7-vs-5
.558	.589	.558	.615	.581	.622	.622	.620	.620	.620	.592	.592	.558	.595	.595	.622	.622	.620	.620	.620	.597	.590	.577	glass-0-1-4-6-vs-2
.567	.582	.582	.527	.534	.542	.536	.542	.604	.582	.527	.527	.527	.530	.533	.533	.551	.558	.584	.584	.508	.555	.519	glass-0-1-5-vs-2
.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994	.984	.984	.994	glass-0-4-vs-5
.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.995	.945	.989	.945	glass-0-6-vs-5
.576	.576	.576	.576	.576	.576	.576	.576	.782	.782	.634	.657	.692	.777	.768	.576	.576	.576	.781	.783	.782	.605	.670	yeast-0-2-5-6-vs-3-7-8-9
.883	.893	.889	.901	.901	.901	.901	.901	.902	.897	.900	.901	.901	.900	.898	.894	.896	.896	.897	.900	.524	.785	.577	yeast-0-2-5-7-9-vs-3-6-8
.597	.632	.606	.620	.621	.606	.605	.606	.605	.605	.601	.607	.589	.619	.633	.611	.612	.600	.605	.605	.539	.633	.557	yeast-0-3-5-9-vs-7-8

Table 2: Balanced accuracy scores obtained using GNB as a base classifier

Without oversampled set										With oversampled set										Dataset			
All members					Reduced members					All members					Reduced members					OS	US	Full	
REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC				
.836	.836	.836	.847	.849	.845	.845	.847	.844	.845	.830	.830	.832	.835	.834	.854	.853	.851	.853	.856	.835	.835	.850	ecoli-0-1-3-7-vs-2-6
.941	.941	.945	.949	.949	.940	.940	.940	.921	.945	.964	.967	.972	.909	.909	.945	.945	.945	.940	.943	.909	.928	.848	ecoli4
.718	.718	.718	.715	.724	.732	.735	.726	.746	.699	.717	.751	.760	.750	.728	.705	.705	.737	.758	.735	.656	.666	.555	glass-0-1-6-vs-2
.880	.880	.880	.883	.883	.830	.877	.880	.880	.880	.836	.839	.853	.879	.879	.833	.833	.836	.833	.841	.933	.852	.739	glass-0-1-6-vs-5
.696	.696	.724	.721	.719	.701	.724	.718	.724	.724	.737	.757	.746	.756	.756	.726	.732	.731	.723	.746	.715	.678	.485	glass2
.861	.861	.861	.878	.875	.861	.861	.861	.863	.863	.868	.868	.888	.905	.913	.861	.861	.861	.888	.898	.925	.865	.781	glass4
.823	.823	.826	.830	.830	.813	.813	.813	.813	.816	.838	.843	.879	.801	.873	.816	.813	.826	.821	.833	.830	.811	.695	glass5
.868	.868	.874	.877	.877	.867	.871	.866	.902	.897	.901	.907	.926	.909	.924	.874	.875	.881	.917	.920	.917	.872	.808	page-blocks-1-3-vs-4
.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	.996	shuttle-c0-vs-c4
.859	.859	.859	.891	.859	.879	.883	.850	.883	.850	.996	.000	.000	.000	.000	.887	.887	.906	.927	.996	.000	.845	.600	shuttle-c2-vs-c4
.942	.942	.942	.944	.943	.943	.942	.943	.945	.945	.952	.953	.963	.997	.996	.948	.948	.954	.980	.986	.999	.939	.977	vowel0
.781	.781	.785	.786	.790	.779	.779	.785	.785	.787	.810	.824	.832	.830	.820	.799	.800	.819	.828	.829	.795	.792	.667	yeast-0-5-6-7-9-vs-4
.644	.645	.662	.665	.663	.659	.660	.655	.645	.659	.702	.695	.673	.625	.626	.655	.660	.684	.692	.693	.627	.652	.499	yeast-1-2-8-9-vs-7
.576	.574	.597	.593	.592	.571	.571	.602	.571	.584	.622	.628	.614	.612	.609	.594	.607	.623	.636	.643	.615	.590	.499	yeast-1-4-5-8-vs-7
.698	.699	.695	.705	.697	.730	.729	.736	.733	.742	.737	.743	.712	.700	.700	.745	.755	.734	.706	.710	.705	.682	.517	yeast-1-vs-7
