Exercise 2.2

Truth table for exclusive or:

P	Q	Xor
T	F	1
F	T	1
T	T	0
F	F	0

Exercise 2.4

Basically the negation of P if and only if Q means that when both values are true, the expression will result in false. In case both P and Q are false, the evaluation of the iff expression will result in true, but will also be negated with this expression. When either P or Q is true (and thus the other false), the expression will be evaluated to true. Therefore the truth table for exclusive or is logically equivalent to $\neg(P \Leftrightarrow Q)$.

Exercise 2.9

$(P \oplus Q) \oplus Q \Leftrightarrow P$ is proven by this truth table:

P	⊕	Q	⊕	Q	⇔	P
1	0	1	*1*	1	*1*	*1*
1	1	0	*1*	0	*1*	*1*
0	1	1	*0*	1	*0*	*0*
0	0	0	*0*	0	*0*	*0*

Exercise 2.11

$P \equiv \neg\neg P$

P	=	¬	¬	P
1	1	*1*	0	1
0	1	*0*	1	0

$P \wedge P \equiv P$

P	^	P	=	P
1	*1*	1	1	*1*
0	*0*	0	1	*0*

$P \vee P \equiv P$

P	∨	P	=	P
1	*1*	1	1	*1*
0	*0*	0	1	*0*

$(P \Rightarrow Q) \equiv \neg P \vee Q;$

P	⇒	Q	=	¬	P	∨	Q
1	*1*	1	*1*	0	1	*1*	1
1	*0*	0	*1*	0	1	*0*	0
0	*1*	1	*1*	1	0	*1*	1
0	*1*	0	*1*	1	0	*1*	0

$\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$

¬	(P	⇒	Q)	=	P	∧	¬	Q
0	1	1	1	*1*	1	*0*	0	1
1	1	0	0	*1*	1	*1*	1	0
0	0	1	1	*1*	0	*0*	0	1
0	0	1	0	*1*	0	*0*	1	0

(¬P ⇒¬Q) ≡ (Q⇒P);

¬	**P**	**⇒**	**¬**	**Q**	**=**	**Q**	**⇒**	**P**
0	1	*1*	0	1	*1*	1	*1*	1
0	1	*1*	1	0	*1*	0	*1*	1
1	0	*0*	0	1	*1*	1	*0*	0
1	0	*1*	1	0	*1*	0	*1*	0

(P ⇒¬Q)≡(Q⇒¬P);

P	**⇒**	**¬**	**Q**	**=**	**Q**	**⇒**	**¬**	**P**
0	*1*	1	0	*1*	0	*1*	1	0
0	*1*	0	1	*1*	1	*1*	1	0
1	*1*	1	0	*1*	0	*1*	0	1
1	*0*	0	1	*1*	1	*0*	0	1

(¬P ⇒ Q) ≡ (¬Q ⇒ P)

¬	**P**	**⇒**	**Q**	**=**	**¬**	**Q**	**⇒**	**P**
0	1	*1*	0	*1*	1	0	*1*	1
1	0	*1*	1	*1*	0	1	*1*	0
0	1	*1*	1	*1*	0	1	*1*	1
1	0	*0*	0	*1*	1	0	*0*	0

(P ⇔Q)≡((P ⇒Q) ∧ (Q⇒P))≡ ((P ∧ Q) ∨ (¬P ∧ ¬Q))

P	**⇔**	**Q**	**=**	**((P**	**⇒**	**Q)**	**∧**	**Q**	**⇒**	**P)**	**=**	**((P**	**∧**	**Q)**	**∨**	**(¬**	**P**	**∧**	**¬**	**Q))**
1	*0*	0	1	1	0	0	*0*	0	1	1	1	1	*0*	0	*0*	0	1	*0*	1	0
0	*0*	1	1	0	1	1	*0*	1	0	0	1	0	*0*	1	*0*	1	0	*0*	0	1
1	*1*	1	1	1	1	1	*1*	1	1	1	1	1	*1*	1	*1*	0	1	*0*	0	1
0	*1*	0	1	0	1	0	*1*	0	1	0	1	0	*0*	0	*1*	1	0	*1*	1	0

