

Topological Sorting

Topological Sorting:

Topological sorting is a linear ordering of the vertices in a directed acyclic graph (DAG) such that for every directed edge $U \rightarrow V$, vertex U comes before vertex V in the ordering.

Use Cases

- **Task Scheduling:** When tasks must be completed in a specific order (e.g., prerequisite tasks).
- **Course Prerequisites:** Determining the order in which courses should be taken based on prerequisites.
- **Dependency Resolution:** Resolving dependencies in package managers.

Key Concepts

- **Directed Acyclic Graph (DAG):** A directed graph with no cycles.
- **Indegree:** The number of incoming edges to a vertex.

Algorithm for Topological Sorting

1. **Kahn's Algorithm (BFS Approach):**
 - **Step 1:** Calculate the indegree of each vertex.
 - **Step 2:** Enqueue all vertices with indegree 0 (i.e., vertices with no incoming edges).
 - **Step 3:** While the queue is not empty:
 - Dequeue a vertex from the queue and add it to the topological order.
 - Decrease the indegree of all its neighboring vertices by 1.
 - If a neighbor's indegree becomes 0, enqueue it.
 - **Step 4:** If all vertices are visited, the graph is a DAG, and the order is the topological sort. If not, the graph contains a cycle.
2. **Depth-First Search (DFS) Approach:**
 - **Step 1:** Mark all vertices as unvisited.
 - **Step 2:** For each unvisited vertex, perform a DFS. On reaching a vertex with no unvisited neighbors, push it onto a stack.
 - **Step 3:** After all vertices have been processed, pop elements from the stack to get the topological order.

Example

Consider a graph with edges:

- $A \rightarrow B$
- $A \rightarrow C$
- $B \rightarrow D$
- $C \rightarrow D$

Topological Sort:

- Possible orderings: `A`, `B`, `C`, `D` or `A`, `C`, `B`, `D` (depends on the order of processing in DFS/BFS).

Kahn's Algorithm Example

- **Indegrees:** A: 0, B: 1, C: 1, D: 2
- Start with `A` (indegree 0), then process `B` and `C`, finally `D`.

This way, you obtain the topological order.

EX:

```

A
/ \
B C
\ /
D

```

Articles

1. **Topological Sorting on GeeksforGeeks:**

Link: <https://www.geeksforgeeks.org/topological-sorting/>

YouTube Videos

1. Topological Sort - Graph Algorithm By Abdul Bari:

Link: <https://www.youtube.com/watch?v=Q9PlxaNGnig>

2. Topological Sort (DFS & Kahn's Algorithm) by take U forward:

Link <https://www.youtube.com/watch?v=5lZ0iJMrUMk>

3. Graph Series by Striver (Kahn's Algorithm & DFS Topological Sort):

Link: <https://www.youtube.com/playlist?list=PLgUwDviBlf0rGEWe64KWas0Nr7SCRWw>