.915	.915	.914	.907	.908	.914	.914	.912	.917	.917	.921	.921	.913	.906	.906	.921	.920	.921	.921	.909	.885	.908	.819	yeast-2-vs-4
.715	.715	.740	.719	.719	.721	.721	.728	.727	.734	.782	.782	.778	.803	.803	.738	.738	.752	.755	.758	.803	.734	.774	yeast-2-vs-8
.819	.819	.819	.839	.838	.840	.841	.841	.841	.843	.838	.829	.839	.771	.774	.844	.844	.843	.841	.852	.749	.835	.574	yeast4
.955	.955	.954	.955	.954	.957	.957	.956	.957	.958	.956	.956	.960	.962	.967	.958	.958	.958	.960	.960	.964	.952	.850	yeast5
.883	.883	.884	.886	.885	.886	.886	.885	.887	.886	.899	.900	.892	.840	.865	.887	.887	.888	.893	.895	.840	.879	.739	yeast6
.890	.890	.890	.892	.892	.892	.892	.892	.892	.890	.894	.894	.896	.904	.906	.892	.892	.892	.892	.892	.917	.886	.898	ecoli-0-1-4-6-vs-5
.888	.888	.888	.890	.885	.893	.895	.891	.896	.896	.869	.869	.886	.869	.886	.886	.886	.889	.904	.902	.856	.882	.847	ecoli-0-1-4-7-vs-2-3-5-6
.886	.886	.888	.868	.868	.883	.885	.881	.913	.891	.874	.876	.887	.892	.894	.893	.893	.871	.887	.889	.899	.883	.838	ecoli-0-1-4-7-vs-5-6
.892	.892	.892	.914	.894	.892	.892	.872	.892	.872	.872	.865	.867	.885	.885	.874	.874	.872	.883	.863	.887	.895	.830	ecoli-0-1-vs-2-3-5
.905	.905	.907	.914	.911	.905	.905	.907	.911	.911	.911	.911	.911	.911	.914	.909	.909	.911	.911	.911	.916	.902	.900	ecoli-0-1-vs-5
.900	.900	.900	.895	.897	.900	.900	.903	.900	.903	.900	.900	.900	.900	.900	.900	.900	.900	.900	.900	.909	.904	.894	ecoli-0-2-3-4-vs-5
.836	.836	.836	.841	.841	.833	.836	.833	.816	.811	.853	.858	.860	.830	.833	.843	.843	.848	.860	.855	.890	.814	.787	ecoli-0-2-6-7-vs-3-5
.887	.887	.874	.890	.890	.887	.887	.879	.890	.890	.893	.893	.895	.901	.901	.893	.893	.890	.895	.895	.911	.881	.875	ecoli-0-3-4-6-vs-5
.904	.904	.902	.917	.917	.904	.904	.902	.921	.915	.901	.901	.901	.890	.890	.913	.911	.897	.885	.888	.894	.887	.876	ecoli-0-3-4-7-vs-5-6
.903	.903	.903	.903	.900	.900	.900	.903	.903	.900	.906	.906	.906	.903	.903	.906	.906	.906	.903	.903	.911	.889	.875	ecoli-0-3-4-vs-5
.890	.890	.890	.887	.884	.884	.884	.884	.890	.890	.890	.890	.890	.898	.895	.890	.890	.890	.892	.895	.914	.889	.900	ecoli-0-4-6-vs-5
.860	.860	.860	.872	.875	.857	.857	.855	.872	.867	.875	.875	.855	.865	.863	.865	.870	.870	.863	.858	.893	.844	.835	ecoli-0-6-7-vs-3-5
.853	.853	.850	.868	.863	.860	.860	.858	.870	.872	.878	.878	.880	.863	.885	.865	.865	.872	.890	.887	.863	.850	.847	ecoli-0-6-7-vs-5
.715	.715	.706	.712	.725	.717	.712	.712	.714	.717	.731	.717	.701	.718	.716	.741	.749	.747	.734	.727	.732	.681	.512	glass-0-1-4-6-vs-2
.684	.650	.653	.663	.653	.674	.674	.650	.653	.650	.705	.728	.748	.767	.740	.676	.695	.712	.745	.758	.656	.651	.527	glass-0-1-5-vs-2
.932	.932	.939	.932	.932	.939	.939	.944	.939	.944	.951	.951	.951	.951	.951	.951	.951	.951	.951	.951	.988	.917	.850	glass-0-4-vs-5
.873	.873	.873	.868	.868	.868	.868	.868	.873	.878	.883	.898	.928	.953	.953	.878	.878	.878	.913	.933	.985	.816	.745	glass-0-6-vs-5
.784	.784	.786	.781	.784	.785	.785	.785	.790	.786	.801	.802	.800	.791	.798	.793	.798	.800	.795	.798	.784	.760	.762	yeast-0-2-5-6-vs-3-7-8-9