P ∧ Q≡Q ∧ P

P	∧	**Q**	**=**	**Q**	∧	**P**
0	*0*	1	*1*	1	*0*	0
1	*0*	0	*1*	0	*0*	1
1	*1*	1	*1*	1	*1*	1
0	*0*	0	*1*	0	*0*	0

P ∨ Q≡Q ∨ P

P	∨	**Q**	**=**	**Q**	∨	**P**
0	*1*	1	*1*	1	*1*	0
1	*1*	0	*1*	0	*1*	1
1	*1*	1	*1*	1	*1*	1
0	*0*	0	*1*	0	*0*	0

P ∧ (Q ∧ R) ≡ (P ∧ Q) ∧ R

P	∧	**(Q**	∧	**R)**	**=**	**(P**	∧	**Q)**	∧	**R**
1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1
1	*0*	1	*0*	0	*1*	1	*1*	1	*0*	0
1	*0*	0	*0*	0	*1*	1	*0*	0	*0*	0
0	*0*	1	*1*	1	*1*	0	*0*	1	*0*	1
1	*0*	0	*0*	1	*1*	1	*0*	0	*0*	1
0	*0*	0	*0*	0	*1*	0	*0*	0	*0*	0
0	*0*	1	*0*	0	*1*	0	*0*	1	*0*	0
0	*0*	0	*0*	1	*1*	0	*0*	0	*0*	1

$P \lor (Q \lor R) \equiv (P \lor Q) \lor R$

P	∨	**(Q**	∨	**R)**	**=**	**(P**	∨	**Q)**	∨	**R**
1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1
1	*1*	1	*1*	0	*1*	1	*1*	1	*1*	0
1	*1*	0	*0*	0	*1*	1	*1*	0	*1*	0
0	*1*	1	*1*	1	*1*	0	*1*	1	*1*	1
1	*1*	0	*1*	1	*1*	1	*1*	0	*1*	1
0	*0*	0	*0*	0	*1*	0	*0*	0	*0*	0
0	*1*	1	*1*	0	*1*	0	*1*	1	*1*	0
0	*1*	0	*1*	1	*1*	0	*0*	0	*1*	1

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$

P	∧	**(Q**	∨	**R)**	**=**	**(P**	∧	**Q)**	∨	**(P**	∧	**R)**
1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1
1	*1*	1	*1*	0	*1*	1	*1*	1	*1*	1	*0*	0
1	*0*	0	*0*	0	*1*	1	*0*	0	*0*	1	*0*	0
0	*0*	1	*1*	1	*1*	0	*0*	1	*0*	0	*0*	1
1	*1*	0	*1*	1	*1*	1	*0*	0	*1*	1	*1*	1
0	*0*	0	*0*	0	*1*	0	*0*	0	*0*	0	*0*	0
0	*0*	1	*1*	0	*1*	0	*0*	1	*0*	0	*0*	0
0	*0*	0	*1*	1	*1*	0	*0*	0	*0*	0	*0*	1

P ∨ (Q ∧ R)≡(P ∨ Q) ∧ (P ∨ R)

P	∨	**(Q**	∧	**R)**	**=**	**(P**	∨	**Q)**	∧	**(P**	∨	**R)**
1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1	*1*	1
1	*1*	1	*0*	0	*1*	1	*1*	1	*1*	1	*1*	0
1	*1*	0	*0*	0	*1*	1	*1*	0	*1*	1	*1*	0
0	*1*	1	*1*	1	*1*	0	*1*	1	*1*	0	*1*	1
1	*1*	0	*0*	1	*1*	1	*1*	0	*1*	1	*1*	1
0	*0*	0	*0*	0	*1*	0	*0*	0	*0*	0	*0*	0
0	*0*	1	*0*	0	*1*	0	*1*	1	*0*	0	*0*	0
0	*0*	0	*0*	1	*1*	0	*0*	0	*0*	0	*1*	1