.909	.909	.909	.908	.911	.911	.904	.904	.908	.906	.910	.913	.913	.902	.900	.909	.913	.912	.896	.895	.904	.902	.902	yeast-0-2-5-7-9-vs-3-6-8
.757	.757	.755	.746	.753	.748	.748	.748	.757	.749	.754	.759	.742	.739	.731	.770	.767	.750	.723	.734	.718	.702	.639	yeast-0-3-5-9-vs-7-8

Table 3: Balanced accuracy scores obtained using kNN as a base classifier

Without oversampled set										With oversampled set										Full			Dataset
All members					Reduced members					All members					Reduced members					OS	US		
REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC	REG	WEI	CON	NOR	NC				
.816	.816	.816	.818	.816	.823	.823	.823	.838	.838	.783	.784	.784	.715	.725	.823	.823	.823	.842	.842	.624	.708	.841	<i>ecoli-0-1-3-7-vs-2-6</i>
.866	.866	.866	.866	.866	.876	.873	.873	.873	.873	.876	.878	.878	.865	.889	.876	.876	.876	.834	.834	.817	.848	.866	<i>ecoli4</i>
.740	.737	.731	.657	.657	.732	.727	.727	.680	.680	.716	.716	.722	.603	.584	.741	.738	.738	.700	.700	.581	.630	.546	<i>glass-0-1-6-vs-2</i>
.929	.929	.929	.937	.934	.934	.934	.937	.937	.937	.934	.937	.937	.943	.943	.934	.934	.934	.940	.940	.859	.886	.936	<i>glass-0-1-6-vs-5</i>
.739	.742	.739	.803	.757	.785	.780	.780	.784	.784	.749	.684	.684	.634	.644	.767	.800	.800	.801	.801	.616	.682	.573	<i>glass2</i>
.885	.885	.885	.898	.893	.900	.897	.897	.907	.907	.898	.898	.895	.868	.868	.905	.903	.903	.860	.860	.819	.835	.804	<i>glass4</i>
.924	.924	.924	.879	.877	.934	.934	.934	.891	.891	.939	.939	.946	.971	.971	.939	.939	.939	.949	.949	.933	.867	.898	<i>glass5</i>
.982	.982	.981	.987	.987	.991	.991	.991	.992	.992	.989	.989	.990	.993	.993	.991	.991	.991	.992	.992	.994	.958	.996	<i>page-blocks-1-3-vs-4</i>
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	<i>shuttle-c0-vs-c4</i>
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.950	.950	.950	.950	.950	.000	.000	.000	.000	.000	.990	.959	.950	<i>shuttle-c2-vs-c4</i>
.955	.955	.955	.956	.956	.957	.956	.956	.956	.956	.959	.959	.958	.951	.951	.957	.957	.957	.961	.961	.921	.940	.936	<i>vowel0</i>
.788	.788	.788	.783	.786	.787	.787	.787	.787	.787	.760	.758	.751	.704	.705	.778	.777	.777	.759	.759	.674	.750	.659	<i>yeast-0-5-6-7-9-vs-4</i>
.731	.733	.729	.722	.737	.736	.736	.736	.732	.732	.733	.742	.738	.659	.647	.749	.748	.748	.750	.750	.621	.624	.630	<i>yeast-1-2-8-9-vs-7</i>
.637	.639	.643	.621	.602	.633	.624	.624	.586	.586	.656	.650	.645	.570	.562	.649	.649	.649	.587	.587	.533	.581	.537	<i>yeast-1-4-5-8-vs-7</i>
.767	.768	.764	.784	.781	.783	.794	.794	.801	.801	.737	.741	.709	.622	.598	.782	.782	.782	.789	.789	.603	.661	.683	<i>yeast-1-vs-7</i>
.950	.950	.951	.955	.955	.948	.948	.948	.954	.954	.958	.958	.959	.895	.888	.960	.958	.958	.931	.931	.822	.900	.843	<i>yeast-2-vs-4</i>
.798	.795	.794	.778	.774	.787	.787	.787	.782	.782	.808	.813	.820	.737	.716	.790	.789	.789	.802	.802	.697	.715	.690	<i>yeast-2-vs-8</i>
.824	.824	.817	.837	.826	.846	.846	.846	.844	.844	.815	.818	.830	.651	.642	.850	.850	.850	.845	.845	.626	.792	.643	<i>yeast4</i>
.962	.962	.961	.965	.964	.964	.964	.964	.965	.965	.966	.967	.956	.931	.932	.964	.964	.964	.967	.967	.845	.936	.845	<i>yeast5</i>