Exercise 2.16
The equation x^2 + 1 = 0 has a solution:
x^2 + 1 /= 0

A largest natural number does not exist:
A largest natural number does exist

The number 13 is prime (use dIn for ‘d divides n’).
The number 21 is not a prime

The number n is prime:
The number n is not prime

There are infinitely many primes:
The amount of primes is finite

Exercise 2.17
Produce a denial for the statement that x < y < z (where x, y, z ∈ R).

x = 10
y = 9
z = 8

Exercise 2.18
Proof of (Φ⇔Ψ)≡(¬Φ⇔¬Ψ):

logEquiv2 (\p q -> p <=> q) (\p q -> ((not) p) <=> ((not) q)) results in True

Proof of (¬Φ⇔Ψ)≡(Φ⇔¬Ψ):

logEquiv2 (\p q -> ((not) p) <=> q) (\p q -> p <=> ((not) q)) results in True

Exercise 2.19
Proof of equivalence (each exercise has also been written in Haskell and executed to make sure the answers matched the tables below):

¬P ⇒Q and P ⇒¬Q:

¬	P	⇒	Q	=	P	⇒	¬	Q
0	1	*1*	1		1	*0*	0	1
0	1	*1*	0		1	*1*	1	0
1	0	*1*	1		0	*1*	0	1
1	0	*0*	0		0	*1*	1	0

These are not equivalent.

¬P ⇒ Q and Q ⇒ ¬P:

¬	P	⇒	Q	=	Q	⇒	¬	P
0	1	*1*	1		1	*0*	0	1
0	1	*1*	0		0	*1*	0	1
1	0	*1*	1		1	*1*	1	0
1	0	*0*	0		0	*1*	1	0

These are not equivalent.

¬P ⇒ Q and ¬Q ⇒ P:

¬	P	⇒	Q	=	¬	Q	⇒	P
0	1	*1*	1		0	1	*1*	1
0	1	*1*	0		1	0	*1*	1
1	0	*1*	1		0	1	*1*	0
1	0	*0*	0		1	0	*0*	0

These are equivalent.

P ⇒(Q⇒R) and Q⇒(P ⇒R):

P	⇒	(Q	⇒	R)	=	Q	⇒	(P	⇒	R)
1	*1*	1	1	1		1	*1*	1	1	1
1	*0*	1	0	0		1	*0*	1	0	0

P	⇒	(Q	⇒	R)	=	Q	⇒	(P	⇒	R)
1	*1*	0	1	0		0	*1*	1	0	0
1	*1*	0	1	1		0	*1*	1	1	1
0	*1*	1	1	1		1	*1*	0	1	1
0	*1*	0	1	1		0	*1*	0	1	1
0	*1*	1	0	0		1	*1*	0	1	0
0	*1*	0	1	0		0	*1*	0	1	0

These are equivalent.

P⇒(Q⇒R) and (P ⇒Q)⇒R:

P	⇒	(Q	⇒	R)	=	(P	⇒	Q)	⇒	R
1	*1*	1	1	1		1	1	1	*1*	1
1	*0*	1	0	0		1	1	1	*0*	0
1	*1*	0	1	0		1	0	0	*1*	0
1	*1*	0	1	1		1	0	0	*1*	1
0	*1*	1	1	1		0	1	1	*1*	1
0	*1*	0	1	1		0	1	0	*1*	1
0	*1*	1	0	0		0	1	1	*0*	0
0	*1*	0	1	0		0	1	0	0	0

These are not equivalent.

(P ⇒ Q) ⇒ P and P:

(P	⇒	Q)	⇒	P	=	P
0	1	1	*0*	0		*0*
0	1	1	*0*	0		*0*
1	1	1	*1*	1		*1*
1	0	0	*1*	1		*1*

These are equivalent.

P ∨ Q ⇒ R and (P ⇒ R) ∧ (Q ⇒ R):

P	∨	Q	⇒	R	=	(P	⇒	R)	∧	(Q	⇒	R)
1	1	1	*1*	1		1	1	1	*1*	1	1	1
1	1	1	*0*	0		1	0	0	*0*	1	0	0
1	1	0	*0*	0		1	0	0	*0*	0	1	0
1	1	0	*1*	1		1	1	1	*1*	0	1	1
0	1	1	*1*	1		0	1	1	*1*	1	1	1
0	1	1	*0*	0		0	1	0	*0*	1	0	0
0	0	0	*1*	1		0	1	1	*1*	0	1	1
0	0	0	*1*	0		0	1	0	*1*	0	1	0

These are logical equivalent.

Exercise 2.21

1. Construct a formula Φ involving the letters P and Q that has the following truth table.

P	Q	Φ
1	1	1
1	0	1
0	1	0
0	0	1

Q ⇒ R should be the correct formula (the formula is always true unless the consequent is false and the antecedent is true).

2. How many truth tables are there for 2-letter formulas altogether?

2^4 = 12 possible evaluations (each combination of truth values can lead to both true and false). The amount of sentences that can be created is infinite though.

3. Can you find formulas for all of them?

P	Q	Φ
1	1	1
0	1	0
1	0	0
0	0	1

The table above represents **P** ⇔ **Q**

P	Q	Φ
1	1	1
0	1	1
1	0	0
0	0	0

The table above represents **P** ∨ **Q**

P	Q	Φ
1	1	0
0	1	0
1	0	0
0	0	1

The table above represents ¬**(P** ∨ **Q)**

P	Q	Φ
1	1	1
0	1	0
1	0	0
0	0	0

The table above represents **P** ∧ **Q**

P	Q	Φ
1	1	0
0	1	1
1	0	1
0	0	1

The table above represents **¬(P ∧ Q)**

P	Q	Φ
1	1	0
0	1	1
1	0	1
0	0	0

The table above represents **P ⊕ Q**

P	Q	Φ
1	1	1
0	1	1
1	0	1
0	0	0

The table above represents **(Q II P)**

P	Q	Φ
1	1	0
0	1	0
1	0	0
0	0	1

The table above represents **¬(Q II P)**

P	Q	Φ
1	1	1
0	1	1
1	0	0
0	0	1

The table above represents **P ⇒ Q**

P	Q	Φ
1	1	1
0	1	0
1	0	1
0	0	1

The table above represents **Q ⇒ P**

P	Q	Φ
1	1	0
0	1	0
1	0	1
0	0	0

The table above represents ¬(**P ⇒ Q)**

P	Q	Φ
1	1	0
0	1	1
1	0	0
0	0	0

The table above represents ¬(**Q ⇒ P)**

4. Is there a general method for finding these formulas?
I am not sure because even if you find formula’s systematically, there are always some formula’s that could have the same meaning (like with double negation), thus there is always more formula’s to construct with two letters that implies the same outcome. I.e. multiple formula’s with the same input can lead to the same output.

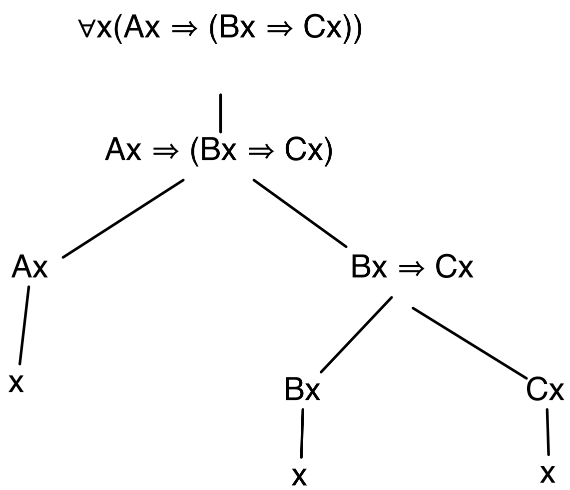
5. And what about 3-letter formulas and more?
It grows exponentially 2^4, 2^5 etc... the output can also differ, hence the formula for the possible truth tables should be: 2^(n+1)