.838	.838	.839	.840	.838	.847	.847	.847	.852	.852	.850	.851	.856	.804	.782	.852	.851	.851	.860	.860	.750	.818	.730	<i>yeast6</i>
.900	.900	.898	.873	.873	.910	.910	.910	.887	.887	.885	.885	.885	.871	.871	.892	.887	.887	.860	.860	.794	.823	.781	<i>ecoli-0-1-4-6-vs-5</i>
.842	.842	.838	.819	.819	.847	.845	.845	.830	.830	.848	.849	.849	.869	.852	.848	.848	.848	.866	.866	.827	.804	.820	<i>ecoli-0-1-4-7-vs-2-3-5-6</i>
.866	.866	.866	.853	.853	.878	.871	.871	.876	.876	.879	.881	.881	.867	.867	.876	.876	.876	.867	.867	.844	.803	.787	<i>ecoli-0-1-4-7-vs-5-6</i>
.873	.871	.871	.873	.873	.878	.878	.878	.865	.865	.840	.840	.840	.838	.838	.820	.820	.820	.831	.831	.764	.802	.760	<i>ecoli-0-1-vs-2-3-5</i>
.873	.873	.873	.873	.873	.848	.848	.848	.850	.850	.850	.850	.850	.855	.830	.852	.850	.850	.855	.855	.805	.841	.857	<i>ecoli-0-1-vs-5</i>
.914	.911	.911	.909	.909	.911	.911	.911	.911	.911	.917	.917	.917	.897	.895	.917	.917	.917	.897	.897	.832	.843	.781	<i>ecoli-0-2-3-4-vs-5</i>
.823	.823	.823	.820	.820	.833	.823	.823	.820	.820	.833	.833	.833	.838	.838	.833	.830	.830	.835	.835	.811	.791	.790	<i>ecoli-0-2-6-7-vs-3-5</i>
.882	.882	.884	.901	.904	.879	.879	.879	.887	.887	.893	.893	.893	.893	.893	.906	.901	.901	.901	.901	.812	.834	.786	<i>ecoli-0-3-4-6-vs-5</i>
.875	.875	.875	.853	.853	.875	.875	.875	.853	.853	.866	.866	.866	.859	.859	.868	.864	.864	.851	.851	.836	.839	.840	<i>ecoli-0-3-4-7-vs-5-6</i>
.928	.928	.928	.914	.914	.928	.928	.928	.911	.911	.936	.936	.936	.944	.944	.911	.936	.936	.947	.947	.869	.862	.831	<i>ecoli-0-3-4-vs-5</i>
.912	.912	.912	.912	.912	.906	.906	.906	.912	.912	.906	.906	.906	.829	.831	.906	.904	.904	.859	.859	.813	.838	.836	<i>ecoli-0-4-6-vs-5</i>
.818	.818	.818	.825	.825	.818	.818	.818	.830	.830	.832	.832	.840	.865	.867	.840	.835	.835	.853	.853	.864	.786	.850	<i>ecoli-0-6-7-vs-3-5</i>
.867	.867	.867	.875	.875	.882	.875	.875	.878	.878	.863	.863	.863	.838	.812	.880	.880	.880	.838	.838	.827	.819	.795	<i>ecoli-0-6-7-vs-5</i>
.835	.833	.830	.805	.810	.835	.827	.827	.746	.746	.847	.813	.796	.711	.681	.739	.739	.739	.715	.715	.676	.675	.610	<i>glass-0-1-4-6-vs-2</i>
.806	.803	.803	.836	.840	.734	.800	.800	.819	.819	.704	.704	.704	.557	.557	.754	.751	.751	.535	.535	.572	.634	.578	<i>glass-0-1-5-vs-2</i>
.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.982	.994	.942	.994	<i>glass-0-4-vs-5</i>
.959	.959	.959	.974	.974	.959	.959	.959	.980	.980	.964	.964	.969	.985	.985	.975	.975	.975	.990	.990	.955	.879	.995	<i>glass-0-6-vs-5</i>
.775	.775	.770	.758	.757	.782	.782	.782	.790	.790	.800	.802	.805	.742	.723	.801	.795	.795	.795	.780	.701	.732	.733	<i>yeast-0-2-5-6-vs-3-7-8-9</i>
.903	.903	.903	.888	.887	.898	.898	.898	.895	.895	.896	.896	.896	.876	.878	.903	.900	.900	.897	.897	.867	.868	.854	<i>yeast-0-2-5-7-9-vs-3-6-8</i>
.736	.736	.737	.709	.716	.729	.729	.729	.724	.724	.728	.724	.724	.627	.628	.720	.715	.715	.651	.651	.599	.635	.688	<i>yeast-0-3-5-9-vs-7-8</i>

Table 4: Balanced accuracy scores obtained using DTC as a base classifier

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