Exercise 2.22
x < y => (x < z < y)

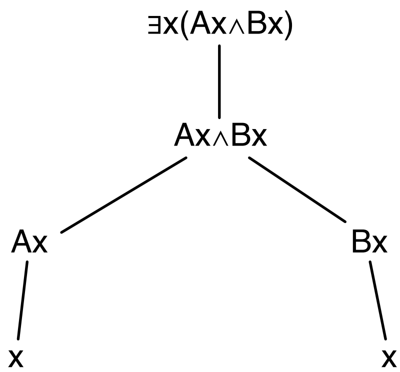
For each x and y there is a z that fits in between. E.g.
x: 2.1, y: 2.2, z: 2.15. This goes on to infinite decimals, e.g.:
x: 2.123456, y: 2.123457, z: 2.1234565

Exercise 2.23
Give structure trees of the following formulas (we use shorthand notation, and write A(x) as Ax for readability).

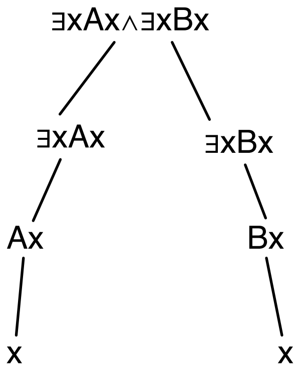
1.



2.



3.



Exercise 2.26

1. $\exists x \exists y(x \in Q \wedge y \in Q \wedge x<y)$:
$\exists x \in Q\ \exists y \in Q\ (x < y)$

2. $\forall x(x \in R \Rightarrow \exists y(y \in R \wedge x<y))$:
$\forall x \in R(x \Rightarrow \exists y \in R(x<y))$

3. $\forall x(x \in Z \Rightarrow \exists m,n(m \in N \wedge n \in N \wedge x=m-n))$.
$\forall x \in Z(x \Rightarrow \exists m \in N \exists n \in N(x = m - n))$

Exercise 2.27

1. $\forall x \in Q \exists m,n \in Z(n \neq 0 \wedge x=m/n)$
$\forall x \exists m,n(x \in Q \wedge m \in Z \wedge n \in Z \Rightarrow n \neq 0 \wedge x = m/n)$

2. $\forall x \in F \forall y \in D(Oxy \Rightarrow Bxy)$
$\forall x \forall y(x \in F \wedge y \in D \wedge Oxy \Rightarrow Bxy)$

Exercise 2.31

1. The equation x2 + 1 = 0 has a solution:
$\exists x(x^2 + 1 <=> 0)$

2. A largest natural number does not exist:
$\neg \exists x \forall y(x > y)$

3. The number 13 is prime (use $d|n$ for ‘d divides n’):
$\neg 13|n$

4. The number n is prime:
$\forall x(\neg x|n)$

5. There are infinitely many primes:
$\exists x \forall y \forall z(x > y \wedge \neg z|y \wedge \neg z|x)$

Exercise 2.32

1. $\forall x(x \Rightarrow L(x,d))$
2. $\forall x(x \Rightarrow L(d,x))$
3. $\forall x(M(x) \Rightarrow M'(x))$
4. $\exists x(B(x) \Rightarrow \neg F(x))$

Exercise 2.33

1.
B(x), x barks
D(x), x is a dog
B'(x), x bites

$\forall x(D(x) \wedge B(x) \Rightarrow \neg B'(x))$

2.
G(x), x glitters
G'(x), x is gold

$\forall x(G(x) \Rightarrow \neg G'(x))$

3.
d, Diana
F(x, y), x is a friend of y

$\forall x \forall y((F(x, d) \wedge F(y, x)) \Rightarrow F(y, d))$

4.
L(x), the limit of x
i, infinity
A(x, y), x approaches y
z, 0
F(x,y), fraction x/y
$\exists x(A(x, i) \Rightarrow L(F(x,y)) \Leftrightarrow z)$

Exercise 2.34

1. $\neg L(c,d) \wedge \forall x(\neg L(x, d) \Rightarrow x = c)$
2. $\forall x(M(x) \Rightarrow \exists y(A(x, y) \wedge \forall z(A(x, z) \Rightarrow z = y) \wedge \exists y'(A(x, y') \wedge \forall z'(L(x, z') \Rightarrow z' = y'))$
3. $\forall x(M(x) \Rightarrow \exists y(M(x,y) \wedge \forall z(M(x,z) \Rightarrow z = y))$

Exercise 2.35

1. $\exists x(King(x) \wedge \forall y(King(y) \Rightarrow y = x) \wedge \neg Raging(x))$.
2. $\exists x(K(x) \wedge \forall y(K(y) \Rightarrow y = x) \wedge \forall z(S(z, x) \Rightarrow L(s, x)))$.

Exercise 2.36

1. There is a rational number n that results in 5 when raising it to the power.
2. There exists a natural number n and a natural number m so that n < m.
3. For any natural number n there does not exist a natural number d so that both 1 < d < (2^n + 1) and d divides (2^n + 1) are possible/valid
4. For any natural number n there exists a natural number m, so that n < m in combination with each natural number being smaller or equal to n, or m is smaller or equal to p.
5. For each positive real number r there exists a natural number that is smaller or equal to any other natural number which implies that the length of real numbers is less or equal to the positive real number r

Exercise 2.37

Formula	a	b	c	d	e	f
∀x∀y(xRy)	f	f	f	f	f	f
∀x∃y(xRy)	t	f	t	t	f	t/f
∃x∀y(xRy)	f	f	f	f	f	t/f
∃x∀y(x = y ∨ xRy)	t	f	f	f	f	t/f
∀x∃y(xRy ∧ ¬∃z(xRz ∧ zRy))	t	t	f	t	f	t/f

The love question is very ambiguous.

Exercise 2.38
Substitutions for x

Formula	a	b	c	d	e	f
∀y(xRy)	n.a.	n.a.	n.a.	n.a.	n.a.	Diana according to the previous texts (she is loved by everyone)
∃y(xRy)	0	any number, e.g. 1000	any ration al: 1/16	any numbe r: 2	any father	anyone that loves someone
∀y(xRy)	n.a.	n.a.	n.a.	n.a.	n.a.	Diana (see row #1)

Formula	a	b	c	d	e	f
$\forall y(x = y \vee xRy)$	0	n.a.	n.a.	n.a.	n.a.	n.a.
$\exists y(xRy \wedge \neg\exists z(xRz \wedge zRy))$	any number , e.g. 10	any number above 0, e.g. 1000	any rational: 1/16	any number: 2	any father of a child that is not a child of the father and another of his children (sounds like most cases :P)	Maybe anyone that has a girl/ boyfriend, depending on the definition of love in this context (lets say relationship wise)

This exercise is a bit ambiguous because it depends on what you believe; do you believe if someone can love everyone (except for him-/herself).

Exercise 2.39

Φ and Ψ are equivalent iff $\Phi \Leftrightarrow \Psi$ is valid:

This means when $\Phi \Leftrightarrow \Psi$ is valid (thus true in every structure) both the formulas should be equivalent (they obtain the same truth value in every structure). So the fact that both formulas have the same truth value in every structure is quite obvious when $\Phi \Leftrightarrow \Psi$ is valid.

Exercise 2.41

$\exists x{\in}R(x^2 = 5)$:
Negation with quantifiers: $\forall x\neg R(x^2 = 5)$

$\forall n{\in}N\exists m{\in}N(n < m)$:
Negation with quantifiers: $\forall n{\in}N\neg\exists m{\in}N(n < m)$:

$\forall n{\in}N\neg\exists d{\in}N(1<d<(2n +1)\wedge d|(2n +1))$:
Negation with quantifiers: $\neg\forall n{\in}N\neg\exists d{\in}N(1<d<(2n +1)\wedge d|(2n +1))$

$\forall n{\in}N\exists m{\in}N(n < m \wedge \forall p \in N(p \leq n \vee m \leq p))$:
Negation with quantifiers: $\forall n{\in}N\neg\exists m{\in}N(n < m \wedge \forall p \in N(p \leq n \vee m \leq p))$

$\forall \varepsilon{\in}R^+\exists n{\in}N\forall m{\in}N(m \geq n\Rightarrow(|a-a_m| \leq \varepsilon))$:

Negation with quantifiers: $\neg\forall \varepsilon{\in}R^+\exists n{\in}N\forall m{\in}N(m \geq n\Rightarrow(|a-a_m| \leq \varepsilon))$

Exercise 2.46

No, the formulas both are read differently:
1. There is not an element of A with the property p

2. There is an element that is not part of A with the property p

Lets say A is the set of students that attends software engineering at the UvA. Lets say none of them have children.
Property p(x) means x has a child.

Formula one would say, no one from the software engineering students has a child.
Formula two would say, there is someone that does not attend software engineering at the UvA, that has a child .

Exercise 2.47
No, the formulas are both read differently:
1. There exists an element that is not part of A and does not hold property p
2. There exists an element that is part of A and does not hold property p

If we fill this in with the same structure (without the assumption no one has children) we would get:
1. There is someone that is not a software engineering student, that does not have a child
2. There is someone that is a software engineering student, that does not have a child

As demonstrated, these both speak about different sets (domains).

Exercise 2.48

Theorem 2.40 Formula's	Restricted formula
$\forall x \forall y \Phi(x,y)$	$\forall x \in N \forall y \in N \Phi(x,y)$
$\exists x \exists y \Phi(x,y)$	$\exists x \in N \exists y \in N \Phi(x,y)$
$\neg \forall x \Phi(x) \equiv \exists x \neg \Phi(x)$	$\neg \forall x \in N \Phi(x) = \exists x \in N \neg \Phi(x)$
$\neg \exists x \Phi(x) \equiv \forall x \neg \Phi(x)$	$\neg \exists x \in N \Phi(x) \equiv \forall x \in N \neg \Phi(x)$
$\neg \forall x \neg \Phi(x) \equiv \exists x \Phi(x)$	$\neg \forall x \in N \neg \Phi(x) \equiv \exists x \in N \Phi(x)$
$\neg \exists x \neg \Phi(x) \equiv \forall x \Phi(x)$	$\neg \exists x \in N \neg \Phi(x) \equiv \forall x \in N \Phi(x)$
$\forall x(\Phi(x) \wedge \Psi(x)) \equiv (\forall x \Phi(x) \wedge \forall x \Psi(x))$	$\forall x \in N(\Phi(x) \wedge \Psi(x)) \equiv (\forall x \in N \Phi(x) \wedge \forall x \in N \Psi(x))$
$\exists x(\Phi(x) \vee \Psi(x)) \equiv (\exists x \Phi(x) \vee \exists x \Psi(x))$	$\exists x \in N(\Phi(x) \vee \Psi(x)) \equiv (\exists x \in N \Phi(x) \vee \exists x \in N \Psi(x))$

Exercise 2.50
$\forall \delta > 0\ \exists n \forall m \geq n(|a - a_m| < \delta)$ means that the sequence converges to a
$\forall \delta > 0\ \forall n \forall m \geq n(|a - a_m| \geq \delta)$ means that the sequence does not converge to a